E C O N O M I C S B U L L E T I N

Fairness under Uncertainty

Thibault Gajdos
CREST − CNRS

Jean−Marc Tallon
EUREQua − CNRS

Abstract

Ever since its introduction by Foley (1967) and Varian (1974), the notion of fairness has been one of the most extensively used notion to evaluate allocations on an ethical basis. Whereas thereis an extensive literature on the efficiency properties of allocations in economies with uncertainty the concept of an envy−free allocation has not been widely studied in economies with uncertainty. We introduce two very natural notions of equity in an economy under uncertainty, namely ex ante and ex post equity, show they can contradict efficiency requirements. In particular, the set of ex ante efficient and ex post envy−free allocations may be empty. We nevertheless show that, under special circumstances, one may prove the existence of allocations that are both ex ante efficient and ex post envy−free. Such is the case, in particular, in an economy with individual risk and no aggregate risk.

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1. **Introduction**

Ever since its introduction by Foley (1967) and Varian (1974), the notion of fairness has been one of the most extensively used notion to evaluate allocations on an ethical basis. An allocation is said to be fair if it is Pareto optimal and envy-free, in the sense that no agent envies the bundle held by any other individual to his own.

The concept of an envy-free allocation has not been widely studied in economies with uncertainty, to the best of our knowledge [1]. Extending notions of fairness to a setup explicitly encompassing uncertainty is obviously related to the well-known timing-effect problem (see for instance Diamond (1967), Myerson (1981), and Yaari and Bar-Hillel (1984) for empirical evidence about the importance of beliefs in distributional issues) : generally speaking, the outcome of an allocation procedure depends on whether individuals' utility levels are evaluated before or after the resolution of uncertainty. Hence, as stated by Myerson,

> "the timing-effect is often an issue in moral debate, as when people argue about whether a social system should be judged with respects to its actual income distribution or with respect to its distribution of economic opportunities" (p.854).

In this note we take up the issue of fairness under uncertainty and exhibit a tension between the concept of envy-freeness and that of efficiency. Recall that in static exchange economies, there is no real tension between efficiency (Pareto optimality) and no-envy: there always exist allocations that are both envy-free and Pareto optimal (although this is not the case any longer if production is introduced). If we now extend the setup to take into account uncertainty on aggregate resources, several issues come up. The main difference with static economies is of course due to the possibility of risk sharing among agents, whose beliefs and tastes are different. One can define two notions of equity, namely *ex ante* and *ex post* no-envy, according to whether the allocation is judged before or after the realization of uncertainty. We show that there is no contradiction between these two notions, in the sense that any *ex post* envy-free allocation is *ex ante* envy-free as well. Thus, the *ex post* no-envy criterion constitutes a refinement of *ex ante* no-envy. However, this simple observation is not the end of the story as it ignores efficiency properties of these allocations. Indeed, it is easy to show that there is a tension, in most cases, between *ex post* no-envy and *ex ante* Pareto optimality. On the other hand, it is also possible to exhibit classes of economies where this tension does not exist. In particular, when risk is purely of the individual type (as in Cass, Chichilnisky and Wu (1996)), it is easy to show that allocation that is both *ex post* envy free and *ex ante* Pareto optimal exists.

The rest of the paper is organized as follows. In section 2 we introduce the framework and different concepts of equity. In section 3 we identify classes of economies in which there exist allocations that are both *ex ante* efficient and *ex post* envy-free. In section 4 we show through examples that, generally speaking, *ex ante* efficient risk sharing might run counter *ex post* equity. Section 5 concludes, showing in particular that the procedure consisting in giving equal right to uncertain resources to all agents is not consistent with the concept of intertemporally fair allocations.

[1] Although Baumol (1986) (p.19, footnote 8) argues that when uncertainty is involved, one should consider *ex ante* non envy.

2. **Equity and Efficiency under Uncertainty: Definitions**

We consider a two-period economy, with no consumption in the first period and S states of nature in the second $(s = 1, ..., S)$. There are H households $(h = 1, ..., H)$ and C commodities $(c = 1, ..., C)$. We define consumption by agent h of good c in state s by $x_h^c(s)$. Let $x_h(s) = \left(x_h^1(s), ..., x_h^C(s)\right)$ and $x_h = (x_h(1), ..., x_h(S))$. Denote aggregate endowment in state s by $e(s) \in \mathbb{R}_+^C$. An allocation $x = (x_1, ..., x_H) \in \mathbb{R}_+^{CHS}$ is feasible if $\sum_h x_h(s) \leq e(s)$ for all s. Agents have subjective expected utility preferences, given by $\sum_s \pi_h(s) u_h(x_h(s))$, with $\sum_s \pi_h(s) = 1$ for all h and $\pi_h(s) > 0$ for all h and s, and $u_h : \mathbb{R}_+^C \to \mathbb{R}$ is supposed to be strictly concave, strictly increasing, twice differentiable on $\mathbb{R}_{++}^C$ and have indifference surfaces whose closures are contained in $\mathbb{R}_{++}^C$.

We now define two notions of efficiency, which are standard in our framework.

Definition 1. *A feasible allocation x is*

- ex ante *Pareto optimal if there is no feasible allocation y such that $\sum_s \pi_h(s) u_h(y_h(s)) \geq \sum_s \pi_h(s) u_h(x_h(s))$, for all h with a strict inequality for at least one h. We denote by P_a the set of* ex ante *Pareto optimal allocations.*
- ex post *Pareto optimal if there is no feasible allocation y such that $u_h(y_h(s)) \geq u_h(x_h(s))$, for all h and all s with a strict inequality for at least one h and s. We denote by P_p the set of* ex post *Pareto optimal allocations.*

The following fact is well-known (see e.g. Laffont (1981)):

Proposition 1. $P_a \subset P_p$

As a counterpart to these notions of efficiency, it seems also natural to define two concepts of equity under uncertainty, namely *ex ante* and *ex post* equity.

Definition 2. *A feasible allocation x is*

- ex ante *envy-free (or* ex ante *equitable) if*

$$\sum_s \pi_h(s) u_h(x_h(s)) \geq \sum_s \pi_h(s) u_h(x_{h'}(s))$$

  for all h and h'. We denote by E_a the set of ex ante *equitable allocations.*
- ex post *envy-free (or* ex post *equitable) if $u_h(x_h(s)) \geq u_h(x_{h'}(s))$, for all h, h' and s. We denote by E_p the set of* ex post *equitable allocations.*

Ex ante no-envy is arguably the natural extension of the static notion of no-envy. However, it misses an important element of uncertainty, as it ignores the situation after uncertainty is revealed. *Ex ante* equity is weaker than *ex post* equity.

Proposition 2. $E_p \subset E_a$

Proof. Let $x \in E_p$. By definition, we have $u_h(x_h(s)) \geq u_h(x_{h'}(s))$, for all h, h' and s. Multiplying each inequality by $\pi_h(s)$ and summing over s yields $\sum_s \pi_h(s) u_h(x_h(s)) \geq \sum_s \pi_h(s) u_h(x_{h'}(s))$, i.e. $x \in E_a$. □

Allocations that are both Pareto optimal *ex ante* and equitable *ex post* are called henceforth *intertemporally fair allocations.* Such allocations, whenever they exist, satisfy the four criteria defined above, and hence are not objectionable on the basis of any of these criteria.

Before turning to formal examples showing that such allocations might or might not exist, we discuss why one would want to impose *ex post* equity, which seems the most demanding concept of the four introduced so far. Recall first that in the present setup, (aggregate) uncertainty is borne by agents independently of any action they might take, or, more precisely, the resources the society has in each state is independent of their actions. Thus, the society as a whole has no influence on the risk borne. Hence, it seems reasonable to ask as much as possible that this exogenous risk be "borne equally" by each agent. In other terms, one would like to somehow neutralize as much as possible the effect of uncertainty on agents' welfare. The notion of *ex post* equity requires a strong form of such "neutralization" as it explicitly requires that for every possible realization of uncertainty, no agent envies the bundle consumed by another one. Thus, no compensation between states is allowed by that concept. The following example is meant to carry that intuition.

Imagine that uncertainty is about unemployment rate. Agents have different probabilities of being unemployed: a state is hence essentially the list of who is employed and who is unemployed. Assume that there are two aggregate states: in the first one, 5% of the population is unemployed, while in the second one, the unemployment rate is 10%. Aggregate resources are thus higher in the first aggregate state than in the second. The government implements a policy of unemployment benefits. To be efficient, this policy must be *ex ante* Pareto optimal. Now, ethically, it is difficult to imagine in this setup what could justify that an agent, in a given state, be envied by another one even though different agents might have different probabilities of being unemployed. Of course, if agents could have an impact on these probabilities or on the aggregate resources (say an agent can take actions to affect his probability of being unemployed, such as education or different levels of effort to find a job), incentive issues would come into the picture and it is not clear then that *ex post* envy-freeness is an appealing notion.

3. **Intertemporally fair allocations: two cases**

3.1. Individual risk

The economies we consider here are such that $e^s = e$ for all s, i.e., there is no aggregate risk. However, each agent separately bears some individual risk. This case corresponds to the case of pure redistribution: the amount to be shared among the households is a constant, and the only way by which states can differ is through how that amount is distributed among agents. Note also that agents might have different exposure to that individual risk, and hence a priori different beliefs. Formally, the class of economies we consider here is one of individual risk, (see Malinvaud (1973) and Cass, Chichilnisky and Wu (1996)).

Before presenting our results, we introduce a piece of notation. Let $x^e \in \mathbb{R}_+^{CH}$ be an equilibrium allocation of the (static) economy $(u_h(.), e_h = e/H)_{h=1,\dots,H}$, i.e., an equilibrium obtained from the egalitarian distribution of endowments in a state in the second period, and let $p^e \in \mathbb{R}_+^C$ be the associated equilibrium prices. We know (see e.g. Kolm (1972)) that x^e is equitable. Denote by x^E the allocation x^e replicated S times, that is, $x^E = (x^e, ..., x^e) \in \mathbb{R}_+^{CHS}$. Interpret now h to be types. There are N_h agents of type h, with $N = \sum_h N_h$. Each household faces individual uncertainty, with S possible individual states $(s = 1, ..., S)$. Each household of type h has certainty preferences u_h and correctly believes that its probability of being in individual state s is given by $\pi_h(s)$, which we allow to be different among types of agents. What makes this a model of individual risk is that, in fact exactly $\pi_h(s) N_h$ households of type h will find themselves in state s. Under these assumptions, the feasibility condition of an allocation x is that

$$\sum_{h=1}^{H} N_h \sum_{s=1}^{S} \pi_h(s)\, x_h(s) = e$$

Proposition 3. *Under the maintained assumptions, there exists an intertemporally fair allocation in economies with individual risk and no aggregate uncertainty.*

Proof. We show that the allocation x^E, which is equitable *ex post* by construction is Pareto optimal *ex ante* as well, i.e. that x^E is the solution to:

$$\begin{array}{l} Max\ \sum_{h=1}^{H} \lambda_h N_h \sum_{s=1}^{S} \pi_h(s)\, u_h(x_h(s)) \\ \text{s.t. } \sum_{h=1}^{H} N_h \sum_{s=1}^{S} \pi_h(s)\, x_h(s) = e \end{array}$$

for some $\lambda = (\lambda_1, ..., \lambda_H) \gg 0$.

x^E is a solution to this program if and only if there exists $\mu = (\mu^1, ..., \mu^C) \gg 0$ such that $\lambda_h \nabla u_h(x_h(s)) = \mu$ for all s. By construction $x_h^E(s)$ is a solution to the following maximization problem:

$$\begin{array}{l} Max\ u_h(x_h(s)) \\ \text{s.t. } p^e x_h(s) = p^e \frac{e}{N} \end{array}$$

and hence, there exists $\gamma_h > 0$ (note that we can take γ_h independent of s) such that

$$\nabla u_h\left(x_h^E(s)\right) = \gamma_h p^e \text{ for all } s$$

Now, take $\lambda_h = 1/\gamma_h$ and $\mu = p^e$ to conclude that x^E satisfies the first order conditions of the *ex ante* Pareto optimality problem. Hence it is intertemporally fair. □

Thus, although different agents might have different exposure to risk, if, as a whole, society does not face any risk, intertemporal fairness imposes that ex post, no differences be made among agents according to their initial exposure to risk. Although this class of economies is of interest, the result clearly rests on the particular structure imposed which makes the risk sharing aspect easy to accommodate. On the other hand, our result clearly indicates that for economies with only microeconomic risk the concept of *ex post* no-envy is particularly relevant. This class of economies are representative of economies in which distributional issues are not contradictory with perfect risk sharing.

3.2. No aggregate risk and identical beliefs

The other, related, class of economies in which intertemporally fair allocations exist is one in which, as above, there is no aggregate risk but in which agents differ only by their utility function and therefore by their risk aversion. In particular they have the same beliefs.

Proposition 4. *Assume $\pi_h(s) = \pi(s)$ for all h and $e^s = e$ for all s. Then, the allocation x^E is intertemporally fair under the maintained assumptions.*

Proof. x^E is equitable *ex post* since it is obtained, state by state, as an equilibrium allocation of an economy in which endowments are distributed in an egalitarian way. It is Pareto optimal *ex post*, being an equilibrium of static, state by state, second-period economies, i.e., it is a solution to the problem:

$$\begin{array}{l} Max \ \sum_{h=1}^{H} \lambda_h u_h (x_h) \\ \text{s.t. } \sum_{h=1}^{H} x_h = e \end{array}$$

for some $\lambda = (\lambda_1, ..., \lambda_H) \gg 0$. Hence, x^E is trivially a solution to the following problem:

$$\begin{array}{l} Max \ \sum_{s=1}^{S} \pi(s) \sum_{h=1}^{H} \lambda_h u_h (x_h(s)) \\ \text{s.t. } \sum_{h=1}^{H} x_h (s) = e \ \ s = 1, ..., S \end{array}$$

and therefore it is Pareto optimal *ex ante* □

When there is only one good, the only allocation that is equitable *ex post* is the egalitarian allocation, giving e/H to each agent in each state. This allocation is Pareto optimal *ex ante*, and hence, the egalitarian allocation is the only intertemporally fair allocation. However, intertemporally fair allocations are in general not unique, as shown in the sequel (see example 3 below). The assumption that agents have identical beliefs is clearly crucial to obtain that result, as shown in the sequel (see Example 1 below).

The interest of the previous result is to show that absent any aggregate risk, different risk aversion among agents does not justify *per se* that agents be treated differently. Indeed, in the single good case, each agent, no matter his degree of risk aversion (that is, whatever his utility function as long as it is concave) will receive the average amount at an intertemporally fair allocation.

4. **Intertemporally fair allocations: open issues**

We now turn to a few examples illustrating how the different concepts introduced interact. The tension between *ex ante* efficiency and *ex post* fairness is illustrated in the first two examples, in which the set $P_a \cap E_p$ is vacuous. The next example illustrates the complexity of the interaction between beliefs and utility functions to assess the existence of intertemporally fair allocation. It is an open issue to characterize conditions under which these allocations exist.

Example 1. *Assume there is only one good ($C = 1$), two states of nature and no aggregate risk, i.e.* $e(1) = e(2)$. *Let* $H = 2$. *Then, the set of* ex post *equitable allocations reduces to* $x_1 = \left(\frac{1}{2}e(1), \frac{1}{2}e(2)\right)$ *and* $x_2 = \left(\frac{1}{2}e(1), \frac{1}{2}e(2)\right)$. *If agents have different beliefs, that is* $\pi_1(1) \neq \pi_2(1)$, *then, it is well-known that the perfect insurance allocations are not in the set of* ex ante *Pareto optimal allocations. Hence,* $P_a \cap E_p = \emptyset$ ♢

Example 2. *Assume that there is only one good ($C = 1$), two agents ($H = 2$) and two states of nature. We also assume that both agents have the same beliefs, i.e.* $\pi := \pi_1(1) = \pi_2(1)$ *,and that* $e(1) < e(2)$. *The agents' certainty utility functions are as follows:* $u_1(x_1) = \log(x_1)$ *and* $u_2(x_2) = \sqrt{x_2}$

An allocation x is ex ante *Pareto optimal if there exists $\lambda \geq 0$ such that:*

$$\begin{cases} \frac{\lambda}{x_1(1)} = \frac{(1-\lambda)}{2\sqrt{e(1)-x_1(1)}} \\ \frac{\lambda}{x_1(2)} = \frac{(1-\lambda)}{2\sqrt{e(2)-x_1(2)}} \end{cases}$$

Now, it is obvious that an allocation x is equitable ex post *if and only if* $x_1(1) = \frac{1}{2}e(1)$ *and* $x_1(2) = \frac{1}{2}e(2)$. *Hence, an allocation x is intertemporally fair if and only if* $e(1) = e(2)$, *a contradiction* ♢

The tension illustrated by these examples is in essence extremely simple: *ex ante* risk efficient sharing requires in most situations that agents bear risk differently, whether because of different beliefs or risk aversion, or both. Hence, once the uncertainty is resolved individuals' situations might be very different and create envy.

The last example shows that the interaction between beliefs, utility functions and their impact on existence of an intertemporally fair allocation can be rather complex.

Example 3. *Assume* $C = 2$ *(x and y) and* $H = 2$ *with:* $u_1(x, y) = \log(x) + 2\log(y)$ *and* $u_2(x, y) = \log(x) + \log(y)$. *Let* $\pi_1 := \pi_1(1)$ *and* $\pi_2 := \pi_2(1)$. *Simple but tedious algebra shows that there exists an intertemporally fair allocation if and only if beliefs satisfy the following property:*

$$\frac{\sqrt{2}}{\sqrt[3]{4}} \leq \frac{\pi_2/\pi_1}{(1-\pi_2)/(1-\pi_1)} \leq \frac{\sqrt[3]{4}}{\sqrt{2}}$$

Thus, there does not exist an intertemporally fair allocation if agents have the same beliefs. Furthermore, the Pareto optimal allocations that are intertemporally fair are the ones that correspond to weights λ *in the social welfare function that satisfy:*

$$\sqrt{2}\max\left(\frac{\pi_1}{\pi_2}, \frac{1-\pi_1}{1-\pi_2}\right) \leq \frac{1-\lambda}{\lambda} \leq \sqrt[3]{4}\min\left(\frac{\pi_1}{\pi_2}, \frac{1-\pi_1}{1-\pi_2}\right)$$

Two observations may be drawn from this example. First, when it exists, an intertemporally fair allocation is not necessarily unique: if $\sqrt{2}\max\left(\frac{\pi_1}{\pi_2}, \frac{1-\pi_1}{1-\pi_2}\right) < \sqrt[3]{4}\min\left(\frac{\pi_1}{\pi_2}, \frac{1-\pi_1}{1-\pi_2}\right)$, there exists a continuum of intertemporally fair allocations. Second, *ex ante* fair and efficient allocations correspond to weights λ that satisfy $\sqrt{2} \leq \frac{1-\lambda}{\lambda} \leq \sqrt[3]{4}$. Therefore, $P_a \cap E_p \subset P_a \cap E_a$, but this inclusion is not necessary strict: the *ex post* no-envy criterion might select a proper subset of the *ex ante* fair and efficient allocations.

5. **Concluding remarks**

Existence of *ex ante* envy-free allocations that are Pareto optimal *ex ante* is a direct consequence of results in the static setup. One may think of a simple procedure, consisting in giving agents equal rights to future uncertain resources and let them trade these rights, as a way to avoid the problem of non-existence. More precisely, using the above framework, assume that agents are all endowed to an equal share of total resources in every state tomorrow. Through this, we model the idea that all agents are put in a position where they all bear the same "objective" risk. Now, this allocation has no reason to be *ex ante* Pareto optimal, as agents might want to share risk differently, according to their beliefs and tastes. However, if we allow agents to trade these equal claims *ex ante*, then the equilibrium allocation will always exist, be *ex ante* Pareto optimal and *ex ante* envy-free. Thus, any envy that would possibly appear *ex post* is the result of efficient risk sharing and voluntary trade. The idea being here that an agent can always stay with his initial endowment, which is the "average risk" in the economy.

Definition 3. *An allocation x is* strongly ex ante fair *if there exists p such that:*

(i) *given p,* x_h *is a solution to :*

$$\begin{array}{l} Max\ \sum_{s=1}^{S} \pi_h(s)\, u_h(x_h(s)) \\ s.t.\ \sum_{s=1}^{S} p(s)\left[x_h(s) - e(s)/H\right] = 0 \end{array}$$

(ii) $\sum_{h=1}^{H}\left[x_h(s) - e(s)/H\right] = 0$ *for all s.*

It is straightforward to see that such allocations exist (under our assumptions), are *ex ante* Pareto optimal (first welfare theorem) and *ex ante* equitable. Thus, this criterion has the advantage of selecting an allocation that has good *ex ante* properties, and also furthermore treats agents in the same manner with regard to the risk they bear: each agent *ex ante* bears the average risk of the economy. If, for efficient risk-sharing reason they choose to deviate from this position, they cannot argue *ex post* that they were at a disadvantage *ex ante* with respect to their exposure to uncertainty.

Intertemporally fair allocations exhibited in the case of individual risk are strongly *ex ante* fair according to our definition. We illustrate what bite this concept has over the simpler concept of *ex ante* fairness.

Example 4. *Consider a two-agent economy, with two states and two goods (x and y to simplify notation). The aggregate endowments are $(1,1)$ in state 1 and $(2,2)$ in state 2. Assume that agents have identical beliefs of $1/2$ for each state. Let $u_1(x,y) = (x+\varepsilon y)^{1/2}$ and $u_2(x,y) = x^{1/2}+y^{1/2}$. Consider, to simplify, the limit case for which $\varepsilon = 0$ (although it does not satisfy the maintained assumptions of the paper). Then, the following allocation is* ex ante *Pareto optimal and* ex ante *envy-free:*

$$\begin{aligned} x_1(1) &= 1/2,\ y_1(1) = 0,\ x_1(2) = 1,\ y_1(2) = 0 \\ x_2(1) &= 1/2,\ y_2(1) = 1,\ x_2(2) = 1,\ y_2(2) = 2 \end{aligned}$$

Actually, it is ex post *envy-free as well. Nonetheless, this allocation treats agent 2 particularly well since he gets the same amount of good x as agent 1, plus all the good y available. Now, it does not satisfy the equal right to uncertain resources criterion. Indeed, the equilibrium allocation when agents are given egalitarian endowments is:*

$$\begin{aligned} x_1(1) &= \sqrt{2}/2,\ y_1(1) = 0,\ x_1(2) = \sqrt{2},\ y_1(2) = 0 \\ x_2(1) &= 1-\sqrt{2}/2,\ y_2(1) = 1,\ x_2(2) = 2-\sqrt{2},\ y_2(2) = 2 \end{aligned}$$

in which agent 1 is better off than at the previous allocation. This allocation is also ex ante *fair and* ex post *equitable.*

In the previous example, the right to uncertain resources criterion is not redundant with *ex post* no-envy and can refine this concept as well. On the other hand, it is possible to exhibit examples in which strong *ex ante* fairness does not select the intertemporally fair allocations when they exist.

Example 5. *Consider the setup of example 3. Let $p(s)$ (resp. $q(s)$) be the price of good x (resp. good y) in state s. With the notation of example 3, the equilibrium allocation corresponds to the Pareto optimum associated with weight $\lambda = \frac{1}{1+\frac{\mu_1}{\mu_2}} = \frac{2}{5}$. Now, assume that the beliefs are given by $\pi_1 = 0.375$ and $\pi_2 = 0.4$. Then the condition for existence of an intertemporally fair allocation established in example 3 is satisfied. However, $\frac{1-\lambda}{\lambda} = \frac{3}{2}$ is not within the bounds identified in example 3 and therefore, the equilibrium we found here is not fair* ex post. *Therefore, the strongly* ex ante *fair allocation is not fair* ex post, *although there exists an intertemporally fair allocation.*

To conclude, the concept of equal rights to uncertain resources is a refinement of *ex ante* fairness, that has meaning in all economies, and is independent of *ex post* fairness.

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