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On strategy and the likelihood of success in marital matchmaking under uncertainty

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Abstract

Individuals wishing to get married have made increasing use of matchmakers. This notwithstanding, economists have paid scant attention to the strategies employed by matchmakers and to the likelihood of success arising from the use of these strategies. Consequently, we first specify a “local” and then a “global” strategy for matching male and female clients and then we compute the expected total cost to a matchmaker from the use of these strategies. Next, we calculate the mean number of successes that our matchmaker can hope for. Finally, we provide an upper bound on the probability that the number of matching successes is at least 1+ ¸ times the mean number, where ¸ is any positive number.

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1. Introduction

Arranged marriages have traditionally been more popular in the East than in the West. As pointed out by Blood (1960), Moore (1994), Batabyal (2001), and Batabyal and Beladi (2002), in arranged marriages, it is common to use matchmakers.[1] Matchmakers typically meet friends, family members, and increasingly the individuals wishing to get married, and they then attempt to pair male and female candidates with similar aspirations, goals, and interests. Clearly, the matchmaker's objective is to ensure that the paired individuals do in fact get married and that this marriage lasts for an appreciable amount of time. It is important to note that matchmaking activities are fundamentally prospective; further, all matchmakers operate in inherently stochastic environments.

Until recently, most marrying individuals in the West took upon themselves the task of finding a suitable mate. Consequently, matchmaking activities in general were rather limited. However, in the past two decades, owing to a variety of reasons not the least of which is a general lack of time, this state of affairs has changed substantially. As a result, today, even in the West, it is quite common to find a plethora of matchmakers advertising their services in newspapers and on the internet.[2] Given the traditional use of matchmakers in the East and the increasing popularity of matchmakers in the West, a number of interesting questions about the activities of these matchmakers emerge. Examples of such questions include the following. What are the properties of alternate matchmaking strategies? Given a particular matching strategy, what is the expected number of successes that a matchmaker can hope for? Finally, given a desired number of successes, is it possible to make a mathematically precise statement about the probability that the number of matching successes will be at least the desired number? Although these questions are both thought-provoking and relevant, unfortunately, economists have paid virtually no attention to them.

Quah (1990) has discussed the phenomenon of matchmaking but the basic focus of his paper is on analyzing the factors influencing the age at first marriage. More recently, Van Raalte and Webers (1998) have studied a two-sided market in which one type of agent needs the services of a matchmaker in order to be matched to the other type. In this setting, these authors analyze a scenario in which matchmakers compete for agents of both types by means of commission fees. Finally, in a model of two-sided search, Bloch and Ryder (2000) have shown that when a matchmaker charges a uniform participation fee, only agents of higher quality participate in the centralized matching procedure. In contrast, if the matchmaker charges a commission on the matching surplus, then only agents of lower quality go to this intermediary. Although these papers have certainly advanced our understanding of issues related to matchmaking, nonetheless, the questions mentioned in the previous

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In the rest of this paper, we suppose that our matchmaker is a single male individual. Even so, we recognize that the matchmaker may be a single female individual or even a firm. In this regard, the point to note is that except for minor stylistic changes, nothing in our analysis is altered by accounting for these last two possibilities.

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See the "Personals" section of the *New York Times*, the *Boston Globe* and internet sites such as www.Udate.com and www.match.com paragraph remain unanswered.

Consequently, our paper has two objectives. First, we specify two desirable strategies—the "local" strategy and the "global" strategy—for matching male and female clients and then we compute the expected total cost to a matchmaker from the use of each of these strategies.[3] Next, we calculate the expected number of matching successes that a matchmaker can hope for and then we provide an upper bound on the probability that the number of matching successes is at least $1+\theta$ times the mean number, where θ is any positive number. The rest of this paper is organized as follows. Section 2 describes the theoretical framework and the two matchmaking strategies that comprise the subject of this paper. Section 3 computes the average total cost to a matchmaker from the use of each of these strategies. Section 4 calculates the mean number of matching successes and then it shows how the upper bound discussed above can be derived. Section 5 concludes and discusses avenues for further research on the subject of this paper.

2. The Theoretical Framework

Consider a matchmaker who has a number of male and female clients who wish to get married. Specifically, there are n male and n female clients and our matchmaker's job is to match each male client to one and only one female client. Now, the task of matching male and female clients involves the expenditure of some—and possibly considerable—effort on the part of the matchmaker. Put differently, every time the matchmaker assigns a male client to a female client, he incurs a cost. To this end, let $c(j,k)$ be the cost incurred by our matchmaker when he matches male client j to female client k, $j,k=1,...,n$.

Clearly, there are many possible strategies that our matchmaker could use to carry out the task of assigning each male client to one female client. However, to fix ideas, we shall consider the following two desirable strategies in this paper. The first or "local" matching strategy works as follows. The matchmaker begins by assigning male client 1 to the female client that results in the lowest cost to him. In other words, male client 1 is matched with female client k_1, where $c(1,k_1)=\min_{\{k\}}c(1,k)$. Female client k_1 is then removed from further consideration. Then, the matchmaker assigns male client 2 to female client k_2 so that $c(2,k_2)=\min_{\{k\neq k_1\}}c(2,k)$. The matchmaker continues in this manner until all male and female clients have been matched. This local strategy is desirable because it always selects the best female match for the male client currently under consideration.

Our matchmaker's second desirable strategy is a "global" version of the previous paragraph's local strategy. Using this global strategy, the matchmaker first considers all n^2 cost values and he selects the pair (j_1,k_1) for which his cost $c(j,k)$ is minimal. The matchmaker then matches male client j_1 to female client k_1. Next, he eliminates from further consideration all cost values that involve either male client j_1 or female client k_1. As a result, $(n-1)^2$ cost values now remain and our matchmaker continues the process of selecting pairs and then matching as just described. Put differently, at every

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These strategies are variants of the so-called "greedy algorithms." For more on these algorithms and related issues such as the assignment problem, see Winston (1997) and Ross (2000, 2002).

stage, he chooses the male and the female clients that result in the lowest cost among all the unmatched male and female clients. We have already explained why the previous paragraph's local strategy is desirable. Simply put, this global strategy is desirable because it is a more thorough version of the local strategy. Our task now is to compute the average total cost incurred by our matchmaker when he uses each of these two strategies.

3. The Local and the Global Expected Total Costs

As indicated in section 1, our matchmaker operates in a stochastic environment. To model this aspect of the problem, we let all the cost values $c(j,k)$ comprise a set of independent random variables. Now, when analyzing greedy type algorithms, it is common to work with exponential random variables.[4] Consequently, in the remainder of this section, we suppose that the n^2 cost values $c(j,k)$ constitute a set of independent, exponentially distributed random variables with rate β.

3.1. The Local Expected Total Cost

Given that our matchmaker is using the local strategy, let $c(j,\cdot)$ denote the cost associated with matching male client j, $j=1,\ldots,n$. It should be clear to the reader that $c(1,\cdot)$ is the minimum of n independent exponential random variables, each of which has rate β. Hence, using equation 5.6 in Ross (2000, p. 249), it follows that $c(1,\cdot)$ is itself exponentially distributed with rate βn. Similarly, $c(2,\cdot)$ is the minimum of $n-1$ independent exponential random variables with rate β and hence $c(2,\cdot)$ is exponentially distributed with rate $\beta(n-1)$. Continuing in this manner we can tell that $c(j,\cdot)$ is exponentially distributed with rate $\beta(n-j+1)$, $j=1,\ldots,n$.

Using the above information, we conclude that the expected total cost to the matchmaker when he uses the local strategy, $E_l[total\ \ cost]$, is

$$E_l[total\ \ cost]=E[c(1,\cdot)+\ldots+c(j,\cdot)+\ldots+c(n,\cdot)]=E_l[c(1,\cdot)]+\ldots+E_l[c(j,\cdot)]+\ldots+E_l[c(n,\cdot)].\ (1)$$

Now, using the properties of the exponential distribution, the n expectations on the right-hand-side (RHS) of equation (1) can be simplified. This simplification yields

$$E_l[total\ \ cost]=\frac{1}{\beta n}+\ldots+\frac{1}{\beta(n-j+1)}+\ldots+\frac{1}{\beta}=\frac{1}{\beta}[\sum_{j=1}^{j=n}\frac{1}{j}]. \qquad (2)$$

The RHS of equation (2) gives us the expected total cost to our matchmaker when he uses the local strategy to match his male and female clients. We see that this cost is the product of the mean of the exponentially distributed cost values and a summation term. We now compute our matchmaker's average total cost when he uses the global strategy.

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See Ross (2000, p. 250; 2002, p. 36) for additional details on this point.

3.2. The Global Expected Total Cost

Let $c(j,\cdot)$ be the cost of the *jth* male-female pair matched by this global strategy. Because $c(1,\cdot)$ is the minimum of all the n^2 $c(j,k)$ cost values, using equation 5.6 in Ross (2000, p. 249) we can tell that $c(1,\cdot)$ is exponentially distributed with rate βn^2. Now, because exponentially distributed random variables have the memoryless property,[5] we reason that the amounts by which the other $c(j,k)$ exceed $c(1,\cdot)$ will be independent and exponentially distributed random variables with rates β. Hence, $c(2,\cdot)$ equals the sum of $c(1,\cdot)$ and the minimum of $(n-1)^2$ independent exponentials with rate β. Similarly, $c(3,\cdot)$ is equal to the sum of $c(2,\cdot)$ and the minimum of $(n-2)^2$ independent exponentials with rate β, and so on.

Now, using the above reasoning and the properties of exponentially distributed random variables, we infer that $E[c(1,\cdot)]=1/(\beta n^2)$, $E[c(2,\cdot)]=E[c(1,\cdot)]+1/\{\beta(n-1)^2\}$, and $E[c(3,\cdot)]=E[c(2,\cdot)]+1/\{\beta(n-2)^2\}$. Continuing this line of reasoning, we get $E[c(k,\cdot)]=E[c(k-1,\cdot)]+1/\{\beta(n-k+1)^2\}$ and finally $E[c(n,\cdot)]=E[c(n-1,\cdot)]+1/\beta$. These expressions for the various cost expectations can be simplified even further. This simplification yields $E[c(1,\cdot)]=1/(\beta n^2)$, $E[c(2,\cdot)]=1/(\beta n^2)+1/\{\beta(n-1)^2\}$, and eventually $E[c(n,\cdot)]=1/(\beta n^2)+1/\{\beta(n-1)^2\}+...+1/\beta$.

Using the above computations, we conclude that the expected total cost to our matchmaker when he uses the global strategy, $E_g[total\ \ cost]$, is

$$E_g[total\ \ cost]=\frac{n}{\beta n^2}+\frac{(n-1)}{\beta(n-1)^2}+\frac{n-2}{\beta(n-2)^2}+...+\frac{1}{\beta}. \tag{3}$$

Further simplifying the RHS of equation (3), we get

$$E_g[total\ \ cost]=\frac{1}{\beta}[\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+...+1]=\frac{1}{\beta}[\sum_{j=1}^{j=n}\frac{1}{j}]. \tag{4}$$

Inspecting equations (2) and (4), it is clear that we have just established
THEOREM 1: *The expected total cost to our matchmaker is the same for both strategies*.

Theorem 1 contains a rather surprising result. Specifically, this theorem tells us that it does not matter which strategy our matchmaker uses because both strategies lead to the same total expected cost. Intuitively, one expects greater thoroughness on the part of the matchmaker to increase his costs. However, in the setting of this paper, this is not the case. Having said this, the reader should note that greater painstakingness does not, however, result in a lower expected total

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5 For more on this, see Ross (2000, pp. 243-245; 2002, p. 33).

cost. Consequently, from the standpoint of the expected total cost criterion, our matchmaker will be indifferent between the local and the global strategies.

What is the expected number of successes that our matchmaker can hope for from either strategy? Moreover, given a desired number of successes, is it possible to say something mathematically precise about the probability that the number of matching successes is at least the desired number? We now address these two questions.

4. Success in Matching: Two Questions

Before we proceed any further with the above two questions we must first delineate the meaning of a success. Recall that the whole point of matchmaking is to ensure that marriages eventually take place. Consequently, in what follows, we shall say that a match is a success if it leads to marriage within T time periods. The actual value of T will typically vary from society to society and, *ceteris paribus*, we expect T to be shorter in Eastern societies than in Western societies. This notwithstanding, it is clear that there has to be a temporal dimension to the meaning of success. Moreover, it is also clear that for it to be interesting, an analysis of the "expected number of matching successes" question must be conducted from an *ex ante* perspective and not T time periods after the n male-female pairings have been made.

4.1. The Expected Number of Successes

Suppose that our matchmaker observes the n matches that he has just made. Also suppose that our matchmaker's skill is such that the probability that each match is a success (will lead to marriage in T time periods) is $p \in (0,1)$. What is the expected number of successes? To answer this question, it is useful to think of the n matches as n independent Bernoulli random variables.[6] Now, let $S(n)$ denote the number of successes from n matches. Then, using the properties of Bernoulli random variables, we see that

$$E[S(n)] = np. \tag{5}$$

In words, the average number of successes is given by the product of the number of matches and the success probability of each match. Inspecting equation (5), it is clear that holding the number of matches fixed, the expected number of successes is an increasing function of the success probability p. Similarly, keeping the success probability p fixed, as we increase the number of matches, the expected number of successful matches rises. Let us now address the question about the probability that the number of matching successes is at least some desired number.

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See Ross (2000, pp. 27-28; 2002, pp. 6-7) for more on Bernoulli random variables.

4.2. Computing the Upper Bound

From section 4.1 we know that the expected number of matching successes is np. Now given this expected number, suppose that our matchmaker desires a certain number of successes. We would like to make a mathematically precise statement about the probability that the actual number of successes is at least the desired number times the expected number of successes np.

To address this question, let us begin by letting $\theta>0$ be any positive number. Further, suppose that our matchmaker's desired number of successes is $1+\theta$ times the expected number of successes. We will now provide an upper bound[7] on the probability that the desired number of successes is $1+\theta$ times the expected number of successes. Using corollary 3.1.2 in Ross (2002, p. 77), we see that

$$Prob\{S(n)-E[S(n)]\geq\theta E[S(n)]\}\leq\exp\{-2(\theta E[S(n)])^2/n\}. \tag{6}$$

Because $E[S(n)]=np,$ the RHS of inequality (6) can be simplified. This gives

$$Prob\{S(n)-E[S(n)]\geq\theta E[S(n)]\}\leq\exp\{-2n(p\theta)^2\}. \tag{7}$$

In words, the probability that the number of matching successes is at least $1+\theta$ times the expected number of successes np is bounded above by the exponential term on the RHS of inequality (7). In particular, this probability is at most as large as the reciprocal of the exponential raised to $2n(p\theta)^2$. Inspecting inequality (7) we see that holding the number of matches n and the success probability p fixed, the probability of interest decreases to zero exponentially fast as θ increases. This tells us that if we use the expected number of successes np as our benchmark, then there is a tradeoff between a higher desired number of matching successes and the probability that these desired successes will in fact materialize.

5. Conclusions

In this paper we analyzed three hitherto unstudied questions about the nature of decision making in marital matchmaking. First, we established the counterintuitive result (see Theorem 1) that the local and the global strategies both lead to the same expected total cost to our matchmaker. Second, we showed that the expected number of successes that our matchmaker can hope for is given by the product of the number of matches n and the success probability p. Finally, we pointed out that given a desired number of matching successes, it is possible to provide an upper bound on the probability that the actual number of successes is at least this desired number.

The analysis in this paper can be extended in a number of directions. In what follows, we suggest two possible extensions. First, in section 3, we studied the expected total cost to our matchmaker resulting from the use of local and global strategies. Although these strategies are

[7] This bound is also called the Chernoff bound. See Ross (2002, pp. 76-78) for additional details.

desirable in the sense indicated in section 3, it would nonetheless be useful to determine the set of matches that minimizes the sum of the n costs that are incurred.

Second, it would also be useful to study the matchmaking function within the context of a model of common agency. In such a model, the matchmaker would be the common agent serving two principals, namely, a representative male client and a representative female client. An analysis of these aspects of the problem will allow richer analyses of the nexuses between alternate pairing strategies and the outcome of matchmaking in stochastic environments.

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