

E C O N O M I C S   B U L L E T I N

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## Approximation bias in estimating risk aversion

Joseph G. Eisenhauer  
*Canisius College*

### *Abstract*

The asymmetric approximation originally employed by Pratt (1964) to construct reduced-form measures of risk aversion creates a downward bias when used for empirical estimation. Calculations based on recent survey data indicate that estimates from a symmetric approximation are generally three times larger than their asymmetric counterparts, a finding that may help to explain the equity premium puzzle.

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## 1. Introduction

One of the most widely cited papers in the economics of uncertainty is John W. Pratt's (1964) "Risk Aversion in the Small and in the Large". Along with Kenneth J. Arrow's (1971) *Essays in the Theory of Risk Bearing*, it lays the foundation for all subsequent analyses of risk taking and risk avoidance by defining and interpreting measures of absolute and relative risk aversion (or risk loving), based on the concavity (or convexity) of the utility function. Of course, because utility is unobservable, empirical implementation of these measures requires expressing them in reduced form as a function of measurable quantities—specifically, risk and the risk premium. In this regard, however, Pratt's measures are biased: given the level of risk and the risk premium, his approximation underestimates risk aversion (or love of risk), and the extent of the bias increases as individuals diverge from risk neutrality. The present note reexamines Pratt's procedure, and then estimates the magnitude of the approximation bias using recent survey data.

## 2. The asymmetric approximation

Pratt (1964) considers the following problem. An individual with a von Neumann-Morgenstern utility function  $U$  and an initial wealth endowment of  $x$  faces a risk denoted by  $\tilde{z}$ . The expected value of the risk is  $E(\tilde{z}) = \mu$ , and the risk may represent a favorable, fair, or unfavorable gamble depending upon the sign (positive, zero, or negative, respectively) of the expected value.<sup>1</sup> The variance of  $\tilde{z}$  around its expected value is  $\sigma^2 = E\{(\tilde{z} - \mu)^2\}$ , and the individual is indifferent between accepting the risk and receiving an additional endowment equal to  $\mu - \pi$ , where  $\pi$  is the risk premium;  $\pi$  may be positive, zero, or negative depending upon whether the individual is risk averse, risk neutral, or risk loving, respectively. Thus,  $\mu - \pi$  is the certainty equivalent of the risk, and the compensated utility of certain wealth equals the expected utility from the gamble:

$$U(x + E(\tilde{z}) - \pi) = E\{U(x + \tilde{z})\}. \quad (1)$$

To investigate the curvature of a tiny segment of the utility function when the risk is small, Pratt constructs a Taylor series expansion of utility around  $y = x + \mu$  on either side of (1), obtaining

$$U(y) - \pi U'(y) + O(\pi^2) = U(y) + E(\tilde{z} - \mu)U'(y) + .5\sigma^2 U''(y) + o(\sigma^2) \quad (2)$$

where  $O(\pi^2)$  denotes a remainder containing second-order terms, and  $o(\sigma^2)$  is a remainder without second-order terms. Assuming the remainders are of negligible magnitude, rearranging (2) yields the approximate relationship

$$\pi = .5\sigma^2 r(y) \quad (3)$$

where  $r(y) = -U''(y)/U'(y)$  denotes the measure of absolute risk aversion evaluated at the expected value of final wealth. "Thus," Pratt (1964, p. 125) notes, "the risk premium for a risk

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<sup>1</sup> Pratt (1964) focuses primarily on actuarially neutral risks, and briefly mentions risks with nonzero expected values. In order to facilitate our empirical estimation, we consider the more general form (positive, zero, or negative expected value), of which  $\mu = 0$  is a special case.

$\tilde{z}$  with arbitrary mean  $E(\tilde{z})$  but small variance is approximately  $r(x + E(\tilde{z}))$  times half the variance of  $\tilde{z}$ .” Equivalently, the measure of absolute risk aversion can be isolated as

$$r(y) = 2\pi / \sigma^2. \quad (4)$$

Clearly,  $r(y)$  should be of the same sign as  $\pi$ , but because the risk premium is measured in the same units as wealth while the variance is measured in the square of those units, absolute risk aversion is sensitive to the units in which wealth is measured, whereas relative (or proportional) risk aversion, defined as  $yr(y)$ , is not.

In practice, of course, an individual’s utility function is unobservable, but risk and the premium one demands in exchange for risk may be more readily quantified. Thus, one of the major applications of Pratt’s analysis is the estimation of  $r(y)$  from observable data. Indeed, equations (3) and (4) continue to be widely employed as the basis for empirical research; see for example Blake (1996), Hartog et al. (2002), and Kirkwood (forthcoming). In addition, they are used in actuarial models (see for example, Frees (1998)), and they pervade the reference and pedagogical literatures (see for example, Laffont (1989) and Gollier (2000)).

Pratt’s derivation of (2), however, is oddly asymmetric: it involves a first-order Taylor series expansion on the left-hand side (LHS) of (1), but a second-order expansion on the right-hand side (RHS) of (1). That is, the derivation explicitly recognizes the existence of  $U''(y)$  on one side but not the other. Thus, the approximation above is partial at best, even if the third and higher derivatives of utility are indeed negligible so that we may safely disregard them.<sup>2</sup> Specifically, another  $U''(y)$  term—central to the definition of risk aversion—is implicitly subsumed in the  $O(\pi^2)$  remainder on the left-hand side of (2). Consequently, equations (3) and (4) are biased even when  $U'''(y) = 0$ .

Indeed, if the imbalance were reversed, so that a second-order polynomial expansion were constructed on the LHS of (1) with a linear expansion on the RHS, the result would be

$$U(y) - \pi U'(y) + .5\pi^2 U''(y) + o(\pi^2) = U(y) + E(\tilde{z} - \mu)U'(y) + O(\sigma^2), \quad (2')$$

yielding, if both remainders were negligible,  $\pi = -2/r(y)$ ; but this would clearly lack a sensible interpretation, as the risk premium would evidently be independent of the risk (as measured by the variance), and would be negative for any positive degree of risk aversion.

### 3. A symmetric approximation

A more balanced or symmetric expansion of (1)—a second-order approximation explicitly recognizing  $U''(y)$  terms on both sides—would yield the following expression:

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<sup>2</sup> The third derivative of utility is well known to determine the demand for precautionary saving (Leland, 1968) and thus has an important place in the theory of uncertainty, but its empirical magnitude has not been widely studied. Dynan (1993) found the coefficient of absolute prudence,  $\eta(y) = -U'''(y)/U''(y)$ , to be near zero; see Merrigan and Normandin (1996), Eisenhauer (2000), and Eisenhauer and Ventura (2003) for other attempts to measure prudence.

$$U(y) - \pi U'(y) + .5\pi^2 U''(y) + o(\pi^2) = U(y) + E(\tilde{z} - \mu)U'(y) + .5\sigma^2 U''(y) + o(\sigma^2) \quad (5)$$

where neither remainder contains second-order terms. Rearranging (5) yields a fundamentally different, nonlinear relationship between risk aversion and the risk premium. Assuming the  $o(\pi^2)$  and  $o(\sigma^2)$  remainders are both negligible, the risk premium in (5) solves the quadratic

$$.5r_s(y)\pi^2 + \pi - .5\sigma^2 r_s(y) = 0; \quad (6)$$

where the subscript  $s$  indicates the use of a symmetric second-order expansion to approximate absolute risk aversion.<sup>3</sup> Alternatively, for a given risk premium, absolute risk aversion can be approximated by

$$r_s(y) = \frac{2\pi}{\sigma^2 - \pi^2}. \quad (7)$$

Indeed, it can easily be shown that (7) gives the *exact* value of absolute risk aversion if the third derivative of utility is nonexistent—i.e., if utility is quadratic; see (9) below, when  $\eta(y) = 0$ .<sup>4</sup>

In order for  $r_s(y)$  and  $\pi$  to have the same sign, it is necessary that  $0 < \pi < \sigma$  among risk averters and  $0 < -\pi < \sigma$  among risk lovers. Certainly among risk averters,  $\pi < \sigma$  appears to be a reasonable restriction. Indeed, if this condition were violated, the risk averter would be willing to pay an insurance premium in excess of the potential loss. To see this, consider a potential loss of  $L$  having probability  $p$ , and a  $(1 - p)$  probability of no loss. Then the expected loss is  $E(L) = pL$ , the variance is  $\sigma^2 = p(1 - p)L^2$ , and  $\sigma = L\sqrt{p(1 - p)}$ . By definition, the insurance premium  $\pi_I$  that an individual is willing to pay to transfer risk is equal to the expected loss plus the risk premium:  $\pi_I = pL + \pi$ . Now if  $\pi \geq \sigma$ , then  $\pi_I \geq pL + \sigma$ , so  $\pi_I \geq L[p + \sqrt{p(1 - p)}]$ . But for  $p \geq .5$ , we have  $[p + \sqrt{p(1 - p)}] \geq 1$ . Thus, if  $\pi \geq \sigma$  and  $p \geq .5$ , then  $\pi_I \geq L$ ; the individual would be willing to pay an insurance premium that equals or exceeds the maximum possible loss, clearly an implausible outcome for any reasonable level of risk aversion. Empirically, the  $\pi^2 < \sigma^2$  condition is upheld for both risk averters and risk lovers in the data used in section 5.

Indeed, the justification for using (4) seems to have been that a small  $\sigma$  implies a small  $\pi$ , so the  $.5\pi^2 U''(y)$  term in (5) may be trivial. But its magnitude, and the distortion of risk aversion caused by its omission, is ultimately an empirical question that has not previously been addressed. We therefore model and measure the extent of the bias in the following sections.

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<sup>3</sup> Because the risk premium must have the same sign as risk aversion, the solution to (6) is

$$\pi = \frac{-1 + \sqrt{1 + r_s^2 \sigma^2}}{r_s}.$$

<sup>4</sup> Pratt (1964) and Arrow (1971) reject quadratic utility because it implies increasing absolute risk aversion (IARA). Some studies, however, have found empirical evidence of IARA; see for example Eisenhauer (1997) and Eisenhauer and Halek (1999).

#### 4. A third-order expansion and the approximation bias

Comparing (4) with (7), it is clear that  $r(y) < r_s(y)$  for all  $0 < \pi < \sigma$ , and  $-r(y) < -r_s(y)$  for all  $0 < -\pi < \sigma$ . If utility is quadratic,  $r_s(y)$  provides an exact measure of absolute risk aversion, and  $r(y)$  is necessarily an underestimate. For all other utility functions, both  $r_s(y)$  and  $r(y)$  are approximations; we shall therefore compare their accuracy by considering an even more general, third-order expansion on each side of (1):

$$\begin{aligned} U(y) - \pi U'(y) + .5\pi^2 U''(y) - (1/6)\pi^3 U'''(y) + o(\pi^3) = \\ U(y) + E(\tilde{z} - \mu)U'(y) + .5\sigma^2 U''(y) + (1/6)E\{(\tilde{z} - \mu)^3\}U'''(y) + o(\sigma^3). \end{aligned} \quad (8)$$

If we now assume that the remainders in (8)—involving fourth and higher-order derivatives of utility—are negligible, then the third-order expansion gives absolute risk aversion as

$$r_3(y) = \frac{2\pi}{\sigma^2 - \pi^2 - (1/3)[\pi^3 + E\{(\tilde{z} - \mu)^3\}]\eta(y)} \quad (9)$$

where the subscript 3 indicates a symmetric, third-order expansion,  $\eta(y) = -U'''(y)/U''(y)$  is the coefficient of absolute prudence as defined by Kimball (1990), and  $E\{(\tilde{z} - \mu)^3\}$ , the third central moment, indicates whether skewness is present in the distribution of risky wealth. Of course, (9) is itself an approximation, but it is clearly more general and, insofar as it contains more information regarding utility, more accurate than either (4) or (7).<sup>5</sup>

Now it is easily seen that (9) reduces to (7) when the third derivative of utility is zero (that is, when utility is quadratic, implying the absence of prudence) or when there is a skew to the distribution of risky wealth such that, by coincidence,  $E\{(\tilde{z} - \mu)^3\} = -\pi^3$ . On the other hand, given a positive risk premium, (9) reduces to (4) only in the unlikely event that

$$\eta(y) = \frac{-3\pi^2}{\pi^3 + E\{(\tilde{z} - \mu)^3\}}. \quad (10)$$

Thus, neither  $r(y)$  nor  $r_s(y)$  is likely to provide an exact representation of absolute risk aversion unless utility is quadratic. However, the comparison of approximations can be simplified by examining an important special case on which Arrow (1971) focused: a symmetric distribution of risk involving equal probabilities of winning or losing. In that case,  $E\{(\tilde{z} - \mu)^3\} = 0$  and (9) becomes

$$r_3(y) = \frac{2\pi}{\sigma^2 - \pi^2 - (1/3)[\pi^3]\eta(y)}. \quad (9')$$

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<sup>5</sup> Hlawitschka (1994) shows that third-order Taylor series expansions of expected utility such as (8) may not be significantly more accurate than second-order expansions such as (5). Thus,  $r_3(y)$  may not be a much better approximation than  $r_s(y)$ . See also Kroll et al. (1984).

Because we examine a gamble with equal probabilities of winning and losing in the empirical estimation below, (9') is the appropriate basis for comparison. Unfortunately, it is not even possible to estimate the magnitude of risk aversion in (9') without knowing the magnitude of absolute prudence. Importantly, however, if  $r_3(y) > 0$  and  $\eta(y) \geq 0$  as usually assumed, then

$$r(y) < r_s(y) \leq r_3(y); \quad (11)$$

Pratt's asymmetric approximation necessarily underestimates risk aversion, and the extent of the bias increases with the magnitude of prudence. Likewise, if  $r_3(y) < 0$  and  $\eta(y) \leq 0$ , then

$$-r(y) < -r_s(y) \leq -r_3(y); \quad (12)$$

Pratt's approximation underestimates the love of risk. Indeed, (11) and (12) show that for all prudent risk averters and imprudent risk lovers, the difference between  $r(y)$  and  $r_3(y)$  is *at least as large* as the difference between  $r(y)$  and  $r_s(y)$ . Consequently, we may treat the latter difference as a conservative estimate of the true approximation bias; expressed as a percentage of (7), this gives<sup>6</sup>

$$\frac{r_s(y) - r(y)}{r_s(y)} = \frac{\pi^2}{\sigma^2}. \quad (13)$$

Note that the restriction  $\pi^2 < \sigma^2$  implies that the ratio in (13) will never equal or exceed 100 percent. Alternatively, the difference can be expressed in terms of the ratio

$$\frac{r_s(y)}{r(y)} = \frac{\sigma^2}{\sigma^2 - \pi^2}. \quad (14)$$

It is clear from (13) and (14) that the magnitude of the approximation bias increases with the square of the risk premium, and therefore increases with the absolute value of risk aversion or risk-loving itself, even when the variance is held constant. Importantly, equations (13) and (14) are independent of the units in which wealth is measured, and consequently, the same ratios could also be interpreted as comparisons of relative risk aversion estimates.

## 5. Empirical estimation

To estimate the approximation bias described above, we make use of the following question, which the Bank of Italy posed in its 1995 Survey of Household Income and Wealth (SHIW).

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<sup>6</sup> Clearly,  $\frac{r_3(y) - r(y)}{r_3(y)} \geq \frac{r_s(y) - r(y)}{r_s(y)}$  for all  $r_3(y) \geq r_s(y) > 0$  and all  $-r_3(y) \geq -r_s(y) > 0$ .

You are offered the opportunity of acquiring a security permitting you, with equal probabilities, either to gain 10 million lire or to lose all the capital invested. What is the most you are prepared to pay for this security?<sup>7</sup>

Certainty-equivalence questions of this type have become popular vehicles for assessing attitudes toward risk; similar questions from different surveys have been used recently by Warneryd (1996), Donkers et al. (1999), Pennings and Smidts (2000), and Hartog et al. (2002), among others. The answer to this particular question, which we denote by  $k$ , is a reservation price, above which the prospective investor rejects the proffered stock. Following Guiso and Paiella (2001), the reservation price is implicitly defined by

$$U(x) = .5U(x - k) + .5U(x + 10), \quad (15)$$

where monetary values are measured in millions of lire.<sup>8</sup> Notice that the certainty equivalent is zero because the respondent is essentially asked to adjust the variance and the expected value of the risk to make it so. Thus  $E(\tilde{z}) = \pi = 5 - .5k$ , and the variance around the expected value is  $\sigma^2 = (5 + .5k)^2$ . Hence,  $0 < \pi < \sigma$  for all  $0 < k < 10$ , and  $0 < -\pi < \sigma$  for all  $k > 10$ ; therefore  $r_s(y)$  and  $\pi$  have the same sign for all  $k > 0$ . And because the probabilities of gaining and losing are equal, we have a symmetric distribution, for which  $E\{(\tilde{z} - \mu)^3\} = 0$ .

Respondents who indicated a reservation price of 10 million lire ( $k = 10$ ) are risk neutral, those for whom  $k < 10$  are risk averse, and those for whom  $k > 10$  are risk-loving. Because monetary values are nonnegative,  $k = 0$  is the lower bound for all responses; thus, even infinitely risk averse individuals report  $k = 0$ , so no single estimate of risk aversion can be made for them. This in itself shows a problem with the asymmetric expansion, since it gives an upper bound of  $r(y) = .4$  for  $k = 0$ , whereas  $r_s(y)$  is undefined at  $k = 0$ . We therefore remove values of  $k = 0$  from our empirical estimation. A similar problem occurs among risk lovers: unlike  $r_s(y)$ ,  $r(y)$  declines as  $k$  rises only until  $k = 30$ , after which,  $r(y)$  rises with  $k$ ; see Table I below.

Of the 3,483 respondents reporting positive reservation prices for the proposed investment, 3,314 or 95.1 percent exhibited risk aversion, another 125 or about 3.6 percent were risk neutral, and the remaining 44 or 1.3 percent displayed a love of risk. After separating risk averters from risk lovers, Table I reports quartile values for  $k$  and the associated values of  $\pi$ ,  $\sigma^2$ ,  $r(y)$  and  $r_s(y)$ ; the differences in risk aversion estimates are calculated according to (13) and (14). To avoid potential measurement problems involved in constructing a wealth variable, we report absolute risk aversion rather than relative risk aversion; but as noted above, the approximation bias measured by either (13) or (14) would be identical for comparisons of relative risk aversion.

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<sup>7</sup> The lira was the unit of currency in Italy prior to conversion to the euro on 1 January 2002. At the time of the survey, the lira had a value of approximately 1/2000 of a U.S. dollar.

<sup>8</sup> Eisenhauer and Ventura (2003) use both this model and an alternative model based on a slightly different interpretation of the survey question to estimate risk aversion, but the results they obtain are virtually identical in the two cases. In contrast to Pratt (1964), both Guiso and Paiella (2001) and Eisenhauer and Ventura (2003) expand utility around initial wealth ( $x$ ) rather than the expected value of final wealth ( $x + \mu$ ).

As the model predicts, the approximation bias increases as individuals diverge from risk neutrality ( $k = 10$ ). Among all respondents exhibiting positive degrees of risk aversion, the asymmetric approximation yields a median value of  $r(y) = .2975$ , while the symmetric approximation yields a median value of  $r_s(y) = .90$ , three times larger than the former. Thus, at the median, the approximation bias is about 67 percent. (The mean value of  $[r_s(y) - r(y)]/r_s(y)$ , not reported in Table I, is 60 percent.) The bias has an interquartile range from 29 percent to 92 percent. Among risk lovers, the approximation bias has an interquartile range from 11 percent to 42 percent and a median value of 25 percent (as well as a mean value of 26.5 percent). It bears repeating that these are conservative estimates of the approximation bias, inasmuch as they assume no prudence, and infinitely risk averse agents were removed from the sample.<sup>9</sup>

## 6. Conclusion

All approximations are, by definition, inaccurate; but some are more accurate than others. Ludvigson and Paxson (2001) have recently shown that empirical studies of consumption and saving patterns based on linearized Euler equations involve substantial approximation bias—typically resulting in estimates of risk aversion that are only 12 to 60 percent of the correct values. In a similar manner, the present study suggests that empirical estimates of risk aversion derived from a partly linear and partly polynomial expansion of expected utility are likely to be just as severely distorted. Using recent survey data, we find that among risk averters, Pratt's reduced-form approximation yields estimates of risk aversion that are, at the median, no more than one-third of the correct values. This has important implications for understanding economic behavior under uncertainty. In particular, the equity premium puzzle first noted by Mehra and Prescott (1985) has confounded researchers for two decades because the observed premium has historically been sufficiently large to justify widespread stock ownership among individuals unless risk aversion is roughly three times higher than commonly believed; see Kocherlakota (1996) for a survey. Our results show that when correctly estimated, risk aversion may indeed be that much larger than conventional estimates indicate. This suggests that numerical simulations and empirical estimates designed to predict behavior under uncertainty should replace the familiar linear relation between risk aversion and the risk premium with an equally simple but more accurate nonlinear relationship derived from a symmetric expansion of expected utility.

It may also be the case that the risks encountered in practice, including those to which the asymmetric approximations have been applied in empirical research, are simply too large for risk aversion in the small to adequately address. The present work may therefore also be of interest to theorists seeking a reduced-form measure of risk aversion in the large that would generalize the measure of risk aversion in the small. The analysis above suggests that a more general measure should reduce to  $r_s(y)$  rather than  $r(y)$ , as the level of risk becomes infinitesimal.

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<sup>9</sup> The Bank of Italy asked essentially the same question on the 2000 SHIW. Out of 1,173 positive responses, all exhibited risk aversion. The median reservation price was  $k = .40$ , implying  $r_s(y) = 2.4$  and  $r(y) = .355$ , and giving a median underestimate of 85 percent. Because the newer sample is smaller and includes no risk lovers, we focus on the 1995 survey in Table I.

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**Table I. Estimates of risk aversion and approximation bias**

Quartile	Risk Averters			Risk Lovers		
	Q1	Q2	Q3	Q1	Q2	Q3
<b>k</b>	0.20	1.0	3.0	20.0	30.0	47.5
<b><math>\pi</math></b>	4.9	4.5	3.50	-5.0	-10.0	-18.75
<b><math>\sigma^2</math></b>	26.01	30.25	42.25	225.00	400	826.56
<b>r(y)</b>	0.3768	0.2975	0.1657	-.0444	-.0500	-.0454
<b>r<sub>s</sub>(y)</b>	4.90	0.90	0.233	-.0500	-.0667	-.0789
$\frac{r_s(y) - r(y)}{r_s(y)}$	.9231	.6694	.2899	.111	.25	.4253
<b>r<sub>s</sub>(y)/r(y)</b>	13.005	3.025	1.4083	1.125	1.333	1.7401