E C O N O M I C S   B U L L E T I N

Ex-ante production, directed search and indivisible money

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Abstract

There always exists a monetary equilibrium when search is directed, money is indivisible and production is on demand (Julien Kennes King 2007). We demonstrate that when production takes place before exchange, forcing sellers to incur a sunk cost, there must be a minimum buyer-seller ratio for the monetary equilibrium to survive.

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1 Introduction

A standard assumption in the monetary search literature is that the amount of goods traded is produced on demand by sellers contingent on meeting an appropriate buyer. If there is no such meeting, sellers carry on to the next period with no production cost. The assumption of production on demand, interpreted as the good being a service, was made initially in the second generation of monetary search models (Shi 1995, Trejos Wright, 1995) to suit the pairwise random matching-Nash bargaining framework in which those models were developed (see Kiyotaki Wright 1989,1991,1993). In those models of indivisible money, if we assume production prior to meeting, the environment becomes one of random matching and price posting (1 unit of money against q units of goods) and the monetary equilibrium unravels: sellers are aware that the arrival rate of buyers is independent of the quantity they produce, and so they produce a quantity such that buyers get no gains from trade–the value of money falls to zero (Curtis and Wright 2004). This is a variant of the Diamond (1971) paradox.

Departing from random search, recent developments in the money search literature make the production on demand assumption no longer necessary. Especially, models in which search is directed by posted terms of trade (Rocheteau and Wright 2005, Julien Kennes and King 2007) imply commitment on the side of sellers that could perfectly accommodate ex-ante production. Still these models assume production on demand. In this note we extend Julien Kennes and King (2007)'s directed search model of indivisible money by considering production prior to trading rather than production on demand. First, Julien Kennes and King (2007) has shown that there *always* exists a monetary equilibrium with directed search when production is on demand. We want to know is this result holds when sellers produce prior to meeting. Second, production in advance suits the goods market better, which makes it an interesting variation to explore. Our conclusion is that by forcing sellers to incur a sunk cost, ex ante production requires a minimum buyer-seller ratio for the monetary equilibrium to exist.

2 The Model

There is a $[0, 1]$ continuum of infinitely lived agents, a fraction $M \in (0, 1)$ of which are given money and act as buyers. The remaining part act as sellers, implying a buyer-seller ratio $\Phi = M/1 - M$. Goods are divisible, money is indivisible, and agents reverse roles upon successful trades as in the Shi-Trejos-Wright model. Sellers produce and post quantities to attract buyers and taking into account competition from other sellers. Buyers observe all advertisements and allocate themselves across sellers. A seller can only serve one buyer at a time so if more than one buyer selects a seller, one is picked at random to trade. The produced quantity is then traded in all matches. For simplicity we assume that any unsold quantity (no match for a seller) entirely depreciates. In any period and for any agent, consuming q units of his consumption good yields $u(q)$ and producing q costs $c(q)$ with standard concavity and convexity assumptions. In equilibrium buyers will be indifferent between all sellers and will allocate themselves randomly. We note $\beta < 1$ the discount factor.

We solve first for the equilibrium posted q in a particular period, and then solve for steady state. Let $\bar{V}_0$ be the value for a seller and $\bar{V}_1$ be the value for a buyer. These values embed the equilibrium quantity, q^e, simultaneously produced and posted by sellers in each period. Let q be the quantity chosen and produced by one seller while all other sellers set Q and let $\phi(q)$ (not necessarily equal to Φ), be the corresponding expected queue length for this seller. The value to a seller of posting q is:

$$V_0\left(q\right) = -c(q) + \xi(q)\beta\bar{V}_1 + [1 - \xi(q)]\beta\bar{V}_0 \tag{1}$$

where $\xi(q) = 1 - e^{-\phi(q)}$ is the probability of attracting at least one buyer when posting q.[1] The value for a buyer selecting a seller producing q is

$$V_1(q) = \psi(q)\left[u(q) + \beta\bar{V}_0)\right] + [1 - \psi(q)]\beta\bar{V}_1, \tag{2}$$

where $\psi(q) = \frac{1-e^{-\phi(q)}}{\phi(q)}$ is the probability a buyer gets served when selecting a seller posting q.

Let $\Delta = \bar{V}_1 - \bar{V}_0$. For a buyer to be willing to trade within a match it requires $u(q) + \beta\bar{V}_0 \geq \beta\bar{V}_1$, or

$$u(q) - \beta\Delta \geq 0. \tag{3}$$

For a seller to be willing to trade at a given posted q, since production costs are sunk, it only requires $\bar{V}_1 > \bar{V}_0$ which has to be true (and actually is as shown below) since his only chance to cover that cost is to sell his production. The relevant incentive constraint for a seller is then whether to take part into this economy or not, that is $V_0\left(q\right) \geq 0$. From (1) this is equivalent to

$$\bar{V}_0\left(1 - \beta\right) = \xi(q)\beta\Delta - c(q) \geq 0. \tag{4}$$

By comparison, in Julien Kennes and King (2007), the incentive constraint for the seller is $\beta\Delta - c(q) \geq 0$. Here production costs are sunk whereas trade is contingent on attracting at least one buyer, which happens with probability $\xi(q)$.

[1] A derivation of the matching probabilities for sellers and buyers is generated by the symmetric equilibrium mixed strategies. As shown in Julien, Kennes and King (2007), these generate an urn-ball (exponential) matching function.

The first participating constraints (3) implies a minimum $\underline{q}$ that a buyer is willing to accept defined by $u(\underline{q}) = \beta\Delta$. The second implies a maximum $\bar{q}$ that a seller is willing to offer defined by $c(\bar{q}) = \xi(\bar{q})\beta\Delta$. When deciding which quantity to produce and advertise, a seller solves the following problem

$$\max_{q\in[\underline{q},\bar{q}]} V_0(q) \quad \text{s.t.} \quad V_1(q) = \bar{V}_1 \tag{5}$$

where the constraint determines how the expected queue length $\phi(q)$ changes with the posted q. The expected queue length is determined by buyers being indifferent between selecting the seller producing q or any other seller in the market producing Q like all others. In large markets, the value of selecting the seller posting q must be equal to the buyer's equilibrium market utility value $\bar{V}_1$.[2] It follows that, for any $q \leq \underline{q}$, we have $V_1(q) \leq \bar{V}_1$ or $\phi(q) = 0$: no buyer would select a seller producing $q < \underline{q}$ even if he were to be served with probability 1. Similarly, for any $q \geq \bar{q}$, we have $V_1(q) \geq \bar{V}_1$ or $\phi(q) = 1$: all buyers select the seller, but the seller is not willing to produce $q > \bar{q}$.

Taking the derivative of (1) yields

$$V_0'(q) = -c'(q) + \beta\Delta\phi'(q)e^{-\phi(q)}. \tag{6}$$

Differentiating and using the implicit function theorem on $V_1(q) = \bar{V}_1$ gives

$$\psi'(q)[u(q) - \beta\Delta] + \psi(q)u'(q) = 0$$

and

$$\phi'(q) = \frac{u'(q)\phi(q)(1 - e^{-\phi(q)})}{[1 - e^{-\phi(q)} - \phi(q)e^{-\phi(q)}][u(q) - \beta\Delta]}. \tag{7}$$

It is easy to show that $V_0(q)$ is concave in q and $\phi'(q) > 0$. Using (7) in (6), and setting (6) equal to zero yields the equilibrium production q in a given period taking the continuation values in Δ as given. It is characterized by

$$\frac{c'(q)}{u'(q)} = g(q)\frac{\xi(q)\beta\Delta}{[u(q) - \beta\Delta]} \tag{8}$$

with $g(q) = \frac{\phi(q)e^{-\phi(q)}}{1-e^{-\phi(q)}-\phi(q)e^{-\phi(q)}}$.

Let $q = q(\Delta)$ be the solution to (8). The solution is a seller's best response to the expected aggregate variable Δ. In order to solve for $\Delta = \bar{V}_1 - \bar{V}_0$, let q^e be the quantity posted by sellers in any period. We have

$$\bar{V}_0 = -c(q^e) + \xi(\Phi)\beta\Delta + \beta\bar{V}_0 \tag{9}$$

[2]To solve for equilibrium q we use the *market utility property* which is known to yield equivalent solutions to considering a subgame perfect equilibrium in large markets. See Peters (2000).

and

$$\bar{V}_1 = \psi(\Phi)\left[u(q^e) - \beta\Delta\right] + \beta\bar{V}_1. \tag{10}$$

Notice that $\bar{V}_0$ and $\bar{V}_1$ are functions of q^e. The matching probabilities are expressed as $\xi(\Phi) = 1 - e^{-\Phi}$ and $\psi(\Phi) = \frac{1-e^{-\Phi}}{\Phi}$ to reflect the fact that in a symmetric equilibrium, buyers select all sellers with the same probability. Hence, $q = Q = q^e$, and $\phi(q) = \Phi = \frac{M}{1-M}$. Using (9) and (10) along with Δ gives

$$\Delta(q^e) = \bar{V}_1(q^e) - \bar{V}_0(q^e) = \frac{\psi(\Phi)u(q^e) + c(q^e)}{(1 - \beta(1 - \psi(\Phi) - \xi(\Phi)))} > 0 \Longleftrightarrow q^e > 0. \tag{11}$$

The relationship $\Delta = \Delta(q^e)$ is an aggregate consistency requirement, determining the value of Δ when all sellers are producing and trading q^e for a unit of fiat money (Shi 2006). A fixed point of $q^e = q^e(\Delta)$ and $\Delta = \Delta(q^e)$ gives the steady state equilibrium.

To show existence, insert $\Delta = \bar{V}_1 - \bar{V}_0$ into (8) and note that the LHS of (8) is continuous, bounded and increasing on $\left[\underline{q}, \bar{q}\right]$ with a maximum equal to

$$\frac{u'(\bar{q})}{c'(\bar{q})}. \tag{12}$$

The RHS of (8) decreases from ∞^+ to

$$g(\bar{q})\frac{c(\bar{q})}{[u(\bar{q}) - \frac{c(\bar{q})}{\xi(\bar{q})}]} \tag{13}$$

where we have used the fact that $c(\bar{q}) = \xi(\bar{q})\beta\Delta(\bar{q})$. From (12) and (13) a sufficient condition for existence is then given by

$$u(\bar{q})c'(\bar{q}) \geq c(\bar{q})\left[u'(\bar{q})g(\bar{q}) + \frac{c'(\bar{q})}{\xi(\bar{q})}\right] \tag{14}$$

It is easy to see that this inequality is satisfied for large Φ. Especially when $\Phi \to \infty^+$ it simplifies into $u(\bar{q}) \geq c(\bar{q})$ which is always true.[3] Conversely, from (14) again, we can derive two sufficient conditions for *non* existence of a monetary equilibrium

$$\begin{aligned} (i) &\ :\ u'(\bar{q})g(\bar{q})c(\bar{q}) \geq u(\bar{q})c'(\bar{q}) \\ (ii) &\ :\ c(\bar{q}) \geq u(\bar{q})\xi(\bar{q}) \end{aligned}$$

both of which are verified for low Φ. Therefore

[3] Another way to see this is to notice that when $\Phi \to \infty^+$ (4) transforms into $\beta\Delta - c(q) \geq 0$, which is the seller's participation constraint in Julien Kennes and King (2007). Whether production takes place before or after meeting does not matter if a seller is certain to meet a buyer.

Proposition 1 *When production costs are sunk, there must be a minimum buyer-seller ratio in a directed search economy for a monetary equilibrium to exist.*

When production takes place before exchange, forcing sellers to incur a sunk cost, sellers have to expect a positive return to cover that cost. In random matching models this is achieved, for instance, by increasing the share sellers receive in the equilibrium of the bargaining game. In directed search, this is achieved by raising the buyer-seller ratio.

3 Conclusion

The assumption that production is on demand proved helpful in developing monetary search models. Especially, no monetary equilibrium is sustainable with production in advance and random matching. In this note, we have shown that a monetary equilibrium exists with production in advance and directed search, but this requires to raise the seller's expected return via a minimum buyer-seller ratio.

When search is directed but money is fully divisible, as in Rocheteau and Wright (2005), there again exists a monetary equilibrium for all buyer-seller ratios when production is on demand. It is not clear, however, that our result would hold in their environment as there are now two margins of adjustment operating ex-ante. As the buyer-seller ratio falls, sellers can respond by increasing the posted price or decreasing the posted quantity. We leave this question open for future research.

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