ECONOMICS BULLETIN

Extending the cost function: A simple method of modeling environmental regulation

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Abstract

The paper shows that a variety of different scenarios in environmental economics can be modeled by one abstract concept, an extended cost function, which takes into account a firm's technological and regulatory constraints. It satisfies the usual properties of a cost function and reasonable properties with respect to the regulatory parameter. An extended cost function represents a simple unified approach which does not depend on the specific form of regulation and the way emissions are modeled.

I thank Martin Duensing, Heinz Welsch and participants of the Economics Seminar at the University of Oldenburg for helpful comments on an earlier version of this paper.
Citation: Ebert, Udo, (2006) "Extending the cost function: A simple method of modeling environmental regulation." *Economics Bulletin*, Vol. 17, No. 6 pp. 1-6
Submitted: November 24, 2006. **Accepted:** November 28, 2006.
URL: http://economicsbulletin.vanderbilt.edu/2006/volume17/EB-06Q20006A.pdf

1. Introduction

This paper deals with a concept useful for environmental economics. Assuming that firms generate a negative externality and are therefore regulated, we describe a representative firm's behavior by means of an extended cost function. A firm's technological constraints can be represented by a production function and – equivalently – by a corresponding (standard) cost function (given the usual properties of the technology). When emissions are regulated a firm has to obey a regulatory constraint in addition. An extended cost function takes into account *both* kinds of constraints in a simple manner.

Furthermore, the present note shows that a variety of different scenarios can be represented by this concept. We consider a model in which the negative externality is interpreted as emissions which are modeled either as an input or as an output. Moreover three instruments are examined: an emission tax and absolute and relative standards; i.e. the concept can be applied to market based instruments and to performance standards. We will demonstrate that in all these cases we obtain the same abstract extended cost function which satisfies the usual properties of a cost function and possesses reasonable properties with respect to a parameter describing the (level of) regulation. Thus we present a unified approach which is independent of the specific form of regulation and of modeling the emissions.

In order to keep the model as simple as possible only one factor of production is considered and the technological constraints are described by the (standard) cost function. Then the extended cost function is derived by means of an additional minimization process. The focus of the analysis is theoretical, i.e. the implications of using an extended cost function for empirical work are not discussed, but the underlying idea and its advantages for comparative statics are brought out.

The paper is organized as follows: Section 2 discusses the model and the technological constraints and introduces the standard cost function(s). Section 3 is concerned with environmental regulation and defines the extended cost function. In section 4 its properties are examined and a simple comparative statics analysis is performed which shows the advantages of employing the concept. Section 5 offers some conclusions.

2. General framework: standard cost function

At first we consider the technological constraint(s) and describe the technology of a (representative) firm. The model chosen is simple. We assume that the firm employs a factor of production Z (like labor or capital or another resource), produces a commodity X (a final product), and generates E units of emissions (leading to pollution). In principle the firm is able to abate emissions. Its possibilities depend on the way emissions are modeled. One can find two different approaches in the literature (cf. e.g. Baumol and Oates (1988), p. 166 and p. 36): Emissions can be modeled as an input or as an output. If they are interpreted as a by-product of production, we have to model it as a joint product. On the other hand we can suppose that the production activity uses the environment as a medium for disposal (of residuals). Then the actual factor is the service provided by the environment and emissions can be modeled as an input. In the first case we assume that emissions can be abated by means of a separate abatement technology; in the other one the firm can substitute emissions by the factor of production. In the following both approaches are examined.

(i) If emissions are modeled as an input, production is characterized by a production function $F(E,Z)$ which is strictly concave (we assume that the production and cost functions considered are twice continuously differentiable). E and Z are supposed to be normal factors, i.e. the derived demand for E and Z increases with X (see Bear (1965)). Moreover, we assume that the production function F exhibits decreasing returns to scale and that for every Z there is $E(Z)$ such that $F_E(E(Z),Z)=0$. $E(Z)$ denotes the level of emissions chosen by the firm in the absence of any regulation given Z.

We will distinguish between the (usual) cost function[1] $C(v,w,X)$ and the conditional cost function $CC(w,E,X)$ where v and w denote the price of E and Z, respectively. They are defined by

$$C(v,w,X)=\min_{E,Z}\ vE+wZ \qquad \text{s.t.} \qquad X=F(E,Z)$$

and

$$CC(w,E,X)=\min_{Z}\ wZ \qquad \text{s.t.} \qquad X=F(E,Z),$$

respectively. The latter describes the minimal costs if the level of emissions is a priori set[2] to E. It can also be expressed explicitly by $CC(w,E,X)=wF^{-1}(E,X)$ where F^{-1} is the inverse of F with respect to the second argument.

Duality theory implies that the cost functions C and CC satisfy

Property C

The cost function is linearly homogeneous, increasing and concave in (factor) prices and increasing and convex in output.

By Shephard's Lemma the derived demand for E and Z can be described by C_v and C_w (CC_w), respectively, where C_v, C_w etc. denote partial derivatives. Furthermore, we obtain $C_{vw}>0$ and $CC_{wE}\le 0$ since E and Z are substitutes. Normality of E and Z yields $C_{vX}>0,\quad C_{wX}>0$ and $CC_{wX}>0,\quad CC_{EX}<0$.

(ii) If emissions are modeled as an output, E and X are joint products. As it is often done in environmental economics we identify both variables and define gross emissions by $E_g := X$. They can be reduced by abatement (think of an end of pipe technology). Supposing that Q units of (gross) emission are abated, net emissions are given by $E=E_g-Q=X-Q$. We assume that production and abatement also require the factor Z. The corresponding technologies are described by the cost function $CG(w,X)$ and, respectively, $AC(w,Q)$ which satisfy Property C.

The technologies determine the firm's behavior when it faces further constraints.

3. Environmental regulation: extended cost function

Now we turn to the topic of regulation and consider three instruments of environmental policy: an emission tax, and an absolute and a relative emission standard. In each case we will describe the regulatory constraint and then derive the corresponding extended cost function, i.e. the function of total costs TC which takes into account the costs of production *and* the costs of reacting to the form and level of regulation imposed. The cost function depends on output X, the factor price w and the instrument I under consideration. Since

[1] See Diewert (1982) and Nadiri (1982) for the theory of cost and production functions.

[2] Here it is implicitly assumed that there is a quantity Z s.t. $X=F(E,Z)$.

emissions can be modeled either as an input or as an output we obtain six cases in total. It will be proved that all these cases can be described by *one* (abstract) cost function $TC(w, X, I)$. Thus this section presents a simple and unified approach of modeling environmental regulation.

At first we discuss the price control and then the quantity restrictions.

a) Emission tax

Suppose that emissions *E* are taxed by the tax *t*. If *E* is a factor the tax *t* represents the price of emissions. Therefore the extended cost function can be defined by means of the standard cost function $TC^f(w, X, t) := C(t, w, X)$ for $I = t$. Here the superscript *f* indicates that emissions are a *factor*.

If *X* and *E* are joint products the extended cost function $TC^p(w, X, t)$ (*p* = *product*) is the solution of the cost minimization problem

$$\min_Q \; CG(w, X) + AC(w, Q) + t(X - Q).$$

Total costs comprise production and abatement cost *and* the tax payment. The solution to this problem requires that the marginal abatement costs equal the tax: $AC_Q(w, Q) = t$ or $Q = AC_Q^{-1}(w, t)$ where AC_Q^{-1} denotes the inverse of AC_Q with respect to the second argument. Therefore we obtain

$$TC^p(w, X, t) = CG(w, X) + AC\left(w, AC_Q^{-1}(w, t)\right) + t\left(X - AC_Q^{-1}(w, t)\right).$$

The emission tax is, of course, levied on final emissions after abatement and is therefore not identical to a commodity tax.

b) Absolute standard

Let the level of emissions be restricted[3] to no more than *E*. Then final emissions have to be equal to $E = X - Q$, given that the restriction is binding. For $I = E$ the extended cost function can therefore be defined directly by $TC^f(w, X, E) := CC(w, E, X)$ and $TC^p(w, X, E) := CG(w, X) + AC(w, X - E)$, respectively, since in the latter case $Q = X - E$ units of emissions have to be abated.

c) Relative standard

A relative standard[4] α restricts (net) emissions per unit of the final product *X*. It is given by $E/X \le \alpha$. If emissions represent a factor, α can be any positive number. When the standard is binding we can define $TC^f(w, X, \alpha) := CC(w, \alpha X, X)$.

If emissions represent a joint product the relative standard α has to be less than unity (since $E_g \equiv X$): it restricts final emissions

$$\frac{E}{X} = \frac{E_g - Q}{X} = \frac{X - Q}{X} \le \alpha$$

3 Helfand (1991) discusses various standards in detail.

4 See also Ebert (1998).

and leads to $Q=(1-\alpha)X$.

We then define $TC^p(w,X,\alpha):=CG(w,X)+AC(w,(1-\alpha)X)$ for $I=\alpha$.

For relative standards we have to impose the condition that the problem of profit maximization is well-defined and that the firm does not abate emissions voluntarily.

Thus in all these cases an extended cost function can be derived from the assumptions made.

4. Discussion and application

Now we discuss the outcome: we obtain an extended cost function $TC(w,X,I)$ for $I=t,E,$ or α and both types of technology (approaches described above). The properties of the cost function can be derived directly and are the usual ones. We obtain:[5]

Result 1

$TC(w,X,I)$ *satisfies Property C and* $TC_{wX}>0$ *for* $I=t,E$ *or* α .

Given additional regulatory constraints it is not clear whether Shephard's Lemma is still valid in its usual form. But in the model considered here we get

Result 2

The derived demand function for the factor Z is given by $TC_w(w,X,I)$.

For the comparative statics analysis performed below the derivatives TC_{XI}, TC_{wI} , and TC_I will play an important role: They describe the reaction of marginal cost, of the derived demand for *Z*, and of total cost to a (marginal) change in the level of the instrument *I*. In order to characterize them (qualitatively) we introduce the term 'strengthening the instrument *I*'. It means that the abatement and reduction requirement becomes more stringent.

Strengthening the regulation *I* marginally can be described by

$$dI>0 \text{ for } I=t \text{ and } dI<0 \text{ for } I=E \text{ or } \alpha .$$

Given these definitions we obtain (see also Table I)

Result 3

Strengthening the regulation I

(i) *increases marginal total costs and the derived demand for Z:*

$$TC_{XI}dI\geq 0,\;\; TC_{wI}dI\geq 0 \;\; for \;\; I=t,E, \; or \; \alpha$$

and

(ii) *increases total costs at a decreasing rate:*

$$TC_I dI>0 \;\; and TC_{II}dI<0 \;\; for \;\; I=t,E \;\; or \;\; \alpha .$$

The implications of tightening the regulation are as expected: marginal costs and the demand for the factor *Z* are increased. If *E* is a factor, it is substituted by *Z*. If it is an output, abate-

[5] The proof of all assertions concerning the extended cost function *TC* is straightforward and can be found in an Appendix available on request from the author.

ment and reduction of *E* raise the demand for *Z*. Total costs are a concave function of the parameter *I*. Thus costs are increasing in *I*, but at a decreasing rate.

Having discussed the properties of an extended cost function we finally consider a firm and its reaction to the forms of environmental regulation introduced. The implications for its behavior can be easily described for the case in which prices do not change. We establish

Result 4

For a price-taking firm strengthening the instrument I decreases the quantity produced, (net) emissions and profits.

The firm has two possibilities to react to the environmental regulation. It can decrease production. Then the level of emissions is also reduced automatically. Moreover, it can intensify abatement. When emissions are interpreted as a factor, they can be substituted by the factor of production. If the final product and emissions are joint products the firm possesses a separate abatement technology and can make more efforts in abating. If it is subject to a performance standard it must react in one (or both) of these ways. If it is regulated by a tax, the reactions described are in the firm's own interest. In all these cases output, emissions and profits are decreased (see also Table I).

The proof of this result is simple: Profit maximization yields the first-order condition $p - TC_X = 0$. The implicit function theorem then implies $dX/dI = -\left(TC_{XI}/TC_{XX}\right) < 0$. Furthermore, the marginal change in profits is given by $d\,profit = -TC_I dI < 0$ since $\frac{d\,profit}{dI} = \left(p - TC_X\right)\frac{dX}{dI} - TC_I$. One similarly obtains

$$\frac{dE}{dI} = E_X \frac{dX}{dI} + E_I < 0 .$$

5. Conclusion

The discussion in section 4 demonstrates the attractiveness and relevance of the concept of an extended cost function. It possesses the usual properties of a cost function with respect to the factor price and output. Furthermore, it reacts appropriately to changes in the regulatory parameter. The qualitative results derived above do not depend on the scenario considered (though things can sometimes be made a little bit more precise – see Table I). Therefore, using the device of an extended cost function, we are able to confine ourselves to performing a single (comparative statics) analysis, the results of which are valid for various scenarios (modeling of emissions and kinds of regulation) and can therefore be interpreted in many different ways.

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Table I: Derivatives of *TC* and *E*

* If the results differ, $[...]$ contains the result for the case in which emissions are modeled as an output.

$TC(w, X, I)$	$I=t$ input/output*	$I=E$	$I=\alpha$
TC_I	>0	<0	<0
TC_{wI}	>0	<0	<0
TC_{XI}	>0[1]	<0	<0
TC_{II}	<0	>0	>0
E	$=TC_t=\left[X-Q(w,t)\right]$	$=E$	$=\alpha X$
E_w	>0	0	0
E_X	>0[1]	0	α
E_I	<0	1	X