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## A Role of public finance in the Ghatak and Jiang model; a comment

Katsunori Yamada

*Japan Society for the Promotion of Science*

### *Abstract*

This paper proposes that a public finance system can Pareto-improve a version of the economy in Ghatak and Jiang (2002). Our discussion relates to the controversial issue of wealth inequality in Japan. Its implications are important for consideration during administrative reform by the Koizumi Cabinet.

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# 1 Introduction

The issue of wealth inequality in Japan is highly controversial.<sup>1</sup> Recently, some economists and the media have asserted that social polarization is occurring in the form of wealth stratification in Japan and call it as the “Kachigumi-Makegumi (winners and losers) problem”. Recently, the Koizumi cabinet initiated administrative reforms, a chief goal of which is to privatize and restructure government services. Among the plans for reformulation, public attention has been paid especially to the restructuring of the Ministry of Internal Affairs and Communications’ (MIACs’) original official operations. This issue is important because MIAC acted as a planning center for operations in supplying public financial services.

Our interest in this paper then is: “Is it appropriate to privatize the financial services of MIAC?”, taking into account the presumption that wealth stratification in Japan is becoming polarized.

To answer this question, we use the model of Ghatak and Jiang (2002). In a non-stochastic version of their model, the wealth distribution might be polarized. In that case, the economy cannot achieve maximized output because of a malfunction in lending activities of private banks. Some Japanese economists, including Ogawa (2003) argue that negative tendency in lending to small businesses by private banks is the reason why Japan has stagnated since the end of the bubble era.<sup>2</sup> Ghatak and Jiang (2002) suggest a redistributive policy using revenue from taxes on bequests of wealthy families to correct that inefficiency. In contrast, we will suggest that the government should correct that inefficiency through a redistributive policy with a public finance system because the government can not only enhance the total output but can also Pareto-improve the economy. We will discuss the conditions under which that is the case.

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<sup>1</sup>See, for example, Ishikawa (1994), Tachibanaki (1998) and Otake (2005).

<sup>2</sup>There is also controversy on the credit crunch hypothesis in Japan. See, for example, Hayashi and Prescott (2002).

## 2 The Model

### 2.1 The model structure

We consider a non-stochastic version of the model presented in Ghatak and Jiang (2002). Time is discrete and extends to infinity. As in Galor and Zeira (1993) and Matsuyama (2000), we consider a small open economy. Infinitely-lived dynasties, which are mutually differentiated only by the amount of wealth, exist in the economy and live one period. The population is large and its size is one. Furthermore, there is no population growth. In period  $t$ , a dynasty  $i$  is endowed with one unit of labor and an initial wealth stock of  $a_{i,t}$ . Distribution of dynasties with wealth *strictly less than*  $a$ , at the head of time  $t$ , is designated as  $G_t(a)$ .

Two decisions can be made in this model. Time  $t$  dynasties choose their occupations at the head of the period according to their level of wealth. In addition, at the end of the period, they consume and make a bequest to their descendant. The utility function is given as  $U^i(c_{i,t}^{1-s}, b_{i,t}^s)$ , where  $c$  is consumption,  $b$  is the bequest and  $s \in (0, 1)$ .

The structure of occupational choice is important in this model. There are three occupations: subsistence, worker and entrepreneur. Among them, only entrepreneurs are required to pay an initial cost of  $I(> 0)$  out of inherited wealth  $a_{i,t}$ . When an entrepreneur starts her project, she must hire a unit of labor that a worker supplies and her labor is used in monitoring that worker. Because the population of the economy is one, the size of entrepreneurs is at most  $1/2$  in every period. Projects initiated by every entrepreneur (and supported by a worker) yield  $q$  units of output. This technology is assumed to be superior to the subsistence one in that the net product per unit of labor is higher than that of the subsistence technology:

$$\frac{q - rI}{2} > \underline{w},$$

where  $r$  is the gross world interest rate and  $\underline{w}$  denotes the productivity in the subsistence sector.

In the model, the only endogenously determined price is the wage rate ( $w_t$ ) that entrepreneurs pay to the workers. The information of wealth distribution and occupational choice is required for determination of  $w_t$ . Following Ghatak and Jiang (2002), we assume that *lending activity* by private banks involves some malfunction, e.g. high agency costs or low screening skills of bankers. This assumption posits that only those who are endowed with more than  $I$  units of wealth can become entrepreneurs.<sup>3</sup>

Assume that, from the participation constraint, numerous agents cannot initiate projects (i.e.  $G_t(I) > 1/2$ ). Consequently, the number of projects becomes small (the work force becomes abundant); therefore, the wage rate decreases to  $\underline{w}$ , at which point poor agents are indifferent between becoming workers and entering the subsistence sector. On the other hand, when relatively many agents are able to initiate projects (i.e.  $G_t(I) < 1/2$ ), the wage rate increases to the point at which rich agents endowed with more than  $I$  units of initial wealth are indifferent between being entrepreneurs and being workers. From the last statement, we can obtain a threshold level of the wage rate  $\bar{w} = (q - rI)/2$ . Consequently, we obtain the equilibrium wage rate of time  $t$  as<sup>4</sup>

$$w_t^* = \begin{cases} \bar{w} & \text{if } G_t(I) < \frac{1}{2} \\ [\underline{w}, \bar{w}] & \text{if } G_t(I) = \frac{1}{2} \\ \underline{w} & \text{if } G_t(I) > \frac{1}{2}. \end{cases}$$

Throughout the analyses in this paper, we will assume that  $w_t = \bar{w}$  when  $G_t(I) = 1/2$ .

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<sup>3</sup>Notice that agents can deposit wealth with private banks and earn capital gains with a gross interest rate of  $r$ . They, however, are assumed to have no access to international capital markets to finance the initial cost of the investment, probably because of some regulation or some fixed cost. This form of credit market imperfection is equivalent to that introduced by Matsuyama (2000).

<sup>4</sup>See section 2.4 of Ghatak and Jiang (2002) for a more detailed explanation of the credit market and the equilibrium conditions of the labor market.

## 2.2 Dynamics of Individual Wealth: a Special Case

Using the setup described above, the difference equations describing the evolution of dynasty  $i$ 's wealth, who will be either an entrepreneur, a worker, or a person in the subsistence sector, can be obtained easily as follows:

$$a_{i,t+1}(a_{i,t} | w_t = \underline{w}) = \begin{cases} s[ra_{i,t} + \underline{w}] & \text{if } a_{i,t} < I \\ s[r(a_{i,t} - I) + q - \underline{w}] & \text{if } a_{i,t} \geq I \end{cases} \quad (1)$$

$$a_{i,t+1}(a_{i,t} | w_t = \bar{w}) = s[ra_{i,t} + \bar{w}] \quad \forall a_{i,t} \quad (2)$$

Ghatak and Jiang (2002) showed, using the assumption that  $sr < 1$ , that a unique stationary wealth distribution exists in this model (Proposition 1). Moreover, the stationary distribution of wealth is shown to depend on the initial distribution as well as some parametric restrictions (Proposition 2).

In this paper, we concentrate our attention on the most interesting case: the case in which initial conditions, which determine the initial wealth distribution, matter. Specifically, assume the following condition:

$$\frac{s\underline{w}}{1-sr} < I \leq \frac{sq}{2-sr}. \quad (3)$$

In that equation,  $\frac{s\underline{w}}{1-sr} (\equiv a^W(\underline{w}))$  is the fixed point in the difference equation of a worker-dynasty (or a dynasty in the subsistence sector) when  $w^* = \underline{w}$ . On the other hand,  $\frac{sq}{2-sr} (\equiv a^A(\bar{w}))$  is the fixed point in the difference equation of all dynasties when  $w^* = \bar{w}$ . Figure (1) and Fig. (2) illustrate the situation. Figure (1) depicts the case in which the wealth level of the median dynasty (designated as  $a_t^M$ ) at the initial period ( $t = 0$ ) is lower than  $I$ . On the other hand, Fig. (2) portrays the case where  $a_0^M \geq I$ .

In Fig. (1), eq.(1) governs the economy. In the initial period ( $t = 0$ ), there are few entrepreneurial projects. As a result, the wage rate declines to  $w_0 = \underline{w}$  and entrepreneurs have a rent. Because the median dynasty cannot accumulate the required level of wealth to invest in the project, eq. (1) continues to govern the economy until the economy settles into the steady state. On the other hand, when the initial period starts with  $a_0^M \geq I$ , eq. (2)

governs the economy and the situation shown in Fig. (2) pertains. In this case, every dynasty earns the same level of income and the wealth distribution degenerates into the steady state.

An important implication of Ghatak and Jiang (2002) is embedded in their proposition 3. They show that under the condition of (3), *the greater the fraction of the population who are initially poor, the lower the steady state [aggregate] income*. Indeed, this proposition is intuitive: the source of inefficiency when  $a_0^M < I$  comes from the fact that all potential entrepreneurial projects are not seized (equivalently, the size of projects is less than 1/2) because of credit market imperfection. If the credit market is perfect, then all the potential entrepreneurial projects can be seized. That is, the size of the project is 1/2 for every period; thereby, the economy attains its maximum aggregate output. Hence, Ghatak and Jiang (2002) suggested that *one shot redistributive policies can raise the total income of the economy permanently* and that revenue from taxes on bequests of rich dynasties will be redistributed to poor dynasties whose wealth is less than  $I$  to change the regime of the economy.

That suggested redistributive policy can enhance the total output of the economy when condition (3) is met and  $a_0^M < I$ . That policy, however, will indeed be difficult to implement, simply because the policy does not Pareto-improve the economy. Dynasties with more than  $I$  units of initial wealth suffer from both the tax and reduced income from projects as a consequence of the higher labor wage rate.<sup>5</sup> Hence, the government should find another way to achieve efficiency and greater equality in the framework of Ghatak and Jiang (2002).

### 2.3 The Role of Government

We can consider the economy of Ghatak and Jiang (2002) as that of small businesses. Furthermore, the model captures some features of the Japanese

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<sup>5</sup>If the policy is determined by majority vote, the redistributive policy is supported.

economy. It is plausible that small businesses have no access to international capital markets, probably because of high agency costs that small businesses must confront. Consequently, the economy might be inefficient in terms of the total output due to malfunctions in lending activities of private banks.

That inefficiency, however, can be corrected through government intervention via a public finance system. Consider a public finance system that has access to the international capital markets and assume that it has perfect information and can reinforce repayments of a loan perfectly. With a public finance system that complements the lending activities of private banks, a condition that ensures Pareto-improvement of the originally inefficient economy is derived as follows.<sup>6</sup>

**Proposition:** Consider the case in which initial conditions matter in the non-stochastic model of Ghatak and Jiang (2002); assume that few rich dynasties exist in the initial period ( $a_0^M < I$ ). Under that condition, wealth stratification is polarized. The public finance system can Pareto-improve the economy if and only if

$$\frac{r}{r-1}s(2G_0(I) - 1)\left(\frac{q-rI}{2} - \underline{w}\right) > \int_{a_0^M}^I (I - a_{i,0})dG_0(a). \quad (4)$$

**Proof:** Consider a public finance system that has access to international capital markets. The system borrows capital at the gross interest rate of  $r$  and then lends it to poor dynasties in the initial period. To change the regime of the economy to that where  $w^* = \bar{w}$  holds (and the total output is maximized), at least  $\int_{a_0^M}^I (I - a_{i,0})dG_0(a) (\equiv K)$  units of capital are required. The policy enhances the income of an originally poor dynasty by  $s\left(\frac{q-rI}{2} - \underline{w}\right) (\equiv \omega)$  units whereas the policy reduces the income of an originally rich dynasty by the same amount from (1) and (2). Thereby, the policy enhances the total output by  $\{2G_0(I) - 1\}\omega (\equiv W > 0)$ .

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<sup>6</sup>We are assuming, for public finance, that  $\pi = 1$  and  $F \geq rI$  hold, in the nomenclature presented in Ghatak and Jiang (2002).

The policy provides Pareto-improvement: (i) when the government redistributes to originally rich dynasties (dynasty  $i$  with  $a_{i,0} \geq I$ ) from originally poor dynasties (dynasty  $i$  with  $a_{i,0} < I$ ) such that all dynasties are at least indifferent as to the regime of the economy; and (ii) when the budget of the government is sustainable.

To investigate whether we can attain (i) and (ii) simultaneously, we have only to compare the discounted net profit of the policy ( $= \frac{r}{r-1}W$ ) to the initial cost of projects that are newly initiated through public finance ( $= K$ ). This comparison yields (4). Q.E.D.

Economic interpretations of eq. (4) are intuitive. The l.h.s. of (4) presents the discounted total net surplus that the government can, at most, seize when the policy changes the regime. The r.h.s. is the total cost of investment projects that are newly initiated through the public finance system. Equation (4) will be satisfied when the cost of capital is low, when the number of originally poor dynasties is large, or when the initial total cost of newly invested projects is low. The last condition requires that the density of initial wealth distribution in  $[a_0^M, I)$  to be low and that  $I - a_0^M$  not be too large. Hence, the policy may be effective when the initial wealth distribution is bi-modal and the median area is low (see, e.g., Fig. (3)). A candidate policy is one that targets the median area between those two peaks.

### 3 Conclusion

This paper has explained that the government can, when eq. (4) is satisfied, Pareto-improve the inefficient economy presented in Ghatak and Jiang through the use of the public finance system. Especially, this will be the case when the initial wealth distribution is one that has two peaks. Therefore, under the presumption that Japanese wealth stratification is now becoming polarized and that negative tendencies in lending to small businesses are one reason for Japan's economic stagnation, privatization of financial services of the MIAC might not be appropriate.



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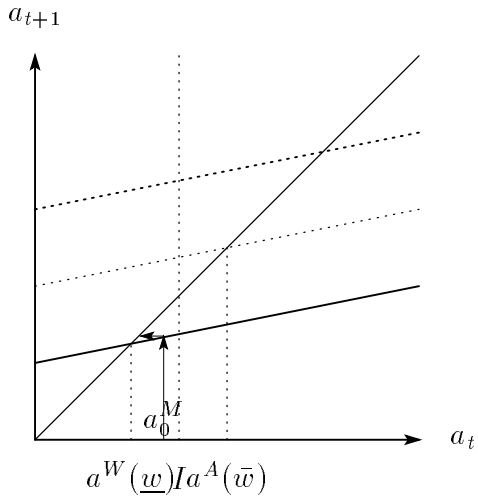


Figure 1: Phase-diagram when  $a_0^M < I$

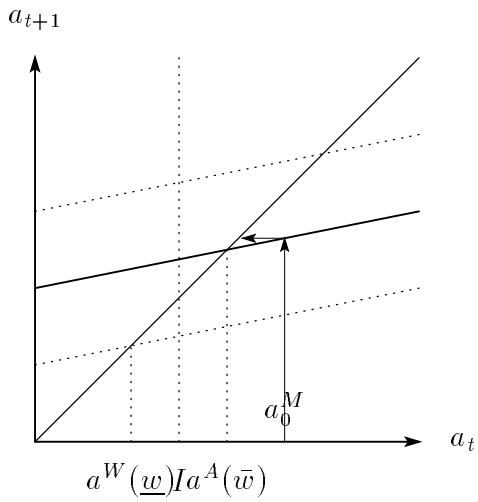


Figure 2: Phase-diagram when  $a_0^M \leq I$

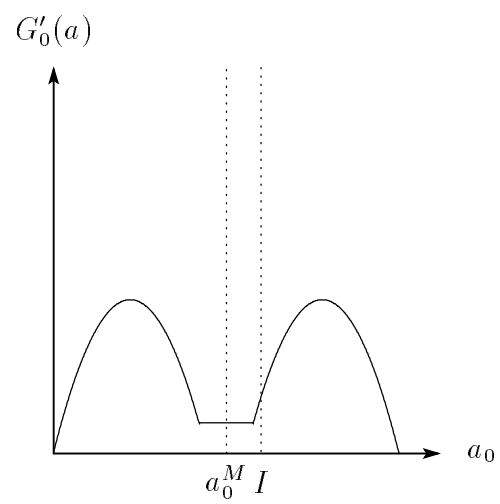


Figure 3: Example of Initial Wealth Distribution