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Coordination games and the option to wait

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Abstract

We take a coordination game and add the option to wait; each player can opt to take an action in the standard game or they can decide to wait. If one player has taken a standard option, the waiting player can adopt their best response to this action. Interpreting the payoff in the final period (when there is no waiting possible) as a outside option or default, we show that a party's equilibrium payoff can be decreasing in their default. Further, a player's role of leader or follower alternates as the number of waiting periods changes.

1 Introduction

Situations in which parties need to coordinate their actions are ubiquitous. If possible, parties will try to avoid coordination failures, for example by engaging in pre-game communication (Farrell 1987). Two potential alternative ways of avoiding a coordination failure is to wait (allowing the party that waited the opportunity to adopt the best response to the other player's action); and, second, having an equilibrium exogenously selected for the players by, for instance an arbitrator or by social precedent. In this paper we examine the interaction between these two coordinating devices. We show that in combination their effect can be somewhat perverse.

2 The model

Take a standard 2-by-2 coordination game. There are two players; each can choose from the set of two possible actions. Let the payoffs be given by the following figure, where $c_i > b_i > 0$ for i = 1, 2.

Player 2
$$\begin{array}{c|cccc} & & \text{Player 2} \\ L & R \\ \hline \text{Player 1} & T & 0,0 & b_1,c_2 \\ \hline c_1,b_2 & 0,0 \\ \hline \end{array}$$

Figure 1: The normal form of the standard coordination game

In this coordination game there are three Nash equilibria: two pure strategy equilibria - (T, R) and (B, L) - and a mixed strategy equilibrium. In many cases the parties will develop institutions to avoid coordination failures. Let us assume that both players believe that the outcome of this game will be (T, R). As mentioned above, this could come about because of precedent or it could be the outcome that is decreed by a third party.

Now, augment the game to allow each party the option to wait - denoted as W - for one period. In this case, in period 1, each player can wait and not select a regular action (either T, B, L, or R, depending on the player). Further, if one player waits and the other selects a regular action, the waiting player can observe their choice. In the second, and final, period, the player that waited can choose their best response to the action that has been taken. If both players wait, the game proceeds to the second period, in which the stage game is represented in Figure 1, with its exogenously given equilibrium outcome (T,R). This can be interpreted as a situation in which, if the players have not resolved the coordination issue by a certain deadline, it is resolved for them.

The relevant normal form of this augmented game is shown in Figure 2. If both players select a standard action, the payoffs are identical to the standard coordination game (Figure 1). If player 1 opts to play W and player 2 chooses L in period 1, in

		Player 2		
		L	R	W
	T	0,0	b_1, c_2	b_1, c_2
Player 1	B	c_1, b_2	0,0	c_1, b_2
	W	c_1, b_2	b_1, c_2	b_1, c_2

Figure 2: The relevant normal form of the augment game

the second period player 1's best response is B - thus the payoff to the parties will be (c_1, b_2) . If player 1 waits and player 2 chooses R, player 1 will opt for T in the second period, yielding payoffs of (b_1, c_2) . A similar logic applies when player 2 waits and player 1 takes a standard action. Finally, note, if both players wait we return to the standard 2-by-2 coordination game in the second and final period, the equilibrium of which is given as (T, R) with its payoffs (b_1, c_2) .

We use the notion of weak domination to solve this augmented game. Player 2 will have a weakly-dominant strategy to wait (W) - she cannot do better by taking either of the other two actions L or R in period 1. Given that we can cancel the first two actions of Player 2, Player 1 has an incentive to preempt the last period outcome and take an action immediately - she will choose B. If player 1 plays B immediately, she guarantees that the eventual outcome will be (B, L), which is preferable to her than (T, R), the outcome that occurs if both players initially wait. Consequently, the unique equilibrium payoffs arising from weak domination are (c_1, b_2) . Notably here, the outcome differs from the exogenous final period outcome of (b_1, c_2) ; the party that has the larger default payoff at the deadline ends up in equilibrium receiving the smaller payoff of the two parties.

This simple idea has several general implications. First, the final-period outcome gives one party a stronger position - we could interpret this position as an outside option or a default. When this is combined with the option to wait, in an attempt to preempt this default outcome, the weaker party acts immediately. Further, because of its seemingly advantageous default, the player with the larger default payoff waits. Taken together, these strategies have the effect that strong default position actually lowers the equilibrium payoff of a player. This is an unexpected result. For example, in the bargaining literature, a party's payoff is often non-decreasing or increasing in their outside option (for example, as in Shaked and Sutton 1984). Here, in the two-period game a player's equilibrium payoff is decreasing in her outside option.

Second, it is also notable that it is seemingly the weaker party that moves first. This is the opposite to the result of van Damme and Hurkens (2004). In their model, the firm that sets their price first in the risk-dominated equilibrium is the strong (low-cost) firm.

Third, if an additional potential waiting period is added to the game the role of the players again switch. With three potential periods, the player with the strong default position in the final period will have an incentive to move immediate (in period 1) to

avoid being locked into the (B, L) outcome by player 1 in the two-period game. The relevant normal form of the 3-period game is illustrated in Figure 3. In this longer game, the payoff from waiting in the first period is the payoff in the two-period game (c_1, b_2) - this is illustrated by the payoff that arises when the actions taken in the first period are (W, W). All of the other payoffs remain the same as they were in Figure 2.

		Player 2		
		L	R	W
	T	0,0	b_1, c_2	b_1, c_2
Player 1	B	c_1, b_2	0,0	c_1, b_2
	W	c_1, b_2	b_1, c_2	c_1, b_2

Figure 3: The relevant normal form of the 3-period augment game

Again using the elimination of weakly-dominated strategies, Player 1 now has a weakly-dominant strategy to wait. Given this, Player 2 will choose R immediately. If the number of periods in the game are extended further, the role of the players switch again; when one party has the strong default position it is the other party that will take an action immediately. Consequently, a player's payoff in equilibrium moves non-monotonically with respect to their default (final-period) payoff as the number of periods in the game changes.

Finally, in the two-period game player 1 is hurt by the fact that she cannot commit not to wait. Similar results arise in other papers in the literature. For example, Solan and Yariv (2004) examined a game in which one party has the option to engage in espionage by purchasing a noisy signal of the other player's action. Anticipating the espionage, the non-spying player uses the opportunity to commit to particular action - in effect the second player becomes the Stackelberg leader. It can be the case that the first player would like to commit not to spy. However, given this is not is possible, the player with the opportunity to spy becomes the Stackelberg follower, and is ultimately disadvantaged by its ability to engage in espionage.

References

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