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Unusual behaviour of Dickey−Fuller tests in the presence of trend mis−specification

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Abstract

This paper analyses the properties of Dickey−Fuller (1979) (DF) unit root tests in the presence of trend mis−specification. It is shown that while the performance of the DF coefficient test is as expected, the DF test in its t−ratio form exhibits unusual behaviour. In particular it is found that the power of the test increases as the autoregressive parameter approaches 1. Interestingly, this increased power is not accompanied by oversizing.

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1 Introduction

It has long been recognised that the Dickey-Fuller (1979) (DF) unit root test can suffer from low power, particularly when applied to nearly-integrated time series.[1] It is equally well known that the DF test can suffer from low power when the testing equation is mis-specified. For example, should a necessary deterministic trend be omitted from the testing equation, the DF test will be biased towards the non-rejection of the unit root null hypothesis. In this paper the behaviour of the DF test is examined when the testing equation contains an intercept and linear trend, but a quadratic trend exists in the Data Generation Process (DGP) under the null. The analysis is performed using the DF test in both its t-statistic (τ_τ test) and coefficient (K test) forms. Crucially, it is found that the results obtained for two tests can differ dramatically. The results for the K test are as expected, with power decreasing as the autoregressive parameter (ρ) tends to 1. In contrast, for a range of values of ρ close to unity, the τ_τ test displays unexpected behaviour, with the power of the test actually increasing as ρ increases. However, it should be noted that this increased power is not accompanied by oversizing, with both tests found to have a size of zero when $\rho = 1$, as suggested by intuition.

In the following section the Monte Carlo experimental design employed is outlined, with the results obtained presented in section [3]. Section [4] contains some concluding remarks and avenues for future research.

2 Monte Carlo simulation

Consider the following DGP:

$$y_t = \mu + y_{t-1} + \epsilon_t \quad \epsilon_t \sim i.i.d.\left(0, \sigma^2\right) \tag{1}$$

It is well known that the presence of the drift parameter (μ) will induce a trend in y_t, with simple algebra showing that $\mathsf{E}[y_t] = y_0 + \mu t$. When performing a unit root test upon a trending series it is therefore suggested that a linear time trend be included in the testing equation, to allow a trend to be present under both the null of a unit root and the alternative hypothesis $|\rho| < 1$. The appropriate testing equation is therefore:

$$y_t = a + bt + \phi y_{t-1} + \xi_t \qquad t = 1, ..., T \tag{2}$$

[1] See Maddala and Kim (1998) for an overview of the literature.

The additional term bt is recommended as it allows subsequent DF tests to be independent of the value of μ in (1) and also ensures similar behaviour of y_t under both the null and alternative hypotheses (see, *inter alia*, Banerjee *et al.* 1993; Hamilton 1994). The unit root hypothesis ($\phi = 1$) can then be tested in (2) using the Dickey-Fuller τ_τ and K tests which are calculated as:

$$\begin{aligned} \tau_\tau &= \frac{\widehat{\phi} - 1}{se\left(\widehat{\phi}\right)} \\ K &= T\left(\widehat{\phi} - 1\right) \end{aligned}$$

Rejection or non-rejection of the unit root hypothesis is then determined by comparison with specifically calculated critical values presented by, *inter alia*, Fuller (1996).

However it must be recognised that in practice the process generating the sample data is unknown. It is therefore possible that the above approach may be followed, with the data assumed to be generated by (1) when this is not the case. In this paper the properties of the DF tests are considered when the testing equation (2) is employed but the data have been generated by the following process which includes both a drift parameter and a trend term:

$$y_t = \alpha + \beta t + \rho y_{t-1} + \xi_t \qquad t = 1, ..., T \tag{3}$$

Under the null of a unit root ($\rho = 1$), y_t can be re-expressed via repeated substitution as:

$$\begin{aligned} y_t &= y_0 + \alpha t + \frac{\beta}{2} t\left(t+1\right) + \sum_{j=1}^{t} \xi_j \\ &= y_0 + \left\{\alpha + \frac{\beta}{2}\right\} t + \frac{\beta}{2} t^2 + \sum_{j=1}^{t} \xi_j \\ &= \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \sum_{j=1}^{t} \xi_j \end{aligned} \tag{4}$$

where $\gamma_0 = y_0$, $\gamma_1 = \alpha + \beta/2$, and $\gamma_2 = \beta/2$. A quadratic trend is therefore apparent under the null, but is not present under the alternative $|\rho| < 1$, in which case y_t can be expressed as:

$$y_t = \left\{ \alpha \sum_{j=0}^{t-1} \rho^j + \beta \sum_{j=1}^{t} j\rho^{t-j} + \rho^t y_0 \right\} + \left\{ \sum_{j=1}^{t} \rho^{t-j} \xi_j \right\} \qquad t = 1, ..., T \tag{5}$$

Note that under the null equation (4) and equation (5) are identical.

Following the above arguments, the appropriate testing equation should employ a quadratic trend term if the investigator knew the true DGP (3). In this sense, a form of trend mis-specification may be said to exist. In the following section, the properties of the DF τ_τ and K tests are examined when the investigator applies the testing equation (2) and the corresponding critical values of Fuller (1996). Implicitly, the investigator is assuming that the data are generated by (1), when they are in fact generated by (3).

3 Monte Carlo results

To analyse the properties of the τ_τ and K tests, a range of values were considered for the design parameters $\{\rho, \alpha, \beta, T\}$ in (3), with all experiments performed over 25,000 replications with an initial 100 observations discarded to minimise the influence of the initial condition $y_0 = 0$. The error process $\{\xi_t\}$ was generated as pseudo *i.i.d.* $\mathsf{N}(0, 1)$ random numbers using the RNDNS procedure in the Gauss programming language version 3.2.13. However, in the interests of brevity, and to ease interpretation, only a subset of these results are presented here. More precisely, results are presented below for the DGP of (3) with $\{\alpha, \beta, T\} = \{0, 0.08, 250\}$, and ρ taking the values $\{0.9600, 0.9605, ..., 0.9995, 1\}$. The values of the design parameters reported here were selected to display clearly the counterintuitive behaviour of the standard DF tests.[2]

As stated previously, it is well known that omission of a required trend term from the DF testing equation will result in low power, causing a bias towards the non-rejection of the unit root null hypothesis. Intuitively, it will be expected that estimation of (2) under the DGP of (3) will also result in low power. Additionally, we also expect power to be reduced as $\rho \to 1$. To examine whether these conjectures hold in the presence of

[2] It must be emphasised that the unusual behaviour of the DF test is exhibited over a range of values of the design parameters. Although unreported here, further results are available from the authors upon request.

mis-classification of the DGP, the experiments outlined above were performed. Following the arguments of Davidson and MacKinnon (1998), the results of these experiments are presented graphically in Figure 1 to ease exposition and interpretation.

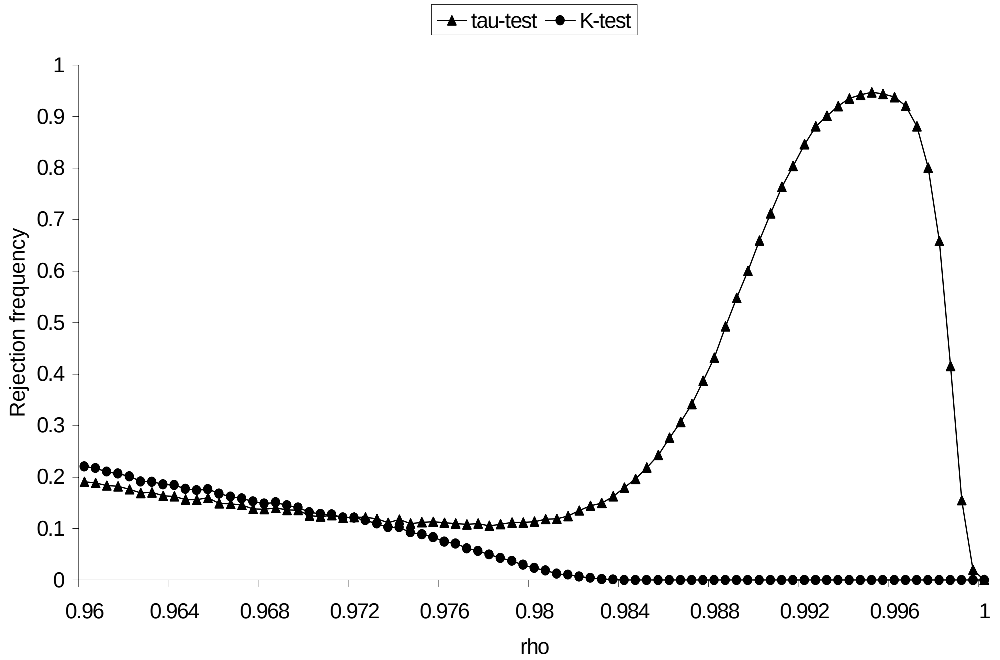

Figure 1: Empirical rejection frequencies for the DF tests

Considering the results for the K test, it can be seen that as expected, the test has low power in rejecting the unit root hypothesis. It is also clear that power of the test falls as $\rho \rightarrow 1$, with the simulations producing a size of the test equal to zero when $\rho = 1$. More precisely, the power of the test is at a maximum of 22.05% for the initial value of $\rho = 0.96$, but falls to zero when $\rho = 0.9865$. This steady fall towards a rejection frequency of zero rather than the nominal size of 5% is as expected given mis-specification of the trend. However, the behaviour of the τ_τ test is in sharp contrast to this intuitive result. Initially the power of the τ_τ test is similar to that of the K test, with a slightly lower rejection frequency of 19.05% observed for $\rho = 0.96$. Whilst the power of the τ_τ test continues to fall as ρ increases over a subsequent range of values of ρ, it becomes more powerful than the K test when $\rho = 0.972$. However, the noticeable feature of the Monte Carlo results is the increasing power of the τ_τ test as ρ increases beyond 0.9875. A dramatic increase in the power of the test is then observed, power reaching a maximum value of 94.72% when $\rho = 0.995$.[3] Further simulation results show that this increased power results from the accelerated convergence of the standard error of ϕ towards zero. For values of ρ greater than this, the power begins to fall. Crucially, when $\rho = 1$ the empirical rejection frequency, or size of the test, is zero as noted above for the K test. The unexpected increased power of the τ_τ test is therefore not associated with oversizing.

4 Concluding remarks and future research

In this paper it has been shown that in the presence of mis-classification of the DGP, the DF τ_τ test displays unusual behaviour, with a positive relationship existing between ρ and the rejection of the unit root hypothesis over a range of values of ρ. It can also be noted that if the investigator were aware of the true DGP of equation (3), it would be recognised that the specification of the testing equation (2) and the DGP coincide. The results of West (1988) show that under these circumstances the τ_τ statistic is asymptotically normally distributed. Further results, unreported here, show that the unusual behaviour of the τ_τ is still present when standard normal critical values are employed, although it is somewhat less pronounced.[4] In contrast, the DF K test does not possess such counterintuitive properties, and this leads us to recommend use of both the K test and the τ_τ test in applied research.

In further research we intend to conduct an analytical examination of the present simulation results using local-to-unity theory, and also to investigate the properties of DF tests under alternative forms of mis-specification. As a consequence of the popularity of the standard unit root tests in the empirical literature, the present results relate purely to individual tests of the unit root hypothesis $\phi = 1$ in equation (2). The current analysis could therefore be extended to examine the behaviour of the Φ_3-test of the joint hypothesis $\phi = b = 0$.

[3]Interestingly, careful reading of Blough (1992, p.306, central figure) shows a similar phenomenon of a positive relationship between ρ and empirical rejection frequencies for the Said-Dickey (1984) test. However, no such result is evident for the standard DF test.

[4]Rejection frequencies for the τ_τ test using standard normal critical values are available from the authors upon request.

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