E C O N O M I C S   B U L L E T I N

A fractionally integrated model for the Spanish real GDP

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Abstract

The annual structure of the Spanish real GDP is investigated in this article by means of fractional integration techniques. The results show that the series can be specified in terms of an I(d) process with d smaller than one and thus showing long memory and mean−reverting behaviour.

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1. Introduction

It is a well-known stylised fact that many macroeconomic and financial time series can be specified in terms of fractionally integrated (I(d)) processes. Examples are the papers of Diebold and Rudebusch (1989); Baillie and Bollerslev (1994); Gil-Alana and Robinson (1997); etc. The distinction between I(d) processes is important from both statistic and economic viewpoints. Thus, if $d \in (0, 0.5)$, the time series is stationary and mean reverting; however, if $d \in [0.5, 1)$, it will be nonstationary but still mean reverting, while $d \geq 1$ implies nonstationarity and non-mean reverting behaviour, with shocks affecting the series persisting forever.

In this article we examine annual data of the Spanish real output by means of the tests of Robinson (1994), which permit us to test I(d) statistical models in a fully parametric way. A model selection criterion is then established to determine the most adequate specification for this series.

2. Testing of I(d) hypotheses in the Spanish real GDP

We consider the regression model:

$$y_t = \alpha + \beta t + x_t, \qquad t = 1, 2, ... \tag{1}$$

$$(1 - L)^d \, x_t = u_t, \qquad t = 1, 2, ..., \tag{2}$$

where u_t is an I(0) process, defined as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d = d_o, \tag{3}$$

in (1) and (2) for any real value d_o and different types of I(0) disturbances u_t. We will call the test statistic $\hat{r}$, and its functional form can be found in Robinson (1994) or in any of the numerous empirical applications of his tests (e.g., Gil-Alana and Robinson, 1997; Gil-Alana, 2000, 2001; etc.). It can be shown that, based on H_o (3), under certain regularity conditions,[1]

$$\hat{r} \;\to_d\; N(0, 1) \qquad as \;\; T \to \infty. \tag{4}$$

Thus, a test of (3) will reject H_o against the alternative: H_a: $d > d_o$ ($d < d_o$) if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α.

Table I reports the results of $\hat{r}$ for the Spanish real GDP for the time period 1900-2000 annually. We evaluate the test statistic for values d_o = 0.50, (0.10), 1.50 and disturbances which are white noise, AR(1) and AR(2), and consider separately the cases of $\alpha = \beta = 0$ a priori (i.e., with no regressors in the undifferenced model (1)); α unknown and $\beta = 0$ a priori (i.e., with an intercept); and both α and β unknown (i.e., with a linear time trend). A noticeable feature observed across this table is that $\hat{r}$ monotonically decreases with d_o. This is something to be expected given that $\hat{r}$ is a one-sided statistic. Thus, for example, if H_o (3) is rejected with d_o = 1 against alternatives of form: H_a: $d > 1$, an even more

[1] These conditions are very mild regarding technical assumptions, which are satisfied by model (1) and (2).

significant result in this direction should be expected when $d_o = 0.75$ or $d_o = 0.50$ are tested. Starting with white noise u_t, we see that the non-rejection values occur when $d_o = 1.40$ or 1.50 for the cases of no regressors and an intercept, while including a linear time trend, $d_o = 1.50$ appears as the only non-rejectable value. However, including weakly parametrically autocorrelated disturbances, the orders of integration are smaller, ranging between 0.70 and 1.30 in all cases.

TABLE I

Testing H_o (3) in the model given by (1) and (2) with the tests of Robinson (1994)

u_t	Regressors / d	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
White noise	$\alpha = \beta = 0$	21.63	21.27	20.21	18.13	14.91	10.98	7.14	3.99	1.68	**0.05'**	**-1.08'**
	$\alpha \neq 0;.\ \beta = 0$	19.22	18.27	18.23	17.42	15.46	12.51	9.09	5.89	3.33	**1.47'**	**0.14'**
	$\alpha \neq 0;.\ \beta \neq 0$	22.32	21.30	19.91	18.04	15.62	12.76	9.66	6.64	3.99	1.89	**0.34'**
AR(1)	$\alpha = \beta = 0$	2.53	1.73	**0.85'**	**-0.08'**	**-0.35'**	**-1.24'**	**-1.46'**	**-1.55'**	-2.32	-2.35	-2.43
	$\alpha \neq 0;.\ \beta = 0$	2.58	2.51	2.33	2.03	**1.55'**	**1.07'**	**0.38'**	**-0.07'**	**-0.20'**	**-0.51'**	-1.85
	$\alpha \neq 0;.\ \beta \neq 0$	2.12	1.91	**0.32'**	**-0.36'**	**-0.69'**	**-1.62'**	-1.90	-2.07	-21.3	-2.42	-2.79
AR(2)	$\alpha = \beta = 0$	5.66	4.47	3.22	**1.05'**	**-0.42'**	**-0.48'**	**-0.59'**	**-0.97'**	-1.74	-1.83	-1.99
	$\alpha \neq 0;.\ \beta = 0$	5.92	5.37	4.90	2.90	**1.56'**	**1.05'**	**0.14'**	**-0.71'**	**-1.52'**	-1.89	-1.93
	$\alpha \neq 0;.\ \beta \neq 0$	4.42	3.35	**0.80'**	**-0.50'**	**-1.45'**	**-1.63'**	-2.64	-2.84	-3.75	-4.01	-4.33

' and in bold: Non-rejection values of the null hypothesis at the 5% significance level.

TABLE II

Impulse response values for the selected model

Impulse	Value	Impulse	Value
0	1.000	11	1.014
1	1.270	12	0.995
2	1.316	13	0.979
3	1.290	14	0.964
4	1.245	15	0.950
5	1.198	16	0.937
6	1.555	17	0.925
7	1.118	18	0.914
8	1.086	19	0.904
9	1.059	20	0.895
10	1.035	30	0.823

In order to decide now which might be the most adequate specification for this series we perform as follows: First, we choose for each type of disturbances and each type of regressors, the value of d_o which produces the lowest $|\hat{r}|$ across d_o. Then, for each of the selected nine models we perform several diagnostic tests on the residuals. In particular, we test for no serial correlation (Durbin, 1970; Godfrey, 1978a, b), homoscedasticity (Koenker, 1981) and functional form (Ramsey, 1969, RESET test), using Microfit. As a result, we only find a single model passing all the diagnostics at the 5% level. The resulting model appears to be:

$$(1 - L)^{0.80} x_t = u_t; \quad u_t = 0.47 u_{t-1} + \varepsilon_t,$$

implying an order of integration smaller than one and thus showing mean reversion.[2]

Table II reports the first twentieth impulse responses for this model. We see that a one-unit shock initially produces an increasing effect that lasts above 1 up to the 11th period, then decreasing slowly and taking a very long time to disappear completely. In fact, we observe that even 20 periods after the initial shock, more than 80% of its effect still remains in the series.

3. Conclusions

The annual structure of the Spanish real GDP has been examined in this article by means of fractional integration techniques. Using a version of the tests of Robinson (1994) for testing I(d) statistical models, we show that if the disturbances are white noise, the order of integration of the series is higher than one, however, allowing autoregressive disturbances, the degree of integration fluctuates between 0.70 and 1.30. A model selection criterion based on diagnostic tests on the residuals was then established to determine the correct model specification of the series, and the resulting model appears to be an ARFIMA(1, 0.80, 0), implying thus nonstationarity but mean reversion. This result is in apparent contradiction with other studies, which implicitly assume that the real output is an I(1) variable. By contrast, we show in this paper that a fractional model with d smaller than one might be a more appropriate way of describing this series. Moreover, Gil-Alana (2003) also performed a fractional model on the same variable, and his conclusion was that the order of integration was higher than 1.[3] However, the results are not directly comparable. In Gil-Alana (2003) the log-transformed series was used, and it was assumed in that paper that the disturbances were autocorrelated throughout the non-parametric model of Bloomfield (1973). Here, we have employed a fully parametric model, describing the short run dynamics through a model selection criterion, and the results support the view that the series is nonstationary but mean reverting.

[2] Sowells' (1992) procedure, based on maximum likelihood estimation of ARFIMA(p,d,q) models was also performed for p, q ≤ 3, and according to the SIC, the best model was an ARFIMA(1, 0.81,0).

[3] Imposing white noise disturbances, the results in Gil-Alana (2003) were completely in line with those reported in this paper.

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