ECONOMICS BULLETIN

The Bi−parameter Smooth Transition Autoregressive model

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Abstract

The present paper introduces the Bi−parameter Smooth Transition Autoregressive (BSTAR) model that generalizes the LSTR2 model, see Terasvirta (1998). In contrast to the LSTR2 model, which features the symmetric transition function, the BSTAR model is characterized by the asymmetric transition function which implies different local dynamics in the neighborhood of the respective location parameters. An empirical example using the time series of the annual growth rates of the Italian industrial production index is provided.

The paper has benefited from helpful comments of Philip Hans Franses, Niels Haldrup, Svend Hylleberg, Hans Christian Kongsted, Timo Teräsvirta, Dick van Dijk, and Allan Würtz as well as the participants in the 58th European Meeting of Econometric Society, ESEM (Stockholm, Sweden), in the 2nd Nordic Econometrics Meeting (Bergen, Norway) and in the ERC/METU V conference in Economics (Ankara, Turkey).
Citation: Siliverstovs, Boriss, (2005) "The Bi−parameter Smooth Transition Autoregressive model." *Economics Bulletin*, Vol. 3, No. 22 pp. 1−11
Submitted: February 13, 2005. **Accepted:** May 4, 2005.
URL: http://www.economicsbulletin.com/2005/volume3/EB−05C50003A.pdf

1 Introduction.

Since the seminal articles of Teräsvirta and Anderson (1992) and Teräsvirta (1994), smooth transition autoregressive (STAR) models have become one of most popular classes of non-linear models in modern applied economics. The STAR models have been employed in modelling the dynamics of various types of economic time series, for example industrial production in Teräsvirta and Anderson (1992), unemployment in Skalin and Teräsvirta (2002), interest rates in van Dijk and Franses (2000), exchange rates in Taylor, Peel, and Sarno (2001), *inter alia*. Also there are a number of surveys that review features of modelling with STAR models such as Granger and Teräsvirta (1993), Teräsvirta (1998), Potter (1999), and most recently, van Dijk, Teräsvirta, and Franses (2002).

A typical univariate STAR model[1] has the following form

$$y_t = \phi' \mathbf{x}_t + \theta' \mathbf{x}_t F_t(\gamma, c; y_{t-d}) + u_t, \tag{1}$$

where $\mathbf{x}_t = \{1, y_{t-1}, ..., y_{t-p}\}'$ is a vector of the lags of the dependent variable including the constant term. The vectors of the autoregressive parameters are $\phi = (\phi_0, \phi_1, ..., \phi_p)'$ and $\theta = (\theta_0, \theta_1, ..., \theta_p)'$. The error term u_t is usually assumed to be the NID random variable with mean zero and variance σ^2. The transition function $F_t(\gamma, c; y_{t-d})$ defines regime-specific dynamics and governs the transition between these regimes, depending on the values of the transition variable y_{t-d} relative to those of the slope γ and of the location c parameters. The magnitude of the slope parameter measures smoothness of transition between the regimes while the value of the threshold parameter indicates the location of the transition. Observe that depending on model complexity, the slope and location parameters can be either scalar or vector. The delay parameter d of the transition variable can take values in the range of $1 \leq d \leq p$ or $p < d$.

Teräsvirta and Anderson (1992) introduce the two types of two-regime STAR models, each having its own attractive features: logistic and exponential STAR models[2]. The former model is based on the first-order logistic transition function

$$F_t(\gamma, c; y_{t-d}) = \frac{1}{1 + \exp(-\gamma(y_{t-d} - c))}, \quad \gamma > 0 \tag{2}$$

and therefore is usually referred to as LSTR1 model. Two different regimes are defined by the small or large values of the transition variable y_{t-d} relative to the threshold parameter c. Such models have been widely applied to capture the business cycle asymmetries, e.g. different time series dynamics during expansions and contractions. Observe that on the one hand when the slope parameter $\gamma \longrightarrow 0$ the LSTR1 model becomes a linear model, and on the other hand when $\gamma \longrightarrow \infty$ the LSTR1 model converts into the Self-Exciting Threshold Autoregressive (SETAR) model, popularized in Tong (1983, 1990).

The latter model utilizes the exponential transition function

$$F_t(\gamma, c; y_{t-d}) = 1 - \exp\left(-\gamma (y_{t-d} - c)^2\right), \quad \gamma > 0 \tag{3}$$

[1] Smooth transition regression (STR) models are not necessary univariate models. The STR models that include other exogenous variables as well as the vector STR models have been already suggested in the literature, see e.g. van Dijk et al. (2002).

[2] Other extensions of the STAR models that involve more than two regimes as well as time varying parameters have been suggested in van Dijk and Franses (1999) and Lundbergh, Teräsvirta, and van Dijk (2003), respectively. In order to save space, we omit discussion of these models.

and therefore is usually referred to as ESTAR model. The ESTAR model defines different regimes in terms of small and large absolute deviations of the transition variable values from the threshold parameter value. Hence, this model has a 'sandwich' structure with the outer regime (defined for large absolute values of the transition variable with respect to the value of the threshold parameter), that is contrasted with the inner regime (defined by the values of the transition variable that are rather close to the threshold parameter value). Observe that the outer regime consists of two phases: lower and upper, defined for the values of the transition variable that are respectively smaller and larger than the threshold parameter value. Such models have proved to be particularly useful for modelling exchange rates, see Taylor, Peel, and Sarno (2001) for an example. The properties of the exponential function suggest that in both cases when the slope parameter $\gamma \longrightarrow \infty$ or $\gamma \longrightarrow 0$, the nonlinear ESTAR model collapses into a linear model. Hence it does not nest the SETAR model as a special case.

The ESTAR model allows for smooth change in the values of coefficients from $(\phi+\theta)$ to ϕ at $y_{t-d} = c$ and then back to $(\phi+\theta)$. As noted in Jansen and Teräsvirta (1996), it may be desirable to allow for possibility of abrupt changes in the values of the coefficients in both directions. This is achieved using the following second-order logistic transition function (LSTR2)

$$F_t(\gamma, c_1, c_2; y_{t-d}) = \frac{1}{1+\exp\left(-\gamma\left(y_{t-d}-c_1\right)\left(y_{t-d}-c_2\right)\right)} \quad , \quad \gamma > 0. \tag{4}$$

The LSTR2 model has been introduced in Jansen and Teräsvirta (1996) and Teräsvirta (1998). Note that contrary to the ESTAR model, the LSTR2 model has two location parameters c_1 and c_2. This fact ensures that for rather large values of the slope parameter, the LSTR2 corresponds to the SETAR model with inner and outer regimes.

Observe that both ESTAR and LSTR2 models only allow for symmetric transition between the inner regime and each of the phases of the outer regime. Clearly, this is a restrictive assumption that can be relaxed. Motivation for relaxing the symmetry assumption comes from the observation, supported by economic theory, that in various situations the economic agents adjust their behavior differently in response to positive and negative shocks, see e.g. Cover (1992) for discussion of the asymmetric effects of positive and negative money-supply shocks on output, Ball and Mankiw (1994) for discussion of asymmetric price responses to positive and negative shocks, Huizinga and Schiantarelli (1992) and Andolfatto (1997) for models describing a cyclical asymmetry in unemployment rate fluctuations, and Jackman and Sutton (1982) for a model of asymmetric effects of interest rate changes on aggregate consumption, *inter alia*.

Anderson (1997) made the first attempt to relax the symmetry assumption in STAR models that define different regimes in terms of small and large absolute deviations of the transition variable values from the threshold parameter value. Modelling investor response to changes in the yield spreads between the US Treasury Bills of different maturity, Anderson (1997) suggests that in the presence of the heterogenous transaction costs, investors adjust their portfolios gradually depending on the size and, possibly, the sign of deviations of the yield spreads from the implied equilibrium levels. Such that the investor response is rather weak for the small values of disequilibrium on the one hand, and it is rather strong for the rather large values on the other hand. Moreover, the model of Anderson (1997) allows for asymmetric investor response to situations when the bill of shorter maturity is either over- or under-priced relative to the bill of longer maturity.

In order to capture this possible asymmetry in the investor response, Anderson (1997)

Table 1: Classification of transition functions.

		Transition function	
		Symmetric	Asymmetric
Collapses as $\gamma \to \infty$	YES	ESTAR	AESTAR
	NO	LSTR2	BSTAR

modifies the symmetric ESTAR transition function as follows

$$\begin{aligned} F_t(\gamma, c, \delta; y_{t-d}) &= 1 - \exp\{-\gamma [y_{t-d} - c]^2 \times h(y_{t-d})\}\ ,\ \gamma > 0, \\ h_t(c, \delta; y_{t-d}) &= \{0.5 + (1 + \exp\{-\delta [y_{t-d} - c])^{-1}\}\ ,\ \delta \neq 0. \end{aligned} \tag{5}$$

The resulting transition function (5) is asymmetric around the location parameter value c, depending on whether the parameter δ takes a positive or negative value. Hence the model with the transition function (5) allows for different speeds of transition between the inner regime and each of the phases of the outer regime. Moreover, the transition function (5) corresponds to the ESTAR transition function when $\delta = 0$. We refer to the model with the transition function (5) as the Asymmetric ESTAR model or AESTAR in short.

Observe that the AESTAR model has a similar feature to the ESTAR model, i.e. in both cases when $\gamma \to 0$ and $\gamma \to \infty$, the model becomes a linear model. In this paper, we suggest a two-regime model with the transition function that has two important features: First, it allows for asymmetric speed of transition between the inner regime and each of the phases of the outer regime. Second, for rather large values of the slope parameter, it becomes the SETAR model with inner and outer regimes. Hence, it extends the LSTR2 model with the symmetric transition function by allowing for asymmetric speed of transition between each of the phases of the outer and middle regimes in a similar way to how the AESTAR model of Anderson (1997) extends the ESTAR model. Similarly to the LSTR2 model, the suggested model has the transition function with *two* location parameters, but the transition function asymmetry is achieved by introducing an additional threshold parameter, such that each of the *two* slope parameters determines the transition speed between the inner regime and the respective phases of the outer regime. Henceforth, the model in question is referred to as the Bi-parameter Smooth Transition Autoregressive model (BSTAR in short).

The paper proceeds as follows. Section 2 introduces the BSTAR model. Section 3 provides an empirical example. The final section concludes.

All computations were performed using the object-oriented programming language Ox 3.30 Professional, see Doornik (2001), and the empirical modelling program package PcGive 10.3, see Hendry and Doornik (2001).

2 BSTAR model

It was stated earlier that the asymmetric transition function of Anderson (1997) provides an extension of the symmetric transition function of the ESTAR model. Therefore it has a similar inherent problem when the slope parameter tends to infinity: then it is problematic to distinguish this non-linear model from a linear one.

In this section, the BSTAR model with following transition function is introduced

$$F_t(\gamma_1, c_1, \gamma_2, c_2; y_{t-d}) = \frac{\exp[-\gamma_1(y_{t-d}-c_1)] + \exp[\gamma_2(y_{t-d}-c_2)]}{1+\exp[-\gamma_1(y_{t-d}-c_1)] + \exp[\gamma_2(y_{t-d}-c_2)]} \quad (6)$$
$$\gamma_1 > 0, \gamma_2 > 0, c_1 < c_2.$$

This transition function has two threshold parameters c_1 and c_2, and two slope parameters γ_1 and γ_2. In the BSTAR model, different values of the slope parameters γ_1 and γ_2 result in the different slopes of the transition function at the corresponding threshold parameters c_1 and c_2. This allows for an asymmetric speed of transition between the inner and the lower and upper phases of the outer regime. Note that when the restriction of equality of the slope parameters is imposed, i.e. $\gamma_1 = \gamma_2 = \gamma$, the BSTAR model imposes the symmetric transition between the outer and inner regimes and thus it replicates the features of the LSTR2 model.

Observe that $F_t(\gamma_1, c_1, \gamma_2, c_2; y_{t-d}) \to 1$ for $y_{t-d} \to \pm\infty$. The minimum value of $F_t(\gamma_1, c_1, \gamma_2, c_2; y_{t-d})$ lies in the interval between 0 and 2/3, the upper limit holds for $c_1 = c_2$ and $\gamma_1 = \gamma_2$. When $\gamma_1 \to \infty$ and $\gamma_2 \to \infty$, $F_t(\gamma_1, c_1, \gamma_2, c_2; y_{t-d}) \to 0$ for $c_1 \leq y_{t-d} \leq c_2$ and $F_t(\gamma_1, c_1, \gamma_2, c_2; y_{t-d}) \to 1$ otherwise. These features of the BSTAR transition function are comparable to those of the LSTR2 transition function with the only difference that the minimum value of the latter transition function lies in the interval between 0 and 1/2, see Teräsvirta (1998) for details.

Notice that the transition function (6) offers a more expository way to capture the asymmetry in the speed of transition when it is compared to the approach of Anderson (1997) and embodied in the transition function of the AESTAR model. In the transition function of the BSTAR model, the values of each slope parameter determine steepness of the transition function at the corresponding transition locations separately one from each other. In contrast, in case of the AESTAR model, both the slope and asymmetry parameters, γ and δ, jointly determine the degree of asymmetry of the AESTAR transition function. This comes from the fact that there is only one slope parameter γ that determines how steep the corresponding AESTAR transition function is.

Table 1 provides a classification of the two (i.e. inner and outer) regime STAR models according to whether they allow for symmetric or asymmetric speeds of transition between the inner regime and each phase of the outer regime, and whether they collapse to a single spike for rather large values of the slope coefficient(s). As seen in Table 1, the suggested BSTAR model fills the appropriate niche in this classification of the STAR models and in so doing, naturally complements the family of nonlinear STAR models.

Observe that since the BSTAR model possesses the basic features of the other STAR models, the steps of the modelling cycle such as specification, model estimation, and model evaluation, suggested in Teräsvirta (1994, 1998), can be easily adapted in order to accommodate the BSTAR model. Hence given that the modelling cycle has already been discussed extensively in the

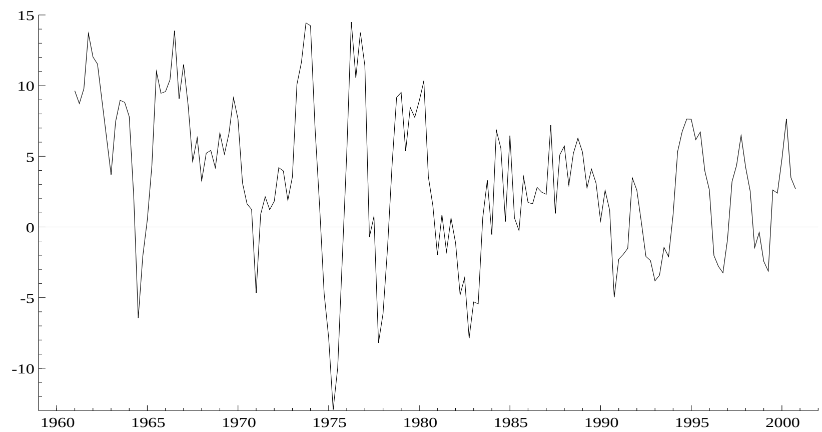


Figure 1: Annual growth rate of IIP of Italy, 1960.1:2000.4, *source:* Main Economic Indicators

Table 2: Linearity tests

	Transition variable											
	y_{t-1}	y_{t-2}	y_{t-3}	y_{t-4}	y_{t-5}	y_{t-6}	y_{t-7}	y_{t-8}	y_{t-9}	y_{t-10}	y_{t-11}	y_{t-12}
H_0:	0.275	0.223	0.065	0.004	0.141	0.030	0.128	0.041	0.015	0.006	0.151	0.194
H_{01}:				0.008						0.020		
H_{02}:				0.011						0.031		
H_{03}:				0.810						0.371		

Results of linearity tests ($p-$values) are based on the following auxiliary regression: $y_t = \beta_1'\mathbf{x}_t + \beta_2'\widetilde{\mathbf{x}}_t \cdot y_{t-d} + \beta_3'\widetilde{\mathbf{x}}_t \cdot y_{t-d}^2 + \beta_4'\widetilde{\mathbf{x}}_t \cdot y_{t-d}^3 + \eta_t$. The null hypotheses H_0, H_{01}, H_{02}, and H_{03} are $\beta_2 = \beta_3 = \beta_4 = 0$, $\beta_4 = 0$, $\beta_3 = 0|\beta_4 = 0$, and $\beta_2 = 0|\beta_3 = \beta_4 = 0$, respectively, see Teräsvirta (1994).

literature and in order to save space, we relegate the details to the technical appendix available upon request.

3 Empirical example

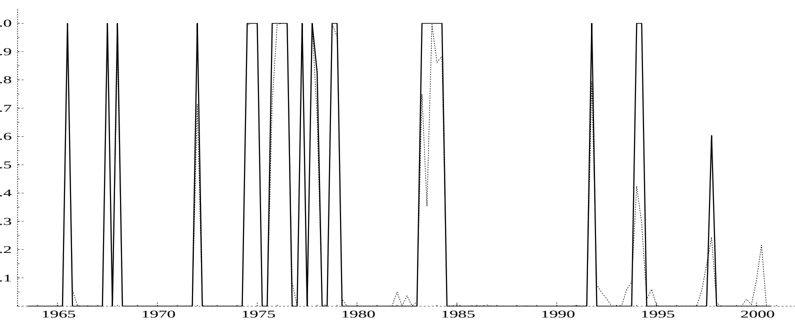
As an illustration of the empirical relevance of the suggested model, the modelling strategy suggested in Teräsvirta (1994, 1998) is applied to the industrial production index of Italy. The data are taken from *Main Economic Indicators* and were adjusted for the effects of the widespread industrial strike in 1969. The seasonally adjusted quarterly data span the period from 1960:1 until 2000:4. The sample size is 164 observations. Following Teräsvirta and Anderson (1992), we model the annual growth rate of the time series in question, displayed in Figure 1. Note that in order to avoid having to deal with small numbers, the transformed time series was multiplied by 100 and it is denoted as y_t.

The results of the sequence of the linearity tests of Teräsvirta (1994) are reported in Table 2. As seen, the strongest rejection occurs at the fourth and the tenth lags of the transition variable. Furthermore, the linearity test results are rather inconclusive regarding the choice between a LSTR1 or a BSTAR model. On the one hand, rejection of H_{01} favors a LSTR1 model, on the other hand, acceptance of H_{03} after rejecting H_{02} points at a BSTAR model. Hence, we estimate both types of models and let the diagnostic tests choose between the two alternatives. The diagnostic tests for the estimated LSTR1 models (not reported to save space) report problems with remaining nonlinearity and autocorrelation indicating inadequacy of the LSTR1 models for the present time series. However, the estimated BSTAR models pass the diagnostic tests as reported below. Next, we find that the BSTAR models with the delay parameters $d = 4$ and $d = 10$ provided a similar in-sample fit, and hence in order to save space, we report the estimation results for the BSTAR models with unrestricted slope coefficients $\gamma_1 \neq \gamma_2$ and with restricted slope coefficients $\gamma_1 = \gamma_2$ only for the delay parameter $d = 4$. These models are presented in equations (7) and (8), respectively. Figure 2 displays the estimated transition functions for both BSTAR models plotted against the time variable (upper panel) and the transition variable (lower panel). The diagnostic tests are reported in Table 3.

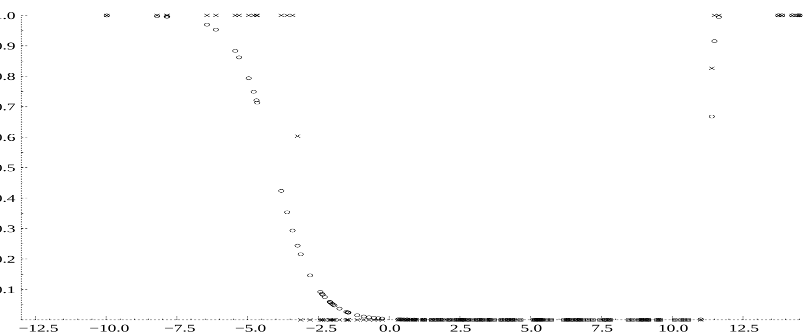
As expected, the nonlinear models provide better in-sample fit than a linear AR(10) model. The ratio of residual variances of either nonlinear model to that of the AR(10) model is 0.831. We also are unable to reject the null hypotheses of no ARCH effects and normality of residuals for both BSTAR model specifications. The long-run properties of the estimated models imply that both the unrestricted and the restricted models have a stable stationary point, which is 2.632 for the former and 2.378 for the latter.

Note that despite the similarity in the autoregressive coefficient estimates of the both BSTAR models, the transition function of the model with unrestrictive slope parameters is much smoother in the neighborhood of the lower location parameter c_1 and it is very similar to the shape of the transition function of the model with equal slope parameters at the other location parameter c_2. Hence, the local behavior of the model near the location parameter c_1 constitutes the difference between these two specifications. Finally, the diagnostic tests for autocorrelation, parameter constancy, and remaining nonlinearity suggest that even though the restricted model displays no autocorrelation in residuals as well as stable parameters, there is some evidence of remaining nonlinearity. In contrast, we are unable to reject the null of no autocorrelation, parameter constancy, and no remaining nonlinearity at the 5% significance level for the unrestricted BSTAR model.

$$\begin{aligned} y_t \;=\;& \underset{(0.341)}{0.625} + \underset{(0.067)}{0.783} y_{t-1} + \underset{(0.078)}{0.259} y_{t-2} - \underset{(0.073)}{0.565} y_{t-4} + \underset{(0.079)}{0.457} y_{t-6} - \underset{(0.077)}{0.355} y_{t-8} + \underset{(0.059)}{0.183} y_{t-10} \\ & + \left[-\underset{(0.120)}{0.315} y_{t-1} - \underset{(0.079)}{0.457} y_{t-6} + \underset{(0.145)}{0.433} y_{t-7} + \underset{(0.250)}{0.560} y_{t-9} - \underset{(0.223)}{0.419} y_{t-10} \right] F_t(\widehat{\gamma}_1, \widehat{c}_1, \widehat{\gamma}_2, \widehat{c}_2; y_{t-4}) + \widehat{e}_t \\ & F_t(\widehat{\gamma}_1, \widehat{c}_1, \widehat{\gamma}_2, \widehat{c}_2; y_{t-4}) = \frac{\exp\left[-\underset{(6.562)}{7.466} \left(y_{t-4} + \underset{(0.924)}{4.025} \right) \right] + \exp\left[\underset{(164.7)}{94.66} \left(y_{t-4} - \underset{(0.11)}{11.360} \right) \right]}{1 + \exp\left[-\underset{(6.562)}{7.466} \left(y_{t-4} + \underset{(0.924)}{4.025} \right) \right] + \exp\left[\underset{(164.7)}{94.66} \left(y_{t-4} - \underset{(0.11)}{11.360} \right) \right]} \\ & Log - Lik = -350.90, T = 150, \widehat{\sigma}^2_{BSTAR} = 6.302, \widehat{\sigma}^2_{BSTAR} / \widehat{\sigma}^2_{AR(10)} = 0.831, \\ & \chi^2_{normality}(2) = 1.429[0.489], F_{ARCH(4)}(4, 141) = 1.153[0.334] \end{aligned} \quad (7)$$



(a) Transition function against time: 'dashed line' - $\gamma_1 \neq \gamma_2$, 'solid line' - $\gamma_1 = \gamma_2$



(b) Transition function against y_{t-10}: '$\circ$' - $\gamma_1 \neq \gamma_2$, '$\times$' - $\gamma_1 = \gamma_2$

Figure 2: Transition functions of estimated unrestricted ($\gamma_1 \neq \gamma_2$) and restricted ($\gamma_1 = \gamma_2$) BSTAR models

$$
\begin{aligned}
y_t \quad = \quad & \underset{(0.331)}{0.502} + \underset{(0.067)}{0.789} y_{t-1} + \underset{(0.078)}{0.258} y_{t-2} - \underset{(0.072)}{0.555} y_{t-4} + \underset{(0.077)}{0.459} y_{t-6} - \underset{(0.076)}{0.345} y_{t-8} + \underset{(0.059)}{0.184} y_{t-10} \\
& + \left[-\underset{(0.108)}{0.281} y_{t-1} - \underset{(0.077)}{0.459} y_{t-6} + \underset{(0.141)}{0.422} y_{t-7} + \underset{(0.205)}{0.441} y_{t-9} - \underset{(0.197)}{0.325} y_{t-10} \right] F_t(\widehat{\gamma}_1, \widehat{c}_1, \widehat{\gamma}_2, \widehat{c}_2; y_{t-4}) + \widehat{e}_t \\
& F_t(\widehat{\gamma}_1, \widehat{c}_1, \widehat{\gamma}_2, \widehat{c}_2; y_{t-4}) = \frac{\exp\left[-\underset{(1592)}{431.2} \left(y_{t-4} + \underset{(0.075)}{3.231} \right) \right] + \exp\left[\underset{(1592)}{431.2} \left(y_{t-4} - \underset{(0.075)}{11.380} \right) \right]}{1 + \exp\left[-\underset{(1592)}{431.2} \left(y_{t-4} + \underset{(0.075)}{3.231} \right) \right] + \exp\left[\underset{(1592)}{431.2} \left(y_{t-4} - \underset{(0.075)}{11.380} \right) \right]} \\
& Log - Lik = -350.90, T = 150, \widehat{\sigma}^2_{BSTAR} = 6.302, \widehat{\sigma}^2_{BSTAR} / \widehat{\sigma}^2_{AR(10)} = 0.831, \\
& \chi^2_{normality}(2) = 1.308[0.520], F_{ARCH(4)}(4, 141) = 1.242[0.296]
\end{aligned} \tag{8}
$$

Table 3: Diagnostic tests

	Tests for qth order serial correlation									
q	1	2	3	4	5	6	7	8	9	10
$\gamma_1 \neq \gamma_2$	0.338	0.617	0.648	0.411	0.413	0.542	0.665	0.108	0.155	0.218
$\gamma_1 = \gamma_2$	0.542	0.838	0.842	0.278	0.226	0.336	0.473	0.073	0.105	0.139
	Tests for parameter constancy									
	All	Linear	Nonlinear					All	Linear	Nonlinear
$\gamma_1 \neq \gamma_2$	0.394	0.229	0.468				$\gamma_1 = \gamma_2$|	0.340	0.289	0.386
Transition	Tests for remaining nonlinearity									
variable	y_{t-1}	y_{t-2}	y_{t-3}	y_{t-4}	y_{t-5}	y_{t-6}	y_{t-7}	y_{t-8}	y_{t-9}	y_{t-10}
$\gamma_1 \neq \gamma_2$	0.225	0.723	0.814	0.547	0.670	0.159	0.531	0.608	0.055	0.213
$\gamma_1 = \gamma_2$	0.340	0.683	0.765	0.679	0.539	0.160	0.664	0.472	0.029	0.288

Table reports $p-$values of the diagnostic tests for estimated unrestricted ($\gamma_1 \neq \gamma_2$) and restricted ($\gamma_1 = \gamma_2$) BSTAR models for the annual growth rates of industrial production of Italy.

4 Conclusion.

The present paper has introduced a nonlinear model that is related to the STAR class of models. The so-called BSTAR model, which here is fitted to the growth rate of the Italian Index of Industrial Production time series, is a generalization of the two regime LSTR2 model, see Teräsvirta (1998). In contrast to the LSTR2 model, which features the symmetric transition function, the BSTAR model is characterized by the asymmetric transition function which implies different local dynamics in the neighborhood of the respective location parameters. In the case when symmetry restriction is imposed, the BSTAR model acts as a close substitute for the LSTR2 model.

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