E C O N O M I C S B U L L E T I N

Political instability, public investment and macroeconomic performance

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Abstract

This paper attempts to provide a framework to explain both the lower share of current spending in large fiscal adjustments and the potential expansionary effects of fiscal contractions. We distinguish between current spending and productivity enhancing public investments and analyze the potential determinants of the policy maker's choice for the composition of overall public spending. Using this framework, we also link the overall macroeconomic performance to the public spending decisions. Our results suggest that raising current spending at the expense of public investment is associated with less favourable performance in terms of not only inflation and output but also, interestingly, future 'current' spending.

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1 Introduction

The 1990s have witnessed concerted efforts on the part of governments in both industrial and developing countries to undertake large fiscal adjustments in response to huge deficits experienced during the previous two decades. Both the US and the UK governments, for example, achieved balanced budgets in the second half of the 1990s. Similarly, tight budgetary requirements specified by the Maastricht entry criteria for the aspiring candidates of the European Monetary Union forced a number of countries to acquire fiscal discipline. Conventional wisdom suggests that such fiscal consolidations would be contractionary. However, some country experiences notably from Denmark and Ireland indicated the opposite. There is by-now more extensive empirical evidence for the expansionary consequences of fiscal contractions due to the so-called non-keynesian effects of fiscal policy.[1] The existing empirical evidence also reveals that the composition of fiscal adjustments greatly determines their likely impact. More specifically, adjustments that entail largely capital expenditure cuts are shown to be contractionary while current expenditure cuts are not [Perotti (1996)]. The link between productive capital spending and productivity growth had previously been established by Aschauer (1989a,b).[2] One clear implication of these findings is that the share of public investment in total public spending should be raised to improve the growth potential of an economy. In reality, however, a large number of countries succeeded in reducing their deficits by lowering the share of investment spending in total spending.[3]

This paper attempts to provide a framework to explain both the lower share of current spending in large fiscal adjustments and the potential expansionary effects of fiscal contractions. In contrast to previous studies of non-keynesian effects of fiscal contractions that focused on private consumption behavior, the key to our framework is the productivity enhancing role of public investments. We distinguish between public consumption (or current) spending and investment spending that raises future productivity. Policy maker's choice for one type of spending over the other is taken to be determined by a series of politico-economic factors. By utilizing this framework, we show that secular reductions in the share of productive public investment similar to those experienced by a number of countries may be the equilibrium

[1] See, for example, Alesina and Ardagna (1998) among others.

[2] Additional evidence was provided by Pereira (2000) among others.

[3] See, for example, De Haan *et al* (1996), p.55.

outcome for a given set of political and institutional factors.

The rest of this paper is organized as follows. Section 2, first develops the basic model by incorporating the productivity enhancing role of public spending, and then goes on to analyze the role of a number of politico-economic factors in determining the composition of public spending and in influencing macroeconomic performance. Finally, Section 3 concludes the paper.

2 Dynamics of Public Spending Decisions: Current Spending versus Public Investments

In order to analyze the dynamics of public spending decisions, we utilize a simple model of discretionary monetary and fiscal policy making.[4,5]

To explore the implications of the policy maker's strategic decision regarding the composition of public expenditure, we distinguish between two broad spending categories; the investment (g^i) and consumption (g^c). Public investment spending consists of spending, for example, on infrastructure, health and education that has a positive impact on overall productivity. However, as these favorable consequences are not realized until future periods this type of spending does not form part of the policy maker's current utility function. Current utility derives from current or consumption spending which consists of public wages, current public spending on goods, and other government spending which may yield instant utility to the policy maker. Taken together these suggest that the loss function may take the following form

$$L_t^G = \frac{1}{2}\sum_{t=1}^{2}\beta_G^{t-1}[\delta_1\pi_t^2 + (x_t - \overline{x}_t)^2 + \delta_2(g_t^c - \overline{g}_t^c)^2] \tag{1}$$

where L_t^G denotes the welfare losses incurred by the government, π_t is inflation rate, x_t and $\overline{x}_t$ are the (log of) actual and desired level of output, g_t^c and $\overline{g}_t^c$ are the actual and desired public consumption spending as shares of output, δ_1 and δ_2 represent, respectively, the government's aversion for the deviations of inflation and public spending from their respective targets and

[4]See Ismihan and Ozkan (2004) for the decentralized version of this model.

[5]It should be noted that this paper analyzes the role of fiscal management on output performance rather than on economic growth. See, Devarajan *et al* (1996) for an example of a growth model incorporating the composition of public expenditure.

β_G is the government's discount factor. Target inflation rate is taken to be zero to indicate the desirability of price stability. Output supply is given by the following equation[6]

$$x_t = \alpha(\pi_t + \psi g^i_{t-1} - \pi^e_t - \tau_t) \tag{2}$$

where π^e_t is expected inflation and ψ is a measure of the productivity of public investment. Distortionary taxes, which are the only form of taxes available to the government, are levied on the total revenue of firms at the rate τ_t. The government budget constraint creates the link between the fiscal and monetary policies, which is formally given by

$$g^c_t + g^i_t = \pi_t + \tau_t \tag{3}$$

To abstract from issues of debt dynamics we exclude borrowing as a source of finance.[7] Thus seigniorage and taxes are the two alternative ways of financing the two types of public expenditure. This implies that the link between the first and the second period is only through the productivity enhancing role of g^i_1 on x_2.

2.1 Characterization of Equilibrium

The policy maker in $t = 2$ minimizes its in-period losses with respect to π_2, τ_2 and g^c_2, for a given level of g^i_1. The outcome is presented in Table I in terms of the deviations of the equilibrium values of output, government spending and inflation. This can be compared with the equilibrium outcome in the absence of the productivity link between public investment and output which is derived and presented in the Appendix, part 2. The latter corresponds to the case where $\psi = 0$ in equation 2, which reduces the basic model to a static set-up with no links between periods. On comparing the outcome presented in Table I with that of the static case in Table A-I (for $t = 2$), it is evident that there is an intratemporal trade-off between output and spending in both cases. However, while higher public consumption and output

[6]Equation (2) is derived from the representative competitive firm's profit maximization behaviour in the Appendix, part 1.

[7]Ismihan and Ozkan (2003) incorporates public borrowing as a third source of financing public expenditure. It is shown, for example, that the net effect of public investment on future macroeconomic performance depends on the benefits of public investment relative to the costs of public borrowing.

targets, $\overline{g}_2^c$ and $\overline{x}_2$ respectively, push π_2, x_2 and g_2^c further away from their respective targets, higher public investment in the previous period reduces the gap between the actual and the target values of these variables in the presence of the productivity link. Proposition 1 formalizes these arguments.

Proposition 1 *The higher the productivity enhancing public spending in* $t = 1$ *the higher the equilibrium values of output and public consumption in* $t = 2$. *As a result, equilibrium inflation is lower.*

Proof. See the Appendix, part 3. ■

2.2 Composition of Public Spending

The decision regarding how much to divert to each category of spending is made by the policy maker who distributes the existing distortions across the available instruments. More formally, this problem amounts to minimizing the government's intertemporal loss function with respect to π_1, τ_1, g_1^i and g_1^c after substituting the equilibrium values from $t = 2$. The equilibrium solution is presented in Table II and a number of interesting issues arise.

Firstly, the beneficial effects of the first period's public investment on the second period's outcome are not limited to inflation and output but also extend to future 'current' spending. In essence, by investing in infrastructure, health, and education the policy maker increases the resources available to him in future, which helps him expand even the consumption spending. This is the source of an asymmetry between the effects of investment and consumption spending. Secondly, an interesting asymmetry arises between the effects of current and future consumption spending targets. From Table II it can be seen that a rise in the current spending target decreases investment spending in $t = 1$ while a rise in its future target raises it (both Θ and $\Gamma > 0$). This is because, expanding current spending in $t = 1$ can only take place at the expense of lower public investment in $t = 1$, while higher 'current' spending in $t = 2$ can be made possible as a result of higher public spending in $t = 1$. Proposition 2 formalizes these arguments.

Proposition 2 *The higher are output and current spending targets in future;* $\overline{x}_2$ *and* $\overline{g}_2^c$ *respectively, the lower (higher) must be the share of consumption (investment) spending today.*

Proof. See Appendix, part 4. ■

A political economy interpretation of these relationships is provided in the next section.

2.2.1 The Role of Political Instability

It is widely argued that the share of public investment versus current public spending is determined by the characteristics of the political structure. Among the features of the political structure highlighted is the existence or otherwise of political instability -which arguably plays a major role on both the level and the composition of public spending. In general, political instability is defined as the 'propensity to observe' constitutional or unconstitutional changes of the executive [Alesina and Perotti (1996)]. While the likelihood of changes in the executive leads to myopic behavior on the part of the incumbent, a number of other aspects of the political scene- such as the existence of weak or strong governments, income and wealth inequality, social fractionalization, political polarization etc.- may have more substantive consequences for political instability and public spending decisions.[8]

As a consequence and by embracing the arguments presented in footnote 9, it is possible to interpret the consumption spending target ($\overline{g}_t^c$) as a measure of political instability. The higher the income and wealth inequality, political polarization and social fractionalization, the higher would be the policy maker's target for consumption spending that could be directed at alleviating their undesirable consequences.[9] Proposition 3 summarizes the implications of high current spending motives such as these.

Proposition 3 *The higher the current spending target ($\overline{g}_1^c$), the higher is actual current spending (g_1^c) and the lower is actual public investment (g_1^i).*

[8]For example, politically weak governments tend to cut public investment rather than current spending relative to politically strong governments [Roubini and Sachs (1989)]. Moreover, income inequality and social or ethnic fractionalization may also lead to an increase in government consumption spending aimed at 'placating opposition' [Easterly and Levine (1997) and Annett (2001)]. Similarly, a high degree of income and wealth inequality, especially in less developed and developing countries, usually brings about an unstable political environment [Alesina and Perotti (1996)]. Therefore, governments in such politically unstable environments have greater incentives to follow populist policies which favor redistributive public spending, for example, in the form of public wage increases.

[9]It is possible to interpret δ_2 in a similar fashion. The greater the political distortions the greater will be the government's aversion to deviations of spending from target.

As a result, the lower is the output performance of the economy.

Proof. See the Appendix, part 5. ■

This proposition provides a possible explanation for the empirical regularity that the composition of fiscal adjustments matters for productivity and growth. It is clear that if the policy maker succeeds in reducing current spending- which relaxes the budget constraint and provides resources for productive uses- the overall effect of fiscal contractions is favorable in terms of output performance.

2.2.2 Elections, Myopia and the Strategic Behavior

The above mentioned constitutional changes of the government take place through the electoral process. Although the consequences of electoral uncertainty are not usually as serious as the implications of other sources of political instability discussed in the previous section, having an election in the horizon inevitably leads to relatively short-sighted policy makers with a high rate of time preference. It is commonly argued that such myopic governments tend to favor current expenditures at the expense of public investment [De Haan *et al* (1996)]. The probability of being out of office in the future obviously alters the costs versus benefits of productive investment spending for the policy maker and is likely to tilt the balance in the composition of public expenditure towards current expenditure.

To analyze the role of an up-coming election on fiscal management, we assume that the incumbent faces the electorate at the beginning of $t = 2$. This suggests that his effective discount factor is equal to $\beta_G^* = p\beta_G$ where p is the incumbent's re-election probability. Elections take place before wage setters negotiate nominal wage settlements. However, when making its spending decisions in $t = 1$ the policy maker faces a non-zero probability of not being in office in $t = 2$ and hence faces an additional predicament in expanding public investment. Such concerns on the part of the incumbent clearly reduces the 'effective rate of return' from public investments to be committed in the current period.[10] The policy maker's resulting choice of public investment in $t = 1$ is as follows

[10]It should be noted that today's incumbents would still derive utility from higher output in future even when they are not in office. However, given that governments already discount the future at a higher rate than does society, electoral uncertainty further increases such impatience and thus effectively reduces the benefits *vis-a-vis* the costs of public investments.

$$g_1^{i,E} = \Theta^*[-\overline{g}_1^c + \Gamma^*\overline{g}_2^c - (1/\alpha)\overline{x}_1 + (\Gamma^*/\alpha)\overline{x}_2] \tag{4}$$

where superscript E is used for outcomes when electoral uncertainty exists and $\Theta^* = \frac{1}{1+\psi\Gamma^*}$, $\Gamma^* = (1+\phi)\Lambda^*$, $\Lambda^* = \psi\beta_G^* D$, $\beta_G^* = p\beta_G$ and all other variables are as defined in Table II.

Proposition 4 *The existence of electoral uncertainty introduces a bias in public spending towards higher consumption spending and away from public investment spending. The lower (higher) the incumbent's probability of re-election, p, the lower (higher) the productive investment spending. As p approaches 1 the outcome is the same as under the absence of elections.*

Proof. See Appendix, part 6. ■

3 Conclusion

Public spending, if directed at productivity enhancing activities reduces distortions in the economy and alleviates the future inflation-output trade-off. However, an increase in current distortions due to, for example, political instability arising from social fractionalization, political polarization and income and wealth inequality, may lower the share of investment spending and hence result in a worse macroeconomic performance in future. Electoral uncertainty also plays a role in determining the equilibrium composition of public spending. As the costs of public investment are borne contemporaneously while the benefits can only be enjoyed in future, the slimmer the chance of re-election for an incumbent the lower is the share of public investment in total public spending.

The dynamic framework developed in this paper provides a political economy explanation for the observed expansionary effects of fiscal contractions. Our analysis suggests that if the cut in spending is accomplished by reducing investment spending, future productivity and output fall and thus the conventional effects are achieved. If, on the other hand, the policy maker succeeds in reducing the current consumption spending, which relaxes the budget constraint and provides resources for productive uses, the overall effect of fiscal contractions may well be an expansion.

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Appendix

1.Derivation of the Output Supply Function in Equation (2)

Consider the following form of the production function: $Y_t = A_t N_t^{\gamma}$, where Y_t represents output in period t, N_t represents labor in period t, A_t represents the level of productivity in period t and $0 < \gamma < 1$. The representative competitive firm's problem is to maximize profits $P_t(1-\tau_t)A_t N_t^{\gamma} - W_t N_t$, where P_t represents price level in period t, W_t represents the wage rate in period t and τ_t is the tax rate on the total revenue of the firm in period t. The representative firm chooses labor to maximize profits by taking P_t,W_t and τ_t as given. The resulting output supply function is $y_t = \alpha(p_t + \frac{1}{\gamma}a_t - w_t - \tau_t) + z$, where lower case letters represent logs, e.g. $y_t = ln(Y_t)$, $\alpha = \gamma/(1-\gamma)$, $\ln(1-\tau) \simeq -\tau$ and $z = \alpha ln(\gamma)$. Furthermore, we incorporate the productivity enhancing role of public investments. More formally, we model productivity as follows: $a_t = a_0 + \zeta g^i_{t-1}$, where $\zeta > 0$. Substituting a_t into the previous equation, then, normalizing output by subtracting the constant term $z' = z + \alpha a_0/\gamma$ for simplicity and utilizing $w_t = p^e_t$, where superscripts e denote expectation, yields the normalized output supply function that appears in equation (2) in the text.

2. The Static Model

This section sets-out the simplified version of the basic model utilized above in the absence of the productivity link, as used by Alesina and Tabellini (1987), Beetsma and Bovenberg (1997), and Ozkan (2000) among others. The equilibrium outcome of this version as presented by Table A-I forms basis of the comparisons provided in section 2.1 in the text.

Set-up

The policy maker's preferences are assumed to be summarized by the following loss function

$$L_t^G = \frac{1}{2}\sum_{t=1}^{T}\beta^{t-1}[\delta_1\pi_t^2 + (x_t - \overline{x}_t)^2 + \delta_2(g_t - \overline{g}_t)^2] \tag{A1}$$

where g_t and $\overline{g}_t$ are the actual and desired public spending as shares of output[11] and all other variables are as defined above.

Output is given by the following production function: $Y_t = N_t^{\gamma}$, where Y_t and N_t represent output and labor respectively, in period t and $0 < \gamma < 1$. The representative competitive firm's problem is to maximize profits $P_t(1 - \tau_t)N_t^{\gamma} - W_tN_t$ with respect to N_t, where all variables are as defined earlier. The resulting output supply function is $y_t = \alpha(p_t - w_t - \tau_t) + z$, where $z = \alpha ln(\gamma)$ and all variables are as defined earlier. Normalizing output by subtracting the constant term z for simplicity and utilizing $w_t = p_t^e$ we obtain the following (normalized) output supply function

$$x_t = \alpha(\pi_t - \pi_t^e - \tau_t) \tag{A2}$$

The budget constraint of the government is given by

$$g_t = \tau_t + \pi_t \tag{A3}$$

This equation suggests that there are two sources of financing for the government outlays; distortionary taxes and seigniorage.

Characterization of Equilibrium

In this model, government controls both fiscal and monetary policies; therefore government spending is residually determined via the budget constraint. More formally, government optimally selects its policies by minimizing the loss function with respect to π and τ. Combining the relevant

[11]Note that in this model, there is only one form of public spending which has no effect on output. Therefore, g_t and $\overline{g}_t$ represent the actual and desired public consumption spending, respectively.

first-order conditions with the budget constraint (and output supply function) and imposing rational expectations condition (i.e. $\pi_t^e = \pi_t$) we arrive at the equilibrium values of inflation, the tax rate, public spending and output contained in Table A-I.

3. Proof of Proposition 1

The derivatives of $(\overline{x}_2 - x_2)$ and $(\overline{g}_2^c - g_2^c)$ with respect to g_1^i are $-\psi\lambda\delta_2/\alpha$ and $-\psi\lambda$, which are both unambiguously negative for all values of ψ, λ, δ_2 and α. The derivative of π_2 with respect to g_1^i is $-2\delta_2\psi\lambda/\delta_1$,which is also unambiguously negative for all values of ψ, λ, δ_2 , α and δ_1.

4. Proof of Proposition 2

The derivative of g_1^c with respect to $\overline{g}_2^c$ is $-\Lambda\Theta$. Given that $\Lambda\Theta$ is unambiguously positive this derivative is negative. The derivative of g_1^c with respect to $\overline{x}_2$ is $-\Lambda\Theta/\alpha$, which is also negative for all values of the above parameters. Similarly, the derivatives of g_1^i with respect to $\overline{g}_2^c$ and $\overline{x}_2$ are respectively, $\Gamma\Theta$ and $\Gamma\Theta/\alpha$, both of which are unambiguously positive.

5. Proof of Proposition 3

The derivative of g_1^c with respect to $\overline{g}_1^c$ is $1-\psi\Lambda\Theta$, which is unambiguously positive for all values of the above parameters since $0 < \psi\Lambda\Theta < 1$. The derivative of g_1^i with respect to $\overline{g}_1^c$ is $-\Theta$ where Θ is unambiguously positive. Thus $\partial g_1^c/\partial\overline{g}_1^c$ is positive. Also $\partial x_1/\partial\overline{g}_1^c$, $\partial x_1/\partial\overline{g}_2^c$, $\partial x_2/\partial\overline{g}_1^c$ and $\partial x_2/\partial\overline{g}_2^c$ are all negative.

6. Proof of Proposition 4

The derivative of $g_1^{i,E}$with respect to p is $\frac{\Gamma^*}{p}\Theta^{*2}[\overline{g}_2^c + (1/\alpha)\overline{x}_2 + \psi(\overline{g}_1^c + (1/\alpha)\overline{x}_1)]$. Given that all parameters in this expression are positive, this derivative is unambiguously positive. The $\lim_{p\to 1}\Gamma^* = \Gamma$ and $\lim_{p\to 1}\Theta^* = \Theta$, so, $\lim_{p\to 1} g_1^{i,E} = g_1^i$.

Table A-I: Output, Public Consumption, Inflation and the Tax Rate in Equilibrium

$x_t = (1 - \lambda\delta_2/\alpha^2)\overline{x}_t - (\delta_2/\alpha)\lambda\overline{g}_t$
$g_t = \lambda[\phi\overline{g}_t - \overline{x}_t/\alpha]$
$\pi_t = (2\delta_2/\delta_1)\lambda[\overline{x}_t/\alpha + \overline{g}_t]$
$\tau_t = (\delta_2/\alpha^2)\lambda\overline{g}_t + [(\delta_2/\alpha^2)\lambda - 1]\overline{x}_t/\alpha$
Or,
$(\overline{x}_t - x_t) = (\delta_2/\alpha)\lambda[\overline{x}_t/\alpha + \overline{g}_t]$
$(\overline{g}_t - g_t) = \lambda[\overline{x}_t/\alpha + \overline{g}_t]$
$(\overline{\pi}_t - \pi_t) = -(2\delta_2/\delta_1)\lambda[\overline{x}_t/\alpha + \overline{g}_t]$

Note: $\lambda = 1/(1+\phi)$ and $\phi = \frac{\delta_2}{\alpha^2} + \frac{2\delta_2}{\delta_1}$.

Table I: Deviations of Output, Public Consumption and Inflation from their Targets in $t = 2$

$(\overline{x}_2 - x_2) = (\delta_2\lambda/\alpha)[\overline{x}_2/\alpha + \overline{g}_2^c - \psi g_1^i]$
$(\overline{g}_2^c - g_2^c) = \lambda[\overline{x}_2/\alpha + \overline{g}_2^c - \psi g_1^i]$
$(\overline{\pi}_2 - \pi_2) = -(2\delta_2\lambda/\delta_1)[\overline{x}_2/\alpha + \overline{g}_2^c - \psi g_1^i]$

Table II: Output, Public Consumption, Inflation and the Tax Rate in Equilibrium

$x_2 = [1 - (\delta_2\lambda/\alpha^2)]\overline{x}_2 - (\delta_2\lambda/\alpha)\overline{g}_2^c + (\delta_2\lambda/\alpha)\psi g_1^i$
$g_2^c = \lambda[\phi\overline{g}_2^c + \psi g_1^i - \overline{x}_2/\alpha]$
$\pi_2 = (2\delta_2\lambda/\delta_1)[\overline{x}_2/\alpha + \overline{g}_2^c - \psi g_1^i]$
$\tau_2 = [1 - (\delta_2\lambda/\alpha^2)]\psi g_1^i + (\delta_2\lambda/\alpha^2)\overline{g}_2^c + [(\delta_2\lambda/\alpha^2) - 1]\overline{x}_2/\alpha$
$x_1 = -\delta_2/\alpha[\Lambda\Theta(\psi\overline{g}_1^c + \overline{g}_2^c + (1/\alpha)\overline{x}_2)] + [1 - (\psi\delta_2\Lambda\Theta/\alpha^2)]\overline{x}_1$
$g_1^c = (1 - \psi\Lambda\Theta)\overline{g}_1^c + \Lambda\Theta[-\overline{g}_2^c - (\psi/\alpha)\overline{x}_1 - (1/\alpha)\overline{x}_2]$
$g_1^i = \Theta[-\overline{g}_1^c + \Gamma\overline{g}_2^c - (1/\alpha)\overline{x}_1 + (\Gamma/\alpha)\overline{x}_2]$
$\pi_1 = 2\delta_2/\delta_1[\Lambda\Theta(\psi\overline{g}_1^c + \overline{g}_2^c + (\psi/\alpha)\overline{x}_1 + (1/\alpha)\overline{x}_2)]$
$\tau_1 = \delta_2/\alpha^2[\Lambda\Theta(\psi\overline{g}_1^c + \overline{g}_2^c + (1/\alpha)\overline{x}_2)] - [(1 - (\psi\delta_2\Lambda\Theta/\alpha^2))/\alpha]\overline{x}_1$

Note: $\Theta = \frac{1}{1+\psi\Gamma}$, $\Gamma = (1+\phi)\Lambda$, $\Lambda = \psi\beta_G D$, $D = \frac{2\delta_2}{\delta_1}\lambda^2 + \lambda, \lambda = 1/(1+\phi)$ and $\phi = \frac{\delta_2}{\alpha^2} + \frac{2\delta_2}{\delta_1}$.