E C O N O M I C S   B U L L E T I N

Optimal income taxation and the shape of average tax rates

Jean HINDRIKS
Université Catholique de Louvain − Department of Economics and CORE

Etienne LEHMANN
ERMES − Université Paris 2, Université Catholique de Louvain − Department of Economics and IZA

Alexis PARMENTIER
Université Catholique de Louvain − Department of Economics

Abstract

We investigate whether the optimal income tax model of Mirrlees (1971) can reproduce the empirically observed increasing average tax rates. We give a necessary condition and exhibit two examples where the optimal average tax rates are increasing.

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1 Introduction

Since Mirrlees (1971), the main results in the optimal income taxation literature concern marginal tax rates. This note focuses on average tax rates (defined as the ratio of taxes minus benefits over gross earnings), a dimension that has drawn little attention yet. Actually, in most OECD countries, average tax rates are increasing over the whole income distribution (see e.g. OECD Tax Database and Immervoll (2004)). This fact is an important property to explain since it characterizes the redistributive power of the tax/benefit system. Our purpose is to investigate whether increasing average tax rates can be justified by the optimal income tax model.

Our analysis points out two analytical results. First, an unbounded distribution function is a necessary condition for increasing optimal average tax rates. Our argument is therefore complementary to the ones put forward by Diamond (1998) and Saez (2001) who shed light on the usefulness of an unbounded distribution function in the analysis of optimal income taxation problems [1]. Second, we emphasize two analytical examples of increasing average tax rates. These examples are characterized by Pareto unbounded distribution functions, a maximin social welfare function and quasi-linearity of the utility function either in consumption (with an isoelastic labor supply) or in leisure.

2 The general framework

We consider the typical Mirrlees' (1971) model. It is a one period model with labor as the only source of income. Individuals differ only in their exogenous productivity denoted by $w \in [\underline{w}, \overline{w}]$ with $0 \leq \underline{w} < \overline{w} \leq +\infty$. $F(.)$ denotes the cumulative distribution function of w in the population and $f(.)$ is its corresponding continuous density. The labor market is perfectly competitive and the different types of labor are perfect substitutes in the aggregate production function. All the agents have the same utility function $U(C, L)$ over a single consumption good C and the labor supply L. For the moment, we only assume that the utility function is "standard": $U(.,.)$ is twice differentiable, strictly quasi-concave and yields $U_C > 0$ and $U_L < 0$.

The government can observe neither agent's productivity w nor their labor supply L. Taxes rely thus on before tax earnings $Y = w \cdot L$. Each agent takes the tax function $T(.)$ as given when she chooses her labor supply:

$$L(w) = \arg \max_L U(w\ L - T(w\ L), L) \tag{1}$$

[1] Their argument relies on the properties of the observed income distribution function and on the observed positive marginal tax rates for top incomes.

The after-tax utility of agent w is given by:

$$\mathcal{U}(w) \equiv U\left(Y(w) - T(Y(w)), \frac{Y(w)}{w}\right)$$

where we define $Y(w) \equiv w \cdot L(w)$. Deriving the average tax rates $T_M(Y) = T(Y)/Y$ with respect to before-tax earnings Y gives:

$$T'_M(Y) = \frac{T'(Y) - T_M(Y)}{Y} \tag{2}$$

where $T'(Y(w))$ denotes the marginal tax rate. Therefore, average tax rates are increasing if and only if marginal tax rates are above average taxes $T'(Y) > T_M(Y)$. The government's budget constraint writes:

$$\int_{\underline{w}}^{\overline{w}} T(Y(w))\ dF(w) \geq 0 \tag{3}$$

Since each individual has the same utility function, inequalities are not due to agent's responsibility. Therefore, the government wishes to redistribute income from high to low productive individuals. For the moment we assume a welfarist and redistributive objective: the government's objective is (weakly) increasing in agents' utility $\mathcal{U}(w)$, and gives a higher weight to less productive agents. These assumptions are consistent with different specification of the government's objective, including a "weighted utilitarianism" objective of the form:

$$W = \int_{\underline{w}}^{\overline{w}} a(w) \cdot \mathcal{U}(w) \cdot dF(w)$$

where $a(w)$ is a non-negative and non-increasing function of w, or a maximin (Rawlsian) objective

$$W = \min \mathcal{U}(w)$$

or a "Bergson-Samuelson" objective of the form:

$$W = \int_{\underline{w}}^{\overline{w}} \Psi(\mathcal{U}(w))\, dF(w)$$

where $\Psi(.)$ is increasing and concave. The government's problem consists in maximizing its objective by choosing the tax schedule $T(.)$ under the constraint of individual reactions given by (1) and the budget constraint (3).

The optimal tax schedule emerges from an efficiency-equity trade off. Consider that the tax schedule has been optimized for all workers up to type w. A rise in the marginal tax rates of agents of productivity w distorts their labor supply and generates an efficiency loss. However, a rise in this marginal tax rate increases the tax level for more productive agents $\tilde{w} > w$. This implies an income transfer from agents of productivity $\tilde{w} > w$ to agents of productivity $\hat{w} \leq w$. Since the formers contribute less to the government's objective than the latters, such a shift is positively valued by the government. This is the equity part of the trade off. The literature has put forward few analytical results on tax schedules emerging from this trade off. Under weak conditions on preferences [2], it has been shown that (see Seade (1982) or Werning (2000) for details and derivation) optimal marginal tax rates lie between 0 and 1 and are not always equal to zero. Henceforth, we assume that these conditions are verified.

3 Bounded productivity distribution

In this section we assume the productivity distribution is bounded, so $\bar{w} < +\infty$. In such a case, the optimal marginal tax rate at the top is nil (i.e. $T'(Y(\overline{w})) = 0$) since a positive marginal tax rate would have no distributional effect but would distort efficiency. Therefore, we get the following result:

Proposition 1 *If the productivity distribution is bounded, optimal average tax rates are decreasing close to the top of the distribution function.*

Proof. *Since marginal tax rates lie between* 0 *and* 1*, the maximum level of tax is* $T(Y(\overline{w}))$ *so from (3) one gets:*

$$T(Y(\overline{w})) > \int_{\underline{w}}^{\overline{w}} T(Y(w))\, dF(w) \geq 0$$

Since $T'(Y(\overline{w})) = 0$, *one gets from (2)* $T'_M(Y(\overline{w})) < 0$. *By continuity, in the neighborhood of* $Y(\overline{w})$, *one still has* $T'_M(Y) < 0$. ■

As a consequence, the optimal tax model is never characterized by an increasing pattern of average tax rates when the productivity distribution is bounded. We now turn to the case with unbounded productivity distribution.

4 Unbounded productivity distribution

In this section, we argue that it is possible to generate analytically increasing optimal average tax rates in a Mirrlees optimal income tax problem when the productivity distribution is unbounded. Diamond (1998) and Saez (2001) argue that an unbounded Pareto distribution represents a good approximation for the upper part of the productivity distribution in the US. We

[2] The two weak conditions are the single crossing property (called also the agent monotonicity condition) and the normality of leisure.

therefore assume an unbounded Pareto productivity distribution. Hence, the ratio

$$\frac{1-F(w)}{w\ f(w)}=p \tag{4}$$

is henceforth constant, $p>0$ denoting this constant. To get closed form solutions, we make further assumptions. First, we assume that the government has maximin preferences. Second, we analyze alternatively two popular assumptions on agents' preferences: quasi-linearity in consumption and quasi-linearity in leisure.

4.1 Quasi-linear preferences in consumption

Following Diamond (1998), Piketty (1997) and Salanié (2003) we omit the income effects in the labor supply by considering quasi-linear preferences in consumption. The utility function writes :

$$U(C,L)=C-b\cdot L^{1+\frac{1}{\varepsilon_L}}$$

where $\varepsilon_L>0$ stands for the constant wage elasticity of the labor supply and $b>0$ is a normalization parameter. The optimal marginal tax rates are given by (see e.g. Piketty (1997) or Salanié (2003, pages 84-87) for a derivation and an interpretation):

$$\frac{T'(Y(w))}{1-T'(Y(\overline{w}))}=\left(1+\frac{1}{\varepsilon_L}\right)\cdot\frac{1-F(w)}{w\ f(w)}$$

Therefore, from (4), the optimal marginal tax rates are constant and positive, the optimal tax function is linear and writes:

$$T^*(Y)=\tau\cdot Y+T_0 \tag{5}$$

and the optimal average tax rate is:

$$T^*_M(Y)=\tau+\frac{T_0}{Y} \tag{6}$$

where:

$$\tau=\frac{p(1+\varepsilon)}{\varepsilon+p(1+\varepsilon)}\in(0,1)$$

>From equation (6), average tax rates are therefore increasing if and only if $T_0<0$. Since the marginal tax rate is constant and positive, the function $T(.)$ is increasing and from the budget constraint (3), one has necessarily $T(0)\leq T(Y(\underline{w}))<0<T(Y(\overline{w}))$. Since $T_0=T(0)$, the whole shape of average tax rate is increasing.

4.2 Quasi-linear preferences in leisure

When preferences are of the form $U(C, L) = v(C) - L$ with $v'(.) > 0$ and $v''(.) < 0$, and the government has maximin preferences, the optimal marginal tax rates are given by (see e.g. Boadway *et alii* (2000), equation (18) page 448):

$$T'(Y(w)) = \frac{1 - F(w)}{w\ f(w)}$$

Again, under a Pareto productivity distribution, the optimal tax function if linear of the form (5) and the shape of average tax rate if of the form (6) with

$$\tau = p \in (0, 1)$$

We therefore replicate the preceding argument to get $T_0 < 0$, which implies that optimal average tax rates are increasing.

Our results are summarized in the following proposition :

Proposition 2 *If the productivity distribution is Pareto unbounded, the government has maximin preferences and agents have a utility function that is either quasi-linear in consumption (with an isoelastic labor supply) or quasi-linear in leisure, optimal average tax rates are increasing.*

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