ECONOMICS BULLETIN

Models of labour demand with fixed costs of adjustment: a generalised tobit approach

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Abstract

Traditional models of factor demand rely upon convex and symmetric adjustment costs: however, the fortune of this highly restrictive model is due more to analytical convenience than to actual empirical relevance. In this note we first examine the model of employment adjustment under the more realistic hypothesis of fixed costs, show that it can be cast in the form of a Double Censored Random Effect Tobit Model, derive its likelihood function, and finally evaluate the empirical performance of the ML estimators through a Monte Carlo experiment. The performances, although strongly dependent on the degree of censoring, appear promising.

Corresponding author: F. Di Iorio, diiorio@unina.it. We have much benefited from discussions with Erich Battistin, Andrea Gavosto, Paolo Sestito and Alessandro Sembenelli, but of course we retain the full responsabilty of all errors. S. Fachin wishes to thank Paul Ryan for his kind hospitality when visiting the Faculty of Economics of the University of Cambridge, where this paper has been drafted. Financial support from the Department of Statistics of the University of Naples Federico II, MIUR and CNR is gratefully acknowledged.
Citation: Di Iorio, Francesca and Stefano Fachin, (2004) "Models of labour demand with fixed costs of adjustment: a generalised tobit approach." *Economics Bulletin,* Vol. 3, No. 31 pp. 1−8
Submitted: July 9, 2004. **Accepted:** September 16, 2004.
URL: http://www.economicsbulletin.com/2004/volume3/EB−04C50002A.pdf

1 Introduction

Traditional models of factor demand rely upon convex and symmetric adjustment costs: the marginal cost of varying the quantity of an input used in the production process is assumed (i) to increase with the size of the adjustment, and (ii) to be the same for positive and negative changes. Adjustment costs of this shape can generate the partial adjustment dynamics assumed in much empirical work. However, the fortune of this highly restrictive model is due more to analytical convenience than to actual empirical relevance: while convex costs implies frequent changes of small size, in practice micro behavior is often characterised by lumpy adjustment, with rare large changes and many periods of inaction (cf.*e.g.*, Davis and Haltiwnger, 1991, Gavosto and Sestito, 1993). Consequently, over the last decade there have been several efforts to explore alternative models. In this note we first examine the model of employment adjustment under the hypothesis of fixed costs, show that it can be cast in the form of a Double Censored Random Effect Tobit Model and derive its likelihood function (section 2), and then evaluate the empirical performance of the ML estimators through a small Monte Carlo experiment (section 3). Some Conclusions are finally drawn (section 4).

2 Modelling Employment Adjustment with Fixed Costs

Consider a single firm i operating with the objective to maximise the discounted expected value of future profits. In presence of fixed costs this implies that the adjustment of labour inputs (L) is carried out if the shadow value of the marginal worker, *i.e.* if the cost of not adjusting (c_{NA}), exceeds in absolute value the fixed cost of the adjustment (c_A), either because hiring costs are smaller than expected revenues net of wage costs or firing costs are smaller than expected losses (we are assuming symmetry of the costs in order to simplify notation, but the extension to non-symmetric costs is trivial). Define as target employment (L^*) the labour inputs needed in order to deliver the desired amount of output in standard operating conditions (no extra-time nor labour hoarding), and measure disequilibrium with the difference between non-adjustment and adjustment costs. Under the assumption of static expectations or, equivalently, rational expectations and target employment following a random walk, a sufficiently large disequilibrium in period t will cause an immediate shift to the new target level of employment (cf. Hammermesh and Pfann, 1996):

$$L_{it+s} = \begin{cases} L_{it-1} & if \quad |c_{At}| > |c_{NAt}| \\ L^*_{it} & else \end{cases} \qquad s = 0, 1, \dots \tag{2.1}$$

The cost of not adjusting is the present value of the difference of the future streams of net total revenues delivered by L^*_{it} and L_{it-1} workers. Define revenues (R), costs per worker

(W) and net revenues $\overline{R} = R - W$. Then, assuming costs and revenues are linear we have that $c_{NAt} = \sum_{s>0}(\frac{1}{1+r})^s \overline{R}(L^*_{it} - L_{it-1}) = \frac{(1+r)}{r}\overline{R}(L^*_{it} - L_{it-1})$, where r is the relevant discount rate, and thus (Hamermesh,1989):

$$L_{it+s} = \begin{cases} L_{it-1} & if \ \ |L^*_{it} - L_{it-1}| < \Delta_{\min} \\ L^*_{it} & else \end{cases} \qquad s = 0, 1, \ldots \tag{2.2}$$

where now the threshold is $\Delta_{\min} = c_A(\frac{1+r}{r}\overline{R})^{-1}$. The target employment can be written as a static function of a set of explanatory variables (*e.g.*, output and costs), say $L^*_{it} = f(x_{it}, \beta, \epsilon_{it})$, as all the intertemporal effects are assumed to be embedded in the adjustment rule (in other words, we are defining target employment as the long-run equilibrium value of employment conditional on the $x's$). For estimation purposes it is convenient to rewrite model (2.2) in terms of the target relative change in employment; defining Δ as the log difference operator, $\Delta^* L_{it} = \ln(L^*_{it}) - \ln(L_{it-1})$. Under the assumption that the firm i in period $t-1$ is in equilibrium, the target change is a function of the log differences of the explanatory variables; dropping the simplifying assumption of symmetric costs and rearranging further we obtain:

$$\Delta^* L_{it} = \Delta x_{it}\beta + \epsilon_{it} \tag{2.3}$$

$$\Delta L_{it} = \begin{cases} \Delta^* L_{it} & if \ \ \Delta^* L_{it} \leq \theta^-_i \\ 0 & if \ \ \theta^-_i < \Delta^* L_{it} < \theta^+_i \\ \Delta^* L_{it} & if \ \ \Delta^* L_{it} \geq \theta^+_i \end{cases} \tag{2.4}$$

where $\Delta L_{it} = \ln(L_{it}) - \ln(L_{it-1})$ is the actual relative change in employment, $x_{it} = (x_{1it}, ...x_{kit})$ are k exogenous variables (per-worker return, factor inputs and so on) observed on each firm i in $t = 1, \ldots, T$, $i = 1, \ldots, n$, and $\beta = (\beta_1, ...\beta_k)$ is the coefficient vector. Two remarks on model (2.4) are in order: (*i*) since this type of model is typically used for panels with a small T, it can also be thought of as a local linearisation of a more complex non linear labour demand function; (*ii*) for our purposes, it is best seen as Double Censored Tobit Model, with the thresholds (θ^+, θ^-) not known. Indeed, these can be conceived as variables likely to depend upon a large number of factors, both economy-wide (*e.g.*, economic cycle) and firm-specific (*e.g.*, size, technology and state of industrial relations), as in the *threshold regression model* (Dagenais, 1975). A different way to take into account the unobserved heterogeneity is to consider a general Chamberlain-like model (for a discussion see Wooldridge, 2002, p. 541). Let (*a*) $\overline{\Delta x}_i$ be the vector of the average over time of the determinants, (*b*) ν^-_i and ν^+_i be independent idiosyncratic noises satisfying $\nu^-_i|\Delta x_i \sim N(0, \sigma_{\nu^-})$, $\nu^+_i|\Delta x_i \sim N(0, \sigma_{\nu^+})$. Then, we can assume $\theta^-_i = \gamma^-\overline{\Delta x}_i + \nu^-_i < 0$, $\theta^+_i = \gamma^+\overline{\Delta x}_i + \nu^+_i > 0$. If x_{it} contains a time-constant variable to make identification possible we assume that its coefficient in γ^+ and γ^- is zero. Model (2.4) may be rewritten as a "friction model". The standard form of these class of models (Rosett, 1959) is:

$$y_i = \begin{cases} y^*_i - \alpha_2 & if \ \ y^*_i > \alpha_2 \\ 0 & if \ \ \alpha_1 < y^\star_t < \alpha_2 \\ y^*_i - \alpha_1 & if \ \ y^*_i < \alpha_1 \end{cases} \tag{2.5}$$

where $y_i^* = x_i\beta + \epsilon_i$, and the thresholds $\alpha_1(< 0)$ and $\alpha_2(> 0)$ are known. The log-likelihood function can be found in Maddala (1983). Rearranging (2.4) according to (2.5) we obtain an extension of the Random Effect Tobit model (Wooldbridge, 2002, p. 541) in case of double censoring "at random". Then, under assumptions (a) and (b), the model assumes the following expression:

$$\begin{aligned} \Delta^* L_{it} &= \Delta x_{it}\beta + \epsilon_{it} \\ \Delta L_{it} &= \begin{cases} \Delta^* L_{it} - \theta_i^- & if \;\; \Delta^* L_{it} \leq \theta_i^- \\ 0 & if \;\; \theta_i^- < \Delta^* L_{it} < \theta_i^+ \\ \Delta^* L_{it} - \theta_i^+ & if \;\; \Delta^* L_{it} \geq \theta_i^+ \end{cases} \end{aligned} \tag{2.6}$$

where the noise ϵ satisfies $(\epsilon_{it} \mid \nu_i^-, \nu_i^+, \Delta x_i) \sim N(0, \sigma_\epsilon)$, for $i = 1, ..., n$ and $t = 1, ...T$. The log-likelihood essentially combines the structure of those of Random Effect Tobit and friction models, and assuming that the first differences $(\Delta L_{i1}, \Delta L_{i2}, ...\Delta L_{iT})$ are mutually indipendent conditional on $(\Delta x_i, \theta_i^+, \theta_i^-)$, it has the following expression:

$$logL(\psi|\Delta L, \Delta x_i, a_i, b_i) = \sum_{i=1}^{n} logF_i \tag{2.7}$$

where $\psi = (\beta, \gamma^-, \gamma^+, \sigma_\epsilon, \sigma_a, \sigma_b)$ is the vector of parameters of interest,

$$\begin{aligned} &F_i(\Delta L_i|\Delta x_i, \nu_i^-, \nu_i^+; \psi) = \\ &\int\int \left[\prod_{t=1}^{T} f_t(\Delta L_{it}|\Delta x_i, \nu_i^-, \nu_i^+; \psi)\right] \frac{1}{\sigma_{\nu_i^-}}\phi\left(\frac{\nu_i^-}{\sigma_{\nu_i^-}}\right) \frac{1}{\sigma_{\nu_i^+}}\phi\left(\frac{\nu_i^+}{\sigma_{\nu_i^+}}\right) d\nu_i^- d\nu_i^+ \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} f_t(\Delta L_{it}|\Delta x_i, \nu_i^-, \nu_i^+; \psi) = \;& \prod_{\Delta L_{it}<0} \frac{1}{\sigma_\epsilon}\phi\left(\frac{\Delta L_{it} - \Delta x_{it}\beta + \gamma^-\overline{\Delta x}_i + \nu_i^+}{\sigma_\epsilon}\right) \\ & \prod_{\Delta L_{it}=0} \left[\Phi\left(\frac{\gamma^+\overline{\Delta x}_i + \nu_i^- - \Delta x_{it}\beta}{\sigma_\epsilon}\right) - \Phi\left(\frac{\gamma^-\overline{\Delta x}_i + \nu_i^- - \Delta x_{it}\beta}{\sigma_\epsilon}\right)\right] \\ & \prod_{\Delta L_{it}>0} \frac{1}{\sigma_\epsilon}\phi\left(\frac{\Delta L_{it} - \Delta x_{it}\beta + \gamma^+\overline{\Delta x}_i + \nu_i^-}{\sigma_\epsilon}\right). \end{aligned} \tag{2.9}$$

The partial effect can be evaluated at the mean value taking the first derivative of

$$m(\hat{\gamma}^-\overline{\Delta x}_i - \Delta x_{it}\hat{\beta}, \hat{\sigma}^2) + m(\Delta x_{it}\hat{\beta} - \hat{\gamma}^+\overline{\Delta x}_i, \hat{\sigma}^2)$$

where $m(c, \sigma^2) = \Phi(c/\sigma)c + \sigma\phi(c/\sigma)$ with respect to the elements of x.

3 Monte Carlo Experiment

To evaluate the empirical performance of the ML estimates we conducted a small Monte Carlo experiment. As already mentioned above, the empirical analysis we are mimicking are typically based on panels with a small time dimension, here fixed at $T = 3$; further, we also make the computationally convenient assumption of a small cross-section dimensions, namely $n = 100$. Hence, in the labour demand function we may assume (i) a constant rate of technical progress; (ii) market conditions homogeneous across firms. Thus,the Monte Carlo Data Generating Process (DGP) is based on model (2.6). The target change in employment is assumed to be a function of two explanatory variables, which may be taken to represent respectively output growth (Δx_1, generated as a standard normal deviate) and wage costs growth (Δx_2, generated as a log normal deviate). We consider two different treshold specifications:

1. $\theta_i^+ = \gamma_0^+ + \nu_i^+$ and $\theta_i^- = \gamma_0^- + \nu_i^-$;
2. $\theta_i^+ = \gamma_0^+ + \gamma_1^+ \overline{\Delta x_{1i}} + \nu_i^+$ and $\theta_i^- = \gamma_0^- + \gamma_1^- \overline{\Delta x_{1i}} + \nu_i^-$;

In case (1) the tresholds are assumed to fluctuate randomly across firms around an unknown mean value, while in case (2), following a Chamberlain-type approach, to be a stochastic function of the average over time of one of the explanatory variables of the target change in employment. Case (1), although obviously not as general as case (2), is worth examining in that it represent a first advance with respect to the standard employment demand models based on convex costs. In case (2) the thresholds are explicitly modelled, but as they are assumed to depend on the same explanatory variables of target employment, the approach is clearly still somehow restrictive. However, the fully general case is technically and computationally extremely heavy, and has thus been left for future research. Since the degree of censoring is likely to affect heavily the performance of the estimators, for each experiments we considered a Low Censure case, with a fraction of censored observation averaging about 25%, and High Censure case, where on the average around 60% of the observations are censored in each period. Thus, in the former case about $3 \times 100 \times 0.75 = 225$ employment changes are observed overall, while in the latter case this figure drops to only about $3 \times 100 \times 0.40 = 120$ changes. The design parameter governing the degree of censoring is the constant in the treshold expression, as obviously the higher this is in absolute value, the higher the will be the degree of censoring. Finally, given also that a number of pilot experiments had to be run, the number of Monte Carlo replications has been fixed at 100 in order to keep within reasonable limits both the computational cost and the calendar time required (on a fast UNIX workstation with programs written in GAUSS, the simulations for the case (1) and (2) require just over 12 and 22 hours respectively).
Let us now discuss the results. The Monte Carlo means and standard errors of the estimates for case (1), i.e. with the tresholds assumed to fluctuate randomly across firms around unknown constants, and case (2), when they are assumed to depend on $\overline{\Delta x_1}$, are reported respectively in Table 1 and 2. In both cases the data are assumed to be standardised so that $\sigma_\epsilon = 1$.

As expected, the performance of the estimators is not entirely satisfactory in the High Censure case both in terms of distance of the point estimates to the DGP values and dispersion. However, the confidence intervals always include the DGP values, while generally excluding zero (exceptions to the latter statement are the estimates of γ_1^+ and γ_1^- with the more general model for the thresholds and high censure, table 2). Given the difficulty of the task, it is thus fair to conclude that the ML estimators manage to fall always reasonably close to the DGP values. Much better results in high censoring cases can be expected with the larger sample sizes typically available in microeconometric studies, difficult to handle in the context of a Monte Carlo experiment but fully feasible in a single estimation run (the computational cost of our n=100, 100 Monte Carlo replications is roughly comparable to the cost of a single estimation with n=10,000).

Table I

	Low Censure (25%)		High Censure (60%)	
	DGP	Estimates (*Monte Carlo s.e.*)	*DGP*	Estimates (*Monte Carlo s.e.*)
β_1	0.40	0.431 (0.051)	0.40	0.573 (0.107)
β_2	−0.30	−0.309 (0.024)	−0.30	−0.393 (0.047)
γ_0^+	0.40	0.477 (0.118)	1.40	1.301 (0.168)
γ_0^-	−0.20	−0.176 (0.066)	−1.20	−1.171 (0.128)
σ_{ν^+}	0.90	1.016 (0.138)	0.90	0.847 (0.225)
σ_{ν^-}	0.70	0.716 (0.112)	0.70	0.882 (0.161)

target change: $\Delta^* L_{it} = \beta_1 \Delta x_{1t} + \beta_2 \Delta x_{2t} + \epsilon_{it}$;
actual change: $\Delta L_{it} = 0$ if $\theta_i^- < \Delta^* L_{it} < \theta_i^+$;
thresholds: $\theta_i^+ = \gamma_0^+ + \nu_i^+, \theta_i^- = \gamma_0^- + \nu_i^-$;
$T = 3, n = 100$;
censure: average fraction of censored observations ($\Delta L_{it} = 0$).

Table II

	Low Censure (25%)		High Censure (62%)	
	DGP	Estimates (*Monte Carlo s.e.*)	*DGP*	Estimates (*Monte Carlo s.e.*)
β_1	0.40	0.424 (0.063)	0.40	0.566 (0.114)
β_2	-0.30	-0.312 (0.038)	-0.30	-0.402 (0.049)
γ_0^+	0.40	0.430 (0.112)	1.40	1.447 (0.213)
γ_1^+	0.30	0.347 (0.117)	0.30	0.218 (0.150)
γ_0^-	-0.20	-0.179 (0.082)	-1.20	-1.198 (0.163)
γ_1^-	-0.20	-0.201 (0.085)	-0.20	-0.145 (0.105)
σ_{ν^+}	0.90	1.034 (0.209)	0.90	0.936 (0.269)
σ_{ν^-}	0.70	0.717 (0.154)	0.70	0.865 (0.162)

target change: $\Delta^* L_{it} = \beta_1 \Delta x_{1t} + \beta_2 \Delta x_{2t} + \epsilon_{it}$;
actual change: $\Delta L_{it} = 0$ if $\theta_i^- < \Delta^* L_{it} < \theta_i^+$;
thresholds: $\theta_i^+ = \gamma_0^+ + \gamma_1^+ \overline{\Delta x}_1 + \nu_i^+, \theta_i^- = \gamma_0^- + \gamma_1^- \overline{\Delta x}_1 + \nu_i^-$;
$T = 3, n = 100$;
censure: average fraction of censored observations ($\Delta L_{it} = 0$).

4 Conclusions

In this note we derived the likelihood function of a model of employment adjustment under the hypothesis of fixed costs cast in the form of a Double Censored Random Effect Tobit Model. The key difference between the two models is the status of the censoring thresholds, which are assumed known in the latter but not in the former (in fact, modelling them as a function of a set of explanatory variables may be an important part of the analysis). Hence, a crucial consequence of adopting the friction model approach is that the thresholds, or rather their expressions in terms of the chosen set of explanatory variables, appear as arguments of the Gaussian distribution function in the likelihood. As there is no reason why the thresholds should be symmetrical or the random noise to be the same for both thresholds (indeed, the opposite is likely to hold), double integrals are involved in the computation of the likelihood and the hypothesis that the idiosyncratic noises ν_i^- and ν_i^+ are independent turns out to be crucial. Although at a first sight this assumption may be appear to be very strong, this may not be the case if we take a vector of explanatory variables general enough to actually explain all the systematic variability of the thresholds across firms (very much the same applies to the question of the dynamic structure of these errors). From the empirical point of view, the computational point about the double integrals can be more important, as in practice maximum likelihood estimates may be difficult to obtain with the large sets of individual data ideally to be used for this type of models. Monte Carlo experiments with standard maximisation techniques and a small sample size have given encouraging and interesting results, which can be summarised as follows: (*i*) the ML estimates do fall reasonably close to the DGP values; (*ii*) the quality of the results depends much more heavily than commonly assumed on the degree of the censoring. Directions for future research include modelling the variances of the $\nu's$ across firms and assessing the performance of alternative estimation strategies, as suggested by Hajivassilou and Ruud (1994) for the Limited Dependent Variable models.

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