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Long horizon regressions with moderate deviations from a unit root

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Abstract

We consider long horizon regressions where the predictor with unknown degree of persistence follows a process of moderate deviations from a unit root. Some asymptotic properties of OLS estimator and of the t statistic are presented.

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1. Introduction

Long horizon regression models have been popular in economics and finance, particularly in the context of return predictability (Ang and Bekaert (2001), Fama and French (1988), Campbell and Shiller (1988) to name a few). In this paper, we consider the predictive regression model where the predictor have unknown degree of persistence. In doing so, we assume that the predictor follows a process of moderate deviation from a unit root (Phillips and Magdalinos (2005)). Moderate deviations process can generate varying degree of persistence, depending on how far the predictor deviates from unit root. In addition, as we construct long horizon variables by taking rolling summation of the regressand and the regressor, it is known that the long horizon variables are nothing but the partial sum processes. Thus, one can view this long horizon regression as the regression of non-stationary regressand on non-stationary predictor.

Given the above setup, we present asymptotic distributions of the OLS estimator and of the t statistic. Standard functional limit theories are applied to obtain the desired convergences (e.g., Phillips and Solo (1992), Valkanov (2003)). It is found that the OLS estimator is consistent only when both the regressor and the regressand are overlapped. In such case, the convergence rate depends on the deviation parameter of the predictor, which determines the degree of persistence. Also, we find that the t test statistic needs to be normalized by $T^{1/2}$, to have a well-defined limit, which results from a property of spurious regression. We provide a brief simulation studies, and investigate the effects of persistence on the finite sample bias of the OLS estimator.

2. Long Horizon Predictive Regression Model

We write a short horizon predictive regression,

$$y_{t+1} = \alpha + \beta x_t + u_{t+1}, \tag{1}$$

where y_{t+1} is the regressand at $t+1$, and x_t is the predictor at t. We assume that u_t follows martingale difference or i.i.d. sequence (e.g., Campbell and Shiller (1988), Hodrick (1992), Ang and Bekaert (2001)). This conventional assumption implies that, for example, the regressand y_t is short memory process under the null of no predictability, $\alpha = \beta = 0$. On the other hand, the predictor variable x_t exhibits a certain degree of persistence. Typical examples of such predictor include the dividend-price ratio or the yield spread in the bond market (e.g., logarithm of the long rate divided by the short rate). As the predictor carries unknown degree of persistence, we assume that x_t is a process of moderate deviation from a unit root (Phillips and Magdalinos (2005))

$$x_t = (1 - \frac{c}{T^\alpha})x_{t-1} + v_t, \quad c < 0,\ \alpha \in (0,1), \tag{2}$$

where v_t is i.i.d. sequence. The parameter α governs the degree of persistence. As α gets close to zero, x_t becomes stationary AR(1) process. When α approaches to one, x_t behaves as a local to unity process. In the context of testing predictability, the predictor x_t is often modelled as a local to unity process (e.g., Valkanov (2003), Rossi (2005)).On the other hand, (2) provides a flexible model for unknown persistence in the predictor, where the local to unity process is a special case of moderate deviations for x_t. For the detailed asymptotic limit theories regarding the correlation coefficient $\rho = (1 - c/T^\alpha)$ in (2), see Phillips and Magdalinos (2005).

We state the assumption in (1) and (2).

Assumption 1: Given the model (1) and (2),
$z_t = \begin{pmatrix} u_t \\ v_t \end{pmatrix}$ is mean-zero i.i.d. and $Ez_t z_t' = \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix}$.

The Assumption 1 is sufficient to apply the functional central limit theorem (e.g., Brown (1971, Theorem 2), Phillips and Solo (1992)).

Here, we note that the short horizon predictive regression in (1) is problematic. First, as the predictor behaves as local to unity process (or as α is close to one), the regression makes little sense due to imbalance of the order of integration between x_t and y_t. Second, it is widely known in empirical studies that the underlying variables in (1) are quite noisy, which typically cause insignificant results of the estimate of β and low value of the R^2. To partly overcome this problem, one can consider a long horizon regression model by aggregating y_t, which can strengthen the signal and make the noise relatively negligible. In doing so, the predictability is interpreted as that of k-th period continuously compounded regressand out of the predictor at the current period. Write

$$y_t^k = \delta + \beta x_t + u_t^k, \tag{3}$$

where $y_t^k = \sum_{i=1}^k y_{t+i}$, and $u_t^k = \sum_{i=1}^k u_{t+i}$.

In order to understand the convergence of the OLS estimator $\widehat{\beta}$, we treat the long horizon variable as the difference in two partial sum processes as in Valkanov (2003), say,

$$u_t^k = \sum_{j=1}^{t+k} u_j - \sum_{j=1}^{t} u_j. \tag{4}$$

Then, we can apply functional limit theorem to obtain convergence results of the partial sum processes. Here, we also assume that the number of overlapping observations grows with T.

Assumption 2: k is a portion of overlapping summations; $k = [\lambda T]$ for $\lambda \in (0, 1)$,

where T is the sample size and $[z]$ is the closest integer to z. In practice, the fraction λ can be chosen as 0.1 or 0.2.

We establish the following lemma useful to deal with some convergences. Denote $\overline{u}_t^k = u_t^k - \overline{u}^k$, where $\overline{u}^k = (T-k)^{-1}\sum_{t=1}^{T-k} u_t^k$.

Lemma 1: Suppose Assumptions 1 and 2 hold under the model (3).

(a) $\sigma_e^{-1}T^{-1/2}u_t^k \to U(r+\lambda) - U(r) = U(r,\lambda)$,

(b) $\sigma_e^{-1}T^{-1/2}\overline{u}_t^k \to U(r,\lambda) - (1-\lambda)^{-1}\int_0^{1-\lambda} U(r,\lambda)dr = U^*(r,\lambda)$,

(c) $\sigma_w^{-1}T^{-\alpha/2}x_t \to J_c(r)$

(d) $\sigma_w^{-1}T^{-\alpha/2}\overline{x}_t \to J_c(r) - (1-\lambda)^{-1}\int_0^{1-\lambda} J_c(r)dr = J_c^*(r)$

where $J_c(r)$ is a linear diffusion process defined as

$$J_c(r) = W(r) + c\int_0^r e^{c(r-z)}dW(z), \text{ for } c < 0,$$

and $(U(r), W(r))'$ are bivariate standard Brownian motions with covariance $\delta = \sigma_{uv}/(\sigma_u\sigma_v)$. Under the null of $\beta = 0$, the lemma 1 (a) and (b) are applied to $\{y_t^k\}$ process. The lemma (c) and (d) make use of the results in Phillips and Magdalinos (2005).

We obtain convergence results for OLS estimator $\widehat{\beta}, t$ statistic, and R^2.

Theorem 1: Suppose Assumptions 1 - 2 hold under the model (3). For $H_0 : \beta = 0$,

(1) $T^{(\alpha-1)/2}\widehat{\beta} \to \left(\sigma_e\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r)dr \Big/ \sigma_w\int_0^{1-\lambda} J_c^*(r)^2dr\right)$,

(2) $T^{-1/2}t \to \left[\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r)dr \,/ G[U^*(r,\lambda), J_c^*(r)]^{1/2}\right]$, where

$G[U^*(r,\lambda), J_c^*(r)] = \left(\int_0^{1-\lambda} U^*(r,\lambda)^2dr\int_0^{1-\lambda} J_c^*(r)^2dr - [\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r)]^2\right)$

(3) $R^2 \to \left[[\int_0^{1-\lambda} U^*(r,\lambda)J_c(r)dr]^2\right]^2 \Big/ \left\{\left[\int_0^{1-\lambda} U^*(r,\lambda)^2dr\right]\left[\int_0^{1-\lambda} J_c^*(r)^2dr\right]\right\}$.

First thing to note is that OLS estimator $\widehat{\beta}$ is not consistent for all values of $\alpha \in (0,1)$. Thus, we need to consider an alternative modelling to obtain consistent estimate, which will be covered in the Theorem 2 below. Second, the t statistic diverges at the rate of $T^{1/2}$, which is expected since the long horizon regressions generate spurious regressions. Thus, we need to consider a scaled test statistic $T^{-1/2}t$ to obtain a well-defined limit, which depends on unknown locality parameter c. For reference, see Valkanov (2003) for detailed analysis. Third, the R^2 does not converge to zero under the null, which is also expected due to the spurious regressions.

Now, we consider a long horizon regressions when both y_t and x_t are overlapped.

$$y_t^k = \delta + \beta x_t^k + u_t^k, \tag{5}$$

where $\overline{x}_t^k = x_t^k - \overline{x}^k$, where $x_t^k = \sum_{i=0}^{k-1} x_{t+i}$, and $\overline{x}^k = (T-k)^{-1} \sum_{t=1}^{T-k} x_t^k$.

In addition to Lemma 1, we further have following convergence results for overlapped predictor.

Lemma 1 (continued):

(e) $\sigma_w^{-1} T^{-(\alpha+2)/2} x_t^k \to \int_r^{r+\lambda} J_c(z)dz \equiv J_c(r,\lambda)$

(f) $\sigma_w^{-1} T^{-(\alpha+2)/2} \overline{x}_t^k \to J_c(r,\lambda) - (1-\lambda)^{-1} \int_0^{1-\lambda} J_c(r,\lambda)dr = J_c^*(r,\lambda)$

Given this, we are led to have convergence results as follows.

Theorem 2: Suppose Assumptions 1 - 2 hold under the model (5). For $H_0 : \beta = 0$,

(1) $T^{(\alpha+1)/2}\widehat{\beta} \to \left(\sigma_e \int_0^{1-\lambda} U^*(r,\lambda) J_c^*(r,\lambda)dr \Big/ \sigma_w \int_0^{1-\lambda} J_c^*(r,\lambda)^2 dr\right)$,

(2) $T^{-1/2}t \to \left[\int_0^{1-\lambda} U^*(r,\lambda) J_c^*(r,\lambda)dr \,/ G[U^*(r,\lambda), J_c^*(r,\lambda)]^{1/2}\right]$, where

$G[U^*, J_c^*]$ is defined as in Theorem 1

(3) $R^2 \to \left[[\int_0^{1-\lambda} U^*(r,\lambda) J_c^*(r,\lambda)dr]^2\right]^2 \Big/ \left\{\left[\int_0^{1-\lambda} U^*(r,\lambda)^2 dr\right]\left[\int_0^{1-\lambda} J_c^*(r,\lambda)^2 dr\right]\right\}$.

It is clear that $\widehat{\beta}$ is consistent for $\alpha \in (0,1)$. The convergence rate depends on α in a sense that the greater the value of α, the faster the convergence rate. Given the correlation between disturbance and the predictor, we expect that as α increases, the finite sample bias decreases. This is shown through simulations in the later section. Convergences of the t statistic and of the R^2 are pretty similar to those in Theorem 1.

3. Simulation

Finally we investigate a finite sample bias of the OLS estimator and empirical distributions of scaled t statistic through simulations. In the long horizon regressions in (5), $\{u_t\}_{t=1}^T$ is generated from $i.i.d.$ $N(0,1)$. The predictor $\{x_t\}_{t=1}^T$ is generated from (2), where $c = -1$ and v_t ~i.i.d. $N(0,1)$. We denote a contemporaneous correlation between u_t and v_t as δ. The values of deviation parameter α are chosen from 0.1 to 0.9 to allow different degree of persistence. Further, the portion of overlapping summation k is set as $k = [0.1T]$ and $= [0.2T]$, to see the effect of k on the finite sample performance. Two sample sizes $n = 200$ and 500 are considered and $5,000$ replications are conducted.

Table 1 shows the finite sample bias of OLS estimator according to different values of α, and of the number of horizon, given $\delta = -0.9$. First, as α increases, the bias reduces, which is well expected from Theorem 2. This bias reduction with increase in α is more pronounced for the larger sample size. Second, the effect of k on the bias is negligible, particularly for larger sample size of $T = 500$.

Next, we present the empirical distributions of the scaled t test $T^{-1/2}t$. In practice, empirical distributions using finite sample are more useful than the asymptotic distributions, specially when there exist correlations between disturbance and the predictor (See Mishkin (1995)). The distributions are simulated under the null of $\beta = 0$ with $k = [0.1T]$ and $T = 500$ in Tables 2. Selective values of percentiles are given according to different values of α and δ. We note some findings. First, when $\delta = 0$, the distribution is shown to be nearly symmetric around zero. As α gets larger, the variance of scaled t test increases, so the distribution becomes more flattened. Second, negative correlation δ shifts the distribution to the negative range as expected. The effect of negative correlations, however, are mitigated as the persistence of the predictor increases. We do not report empirical distributions with other choices of k, δ, and T, which are available upon request.

4. Conclusion

We present some asymptotic properties of OLS estimator and of the test statistic in long horizon regressions when the predictor follows a process of moderate deviation from unity. It is shown that OLS estimator is consistent under the regression of overlapped regressand on the overlapped predictor. In this case, the convergence rate of the estimator depends on how far the predictor variable deviates from unit root process. Our simulation studies verify the theoretical conjecture.

Appendix

Proof of Theorem 1: We write

$$\widehat{\beta}-\beta=\frac{\sum_{t=1}^{T-k}\overline{u}_t^k\overline{x}_t}{\sum_{t=1}^{T-k}(\overline{x}_t)^2}, \tag{A.1}$$

where $\overline{u}_t^k=u_t^k-\overline{u}^k$, $\overline{x}_t=x_t-\overline{x}$ with $\overline{u}^k=(T-k)^{-1}\sum_{t=1}^{T-k}u_t^k$.

Using the lemma 1, we obtain

$$\begin{aligned}\widehat{\beta}-\beta &= \frac{T^{(3+\alpha)/2}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{u}_t^k)(T^{-\alpha/2}\overline{x}_t^k)]}{T^{1+\alpha}[T^{-1}\sum_{t=1}^{T-k}(T^{-\alpha/2}\overline{x}_t)^2]}\\ &= T^{(1-\alpha)/2}\frac{[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{u}_t^k)(T^{-\alpha/2}\overline{x}_t^k)]}{[T^{-1}\sum_{t=1}^{T-k}(T^{-\alpha/2}\overline{x}_t)^2]},\end{aligned} \tag{A.2}$$

thus

$$T^{(\alpha-1)/2}(\widehat{\beta}-\beta)\to\frac{\sigma_e\int_0^{1-\lambda}U^*(r,\lambda)J_c^*(r)dr}{\sigma_w\int_0^{1-\lambda}J_c^*(r)^2dr}, \tag{A.3}$$

under the $H_0:\beta=0$.

For the t statistic, we can write

$$\begin{aligned}t &= \frac{\widehat{\beta}[\sum_{t=1}^{T-k}(\overline{x}_t)^2]^{1/2}}{\left[T^{-1}\sum_{t=1}^{T-k}(\overline{y}_t^k-\widehat{\beta}\overline{x}_t)^2\right]^{1/2}}\\ &= \frac{\widehat{\beta}[\sum_{t=1}^{T-k}(\overline{x}_t)^2]}{\left[T^{-1}\sum_{t=1}^{T-k}(\overline{y}_t^k)^2\sum_{t=1}^{T-k}(\overline{x}_t)^2-T^{-1}(\sum_{t=1}^{T-k}\overline{y}_t^k\overline{x}_t)^2\right]^{1/2}}.\end{aligned} \tag{A.4}$$

Making use of lemma 1 for the denominator, we have

$$t=\frac{T^{(3+\alpha)/2}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)(T^{-\alpha/2}\overline{x}_t)]}{(A_T-B_T)^{1/2}}, \tag{A.5}$$

where

$$\begin{aligned}A_T &= T^{-1}\sum_{t=1}^{T-k}(\overline{y}_t^k)^2\sum_{t=1}^{T-k}(\overline{x}_t)^2\\ &= T^{2+\alpha}\left\{[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)^2][T^{-1}\sum_{t=1}^{T-k}(T^{-\alpha/2}\overline{x}_t^k)^2]\right\},\end{aligned}$$

and

$$\begin{aligned}B_T &= T^{-1}[\sum_{t=1}^{T-k}\overline{y}_t^k\overline{x}_t]^2\\ &= T^{-1}\left\{T^{(3+\alpha)/2}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)(T^{-\alpha/2}\overline{x}_t)]\right\}^2\\ &= T^{2+\alpha}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)(T^{-\alpha}\overline{x}_t^k)]^2.\end{aligned}$$

It follows that

$$T^{-1/2}t \to \frac{\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r)dr}{\left[\int_0^{1-\lambda} U^*(r,\lambda)^2dr \int_0^{1-\lambda} J_c^*(r)^2dr - [\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r)]^2\right]^{1/2}}. \tag{A.6}$$

By similar reasoning, we further obtain

$$R^2 = \frac{\widehat{\beta}^2 \sum_{t=1}^{T-k}(\overline{x}_t)^2}{\sum_{t=1}^{T-k}(\overline{y}_t^k)^2} \to \frac{[\int_0^{1-\lambda} U^*(r,\lambda)J_c(r)dr]^2}{[\int_0^{1-\lambda} U^*(r,\lambda)^2dr][\int_0^{1-\lambda} J_c^*(r)^2dr]}. \tag{A.7}$$

Proof of Theorem 2: Under the lemma 1 (e)-(d), we have

$$\widehat{\beta} - \beta = \frac{\sum_{t=1}^{T-k} \overline{u}_t^k \overline{x}_t^k}{\sum_{t=1}^{T-k}(\overline{x}_t^k)^2}, \tag{A.8}$$

where $\overline{x}_t^k = x_t^k - \overline{x}^k$ with $\overline{x}^k = (T-k)^{-1}\sum_{t=1}^{T-k} x_t^k$.

By similar fashion as in (A.1) to (A.4), we obtain

$$\widehat{\beta} - \beta = \frac{T^{(5+\alpha)/2}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{u}_t^k)(T^{-(\alpha+2)/2}\overline{x}_t^k)]}{T^{3+\alpha}[T^{-1}\sum_{t=1}^{T-k}(T^{-(\alpha+2)/2}\overline{x}_t^k)^2]},$$

then

$$T^{(\alpha+1)/2}(\widehat{\beta} - \beta) \to \frac{\sigma_e \int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r,\lambda)dr}{\sigma_w \int_0^{1-\lambda} J_c^*(r,\lambda)^2dr}, \tag{A.9}$$

under the $H_0 : \beta = 0$.

For the convergence of the t statistic, we write

$$t = \frac{T^{(5+\alpha)/2}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)(T^{-\alpha/2}\overline{x}_t)]}{(A_T - B_T)^{1/2}},$$

where

$$A_T = T^{4+\alpha}\left\{[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)^2][T^{-1}\sum_{t=1}^{T-k}(T^{-(\alpha+2)/2}\overline{x}_t^k)^2]\right\},$$

and

$$B_T = T^{4+\alpha}[T^{-1}\sum_{t=1}^{T-k}(T^{-1/2}\overline{y}_t^k)(T^{-(\alpha+2)/2}\overline{x}_t^k)]^2.$$

Thus,

$$T^{-1/2}t \to \frac{\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r,\lambda)dr}{\left[\int_0^{1-\lambda} U^*(r,\lambda)^2dr \int_0^{1-\lambda} J_c^*(r,\lambda)^2dr - [\int_0^{1-\lambda} U^*(r,\lambda)J_c^*(r,\lambda)]^2\right]^{1/2}}. \tag{A.10}$$

The convergence of R^2 is the same as that in Theorem 1.

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Table 1: Finite sample bias of OLS estimator:

DGP: The model (3) under the null $\beta = 0$.

T=200					
$k\backslash\alpha$	0.1	0.3	0.5	0.7	0.9
$[0.1T]$	-0.2793	-0.0971	-0.0335	-0.0118	-0.0040
$[0.2T]$	-0.2819	-0.0981	-0.0341	-0.0123	-0.0045
T=500					
$[0.1T]$	-0.2624	-0.0760	-0.0221	-0.0067	-0.0020
$[0.2T]$	-0.2645	-0.0766	-0.0222	-0.0067	-0.0021

Note: (a) 5000 replications.

(b) k is the portion of rolling summations of x_t and y_t.

(c) The locality parameter c is set to -1.

(d) The correlation δ is set to -0.9.

Table 2: Percentiles of the simulated distribution of the scaled t statistic: DGP: The model (5) with $k = [0.1T], T = 500$ under the null $\beta = 0$.

	1%	2.5%	5%	50%	95%	97.5%	99%
α				$\delta = 0$			
0.1	-0.7046	-0.5796	-0.4785	0.0031	0.4843	0.5798	0.6751
0.3	-0.7402	-0.6237	-0.4950	0.0056	0.5061	0.6021	0.7114
0.5	-0.7822	-0.6625	-0.5440	0.0005	0.5349	0.6395	0.7597
0.7	-0.8419	-0.7010	-0.5821	-0.0092	0.5637	0.6891	0.8168
0.9	-0.8606	-0.7154	-0.5910	-0.0122	0.5684	0.6996	0.8273
				$\delta = -0.9$			
0.1	-1.8737	-1.7000	-1.5393	-0.8378	-0.3153	-0.2274	-0.1277
0.3	-1.7559	-1.5838	-1.4287	-0.7586	-0.2496	-0.1630	-0.0556
0.5	-1.4292	-1.2728	-1.1293	-0.5441	-0.0759	0.0121	0.1051
0.7	-1.0762	-0.9238	-0.8108	-0.2929	0.1539	0.2334	0.3293
0.9	-0.8650	-0.7235	-0.6139	-0.1186	0.3285	0.3908	0.4977

Note: (a) 5000 replications.

(b) k is the portion of rolling summations of x_t and y_t.

(c) The locality parameter c is set to -1.