E C O N O M I C S  B U L L E T I N

# Do budget deficits follow a linear or non-linear path?

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## *Abstract*

One method to test whether any government's fiscal policy has been effective in dealing with budget deficits is to test for mean reverting properties of deficits. If deficits-GDP ratio reverts to their mean, or are stationary, that will be an indication of taking corrective measure. Previous studies using standard linear ADF test did not find much support for the stationarity of deficits/GDP ratio. However, when we employ non-linear ADF test and data from 28 countries, we find support for stationarity of the deficits/GDP ratio in 50% of the countries in the sample.

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## 1. Introduction

The U.S. government has run a chronic deficit since 1930, causing many to question how long government budget deficits can continue unchecked. Is it even feasible for government to run a budget deficit forever? The Congressional Budget Office reported the federal budget deficit for 2005 at $477 billion dollars. Given Congress's apparent inability to control total spending, coupled with the Bush administration's efforts to increase defense expenditures while making the ten-year tax cut permanent, future budget forecasts are not at all encouraging. Government deficits have become a focus of professional interest and political debate. Record-breaking deficits have produced a vast amount of literature dealing with the possible impact it has on the rest of the economy.

Although different studies related to budget deficits have tried to address different budget-related issues, many of them have been concerned with the sustainability of the recently large budget deficits in the U.S. For example, Hamilton and Flavin (1986) show that historical data provides a basis for expecting the need for a present-value borrowing constraint. They find that the proposition that the government must promise creditors that it will balance the budget in expected present-value terms is largely consistent with postwar U.S. data. They also conclude that the sentiment that current deficits can continue forever is wrong due to the limitations of government borrowing. Therefore, the series of deficits must soon turn to surplus. They find that when government runs a deficit, it is making an implicit promise to creditors that it will run offsetting surpluses in the future. If past deficits are to be offset with future surpluses, then it is possible for budget deficits to revert to their mean.

Indeed, several studies have concentrated on the mean reverting property of budget deficits. If they are mean-reverting, which implies stationary behavior, governments are said to take

corrective measures. Bohn (1998) provides substantial evidence that the U.S. does in fact take corrective action and that the debt/GDP ratio is stationary if war-time spending and cyclical fluctuations are controlled for. However, in general, Trehan and Walsh (1988, 1991), Bohn (1991), Kremers (1991), Corsetti and Roubini (1991), and Bohn (1998) argue that due to the fact that every economy is faced with both a continuously growing tax base and growing government spending, we are presented with a high and growing debt-GDP ratio that is non-stationary over time, making it hard to reject a unit root in the debt-GDP ratio, implying that corrective measures and macro policies are necessary.[1]

In testing for mean-reverting property of deficit-GDP ratio, previous researchers have used the standard Augmented Dickey-Fuller (ADF) test, which tests the null of non-stationarity of a time-series variable against an alternative of linear stationarity. However, due to business cycles that affect the deficit-GDP ratio, it is possible that the ratio follow a non-linear path. Indeed, Barro (1979) has argued that there is a countercyclical response of debt to temporary income movements, and a one-to-one effect of expected inflation on nominal debt growth. Thus, in this paper we consider the mean-reverting properties of the budget deficits-GDP ratio in as many countries as data permits, one more time. More precisely, in addition to using the standard ADF test, we employ a new test by Kapetanios, Shin and Snell (2003) that accounts for non-linearity in the mean-reversion process of a time series variable such as deficit-GDP ratio. Section 2 introduces the Kapetanios, Shin and Snell (2003) test, which we will refer to as the KSS test hereafter. Section 3 discusses the results with a summary in section 4.

[1] Note that an alternative to unit-root testing is to consider the movement in revenue and spending. Aschauer (1985) and Seater and Mariano (1985) tested and accepted the hypothesis that the government's revenues must equal its expenditures in present-value terms jointly with a permanent income hypothesis. Evans (1985) found evidence in support of this same joint hypothesis. Barro (1984) tested and accepted the hypothesis that the government is subject to the present-value borrowing constraint jointly with the assumption that taxation and deficit policies have historically been optimal.

## 2. The KSS Test[2]

The standard ADF test assumes the null hypothesis to be unit root and the alternative hypothesis to be stationarity of a variable that follows a linear path. Kapetanios, Shin and Snell (2003) extend the standard ADF test and introduce a new test. In this new test the null hypothesis is again unit root but the alternative hypothesis is nonlinear stationary smooth transition autoregressive (STAR) process. Consider a time series variable Y. The KSS test is based on the following exponential smooth transition autoregressive (ESTAR) specification:

$$\Delta Y_t = \lambda Y_{t-1}[1-\exp(-\vartheta Y_{t-1}^2] + \mu_t \qquad (1)$$

In (1) $Y_t$ could be a de-meaned or de-trended variable and μ is an error term with usual properties. In this set up, since λ is not identifiable, Kapetanios *et al.* (2003) propose using Taylor series so that specification (1) could be proxied by (2):

$$\Delta Y_t = \delta Y_{t-1}^3 + \varepsilon_t \qquad (2)$$

Following the same procedure as ADF, they show that (2) could also be augmented by several lags of the dependent variable as in (3):

$$\Delta Y_t = \delta Y_{t-1}^3 + \sum_{k=1}^{n} \rho_k \Delta Y_{t-k} + \varepsilon_t \qquad (3)$$

[2] For another application of these tests and additional explanation, see Cerrato and Sarantis (2006) in this journal.

Note that if the first term on the right hand-side of (3) was raised to power one, (3) would resemble exactly the standard ADF test. Thus, non-linearity is introduced by raising the lagged level of *Y* to the third power. Whether standard linear ADF or non-linear ADF (i.e. KSS), the null of unit root, i.e., $\delta = 0$ is tested against the alternative of $\delta > 0$ by familiar t ratio obtained for $\delta$. However, just like the standard ADF test which has its own critical values for the t-ratio, the KSS test also has new critical values for the t-ratio. These new critical values are tabulated by Kapetanios, Shin and Snell (2003). Note that in selecting the optimum number of lags, we closely follow KSS who recommend relying upon the significance of augmented terms (KSS, p. 365).

**3. The Results**

Table 1 reports the results of unit root tests applied to the budget deficits/GDP ratio for a total of 28 countries for which the data were available from the International Financial Statistics of the International Monetary Fund. We report in Table 1 a total of five t statistics. When the standard linear ADF test that included only a constant term was applied, the resulting t-ratio is denoted by $ADF_C$. However, when the same test included a constant and a trend, the statistic is denoted by $ADF_t$. As mentioned before, these two statistics are reported not only to determine linear stationarity of the deficits/GDP ratio but also to compare the results of linear tests to those obtained from non-linear tests. For non-linear KSS test, we report three statistics. Following KSS, we first employ the raw data on deficits/GDP variable without any adjustment and apply the non-linear test outlined by equation (3). The resulting t-ratio is denoted by $t_{NL1}$. Next, since there is no constant in (3), again we follow KSS and subtract the mean of deficits/GDP ratio from the raw data and apply (3) to this newly generated de-meaned data. Once (3) is applied to

de-meaned data, we denote the resulting t-ratio by $t_{NL2}$ in Table 1. Finally, we de trend the raw data following the procedure in KSS (2003, p. 364) and apply (3) to the de-trended data and report the t-ratio as $t_{NL3}$.

Table 1 goes here

Identifying cases in which stationarity is supported by a *, Table 1 reveals that from at least one of the standard linear ADF tests ($ADF_C$ or $ADF_t$ ), there are 10 countries in which the null of unit root is rejected in favor of stationarity of deficits/budget ratio. This is because our calculated statistic is larger than critical value in absolute term. These 10 countries are Cyprus, Finland, Germany, Korea, Luxembourg, China, Spain, Sweden, Switzerland and the U.S. Thus, based on these results, the deficits/GDP ratio in these countries is stationary, implying that each country has taken corrective steps in controlling budget deficits. Since the data for the U.S. excludes the war periods and the Great Depression, our finding for the U.S. is consistent with Bohn (1998). Turning to the results of non-linear tests we gather from Table 1 that at least one of the three $t_{NL}$ statistics support stationarity of deficits/GDP ratio in 11 countries. However, there are only four countries that stationarity is rejected by linear ADF tests but not by non-linear tests. These are Australia, Belgium, Greece, and Israel.[3] Adding these four to the list of 10 whose deficits/GDP ratio was found to be stationary by one of the linear ADF tests, we provide support for the stationarity of the deficits/GDP ratio in total of 14 out of 28 countries.

## 4. Summary and Conclusion

[3] These could be countries that have faced business cycles more often than others. As mentioned before, these business cycles could be source of non-linearity that is detected by the KSS test in these countries.

One way of testing whether a government's macroeconomic policies have been successful in curbing budget deficits is to test and determine whether deficits revert to their means. If they do, they are said to be stationary, implying that macro policies have been successful.

Previous studies that tested for unit root in deficits/GDP ratio employed standard ADF test in which the null hypothesis of unit root is tested against linear stationarity. However, due to wars and business cycles, it is possible for the deficits/GDP ratio to follow a non-linear path. Therefore, in this paper we employ a relatively new unit root testing procedure by Kapetanios *et al.* (2003) which accounts for non-linearity in a time series variable, i.e., budget deficits/GDP variable. After applying standard linear ADF test as well as non-linear ADF test to the deficit-GDP ratio of 28 countries for which data was available, we were able to show that there were only four countries in which stationarity of the ratio was supported by non-linear test but not by linear test. These countries were Australia, Belgium, Greece, and Israel. In addition to these countries there were 10 other countries in which stationarity was supported by both linear ADF and non-linear ADF tests. These countries were Cyprus, Finland, Germany, Korea, Luxembourg, China, Spain, Sweden, Switzerland and the U.S. Thus, it appears that stationarity of the deficits/GDP ratio is supported in 50% of the cases.

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**Table 1: Unit Root Test Results for the Deficit/GDP ratio.**

| Country | Study Period | $ADF_C$ | $ADF_t$ | $t_{NL1}$ | $t_{NL2}$ | $t_{NL3}$ |
|---|---|---|---|---|---|---|
| Australia | 1949-2002 | -2.12[6] | -1.38[6] | -1.30[6] | -2.76[6]* | -2.72[6] |
| Austria | 1948-1996 | -0.52[6] | -1.91[6] | 0.06[6] | -0.80[6] | -0.93[6] |
| Belgium | 1954-1998 | -1.27[1] | -1.34[1] | -2.22[1]* | -2.30[1] | -3.60[1]* |
| Canada | 1948-2001 | -1.88[1] | -1.34[1] | -1.20[1] | -1.73[1] | -0.43[1] |
| Cyprus | 1966-2003 | -2.67[7]* | -3.73[1]* | -2.14[7]* | -3.00[7]* | -2.94[7] |
| Denmark | 1950-2000 | -2.47[6] | -2.67[6] | -1.73[6] | -1.79[6] | -1.89[6] |
| Finland | 1950-1998 | -3.20[1]* | -3.59[1]* | -2.34[1]* | -2.46[1] | -2.93[1] |
| France | 1950-1977 | -2.22[7] | -2.48[7] | -0.90[7] | -2.12[7] | -2.09[7] |
| Germany | 1950-1998 | -3.52[1] | -3.86[1]* | -3.24[1]* | -3.73[1]* | -3.93[1]* |
| Greece | 1951-1999 | -2.41[7] | -1.85[7] | -4.09[7]* | -4.90[7]* | -5.17[7]* |
| Iceland | 1948-2005 | -0.08[5] | 0.34[5] | -1.69[5] | -0.36[5] | -1.04[5] |
| Ireland | 1948-1999 | -1.02[1] | -1.09[1] | -1.32[1] | -0.88[1] | -1.03[1] |
| Israel | 1957-2001 | -1.95[1] | -2.89[1] | -2.02[1]* | -3.25[1]* | -3.62[1]* |
| Italy | 1951-1998 | -1.69[7] | 0.27[7] | -1.49[7] | -1.83[7] | -1.15[7] |
| Japan | 1955-1993 | -2.27[3] | -1.92[3] | -1.09[3] | -1.95[3] | -2.59[3] |
| Korea | 1954-1997 | -4.51[3]* | -4.36[3]* | -4.01[3]* | -4.27[3]* | -3.51[3]* |
| Luxembourg | 1966-1997 | -3.60[8]* | -3.51[8]* | -0.89[8] | -0.73[8] | -0.78[8] |
| Netherlands | 1956-1998 | -1.88[8] | -1.62[8] | -1.24[8] | -1.32[8] | -1.78[8] |
| New Zealand | 1950-2000 | -1.96[9] | -2.08[9] | -1.64[9] | -2.63[9] | -2.22[9] |
| Norway | 1954-2003 | -2.43[5] | -2.89[5] | -1.54[5] | -1.40[5] | -2.22[5] |
| Portugal | 1970-1998 | -1.56[1] | -1.75[1] | -1.18[1] | -1.77[1] | -1.75[1] |
| P.R.of China | 1978-2003 | -2.53[4] | -3.60[4]* | -1.96[4]* | -3.41[4]* | -3.52[4]* |
| Singapore | 1963-2005 | -1.84[1] | -2.00[1] | -1.14[1] | -2.03[1] | -2.52[1] |
| Spain | 1962-1999 | -2.78[3]* | -3.89[3]* | -1.82[3] | -2.53[3] | -3.68[3]* |
| Sweden | 1950-2005 | -3.68[2]* | -3.59[2]* | -4.39[2]* | -5.24[2]* | -5.33[2]* |
| Switzerland | 1948-2005 | -2.95[4]* | -3.76[4]* | -1.01[4] | -0.96[4] | -0.89[4] |
| U.K. | 1948-1999 | -2.35[8] | -2.65[8] | -1.02[8] | -2.65[8] | -2.71[8] |
| U.S. | 1959-2005 | -2.75[1]* | -2.69[1] | -1.58[1] | -2.37[1] | -2.09[1] |
| **10% Critical Value** | | **-2.57** | **-3.12** | **-1.92** | **-2.66** | **-3.13** |

Note: Critical values come from Kapetanios et al. (2003, p. 364).
Numbers inside the brackets are number of augmented lags.