E C O N O M I C S   B U L L E T I N

Strategic Targeted Advertising and Market Fragmentation

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Abstract

This paper proves that oligopolistic price competition with both targeted advertising and targeted prices can lead to a permanent fragmentation of the market into a local monopoly. However, compared to mass advertising, targeting increases social welfare and turns out to be more beneficial for consumers than for firms.

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1 Introduction

The current proliferation of new and highly specialized communication channels (the Internet, cable TV, specialized press, etc.) is leading firms to progressively abandon the use of indiscriminate mass advertising to inform consumers about their new products in favor of targeted advertising, which allows sellers to concentrate their ads on particular segments of the potential demand, thus saving advertising costs. This change in advertising technology has important implications for the pattern of price competition between firms. Under mass advertising, information is uniformly spread throughout the market, which induces firms to actively compete in prices. By contrast, targeting may allow a seller to reach uninformed consumers who are not in the target set of the competitors, thus obtaining a captive market which the firm can try to monopolize. The key issue then is to what extent the transition from mass to targeted advertising can lead to a *fragmentation* of the market, that is to say, to the formation of local monopolies. The answer to this question will help to explain how the proliferation of new advertising technologies can affect consumers, firms and social welfare.

The existing literature on strategic targeted advertising claims that a price competition game with either homogeneous (Galleotti *et. al.*, 2004) or horizontally differentiated products (Iyer *et. al.*, 2005) only has a Nash equilibrium in *mixed* strategies and, therefore, that targeting can fragment the market *only from time to time*. We address the problem by considering that products are vertically differentiated and that targeting is closely related to the feasibility of price discrimination through discount coupons. In this alternative framework, we show that the joint consideration of targeted advertising and targeted prices can indeed lead to *permanent* market fragmentation. However, in the price-discrimination equilibrium, the use of targeted advertising increases social welfare, whereas the impact on consumers' surplus (firms' profits) turns out to depend on the type of product quality that they buy (sell).

2 The Model

Consider a market with two firms competing simultaneously in prices and using informative advertising to promote sales. Consumers are unaware of the existence of the goods[1] and sellers can inform them about their existence, price and product specifications by using either mass advertising, which reaches the whole potential market, or specialized advertising, which targets the ads on

[1] We assume that consumers' search cost is high relative to the expected surplus offered by the goods, in such a way that potential buyers are passive and, in the absence of information, do not purchase any good (see, for example, Grossman and Shapiro, 1984 or Stahl, 1994).

a particular segment of the market. In a model of targeting, the fundamental issue is on which segment of the market the specialized media concentrate the ads. In this regard, we follow Esteban *et. al.* (2001, 2006), who note that the degree of media specialization is often positively correlated with consumers' valuation of the goods, in such a way that firms can frequently target their ads *only* on the most eager consumers. One example of this targeting technology is the case of "*specialized magazines with nested readerships.*" There are magazines containing general information on sports, medicine, computers, family matters, etc., while there are others specialized in particular sports (soccer, basketball, golf, etc.), medical specialities (surgery, radiology, dermatology, etc.), computer issues (video-games, Internet, etc.) or leisure activities (fitness, decoration, gardening, etc.) which reach high valuation consumers. Accordingly, firms can target the most eager consumers by using the specialized media. However, to reach low-valuation consumers, sellers can only use general magazines which reach the whole potential market.[2]

In order to accommodate this type of targeting into a price competition model, it is necessary to impose a particular structure on consumers' preference ordering. Both firms can be in accordance about who the "*most eager customers*" are only if all consumers agree on the preference ordering, that is to say if, when products are offered at the same price, all customers choose to purchase the same one. Consequently, we think that a natural way to study the effects of targeted advertising on firms' pricing strategies is in the context of vertically differentiated products, in the spirit of Shaked and Sutton (1982, 1983). Therefore, we will consider that each firm supplies a good with a different level of quality and that consumers are heterogeneous in their taste for quality. In this framework, targeting means that there is a given specialized advertising media which allows firms to concentrate their ads on a subset of consumers who value quality most.[3]

In accordance with these ideas, we consider a market with a unitary mass of consumers who demand, at most, one unit of a product. A consumer's utility is $U = v + \theta s - p$, when he buys a

[2] An example might help to understand this point. Consider that a firm is introducing a product, for example, a new computer video game, and that it can classify its potential consumers into those who are regular users of video games, with a high valuation of the good, and those who are only regular users of computers, with a lower valuation. A careful selection of magazines specializing in video games (such as Computer Gaming World, PC Top Player, etc.), would allow the firm to reach exclusively high-demand consumers. However, if the firm wants to reach low-demand consumers, it has to use the class of general computer magazines (such as Computer World, Home PC, etc.), which spread the ads across the whole potential market. As a result, the targeting technology allows firms to segment the market *only* by isolating the most eager customers.

[3] Going back to the example of computer video games, our model of targeting implies that there is one firm producing a high-quality video game while the other sells a low-quality video game, and that each seller can advertise its product in highly specialized video games magazines, which reach exclusively high valuation consumers, or in general computer magazines, which also reach low valuation consumers.

good of quality s at price p, and 0 if he does not buy. The parameter $v > 0$ represents consumers' common valuation of the product. The parameter θ of taste for quality is uniformly distributed across the population of consumers in the interval $[a, b]$, with $b - a = 1$. The two firms in the market, $i = 1, 2$, produce two goods of a given quality $s_i > 0$, with $\Delta s = s_2 - s_1 > 0$, at a cost $C(s_i) = cs_i$. Firms can inform consumers by using either the mass media, which distributes the ads to the entire population of potential buyers $[a, b]$, or the specialized media, which reaches only those consumers in the segment $[z, b]$, with $b > z > a$. Thus, if t denotes the target of the campaign, firm i can insert the ads in the mass media, $t_i = a$, and/or in the specialized media[4] $t_i = z > a$, and we assume that the value of z is exogenous. We further consider that when a firm advertises the product in a segment of the market, all consumers in that segment become informed about the existence, price and characteristics of the good. Advertising is costly, and the cost of a campaign depends on the size of the target market. If A_0 denotes the cost of informing all consumers in $[a, b]$, and A_1 denotes the cost of a campaign targeted on $[z, b]$, then, given that targeting reduces the number of consumers reached by the campaign, we consider[5] that $\Delta A = A_0 - A_1 > 0$.

Having specified the fundamentals of the model, we now analyze the simultaneous move game in which both firms decide their pricing-targeted advertising strategies, (p_i, t_i). For future reference, let (p_1^m, p_2^m) denote the unique equilibrium price strategies when both firms can use *only* mass advertising, $(t_1 = a, t_2 = a)$, and compete for the fully informed marginal consumer, $\theta^m = \frac{p_2^m - p_1^m}{\Delta s}$, with firm 1 (the low-quality firm) and firm 2 serving the market segments $[a, \theta^m]$ and $[\theta^m, b]$, respectively (see the Appendix for details of this equilibrium). This solution, which is equal to the full information outcome, constitutes a reasonable benchmark against which we can compute the impact of targeting on market prices. Next, let us assume that there is a specialized advertising media which allows firms to target the ads on a given subset of high valuation consumers, $[z, b]$. We first note that if z is sufficiently high, $z > \theta^m$, both firms will have a low incentive to target their campaigns, given that firm 1 could not reach any consumer in $[a, \theta^m]$, whereas firm 2 could reach only a fraction of $[\theta^m, b]$. Therefore, it make sense to focus our analysis on the case in which $z \leq \theta^m$. Under this condition, we first prove that a pure strategy Nash equilibrium of the pricing-targeted advertising game in which both firms use mass advertising does not exist (all the proofs are relegated to the Appendix).

[4] Note that the simplifying assumption of a binary targeting choice is not restrictive. Esteban *et. al.* (2006) show that this type of analysis can be extended to the case in which a firm simultaneously uses multiple advertising media with different target audiences.

[5] Esteban *et. al.* (2001, 2006) provide empirical evidence confirming this intuition for the case of "*specialized magazines with nested readerships.*" For example, for the case of computer magazines in the Dutch market, they claim that moving an advertising campaign from less to more specialized media yields a cost saving of 44%.

Lemma 1 *If $z \leq \theta^m$, the strategy profile $s = [(p_1, t_1 = a); (p_2, t_2 = a)]$ cannot be an equilibrium.*

Lemma 1 simply states that using mass advertising cannot be optimal, given that firms can benefit from the higher cost efficiency of targeting, and so we must look for equilibria in which some firm targets its campaign. Taking into account that nowadays the number of specialized media is growing very fast, it makes sense to consider that some firms may be able to implement precise advertising campaigns. Accordingly, we concentrate our attention on the most interesting targeting scenario, namely, the case in which the high-quality firm can approximately target its potential customers, i.e. $z \longrightarrow \theta^m$. In this set up, and given the natural *asymmetry of* vertically differentiated markets, we note that firms have different incentives to use mass or targeted advertising. Thus, firm 2 has a particularly high incentive to use specialized advertising, given that this media allows the firm to reach its clients at a lower advertising cost. By contrast, under uniform pricing, firm 1 has a clear incentive to set $t_1 = a$, since this is the natural way in which the seller can reach its potential market, whereas setting $t_1 = z$ would basically trigger more intense price competition. Therefore, it seems reasonable to look for an equilibrium strategy profile with $(t_1 = a, t_2 = z)$. However, Lemma 2 sates that, under these conditions, the pricing-targeted advertising game does not have a pure strategy Nash equilibrium.

Lemma 2 *If $z \longrightarrow \theta^m$, the pricing-targeted advertising game does not have a pure strategy Nash equilibrium in which $(t_1 = a, t_2 = z)$.*

The intuition of this Lemma is as follows. Starting from the benchmark, where firms can use only mass advertising, when sellers can target their ads on $z \longrightarrow \theta^m$, and given $s_1 = (p_1^m, t_1 = a)$, firm 2's best response is $s_2 = (p_2^m, t_2 = z)$, which causes products to be differentiated along two dimensions, quality and information, thus substantially changing the pattern of price competition in the market. In particular, given this s_2, firm 1's best response is $s_1 = (p_1^M, t_1 = a)$, that is to say, to charge the monopoly price, $p_1^M > p_1^m$, to the segment of imperfectly informed consumers, $[a, z]$. The problem with this outcome is that both products are strategic complements and so, if $t_2 = z$, firm 2 would respond to the monopolization of $[a, z]$ by raising the price, $p_2 > p_2^m$. This will lead firm 1 to compete for the segment of the market $[z, b]$ by lowering the price $p_1 < p_1^M$ which, in turn, will induce firm 2 to also lower the price. Finally, firm 1's best response to this latter strategy would be, once more, to monopolize $[a, z]$ by raising the price to p_1^M, thus starting the same price-cycle again, which implies that, under uniform pricing, a pure strategy Nash equilibrium with *permanent* market fragmentation does not exist.

This result is in line with the existing literature on strategic targeting, which claims that this type of advertising can *fragment* the market *only sporadically.* Next, we show that this conclusion changes when firms compete with vertically differentiated products and when we take into account that targeting is closely related to price discrimination. More precisely, the following Proposition states that there exists a *pure strategy* Nash equilibrium in which firm 2 advertises a price p_2^t to the segment of the market $[\theta^m, b]$ whilst firm 1 uses both the mass media, $t_1 = a$, where it announces a high price, p_1^t, and the specialized media, $\widehat{t_1} = \theta^m$, where it inserts discount coupons which allow consumers in $[\theta^m, b]$ to purchase the low-quality good at a reduced price, $\widehat{p}_1^t$.

Proposition 1 *If (i)* $A_1 < \frac{[b-2a+c]^2 \Delta s}{81}$, *(ii)* $\Delta A > \frac{[(13b+a-14c)s_2-(13b-8a-5c)s_1+9v]^2}{648\Delta s} - \frac{[5b-a-4c]^2\Delta s}{81}$, *and (iii)* $s_2 > \frac{11s_1(a-c)+7s_1+9v}{7+2(a-c)}$, *then the following pure strategy Nash equilibrium exists:*

(a) $\left[t_2 = \theta^m;\ p_2^t = \frac{(5b-a)\Delta s+4cs_1+5cs_2}{9}\right]$, *with* $p_2^t < p_2^m$.

(b) $\left[(t_1 = a,\ p_1^t = v + as_1); \left(\widehat{t_1} = \theta^m,\ \widehat{p}_1^t = \frac{(b-2a)\Delta s+8cs_1+cs_2}{9}\right)\right]$, *with* $p_1^t = p_1^M$, *and* $\widehat{p}_1^t < p_1^m$.

To understand why the game has an equilibrium, note that, when price discrimination is feasible, and given $t_2 = \theta^m$, an informational asymmetry arises across consumers allowing firm 1 to segment the market and charge different prices to two potential demands: (i) a captive demand, $[a, \theta^m]$, which stems from *uninformed* consumers that can be reached only by mass advertising, and (ii) a competitive demand, which stems from those consumers who have *imperfect* information and, therefore, who are more price sensitive, and that can be isolated by the specialized media. Proposition 1 states that firm 1 can monopolize the captive market and, simultaneously, "invade" the rival's market by inserting discount coupons in the specialized media in order to capture a fraction of fully informed consumers in $[\theta^m, b]$. The *key point* is that, although the segment $[a, \theta^m]$ is fully monopolized, the use of coupons yields more intense competition for consumers in $[\theta^m, b]$ and so, instead of raising the price, firm 2 finds it optimal to lower it below p_2^m and to set $t_2 = \theta^m$ in order to benefit from the higher cost efficiency of the specialized media. As a result, there exists a Nash equilibrium which involves a local monopoly and, therefore, our model proves that the current proliferation of new advertising technologies can yield a *permanent fragmentation* of the market.

Regarding the existence conditions of the game, it is clear that, in equilibrium, firm 1 will be able to capture some fully informed consumers, who have a relatively high taste for quality, only if it offers the low-quality good at a substantially low price, $\widehat{p}_1^t$. Taking this low price into account, the profit obtained by firm 1 from price discrimination will be positive only if the cost of targeting is sufficiently low (restriction (i)). In this setting, and under uniform pricing, firm 2 has two possible responses: first, it can accommodate the more intense competition induced by the use of coupons by advertising p_2^t in the specialized media and, secondly, it can deviate by using mass advertising to compete for the segment of the market $[a, \theta^m]$, thus achieving a larger market share. Obviously, firm

2 will find it optimal to set $t_2 = \theta^m$ only if the shift from mass to targeted advertising generates sufficiently high savings in advertising costs (restriction (ii)). Finally, firm 2 could also deviate by using both mass and specialized advertising to price discriminate with discount coupons. If s_2 is sufficiently high, this strategy is not feasible because the optimal price set in the coupon (which is increasing in s_2) turns out to be *greater* than the regular price announced in the mass advertising campaign (restriction (iii)). Accordingly, we can expect that, for a sufficiently high degree of product differentiation, the equilibrium described in Proposition 1 will exist if the cost of the specialized (mass) media is sufficiently low (high). Thus, for example, it is straightforward to check that for the market scenario $[v = 100, s_1 = 38, c = 0.75, a = 0, A_0 = 10]$, if $s_2 = 160$, the set of existence restrictions holds for any $A_1 < 4.6$. Further, the equilibrium exists for a wide set of market scenarios, some of which are shown in Table 1.

Insert Table 1 here.

This table can also be used to discuss how oligopolistic price discrimination, based on targeted coupons, can affect both consumers and firms. The existing literature on informative advertising targeted on high valuation consumers (Esteban *et. al.*, 2001, 2006) states that this type of advertising yields higher prices,[6] thus increasing firms' profits (Π_i) and lowering consumers' surplus (CS). By contrast, the literature on oligopolistic price discrimination with coupons targeted on low valuation consumers (Bester and Petrakis, 1996) yields the opposite results. Given the asymmetry of vertically differentiated markets, in our model the two firms use specialized advertising for quite different purposes. Firm 2 targets the advertising simply to better reach those consumers with a higher valuation of its product. By contrast, given that firm 1 can use mass advertising to fully monopolize its captive market, it uses the targeting to reach the segment of the market with a more price sensitive demand and, in this sense, the targeted ads reach "low valuation" customers. Accordingly, it is worth noting that our work brings together the two strands of the above literature by combining, on the one hand, informative advertising targeted on high valuation consumers, ($t_2 = z$), and, on the other, price discrimination with coupons targeted on low valuation consumers, ($\widehat{t}_1 = z$). This explains why our model yields the following variety of results.

Clearly, the transition from mass to targeted advertising benefits firm 1, given that this firm can monopolize the captive market $[a, \theta^m]$ and achieve an additional profit from the use of coupons. By contrast, the impact on firm 2's profit depends on whether the savings on advertising costs exceed the losses from the more intense price competition induced by couponing. In this respect,

[6] Notice that, under uniform pricing, the targeting models of Galleotti *et. al.* (2004) and Iyer *et. al.* (2005), where the ads cannot be targeted on high valuation consumers, also yield higher prices.

the calibration of our model suggests that, under a wide variety of market conditions, firm 2's profit tends to decrease with targeting. Indeed, this profit increases only if ΔA is very high ($A_0 = 20, A_1 = 4.6$). As regards consumers, those who have imperfect information are worse off with targeting, given that they end up paying the monopoly price. However, *all* consumers who are fully informed pay a lower price and, therefore, are better off, due to either the use of coupons or the more intense price competition in which both firms engage. Regarding these trade-offs, extensive simulation of our model yields two interesting results (see Table 1): (i) even though specialized advertising generates a local monopoly, targeting increases *aggregate* consumer surplus and (ii) even though specialized advertising generates substantial saving on advertising costs, targeting is likely to yield lower *aggregate* profits. Therefore, it seems that, at the aggregate level, the effects associated with price discrimination dominate the effects related to the higher market power induced by targeting, and so targeted advertising is likely to be more beneficial for consumers than for firms.

Finally, under reasonable market conditions (v not very small), in equilibrium, the market is fully covered, and so the impact of targeting on social welfare depends essentially on the advertising costs. In this regard, the transition from mass to targeted advertising reduces the advertising budget of firm 2 by ΔA and increases the advertising cost of firm 1 by A_1. Extensive simulations of our model yield that, in equilibrium, $\Delta A > A_1$, which explains why the level of social welfare (computed as $CS + \Pi_1 + \Pi_2$) in Table 1 is always higher with targeting than with mass advertising. This suggests that, even though targeting generates a local monopoly, this advertising technology is welfare-improving.

3 Conclusions

This paper formulates a simple model of oligopolistic price competition, with both targeted advertising and targeted prices, to prove that the transition from mass advertising to targeting can lead to a *permanent fragmentation* of the market into a local monopoly. Further, we find that, although targeting is more efficient than mass advertising, the use of specialized media tends to reduce the profits achieved by the high-quality firm. Finally, a particularly noteworthy result is that, even though the market is fragmented, targeting increases both consumer surplus and social welfare. These results contribute to the ongoing research on targeted advertising by providing some new insights about how the proliferation of new advertising technologies can affect market performance.

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Appendix:

Proof of Lemma 1: If $t_1 = t_2 = a$ and firms compete for the fully informed marginal consumer $\theta = \frac{p_2 - p_1}{\Delta s}$, profits are given by[7]

$$\Pi_1 = (p_1 - cs_1)\,(\frac{p_2 - p_1}{\Delta s} - a) - A_0, \quad (1)$$

$$\Pi_2 = (p_2 - cs_2)\,(b - \frac{p_2 - p_1}{\Delta s}) - A_0. \quad (2)$$

Differentiating these expressions with respect to p_1 and p_2 we obtain the reactions functions:

$$p_1^m(p_2) = \frac{p_2 - a\Delta s + cs_1}{2}, \quad (3)$$

$$p_2^m(p_1) = \frac{p_1 + b\Delta s + cs_2}{2}. \quad (4)$$

The intersection of (3) and (4) yields the unique Nash equilibrium of the pricing game: $p_1^m = \frac{\Delta s(1-a)+2cs_1+cs_2}{3}$, $p_2^m = \frac{\Delta s(2+a)+2cs_2+cs_1}{3}$, with profits $\Pi_1^m = \frac{[\Delta s(1-a)+cs_2-cs_1)]^2}{9\Delta s} - A_0$, and $\Pi_2^m = \frac{[\Delta s(2+a)+cs_1-cs_2)]^2}{9\Delta s} - A_0$. Next, let us assume that firms can target their ads on $z \leq \theta^m = \frac{p_2^m - p_1^m}{\Delta s}$. Then, given ($t_1 = a$, p_1^m), firm 2's profits under targeted advertising are: $\Pi_2(p_2, z) = (p_2 - cs_2)$ $Min\left\{b - \frac{p_2 - p_1^m}{\Delta s}, b - z\right\} - A_1(z)$. Assume, for the moment, that $Min\left\{b - \frac{p_2 - p_1^m}{\Delta s}, b - z\right\} = b - \frac{p_2 - p_1^m}{\Delta s}$. In this case, the price that maximizes $\Pi_2(p_2, z) = (p_2 - cs_2)\,(b - \frac{p_2 - p_1^m}{\Delta s}) - A_1(z)$ is $p_2 = p_2^m$, and so it holds that $b - \frac{p_2^m - p_1^m}{\Delta s} \leq b - z$. Finally, $\frac{\partial \Pi_2(p_2^m, z)}{\partial z} = -A_1'(z) > 0$ which implies that, given the pricing-advertising strategy of firm 1, firm 2's best response is $t_2 = z$. Therefore, $t_1 = t_2 = a$ cannot be part of an equilibrium. Q.E.D.

Proof of Lemma 2: For the targeting strategy ($t_1 = a, t_2 = z$), if firm 1 competes for fully informed consumers in $[z, b]$, then the optimal prices are (p_1^m, p_2^m), with $\theta^m = \frac{p_2^m - p_1^m}{\Delta s}$. However, this is not an equilibrium strategy, given that if $t_2 = z \longrightarrow \theta^m$ and $p_2 = p_2^m$, then it is clear that firm 1's best response will be to monopolize the captive market by setting $p_1^M = Max\left[v + as_1, \frac{v + s_1(z+c)}{2}\right] > p_1^m$, in such a way that $\theta^M = \frac{p_2^m - p_1^M}{\Delta s} < \theta^m$ and the demand served by firm 1 is $D_1^M = Min\left[(z - a), \frac{v + s_1(z-c)}{2s_1}\right]$. As a result of this monopolization strategy, if $t_2 = z$, firm 2 faces a demand: $D_2 = Min[b - z, b - \frac{p_2 - p_1^M}{\Delta s}]$. Given that from (4) we know that $\frac{\partial p_2}{\partial p_1} < 1$, the solution of $Max_{p_2}\ (p_2 - cs_2)\,(b - \frac{p_2 - p_1^M}{\Delta s}) - A_1$ yields $\frac{p_2 - p_1^M}{\Delta s} < \theta^m$. Therefore, $D_2 = b - z$, and firm 2 will respond to p_1^M by charging the maximum price that the marginal consumer, $z = \theta^m$, is willing to pay, i.e. $p_2' = p_1^M + \Delta s\ \theta^m$, in such a way that $\frac{p_2' - p_1^M}{\Delta s} = \theta^m$. Given $(t_2 = z, p_2')$, firm 1 can either monopolize $[a, z]$, which yields a benefit $\Pi_1^M = (p_1^M - cs_1)\ D_1^M - A_0$, or compete for $[z, b]$, which implies to maximize $\Pi_1' = (p_1 - cs_1)\ (\frac{p_2' - p_1}{\Delta s} - a) - A_0$. The solution of this problem yields

[7] For details of this equilibrium, see Tirole (1988).

$p_1' = \frac{p_2' - a\Delta s + cs_1}{2}$ and profits $\Pi_1' = \frac{\left[p_1^M + (z-a)\Delta s - cs_1\right]^2}{4\Delta s} - A_0$. Some calculations yield that $\Pi_1^M > \Pi_1'$ implies $(p_1^M - cs_1)^2 + (z-a)^2\Delta s^2 + (p_1^M - cs_1)\ \Delta s\ \left[2(z-a) - 4D_1^M\right] < 0$. Taking into consideration that $D_1^M \leq z - a$, we have that $2(z-a) - 4D_1^M \geq -2(z-a)$, and so

$$\begin{aligned} 0 &> (p_1^M - cs_1)^2 + (z-a)^2\Delta s^2 + (p_1^M - cs_1)\Delta s\left[2(z-a) - 4D_1^M\right] \geq \\ &\geq (p_1^M - cs_1)^2 + (z-a)^2\Delta s^2 - 2(p_1^M - cs_1)\Delta s(z-a) = \left[(p_1^M - cs_1) - \Delta s(z-a)\right]^2, \end{aligned}$$

which constitutes a contradiction. This shows that firm 1's best response to $(t_2 = z, p_2')$ is to compete for $[z, b]$. Finally, if both firms compete for the fully informed consumers in $[z, b]$, the unique Nash equilibrium of the pricing game is (p_1^m, p_2^m) which, as we have already shown, cannot be part of an equilibrium. This completes the proof.

Proof of Proposition 1: To prove that these strategy profiles indeed constitute an equilibrium and to prove existence, we need to verify that neither firm has an incentive to deviate from the strategies prescribed and that expected profits are strictly positive.

• We first analyze firm 1's optimal strategy. Given $\left(t_2 = \theta^m, p_2^t\right)$, firm 1 can monopolize $[a, z]$ according to the demand $D_1^M = Min[\theta^m - \frac{p_1 - v}{s_1}, \theta^m - a]$ and, at the same time, this firm can compete for those consumers in $[z, b]$ by advertising a low price, $\widehat{p}_1^t$, in the specialized media, i.e. $\widehat{t}_1 = \theta^m$, in order to capture an additional demand $\widehat{D}_1 = Min[\frac{p_2^t - \widehat{p}_1}{\Delta s} - \theta^m, b - \theta^m]$. To focus the analysis on the most interesting case, we assume that $D_1^M = \theta^m - a = \frac{b-2a+c}{3}$, which implies $v \geq \frac{(b-5a+4c)s_1}{3}$, and that $\widehat{D}_1 = \frac{p_2^t - \widehat{p}_1}{\Delta s} - \theta^m$. Under these conditions, $p_1^t = v + as_1$ and $\widehat{p}_1$ is determined by the maximization of: $(\widehat{p}_1 - cs_1)\left[\frac{p_2^t - \widehat{p}_1}{\Delta s} - \theta^m\right] - A_1$, which yields $\widehat{p}_1^t = \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}$. Given this result, it is straightforward to see that $\widehat{p}_1^t < p_1^m < p_1^t$. Further, firm 1's profits are $\Pi_1^t = [v + (a-c)s_1]\left(\frac{b-2a+c}{3}\right) - A_0 + \frac{[b-2a+c]^2\Delta s}{81} - A_1$, and the equilibrium can exist only if price discrimination is profitable, i.e. $\frac{[b-2a+c]^2\Delta s}{81} - A_1 > 0$.

• Next, we analyze firm 2's optimal strategy. Given $[\left(t_1 = a,\ p_1^t = v + as_1\right); \left(\widehat{t}_1 = \theta^m,\ \widehat{p}_1^t = \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}\right.$ if firm 2 sets $t_2 = \theta^m$, then p_2^t is obtained from $Max_{p_2}\ \Pi_2 = (p_2 - cs_2)\left(b - \frac{p_2 - \widehat{p}_1^t}{S}\right) - A_1$, which yields $p_2^t = \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9} < p_2^m$, $D_2^t = \frac{5b-a-4c}{9}$, and $\Pi_2^t = \frac{[5b-a-4c]^2\Delta s}{81} - A_1$. Starting from this solution, firm 2 has two possible deviations:

(i) It can price discriminate by advertising $\overline{p}_2$ in $t_2 = a$ in order to attract consumers in $\left[a,\ \theta^m\right]$, according to the demand $\overline{D}_2 = Min[\theta^m - \frac{p_2 - (v + as_1)}{\Delta s}, \frac{b-2a+c}{3}]$, and $\widehat{p}_2 < \overline{p}_2$ in $\widehat{t}_2 = \theta^m$ in order to attract consumers in $\left[\theta^m, b\right]$, according to the demand $\widehat{D}_2 = Min[b - \frac{\widehat{p}_2 - \widehat{p}_1^t}{\Delta s}, \frac{2b-a-c}{3}]$. Under this strategy, $\overline{p}_2$ and $\widehat{p}_2$ are determined by

$$Max_{\overline{p}_2, \widehat{p}_2}\ \Pi_2 = (\widehat{p}_2 - cs_2)\left(b - \frac{\widehat{p}_2 - \widehat{p}_1^t}{\Delta s}\right) - A_1 + (\overline{p}_2 - cs_2)\left(\theta^m - \frac{\overline{p}_2 - (v + as_1)}{\Delta s}\right) - A_0$$

which yields $\widehat{p}_2 = \frac{(5b-a)\Delta s+4cs_1+5cs_2}{9} = p_2^t$, and $\overline{p}_2 = \frac{(a+b)\Delta s+3as_1+4cs_2-cs_1+3v}{6}$. This type of deviation will be possible only if $\widehat{p}_2 < \overline{p}_2$. Therefore, if $\widehat{p}_2 > \overline{p}_2$ (or, more generally, if profits from price discrimination are negative, i.e. $\frac{[(a+b-2c)s_2+3v-(b-2a+c)s_1]^2}{36\Delta s} - A_0 < 0$), firm 2 will not deviate.

(ii) Firm 2 can also advertise a uniform price p_2 in $t_2 = a$. In this case, it faces two demands: first, the demand from consumers in $\left[a, \theta^m\right]$ is given by $D_{21} = Min[\theta^m - \frac{p_2-(v+as_1)}{\Delta s}, \frac{b-2a+c}{3}]$, with inverse demand $p_2 = Max[\theta^m \Delta s + v + as_1 - \Delta s\ D_{21}, v + as_2]$ and, second, the demand from consumers in $\left[\theta^m, b\right]$ is given by $D_{22} = Min[b - \frac{p_2-\widehat{p}_1^t}{\Delta s}, \frac{2b-a-c}{3}]$, with inverse demand $p_2 = Max[\frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} - \Delta s\ D_{22}, \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9}]$.

It is straightforward to show that: (a) $\widehat{p}_2 > \overline{p}_2$, i.e. $\frac{(5b-a)\Delta s+4cs_1+5cs_2}{9} > \frac{(a+b)\Delta s+3as_1+4cs_2-cs_1+3v}{6}$, implies $\frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} > \theta^m \Delta s + v + as_1$, (b) that $p_1^m < v + as_1$ implies $\theta^m \Delta s + v + as_1 > \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9}$ and, finally, (c) that $D_1^m > 0$ implies $\theta^m \Delta s + v + as_1 > v + as_2$. Accordingly, for the case in which $\frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} > v + as_2$, firm 2 faces an aggregate demand:

$$D_2 = \begin{cases} 0 & \text{if} \quad p_2 > \frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} \\ b - \frac{p_2 - \frac{(b-2a)\Delta s+8cs_1+cs_2}{9}}{\Delta s} & \text{if} \quad \frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} > p_2 > \theta^m \Delta s + v + as_1 \\ \frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s} & \text{if} \quad \theta^m \Delta s + v + as_1 > p_2 > \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} \\ b + \frac{as_1+v}{\Delta s} - \frac{p_2}{\Delta s} & \text{if} \quad \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} > p_2 > v + as_2 \\ 1 & \text{if} \quad p_2 < v + as_2 \end{cases}$$

whereas if $\frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} < v + as_2$, the demand is

$$\widetilde{D}_2 = \begin{cases} 0 & \text{if} \quad p_2 > \frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} \\ b - \frac{p_2 - \frac{(b-2a)\Delta s+8cs_1+cs_2}{9}}{S} & \text{if} \quad \frac{2(5b-a)\Delta s+8cs_1+cs_2}{9} > p_2 > \theta^m \Delta s + v + as_1 \\ \frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s} & \text{if} \quad \theta^m \Delta s + v + as_1 > p_2 > v + as_2 \\ \frac{13b-8a+3c}{9} + \frac{8cs_1+cs_2}{9\Delta s} - \frac{p_2}{\Delta s} & \text{if} \quad v + as_2 > p_2 > \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} \\ 1 & \text{if} \quad p_2 < \frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} \end{cases}$$

Regardless of $\frac{(4b+a+3c)\Delta s+8cs_1+cs_2}{9} \lesseqgtr v + as_2$, the deviation related to the second section of these demands cannot be profitable, given that the solution is dominated by $t_2 = \theta^m$. Further, the optimal price corresponding to the third section is given by the solution of:

$$Max_{p_2^d}\ \Pi_2 = (p_2 - cs_2)\left(\frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s}\right) - A_0,$$

which yields $p_2^d = \frac{(13b+a)\Delta s+9as_1+22cs_2+5cs_1+9v}{36}$, $\Pi_2^d = \frac{[(13b+a-14c)s_2-(13b-8a-5c)s_1+9v]^2}{648\Delta s} - A_0$. Thus, the strategy $\left(t_2 = a,\ p_2^d\right)$ does not constitute a profitable deviation from $(t_2 = \theta^m, p_2^t)$, if $\Pi_2^d < \Pi_2^t$. Therefore, the equilibrium described in Proposition 1 exists only if the following three conditions hold: (i) $\frac{[b-2a+c]^2\Delta s}{81} - A_1 > 0$, (ii) $\Pi_2^d < \Pi_2^t$, and (iii) $\widehat{p}_2 > \overline{p}_2$, and the existence conditions formulated in the Proposition are directly obtained from these inequalities. We have computed the model for different market scenarios and have found that for the third section of D_2 and $\widetilde{D}_2$ (and *only* for this section) the parameter space which satisfies all these restrictions is not empty (see Table 1) and, therefore, the equilibrium described in Proposition 1 exists.

a	s_1	s_2	A_0	A_1	$p_1^m; p_2^m$	$\Pi_1^m; \Pi_2^m$	p_1^t	$\widehat{p}_1^t$	p_2^t	$\Pi_1^t; \Pi_2^t$	CS^m	CS^t
0	38	160	10	4.6	99.6; 170.8	31.5; 11.1	100	52.2	147.1	31.7; 1.4	29.9	41.9
0	20	160	10	3.2	96.6; 178.3	37, 6; 14.3	100	42.2	151.1	41.6; 3.7	25.4	37.5
0	74	147	10	2.4	98.0; 140.6	14.8; 2.6	100	69.6	126.4	16.3; 1.2	45.2	51.4
0	34	156	20	4.6	96.6; 167.8	21.5; 1.1	100	49.2	144.1	23.4; 1.4	30.9	41.1
0.2	84	168	10	2.2	106.4; 166.6	12.4; 9.6	116.8	77.4	152.1	18.0; 5.9	62.2	82.4

Table 1. Equilibria ($v = 100, c = 0.75$).