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On Stickiness, Cash in Advance, and Persistence

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On Stickiness, Cash in Advance, and Persistence *

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Abstract

This paper shows that a model which combines sticky price and sticky wages with investment in the cash-in-advance constraint generates business cycle dynamics consistent with empirical evidence. The model reproduces the responses of the key macroeconomic variables to technology and money supply shocks. In particular, the model generates enough outuput and inflation persistence with standard stickiness parameters. This setup is also able to generate the liquidity effect after a money injection, overcoming other standard new Keynesian models.

Keywords: sticky prices, sticky wages, monetary facts, labor market facts, cash-in-advance.

JEL Class.: E32, E41, E52

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1 Introduction

This paper argues that a cash-in-advance (henceforth, CIA) model is able to account for output and inflation persistence. It also shows that a monetary model with investment in the CIA constraint generates some key monetary and technology stylized facts, overcoming standard new Keynesian models.

Previous literature has emphasized the inability of sticky prices alone to generate business cycle fluctuations, mainly inflation and output persistence. The main challenge facing dynamic stochastic general equilibrium models (henceforth DSGE models) is how much the mechanism with nominal rigidities can deliver in transmitting business cycle shocks. Standard DSGE models have so far achieved mixed success along this dimension. For example, it remains a challenge to account for output persistence (e.g., Chari, Kehoe and McGrattan, 2000) or inflation persistence (e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999). On the other hand, DSGE models can do well in explaining some labor market dynamics, such as the cyclical behavior of employment and real wages (e.g., Huang, Liu and Phaneuf, 2004; Galí, 1999; Liu and Phaneuf, 2006); while a new strand of the literature attempts to reproduce stylized monetary facts by constructing a dual stickiness framework: sticky prices and sticky information (e.g., Collard and Dellas, 2006; Dupor, Kitamura and Tsuruga, 2006).

Christiano, Eichenbaum and Evans (2005) present a monetary model with both nominal and real rigidities to analyze inflation inertia and output persistence after a monetary shock. They find that the key factors driving the results are those rigidities preventing marginal costs from reacting too much after the shock, in particular, wage stickiness and variable capital utilization. One important additional factor is the use of price indexation for those firms not adjusting prices. This fact implies a lagged inflation term in the new Phillips curve, inducing more persistence in the response of inflation. However, this assumption is not completely supported by the data (see for example, Dhyne et al. (2005) for some evidence on Euro area data). This paper shows that a sticky-price, sticky-wage model with investment in the CIA constraint can generate enough output and inflation persistence without the need of price indexation.

In addition, this framework allows to closely reproduce monetary and labor market facts.

Wang and Wen (2006) analyze output persistence in a sticky price model with investment in the CIA constraint. They find that introducing investment as a cash good is crucial for generating output persistence otherwise missing in a standard sticky price model. Our setup is similar to theirs in that we also consider sticky prices and investment as a cash good. However, we go further in their analysis in three main aspects. First, we also consider sticky wages as a more important mechanism in generating persistence than sticky prices. Adding wage stickiness to a sticky price model has shown to be quite successful in recent literature, in particular, in generating output persistence (Christiano, Eichenbaum and Evans, 2005; Liu and Phaneuf, 2006). And second, not only do we focus on output but also on inflation persistence, one of the main failures of new Keynesian models. Third, we study the dynamics properties of this framework with regard to some key monetary and labor market stylized facts.

In contrast to previous new Keynesian models where the role of monetary holdings is usually modelled as real balances in the utility function, we introduce money through a CIA constraint. In spite of the different setup, the timing is equivalent to that of a model with money in the utility function, but at the same time it allows for extensions of interest such as making investment a cash good. Previous research stressed the role of inflation on investment demand, and introduced investment decisions constrained that way (Stockman, 1981; Abel, 1985). Empirically, although it is still topic of debate, there seems to be some evidence regarding the effects of firms' internal cash flows on investment demand in a context of capital market imperfections (Fazzari, Hubbard and Peterson, 1988). In this sense, cash flows are often used as a proxy for net worth in determining investment. Recently, some studies for the US and countries in the Euro area reveal a significant effect of cash flows on investment demand, although the strength of the effect varies across countries (Chirinko, Fazzari and Meyer, 1999; Angeloni, Kashyap and Mojon, 2003). The relevance of cash flows for investment demand, and therefore, the ability of firms to react to shocks can be addressed in our model by including investment in the CIA constraint.

Our framework is a general equilibrium monetary model of the business cycle in which firms set prices in a staggering way, à-la-Calvo. Besides, we introduce wage stickiness by allowing individuals to have some market power in their supply of labor, as in Erceg, Henderson and Levin (2000), and we introduce investment in the CIA constraint. We analyze three separate scenarios. First, we check the ability of a sticky price model to reproduce some stylized facts of US business cycles. More concretely, we focus on the dynamics after technology and money supply shocks. Then, we analyze the properties of inflation and output dynamics generated by this model, with particular emphasis on persistence, by considering also sticky wages and alternative degrees of investment in the CIA constraint.

We find that a model which combines both sticky wages and sticky prices and investment as a cash good reproduces the stylized facts after a technology shock and also after a money injection. Our model generates enough inflation and output persistence compared to that observed in the data, with reasonable degrees of stickiness. The key factor driving these results is the inclusion of investment in the CIA constraint, without the need of price indexation or variable capital utilization. Finally, our setup is able to generate the liquidity effect. This result stresses the relevance of sticky wages versus sticky prices in modeling the monetary transmission mechanism. Also, we need investment completely financed with cash to obtain the liquidity effect. The mechanism behind these results is the delayed response of aggregate demand to shocks, due to the CIA constraint, together with marginal costs being affected by the interest rate.

The paper is structured as follows. First, we present the model and make special emphasis on the introduction of sticky wages and a CIA constraint on aggregate demand. In sections 3 and 4, we calibrate the model and solve for the equilibrium. We proceed to analyze the dynamics of the model after a positive technology shock and a monetary injection in Section 5. Section 6 focuses on the persistence generating by the model. Section 7 closes the paper.

2 The Model

The economy is populated by a large number of identical infinitely-lived households and consists of two sectors: one producing intermediate goods and the other final goods. The intermediate good is produced with capital and labor, and the final good with intermediate goods. The final good is homogeneous and can be used for consumption and investment purposes.

2.1 Final goods sector

The final good is produced by combining intermediate goods. This process is described by the following CES function:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1}{\lambda_f}} dj \right)^{\lambda_f}, \tag{1}$$

where $\lambda_f \in [1, \infty)$ determines the elasticity of substitution between the various inputs. Producers in this sector are assumed to behave competitively, and to determine their demand for each good, $Y_t(j)$, $j \in (0,1)$ by maximizing the static profit equation

$$\max_{\{Y_t(j)\}_{j\in(0,1)}} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

subject to (1), where $P_t(j)$ denotes the price of the intermediate good j. This yields inputs demand functions of the form

$$Y_t(j) = \left(\frac{P_t}{P_t(j)} \right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t,$$

and the following aggregate price index:

$$P_t = \left(\int_0^1 P_t(j)^{\frac{1}{1-\lambda_f}} dj \right)^{1-\lambda_f}.$$

2.2 Intermediate goods producers

Each firm $j \in (0,1)$ produces an intermediate good by means of capital and labor according to the following constant returns-to-scale production function:

$$Y_t(j) = a_t K_t(j)^{\alpha} L_t(j)^{1-\alpha} \quad \text{with} \quad \alpha \in (0,1), \tag{2}$$

where $K_t(j)$ and $L_t(j)$, respectively, denote the physical capital and the labor input used by firm j in the production process; a_t is an exogenous stationary stochastic technology shock, whose properties will be defined later. Assuming that each firm j operates under perfect competition in the input markets, the firm determines its production plan to minimize its total cost

$$\min_{\{K_t(j),L_t(j)\}} P_t w_t L_t(j) + P_t r_t^k K_t(j),$$

subject to (2). This yields to the following expression for total costs:

$$P_t \phi_t Y_t(j),$$

where the real marginal cost, ϕ_t, is given by $\dfrac{w_t^{1-\alpha}(r_t^k)^\alpha}{\chi a_t}$, with $\chi = \alpha^\alpha(1-\alpha)^{1-\alpha}$.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo (1983) in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability $1-\xi_p$) or it does not. When the firm does not reset its price, it just applies steady state inflation, π^*, to the price it charged in the last period such that $P_t(j) = \pi^* P_{t-1}(j)$. When it gets a chance to do it, firm j resets its price, $\tilde{P}_t(j)$, in period t in order to maximize its expected discounted profit flow this new price will generate. In period t, the profit is given by $\Pi(\tilde{P}_t(j))$. In period $t+1$, either the firm resets its price, such that it will get $\Pi(\tilde{P}_{t+1}(j))$ with probability $1-\xi_p$, or it does not and its $t+1$ profit will be $\Pi(\pi^*\tilde{P}_t(j))$ with probability ξ_p. Likewise in $t+2$. Such that the expected profit flow generated by setting $\tilde{P}_t(j)$ in period t writes

$$\max_{\tilde{P}_t} E_t \sum_{\tau=0}^{\infty} \Phi_{t+\tau} \left(\xi_p\right)^{\tau-1} \Pi(\pi^{*\tau}\tilde{P}_t(j)),$$

subject to the total demand it faces

$$Y_t(j) = \left(\frac{P_t}{P_t(j)}\right)^{\frac{\lambda_f}{\lambda_f-1}} Y_t,$$

and where $\Pi(\pi^{*\tau}\tilde{P}_{t+\tau}(j)) = \left(\pi^{*\tau}\tilde{P}_t(j) - P_{t+\tau}\phi_{t+\tau}\right) Y_{t+\tau}(j)$; and $\Phi_{t+\tau}$ is an appropriate discount factor related to the way the household value future as opposed to current consumption, such that

$$\Phi_{t+\tau} \propto \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t}.$$

This leads to the price setting equation

$$\frac{1}{\lambda_f} \tilde{P}_t(j) E_t \sum_{\tau=0}^{\infty} (\beta\pi^* \xi_p)^{\tau} \Lambda_{t+\tau} \left(\frac{\pi^{*\tau} \tilde{P}_t(j)}{P_{t+\tau}} \right) Y_{t+\tau} = E_t \sum_{\tau=0}^{\infty} (\beta\xi_p)^{\tau} \Lambda_{t+\tau} \left(\frac{\pi^{*\tau} \tilde{P}_t(j)}{P_{t+\tau}} \right)^{\frac{\lambda_f}{1-\lambda_f}} P_{t+\tau} \phi_{t+\tau} Y_{t+\tau} \tag{3}$$

from which it shall be clear that all firms that reset their price in period t set it at the same level ($\tilde{P}_t(j) = \tilde{P}_t$, for all $j \in (0,1)$).

Recall now that the price index is given by

$$P_t = \left(\int_0^1 P_t(j)^{\frac{1}{1-\lambda_f}} dj \right)^{1-\lambda_f}.$$

In fact, it is composed of surviving contracts and newly set prices. Given that in each an every period a price contract has a probability $1-\xi_p$ of ending, the probability that a contract signed in period $t-s$ survives until period t, and ends at the end of period t is given by $(1-\xi_p)\xi_p^s$. Therefore, the aggregate price level may be expressed as the average of all surviving contracts

$$P_t = \left(\sum_{s=0}^{\infty} (1-\xi_p)\xi_p^s \left(\pi^{*j} \tilde{P}_{t-s} \right)^{\frac{1}{1-\lambda_f}} \right)^{1-\lambda_f},$$

which can be expressed recursively as

$$P_t = \left((1-\xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left(\pi^* P_{t-1} \right)^{\frac{1}{1-\lambda_f}} \right)^{1-\lambda_f}. \tag{4}$$

A log-linear approximation of (3) around a zero inflation steady state yields the new Keynesian Phillips curve in this model

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \hat{\phi}_t,$$

where current inflation depends on expected future inflation and on marginal costs.

2.3 The household

There is a continuum of households in the interval $[0,1]$. Household preferences are characterized by the lifetime utility function:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{h_{it}^{1+\psi}}{1+\psi} \right) \tag{5}$$

where $0 < \beta < 1$ is a constant discount factor, c denotes consumption and h is labor supply.

Consumption and investment purchases have to be made in cash, therefore the household is subject to the following CIA constraint:

$$c_t + \varphi x_t \leq \frac{M_t}{P_t},$$

with capital accumulating according to the law of motion

$$x_t = k_{t+1} - (1-\delta) k_t,$$

where $\delta \in [0,1]$ denotes the rate of depreciation. Notice that investment enters with a coefficient φ in the CIA constraint. In the simulations below, we will set $\varphi \in [0,1]$, allowing for investment in or out the CIA constraint. As shown in Wang and Wen (2006) this extension of the model ends up having important implications in terms of persistence.

In each and every period, the representative household faces a budget constraint of the form

$$\frac{\frac{B_t}{R_t} + M_t}{P_t} + c_t + x_t \leq \frac{B_{t-1} + M_{t-1}}{P_t} + \Pi_t + w_{it} h_{it} + r_t^k k_t, \tag{6}$$

where B_t and M_t are nominal bonds and money holdings acquired during period t, P_t is the nominal price of the final good, R_t is the gross nominal interest rate, w_{it} and r_t^k are the real wage rate and real rental rate of capital, respectively. In this economy, bonds are in zero net supply, that is, $B_t = 0$ in equilibrium. The household owns k_t units of physical capital which is rented to the firm at a price r_t^k. He also makes an additional investment of x_t, consumes c_t and supplies h_{it} units of labor. It receives the profits, Π_t earned by the firms.

The representative household maximizes utility subject to the CIA and the budget constraint by choosing the paths of c_t, k_{t+1}, M_t and B_t. The first order conditions are

$$u'(c_t) - \lambda_t - \gamma_t = 0, \tag{7}$$

$$-\varphi\gamma_t - \lambda_t + \beta E_t\left[\lambda_{t+1}\left(r^k_{t+1} + 1 - \delta\right) + \varphi(1-\delta)\gamma_{t+1}\right] = 0, \tag{8}$$

$$-\frac{\lambda_t}{P_t} + \frac{\gamma_t}{P_t} + \beta E_t\frac{\lambda_{t+1}}{P_{t+1}} = 0, \tag{9}$$

$$-\frac{\lambda_t}{R_t} + \beta E_t\lambda_{t+1} = 0, \tag{10}$$

$$M_t - P_tC_t - \varphi P_t\left[K_{t+1} - (1-\delta)K_t\right] = 0, \tag{11}$$

$$B_{t-1} + M_{t-1} + P_t\Pi_t + P_tw_{it}h_{it} + P_tr^k_tk_t - \left(\frac{B_t}{R_t} + M_t\right) - P_tc_t - P_t\left[k_{t+1} - (1-\delta)k_t\right] = 0, \tag{12}$$

where λ_t denotes the Lagrange multiplier associated to the budget constraint, and γ_t is the Lagrange multiplier associated to the CIA constraint.

2.3.1 Sticky wages

In addition, we follow Erceg, Henderson, and Levin (2000), and assume that each household $i \in (0,1)$ is a monopolistic supplier of a differentiated labor service, h_{it}. Each household sells this service to a representative, competitive firm which transforms it into an aggregate labor input, L_t, using the following technology:

$$h_t = \left[\int_0^1 h_{it}^{\frac{1}{\lambda_w}}\,di\right]^{\lambda_w},$$

with $\lambda_w > 1$ being the Dixit elasticity of substitution among differentiated labor services.

Following the same procedure as with final firms, it can be shown that the demand curve for h_{it} is given by

$$h_{it} = \left(\frac{W_t}{W_{it}}\right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t. \tag{13}$$

The aggregate nominal wage index is given by

$$W_t = \left[\int_0^1 W_{it}^{\frac{1}{1-\lambda_w}}\,di\right]^{1-\lambda_w},$$

where W_{it} denotes individual household nominal wage.

To introduce sticky wages, Erceg, Henderson and Levin (2000) assume that households reset nominal wages with a probability $1-\xi_w$ and choose a new wage $\tilde{W}_{it}$; and with probability ξ_w nominal wages are set according to

$$W_{i,t+l} = \pi^* W_{i,t}.$$

We also assume that households have access to a complete set of state contingent contracts. This ensures the same marginal utility of consumption for all workers in equilibrium (Erceg, Henderson and Levin, 2000; Sbordone, 2001).

The representative household chooses the optimal nominal wage $\tilde{W}_{i,t}$ to maximize utility (5) subject to the budget constraint (6) and the labor demand (13) under the scenario of being unable to reset wages, taking h_t, P_t and W_t as given.

The first order condition is

$$E_t \sum_{l=0}^{\infty} \beta^l \xi_w^l \left[-z'(h_{i,t+l}) \frac{\partial h_{i,t+l}}{\partial W_{i,t+l}} + \lambda_{t+l} \left(h_{i,t+l} + W_{i,t+l} \frac{\partial h_{i,t+l}}{\partial W_{i,t+l}} \right) \right] = 0, \tag{14}$$

that is, the present discounted value of the disutility of working $h_{i,t}$ hours at the new wage must equal the benefit of working, measured in terms of the marginal utility of consumption.

Plugging (13) into (14) and taking into account that

$$\frac{\partial h_{i,t+l}}{\partial W_{i,t+l}} = \left(\frac{W_{t+l}}{W_{i,t+l}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} h_{t+l} \frac{1}{W_{i,t+l}} \left(\frac{-\lambda_w}{\lambda_w - 1} \right)$$

and the utility function on labor, the FOC becomes

$$E_t \sum_{l=0}^{\infty} \beta^l \xi_w^l \left\{ \begin{array}{c} -\Psi h_{i,t+l}^{\psi} \left(\frac{W_{i,t+l}}{W_{t+l}} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} h_{t+l} \frac{1}{W_{t+l}} \left(\frac{\lambda_w}{1-\lambda_w} \right) + \\ \lambda_{t+l} \left[h_{i,t+l} + W_{i,t+l} \left(\frac{W_{i,t+l}}{W_{t+l}} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} h_{t+l} \frac{1}{W_{t+l}} \left(\frac{\lambda_w}{1-\lambda_w} \right) \right] \end{array} \right\} = 0,$$

using the fact that for those who can adjust, the optimal price will be W_t^*

$$W_{i,t+l} = W_t^*,$$

and that

$$h_{it} = \left(\frac{W_{i,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} h_t = \left(\frac{W_t^*}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} h_t,$$

we obtain

$$E_t \sum_{l=0}^{\infty} \beta^l \xi_w^l \left\{ \Psi \lambda_w h_{t+l}^{1+\psi} \frac{1}{W_{t+l}} \left(W_t^*\right)^{\frac{\psi \lambda_w}{1-\lambda_w}-1} \right\} \left(W_t^*\right)^{1-\frac{\psi\lambda_w}{1-\lambda_w}} \left(\frac{W_t^*}{W_{t+l}}\right)^{\frac{(1+\psi)\lambda_w}{1-\lambda_w}-1} = \tag{15}$$

$$= E_t \sum_{l=0}^{\infty} \beta^l \xi_w^l \left\{ \lambda_{t+l} h_{t+l} \left(\frac{W_t^*}{W_{t+l}}\right)^{1-\frac{\psi\lambda_w}{1-\lambda_w}} \right\} \left(\frac{W_t^*}{W_{t+l}}\right)^{\frac{(1+\psi)\lambda_w}{1-\lambda_w}-1}. \tag{16}$$

After some algebra we obtain the wage-inflation equation

$$\widehat{\pi}_t^w = \frac{\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\xi_w \left(1+\frac{\psi\lambda_w}{\lambda_w-1}\right)} \left\{ \psi \widehat{h}_t - \widehat{\lambda}_t - \widehat{w}_t \right\} + \beta E_t \widehat{\pi}_{t+1}^w.$$

Finally, recall that $\widehat{mrs}_t = \psi \widehat{h}_t - \widehat{\lambda}_t$, then

$$\widehat{\pi}_t^w = \frac{\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\xi_w \left(1+\frac{\psi\lambda_w}{\lambda_w-1}\right)} \left\{ \widehat{mrs}_t - \widehat{w}_t \right\} + \beta E_t \widehat{\pi}_{t+1}^w,$$

where current wage-inflation depends on future wage-inflation and on the marginal rate of substitution between consumption and labor derived in this model. Notice that sticky wages introduce a wedge between the marginal product of labor and the marginal cost of firms in hiring workers: marginal costs now depend on the aggregate wage index, which is affected by wage stickiness, and the marginal rate of substitution between labor and consumption.

Following the same reasoning as with sticky prices, the aggregate wage index can be expressed recursively as a weighted average of reset and old wages

$$\overline{W}_t = \left[(1-\xi_w)(\widetilde{W}_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w (\pi^* \overline{W}_{t-1})^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{17}$$

2.4 The monetary authority

Money is exogenously supplied by the central bank according to the following money growth rule:

$$M_t = \mu_t M_{t-1},$$

where $\mu_t \geqslant 1$ is the exogenous gross rate of money growth, such that

$$N_t = M_t - M_{t-1} = (\mu_t - 1) M_t.$$

The growth rate of money is assumed to be an exogenous stochastic process, which follows an AR(1), with autoregressive coefficient ρ_μ.

3 Equilibrium

Given the description of the model, we proceed to define an equilibrium in this setup.

Definition 1 *A competitive general equilibrium in this model is given by a set of allocations* $\left\{Y_t, Y_{jt}, K_{t+1}, K_{jt+1}, H_t, H_{it}, C_t, M_t, B_t, W_t, W_{it}, P_t, P_{jt}, r_t^k, R_t\right\}$ *such that:*

i) taking prices and shocks as given, the household's problem is optimally solved, and the F.O.C. (7)-(12) are satisfied, and the CIA constraint holds with equality

$$C_t + \varphi X_t = \frac{M_t}{P_t};$$

ii) taking prices, wages and shocks, the final good firm's problem is optimally solved;

iii) taking wages and shocks, the intermediate firm's problem is optimally solved;

iv) markets clear, that is,

$$\begin{aligned}
Y_t &= C_t + X_t, \\
X_t &= K_{t+1} - (1-\delta) K_t, \\
M_t &= M_{t-1} + N_t, \\
h_t &= \left[\int_0^1 h_{it}^{\frac{1}{\lambda_w}} di\right]^{\lambda_w} = L_t, \\
Y_t &= \left(\int_0^1 Y_t(j)^{\frac{1}{\lambda_f}} dj\right)^{\lambda_f};
\end{aligned}$$

v) prices satisfy equations (3) and (4);

vi) and wages satisfy equations (15) and (17).

The model is log-linearized around a nonstochastic steady state and then simulated to analyze the responses under technology and money supply shocks.

4 Calibration

When possible we follow parameter values which are standard in the literature (Christiano, Eichenbaum and Evans, 2005; and Collard and Dellas, 2006). The baseline parameter values are given in Table 1. The model is parameterized using US quarterly data for the postwar WWII period.

Preferences

The subjective discount factor, β, is equal to 0.988 implying a 5% annual rate of discount for households. The intertemporal elasticity of substitution for consumption is $\sigma = 2$. The inverse of the labor supply elasticity with respect to wages is $\psi = 1$.

Technology

The capital share of output, α, is standard and equals 0.36. Capital depreciates at an annual rate of 10%, that is, $\delta = 0.025$. Monopolistically competitive firms charge a 10% markup on prices, and households charge a 20% on wages, implying λ_p, and λ_w equal to 0.85 and 0.80 respectively, consistent with estimates provided by Christiano, Eichenbaum and Evans (2005). Regarding price and wage setting, we assume that there is $\xi_p = \frac{1}{4}$ probability or resetting prices, and $\xi_w = \frac{1}{5}$ of resetting wages every period, (implying an average contract duration of 4 and 5 quarters, respectively), which are close to those employed by Erceg, Henderson and Levin (2000).

Shock processes

The productivity shock is assumed to follow an AR(1) process with autocorrelation $\rho_a = 0.99$ and standard deviation $\sigma_a = 0.008$. We assume that gross money growth (measured as M0) follows the same autorregressive process with autocorrelation $\rho_\mu = 0.6$, with standard deviation $\sigma_\mu = 0.006$, which is the same as that employed by Wang and Wen (2006).

5 Dynamics of the model

5.1 Labor market and technology shocks

In this section, we analyze the dynamics of the model under three specifications of stickiness (price, wage or both) to a one percent technology shock at time $t = 1$.

Figure 1 displays the response of the model with sticky price and wages in three scenarios. The solid line depicts the case when investment is a credit good $(\varphi = 0)$; the dashed line refers to investment as a partially cash good ($\varphi = 0.6$); and the dotted line denotes investment fully financed with cash $(\varphi = 1)$. This figure shows that to reproduce the labor market dynamics after a technology shock, both rigidities and investment either completely or partially financed with cash are needed. In particular, hours fall after a technology shock (as in Galí, 1999), and real wages are also consistent with the data: combining both sticky wages and sticky prices makes nominal wages hardly react on impact and prices fall, driving real wages up. This is in line with Liu and Phaneuf (2006), who use a model which combines sticky price and sticky wages with habit formation to reproduce the labor market dynamics after a technology shock. Their findings after a rise in productivity are replicated in Figure 1: a weak response in nominal wage inflation, mild decline in price inflation and modest rise in real wage.

Notice that we need both nominal rigidities and φ positive to reproduce both the dynamics of hours and real wages. However, for all the degrees of rigidities considered we do obtain a fall in hours after a positive technology shock as long as investment is a cash good, with the exception of the sticky price model with $\varphi = 1$. Figure 2 shows the responses for the pure sticky price and pure sticky wage models with $\varphi = \{0, 1\}$.

In spite of the similar setup, our model outperforms that in Liu and Phaneuf (2006). In contrast to their paper, we find that in a pure sticky wage model hours do fall after a technology shock as long as investment is financed with cash (either completely or partially), which is consistent with Galí (1999) and the literature thereafter. The intuition behind this result is that the response of consumption and investment is subject to agents holding real balances in advance. In this case, the rise in output is smoothed with respect to the rise in productivity, and hours fall. Our model also differs from Liu and Phaneuf (2006) in that the sticky price model generates a rise in the real wage (with nominal wages falling) as long as $\varphi = 1$. In general, sticky price models cannot account for nominal wage dynamics after a technology shock. In our pure sticky price model we obtain the same: either nominal wages go up (when $\varphi = 1$), or they fall considerably for just one period, which is not consistent with the data. As a result, we obtain that real wages do go up, but due to the positive reaction of nominal wages and the great fall in prices. This result is reversed when sticky wages are considered whenever investment appears in the CIA constraint.

5.2 Money supply shocks and the liquidity effect

In this section we show that a sticky price-sticky wage model with investment in the CIA constraint generates a fall in nominal interest rates after a money injection, that is, the liquidity effect, which has been a failure for most standard new Keynesian models (Galí, 2003).

Figure 3 plots the responses of the sticky price-sticky wage model to a one percent rise in money supply at time $t = 1$. As in the previous section, we consider alternative specifications for investment in the CIA constraint $(\varphi = \{0, 0.6, 1\})$.

The model with both frictions and investment as a cash good generates a rise in output and inflation, with a fall in the nominal interest rate after a money injection, which is consistent with the empirical evidence documented in Christiano, Eichenbaum and Evans (1997), among others. In a recent paper, Christiano, Eichenbaum and Evans (2005) argue that having working capital (mainly, firms borrowing to pay the wage bill) together with variable capital utilization is key for getting the liquidity effect after a money injection, since changes in the nominal interest rate will have a direct effect on marginal costs. Notice that in our setup making investment a cash good introduces the nominal interest rate into marginal costs of the firm, in a similar way as in Christiano, Eichenbaum and Evans (2005). Sticky price and sticky wages combined with both consumption and investment being cash goods reduce the response of aggregate demand to the money injection, resulting in a falling nominal interest rate.

The advantage of the setup presented here is that no extra frictions are needed (habit formation, variable capital utilization, ...) in contrast with Christiano, Eichenbaum and Evans (2005), nor specific assumptions on the consumer's preferences, as suggested in Andrés, López-Salido and Vallés (1999), to generate the liquidity effect. Just modeling money demand with a CIA constraint that affects all aggregate demand (that is, when both consumption and investment are fully financed with cash) and sticky wages in an otherwise standard sticky price model is enough to generate the fall in interest rates, as shown in Figure 3. This result overcomes the failure reported by Huang and Liu (2002) for a staggered wage setting model, and by Wang and Wen (2006) for a pure sticky price setup. Both failures are captured in Figure 4. In particular, the well-known failure of sticky price models to generate the liquidity effect, independently on the proportion of investment financed with cash.

6 Output and inflation persistence

As mentioned in the introduction, the inability of new Keynesian models to generate persistence is one of the workhorses of recent business cycle literature (Mankiw, 2001; Huang and Liu, 2002). Empirical studies show the long-lasting effects of monetary policy shocks on aggregate variables, as well as for technology shocks (e.g., Christiano, Eichenbaum and Evans, 2005).

In addition to generating real and nominal dynamics close the stylized facts after technology and money supply shocks, our model also generates persistence in output and inflation. This is in line with Wang and Wen (2006), who also consider the role of investment as a cash good in generating output persistence and obtain that such a framework is key in generating output persistence. The reason is the delayed response of aggregate demand to any impulse of the economy due to the CIA constraint.

In Figure 5, we show that combining sticky prices and sticky wages can generate enough output and inflation persistence and hump-shape reaction in output as long as investment is included in the CIA constraint. We also find the well known result that inflation dynamics fail in a pure sticky price model, whereas output dynamics can be replicated as long as investment is a cash good (Wang and Wen, 2006). However, in response to a technology shock, the pure sticky wage model cannot generate a hump-shaped response in output in any of the cases considered.

In order to quantify how close these results are to the persistence found in the data, we compare the impulse response functions generated by our qualitatively best model (sticky price-sticky wage with investment in the CIA) with those obtained from an estimated VAR[1] and with those from by Wang and Wen (2006). Figure 6 reports the results for a positive technology shock. We find that impulse responses generated by our model fall within the confidence intervals of the VAR estimation. It is worth noticing that the model generates inflation dynamics which are close to those in the data, both on impact and on persistence: after a rise in productivity, inflation falls and returns to steady state after five quarters, approximately. The dynamics implied for output, though still consistent with the estimation, denote more persistence than in the data. Notice however, that the model by Wang and Wen (2006) still generates further persistence.

That sticky wages and investment in the CIA constraint add persistence in the case of money supply shocks is shown in Figure 7. Considering only sticky wages or sticky prices and sticky wages with investment in the CIA constraint outperforms the sticky price model regarding both output and inflation dynamics. In this case, when compared to the VAR estimation and to Wang and Wen (2006) (Figure 8), we can see that the specification that best qualitative results generates (sticky price, sticky wages and investment fully financed with cash), provides output dynamics close to those in the data. Regarding inflation dynamics, although the best model generates more inflation persistence than the standard sticky price setup, it reports an initial rise in inflation (which is much higher for Wang and Wen, 2006), contrary to VAR evidence. This means that there is still room for improvement regarding inflation dynamics.

[1] The impulse responses for the VAR are reproductions of those in Altig et al. (2004). Data are quarterly and the period considered is 1950:2-2001:4. For a detailed description of the data employed see Altig et al. (2004).

7 Conclusions

In this paper, we present a model with sticky prices, sticky wages and investment in the CIA constraint which generates business cycle dynamics consistent with empirical evidence. First, it is worth emphasizing that our setup generates enough output and inflation persistence with standard stickiness parameters. The key factor driving these results is the inclusion of investment in the CIA constraint, rather than introducing any other nominal or real rigidity. And second, the model reproduces the responses of the key macroeconomic variables to technology and money supply shocks. As for technology shocks, our model reproduces the labor market dynamics after a positive increase in productivity: hours fall, nominal wages hardly react, and real wages go up. Regarding the money supply shock, our model specification generates the liquidity effect, a fall in the nominal interest rate after a money injection, a fact which is absent in most sticky price models. Therefore, including investment in the CIA constraint seems to be a simple modelling way to deeply improve the qualitative and quantitative properties of new Keynesian models.

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Appendix

Set of linearized equations

After deflating and log-linearizing the equilibrium equations around the nonstochastic steady state, the model reduces to the following set of equations.
The new Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\hat{\phi}_t, \tag{18}$$

F.O.C. on consumption

$$(2-\beta)\,\hat{\lambda}_t + \beta \hat{r}_t + \sigma\,(2-\beta)\,\hat{c}_t = 0, \tag{19}$$

F.O.C. on capital

$$\left[r^k + (1-\delta)(1-\varphi\beta)\right] E_t\hat{\lambda}_{t+1} + \varphi\beta(1-\delta)E_t\hat{r}_{t+1} + r^k E_t\hat{r}^k_{t+1} = \varphi\beta\hat{r}_t + (1-\varphi\beta)\hat{\lambda}_t. \tag{20}$$

This last equation will change depending on the specification considered:

- if $\varphi = 0$, investment is fully financed with credit, and does not appear in the cash-in-advance constraint. Therefore, the equation becomes

$$\left[r^k + (1-\delta)\right] E_t\hat{\lambda}_{t+1} + r^k E_t\hat{r}^k_{t+1} = \hat{\lambda}_t, \tag{21}$$

- if $\varphi = 1$, investment is fully financed with cash, and is, therefore, affected by the nominal interest rate. Then, the equation becomes

$$\left[r^k + (1-\delta)(1-\beta)\right] E_t\hat{\lambda}_{t+1} + \beta(1-\delta)E_t\hat{r}_{t+1} + r^k E_t\hat{r}^k_{t+1} = \beta\hat{r}_t + (1-\beta)\hat{\lambda}_t, \tag{22}$$

Goods market clearing

$$\frac{y}{k}\hat{y}_t - \frac{c}{k}\hat{c}_t - \hat{k}_{t+1} = -(1-\delta)\hat{k}_t. \tag{23}$$

Rental price of capital

$$\hat{r}^k_t - \hat{\phi}_t - \hat{y}_t = -\hat{k}_t. \tag{24}$$

Real wage

$$\hat{w}_t - \hat{\phi}_t - \hat{y}_t + \hat{h}_t = 0. \tag{25}$$

Law of motion of capital (definition of investment)

$$\hat{k}_{t+1} - \delta \hat{x}_t = (1-\delta)\hat{k}_t. \tag{26}$$

Law of motion of money

$$\hat{m}_t - \hat{\mu}_t + \hat{\pi}_t = \hat{m}_{t-1}. \tag{27}$$

Cash-in-advance constraint

$$\frac{c}{k}\hat{c}_t + \varphi \hat{k}_{t+1} + \frac{m}{k}\hat{m}_t = \varphi(1-\delta)\hat{k}_t. \tag{28}$$

Real marginal costs

$$\hat{\phi}_t - \alpha \hat{r}_t^k - (1-\alpha)\hat{w}_t + \hat{a}_t = 0. \tag{29}$$

Law of motion of real wage

$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t. \tag{30}$$

Flexible-price output

$$\hat{y}_t^f - \alpha \hat{k}_t - (1-\alpha)\hat{h}_t - \hat{a}_t = 0. \tag{31}$$

Definition of output gap

$$\hat{y}_t - \hat{y}_t^f. \tag{32}$$

Real wage inflation

$$\widehat{\pi}_t^w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w\left(1+\frac{\psi\lambda_w}{\lambda_w - 1}\right)}\left\{\widehat{mrs}_t - \widehat{w}_t\right\} + \beta E_t \widehat{\pi}_{t+1}^w. \tag{33}$$

Marginal rate of substitution

$$\widehat{mrs}_t = \psi \widehat{h}_t - \hat{\lambda}_t. \tag{34}$$

Plus shock processes.

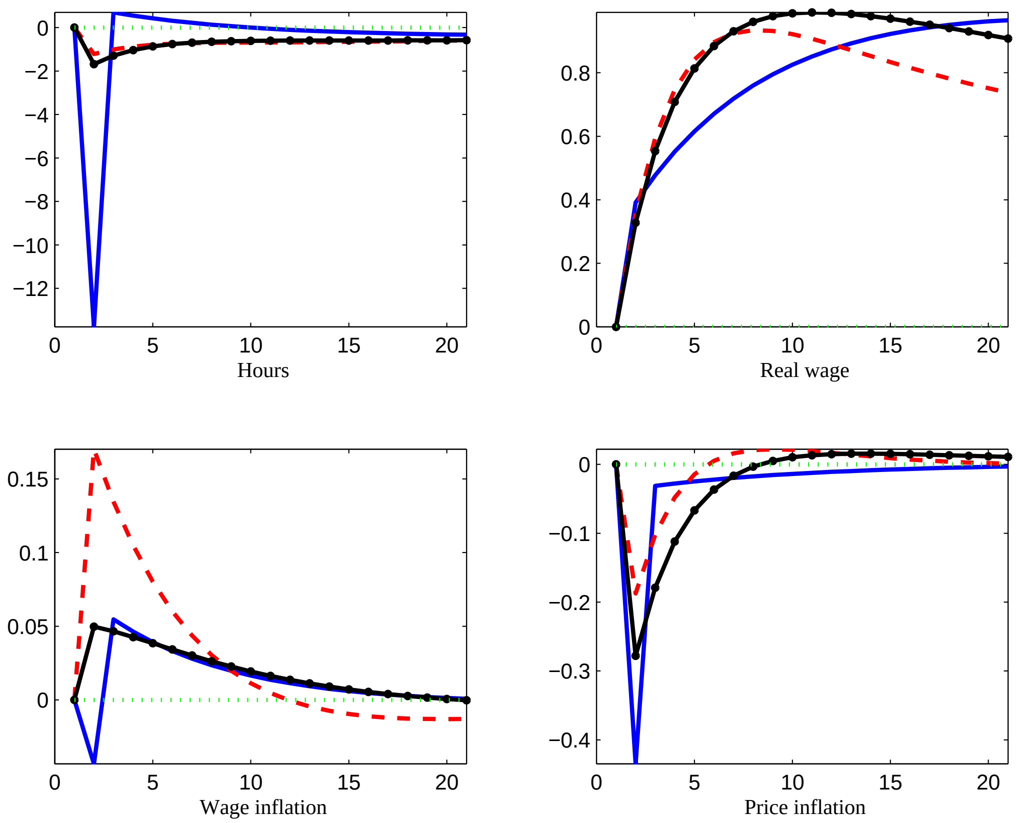
Tables

Table 1: Baseline calibration

Preferences		
Discount factor	β	0.988
Intertemporal elasticity of substitution	σ	2.000
Inverse labor supply elasticity	ψ	1.000
Technology		
Capital share	α	0.360
Depreciation rate	δ	0.025
Elasticity of substitution across goods	λ_p	10/9
Elasticity of substitution across labor inputs	λ_w	6/5
Probability of resetting prices	ξ_p	0.250
Probability of resetting wages	ξ_w	0.200
Shock processes		
Persistence of productivity shock	ρ_a	0.990
Standard deviation of productivity shock	σ_a	0.008
Persistence of monetary shock	ρ_μ	0.600
Standard deviation of monetary shock	σ_μ	0.006

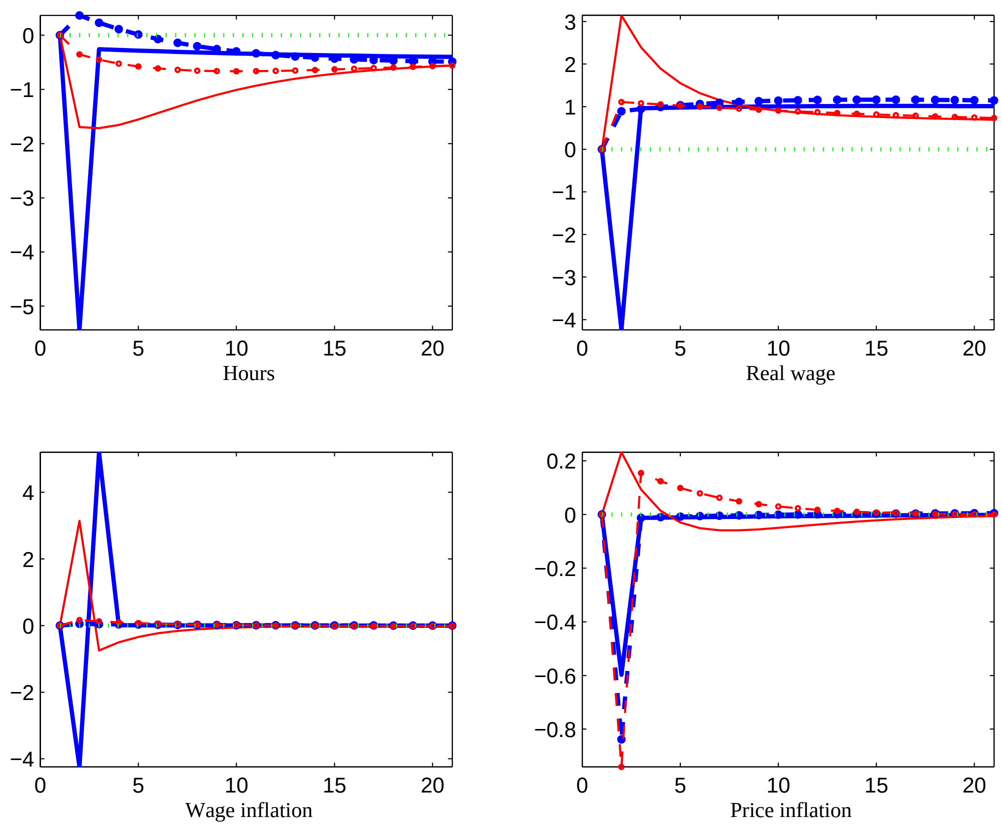
Figures

Figure 1: Impulse response functions of the model with sticky prices and sticky wages to technology shock.



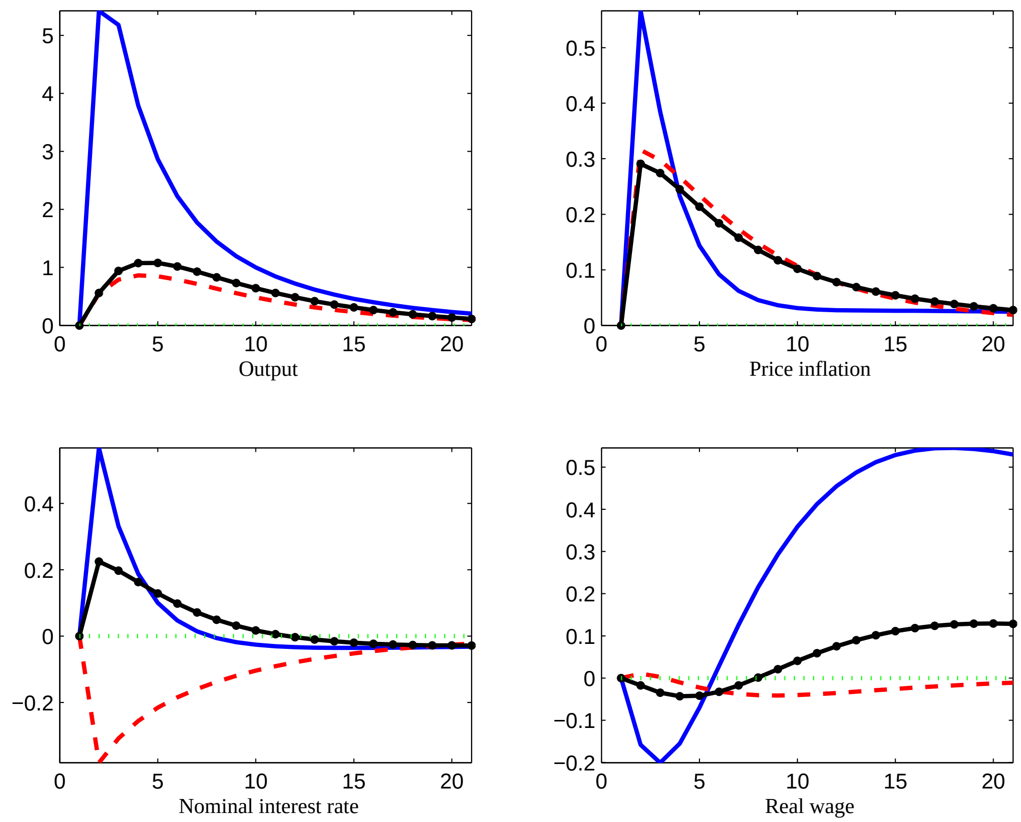
Note: Plots depict the model without capital in the CIA (solid line), with capital in the CIA (dashed line), and with capital partially financed with cash (dotted line).

Figure 2: Impulse response functions to a technology shock.



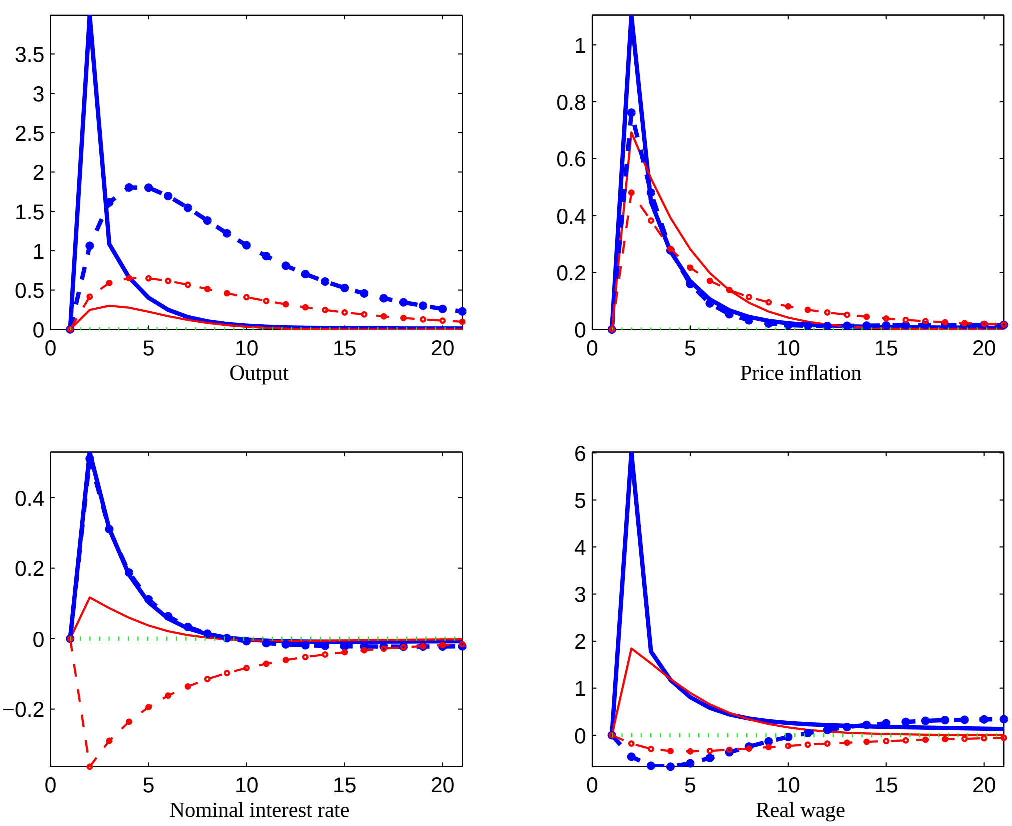
Note: Pure sticky price model without capital in the CIA (strong solid line), and with capital in the CIA (strong dotted line). Pure sticky wage model without capital in the CIA (solid line), with capital in the CIA (dotted line).

Figure 3: Impulse response functions of the model with sticky prices and sticky wages to a money supply shock.



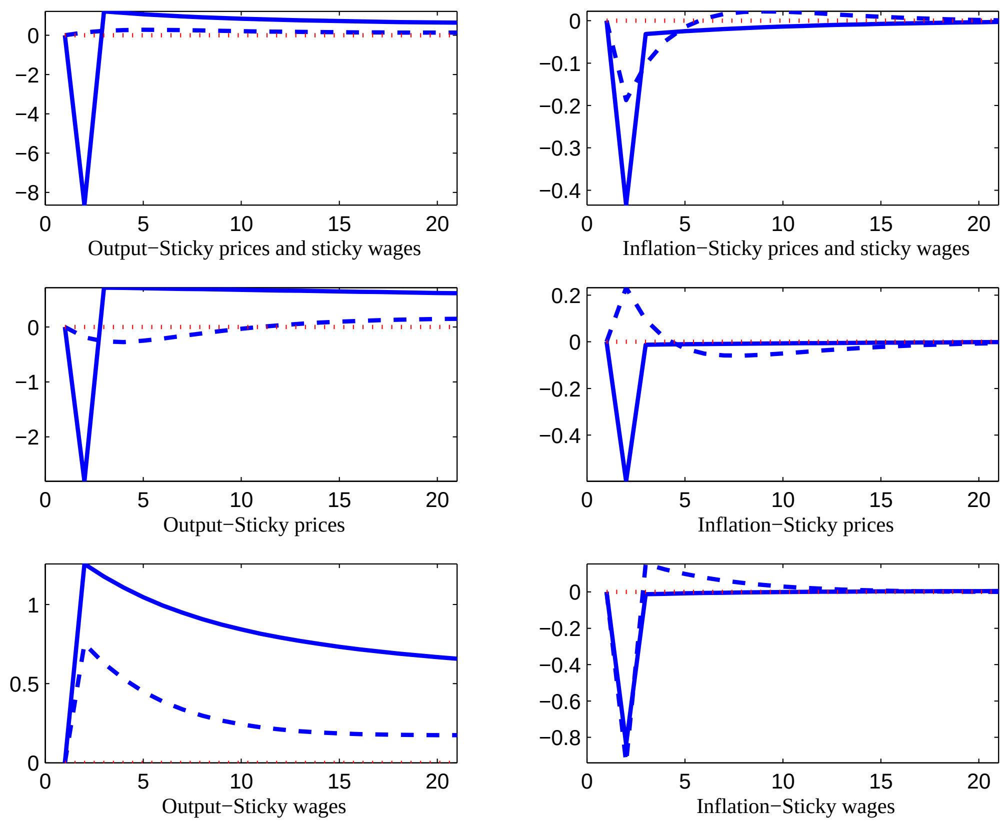
Note: Plots depict the model with capital as a credit good (solid line), with capital fully finance with cash (dashed line), and with capital partially financed with cash (dotted line).

Figure 4: Impulse response functions to a money supply shock.



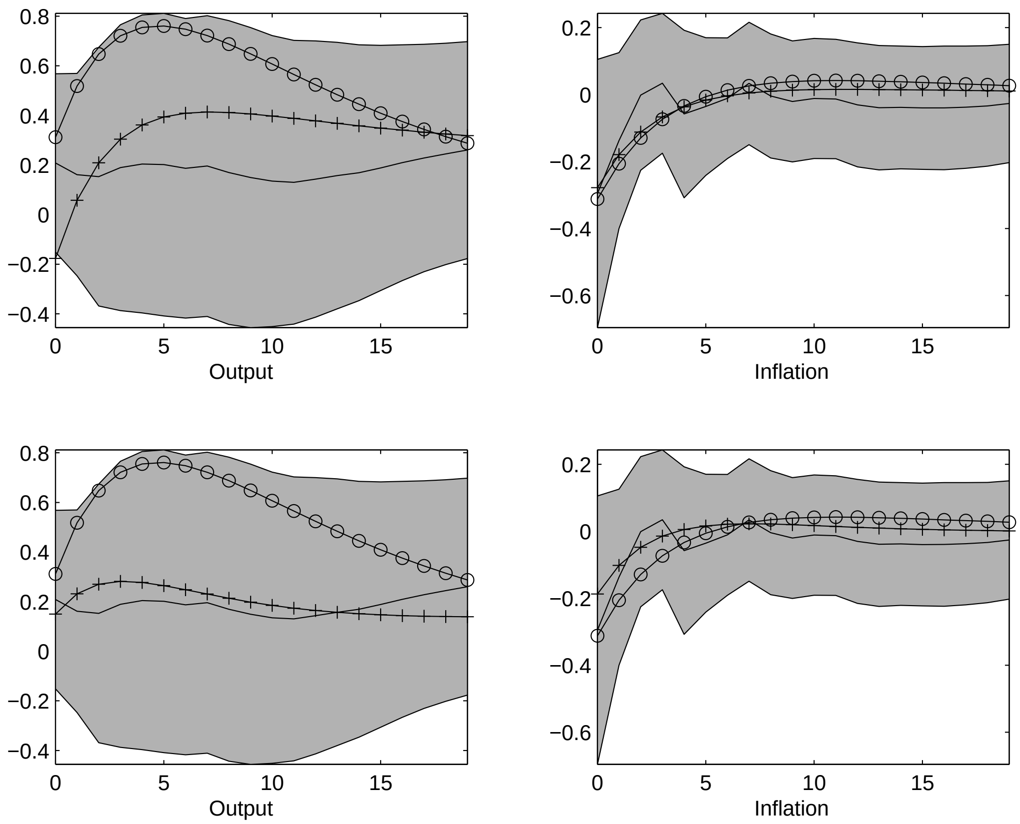
Note: Pure sticky price model without capital in the CIA (strong solid line), and with capital in the CIA (strong dotted line). Pure sticky wage model without capital in the CIA (solid line), with capital in the CIA (dotted line).

Figure 5: Impulse response functions to a positive technology shock: output and inflation dynamics.



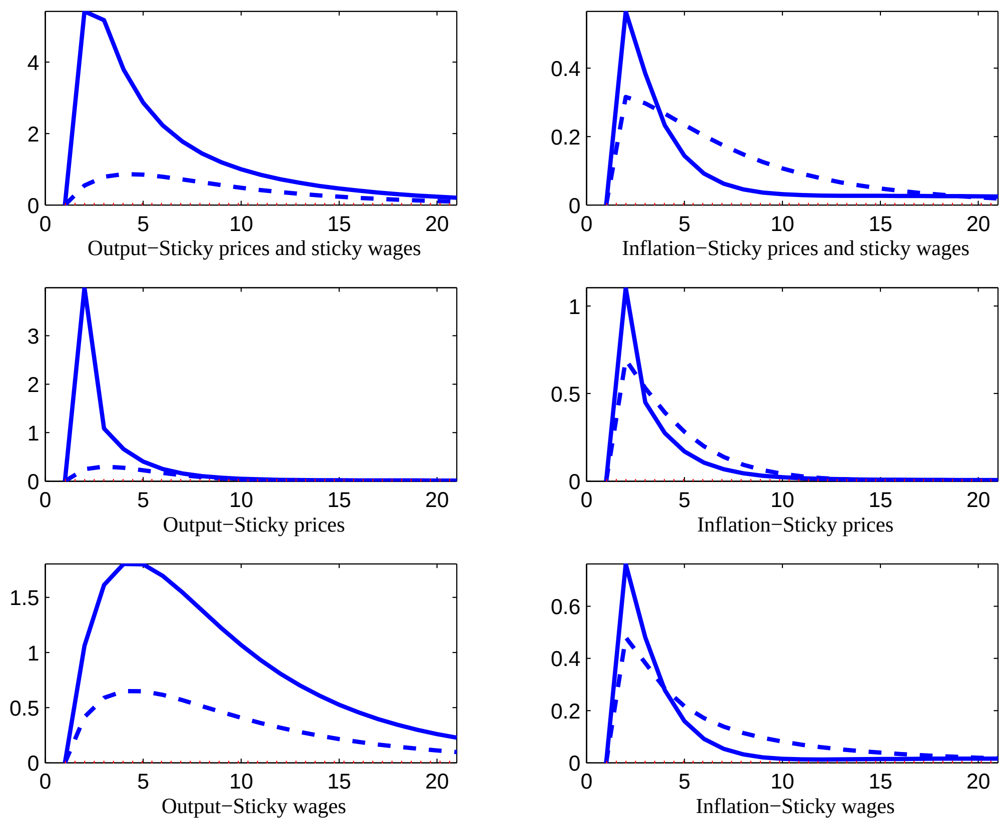
Note: Left column is for output, right column for inflation. All three setups considered. Solid line denotes no capital in the CIA, dashed line stands for capital in the CIA constraint.

Figure 6: Impulse response functions to a positive technology shock.



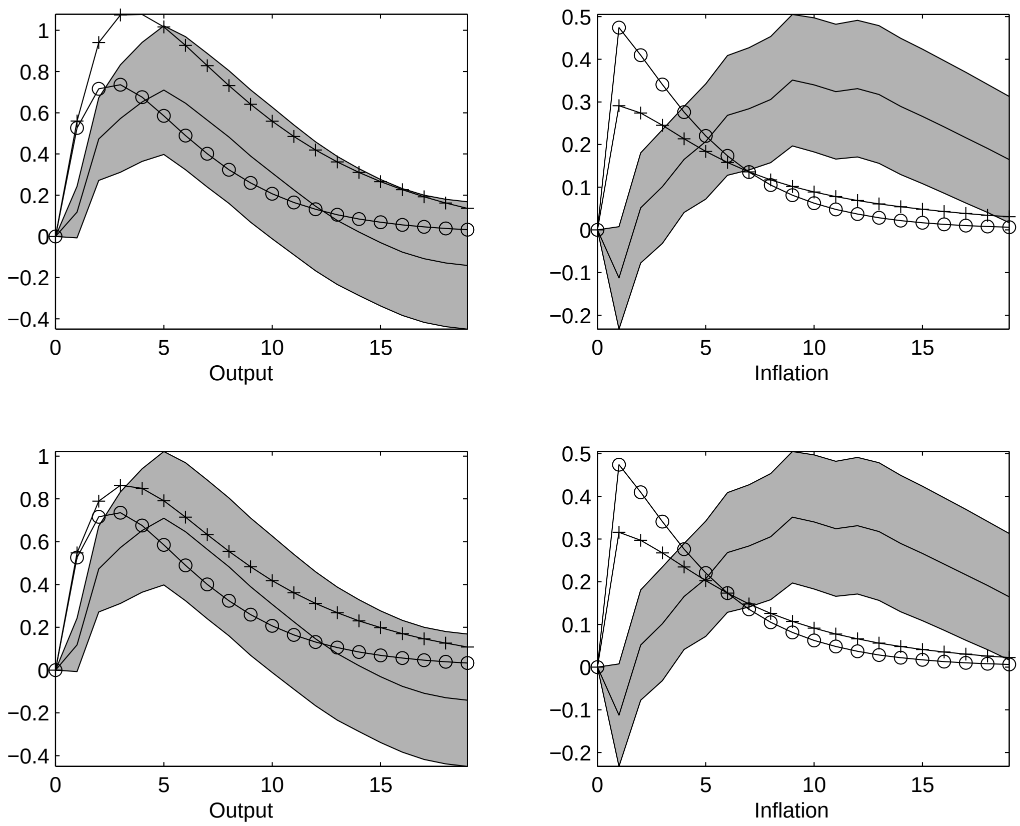
Note: Solid line stands for VAR estimation from Altig et al. (2004), circled line stands for Wang and Wen (2006), and crossed line stands for the sticky price-sticky wage model. Top panels: model with investment partially financed with cash ($\varphi = 0.6$), bottom panels: model with investment fully financed with cash ($\varphi = 1$).

Figure 7: Impulse response functions to a positive money supply shock: output and inflation dynamics.



Note: Left column is for output, right column for inflation. All three setups considered. Solid line denotes no capital in the CIA, dashed line stands for capital in the CIA constraint.

Figure 8: Impulse response functions to a positive money supply shock.



Note: Solid line stands for VAR estimation from Altig et al. (2004), circled line stands for Wang and Wen (2006), and crossed line stands for the sticky price-sticky wage model. Top panels: model with investment partially financed with cash ($\varphi = 0.6$), bottom panels: model with investment fully financed with cash ($\varphi = 1$).