E C O N O M I C S   B U L L E T I N

Strategic R and D investments with uncertainty

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Abstract

I introduce uncertainty into the model of strategic cost−reducing R and D investments and reexamine welfare implications. I discuss two models. In one model an increase in expenditure decreases production costs when R\Dsucceeds, and in the other model it increases probability of success. I show that two models yield completely different implications for tax−subsidy policies on R and D investments. In the former model equilibrium investment level is always too low from the viewpoint of social welfare, while in the latter model it can be either too low or too high and relatively risky (safe) investments should be subsidized (taxed).

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1 Introduction

I reexamine welfare effect of strategic cost-reducing R&D investments in oligopoly markets by introducing uncertainty of success. In contrast to a huge body of literature on R&D competition models of patent race, quite a smaller body of literature on strategic cost-reducing R&D deals with uncertainty. The literature with strategic R&D competition is fairly large.[1] Most studies assume that R&D succeeds with probability one. Without any doubt, most R&D investments fail with a positive probability. Thus, investigating the effect of uncertainty is important.

In this paper I introduce into the model of Brander and Spencer (1983) and Lahiri and Ono (1999) uncertainty, where R&D investments fail with a positive probability, and discuss whether or not the equilibrium investment level exceeds the efficient one. I consider two models. One is the model where an increase in investments decreases production cost when R&D succeeds, and another is the model where an increase in investments increases the probability of success. I find that two models yield quite different welfare implications. In the former model the equilibrium investments are insufficient from the viewpoint of social welfare. In the latter model the equilibrium investments are insufficient (excessive) if the equilibrium probability of success is low (high). In other words, the private incentive of R&D investments for relatively risky (safe) projects is insufficient (excessive). These results indicate that whether or not subsidies for R&D investments improve welfare depends on whether firms spend money on increasing innovation size or the probability of success, and on the property of the project (risky or safe).

I use a standard strategic commitment games. In the first stage firms engage in cost-reducing R&D investments, and in the second stage they compete à la Cournot. A lot has been written on such two-stage models. Recently Lahiri and Ono (1999) point out that, very few studies analyze a question of R&D subsidies, while they are widely introduced in many countries. They also emphasize the importance of *ex ante* asymmetries between firms. In this paper I consider *ex ante* symmetric games. Introducing uncertainty, asymmetries between two firms appear in the second stage with a positive probability. I show that this *ex post* asymmetries are quite important for analyzing optimal R&D subsidies.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 investigates the equilibrium investment level. Section 4 discusses the welfare implication by comparing the equilibrium investment level with the socially efficient one.

[1] See, among others, Brander and Spencer (1983), Spence (1984), d'Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Suzumura (1992), Okuno-Fujiwara and Suzumura (1993), Matsumura (1995), and Lahiri and Ono (1999). See also Martin (2000), which is a unique work discussing strategic cost-reducing investments in the context of a patent race.

Section 5 concludes the paper.

2 The Model

I formulate two-stage duopoly games. Two firms are symmetric before the game. In the first stage each firm engages in cost-reducing R&D activities. Firm i $(i = 1, 2)$ succeeds in reducing its cost with probability q_i, independently of whether or not firm j succeeds $(j = 1, 2,\ j \neq i)$. Let c_i denote the (constant) marginal production costs. If firm i succeeds in cost-reducing, its marginal production cost becomes $c_i = c - \Delta_i$. Otherwise, its marginal production cost is $c_i = c$.

I consider two games. One is the game where each firm i chooses Δ_i given $q_1 = q_2 = q$. In this game an increase in R&D expenditure decreases the production costs when it succeeds. The other is the game where each firm i chooses q_i given $\Delta_1 = \Delta_2 = \Delta (> 0)$. In the second game, an increase in R&D expenditure increases the probability of the success of R&D. Let $I(\Delta_i, q_i)$ denote the investment costs. I assume that $I(\Delta_i, q_i)$ is twice continuously differentiable, increasing in q_i and Δ_i, and sufficiently convex.[2] I also assume that

$$\lim_{q_i \to 0} \frac{\partial I}{\partial q_i} = 0,\ \lim_{\Delta_i \to 0} \frac{\partial I}{\partial \Delta_i} = 0,\ \lim_{q_i \to 1} \frac{\partial I}{\partial q_i} = \infty,\ \lim_{\Delta_i \to c} \frac{\partial I}{\partial \Delta_i} = \infty$$

so as to ensure the interior solution.

At the beginning of the second stage each firm observes its rival's cost. Then the duopolists produce perfectly substitutable commodities for which the market demand function is given by $p = a - Y$ (price as a function of quantity), where Y is the total output of duopolists. Let y_i denote the output of firm i. Each firm i chooses y_i independently.

3 Equilibrium

I use subgame perfect Nash equilibrium as equilibrium concept. Let superscript 'E' denote the equilibrium outcome. In the Cournot game at the second stage, given c_1 and c_2, the equilibrium output y_i^E and the profit π_i^E of firm i are given by

$$y_i^E = \frac{(a - 2c_i + c_j)}{3}, \quad \pi_i^E = \frac{(a - 2c_i + c_j)^2}{9} - I(\Delta_i, q_i) \ \ (i, j = 1, 2,\ i \neq j). \tag{1}$$

In the first stage the expected profit of firm i is given by

$$E[\pi_i] \ = \ \frac{1}{9}[q_i q_j (a - c + 2\Delta_i - \Delta_j)^2 + q_i(1 - q_j)(a - c + 2\Delta_i)^2$$

[2] For the sufficient convexity of investment cost functions, see footnotes (5) and (9).

$$+ \quad (1-q_i)q_j(a-c-\Delta_j)^2 + (1-q_i)(1-q_j)(a-c)^2] - I(\Delta_i, q_i). \qquad (2)$$

First, consider the game where each firm chooses Δ_i, given $q_1 = q_2 = q$. The first order condition is given by

$$\frac{4q}{9}(-q\Delta_j + 2\Delta_i + a - c) - \frac{\partial I}{\partial \Delta_i} = 0. \qquad (3)$$

Henceforth, I restrict attentions to the symmetric equilibrium.[3] Substituting $\Delta_1 = \Delta_2 = \Delta$ into (3) yields

$$\frac{4q}{9}(-\Delta q + 2\Delta + a - c) = \frac{\partial I}{\partial \Delta}. \qquad (4)$$

Let $\Delta^E(q)$ denote the equilibrium innovation size. It is derived from (4).

Next, consider the game where each firm chooses q_i, given $\Delta_1 = \Delta_2 = \Delta$. The first order condition is given by

$$\frac{4\Delta}{9}(-q_j\Delta + \Delta + a - c) - \frac{\partial I}{\partial q_i} = 0. \qquad (5)$$

Substituting $q_1 = q_2 = q$ into (5) yields

$$\frac{4\Delta}{9}(-q\Delta + \Delta + a - c) = \frac{\partial I}{\partial q}. \qquad (6)$$

Let $q^E(\Delta)$ denote the equilibrium q. It is derived from (6).[4]

4 Results

In this section I discuss the following second best problem so as to compare the equilibrium outcomes discussed in the previous section to the efficient one. The welfare-maximizing social planner chooses the action of each firm in the first stage, given the Cournot competition in the next stage.

[3] Since the reaction curve of the first stage game has a negative slope (i.e., strategic substitutes) and is continuous, I can show that the symmetric equilibrium is unique. However, it is possible that asymmetric equilibria also exist. A sufficient condition for the uniqueness of equilibrium is $\partial^2 I/\partial\Delta^2 > 4q(2+q)/9$, which is a more restrictive condition than the second order condition. For the discussion of asymmetric equilibria, see Amir and Wooders (1998).

[4] Since the reaction curve of the first stage game has a negative slope (i.e., strategic substitutes) and is continuous, the symmetric equilibrium is unique. However, it is possible that asymmetric equilibria also exist. A sufficient condition for the uniqueness of equilibrium is $\partial^2 I/\partial q^2 > 4\Delta^2/9$, which is a more restrictive condition than the second order condition. See also footnote (3).

First, I consider the first game where innovation size of each firm is determined. Suppose that the social planner chooses $\Delta_1 = \Delta_2 = \Delta$, given $q_1 = q_2 = q$.[5] Then each firm faces Cournot competition discussed in the previous sections. Consumer surplus is given by

$$CS = \frac{1}{2}Y^2 = \frac{(2a - c_1 - c_2)^2}{18}, \tag{7}$$

and social welfare (total surplus) is given by

$$W = CS + \pi_1 + \pi_2. \tag{8}$$

The expected welfare is given by

$$\begin{aligned} E[W] &= \frac{1}{9}[4q^2(a-c+\Delta)^2 + 4(1-q)^2(a-c)^2 \\ &+ q(1-q)(8a^2 + 8c^2 + 11\Delta^2 - 16ac + 8a\Delta - 8c\Delta)] - 2I(\Delta, q). \end{aligned} \tag{9}$$

The first order condition is

$$\frac{q}{9}(-7q\Delta + 11\Delta + 4a - 4c) = \frac{\partial I}{\partial \Delta}. \tag{10}$$

Let $\Delta^*(q)$ denote this efficient innovation size Δ. It is derived from (10).

Proposition 1: $\Delta^*(q) \geq \Delta^E(q)$ $\forall q \in [0,1]$ *and the equality is satisfied if and only if* $q = 0$ *or* $q = 1$.

Proof: The left-hand side of (10) − the left-hand side of (4) = $\Delta(1-q)q/3 \geq 0$, and the equality is satisfied if and only if $q = 0$ or $q = 1$. Note that $0 \leq q \leq 1$. From the convexity of $I(\Delta)$, I obtain Proposition 1. **Q.E.D.**

The result that $\Delta^*(1) = \Delta^E(1)$ has already been shown by Brander and Spencer (1983).[6] Proposition 1 implies that introducing uncertainty induces the deviation of the equilibrium innovation size from the efficient one. I now discuss the intuition behind this result.

[5] This assumption is not innocuous. Suppose that cost is given exogenously. Then the total profits of two firms is jointly convex in costs. Thus it is possible that the social planner prefers the asymmetric outcome to the symmetric one. A sufficient condition under which the symmetric outcome is efficient is $\partial^2 I/\partial\Delta^2 > q(11+7q)/9$, which is a more strict condition than what appears in footnote (3). This is a reason why we assume that $\partial^2 I/\partial\Delta^2$ is sufficiently large. For the related works pointing out this convexity in costs and possible efficient asymmetric equilibria, see Amir and Wooders (2000) and Salant and Shaffer (1998).

[6] They also show that $\Delta^*(1) > (<)\Delta^E(1)$ if $p'' > (<)0$. I can show that uncertainty decreases Δ^E more significantly than Δ^*, so Δ^* can be larger than Δ^E even if $p'' < 0$, and Δ^* is always larger than Δ^E if $p'' > 0$.

An increase in Δ increases firm 1's profit and affects welfare when firm 1 is successful in the project. It induces the production substitution from firm 2 to firm 1. With probability $1-q$, the cost of firm 2 is c, which is higher than $c-\Delta_i$. Thus, the above production substitution economizes the total production costs because it reduces the production of the less efficient firm and increases that of the more efficient firm. Firm i chooses Δ_i without considering this welfare-improving production substitution effect, so the incentive for increasing the innovation size becomes insufficient. This effect disappears only if $1-q=0$ (i.e., firm 2 is always as efficient as firm 1).[7]

This result is closely related to that of Lahiri and Ono (1988). They show that a reduction of the output of the firm with the higher marginal cost improves welfare.[8] In my model an increase in Δ increases this welfare-improving production substitution when *ex post* asymmetries appear, so it should be promoted by subsidies. My result shows that the principle of Lahiri and Ono is important even when firms are symmetric *ex ante*.

Next, I consider the second game where the probability of success is determined. Suppose that the social planner chooses $q_1=q_2=q$, given $\Delta_1=\Delta_2=\Delta$.[9] Then each firm faces Cournot competition discussed in the previous sections. The social planner maximizes (9) with respect to q. The first order condition is

$$\frac{\Delta}{18}(11\Delta+8a-8c-14\Delta q)=\frac{\partial I}{\partial q}. \tag{11}$$

Let $q^*(\Delta)$ denote this efficient probability of the success. It is derived from (11).

Proposition 2: *$q^*(\Delta)\le q^E(\Delta)$ if and only if $q^E(\Delta)\ge 1/2$.*

Proof: The left-hand side of (11) – the left-hand side of (6) $=\Delta(1-2q)/6$. Since $\Delta>0$, it is non-positive if and only if $q\ge 1/2$. From the convexity of $I(q)$, I obtain Proposition

[7] The essential point in this paper is *ex post* asymmetries rather than uncertainty. Suppose that I drop the assumption of no-correlation and consider the following model: The probability that both firms succeed or fail is $(1+r)/4$, and the probability that only firm 1 succeeds or fails is $(1-r)/4$, where r is the correlation coefficient. Then I can show that $\Delta^*\ge\Delta^E$ and that the equality is satisfied if and only if $r=1$. Note that, if the projects of two firms are perfectly correlated, the welfare-improving production substitution does not occur in the symmetric equilibrium. For the discussion of correlation in the context of patent race, see Cardon and Sasaki (1998).

[8] For other discussions of welfare-enhancing production substitution effects, see also Brander (1981), Ono (1990), Riordan (1998), Matsumura (1998, 2003), Lahiri and Ono (1998), Ushio (2000), and Matsushima and Matsumura (2003).

[9] A sufficient condition under which the symmetric outcome is efficient is $\partial^2 I/\partial q^2>7\Delta^2/9$, which is a more strict condition than what appears in footnote (4). This is a reason why we assume that $\partial^2 I/\partial q^2$ is sufficiently large.

2. **Q.E.D.**

Proposition 2 states that the equilibrium R&D expenditure is too small from the viewpoint of social welfare if the equilibrium probability of success is less than $1/2$. In other words, each firm has an insufficient incentive for increasing the probability of success if the project is relatively hard to result in a success.

I explain the intuition behind the result. Consider the following situation. The initial probability of success is 0.1 and it increases up to 0.2 by additional investments. Suppose that firm 2 has already failed in the project and its cost is c. Suppose that the cost of firm 1 becomes $c - \Delta$ from c. The reduction of the cost increases the profit of firm 1, and it also induces the production substitution from firm 2 to firm 1. Since firm 1 is more efficient than firm 2, this production substitution improve welfare. Since firm 1 does not fully care about this welfare-improving production substitution effect, firm 1's incentive for investments becomes insufficient. Suppose that firm 2 has already succeeded in the project and its cost is $c - \Delta$. Suppose that the cost of firm 1 becomes $c - \Delta$ from c. The reduction of the cost induces the production substitution from firm 2 to firm 1. Since firm 2 is efficient whether or not firm 1's project is successful, this production substitution from firm 2 to firm 1 is not desirable. Thus, in contrast to the case where firm 2 fails, firm 1's incentive for investments becomes excessive when firm 2 succeeds. In short, given that firm 2 fails (succeeds), firm 1's incentive for investments becomes insufficient (excessive). Therefore, if equilibrium q_2 is low (high), firm 1's incentive for investments becomes insufficient (excessive).

This result makes a sharp contrast with Proposition 1. For example, as is shown in the proof of Proposition 1, in the first game the difference between marginal social and marginal private benefits of R&D expenditure is proportional to $q(1-q)$. It is maximized when $q = 0.5$. On the other hand, Proposition 2 states that, in the second game social benefits of R&D expenditure is equal to private one when $q = 0.5$, so neither taxes nor subsidies are required in this case. These results imply that the optimal tax-subsidy for R&D investments crucially depends on what kind of investments firms practice. Another example is the welfare effect of R&D subsidies for relatively safe project. For example, if $q = 0.6$, subsidies on R&D expenditure improve welfare if they expand the innovation size, while they are harmful if they raise the probability of success. The desirable policy crucially depends on the purpose of R&D investment.

5 Concluding Remarks

In this paper I investigate the optimal tax-subsidy policies on R&D investments. I introduce uncertainty into the standard strategic cost-reducing R&D investments. I discuss two duopoly models. In model 1 (2) an increase in expenditure decreases production costs when R&D succeeds (probability of failure).

I show that two models yield completely different implications for tax-subsidy policies on R&D investments. In model 1 the equilibrium investment level is always too low from the viewpoint of social welfare. On the other hand, in model 2 it can be either too low or too high. I also find that risky (safe) investments should be highly subsidized (taxed), which is never derived in model 1. These results indicate that the optimal tax-subsidy for R&D investments depends on what kind of investment the firms practice. In many countries both direct and indirect subsidies for R&D investments are widely adopted. This paper indicates that the government should carefully choose what kind of R&D should be prompted.

In this paper spillover effects of R&D and the associated free-rider problem are ignored. In order to derive richer policy implications, considering spillover effects are indispensable. Without them, the policy implication becomes limited. This important extension remains for future research.

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