E C O N O M I C S   B U L L E T I N

On The Power to Hurt: Costly Conflict with Completely Informed States

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Abstract

Slantchev (2003, American Political Science Review, 97) studies a class of negotiation models to explain costly conflict between two completely informed nations. In one of his main propositions (Proposition 2.3), Slantchev provides a strategy profile to support the so−called extremal subgame perfect equilibrium, where one nation receives its lowest equilibrium payoff. By means of a counter example, we demonstrate the existence of an equilibrium with one nation's payoffs below the strategy profile provided in his Proposition 2.3 (Case 2).

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1 Introduction

In a recent article, Slantchev (2003) discusses the merits of analyzing warfare as a negotiation process between two nations in the shadow of fighting. Wielding actions of warfare is an intangible part of such processes that may influence the final resolution. This negotiation process is modeled as a bargaining game with endogenous threats, also known as the negotiation model pioneered by Haller and Holden (1990), Fernandez and Glazer (1991), Haller (1991), Bolt (1995) and Houba and Bolt (2000) in the context of wage contract negotiations and followed by Busch and Wen (1995) and Houba (1997) in a general context of the disagreement game. Slantchev (2003) analyzes a class of negotiation games where there is one disagreement outcome dominating any other disagreement outcome while the two nations have different time preferences that are modelled as discount factors. When discount factors are sufficiently large, this negotiation model generally admits many equilibrium outcomes featuring costly fighting before a delayed peace treaty can be reached, despite many restrictions to "stack the model against fighting". The strategy profiles supporting those equilibria, however, critically depend upon every nation's worst or *extremal* equilibrium. These extremal equilibria represent a nation's worst fear of what might happen if it would unilaterally cease fire and, therefore, what keeps nations trapped in continuing the warfare.

The purpose of this note is to alert the scientific community of the complications involved with extremal SPE in the negotiation model when two nations have different time preferences. For that purpose, we critically re-examine a claim in Slantchev (2003) about the extremal equilibrium: By means of a simple counter example, we demonstrate that the supposedly extremal strategy profile provided in Proposition 2.3 (Case 2) of Slantchev (2003) is not necessarily the extremal equilibrium to the nation with a lower discount factor, no matter how close the two discount factors are to 1. The proof of Proposition 2.3 in Slantchev (2003) merely verifies that the stated strategy profile constitutes a subgame perfect equilibrium (SPE). The strategy profile extends the extremal SPE when the two nations have the same discount factor, with appropriate modifications to the case where the two nations differ in their discount factors. Under the framework of Slantchev's model, our counter example demonstrates a simple SPE where the nation with a lower discount factor receives less than from what Slantchev claims to be this nation extremal SPE.

Our counter example is presented in the next section and a concluding remark is put in a separate section.

2 The Counter Example

We now provide an example to demonstrate that the SPE provided in Part 2 of Proposition 2.3 in Slantchev (2003) fails to be the extremal SPE to the nation whose discount factor is lower than the other nation's, no matter how small both discount factors are. Consider the warfare negotiation model with the following symmetric disagreement game:

$1 \setminus 2$	L	M	R
T	$\frac{1}{2}, \frac{1}{2}$	$0, 0$	$0, 0$
M	$0, 0$	$0, 0$	$0, \beta$
B	$0, 0$	$\beta, 0$	$0, 0$

where $0 \leq \beta < \frac{1}{2}$. As constructed, this disagreement game has one Nash equilibrium $a^* = (T, L)$ that Pareto dominates any other disagreement outcome and it is on the bargaining frontier. In order to reduce the number of equilibrium conditions, we assume three other Nash equilibria: (B, R), (M, R) and (B, M), of which Nash equilibrium (M, R) turns out to be crucial. This disagreement game satisfies all the assumptions imposed in Slantchev (2003).

In order to support nation 1's extremal SPE, Slantchev suggests to maximize nation 2's disagreement payoff by describing (T, L) in all odd periods and minimize nation 1's disagreement payoff by describing (B, R) in all even periods. Since both disagreement outcomes, (T, L) and (B, R), are Nash equilibria in the disagreement game, a SPE can be derived based on the infinite sequence of alternating between these two disagreement outcomes. Proposition 2.3 of Slantchev (2003) identifies the following two equilibria:

A. When $\delta_1 \geq \delta_2$, both nations will make acceptable proposals. Accordingly, nation 1 receives $\frac{1}{2}\frac{1-\delta_2}{1-\delta_1\delta_2}$ in any odd period and $\frac{1}{2}\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$ in any even period.

B. When $\delta_1 < \delta_2$, only nation 2 makes an acceptable proposal. Accordingly, nation 1 receives $\frac{1}{2}\frac{1}{1+\delta_1}$ in any odd period and $\frac{1}{2}\frac{\delta_1}{1+\delta_1}$ in any even period.

It is easy to see why nation 1 behaves differently in these two cases. Given the alternating disagreement outcomes, nation 1 can secure a payoff of $\frac{1}{2}\frac{1}{1+\delta_1}$ in any odd period and $\frac{1}{2}\frac{\delta_1}{1+\delta_1}$ in any even period from simply collecting its alternating disagreement payoffs $\frac{1}{2}$ and 0 forever, called the *no-concession strategy* by Bolt (1995) in the wage negotiation model. Nation 1 will make an acceptable proposal if and only if

$$\frac{1}{2}\frac{1-\delta_2}{1-\delta_1\delta_2} \geq \frac{1}{2}\frac{1}{1+\delta_1} \quad \Leftrightarrow \quad \delta_1 \geq \delta_2. \tag{1}$$

Slantchev (2003) claims that Proposition 2.3 characterizes nation 1's extremal SPE in these two cases.

Now we present a SPE strategy profile from which nation 1 receives strictly less than $\frac{1}{2}\frac{1}{1+\delta_1}$ in any odd period and $\frac{1}{2}\frac{\delta_1}{1+\delta_1}$ in any even period for most of $\delta_1 < \delta_2$. Consider the following strategy profile:

C. Both nations make acceptable proposals that are calculated from the constant sequence of disagreement outcomes (M, R) in all periods, and the two nations would play the Nash equilibrium (M, R) in any disagreement game.

It is obvious that no nation has an incentive to deviate in the disagreement game. Unlike the SPE of described by strategy profile B, forever collecting the disagreement payoffs yields nation 1 a payoff of 0. Suppose that nation 1 proposes $(x_1, 1-x_1)$ and nation 2 proposes $(1-x_2, x_2)$. After rejecting a proposal, the responding nation will receive his disagreement payoff during the current period and his equilibrium share in the following period. A responding nation should be indifferent between accepting and rejecting the standing offer:

$$1 - x_1 = (1 - \delta_2)\beta + \delta_2 x_2 \quad \text{and} \quad 1 - x_2 = \delta_1 x_1,$$

which yield nation 1's payoffs in an odd period and an even period as

$$x_1 = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}(1 - \beta) \quad \text{and} \quad 1 - x_2 = \frac{\delta_1 (1 - \delta_2)}{1 - \delta_1 \delta_2}(1 - \beta). \tag{2}$$

If nation 1 makes an unacceptable proposal in an offer period, its payoff will be $\delta_1 (1 - x_2) = \delta_1^2 x_1 < x_1$. Therefore, nation 1 cannot benefit from the no-concession strategy in strategy profile C. This establishes that the strategy profile C is a SPE.

Now the issue is whether x_1 and $\delta_1 x_1$ in (2) are less than nation 1's SPE payoffs predicted by Proposition 2.3 of Slantchev (2003). First observe that

$$x_1 = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}(1 - \beta) \quad > \tfrac{1}{2} \frac{1 - \delta_2}{1 - \delta_1 \delta_2}$$

for all $\beta \in [0, \frac{1}{2})$. This implies that when $\delta_1 \geq \delta_2$, the SPE resulted from strategy profile A yields nation 1 a lower payoff then our SPE of strategy profile C. When $\delta_1 < \delta_2$, however, we have

$$x_1 = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}(1 - \beta) < \frac{1}{2} \frac{1}{1 + \delta_1}, \tag{3}$$

which yields

$$\beta > \beta^* \equiv \frac{1}{2} - \frac{1}{2} \frac{\delta_2 - \delta_1}{(1 + \delta_1)(1 - \delta_2)}.$$

Obviously, $\delta_1 < \delta_2$ implies $\beta^* < \frac{1}{2}$. In other words, when $\delta_1 < \delta_2$, (3) holds for all $\beta \in \left(\beta^*, \frac{1}{2}\right)$. Moreover, it is even possible to obtain $\beta^* < 0$, which requires $\delta_2 > \frac{1+2\delta_1}{2+\delta_1}$ which is larger than δ_1. This last result means that for a large set of parameter values $\delta_1 < \delta_2$, no value of β can be found such that the SPE of strategy profile B is nation 1's worst SPE.

Figure 1 illustrates the region of δ_1 and δ_2 for which the strategy profile C yields nation 1 a lower equilibrium payoff than the SPE resulted from strategy profile B. Since this requires that nation 2 is relatively more patient than nation 1, it follows that strategy profile A is not a SPE when $\delta_1 < \delta_2$. The curved boundary of region C depends upon β and this curve shifts downward as β increases with the forty-five degree line at the border case $\beta = \frac{1}{2}$. So, region C expands as β increases and covers about one quarter to half of the unit square. So, region C is quite large.

3 Concluding Remark

The key implication of our examples is that when nations have different time preferences, the current game theoretic literature fails to characterize the lowest equilibrium payoffs, not only in the warfare negotiation model of Slantchev (2003), but also in the general negotiation model of Busch and Wen (1995). Thus far, only a few have analyzed different time preferences

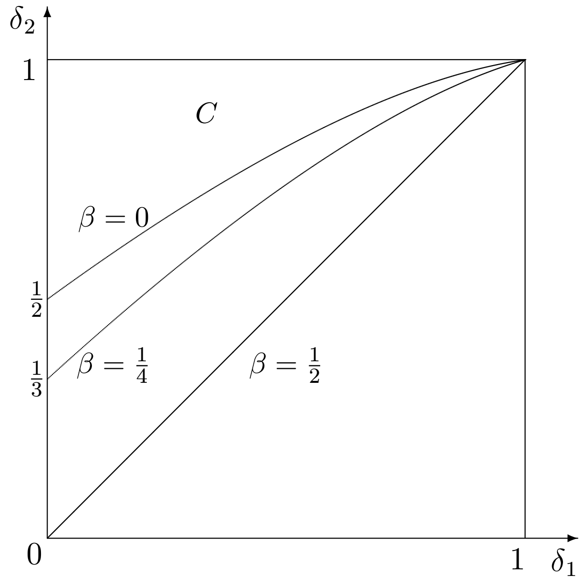

Figure 1: Condition (3) for $\beta = 0$, $\beta = \frac{1}{4}$ and $\beta = \frac{1}{2}$. The region where the strategy profile C yields nation 1 a payoff lower than strategy profile B is indicated by C.

in the negotiation model (often for some restrictive class of disagreement games). The characterization of extremal equilibrium payoffs requires a thorough analysis of equilibrium bounds using the method of Shaked and Sutton (1984), see Osborne and Rubinstein (1990), Muthoo (1999) or Houba and Bolt (2002) for surveys. However, since strategy profiles may involve unacceptable proposals it also implies that the method of Shaked and Sutton (1984) should also be adapted to allow for such deliberate delays.

It is worthwhile to point out that our finding in this note does not change the qualitative aspect of the main message of Slantchev (2003): The existence of equilibria with wasteful fighting and delayed peace treaties. Since these equilibrium strategies revert to each nation's worst or extremal equilibrium as the most effective punishments, downward modifications in the nations' extremal equilibrium payoffs imply a weakening of conditions under which such equilibria can be sustained in equilibrium. We hope our example raises the awareness of the complications involved with extremal SPE in the negotiation model when two nations have different time preferences.

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