

ECONOMICS BULLETIN

Firm growth and scaling of growth rate variance in multiplant firms

Alex Coad

Max Planck Institute of Economics, Jena, Germany

Abstract

While Gibrat's Law assumes that growth rate variance is independent of size, empirical work has usually found a negative relationship between growth rate variance and firm growth. Using data on French manufacturing firms, we observe a relatively low, but statistically significant, negative relationship between firm size and growth rate variance. Furthermore, we observe that growth rate variance does not decrease monotonically the more plants a firm possesses, which is at odds with a number of theoretical models.

1 Introduction

Hymer and Pashigian (1962) were among the first to draw attention to the negative relationship between growth rate variance and firm size. If firms can be seen as a collection of 'components' or 'departments', then the overall variance of the growth rate of the firm is a function of the growth rate variance of these individual departments. In many cases, the variance of the firm's growth rate will decrease with firm size. For example, in the case there these departments (i) are of approximately equal size, such that the size of the firm is roughly proportional to the number of components; and (ii) have growth rates that are perfectly independent from each other, then Central Limit Theorem leads us to expect a decrease in growth rate variance that is proportional to the inverse square root of the firm's size. However, Hymer and Pashigian (1962) were puzzled by the fact that the rate of decrease of growth rate variance with size was lower than the rate that would be observed if large firms were just aggregations of independent departments. At the same time, they found no evidence of economies of scale. They saw this as an anomaly in a world of risk-averse agents. Why would firms want to grow to a large size, if there are no economies of scale, and if the growth rate variance of a large firm is higher than the corresponding variance of an equivalent group of smaller firms? Subsequent studies provided no conclusive answer to this question, although they did bear in mind the existence of a negative relationship between growth rate variance and firm size. As a consequence, empirical analyses of Gibrat's law began to correct for heteroskedasticity in firm growth rates (e.g. Hall (1987), Evans (1987a), Evans (1987b), Dunne and Hughes (1994), Hart and Oulton (1996), Harhoff et al. (1998)).

In recent years efforts have been made to quantify the scaling of the variance of growth rates with firm size. This scaling relationship can be summarized in terms of the following power law: $\sigma(g_i) \sim e^{\beta s_i}$; where $\sigma(g_i)$ is the standard deviation of the growth rate of firm i, β is a coefficient to be estimated, and s_i is the size (total sales) of firm i. Values of β have consistently been estimated as being around -0.2 for large US manufacturing firms (Amaral et al. (1997), Amaral et al. (1998), Bottazzi and Secchi (2003)) and also for large firms in the worldwide pharmaceutical industry (Bottazzi et al. (2001), Matia et al. (2004), Bottazzi and Secchi (2006)). Lee et al. (1998) find that a scaling exponent of -0.15 is able to describe the scaling of growth rate variance for both quoted US manufacturing firms and the GDP of countries.

The discussion in (Lee et al., 1998, p. 3277) gives us a better understanding of the values taken by β , the scaling exponent. If the growth rates of divisions of a large diversified firm are perfectly correlated, we should expect a value of $\beta = 0$. On the other hand, if a firm can be viewed as an amalgamation of perfectly independent subunits, we expect a value of $\beta = -0.5$. The fact that the estimated exponents are between these extreme values of 0 and -0.5

suggest that the constituent departments of a firm have growth patterns that are somewhat correlated.

Virtually all of the proposed explanations of the scaling relation assume that firms can be decomposed into a number of smaller entities, and that some sort of central limit theorem is at work at the level of these subunits.¹ Amaral et al. (1998), Amaral et al. (2001) and Sutton (2002) propose explanations for the observed scaling relation by suggesting that firms are composed of divisions or business lines that are of different sizes. The size of the divisions composing each firm are then assumed to evolve according to a Gibrat-type random multiplicative process, and given that the divisions are of different sizes the scaling coefficient resembles those observed for US data.

Further possible explanations for the scaling relation are offered by Matia et al. (2004), Bottazzi and Secchi (2006), and Klepper and Thompson (2006) who consider firms as being composed of a certain number of independent *submarkets*. The average size of the submarkets increases with firm size, but the growth rates are independent across submarkets. Matia et al. (2004) and Bottazzi and Secchi (2006) provide support for their model by examining evidence from the worldwide pharmaceutical industry, where a firms portfolio of activities can be decomposed to a fine level of aggregation. As a result, "the explanation of the relationship between the variance of the growth rates distribution and the size of the firm based on the Central Limit Theorem is valid, as long as one considers the actual number of sub-markets a firm operates in, instead of assuming that this number is somehow proportional to the size of the firm" ((Bottazzi and Secchi, 2006, p. 860)).

Recent empirical evidence from Italian data, however, has proven to be a stumbling-block to these theories of firm growth. Bottazzi et al. (2007) fail to find any significant relationship between firm size and growth rate variance in their analysis of Italian manufacturing firms. This could well be due to the fact that the firms analyzed in Bottazzi et al. (2007) are smaller than those firms in the empirical analyses discussed above. In any case, this evidence nourishes skepticism on how far the previous models can be generalized.

The present investigation seeks to complement the existing literature in a number of ways. First, we provide detailed results on the relationship between size and growth rate variance for the case of French manufacturing firms, which complements studies using data for other countries. Although data on the internal composition of firms is not always easy to get, our database contains information on the number of production plants operated by each firm. Second, we explore a new channel relating the scaling relation to a firm's multiplant structure. The peculiarities of multiplant firms has aroused considerable interest in the old industrial

¹An alternative explanation for the decrease of growth rate variance with size, however, could be that firm growth is a lumpy process that is achieved through the addition of indivisible assets or 'resources'. Since the relative size of these indivisibilities will decrease with firm size, it follows that growth rate variance will decrease with firm size (Coad (2007b)).

organization literature,² but to my knowledge this has not yet been linked specifically to the relationship between a firm's size and its growth rate variance.

The layout of the paper is as follows. We begin by presenting the dataset and some summary statistics (Section 2). We then undertake some growth rate regressions and observe that multiplant firms have, ceteris paribus, expected higher growth rates (Section 3.1). We then estimate the scaling coefficient that determines the relationship between size and growth rate variance (Section 3.2.1). Finally, we graph the relationship between growth rate variance and number of plants in a firm (Section 3.2.2). We conclude in Section 4.

2 Database description and summary statistics

2.1 Database

This research draws upon the EAE databank collected by SESSI and provided by the French Statistical Office (INSEE).³ This database contains longitudinal data on a virtually exhaustive panel of French firms with 20 employees or more over the period 1989-2004. We restrict our analysis to the manufacturing sectors.⁴ Since data reporting norms changed over the period, we maintain statistical consistency by only utilizing the period 1996-2004 and we consider only continuing firms over this period. Firms that entered midway through 1996 or exited midway through 2004 have been removed. Since we want to focus on internal, 'organic' growth rates, we exclude firms that have undergone any kind of modification of structure, such as merger or acquisition.

In keeping with previous studies, our measure of growth rates is calculated by taking the differences of the logarithms of size: $g_{it} = log(S_{it}) - log(S_{i,t-1})$; where, to begin with, S is measured in terms of total sales for firm i at time t. The growth rate distributions have been normalized around zero in each year which effectively removes any common trends such as inflation.⁵ In some rare cases we have (continuing) firms that report zero plants in some years – these firms are removed. To start with we had observations for around 22 000 firms per year

²The multiplant structure of firms has traditionally been associated with the desire of firms to reduce volatility of their operations. An early empirical study by Scherer and colleagues reports that "some (of the respondents) viewed the hedge multiple plants afford against ... disasters as one of the most important benefits of multiplant operation" (Scherer et al., 1975, p. 278). Relatedly, Wahlroos (1981) presents a theoretical model where firms choose the number of plants they operate as a trade-off between scale economies and relative stability.

³The EAE databank has been made available to the author under the mandatory condition of censorship of any individual information.

⁴More specifically, we examine firms in the two-digit NAF sectors 17-36, where firms are classified according to their sector of principal activity (the French NAF classification matches with the international NACE and ISIC classifications). We do not include NAF sector 37, which corresponds to recycling industries.

⁵In fact, this method of deflating our variables was to some extent imposed upon us, since I was unable to find a suitable sector-by-sector series of producer price indices to be used as deflators.

for each year of the period,⁶ but we now end up with 8496 firms over the period 1996-2004.

Our focus on a balanced panel means that our results should not be seen as representative of all of French industry. Instead, our results should be seen as focusing on continuing firms that survive over the 9-year period. This will mean that we exclude many small, single-plant firms that enter and exit shortly afterwards. Multiple plant firms (which are the main object of analysis) will presumably be less affected by virtue of their larger size.

2.2 Summary statistics

Summary statistics are presented in Table 1 and Figure 1. The second column of Table 1 shows the size differences between multiplant firms within the cross-section. We observe that, for both years, average firm size generally increases with the number of plants, although this increase is not monotonic. Analytical rigour (pursued in the following section) requires that we separate multiplant effects from sheer size effects (as well as controlling for other factors). A first, naïve look at the data, however, indicates that the growth rate variance of firms with two or three plants may actually be higher than in the case of single-plant firms.

3 Analysis

3.1 Growth rate regressions

We begin with some standard growth rate regressions, where the dependent variable is sales growth and the explanatory variables are number of plants as well as lagged sales growth, lagged size, export intensity (exports/sales) and a full set of 3-digit industry dummies. These regressions are estimated on a year-by-year basis using both a heteroskedasticity-consistent OLS estimator and a Least Absolute Deviation (LAD) estimator. Given the fat-tailed nature of firm growth rate distributions, we prefer the LAD estimates which are more robust to extreme observations (Bottazzi et al. (2005), Coad (2007a)). The results are presented in Table 2. While these regressions provide several interesting results, we focus here on the association of multiplant structure with sales growth. If anything, our results suggest that, ceteris paribus, multiplant firms enjoy slightly higher growth rates. This finding of a positive influence of number of plants on expected growth rate is in line with evidence for US small businesses (Variyam and Kraybill (1992); Audretsch and Mahmood (1994)), large European

⁶22 319, 22 231, 22 305, 22 085, 21 966, 22 053, 21 855, 21 347 and 20 723 firms respectively.

⁷Among other results, we observe a rather small but statistically significantly negative influence of size on growth. In addition, there appears to be a negative autocorrelation in the annual sales growth series, although the coefficients differ considerably between the OLS and the LAD specifications (more on this in Coad (2007a). The R^2 values are low but this is to be expected in regressions of this type (see Coad (2007c), especially Table 2 therein).

corporations (Geroski and Gugler (2004)), and also Italian manufacturing firms (Fagiolo and Luzzi (2006)).

3.2 Scaling of growth rate variance

3.2.1 Parametric regressions

We now use parametric regression techniques to assess the relationship between firm size and growth rate variance. Following on from previous work⁸ we estimate the model:

$$g_{i,t} = e^{\alpha s_{i,t-1}} \varepsilon_{i,t} \tag{1}$$

where $s_{i,t-1}$ is the log of firm size and where $\varepsilon_{i,t}$ is the residual term. α is the parameter of interest, and we estimate it using the LAD regression method.⁹

Results are reported in Table 3. Our results vary for different years,¹⁰ with the estimated values for α are between -0.05 and -0.1. Although we observe that growth rate variance does appear to decrease with firm size, the magnitude of this effect does not resemble the magnitudes found using other datasets. Our coefficient estimates are considerably lower than the values obtained from data on US manufacturing firms and the worldwide pharmaceutical industry, surveyed above.

In unreported regressions we repeated the analysis with Value Added growth instead of Sales growth, as a means of verifying the robustness of our results, and we obtained similar findings.

3.2.2 Scaling of variance and multiplant structure

In this section we put firms into categories according to the number of plants they operate, and compare the variance of growth rates across these categories. We compare the variance of the 'raw' growth rates $(g_{i,t})$ as well as the variance of the 'cleaned' growth rates, where these latter correspond to $\varepsilon_{i,t}$. Taking the cleaned growth rates, we effectively remove any size effects that may affect firms in different multiplant categories, and thus we facilitate a more accurate comparison across categories.

The results are presented in Figure 2. To begin with, we notice that there is little difference between the raw and the cleaned growth rates. Although growth rate variance is negatively associated with firm size, the magnitude of this relationship is not very large. While there

 $^{^8\}mathrm{See}$ among others Amaral et al. (1997), Bottazzi et al. (2002), Bottazzi and Secchi (2003), and Bottazzi et al. (2005)

⁹Our estimates of Equation (1) made use of the gbutils 5.1 software package developed by Giulio Bottazzi.

¹⁰Our results offer some admittedly 'shaky' support to the conjecture that the α coefficient is of a slightly larger (smaller) magnitude during periods of economic growth (recession).

appears to be a negative relationship between number of plants and growth rate variance, the relationship is not monotonically decreasing. Furthermore, the relationship between number of plants and growth rate variance changes considerably from year to year, and especially for the firms with the largest number of plants (this latter may well be because of the smaller number of observations for firms with the most plants).

It is rather interesting to observe that, in each year (apart from 2000), firms with two plants have a higher growth rate variance than single-plant firms. In addition, there are several instances whereby firms with three plants often have a higher growth rate variance than monoplant or two-plant firms. By way of further confirmation of these results, we refer the reader to Figure 3 in Bottazzi et al. (2005) who show how the standard deviation of firm growth rates displays a similar negative but non-monotonic relationship between firm growth rates and firm size. By grouping firms together into 15 equipopulated bins, it appears that the smallest firms do not have the lowest growth rate variance. Our results would thus appear to be somewhat different from predictions emerging from theoretical models.

4 Discussion

This paper offers some results that pose a challenge to a number of theoretical models. First, we observe that the scaling relation can be described by a value of α which is much lower than many (though not all) previous findings, taking values between -0.05 and -0.1. This provides further evidence of considerable heterogeneity in the scaling relation across countries. Second, we observed that, ceteris paribus, firms with more plants tend to have higher expected growth rates. Third, in the great majority of cases we observe that two-plant firms have higher growth rate variance than single-plant firms, even after controlling for effects of sheer size. In fact, in none of the years considered do single-plant firms have the highest growth rate variance.

How can these results be explained? It is worth reconsidering the nature of multiplant firms. We submit that these firms are often run by professional managers, who have only a limited liability for the firm. A main prediction of managerial economics literature suggests that professional managers will have a predisposition towards the growth of their company. This is true because incentives such as remuneration, likelihood of promotion, prestige and also power are linked to the size of the firm. Professional managers are also likely to have received a formal training and presumably will have a relatively high level of managerial skill. In addition, since larger firms are more likely to have a limited liability legal form, they are more prone to risk-taking behaviour. It has also been suggested that competition is more fierce between larger firms than smaller firms.¹¹ These factors can be expected to increase both the

¹¹Boone et al. (2007) measure competition using a firm-specific 'profit elasticity' measure, which corresponds to the elasticity of a firm's profits with respect to its cost level. They observe that larger firms operate in a

growth rates and the variance of growth rates of multiplant firms. Small firms, on the other hand, are often run by 'lifestyler' managers with little by way of growth ambitions, who see their enterprise as a means to an independent lifestyle and a source of stable revenue (Hay and Kamshad (1994)). It is unfortunate that these 'organizational' or 'sociological' perspectives are frequently overlooked in the industrial economics literature, where all too often firms of different sizes are seen as 'independent realizations of the same stochastic process' in the spirit of Gibrat's Law.

more competitive environment than smaller firms.

References

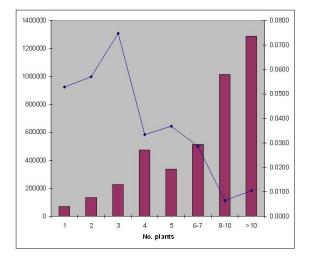
- Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Salinger, M. A., and Stanley, H. E. (1998). Power law scaling for a system of interacting units with complex internal structure. *Physical Review Letters*, 80(7):1385–1388.
- Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Salinger, M. A., Stanley, H. E., and Stanley, M. H. R. (1997). Scaling behavior in economics: the problem of quantifying company growth. *Physica A*, 244:1–24.
- Amaral, L. A. N., Gopikrishnan, P., Plerou, V., and Stanley, H. E. (2001). A model for the growth dynamics of economic organizations. *Physica A*, 299:127–136.
- Audretsch, D. B. and Mahmood, T. (1994). Firm selection and industry evolution: the postentry performance of new firms. *Journal of Evolutionary Economics*, 4:243–260.
- Boone, J., van Ours, J. C., and van der Weil, H. (2007). How (not) to measure competition. CEPR discussion paper 6275.
- Bottazzi, G., Cefis, E., and Dosi, G. (2002). Corporate growth and industrial structure: Some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 11:705–723.
- Bottazzi, G., Cefis, E., Dosi, G., and Secchi, A. (2007). Invariances and diversities in the patterns of industrial evolution: Some evidence from Italian manufacturing industries. *Small Business Economics*, 29(1):137–159.
- Bottazzi, G., Coad, A., Jacoby, N., and Secchi, A. (2005). Corporate growth and industrial dynamics: Evidence from French manufacturing. Pisa, Sant'Anna School of Advanced Studies, LEM Working Paper series 2005/21.
- Bottazzi, G., Dosi, G., Lippi, M., Pammolli, F., and Riccaboni, M. (2001). Innovation and corporate growth in the evolution of the drug industry. *International Journal of Industrial Organization*, 19:1161–1187.
- Bottazzi, G. and Secchi, A. (2003). Common properties and sectoral specificities in the dynamics of US manufacturing companies. *Review of Industrial Organization*, 23:217–232.
- Bottazzi, G. and Secchi, A. (2006). Gibrat's law and diversification. *Industrial and Corporate Change*, 37(2):235–256.
- Coad, A. (2007a). A closer look at serial growth rate correlation. Review of Industrial Organization, 31(1):69–82.

- Coad, A. (2007b). Explaining the Laplace distribution of firm growth rates. Paper presented at the 'Econophysics Colloquium' in Ancona, Italy.
- Coad, A. (2007c). Firm growth: A survey. Papers on Economics and Evolution 2007-03, Max Planck Institute of Economics, Evolutionary Economics Group.
- Dunne, P. and Hughes, A. (1994). Age, size, growth and survival: UK companies in the 1980s. Journal of Industrial Economics, 42(2):115–140.
- Evans, D. S. (1987a). The relationship between firm growth, size and age: Estimates for 100 manufacturing industries. *Journal of Industrial Economics*, 35:567–581.
- Evans, D. S. (1987b). Tests of alternative theories of firm growth. *Journal of Political Economy*, 95(4):657–674.
- Fagiolo, G. and Luzzi, A. (2006). Do liquidity constraints matter in explaining firm size and growth? some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 15(1):1–39.
- Geroski, P. A. and Gugler, K. (2004). Corporate growth convergence in Europe. Oxford Economic Papers, 56:597–620.
- Hall, B. H. (1987). The relationship between firm size and firm growth in the US manufacturing sector. *Journal of Industrial Economics*, 35(4):583–600.
- Harhoff, D., Stahl, K., and Woywode, M. (1998). Legal form, growth and exits of West German firms empirical results for manufacturing, construction, trade and service industries. *Journal of Industrial Economics*, 46(4):453–488.
- Hart, P. E. and Oulton, N. (1996). The growth and size of firms. *Economic Journal*, 106(3):1242–1252.
- Hay, M. and Kamshad, K. (1994). Small firm growth: Intentions, implementation and impediments. *Business Strategy Review*, 5(3):49–68.
- Hymer, S. and Pashigian, P. (1962). Firm size and rate of growth. *Journal of Political Economy*, 70(6):556–569.
- Klepper, S. and Thompson, P. (2006). Submarkets and the evolution of market structure. Rand Journal of Economics, 37(4):861–886.
- Lee, Y., Amaral, L. A. N., Canning, D., Meyer, M., and Stanley, H. E. (1998). Universal features in the growth dynamics of complex organizations. *Physical Review Letters*, 81(15):3275–3278.

- Matia, K., Fu, D., Buldyrev, S. V., Pammolli, F., Riccaboni, M., and Stanley, H. E. (2004). Statistical properties of business firms structure and growth. *Europhysics letters*, 67(3):498–503.
- Scherer, F. M., Beckenstein, A., Kaufer, E., Murphy, D. R., and Bougeon-Massen, F. (1975). The Economics of Multi-Plant Operation. Harvard University Press, Cambridge, MA.
- Sutton, J. (2002). The variance of firm growth rates: the 'scaling' puzzle. *Physica A*, 312:577–590.
- Variyam, J. N. and Kraybill, D. S. (1992). Empirical evidence on determinants of firm growth. *Economics Letters*, 38:31–36.
- Wahlroos, B. (1981). On the economics of multiplant operation: Some concepts and an example. *Journal of Industrial Economics*, 29(3):231–245.

Table 1: Summary statistics with firms sorted according to number of plants. Average sales is in FF'000s in 1997 and €'000s in 2004.

No. plants	Ave. Sales	Ave. Gr. Sales	Gr. variance	No. Obs			
1997							
1	68960	0.0011	0.0529	6417			
2	132042	-0.0022	0.0571	1413			
3	229615	-0.0020	0.0748	362			
4	475979	-0.0192	0.0333	131			
5	337019	0.0197	0.0368	63			
6-7	515227	-0.0137	0.0285	49			
8-10	1012686	-0.0100	0.0065	26			
>10	1287426	-0.0185	0.0104	35			
2004							
1	13543	0.0007	0.0475	6170			
2	23319	-0.0107	0.0564	1527			
3	42045	0.0170	0.0456	418			
4	79035	-0.0008	0.0256	171			
5	80907	0.0128	0.0084	66			
6-7	83718	0.0312	0.0307	73			
8-10	246586	0.0412	0.0348	32			
>10	157098	0.0089	0.0055	39			



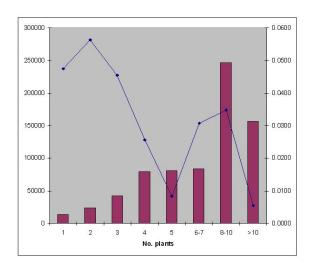


Figure 1: Summary statistics for firms with different numbers of plants, 1997 (left) and 2004 (right). Solid blocks and left axis refers to a firm's total sales. Square dots and right axis refer to a firm's growth rate variance.

Table 2: OLS and bootstrapped LAD regression results, where the dependent variable is sales growth (t). Coefficients significant at the 5% level are in bold ink. t-stats appear below coefficient estimates.

OLS						
1998	-0.2430	-0.0156	0.0002	0.0295	0.0996	8496
	-8.66	-6.97	0.26	2.39		
1999	-0.1732	-0.0147	0.0021	-0.0380	0.0699	8496
	-6.76	-6.21	2.28	-2.95		
2000	-0.1940	-0.0069	0.0008	0.0286	0.0708	8496
	-7.09	-2.79	0.84	2.10		
2001	-0.2087	-0.0106	0.0025	-0.0035	0.0688	8496
	-8.15	-4.46	2.38	-0.28		
2002	-0.2248	-0.0057	0.0035	-0.0023	0.0768	8496
	-6.55	-2.46	3.64	-0.19		
2003	-0.2172	-0.0085	0.0015	-0.0045	0.0676	8496
	-8.05	-3.41	1.60	-0.33		
2004	-0.1777	-0.0048	0.0009	-0.0190	0.0715	8496
	-5.70	-1.83	1.15	-1.38		
$\overline{\text{LAD }(t)}$	-statistics obtai	ined after 50	0 bootstrap	replication	s)	
1998	-0.0710	-0.0069	0.0002	0.0078	0.0253	8496
	-5.26	-4.36	0.50	0.87		
1999	-0.0159	-0.0036	0.0007	-0.0289	0.0272	8496
	-1.41	-2.25	2.07	-3.19		
2000	-0.0448	-0.0002	0.0000	0.0311	0.0334	8496
	-2.76	-0.17	0.04	3.27		
2001	-0.0708	-0.0031	0.0002	-0.0013	0.0196	8496
	-4.84	-1.98	0.49	-0.13		
2002	-0.0498	0.0000	0.0003	0.0035	0.0215	8496
	-4.12	0.02	0.98	0.44		
2003	-0.0463	0.0013	-0.0001	-0.0195	0.0218	8496
	-3.18	0.98	-0.34	-2.73		
2004	-0.0216	0.0040	0.0000	-0.0050	0.0343	8496
	-1.71	2.90	0.08	-0.58		

Table 3: LAD estimation of Equation (1), 8496 observations in each year.

Year	α	Std. Error
1997	-0.1034	0.0043
1998	-0.0807	0.0041
1999	-0.0748	0.0041
2000	-0.0762	0.0042
2001	-0.0518	0.0041
2002	-0.0581	0.0040
2003	-0.0721	0.0042
2004	-0.1010	0.0039

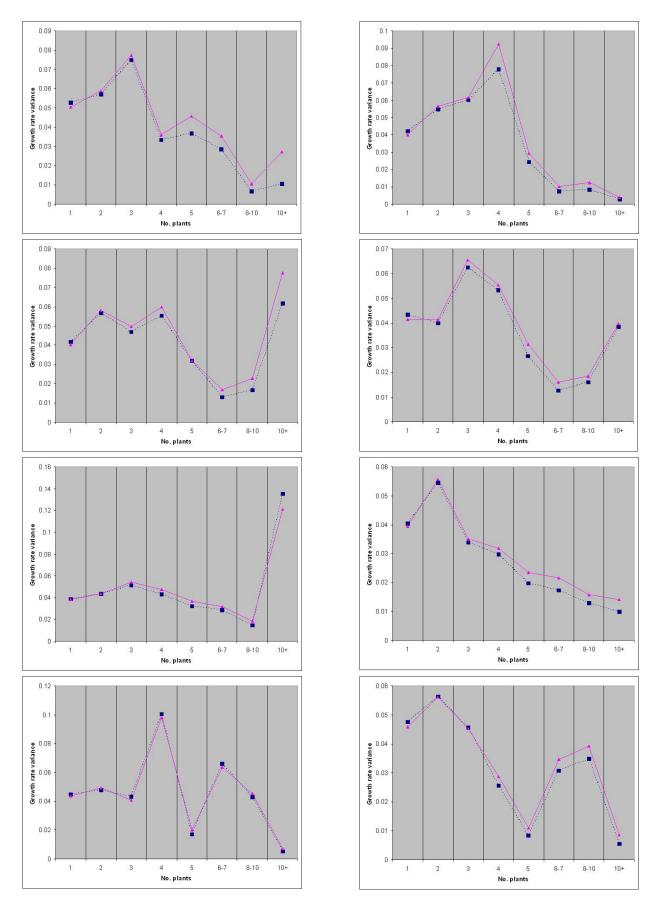


Figure 2: A comparison of growth rate variance across multiplant categories. Top left: 1997; top right: 1998; 2nd row left: 1999; 2nd row right: 2000; 3rd row left: 2001; 3rd row right: 2002; bottom left: 2003; bottom right: 2004. Triangles and solid line correspond to the 'cleaned data' (i.e. size effects removed), squares and dotted line correspond to the raw data.