E C O N O M I C S   B U L L E T I N

Implications of within-period timing in models of speculative attack

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Abstract

Speculative attacks are often modeled as decreases in money demand before currency crises. I discuss how, in models with microfoundations, within-period timing affects whether attacks arise in equilibrium. "Cash-when-I'm-done" timing always generates attacks, but is controversial because it assumes that end-of-period money balances buy current consumption. Cash-in-advance timing, theoretically more appealing, generates attacks only under restrictive assumptions. These issues arise when money is introduced via liquidity constraints, the utility function, or a transactions technology. Modeling attacks via reductions in demand for domestic bonds, instead of reductions in money demand, helps avoid these issues, and may be more realistic.

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1. Introduction

This paper discusses implications of within-period timing assumptions in models of currency crises. In particular, the choice of which money balances are available for transactions has important consequences for a model's ability to generate speculative attacks.

In the seminal papers of Krugman (1979), and Flood and Garber (1979), a speculative attack is modeled as a decline in demand for real balances. These early models assume that real money demand is a decreasing function of expected inflation. An instant before a currency peg collapses, expected inflation increases and money demand falls. The speculative attack is the act of consumers exchanging excess domestic currency for foreign reserves. In models with microfoundations, this pre-crisis drop in money demand has to be utility-maximizing. In a simple, perfect-foresight environment, I analyze the optimal response of consumers to a crisis under alternative timing setups. Implications of within-period timing are often not discussed, because currency crisis papers featuring microfoundations frequently model time as a continuous variable, and thus omit these details.[1] Nevertheless, timing has important implications regardless of whether money is introduced via liquidity constraints, in the utility function, or as input in a transactions technology.[2]

A timing setup that is relatively common, not only in crisis models such as Obstfeld (1986) and Burnside et al. (2001), but also in other areas of macroeconomics, is one that Carlstrom and Fuerst (2001) label "cash-when-I'm-done" (CWID) timing. This timing assumes that money available for transactions is end-of-period money, i.e. money held after receiving transfers, selling endowments, purchasing consumption, and dividing the remaining wealth between money and bonds. Under CWID timing, the result that consumers find it optimal to launch an attack by reducing money holdings is robust to many different specifications of utility. Nevertheless, as Carlstrom and Fuerst (2001) point out, CWID is unsatisfactory from a theoretical point of view, as it supposes that consumption expenditures are restricted, not by the money held as the consumer enters those transactions, but instead by the money held after these transactions in the goods market, and after even more transactions in an asset market.

An obvious candidate for an alternative timing that is immune to this critique is cash-in-advance (CIA) timing, under which only money accumulated in previous periods can help in current-period transactions (see, among many others Clower (1967), Lucas (1980), and Svensson (1985)). But this theoretically more appealing timing does not generate speculative attacks with the same generality as CWID. Restrictive assumptions on utility are required for consumers to find it optimal to exchange domestic money for reserves in the last pre-crisis period. For instance, in models with liquidity constraints and CES utility, consumers reduce domestic money demand only if the intertemporal elasticity of substitution for consumption is above one. In the unit-elasticity, i.e. logarithmic case, consumers keep their nominal money holdings unchanged, even though the nominal interest rate increases. And if the intertemporal elasticity is below unity, consumers actually increase their money holdings. The assumption of intertemporal elasticity of substitution above unity is by no means innocuous, since empirical estimates of this number usually hover around zero (see, for example, Hall (1988)) or around one (Beaudry and van Wincoop (1996)).

[1] Examples of continuous-time models featuring microfoundations for money include Calvo (1987), Burnside, Eichenbaum and Rebelo (2001), and Lahiri and Végh (2003). Obstfeld (1986) analyzes a model both in discrete time, and in continuous time. Burnside, Eichenbaum and Rebelo (2001) analyze a continuous-time model, but, in a footone, show the discrete-time model that corresponds to it when taking the limit as period length goes to zero. Burnside, Eichenbaum and Rebelo (2004) analyze a discrete-time model, but they assume that in the crisis period, there is rationing in the foreign exchange market. Consumers demand unbounded amounts of reserves, but the central bank only gives them an exogenously limited quantity. This makes it difficult to compare results regarding the attack with models in which prices clear the currency market.

[2] I use the term liquidity constraints instead of cash-in-advance constraints to avoid confusion, since I use cash-in-advance as the name of a within-period timing structure.

The reason for the difference in results becomes evident once we consider the consumer's trade-off when choosing money holdings in the presence of liquidity constraints. Under both timings, money provides liquidity services but has an opportunity cost in the form of forgone nominal interest. Thus, under both timings the inflation and nominal interest rate increase that comes with the crisis causes a decline in post-crisis consumption, and thus in post-crisis real balances. However, with CIA timing, this does not necessarily imply that nominal money holdings fall in the last pre-crisis period, because the increase in inflation already erodes the real value of the money held between periods. Hence, under CIA timing, reducing nominal money holdings, i.e. launching a speculative attack, is optimal only if the intertemporal elasticity of substitution is above one, in which case the drop in desired consumption outweighs the effect of inflation. Under CWID money held between the current period and the next provides liquidity in the current period. Thus, in the last pre-crisis period, the opportunity cost of holding money goes up, but the price of goods bought with that money remains fixed. Hence, for any positive intertemporal elasticity of substitution, consumption and nominal money holdings fall. A similar argument applies to models with money in the utility function (MIUF). With CIA timing higher nominal interest rates reduce real money demand, but since money is held for one period before yielding utility, higher inflation already implies lower real balances even if nominal balances are held constant. Wanting real balances to fall even more is only optimal under certain conditions regarding the degree to which utility is concave with respect to real balances. A very similar argument applies when money is an input in a transactions technology. CWID guarantees attacks as long the reductions in transaction costs are a strictly increasing and strictly concave function of real balances, while CIA generates attacks only if the function linking real balances to transaction costs satisfies particular curvature requirements.

In addition to these theoretical issues, there are empirical reasons to focus on variables other than money as a source of attacks. In actual crises, it is a stylized fact that before an exchange rate collapses, reserves fall significantly. But this is usually not accompanied by a decline in money, as exemplified by Mexico (1994), Korea (1997/98), Brazil (1999), and many others. It appears, thus, that the pre-crisis fall in reserves is due to private agents reducing the fraction of savings that they invest in domestic-currency denominated assets other than money, such as bonds. Thus, speculative attacks could be modeled by highlighting the fact that, for given amounts of debt sold by the government, when expected inflation increases, revenue falls.[3] This forces the government to sell reserves in order to pay for expenditures, provided that neither taxes nor expenditures are adjusted, which in turn is not an unreasonable assumption since in reality expectations in financial markets can change very quickly while fiscal variables change only infrequently.

The remainder of the paper is organized as follows. Section 2 sets up a basic model with liquidity constraints and discrete time, and discusses its equivalent continuous-time version. Section 3 proves the main results regarding CWID and CIA timing. Section 4 shows that similar results hold with MIUF and money in a transactions technology. Section 5 proposes reductions in revenue from government bond auctions as a source of attacks and section 6 concludes.

2. The Model in Discrete Time and in Continuous Time

The basic model features a deterministic, discrete-time environment in which periods are denoted by $t = 0,1,\ldots$. The home country is a small open economy that produces and consumes only one good. This good sells at home for p_t units of domestic currency and in the rest of the world for $p_{w,t}$ units of foreign currency. The (domestic for foreign) nominal exchange rate is given by s_t. PPP holds

[3] Similar arguments highlighting the role of nominal interest rates and domestic bonds have been made, for example, by Obstfeld (1994). In that model, however, consumer behavior was not derived from microfoundations.

$$p_t = s_t p_{w,t} \qquad \text{for all } t \geq 0, \tag{1}$$

and $p_{w,t} = 1$ at all times, so that p_t and s_t always coincide. The representative household has a constant endowment y and preferences over consumption sequences $\{c_t\}_{t\geq 0}$ represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1. \tag{2}$$

The period-utility function u satisfies usual strict monotonicity, strict concavity and Inada conditions. There are two assets. One of them is a foreign bond b_t with constant gross real return R. As is usual in the currency crisis literature, $R\beta = 1$ is assumed. The other asset is domestic money, which pays no interest. The representative household starts period t with M_{t-1} units of domestic money and with foreign bond holdings worth b_{t-1} units of time-t consumption. Over the course of period t, households harvest and sell their endowment y, receive a nominal lump-sum transfer τ_t (or pay a tax if τ_t is negative), and purchase c_t units of the consumption good. Finally, the household chooses how much money M_t and bonds b_t to hold between periods t and $t+1$. Thus, the period-by-period budget constraint is

$$\frac{M_t}{p_t} + \frac{b_t}{R} \leq \frac{M_{t-1}}{p_t} + b_{t-1} + \frac{\tau_t}{p_t} + y - c_t. \tag{3}$$

Under CWID timing, consumption cannot exceed end-of-period real balances:

$$c_t \leq \frac{M_t}{p_t}. \tag{4}$$

This assumption is problematic, since it allows cash earned selling the current-period endowment to buy consumption in the same period. According to Carlstrom and Fuerst (2001) *"It is very difficult to justify CWID timing on theoretical grounds …* [This timing] *implies that … what aids in current transactions is the money I leave the supermarket with, not the money I entered the market with."* Furthermore, they observe that "… [CWID timing] *violates Clower's dictum that 'money buys goods and goods buy money, but goods do not buy goods'"*.

An obvious way to avoid the Clower-Carlstrom-Fuerst critique is simply to assume cash-in-advance (CIA) timing, changing the liquidity constraint in order to let M_{t-1} restrict current consumption:

$$c_t \leq \frac{M_{t-1}}{p_t}. \tag{5}$$

It continuous-time models, it is generally impossible to see whether CIA or CWID timing is assumed. At times in which asset holdings do not jump discretely, the continuous-time version of (3) is given by[4]

$$\dot{m}_t + \pi_t m_t \leq y + \frac{\tau_t}{p_t} - c_t + (R-1)b_t - \dot{b}_t, \tag{6}$$

[4] To see this, note that (3) is a special case for $\Delta t = 1$ of

$$\frac{M_t}{p_t} - \frac{M_{t-\Delta t}}{p_t} \leq \Delta t\left(y + \frac{\tau_t}{p_t} - c_t + (R-1)b_{t-\Delta t}\right) - (b_t - b_{t-\Delta t}).$$

Dividing both sides by Δt and letting Δt approach zero yields (6) . Intermediate steps on the left are:

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\frac{M_t}{p_t} - \frac{M_{t-\Delta t}}{p_t}\right) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\frac{M_t}{p_t} - \frac{M_{t-\Delta t}}{p_{t-\Delta t}}\left(\frac{p_{t-\Delta t}}{p_t} + 1 - 1\right)\right) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\left(\frac{M_t}{p_t} - \frac{M_{t-\Delta t}}{p_{t-\Delta t}}\right) + \frac{M_{t-\Delta t}}{p_{t-\Delta t}}\left(1 - \frac{p_{t-\Delta t}}{p_t}\right)\right] = \dot{m}_t + \pi_t m_t$$

where $m_t = M_t / p_t$, $\pi_t = \dot{p}_t / p_t$ is the inflation rate, and a dot over a variable denotes its time derivative. When asset holdings jump, $\dot{m}_t$ and $\dot{b}_t$ do not exist, and instead of (6), we have $\Delta m_t = -\Delta b_t$. Liquidity constraints (4) and (5) become $c_t \leq m_t$ when $\dot{m}_t$ and $\dot{b}_t$ exist, and when they do not, the CWID liquidity constraint (4) is $c_t \leq m_t$ and the CIA liquidity constraint (5) is $c_t \leq (\lim_{\Delta t \to 0} M_{t-\Delta t}) / p_t$. While at first this difference appears to be enough to distinguish CWID from CIA, this is not the case. If start- and end-of-period assets are simply relabeled (M_t, b_t) and (M_{t+1}, b_{t+1}), instead of (M_{t-1}, b_{t-1}) and (M_t, b_t), then, taking limits leaves budget constraints (6) and $\Delta m_t = -\Delta b_t$ unchanged, but liquidity constraints become, under CIA $c_t \leq m_t$ at all times, and under CWID $c_t \leq m_t$ when $\dot{m}_t$ and $\dot{b}_t$ exist, and $c_t \leq (\lim_{\Delta t \to 0} M_{t+\Delta t}) / p_t$ otherwise. Thus, given a constraint (6) (with $\Delta m_t = -\Delta b_t$ for discontinuous times) and liquidity constraints $c_t \leq m_t$, it is impossible to tell whether CIA or CWID is assumed.

3. Currency Crisis and Speculative Attack

The role of within-period timing in generating attacks can be explored in a very simple perfect-foresight environment. For consumers, all that is relevant is that the exchange rate s_t, and thus p_t, stay fixed until time T, and increase at time $T+1$. For simplicity's sake, I will follow the usual assumption that from time $T+1$ onward s_t and p_t grow at the constant rate $\pi > 0$:

$$p_t = \begin{cases} p_0 & \text{for all } t \in \{0,1,\ldots,T\} \\ p_0(1+\pi)^{t-T} & \text{for all } t > T. \end{cases} \tag{7}$$

Of course, this implies that the nominal interest rate, defined as $i_t \equiv R p_{t+1} / p_t - 1$, equals $R-1$ for *t* from zero to $T-1$ and then jumps to $R(1+\pi)-1$ from period *T* onwards.

A speculative attack in this economy is defined as a drop in nominal money demand at time T, $M_{T-1} - M_T$. In other words, an attack occurs if the fall in reserves exceeds τ_T / p_T. In continuous time, τ_t is infinitesimal and the decline in money demand coincides with the reserves lost. But in discrete time one should be careful to distinguish the gradual loss of reserves that happens if transfers are positive up to time *T* (every period, consumers exchange those transfers for reserves as soon as they receive them) from the loss of reserves due to consumers anticipating the collapse of the fixed exchange rate. Having clarified this, I next proceed to state and prove the paper's main results in propositions 1 and 2.

Proposition 1 *In models with liquidity constraints, under CIA timing, consumers reduce nominal money holdings at time T only under restrictive assumptions regarding utility. For* $u(c_t) = (c_t^{1-\sigma} - 1)/(1-\sigma)$ *nominal money holdings fall if* $\sigma < 1$, *stay constant if* $\sigma = 1$ *and increase if* $\sigma > 1$.

Proof Given initial asset holdings b_{-1} and $M_{-1} > 0$, and prices $R = 1/\beta$ and $\{p_t\}_{t \geq 0}$ given by (7), the consumer chooses $\{c_t, b_t, M_t\}_{t \geq 0}$ to maximize (2) subject to (3), (5), and $b_t \geq -B, \forall t \geq 0$, where *B* rules out Ponzi schemes but otherwise does not bind in equilibrium. Taking first order conditions with respect to c_t, b_t, and M_t, and combining them (see appendix A for details) yields

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{p_{t+1} / p_t}{p_t / p_{t-1}}. \tag{8}$$

Given (7), the right-hand side of (8) is one if $t \neq T$ and $(1+\pi)$ if $t = T$. It follows that one consumption level c_T is optimal for all $t \in \{1,\dots,T\}$ (c_0 may be different since M_{-1} is not chosen) and c_{T+1} is optimal for all $t \geq T+1$. Strict concavity of u implies $c_{T+1} < c_T$. But $M_T < M_{T-1}$ holds only if the intertemporal elasticity of substitution is high enough to make consumption fall by more than inflation rises. In the CES case, (8) becomes

$$\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} = \frac{p_{t+1}/p_t}{p_t/p_{t-1}}. \tag{9}$$

Setting $t = T$, using (5) with equality to eliminate c_T and c_{T+1} and rearranging terms yields the desired link between the elasticity of substitution $1/\sigma$ and money holdings

$$M_T = (1+\pi)^{1-\frac{1}{\sigma}} M_{T-1}. \text{ Q.E.D.} \tag{10}$$

Proposition 2 *With CWID timing, consumers find it optimal to launch a speculative attack at time T for any u that is strictly increasing and strictly concave.*

Proof Everything is the same as in the proof of proposition 1, but liquidity constraints are given by (4) instead of (5). Again, taking first order conditions with respect to c_t, b_t, and M_t, and combining them (details, once more, are available in appendix A) yields

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{2-\dfrac{1}{1+i_{t+1}}}{2-\dfrac{1}{1+i_t}}. \tag{11}$$

Since $i_T > i_{T-1}$, strict concavity of u implies that consumption falls between periods $T-1$ and T. Because in both periods the price is still fixed, it follows that $M_T < M_{T-1}$. Q.E.D.

4. Robustness: Money in the Utility Function and Money in a Transactions Technology

The main message of propositions 1 and 2, namely that CWID timing always generates attacks, while CIA timing requires restrictive assumptions, also applies in MIUF models and in models where, as in Lahiri and Végh (2003), money is an input in a transactions technology.

In the MIUF case, with separable utility from real balances, the utility function is

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \varpi(M_t / p_t) \right] \tag{12}$$

in the CWID case, with M_{t-1} replacing M_t in the CIA case. The function ϖ is assumed to satisfy strict monotonicity, strict concavity and Inada conditions. Once more, given b_{-1} and $M_{-1} > 0$, and prices $R = 1/\beta$ and $\{p_t\}_{t\geq 0}$ given by (7) the consumer chooses $\{c_t, b_t, M_t\}_{t\geq 0}$ to maximize (12) subject to (3) and $b_t \geq -B, \forall t \geq 0$. (Of course (12) is modified in the CIA case.) First-order conditions (see appendix B) imply constant consumption $c_t = \bar{c}$ for all $t \geq 0$. Money demand, in the CIA case, is determined by

$$i_t u'(\bar{c}) = \varpi'\left(\frac{M_t}{p_{t+1}}\right) \tag{13}$$

and in the CWID case by

$$u'(c_t)\frac{i_t}{1+i_t}=\varpi'\left(\frac{M_t}{p_t}\right). \tag{14}$$

At time $t=T$ the nominal interest rate i_T increases, i.e. $i_T > i_{T-1}$. Equation (13) implies that $\varpi'(M_T / p_{T+1}) > \varpi'(M_{T-1} / p_T)$, and strict concavity of ϖ implies $M_T / M_{T-1} < p_{T+1} / p_T = (1+\pi)$. In the CIA case, this is all that can be established without further assumptions on how sensitive ϖ' is to changes in its argument. In the CWID case (14) unambiguously implies that, since $p_T = p_{T-1}$ and $i_T > i_{T-1}$, for any strictly concave ϖ, $M_T < M_{T-1}$.

When money is an input in a transactions technology, every period consumers pay a transaction cost ϕ which is decreasing in M_t / p_t and in H_t / p_t. H_t is a domestic asset that can be thought of as, for example, a money market account, in that it provides more liquidity than the foreign bond, but less than money, and it earns nominal interest at the rate $i_t^g \in (0, i_t)$. For clarity of exposition, I will focus on the case in which the government does not increase i_t^g when i_t increases, and just set $i_t^g = i^g$ for all $t \geq 0$. The transaction cost, in the case of CIA timing, is given by

$$\phi(H_{t-1} / p_t, M_{t-1} / p_t) = K - v(H_{t-1} / p_t) - w(M_{t-1} / p_t), \tag{15}$$

where $K > 0$, and v and w satisfy Inada conditions, are strictly increasing, strictly concave, and are such that ϕ is always positive. In this setting, the budget constraint is given by

$$\frac{M_t}{p_t}+\frac{b_t}{R}+\frac{H_t}{(1+i^g)p_t} \leq \frac{M_{t-1}}{p_t}+b_{t-1}+\frac{H_{t-1}}{p_t}+y+\frac{\tau_t}{p_t}-c_t-\phi\left(\frac{H_{t-1}}{p_t},\frac{M_{t-1}}{p_t}\right). \tag{16}$$

Under CWID timing, ϕ is a function of H_t / p_t and M_t / p_t instead of H_{t-1} / p_t and M_{t-1} / p_t. This model turns out to be very similar to the MIUF model. With CIA timing, it is easy to verify that the choice of H_t that maximizes (2) subject to (16) has to satisfy

$$\frac{1+i_t}{1+i^g}=1+v'\left(\frac{H_t}{p_{t+1}}\right). \tag{17}$$

As in the MIUF case, (7) and (17) imply that $H_T < H_{T-1}$ only if v' is relatively insensitive to changes in H_t / p_{t+1}. Under CWID, in contrast, the optimal H_t is dictated by

$$1+i_t=\frac{1}{\dfrac{1}{1+i^g}-v'\left(\dfrac{H_t}{p_t}\right)}, \tag{18}$$

which implies that H_t must fall at $t=T$ since v is strictly concave. The analysis of whether M_t falls at $t=T$ is analogous, just modifying (17) and (18) by setting $i^g=0$ and using w instead of v.

5. Attacks as Reductions in Domestic-Currency Denominated Savings

The analysis so far suggests that there may be theoretical difficulties in modeling speculative attacks as drops in money demand. CWID timing is, per se, controversial, while CIA timing requires assumptions on preferences or technology that may be difficult to support with evidence. In addition to these theoretical issues, modeling speculative attacks as declines in money demand is also subject to empirical challenges. The large drop in reserves that happens in the days, or weeks, prior to currency crises, does not, in general, coincide with reductions in the money supply. This indicates that the drop in reserves may be due to the private sector reducing the amount of resources they invest in assets such as domestic-currency denominated bonds. This idea can be modeled, and has been modeled, in many ways, see for instance Obstfeld (1994), and in the context of debt crises, Cole and Kehoe (2000). In the simple model studied in this paper, this idea can be incorporated by adding domestic debt D_t, which, by no-arbitrage must pay nominal interest i_t, and rewrite the consumer's budget constraint as

$$\frac{M_t}{p_t}+\frac{b_t}{R}+\frac{D_t}{p_t(1+i_t)}\leq\frac{M_{t-1}}{p_t}+b_{t-1}+\frac{D_{t-1}}{p_t}+\frac{\tau_t}{p_t}+y-c_t. \tag{19}$$

Obviously, in period $t=T$ the increase in the nominal interest rate reduces the real resources that consumers need to buy the domestic debt. If consumers do not increase money demand in this last pre-crisis period, the remaining resources are invested in foreign bonds. The decline in reserves also becomes evident once we examine the government's budget constraint

$$g_t+\frac{\tau_t}{p_t}+\frac{D_{t-1}}{p_t}+\frac{f_t}{R}\leq f_{t-1}+\frac{M_t-M_{t-1}}{p_t}+\frac{D_t}{p_t(1+i_t)}. \tag{20}$$

where f_t denotes foreign reserves, and g_t real government purchases. If we suppose, for instance, that the government maintains the debt constant over time, the decrease in the price at which consumers buy the debt reduces government revenues. If g_t and τ_t are not adjusted, reserves fall.[5]

Finally, focusing on bonds instead of money demand has at least two more advantages. First, by allowing bonds to have maturities longer than one period, the decline in reserves prior to crises can be made gradual instead of sudden. This feature seems desirable since in reality speculative pressure often accrues over several weeks or months. Second, the price consumers are willing to pay for government bonds may also reflect (in addition to expectations of devaluation) a probability of default, which is useful since currency collapses and fears of default occur simultaneously in many financial crises.

6. Conclusion

Speculative attacks in which private agents acquire large amounts of reserves from central banks are a central feature of currency crises. In fact, in second-generation models, there are instances in which crises would never happen if private agents did not launch a speculative attack against the currency. Starting with Krugman (1979) and Flood and Garber (1984), attacks have often been modeled as decreases in money demand. This paper argues that in models with microfoundations for money, consumers do not always find it optimal to reduce their holdings of domestic currency in the last period before a crisis. In the environment studied in this paper, controversial assumptions regarding either within-period timing or consumer preferences are needed to guarantee the optimality of these speculative attacks. Since, in addition to these theoretical issues, there are many actual crises in which money did not fall, it may be preferable to focus on the demand for domestic-currency denoinated bonds, rather than money, as a source of attacks.

[5] Of course, this need not hold if government purchases, transfers, or the amount of debt auctioned are adjusted. But, since fiscal policy can always be adjusted so that crises never happen, all currency crisis models must implicitly or explicitly assume that these adjustments are not made.

Appendix A – Intermediate Steps in the Proofs of Propositions 1 and 2

In the proof of proposition 1, letting λ_t and ξ_t denote Lagrange multipliers associated with (3) and (4), respectively, first order necessary conditions with respect to c_t, b_t, and M_t, are:

$$\beta^t u'(c_t) = \lambda_t + \xi_t \tag{A1}$$

$$\frac{\lambda_t}{R} = \lambda_{t+1} \tag{A2}$$

$$\frac{\lambda_t}{p_t} = \frac{\lambda_{t+1}}{p_{t+1}} + \frac{\xi_{t+1}}{p_{t+1}}. \tag{A3}$$

Solving for λ_t in (A3) and substituting the result into (A2) yields

$$\lambda_{t+1}\frac{p_t}{p_{t+1}} + \xi_{t+1}\frac{p_t}{p_{t+1}} = \lambda_{t+1}R.$$

From here, solving for ξ_{t+1} and recalling that $i_t \equiv \left(Rp_{t+1}/p_t\right)-1$, we obtain

$$\xi_{t+1} = \lambda_{t+1}\left(\frac{p_{t+1}}{p_t}R-1\right) = \lambda_{t+1}i_t. \tag{A4}$$

Since t is arbitrary, it is also true that $\xi_t = \lambda_t i_{t-1}$. Combining with (A1), we see that

$$\beta^t u'\left(c_t\right) = \lambda_t(1+i_{t-1}). \tag{A5}$$

Finally, dividing the $t+1$-analog of (A5) by (A5), using (A2) and $R\beta = 1$, (8) obtains.

In the proof of proposition 2, first-order onditions with respect to c_t and b_t are still (A1) and (A2) but with respect to M_t we have

$$\frac{\lambda_t}{p_t} = \frac{\lambda_{t+1}}{p_{t+1}} + \frac{\xi_t}{p_t}. \tag{A6}$$

Solving for ξ_t yields

$$\xi_t = \lambda_t - \frac{p_t}{p_{t+1}}\lambda_{t+1}.$$

Substituting this into (A1) and using (A2) to eliminate λ_{t+1} we obtain

$$\beta^t u'\left(c_t\right) = \lambda_t\left(2-\frac{1}{1+i_t}\right). \tag{A7}$$

From here, (11) is derived following steps analogous to those in the proof of proposition 1.

Appendix B – First-Order Conditions in the MIUF case

The first order condition with respect to consumption is simply given by

$$\beta^t u'(c_t) = \lambda_t \tag{B1}$$

Combining this with (A2) and using the fact that $R\beta = 1$, $c_t = \overline{c}$ for all $t \geq 0$ follows.

Under CIA timing, the first order condition with respect to M_t is

$$\frac{\lambda_t}{p_t} = \frac{\lambda_{t+1}}{p_{t+1}} + \beta^{t+1}\frac{\varpi'(M_t/p_{t+1})}{p_{t+1}} \tag{B2}$$

Multiplying by p_{t+1}, using (A2) and (B1), and solving for $\varpi'(M_t / p_{t+1})$ yields (13).

In the CWID case, the first order condition with respect to M_t is

$$\frac{\lambda_t}{p_t} = \frac{\lambda_{t+1}}{p_{t+1}} + \beta^t \frac{\varpi'(M_t / p_t)}{p_t} \tag{B3}$$

Multiplying by p_t, using (A2) to eliminate λ_{t+1}, using (B1) to eliminate λ_t, solving for $\varpi'(M_t / p_t)$ and rearranging terms yields (14).

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