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On potential maximization as a refinement of Nash equilibrium

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Abstract

We specify an adjustment process that converges to the set of potential−maximizing strategy profiles for 3−player cooperation−formation games or n−player cooperation−formation games based on a superadditive characteristic function. Our analysis provides a justification for potential maximization as a refinement of Nash equilibrium in these settings.

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1. Introduction

Monderer and Shapley (1996) specify a simple adjustment process that converges to a Nash equilibrium of a potential game in a finite number of steps. In their process, whenever the strategy profile is not a Nash equilibrium, it is assumed that one player deviates to a strategy that makes her better off. Unilateral deviations that increase the payoff of the deviator raise the value of the potential, while unilateral deviations that decrease the payoff of the deviator lower it. Hence, once a Nash equilibrium is reached (there are no more self-improving, unilateral deviations) the process terminates and the potential function will be at a maximum in the sense that its value cannot be increased by varying *any single player's strategy*.

While useful for interpreting Nash equilibrium, this result does not support the use of potential maximization as a refinement. The Nash equilibrium reached by the simple adjustment process of Monderer and Shapley (1996) might not maximize the potential function over its entire domain of strategy profiles. To this end, we consider a modification of the simple adjustment process that allows for experimentation. We assume that starting from any strategy profile, players are willing to deviate to alternative strategies that are at least as good as their present ones. Our process is described formally below. We refer to it as a weak adjustment process because players may deviate to strategies to which they are indifferent.

The weak adjustment process operates on a constantly changing set of deviators who are able to switch to a different strategy and be at least as well off, holding fixed the strategies of the other players, as they were before the deviation. The nonemptiness of this set, at each step of the process, is crucial. For 3-player cooperation-formation games the set of deviators is nonempty at any non-potential maximizing profile. This will not be true for games that have strict local maxima of the potential function (i.e., strict Nash equilibria). A strict local maximum of the potential function exists if there is a strategy profile that does not globally maximize the potential, but from which all unilateral deviations make the deviators strictly worse off. An example of such a game is a standard, two-player coordination game (shown below with its potential function).

	L	R
T	1, 1	0, 0
B	0, 0	2, 2

Potential Function =

	L	R
T	1	0
B	0	2

If players find themselves at the Nash equilibrium (T,L) then neither is willing to consider a unilateral deviation, even though a sequence of unilateral deviations would lead (in two steps) to the mutually preferred, potential-maximizing outcome.[1] In what follows, maximum or maximizing is used in the global sense, over all strategy profiles.

The weak adjustment process may involve random selection of both deviators and deviations. We show that the process converges with probability one to the set of potential-maximizing strategy profiles of 3-player cooperation-formation games. The weak adjustment process can be applied to n-player cooperation-formation games ($n > 3$) under mild conditions, as we explain below. The weak adjustment proces1,1s converges to the set of potential-maximizing strategy profiles for cooperation-formation games based on a superadditive characteristic function. However, we can neither establish nor rule out convergence of the weak adjustment process to the set of potential maximizers for non superadditive games with more than 3 players.

2. Notation and definitions

Let $N = \{1, 2, ..., n\}$ and define the strategy set of player i to be $\Pi_i = \{S \subseteq N \mid i \in S\}$. A strategy $\pi_i \in \Pi_i$ is a set of players (including him or herself) with whom player i wishes to form links. Let $\Pi = \times_{i \in N} \Pi_i$. Given $\pi = (\pi_1, \pi_2, ..., \pi_n) \in \Pi$, a link between players i and j forms if $i \in \pi_j$ and $j \in \pi_i$.[2] The undirected bilateral link between players i and j is denoted $i : j$. The set of all (undirected) bilateral links between players is $L = \{i : j \mid i, j \in N\}$. A cooperation structure is a list of undirected bilateral links in L. Given a strategy profile π, the resulting cooperation structure is given by the function $g : \Pi \to L$ where $g(\pi) = \{i : j \mid i \in \pi_j \text{ and } j \in \pi_i\}$.

Given a strategy profile the payoffs are determined as follows. First, it is assumed that any coalition has a value that is expressed by a characteristic function $v : 2^N \to \mathbb{R}$ that is zero-normalized (i.e., $v(\{i\}) = 0$ for all $i \in N$). Second, players that are either directly or indirectly connected by the cooperation structure resulting from the strategy profile can cooperate. Players' payoffs are then determined by the Myerson values (see Myerson, 1977). The Myerson value for the coalitional game (N, v) with cooperation structure g is denoted $\psi(v, g) = (\psi_i(v, g))_{i \in N}$.[3] It can be constructed using the Shapley value (see Shapley, 1953). The Shapley value of the coalitional

[1] Stochastic processes of the type considered by Kandori, Mailath, Rob (1993) and Young (1993) will converge to the mutually preferred outcome in this simple example. Our goal is to see what outcomes can be reached without random mistakes.

[2] This method of determining a cooperation structure follows Myerson (1991, pp. 448) and Qin (1996).

[3] We use g both as a mapping and a generic cooperation structure.

game (N, v) is denoted $\phi(v) = (\phi_i(v))_{i \in N}$, where

$$\phi_i(v) = \sum_{S:S \ni i} \frac{(n - |S|)!(|S| - 1)!}{n!}[v(S) - v(S \backslash i)].$$

Let $\phi(v) = (\phi_i(v))_{i \in N}$. Given a cooperation structure $g \subseteq L$ and a coalition $S \subseteq N$, S/g denotes the partition of S into connected components. Let v^g denote the characteristic function determined according to

$$v^g(S) = \sum_{R \in S/g} v(R), \; S \subseteq N.$$

Myerson (1977) shows that $\psi(v, g) = \phi(v^g)$. Player i's payoff function in the cooperation-formation game is $U_i(\pi) = \psi_i(v, g(\pi))$, for $\pi \in \Pi$. That is,

$$U_i(\pi) = \sum_{S:S \ni i} \frac{(n - |S|)!(|S| - 1)!}{n!}[v^{g(\pi)}(S) - v^{g(\pi)}(S \backslash i)]. \tag{1}$$

Note that $U_i(\pi) = U_i(\pi')$ if $g(\pi) = g(\pi')$. The cooperation-formation game is the strategic-form game $\Gamma = \{\Pi_i, U_i\}_{i \in N}$.

Definition 1 (Monderer and Shapley) *A potential for a game* Γ *is a function* $P : \Pi \to \mathbb{R}$ *such that for any* $i \in N$, $\pi \in \Pi$, *and* $\pi'_i \in \Pi_i$,

$$U_i(\pi'_i, \pi_{-i}) - U_i(\pi) = P(\pi'_i, \pi_{-i}) - P(\pi).$$

Γ *is a potential game if it has a potential.*

Remark 1. An assignment, $(U_i)_{i \in N}$ of players' payoffs satisfies *component efficiency* if for any $\pi \in \Pi$ and for any $S \in N/g(\pi)$, $\sum_{i \in S} U_i(\pi) = v(S)$. By Theorem 1 and Proposition 2 of Qin (1996), the Myerson values are the only way of assigning the players' payoffs that satisfy component efficiency and make the cooperation-formation game a potential game.

3. Weak adjustment process

We seek a simple dynamic adjustment process that converges to potential-maximizing strategy profiles. The process we specify is a modification of the simple adjustment process specified by Monderer and Shapley (1996). Our modification is to allow players to experiment in cases where no strictly self-improving unilateral deviation can be found and try other strategies that are at least as good as the current one. (This is sometimes referred to as adding "drift.")

At each stage of the process define the set of *deviators* to be all those players who can deviate to a different strategy that, holding fixed the strategies of the other players, would leave them at least as well off as they were before the deviation. Then we have the following:

Weak Adjustment Process: Suppose the initial strategy profile is not a potential maximizer. Randomly select one player from the set of deviators. Allow her to deviate to a different strategy that gives her a payoff that is at least as good as her current one. If she has more than one such strategy, have her select one at random. Repeat.

Under the weak adjustment process, the transition probability from a given strategy profile to any other one depends on the given strategy profile, but not on the strategy profiles reached in the previous steps. This means that the process is a Markov chain whose state space is the set of strategy profiles. Thus to show that the process converges with probability one from any initial strategy profile to the set of potential maximizers, it suffices to show that together potential maximizers form the only essential class of the underlying Markov chain. The weak adjustment process does not require that deviators select optimal strategies. Making this assumption would be fine for the convergence result shown below.

4. Result for 3-player cooperation-formation games

The following definitions are useful for establishing convergence of the weak adjustment process.

Definition 2 (Monderer and Shapley) *A path in* Π *is a sequence* $\gamma = (\pi^0, \pi^1, ..., \pi^m)$ *of strategy profiles such that for every* $\ell \geq 1$ *there exists a unique player denoted* i_ℓ *such that* $\pi^\ell = (\pi^\ell_{i_\ell}, \pi^{\ell-1}_{-i_\ell})$ *for some* $\pi^\ell_{i_\ell} \in \Pi_{i_\ell}$ *with* $\pi^\ell_{i_\ell} \neq \pi^{\ell-1}_{i_\ell}$ *(player* i_ℓ *is the only deviator from* $\pi^{\ell-1}$ *in* π^ℓ*).*

Definition 3 *A pure improvement path (PIP) is a path* $\gamma = (\pi^0, \pi^1, .., \pi^m)$ *such that for* $\ell \geq 1, U_{i_\ell}(\pi^\ell) \geq U_{i_\ell}(\pi^{\ell-1})$*. If the inequality is strict for at least one deviator, the path is called a strict PIP.*

Remark 2(i). By the selection criteria of the weak adjustment process, if the initial strategy profile of a finite PIP is reached by the adjustment process, then the entire PIP will be realized with positive probability.

Remark 2(ii). By Theorem 2.8 of Monderer and Shapley (1996), if there is a strict PIP from strategy profile π to another strategy profile π', then there cannot be any PIP from π' to π.

Theorem 1. *For zero-normalized, 3-player cooperation-formation games the weak adjustment process converges from any strategy profile to the set of potential-maximizing strategy profiles with probability one.*

Proof. The proof uses the following three facts that apply to any 3-player cooperation-formation game. Additional notation is needed. Given any cooperation structure g, $\pi(g) = (\pi_1(g), \pi_2(g), \pi_3(g))$ denotes a strategy profile such that $\pi_i(g) = \{j \mid i : j \in g\} \cup \{i\}$, for $i = 1, 2, 3$. That is, $\pi(g)$ is the strategy profile such that every player only proposes to form links with those who are linked with the player in g.

Fact 1. *For any $\pi \in \Pi$ with $\pi \neq \pi(g(\pi))$, there exists a finite PIP from π to $\pi(g(\pi))$.*

Proof of Fact 1. Without loss of generality, assume that $\pi_i \neq \pi_i(g(\pi))$, for all $i \in \{1, 2, 3\}$. Haeringer (2002) shows a potential function for the cooperation-formation game is

$$P(\pi) = \sum_{S \subseteq N} \frac{(n - |S|)!(|S| - 1)!}{n!} v^{g(\pi)}(S), \tag{2}$$

for $\pi \in \Pi$.[4] Observe that $P(\pi) = P(\pi(g(\pi)))$. Set $\pi_1^1 = \pi_1(g(\pi))$, $\pi_2^2 = \pi_2(g(\pi))$, and $\pi_3^3 = \pi_3(g(\pi))$. Then, $g(\pi^1) = g(\pi^2) = g(\pi^3)$, which implies that $\gamma = \{\pi^0, \pi^1, \pi^2, \pi^3\}$ with $\pi^0 = \pi$ is a PIP.[5]

Fact 2. *There exists a strict PIP from any non potential-maximizing strategy profile to a potential-maximizing one.*

Proof of Fact 2. Let π be any strategy profile that does not maximize the potential. By Fact 1, we may assume $\pi = \pi(g(\pi))$. When $|g(\pi)| = 1$, say $g(\pi) = \{i : j\}$, the existence of a strict PIP from π to a potential maximizer is straightforward unless strategy profiles producing $\{i : k,\ j : k\}$ are the only potential-maximizers.[6]

[4] By Lemma 2.7 of Monderer and Shapley (1996), potentials for a given potential game are unique up to an additive constant.

[5] We will continue to assume π^0 is the given strategy profile π in the paths specified below.

[6] This is so because for all other cases a path can be found so that only the last deviator changes the cooperation structure and hence increases the potential.

Suppose this is the case. By (2), the difference between the potential at strategy profiles producing the cooperation structure $\{i : k,\ j : k\}$ and $\pi^N = (N, N, N)$ is $\frac{1}{6}v(\{i, j\})$. Since π^N is not potential maximizing this implies $v(\{i, j\}) < 0$. It follows that $\gamma = \{\pi^0, \pi^1, \pi^2, \pi^3\}$ with $\pi_i^1 = \{i, k\}$, $\pi_j^2 = \{j, k\}$, and $\pi_k^3 = N$ is a strict PIP from π to a potential maximizer.

When $|g(\pi)| = 0$ the existence of a strict PIP from π to a potential maximizer follows from the reason stated in Footnote 6 unless $\pi^N = (N, N, N)$ is the only potential-maximizing strategy profile. Suppose it is. By (2), the difference between the potential at the strategy profile π^N and strategy profiles producing the cooperation structure $\{i : k,\ j : k\}$ is $\frac{1}{6}v(\{i, j\})$, for $i \neq j \neq k$. Since π^N is the only potential-maximizing strategy profile, $v(\{i, j\}) > 0$ for $i \neq j$. It follows that $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_i^1 = \{i, j\}$ and $\pi_j^2 = \{i, j\}$ is a strict PIP from π to π^2. Since $g(\pi^2) = \{i : j\}$, by the above result for $|g(\pi)| = 1$, there exists a strict PIP from π^2 to a potential maximizer. By combining the strict PIP from π to π^2 with one from π^2 to a potential maximizer, we obtain a strict PIP from π to a potential maximizer.

When $|g(\pi)| = 2$, for instance suppose $g(\pi) = \{i : j,\ i : k\}$, the existence of a strict PIP from π to a potential maximizer follows from the reason stated in Footnote 6, unless the potential-maximizing strategy profiles only produce one or more of the following cooperation structures: $\{j : k\}$, $\{i : j, j : k\}$, $\{i : k, j : k\}$. Suppose first that $\{i : k,\ j : k\}$ or $\{i : j,\ j : k\}$ are included. In the former case, $v(\{i, j\}) < v(\{j, k\})$. Thus $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_k^1 = N$ and $\pi_j^2 = \{j, k\}$ is a strict PIP from π to a potential maximizer. In the latter case, we have $v(\{i, k\}) < v(\{j, k\})$, and hence $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_j^1 = N$ and $\pi_k^2 = \{j, k\}$ is a strict PIP from π to a potential maximizer. The remaining difficult case is where strategy profiles producing the cooperation structure $\{j : k\}$ are the only potential maximizers. In this case, zero-normalization and (2) imply $v(\{j, k\}) > 0$. Now consider a path $\gamma = \{\pi^0, \pi^1, \pi^2, \pi^3\}$ with $\pi_k^1 = N$, $\pi_j^2 = N$, and $\pi_i^3 = \{i\}$. Then, $g(\pi^1) = g(\pi)$ which, by (1), implies $U_k(\pi^1) = U_k(\pi)$; $g(\pi^2) = \{1 : 2, 1 : 3, 2 : 3\}$ which, by (1) and the fact that $v(\{j, k\}) > 0$, implies $U_j(\pi^2) > U_j(\pi^1)$; and $g(\pi^3) = \{j : k\}$ which, since π^3 is a potential maximizer and π^2 is not, implies $U_i(\pi^3) - U_i(\pi^2) = P(\pi^3) - P(\pi^2) > 0$. Thus, $\gamma = \{\pi^0, \pi^1, \pi^2, \pi^3\}$ is a strict PIP from π to a potential maximizer.

Suppose $|g(\pi)| = 3$, which means $\pi = \pi^N$. First consider the case in which strategy profiles producing a two-link structure, say $\{i : j, i : k\}$, are potential maximizing. By (2), the difference in potential at π and any strategy profile π' producing $\{i : j, i : k\}$ is $\frac{1}{6}v(\{j, k\})$. Since π is not potential maximizing and π' is, $v(\{j, k\}) < 0$. Thus, $\gamma = \{\pi^0, \pi^1\}$ with $\pi_j^1 = \{i, j\}$ is a strict PIP from π to a potential maximizer. Next consider the case where strategy profiles producing a one-link structure are potential maximizing. Choose $i, j \in N$ such that $v(\{i,j\}) = \max_{|S|=2} v(S)$ and let $\hat{\pi} \in \Pi$ be such that $g(\hat{\pi}) = \{i : j\}$. Then, $\hat{\pi}$ is potential maximizing and thus $P(\hat{\pi}) > P(\pi)$. It follows from (1) and (2) that $U_k(\pi) = P(\pi) - P(\hat{\pi}) < 0$. Since $U_k(\hat{\pi}) = 0$ we conclude that $\gamma = \{\pi^0, \pi^1\}$ with $\pi_k^1 = \{k\}$ is a strict PIP from π to a potential maximizer. Finally, consider the case where strategy profiles producing the zero-link structure are the only potential maximizers. In this case, (2) and zero-normalization imply that $P(\pi') = 0$ for all $\pi' \in \Pi$ such that $g(\pi') = \emptyset$ and $v(S) < 0$ for $|S| = 2$. Consider a path $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_i^1 = \{i, k\}$ and $\pi_k^2 = \{k\}$. Then, $g(\pi^1) = \{i : k, j : k\}$ and $g(\pi^2) = \emptyset$. Since $v(\{i,j\}) < 0$ and $U_i(\pi^1) - U_i(\pi^0) = P(\pi^1) - P(\pi^0) = -\frac{1}{6}v(\{i,j\})$, player i is made strictly better off in the move from π^0 to π^1. Furthermore, since $P(\pi^2) > P(\pi^1)$ and $U_k(\pi^2) - U_k(\pi^1) = P(\pi^2) - P(\pi^1)$, player k is made strictly better of in the move from π^1 to π^2. This establishes that γ is a strict PIP from π to a potential maximizer.

Fact 3. *There exists a finite PIP between any two potential-maximizing strategy profiles.*

Proof of Fact 3. Let π and π' be any two potential-maximizing strategy profiles. We will assume $g(\pi) \neq g(\pi')$ since otherwise, by Fact 1, it is always possible to specify a path from π to π' that keeps the potential constant. Note first that because both π and π' are potential maximizers a PIP, if one exists, from π to π' cannot involve a strict improvement for any deviator. Since the inverse of a PIP without strict improvement is also a PIP, we need only consider the case where $|g(\pi)| \leq |g(\pi')|$. When $|g(\pi)| = |g(\pi')|$ there are only two possibilities; $g(\pi) = \{i : j\}$ and $g(\pi') = \{i : k\}$ or $g(\pi) = \{i : j, i : k\}$ and $g(\pi') = \{i : j, j : k\}$. By (2), $v(\{i,j\}) = v(\{i,k\})$ in the former case and $v(\{i,k\}) = v(\{j,k\})$ in the latter case. Thus, a PIP from π to π' is given by $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_k^1 = \{i, k\}$ and $\pi_i^2 = \{i, k\}$ in the former case, and $\pi_j^1 = N$ and $\pi_k^2 = \{j, k\}$ in the latter case.

When $|g(\pi)| < |g(\pi')|$, four cases deserve special attention: (i) $g(\pi) = \{i : j\}$, $g(\pi') = \{i : j,\, i : k\}$; (ii) $g(\pi) = \{i : j\}$, $g(\pi') = \{i : k,\, j : k\}$; (iii) $g(\pi) = \{i : j,\, i : k\}$, $g(\pi') = \{1 : 2,\, 2 : 3,\, 1 : 3\}$; and (iv) $g(\pi) = \emptyset$, $g(\pi') = \{1 : 2,\, 2 : 3,\, 1 : 3\}$. In case (i), $U_k(\pi) - U_k(\pi') = P(\pi) - P(\pi')$. Thus, since $P(\pi) = P(\pi')$, $U_k(\pi) = U_k(\pi')$. By (1) and zero-normalization $U_k(\pi) = 0$ and thus $U_k(\pi') = 0$. Hence, $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_i^1 = N$ and $\pi_k^2 = \{i, k\}$ is a PIP from π to π'. In case (ii), for both π and π' to be potential maximizing it must be true that $v(\{i,j\}) = 0$; otherwise, by (2), the potential can be increased from π' by adding the link $i : j$. Thus, $P(\pi) = P(\pi') = 0$, and $\gamma = \{\pi^0, \pi^1, \pi^2, \pi^3\}$ with $\pi_i^1 = \{i, k\}$, $\pi_j^2 = \{j, k\}$, and $\pi_k^3 = N$ a PIP. In case (iii), $v(\{j,k\}) = 0$ and so $\gamma = \{\pi^0, \pi^1, \pi^2\}$ with $\pi_j^1 = N$ and $\pi_k^2 = N$ is a PIP. Finally, in case (iv), zero-normalization and (2) imply all coalitions have a value of zero. Hence, any path from π to π' is a PIP.

Fact 2 establishes that from any non potential-maximizing strategy profile there is a finite, strict PIP to a potential-maximizing strategy profile. Fact 3 shows that there is a PIP between any two potential-maximizing strategy profiles. Combining Facts 2 and 3, there is a strict PIP from any non potential-maximizing strategy profile to any given potential-maximizing strategy profile. By Remark 2(ii), starting from any potential-maximizing strategy profile there does not exist any PIP to a non potential-maximizing strategy profile. Because of this and by Remark 2(i), we conclude that every potential maximizer is accessible from any strategy profile, but strategy profiles that are not potential maximizers are not accessible from any potential maximizer. This implies that the set of potential-maximizing strategy profiles is the only essential class of the homogeneous Markov chain given by the weak adjustment process. By Theorem 4.7 of Seneta (1981), the Markov chain and hence the adjustment process converges from any given strategy profile to the set of potential-maximizing strategy profiles with probability 1. ■

5. n players

We conclude with a discussion on applying the weak adjustment process to n-player cooperation-formation games. First, we establish a result for superadditive games.

Theorem 2. *For superadditive, zero-normalized, n-player cooperation-formation games the weak adjustment process converges from any strategy profile to the set of potential-maximizing strategy profiles with probability one.*

Proof. In this case, all potential-maximizing strategy profiles are payoff equivalent to the potential-maximizing strategy profile $\pi^N = (N, ..., N)$ (see Qin, 1996 and Slikker et al., 2000), which is itself a potential maximizer. Moreover, because the addition of new links is never detrimental to the deviating player in superadditive games, the sequence of unilateral deviations $\pi_1^1 = N$, $\pi_2^2 = N, ..., \pi_n^n = N$ (ignoring cases where π_i^i already equals N) defines a PIP to π^N from any strategy profile. Hence, π^N is accessible from any starting strategy profile with strict improvement if the starting strategy profile is not potential maximizing. Furthermore, π^N is accessible from any starting strategy profile without strict improvement if the starting profile is already potential maximizing. This implies that the set of potential-maximizing strategy profiles is the only essential class of the homogeneous Markov chain given by the weak adjustment process. Hence, the argument used in the proof of Theorem 1 for Markov processes applies. ■

For non superadditive games, we have no general convergence results, but we are able to state conditions for the non emptiness of the set of deviators at each non potential-maximizing strategy profile. This is crucial for the application of the weak adjustment process. If the initial strategy profile is such that there exists $i \in N$ with $\pi_i \neq N$ then some player $j \in N \backslash \pi_i$ can either change her strategy profile to $\pi'_j = \pi_j \backslash \{i\}$ (if $i \in \pi_j$) or change her strategy profile to $\pi'_j = \pi_j \cup \{i\}$ (if $i \notin \pi_j$) and her payoff will not change. Starting from the (non potential-maximizing) strategy profile π^N things are more complicated. We require that someone can break a link (or multiple links) and be no worse off. This will be true provided there exists a player i and a coalition $S_i \supsetneqq \{i\}$ such that

$$\sum_{S \subseteq S_i : S \ni i} \frac{(n - |S|)!(|S| - 1)!}{n!} [v(\{i\}) + v(S \backslash \{i\}) - v(S)] \geq 0. \qquad (3)$$

The change in i's payoff from deviating from π^N to the strategy $\pi'_i = (N \backslash S_i) \cup \{i\}$ is given by the difference in player i's Myerson value under the complete graph $g(\pi)$ and the graph created by the deviation, $g(\pi'_i, \pi_{-i})$. Since $g(\pi) = g(\pi') \cup \{i : j \mid j \in S_i\}$ the difference between the Myerson values for player i reduces to the expression in (3). The inequality in (3) is a form of weighted subadditivity that need only exist for one player and one coalition.

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