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On the Impossibility of Strategy−Proof Coalition Formation Rules

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Abstract

We analyze simple coalition formation problems in which a group of agents is partitioned into coalitions and agents' preferences only depend on the identity of the members of the coalition to which they belong. We study coaltion formation rules that associate to each profile of agents' preferences a partition of the group of agents. Assuming that agents' preferences are separable, we show that no coalition formation rule can satisfy the joint requirements of strategy−proofness, individual rationality, non−bossiness, and voters' sovereignty.

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1 Introduction

In this note we analyze simple coalition formation problems in which a group of agents is partitioned into coalitions and agents have preferences over the coalitions they are members of. Following the terminology proposed by Drèze and Greensberg [7], we focus on problems characterized by the "hedonic" aspect of coalition formation. Agents' preferences only depend on the identity of the members of the coalition to which they belong. Hence, we exclude the existence of externalities among different coalitions. The most relevant examples of such problems are matching problems as the roommate problem, or the formation of social clubs, organizations, teams or societies.

More specifically, we focus on problems in which agents' preferences over coalitions are separable. Namely, we assume that whenever an agent i prefers to join agent j rather than staying on her own, then agent i prefers to join agents j and k rather than joining only agent k. Hence, we also exclude the possibility of (negative or positive) complementarities among the members of a coalition.

The literature of Coalitional Game Theory has extensively analyzed the existence of stable partitions in hedonic coalition formation problems.[1] Instead, we propose a social choice approach. We study coalition formation rules that associate to each profile of agents' (separable) preferences a partition of the group of agents. Our main concern is that our rules satisfy *strategy-proofness*. Strategy-proofness is the strongest decentrability property. Each agent needs to know only her own preferences to compute her best choice.

It is well known that the requirements of strategy-proofness are hard to meet. In the abstract model of social choice, Gibbard [8] and Satterthwaite [11] show that –provided there are more than two alternatives at stake– every strategy-proof social choice rule is dictatorial. However, reasonable strategy-proof rules do exist if appropriate restrictions are imposed on agents' preferences. This is the case of separable preferences. In the context of a group of agents choosing a subset from a set of objects (that represent, for instance, candidates who opt to some number of available positions), when agents' preferences over sets of objects are separable, then strategy-proof rules can be decomposed into independent rules, one for each object.[2]

Besides strategy-proofness, we would like our rule to satisfy three additional properties that are natural in the context of coalition formation problems. Our rules should be *individually rational, non-bossy* and *voters' sovereign*. Individual rationality is a minimal participation constraint. It means that no agent should be ever worse off than she would be in her own. Non-bossiness is a collusion-proof requirement. It says that if a change in an agent's preferences does not affect the coalition to which this agent is assigned, then the

[1]For further references, see the recent works by Banerjee, Konishi, and Sönmez [2], Barberà and Gerber [3], Bogomolnaia and Jackson [5], and Pàpai [10].

[2]See Barberà, Sonnenschein, and Zhou [4] and Le Breton and Sen [9] for further details.

remaining agents are also unaffected by this change of preferences. Voters' sovereignty is a weak version of unanimity. Basically, it implies that all possible partitions of the group of agents are feasible.

Our main result shows that, even when agents' preferences are restricted to be separable, no rule satisfies simultaneously the requirements of our four axioms. Then, this note provides further evidence on the difficulties of devising non manipulable rules.

The works by Alcalde and Revilla [1], Cechlárová and Romero-Medina [6], and Sönmez [12] are closely related to the present note. All these works study strategy-proof coalition formation rules. However, they focus on environments in which agents' preferences are not separable. More specifically, Alcalde and Revilla [1], Cechlárová and Romero-Medina [6] assume that agents' preferences over coalitions are based on the best or the worst group of agents in each coalition. In these environments, they prove the existence of strategy-proof rules that always select core-stable partitions.[3] On the other hand, Sönmez [12] proposes a general model of allocation of indivisible goods that can be applied to coalition formation problems. He focuses on preference domains for which there always exist core-stable partitions. His main result states that there exist strategy-proof, individually rational, and Pareto efficient rules only if the set of core-stable partitions is always essentially single-valued.[4]

The remainder of the note is organized as follows. In Section 2, we present the model, basic notation, the main axioms, and the impossibility Theorem. In Section 3, we prove the Theorem. In Section 4, we show the independence of the axioms.

2 The Model and the Impossibility Theorem

Let N be a society consisting of a finite set of at least 3 agents. We call a non-empty subset $C \subseteq N$ a **coalition**. Let $\mathcal{N}$ be the set of all non-empty subsets of N. A **collection of coalitions** is a set of coalitions $\Pi \subseteq \mathcal{N}$. We denote by σ a partition of N and by Σ the set of all partitions of N. For each $i \in N$ and each $\sigma \in \Sigma$, $\sigma_i \in \sigma$ denotes the coalition in σ to which i belongs.

For each $i \in N$, let $\mathcal{C}_i \subset \mathcal{N}$ be the set of all coalitions i may belong to. A **preference** for i, $\succsim_i$, is an asymmetric ordering on $\mathcal{C}_i$.[5] A preference for i, $\succsim_i$, is **separable** if for each $j \in N$ and each $C \in \mathcal{C}_i$ such that $j \notin C$, $\{i, j\} \succ_i \{i\}$ if and only if $(C \cup \{j\}) \succ_i C$. Let $\mathcal{S}^i$ be the set of all agent i's separable preferences.

[3] A partition is core-stable if no coalition of agents unamimously prefer to join rather than to belong to the coalition they are assigned in the original partition.

[4] Takayima [13] proves that the converse result also holds under the assumptions of strict preferences and no externalities in consumption. Indeed, in our coalition formation model, those assumptions are satisfied by agents' preferences.

[5] An *ordering* is a complete and transitive binary relation.

For each $i \in N$ and each $\succsim \in \mathcal{S}^i$, i's preferences over partitions are completely defined by i's preferences over coalitions she belongs to. Then, abusing notation, for each σ, $\sigma' \in \Sigma$, σ is at least as good as σ', $\sigma \succsim_i \sigma'$, if and only if $\sigma_i \succsim_i \sigma'_i$.

For each $i \in N$, each collection of coalitions Π, and each $\succsim_i \in \mathcal{S}^i$, let $\text{top}(\Pi, \succsim_i)$ be the coalition in $\Pi \cap \mathcal{C}_i$ that is ranked first according to $\succsim_i$. Let $\mathcal{S} = \times_{i \in N} \mathcal{S}^i$. We call $\succsim \in \mathcal{S}$ a preference profile. For each $C \subset N$ and each $\succsim \in \mathcal{S}$, $\succsim_C \in \times_{i \in C} \mathcal{S}^i$ denotes the restriction of profile $\succsim$ to the preferences of the agents in C.

We are interested in rules that associate a partition to each profile of agents' preferences.

A **(coalition formation) rule** is a mapping $\varphi : \mathcal{S} \to \Sigma$. For each $i \in N$ and each $\succsim \in \mathcal{S}$, $\varphi_i(\succsim)$ denotes the coalition in $\varphi(\succsim)$ to which i belongs.

Now, we introduce four properties that rules may satisfy. First, agents should never have incentives to misrepresent their preferences.

Strategy-Proofness. For each $i \in N$, each $\succsim \in \mathcal{S}$, and each $\succsim'_i \in \mathcal{S}^i$, $\varphi_i(\succsim) \succsim_i \varphi_i(\succsim_{\mathcal{N} \setminus \{i\}}, \succsim'_i)$. Conversely, φ is **manipulable** if it is not strategy-proof.

The Gibbard-Satterthwaite Theorem states that every *strategy-proof* social choice rule on an unrestricted domain either is dictatorial or its range contains only two elements.[6] As agents' preferences over social outcomes (partitions) are restricted to depend on the coalition they belong to and to be separable, the negative consequences of the Gibbard-Satterthwaite Theorem do not apply to our framework.

The next axiom implies that agents should have incentives to participate in the society.

Individual Rationality. For each $i \in N$ and each $\succsim \in \mathcal{S}$, $\varphi_i(\succsim) \succsim_i \{i\}$.

We also consider rules such that whenever a change in an agent's preference does not change the coalition she is assigned to, then the social choice for the remaining agents does not change.

Non-Bossiness. For each $i \in N$, each $\succsim \in \mathcal{S}$, and each $\succsim'_i \in \mathcal{S}^i$, $\varphi_i(\succsim) = \varphi_i(\succsim_{\mathcal{N} \setminus \{i\}}, \succsim'_i)$ implies $\varphi(\succsim) = \varphi(\succsim_{\mathcal{N} \setminus \{i\}}, \succsim'_i)$.

Finally, we assume that any partition may be the result of the social choice.

Voters' Sovereignty. For each $\sigma \in \Sigma$ there is $\succsim \in \mathcal{S}$ such that $\varphi(\succsim) = \sigma$.

Our main result shows the incompatibility of the four axioms.

[6] A rule is **dictatorial** if there is $i \in N$ (a dictator) such that for each $\succsim \in \mathcal{S}$, $\varphi_i(\succsim) = \text{top}(\mathcal{N}, \succsim_i)$.

Theorem *There does not exist any rule satisfying strategy-proofness, individual rationality, non-bossiness, and voters' sovereignty.*

3 The Proof

We begin this section by introducing some properties that are implied by our axioms. These properties incorporate the idea that a rule cannot be against the preferences of the members of the society. When there is a partition that each agent considers at least as good as every other partition, a rule should choose that best-preferred partition. A stronger requirement would be that whenever the members of a coalition consider this coalition as the best coalition, a rule should allow them to join, independently of the preferences of the remaining agents in society.

Unanimity. Let $\sigma = \{C_1, \ldots, C_m\} \in \Sigma$. For each $\succsim \in \mathcal{S}$, $\text{top}(\mathcal{N}, \succsim_i) = C_t$ for each $t = 1, \ldots, m$ and each $i \in C_t$, implies $\varphi(\succsim) = \sigma$.

Top-Coalition. For each $C \in \mathcal{N}$ and each $\succsim \in \mathcal{S}$, $\text{top}(\mathcal{N}, \succsim_i) = C$ for each $i \in C$ implies $\varphi_i(\succsim) = C$ for each $i \in C$.

It is clear that *top-coalition* implies *unanimity*. Note that *top-coalition* is a property of rules. Banerjee *et al.* [2] use the term top coalition to name a a property of preference profiles. These authors say that a preference profile satisfies the top-coalition property if for every group of agents $V \subseteq N$ there is a coalition $C \subseteq V$ that is mutually the best for all the members of C. Basically, our *top-coalition* implies that if a preference profile satisfies the Banerjee *et al.*'s top-coalition property, then the rule selects a partition in which the coalition that all its members consider the best forms.

Lemma 1. *Let φ satisfy strategy-proofness, non-bossiness, and voters' sovereignty, then φ satisfies unanimity.*

Proof. Let $\sigma = \{C_1, \ldots, C_t\} \in \Sigma$. Let $\succsim \in \mathcal{S}$ be such that for each t and each $i \in C_t$, $\text{top}(\mathcal{N}, \succsim_i) = C_t$. By *voters' sovereignty*, there is $\succsim' \in \mathcal{S}$, such that $\varphi(\succsim') = \sigma$. Let $i \in N$. Let $\succsim'' \in \mathcal{S}$ be such that $\succsim''_i = \succsim_i$ while for each $j \in N \setminus \{i\}$, $\succsim''_j = \succsim'_j$. By *strategy-proofness*, $\varphi_i(\succsim'') \succsim_i \varphi_i(\succsim')$. Then, $\varphi_i(\succsim'') = \varphi_i(\succsim') = \text{top}(\mathcal{N}, \succsim_i)$. By *non-bossiness*, $\varphi(\succsim'') = \varphi(\succsim')$. Repeating the argument as many times as necessary, we get $\varphi(\succsim) = \varphi(\succsim')$. ■

Lemma 2. *Let φ satisfy strategy-proofness, individual rationality, non-bossiness, and voters' sovereignty, then φ satisfies top-coalition.*

Proof. Let $C \in \mathcal{N}$. Let $\succsim \in \mathcal{S}$ be such that for each $i \in C$, $\text{top}(\mathcal{N}, \succsim_i) = C$. Note that if $C = N$, the result is immediate by *unanimity*. If $\#C = 1$, the result follows from *individual rationality*. Let $\succsim' \in \mathcal{S}$ be such that for each $i \in C$, $\text{top}(\mathcal{N}, \succsim'_i) = C$, and for each $C' \subseteq N$ such that there is $j \in (C' \setminus C)$, $\{i\} \succ_i C'$; while for each $k \in (N \setminus C)$, $\succsim_k = \succsim'_k$. By *individual rationality*, for each $i \in C$, $\varphi_i(\succsim) \subseteq C$. Let $\succsim'' \in \mathcal{A}$ be such that for each $i \in C$, $\succsim'_i = \succsim''_i$; while for each $k \in (N \setminus C)$, $\varphi_k(\succsim') = \text{top}(\mathcal{N}, \succsim''_k)$. Let $k' \in (N \setminus C)$. By *strategy-proofness*, $\varphi_{k'}(\succsim'_{N\setminus\{k'\}}, \succsim''_{k'}) = \varphi_{k'}(\succsim')$. By *non-bossiness*, $\varphi(\succsim'_{N\setminus\{k'\}}, \succsim''_{k'}) = \varphi(\succsim')$. Repeating the arguments for each $k \in (N \setminus C)$, $\varphi(\succsim') = \varphi(\succsim'')$. By *unanimity*, for each $i \in C$, $\varphi_i(\succsim'') = C$. Then, $\varphi_i(\succsim') = C$. Finally, let $i \in C$. By *strategy-proofness*, $\varphi_i(\succsim'_{N\setminus\{i\}}, \succsim_i) \succsim_i \varphi_i(\succsim')$. Then, $\varphi_i(\succsim'_{N\setminus\{i\}}, \succsim_i) = C$. Repeating the argument as many times as necessary, we get that for each $i \in C$, $\varphi_i(\succsim) = C$. ■

Proof of the Theorem.
We prove the Theorem by way of contradiction. Assume, to the contrary, that there is a rule φ satisfying *strategy-proofness*, *individual rationality*, *non-bossiness*, and *voters' sovereignty*. Then, by Lemma 2, φ satisfies *top-coalition*.

Let $i, j, k \in N$. For each $i' \in \{i, j, k\}$, define

$$\bar{\mathcal{S}}^{i'} \equiv \{\succsim_{i'} \in \mathcal{S}^{i'} \text{ such that for each } C \in \mathcal{C}_{i'} \text{ with } C \cap (N \setminus \{i,j,k\}) \neq \{\varnothing\},\ \{i'\} \succ_{i'} C\}.$$

Let $\bar{\mathcal{S}} \equiv \bar{\mathcal{S}}^i \times \bar{\mathcal{S}}^j \times \bar{\mathcal{S}}^k$. By *individual rationality*, for each $\succsim \in \mathcal{S}$ such that for each $i' \in \{i, j, k\}$, $\succsim_{\{i,j,k\}} \in \bar{\mathcal{S}}$, $\varphi_{i'}(\succsim) \subseteq \{i, j, k\}$. Abusing notation, for each $\succsim \in \mathcal{S}$, such that $\succsim_{\{i,j,k\}} \in \bar{\mathcal{S}}$, let $\varphi_{\{i,j,k\}}(\succsim)$ denote the restriction of $\varphi(\succsim)$ to the members of $\{i, j, k\}$.

Let $\succsim^1 \in \mathcal{S}$ be such that $\succsim^1_{\{i,j,k\}} \in \bar{\mathcal{S}}$ and

$\succsim^1_i$:	$\succsim^1_j$:	$\succsim^1_k$:
$\{i,j\}$	$\{i,j\}$	$\{j,k\}$
$\{i\}$	$\{j\}$	$\{i,j,k\}$
$\{i,j,k\}$	$\{i,j,k\}$	$\{k\}$
$\{i,k\}$	$\{j,k\}$	$\{i,k\}$

By *top-coalition*, $\varphi_{\{i,j,k\}}(\succsim^1) = (\{i,j\}, \{k\})$.

Let $\succsim^2 \in \mathcal{S}$ be such that $\succsim^2_{N\setminus\{i\}} = \succsim^1_{N\setminus\{i\}}$, while $\succsim^2_i \in \bar{\mathcal{S}}^i$ and $\{i,j,k\} \succ^2_i \{i,j\} \succ^2_i \{i,k\} \succ^i_2 \{i\}$. By *strategy-proofness*, $\varphi_i(\succsim_2) \succsim^2_i \varphi_i(\succsim^1)$. Then, $\varphi_i(\succsim_2)$ is either $\{i,j,k\}$ or $\{i,j\}$. As $\{j\} \succ^2_j \{i,j,k\}$, by *individual rationality*, $\varphi_i(\succsim^2) = \{i,j\}$. Finally, by *non-bossiness*, $\varphi(\succsim^2) = \varphi(\succsim^1)$.

Let $\succsim^3 \in \mathcal{S}$ be such that $\succsim^3_{N\setminus\{j\}} = \succsim^2_{N\setminus\{j\}}$, while $\succsim^3_j \in \bar{\mathcal{S}}^j$ and $\{i,j\} \succ^3_j \{i,j,k\} \succ^3_j \{j\} \succ^3_j \{j,k\}$. By *strategy-proofness*, $\varphi_j(\succsim^3) \succsim^3_j \varphi_j(\succsim^2)$. Then, $\varphi_j(\succsim^3) = \{i,j\}$. By *non-bossiness*, $\varphi(\succsim^3) = \varphi(\succsim^2)$.

Now, let $\succsim^4 \in \mathcal{S}$ be such that $\succsim^4_{N\setminus\{i\}} = \succsim^3_{N\setminus\{i\}}$, while $\succsim^4_i \in \bar{\mathcal{S}}^i$ and $\{i,k\} \succ^4_i \{i,j,k\} \succ^4_i \{i\} \succ^4_i \{i,j\}$. Then,

$\succsim^4_i$:	$\succsim^4_j$:	$\succsim^4_k$:
$\{i,k\}$	$\{i,j\}$	$\{j,k\}$
$\{i,j,k\}$	$\{i,j,k\}$	$\{i,j,k\}$
$\{i\}$	$\{j\}$	$\{k\}$
$\{i,j\}$	$\{j,k\}$	$\{i,k\}$

By *individual rationality*, $\varphi_{\{i,j,k\}}(\succsim^4)$ is either $\{i,j,k\}$ or $(\{i\},\{j\},\{k\})$. Note that, $\{i,j,k\} \succ^3_i \varphi_i(\succsim^3)$. Then, by *strategy-proofness*, $\varphi_{\{i,j,k\}}(\succsim^4) = (\{i\},\{j\},\{k\})$.

Let $\succsim^5 \in \mathcal{S}$ be such that $\succsim^5_{N\setminus\{j,k\}} = \succsim^4_{N\setminus\{j,k\}}$, while $\succsim^5_{\{j,k\}} \in \bar{\mathcal{S}}^j \times \bar{\mathcal{S}}^k$, $\{j,k\} \succ^5_j \{j\} \succ^5_j \{i,j,k\} \succ^5_j \{i,j\}$, and $\{i,j,k\} \succ^5_k \{j,k\} \succ^5_k \{i,k\} \succ^5_k \{k\}$. By *top-coalition*, we have $\varphi_k(\succsim^5_{N\setminus\{k\}}, \succsim^4_k) = \{j,k\}$. By *strategy-proofness*, $\varphi_k(\succsim^5) \succsim^5_k \{j,k\}$. As $\{j\} \succ^5_j \{i,j,k\}$, by *individual rationality*, $\varphi_{\{i,j,k\}}(\succsim^5) = (\{i\},\{j,k\})$.

Let $\succsim^6 \in \mathcal{S}$ be such that $\succsim^6_{N\setminus\{j\}} = \succsim^5_{N\setminus\{j\}}$, while $\succsim^6_j \in \bar{\mathcal{S}}^j$ and $\{i,j,k\} \succ^6_j \{j,k\} \succ^6_j \{i,j\} \succ^6_j \{j\}$. By *strategy-proofness*, $\varphi_j(\succsim^6) \succsim^6_j \{j,k\}$. On the other hand, by *top-coalition*, $\varphi_{\{i,j,k\}}(\succsim^6_{N\setminus\{i\}}, \succsim^2_i) = \{i,j,k\}$. By *strategy-proofness*, $\varphi_i(\succsim^6) \succsim^6_i \{i,j,k\}$. Hence, $\varphi_{\{i,j,k\}}(\succsim^6) = \{i,j,k\}$.

Finally, let $\succsim^7 \in \mathcal{S}$ be such that $\succsim^7_{N\setminus\{j\}} = \succsim^6_{N\setminus\{j\}}$ and $\succsim^7_j = \succsim^4_j$. Then

$\succsim^7_i$:	$\succsim^7_j$:	$\succsim^7_k$:
$\{i,k\}$	$\{i,j\}$	$\{i,j,k\}$
$\{i,j,k\}$	$\{i,j,k\}$	$\{j,k\}$
$\{i\}$	$\{j\}$	$\{i,k\}$
$\{i,j\}$	$\{j,k\}$	$\{k\}$

Note that $\succsim^7$ only differs from $\succsim^4$ in k's preferences. By *strategy-proofness*, $\varphi_j(\succsim^7) \succsim^7_j \varphi_j(\succsim^6) = \{i,j,k\}$. By *individual rationality*, if $j \in \varphi_i(\succsim^7)$, then $\varphi_i(\succsim^7) = \{i,j,k\}$. Hence, $\varphi_{\{i,j,k\}}(\succsim^7) = \{i,j,k\}$. However, $\varphi_k(\succsim^7) \succ^4_k \varphi_k(\succsim^4)$, which violates *strategy-proofness.* ■

4 Independence of the Axioms

The following examples prove the independence of the axioms. If we drop any of the four axioms, we can find rules that satisfy the remaining three axioms.

Example 1 (*Strategy-Proofness*). *Let $N = \{1,2,3\}$. For each $i \in N$ and each $\succsim \in \mathcal{S}$, let*

$$IR_i(\succsim) \equiv \{C \in \mathcal{C}_i, \text{ such that for each } j \in C, C \succsim_j \{j\}\}.$$

Let the rule φ^{-SP} *be such that for each* $\succsim \in \mathcal{S}$, $\varphi_1^{-SP}(\succsim) \equiv \text{top}(IR_1(\succsim), \succsim_1)$ *and for each* $\succsim \in \mathcal{S}$ *such that* $\text{top}(IR_1(\succsim), \succsim_1) = \{1\}$, $\varphi_2(\succsim) \equiv \{2,3\}$ *if* $\{2,3\} \succ_2 \{2\}$ *and* $\{2,3\} \succ_3 \{3\}$, $\varphi_2(\succsim) = \{2\}$, *otherwise . Note that* φ^{-SP} *satisfies individual rationality, non-bossiness, and voters' sovereignty. However,* φ^{-SP} *is manipulable.*[7]

Example 2 (*Individual Rationality*). *Let* $N = \{1,2,3\}$. *Define* φ^{-IR} *in such a way that for each* $\succsim \in \mathcal{A}$, $\varphi_1(\succsim) \equiv \text{top}(\mathcal{N}, \succsim_1)$ *and for each* $\succsim \in \mathcal{S}$ *such that* $\text{top}(\mathcal{N}, \succsim_1) = \{1\}$, $\varphi_2(\succsim) \equiv \text{top}(\mathcal{N} \setminus \mathcal{C}_1, \succsim_2)$. *The rule* φ^{-IR} *is a* ***serial dictatorship***. *Clearly,* φ^{-SP} *satisfies strategy-proofness, non-bossiness, and voters' sovereignty. However,* φ^{-SP} *does not satisfy individual rationality.*

Example 3 (*Non-Bossiness*). *Let* $N = \{1,2,3\}$. *Let* φ^{-NB} *be such that for each* $\succsim \in \mathcal{S}$, *and for each* $i, j, k \in N$

$$\varphi^{-NB}(\succsim) = \begin{cases} N & \text{if for each } i \in N,\ N \succsim_i \{i\}, \\ (\{i,j\}, \{k\}) & \text{if } \{i,j\} \succ_i \{i\}, \{i,j\} \succ_j \{j\} \text{ and } \text{top}(\mathcal{N}, \succsim_k) = \{k\}, \\ (\{1\}, \{2\}, \{3\}) & \text{otherwise.} \end{cases}$$

It is not difficult to check that φ^{-NB} *satisfies strategy-proofness, individual rationality, and voters' sovereignty. However,* φ^{-NB} *is bossy.*[8]

Example 4 (*Voters' Sovereignty*). *Let* $N = \{1,2,3\}$. *Let* φ^{-VS} *be such that for each* $\succsim \in \mathcal{S}$,

$$\varphi(\succsim) = \begin{cases} N & \text{if for each } i \in N,\ N \succsim_i \{i\}, \\ (\{1\}, \{2\}, \{3\}) & \text{otherwise.} \end{cases}$$

It is immediate to check that φ^{-VS} *satisfies strategy-proofness, individual rationality, and non-bossiness. However,* φ^{-VS} *does not satisfy voters' sovereignty.*

[7]In order to check that φ^{-SP} is manipulable, let $\succsim \in \mathcal{S}$ and $\succsim'_2 \in \mathcal{S}^2$ be such that $\{1,2\} \succ_1 \{1,2,3\} \succ_1 \{1\}$, $\{1,2,3\} \succ_2 \{1,2\} \succ_2 \{2,3\} \succ_2 \{2\}$, and $\{1,3\} \succ_3 \{1,2,3\} \succ_3 \{3\}$; while $\{2,3\} \succ'_2 \{1,2,3\} \succ'_2 \{2\}$. Note that $\varphi^{-SP}(\succsim) = (\{1,2\}, \{3\})$, while $\varphi^{-SP}(\succsim_{N\setminus\{2\}}, \succsim'_2) = \{1,2,3\}$. Then, $\varphi_2^{-SP}(\succsim_{N\setminus\{2\}}, \succsim'_2) \succ_2 \varphi_2^{-SP}(\succsim)$.

[8]In order to check that φ^{-NB} violates *non-bossiness*, let $\succsim \in \mathcal{S}$, $\succsim'_3 \in \mathcal{S}^3$ be such that $\{1,2\} \succ_1 \{1\}$, $\{1,2\} \succ_2 \{2\}$, $\text{top}(\mathcal{N}, \succsim_3) = \{3\}$, while $\{2,3\} \succ'_3 \{3\} \succ'_3 \{1,2,3\}$. Note that $\varphi(\succsim) = (\{1,2\}, \{3\})$ and $\varphi(\succsim_{N\setminus\{3\}}, \succsim'_3) = [\{1\}, \{2\}, \{3\}]$.

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