ECONOMICS BULLETIN

The Difficulty of Income Redistribution with Labour Supply

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Abstract

Two common principles in distributional analysis are that (i) a progressive transfer moves the Lorenz curve upwards, and (ii) progressive [neutral] taxation reduces [leaves unchanged] inequality. In order to establish these results it is currently assumed that the distribution of income is exogenously given. The relevance of these results is therefore limited in practice where incomes are determined by the working decisions of the agents in the economy. Considering a simple economy with two goods and two agents we indicate sufficient conditions for inequality in net income to decrease as a result of rich to poor transfers or progressive taxation. By means of simple examples we show that, when one incorporates labour supply responses, the fulfillment of these conditions is highly hypothetical and that everything can happen.

The paper forms part of the research programme of the TMR network Living Standards, Inequality and Taxation [Contract No. ERBFMRXCT980248] of the European Communities whose financial support is gratefully acknowledged. We are grateful to Peter Lambert and the Editor of the Bulletin for helpful comments.
Citation: Ebert, Udo and Patrick Moyes, (2003) "The Difficulty of Income Redistribution with Labour Supply." *Economics Bulletin*, Vol. 8, No. 2 pp. 1−9
Submitted: September 20, 2002. **Accepted:** January 9, 2003.
URL: http://www.economicsbulletin.com/2003/volume8/EB−02H20004A.pdf

1. Introduction[1]

The normative theory of income distribution and redistribution assumes to a large extent that individual incomes are exogenously given.[2] This first round analysis results in neat and elegant theoretical conclusions at the cost of limited applicability since real world incomes are determined by a complex system of interactions where agents' behaviour plays a substantive role. This paper reconsiders two well-established principles in the standard theory of income distribution analysis that prove to rely crucially on the assumption that individuals do not adjust their behaviour to the modification of the institutional environment.

Following Atkinson (1970) (see also Kolm 1969) it is now a well-established practice to measure inequality by means of Lorenz curves: a distribution is less unequal than another distribution if its Lorenz curve lies nowhere below that of the other distribution. According to the *principle of transfers*, a redistribution of income from a richer individual to a poorer individual – a so-called progressive transfer – is typically considered as reducing inequality as it moves upwards the Lorenz curve of the original distribution. It is so the case that most of the profession assimilates inequality reduction with a combination of such progressive transfers.[3] A second important principle is that of *redistributive taxation* according to which a progressive tax schedule – understood as an increasing average tax rate – implies more equally distributed incomes in the sense that the Lorenz curve of post-tax incomes lies above that of pre-tax incomes (Jakobsson 1976).

The aim of this note is to examine to which extent the recognition that individuals react to progressive transfers and income taxation calls into question the two principles above. We present simple examples demonstrating that these two principles do no longer hold when labour supply is fully accounted for in the model. Considering non-pathological and sufficiently flexible preferences that lead to linear labour supply, we show that the change in inequality due to (i) progressive transfers, and (ii) neutral or progressive taxation is indetermined: inequality can decrease, increase or be unchanged. Although this observation is certainly not original, we claim that there does not appear to be papers in the literature – to the best of our knowledge – demonstrating it in such a simple way. It is our trust that these examples might provide useful starting points for extending the theory.

The paper is organized as follows. Section 2 is devoted to the presentation of the economy we consider in the paper. We investigate the impact on effective income inequality of progressive transfers in Section 3. The implication for the distribution of net income of taxation is examined in Section 4 considering successively proportional and progressive taxes. Finally Section 5 concludes the paper.

[1] The paper forms part of the research programme of the TMR network **Living Standards, Inequality and Taxation** [Contract No. ERBFMRXCT980248] of the European Communities whose financial support is gratefully acknowledged.

[2] For instance Lambert's (1993) survey devotes 250 pages to the case where incomes are exogenously given while less than 20 pages consider the case where incomes are endogenous to the model. In the third edition (Lambert 2001) the respective chapter has even completely disappeared.

[3] Actually this is not so true as a number of experimental studies by means of questionnaires have demonstrated that a significant fraction of the respondents do not subscribe to this view (see e.g. Amiel and Cowell 1999).

2. Framework

At first we describe the underlying model and introduce the notation. There are two commodities, consumption C and time L spent on work. We consider two individuals i $(i=1,2)$. Both have the same preference ordering on consumption and labour which can be represented by an ordinal utility function $U(C,L)$. We assume that U is increasing in consumption, decreasing in labour and twice differentiable. Furthermore the preference ordering is supposed to be convex. Individual i $(i=1,2)$ possesses the wage rate w_i and an exogenous income M_i. The price of consumption (demand) is equal to unity. Each individual maximizes her utility function subject to the budget constraint which is given by $C_i = w_i L_i + M_i$. Then $L(w_i, M_i)$ denotes the labour supply function. In what follows we always assume that $w_1 < w_2$ and that individual 1's (gross or net) income is strictly lower than individual 2's one.

For a society consisting of two individuals the measurement of inequality is very simple and unambiguous. Given an income distribution $\mathbf{X}=(X_1,X_2)$, the Lorenz curve $LC(s;\mathbf{X}), s\in[0,1]$, can be described by three points:

$$LC(0;\mathbf{X})=0,\ LC(1;\mathbf{X})=1,\text{ and } LC(\tfrac{1}{2};\mathbf{X})=\frac{X_1}{X_1+X_2}\ \text{ for } X_1<X_2. \tag{1}$$

Two neighbouring points are connected by straight lines. The distribution **X** Lorenz dominates **Y**, which we abbreviate by $\mathbf{X}\succsim_L \mathbf{Y}$, if and only if $LC(\tfrac{1}{2};\mathbf{X})\geq LC(\tfrac{1}{2};\mathbf{Y})$.

We note that for two individual-populations the Lorenz ordering is complete.[4]

In the examples presented we will assume that the underlying preference ordering is represented by the direct utility function

$$U(C,L)=\left(\frac{L-b}{\beta}\right)\exp\left(-\left(1+\frac{\beta(C+a)}{b-L}\right)\right) \tag{2}$$

where $a=\frac{\gamma}{\beta}-\frac{\alpha}{\beta^2}, b=\frac{\alpha}{\beta}$. At first sight the form of the utility function seems to be unusual. But it leads to a linear labour supply function

$$L(w,M)=\alpha w+\beta M+\gamma \tag{3}$$

[4] For $n=2$ every relative inequality measure can be expressed by a strictly increasing transformation of a ratio relating the absolute difference of incomes to total income (see Ebert 1988). It can be defined as $I(\mathbf{X})=f\left(\frac{X_2-X_1}{X_1+X_2}\right)$ for $X_1<X_2$ where f is strictly increasing. $I(\mathbf{X})$ is directly related to the Lorenz curve of **X** since $I(\mathbf{X})=f\left(1-LC(\tfrac{1}{2};\mathbf{X})\right)$.

which is very well suited for the construction of examples. The ordering is well-defined and possesses the properties stated above if $\alpha \geq 0, \beta \leq 0$, and $L \geq 0$. Then the Slutsky matrix is symmetric and negative semidefinite (see Stern 1986 for details).

In the following examples we always suppose that

$$w_1 = 1 < w_2 = 2,\ M_1 = M_2 = 0,\ \text{and}\ \alpha = 3. \tag{4}$$

3. Progressive transfers

Now we consider transfers from the richer to the poorer individual. Let $\Delta > 0$ denote the amount of income redistributed. The amount Δ has to be small enough such that the ranking of incomes is not reversed.

Before the transfer is made income is given by $X_i = w_i L(w_i, 0), i = 1,2,$ and $\mathbf{X} = (X_1, X_2)$ where we set $M_i = 0$ for simplicity. For any exogenous income M we obtain $Y_i(M) := w_i L(w_i, M) + M$ and define the elasticity of (net) income with respect to exogenous income M by

$$\varepsilon_M(Y_i(M)) = \frac{\partial Y_i(M)}{\partial M} \frac{M}{Y_i(M)}. \tag{5}$$

It should be mentioned that in a model having only two commodities $Y_i(M)$ corresponds to expenditure for consumption. Therefore $\varepsilon_M(Y_i(M))$ can also be interpreted as the income elasticity of consumption if the price of consumption is equal to unity.

Supposing that $X_1 < X_2$ a progressive transfer of an amount $\Delta > 0$ takes place if $Y_1(\Delta) < Y_2(-\Delta)$. It yields the income distribution $\mathbf{Y}(\Delta) := (Y_1(\Delta), Y_2(-\Delta))$. Then $\mathbf{X} = \mathbf{Y}(0)$ and the Lorenz curve of $\mathbf{Y}(\Delta)$ is characterized by

$$LC(1/2; \mathbf{Y}(\Delta)) = \frac{Y_1(\Delta)}{Y_1(\Delta) + Y_2(-\Delta)}. \tag{6}$$

We are interested in the change of inequality due to a change in Δ which can be determined by the derivative of $LC(1/2; \mathbf{Y}(\Delta))$ with respect to Δ. Simple calculation and rearrangement yields

$$\frac{d\,LC(1/2; \mathbf{Y}(\Delta))}{d\,\Delta} = \frac{Y_1(\Delta) Y_2(-\Delta)}{\Delta} \left(\frac{d\,Y_1(\Delta)}{d\,\Delta} \frac{\Delta}{Y_1(\Delta)} - \frac{d\,Y_2(-\Delta)}{d(-\Delta)} \frac{(-\Delta)}{Y_2(-\Delta)} \right) \Big/ \left(Y_1(\Delta) + Y_2(-\Delta)\right)^2 \tag{7}$$

for $\Delta > 0$. The sign of this derivative depends on the magnitude of the elasticities involved.

We obtain a necessary and sufficient local condition:

Proposition 1

$$\frac{LC\left(1/2;\mathbf{Y}(\Delta)\right)}{d\,\Delta} \gtreqless 0 \Leftrightarrow \varepsilon_M\left(Y_1(M)\right)\big|_{M=\Delta} \gtreqless \varepsilon_M\left(Y_2(M)\right)\big|_{M=-\Delta} \tag{8}$$

Thus the income effects are crucial. This result allows us to formulate a sufficient condition for Lorenz domination, i.e. for a global inequality decrease:

$$\varepsilon_M\left(Y_1(M)\right)\big|_{M=\tilde{\Delta}} > \varepsilon_M\left(Y_2(M)\right)\big|_{M=-\tilde{\Delta}} \text{ for } \tilde{\Delta}\in(0,\Delta), \tag{9}$$

but, of course, this condition is not necessary.

Now we present an example demonstrating that the effects of a progressive transfer depend on the individuals' reaction (and preference ordering).

Example 1

We employ the linear labour supply function $L(w,M)=\alpha w+\beta M+\gamma$ introduced above and set $\gamma=0$. The parameter β is varied. Then we get the results collected in Table I:

Table I: Progressive transfer of Δ. $\gamma=0$

	$\Delta=0$			$\Delta=1$		
β	X_1	X_2	$LC(1/2;\mathbf{X})$	Y_1	Y_2	$LC(1/2;\mathbf{Y})$
$-2/3$	3	12	0.2	$10/3=3.\overline{3}$	$37/3=12.\overline{3}$	$10/47=0.21$
$-5/6$	3	12	0.2	$19/6=3.1\overline{6}$	$38/3=12.\overline{6}$	$19/95=0.2$
-1	3	12	0.2	3	13	$3/16=0.19$

Here we have to compare the no-redistribution case $\Delta=0$ $\left(\mathbf{X}=\mathbf{Y}(0)\right)$ with the situation after redistribution $\left(\Delta=1 \text{ and } \mathbf{Y}(\Delta)\right)$. Since the exogenous incomes M_i are zero by assumption the parameter β has no influence on the pre-redistribution income, but on the individuals' reaction. The income elasticity is given by

$$\varepsilon_M\left(Y_i(M)\right)=\left(\beta w_i+1\right)\frac{M}{Y_i(M)} \tag{10}$$

The condition (9) is satisfied for $\Delta=1$ and $\beta=-2/3$. For $\beta=-5/6$ and -1 it is not fulfilled. Then inequality is unchanged and increased, respectively. Thus labour supply responses are able to reverse a progressive transfer effectively.

4. Redistributional effects of taxation

In this section it is assumed that the government wants to raise some tax revenue by income taxation. We are interested in its implications for the income distribution (its redistributional effects). At first we examine taxation of income by a proportional tax. Tax liability can be described by $T(X) = tX$ for $X \geq 0$ where t denotes the tax rate. Net income is given by $X - T(X) = (1-t)X$.

Again we have to consider (gross) income before taxation: $\mathbf{X} = (X_1, X_2)$. It corresponds to the no-tax situation and has to be compared to post-tax incomes when the individuals have adapted to $T(X)$. Given proportional taxation and the fact that exogenous income is assumed to be zero the (new) gross (pre-tax) incomes are equal to $Z_i(t) = w_i L((1-t)w_i, 0)$, the (new) net (post-tax) incomes to $Y_i(t) = (1-t)Z_i(t)$.

We introduce the tax elasticity of gross income $Z_i(t)$:

$$\varepsilon_t(Z_i(t)) = \frac{\partial Z_i(t)}{\partial t} \frac{t}{Z_i(t)} \text{ for } t > 0. \tag{11}$$

Using a well-known rule for elasticities (see e.g. Berck and Sydsaeter (1993)) we obtain

$$\varepsilon_t(Z_i(t)) = \left(-\frac{t}{1-t}\right) \varepsilon_w(L_i(w,0))\Big|_{w=(1-t)w_i} \tag{12}$$

where $\varepsilon_w(L(w,0))$ denotes the wage elasticity of labour supply.

Now we investigate the implications for inequality. Defining $\mathbf{Z}(t) = (Z_1(t), Z_2(t))$ and $\mathbf{Y}(t) = (Y_1(t), Y_2(t))$ we get $LC(½; \mathbf{Y}(t)) = LC(½; \mathbf{Z}(t))$ since the terms $(1-t)$ in the nominator and denominator cancel. The (marginal) change in inequality is reflected by $\frac{d\, LC(½; \mathbf{Z}(t))}{d\, t}$. As above it can be derived directly:

$$\frac{d\, LC(½; \mathbf{Z}(t))}{d\, t} = \frac{Z_1(t) Z_2(t)}{t} \left(\frac{\partial Z_1(t)}{\partial t} \frac{t}{Z_1(t)} - \frac{\partial Z_2(t)}{\partial t} \frac{t}{Z_2(t)} \right) \Big/ \left(Z_1(t) + Z_2(t)\right)^2 \tag{13}$$

for $t > 0$. Therefore the sign of the derivative is determined by the magnitude of the respective elasticities. We get a necessary and sufficient local condition

Proposition 2

$$\frac{d\,LC\left(\tfrac{1}{2};\mathbf{Z}(t)\right)}{d\,t} \gtreqless 0 \Leftrightarrow \varepsilon_t\left(Z_1(t)\right) \gtreqless \varepsilon_t\left(Z_2(t)\right)$$

$$\Leftrightarrow \varepsilon_w\left(L(w,0)\right)\Big|_{w=w_1} \lesseqgtr \varepsilon_w\left(L(w,0)\right)\Big|_{w=w_2} \tag{14}$$

for $t > 0$.

It also allows to formulate a sufficient global condition for an increase or decrease of inequality. As expected, however, everything can happen:

Example 2

It is again based on the linear supply function. We set $\beta = -1$. The parameter γ is varied. We get the results shown in Table II:

Table II: Proportional taxation. $\beta = -1$

	$t=0$			$t=\tfrac{1}{2}$		
γ	X_1	X_2	$LC\left(\tfrac{1}{2};\mathbf{X}\right)$	Y_1	Y_2	$LC\left(\tfrac{1}{2};\mathbf{Y}\right)$
-1	2	10	$\tfrac{1}{6}=0.1\overline{6}$	$\tfrac{1}{4}$	2	$\tfrac{1}{9}=0.\overline{1}$
0	3	12	$\tfrac{1}{5}=0.2$	$\tfrac{3}{4}$	3	$\tfrac{1}{5}=0.2$
1	4	14	$\tfrac{2}{9}=0.\overline{2}$	$1\tfrac{1}{4}$	4	$\tfrac{5}{21}=0.24$

In this case the tax elasticity of post-tax income is given by

$$\varepsilon_t\left(Y_i(t)\right) = -\frac{\alpha\, t w_i}{\alpha(1-t)\,w_i + \gamma} \tag{15}$$

For the situations considered in Table II the elasticity is always negative, but its magnitude depends on the preference ordering considered. Since ε_t is proportional to the wage elasticity of labour supply, for proportional taxation the income and substitution effect are decisive. Obviously inequality can increase, decrease, or be unchanged. The individuals' working decisions are able to yield any result. Proportional taxation is no longer automatically neutral.

A proportional tax schedule can be turned into a progressive or regressive schedule by subtracting or adding a (small) amount Δ:

$$T(X) = t\,X + \Delta. \tag{16}$$

When $\Delta < 0$, $T(X)$ is progressive, when $\Delta > 0$ it is regressive. This is even true for very small amounts Δ. Therefore Example 2 can be used to argue that progressive taxation can increase inequality and regressive taxation can decrease inequality. The argument is simple: Starting with the case in which proportional taxation increases inequality we can change the tax schedule slightly to become regressive without changing the result (the process is entirely continuous!). One can similarly argue for progressive taxation.

But for completeness we present a further example demonstrating this outcome.

Example 3

Again we suppose that for the linear labour supply function $\beta = -1$. Then we obtain the results shown in Table III.

Table III: (a) Progressive taxation, (b) Regressive taxation. $\beta = -1$

(a)	$t=0,\Delta=0$			$t=\frac{1}{2},\Delta=-\frac{1}{8}$		
γ	X_1	X_2	$LC(\frac{1}{2};\mathbf{X})$	Y_1	Y_2	$LC(\frac{1}{2};\mathbf{Y})$
-1	2	10	$\frac{1}{6}=0.1\overline{6}$	$\frac{5}{16}$	2	$\frac{5}{37}=0.14$
(b)	$t=0,\Delta=0$			$t=\frac{1}{2},\Delta=\frac{1}{8}$		
γ	X_1	X_2	$LC(\frac{1}{2};\mathbf{X})$	Y_1	Y_2	$LC(\frac{1}{2};\mathbf{Y})$
1	4	14	$\frac{2}{9}=0.\overline{2}$	$1\frac{3}{16}$	4	$\frac{19}{83}=0.23$

5. Conclusion

When the unrealistic assumption of the exogeneity of labour supply is dropped every outcome is possible: Progressive transfers and progressive (regressive) taxation may increase or decrease inequality or may leave it unchanged. The respective result depends on the income or wage elasticities as demonstrated above. The examples presented consider only two individuals and a simple (linear) labour supply function. It is clear that analogous results can also be obtained in more general settings.

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