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The Johansen Test and the Transitivity Property

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Abstract

Sometimes two variables Y and Z are each cointegrated with another variable X, but Y and Z do not appear to be cointegrated with each other. This article provides a possible explanation why this might happen.

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1. Introduction.

Looking for cointegration between variables is currently widespread in empirical economics, e.g. to find relationships among non-stationary variables, to test for convergence, to look at causality among variables, etc. In such research, the Johansen technique has been accepted as a powerful way to test for cointegration –justified by the works of Phillips (1991) and Gonzalo (1994), among others. Nevertheless, this technique can sometimes produce results that appear to be counter-intuitive. One of these outcomes is related to the transitivity property.

Intuitively, one would expect that if two variables Y and Z are each cointegrated with another variable X, then the variables Y and Z should be cointegrated with each other. However, the Johansen test for cointegration does not always fulfil this transitivity property as sometimes the variables Y and Z will not appear to be cointegrated according to this test. In this article we attempt to offer a possible explanation for such a result. In section 2 we provide an example where the above mentioned paradox arises and in section 3 we discuss possible interpretations of this result. Section 4 draws the conclusions.

2. An illustration of the paradox.

In this section we look at the cointegration trace tests for weekly exchange rates of three currencies that belonged to the European Monetary System (EMS) and that have recently been replaced by the euro. The data consists of weekly exchange rates of the Belgian franc (BF), the French franc (FF) and the German mark (DM), all expressed in terms of the US dollar, in natural logarithms. The data runs from January 1980 to May 1996[1], totalling 856 observations. By carrying out the analysis of these exchange rates in terms of the US dollar we avoid the possibility of structural breaks that would no doubt be present if we expressed the exchange rates in ECU or any other EMS rate, given the realignments that occurred throughout the 1980s in the EMS.

The first step in the analysis is to pre-test each variable to determine its order of integration. Augmented Dickey-Fuller (ADF) tests for a lag length of one have been carried out on the variables in levels and in first differences. ADF tests and Phillips Perron (PP) tests have also been carried out on the variables, with the optimal lag length for the ADF test chosen to minimise the AIC - which gives a lag length for all the variables equal to 2. The results are shown in table 1 below. It can be seen that in all the tests, the null hypothesis of a unit root could not be rejected. Therefore, the series appear to be I(1).

The next step is to test for cointegration with Johansen´s tests. In order to do this, we need first to determine what will be the order of the VAR with which we will test for cointegration. There are various criteria designed to aid in choosing the order of a VAR, such as the Akaike’s information criterion (AIC) or the Schwartz criterion (SC), which look at the goodness of fit of the VAR after a correction for degrees of freedom. We followed the Schwartz criterion (SC), which was applied to the unconstrained VAR in levels. This criterion suggested the use of one lag in the analysis[2].

[1] This data provides an example of the paradox. The currencies used were irrevocably fixed from 1997.
[2] The AIC criterion indicated the use of three lags in the analysis. We carried out the cointegration tests using three lags, and the cointegration results were similar to those shown here using one lag.

The bilateral test statistics reported in Table 2 indicate that the BF and FF are each cointegrated with the DM at the 5 % significance level or better, but the BF and the FF do not appear to be cointegrated. Further, tests for the three variables were also carried out (not reported here) and there was evidence at the 5% significance level of the presence of two cointegrating vectors.

3. Interpretation.

The results from the cointegration tests may appear counter-intuitive. Table 2 provides evidence that the BF and the FF are each cointegrated with the DM, but they are not bilaterally cointegrated with each other. Intuitively, one would expect that if two variables *Y* and *Z* (i.e., FF and BF) are each cointegrated with another variable *X* (i.e., the DM), then *Y* and *Z* should be cointegrated with each other. Our interpretation of these results is that the DM has been the causal stochastic trend for the whole sample. In other words, the BF and FF have individually converged to the DM, this latter currency being the common stochastic trend. However, the interplay of the error terms of the relationships between the DM and the FF on one side, and between the DM and the BF on the other side may explain the different behaviour of the test when analysing the relationship between the FF and the BF. To illustrate this, let us represent the relations between the exchange rates in the following manner:

$$FF_t = \alpha_1 DM_t + \varepsilon_{1t} \quad \text{with } \varepsilon_{1t} \sim N(0,\sigma_1^2) \tag{1}$$

and

$$BF_t = \alpha_2 DM_t + \varepsilon_{2t} \quad \text{with } \varepsilon_{2t} \sim N(0,\sigma_2^2) \tag{2}$$

Let us assume that ε_{1t} and ε_{2t} are not contemporaneously correlated. Solving for DM_t in (2) and substituting this in (1) we obtain:

$$FF_t = \frac{\alpha_1}{\alpha_2} BF_t + \varepsilon_{1t} - \frac{\alpha_1}{\alpha_2}\varepsilon_{2t} = \frac{\alpha_1}{\alpha_2} BF_t + \omega_t \tag{3}$$

where ω_t is the linear combination of the two error terms ε_{1t} and ε_{2t}. The term ω_t follows a normal distribution with mean 0:

$$E\left(\varepsilon_{1t} - \frac{\alpha_1}{\alpha_2}\varepsilon_{2t}\right) = 0 \tag{4}$$

and variance:

$$E\left(\varepsilon_{1t} - \frac{\alpha_1}{\alpha_2}\varepsilon_{2t}\right)^2 = \sigma_1^2 + \left(\frac{\alpha_1}{\alpha_2}\right)^2 \sigma_2^2 \tag{5}$$

and, thus, FF and BF must cointegrate. Note that the variance of the error term in the cointegrating relation between FF and BF (5) will always be bigger than the variance of the error terms in expressions (1) and (2) if the ratio $\frac{\alpha_1}{\alpha_2}$ is equal -or bigger- than 1. In the particular example used in the paper, we cannot reject the hypothesis that $\frac{\alpha_1}{\alpha_2}$ = 1[3].

In this case, the fact that the variables FF and BF are both cointegrated with the DM and the coefficient is unity, implies that the variance of the relationship between the FF and BF is affected, increasing in magnitude, and, thus, affecting the power of the test.

In order to support this interpretation, we carried out a Monte Carlo simulation where three variables were created to simulate the behaviour of the DM, FF and BF, each with 850 observations. The variable simulating the behaviour of the DM, X_t, was created as a random walk:

$$X_t = X_0 + \sum_{i=1}^{850} \varepsilon_i \tag{6}$$

with $\varepsilon_i \sim N(0, 0.017^2)$ and $X_0 = 0.5$. The variance of the error term and the initial value for X_t were chosen to approximate the DM series.

The variable simulating the FF, Y_t, was created from the following expression:

$$\Delta Y_t = 0.016 - 0.047 \times (0.28 Y_{t-1} - 0.27 X_{t-1}) + \varepsilon_{2t} \tag{7}$$

with $\varepsilon_{2t} \sim N(0, 0.0172^2)$. This expression corresponds to the ECM for the variable FF estimated by OLS based on cointegrating VAR(1) with the DM. The variance of the error term ε_{2t} was obtained from the variance of the residuals from the actual estimation of (7) with FF and DM.

The variable simulating the BF, Z_t, was obtained from the following expression:

$$\Delta Z_t = 0.049 - 0.041 \times (0.4 Z_{t-1} - 0.43 X_{t-1}) + \varepsilon_{3t} \tag{8}$$

with $\varepsilon_{3t} \sim N(0, 0.0171^2)$. Expression (8) corresponds to the ECM for the variable BF estimated by OLS based on a cointegrating VAR(1) with the DM. The variance of the error term ε_{3t} was obtained from the variance of the residuals from the actual estimation of (8) with BF and DM.

Figures 1, 2 and 3 represent the histograms of the trace test statistics obtained with 5000 repetitions for the bivariate cointegration between the variables X_t, Y_t and Z_t. The results from the Monte Carlo simulation show that Johansen cointegration trace tests for BF-DM would provide evidence of cointegration in 69% of the cases, and for FF-DM in 54% of the cases. However, the FF and the BF would appear not to be cointegrated 74,5% of the times, indicating that the power of cointegration trace tests may in fact be affected by the behaviour of the error terms and their variances.

[3] We carried out tests for pairs of the cointegrated variables and imposed the restriction that the coefficients α_1 and α_2 are equal to one. This would imply that, in the ECM expression for the cointegrating VAR(1) between the FF and DM, and between the BF and DM, the non-zero coefficients in the cointegrating vector should be 1 and -1. The $\chi^2(1)$ result for the FF-DM relationship is 0.05, and for the BF-DM relationship is 0.42, which are well below the 95% critical value of the χ^2 distribution with one degree of freedom. Therefore, we cannot reject the hypothesis that $\frac{\alpha_1}{\alpha_2}$ = 1.

Another possible interpretation of the results obtained is related to the concept of common stochastic trends[4]. Note that, as the coefficient between the FF and DM and between the BF and DM is unity, this would also imply long-run convergence in the sense of Bernard and Durlauf (1995). According to these authors, two series X_{1t} and X_{pt} converge pointwise if:

$$\lim_{t \to \infty} E(X_{1t} - X_{pt}) = \varepsilon_p, \forall p \neq 1 \tag{9}$$

What this implies in our particular case is that the variable X (DM) is providing the stochastic common trend for the variables Y (FF) and Z (BF). These two variables have converged toward the DM variable, that is, their difference with the DM over time has elapsed or tended to a constant. Nonetheless, the FF and the BF might not have converged to each other. An intuitive illustration is provided in figures 4, 5 and 6, which show the difference between the BF and the DM, the FF and the DM, and the BF and the FF, respectively. Figures 4 and 5 show how the differences of the BF and the FF with the DM stabilise over time, whereas the difference between the BF and the FF does not.

4. Conclusion.

In this article we looked at a paradoxical result of the Johansen test: two variables were bilaterally cointegrated with a third one, but the first two variables did not appear to be cointegrated with each other. By carrying out a Monte Carlo simulation we were able to show that, even though the two variables were in fact cointegrated, the test for cointegration was not able to pick this up due to the interplay of the error terms of the relationships between the variables. In particular, we showed that the power of the cointegration trace tests might be affected by the behaviour of the error terms and their variances.

Bibliography.

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Gonzalo, J. (1994) "Five Alternative Methods of Estimating Long-Run Equilibrium Relationships" *Journal of Econometrics* **60**, 203 – 233.

Johansen, S. (1988) "Statistical Analysis of Cointegration Vectors" *Journal of Economic Dynamics and Control* **12**, 231 – 254.

[4] Stock and Watson (1988) observed that cointegrated variables share common stochastic trends. If *n* variables are not cointegrated, then they are subject to *n* independent permanent shocks. If there is cointegration, and there are *r* cointegrating vectors, there are *n-r* independent permanent shocks, called the stochastic shocks. In our example, we believe that there is only one common trend, provided by the DM currency. This is further proved by the cointegration results for the 3 variables, as we find evidence of two cointegrating vectors.

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Phillips, P. (1991) “Optimal Inference in Cointegrated Systems” *Econometrica* **59**, 283 – 306.

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Tables.

Table 1. Unit Root Tests.

	BF	**DM**	**FF**	**95% significance**
ADF(1) tests:				
With intercept and no trend	-1.42	-0.94	-1.98	-2.86
With intercept & linear trend	-2.7	-2.7	-2.7	-3.42
First difference	-21	-21.3	-21.2	-2.86
ADF and PP tests:				
ADF(2)	-2.69	-2.7	-2.69	
Phillips-Perron	-6.9	-8.9	-6.9	-21.8

Table 2. Trace Tests.

DM – FF	DM – BF	FF – BF	95% significance	90% significance
19.5*	19.4*	15.2	17.9	15.75

We indicate a significance level at the 5% by adding a (*) to the tests numbers.

Figures.

Figure 1. Histogram of the trace test for X_t and Y_t (DM and FF).

Number of rejections of cointegration: 2314 (46.3%). Number of acceptances: 2686 (53.7%).

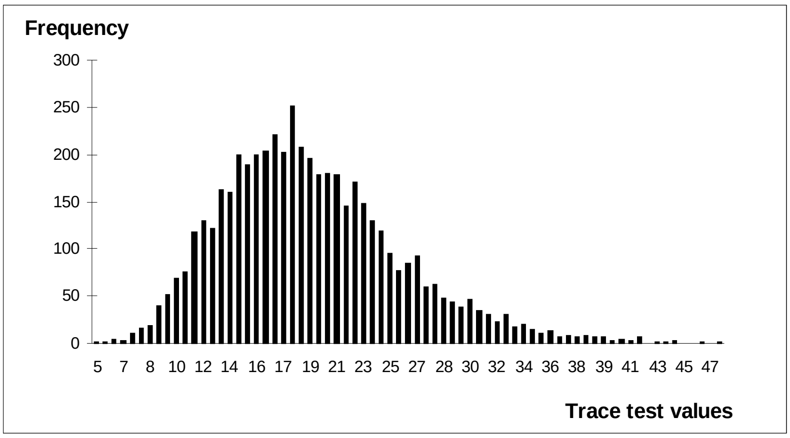

Figure 2. Histogram of the trace test for X_t and Z_t (DM and BF).

Number of rejections of cointegration: 1528 (30.6%). Number of acceptances: 3472 (69.4%).

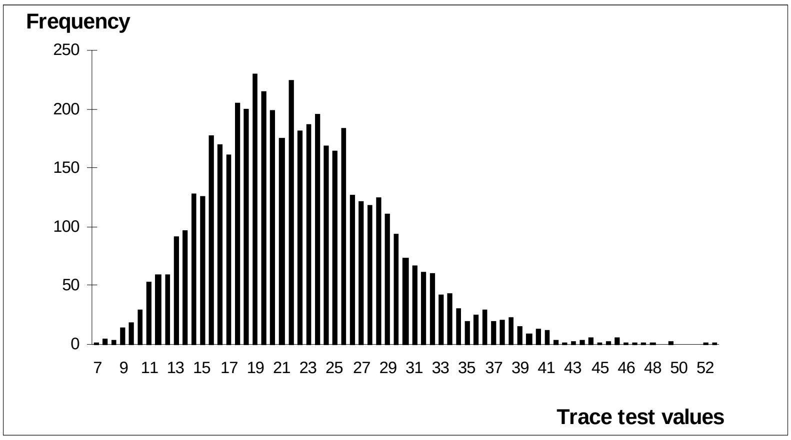

Figure 3. Histogram of trace test for Y_t and Z_t (FF and BF).

Number of rejections of cointegration: 3727 (74.5%). Number of acceptances: 1273 (25.5%).

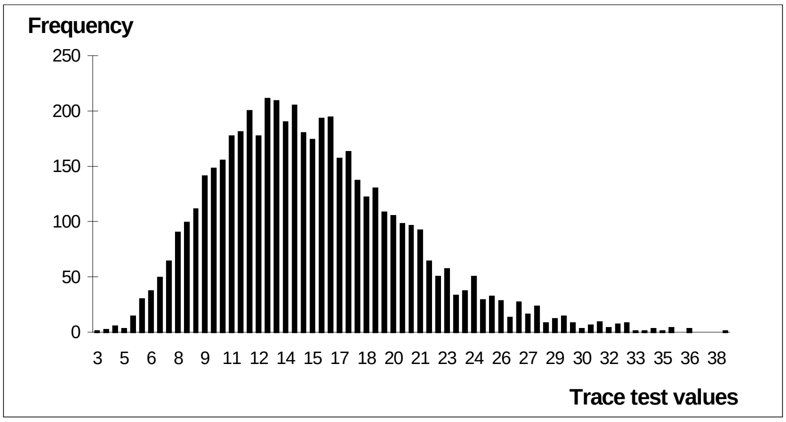

Figure 4. Difference between Belgian Franc and Deutsche Mark.

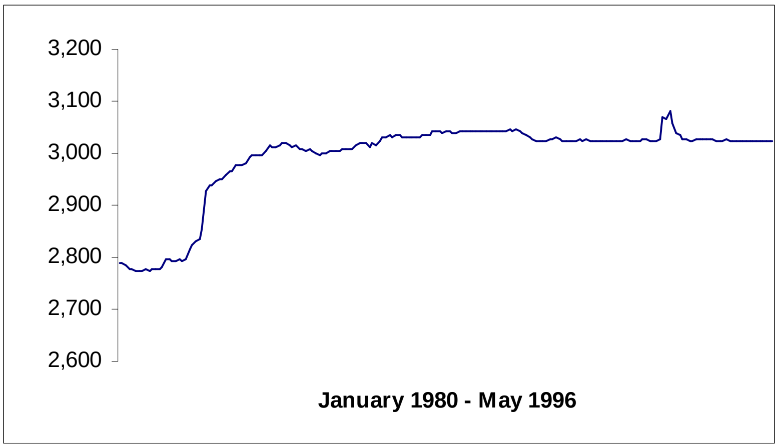

Figure 5. Difference between French Franc and Deutsche Mark.

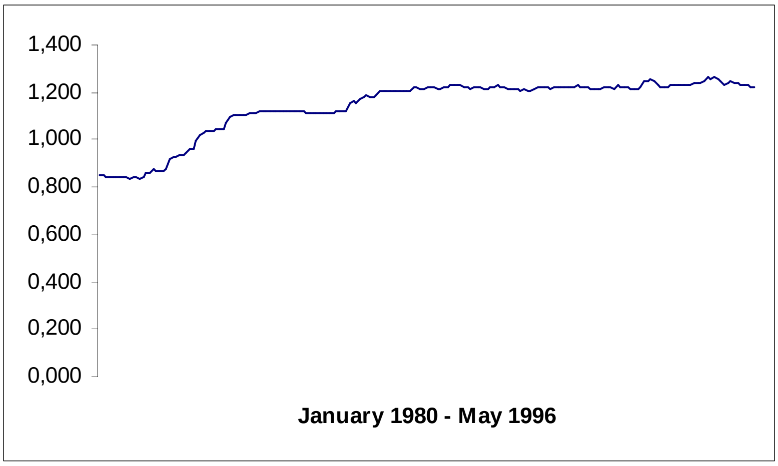

Figure 6. Difference between Belgian Franc and French Franc.