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The Basic Dynamics of the Stock of Money and Capital

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Abstract

We present in this paper a model that explains the role of the external financing of capital in the evolution of a primitive economy constituted by families and firms. Our aim is to clarify which are the essential financial elements that in more complex and advanced economies could explain the evolution of their financial structures towards fragility. Now we begin studying the stability of the financial structure of a rudimentary economy with endogenous money.

I. Introduction: Objective and Method. Money in Space and Time.

1.1 Objective and Method

The original theory of financial fragility and economics instability can be found in the many and heterogeneous works of Hyman P. Minsky (1919–1996). We can see from them how broad his vision of the role of finance in modern capitalist economies was. As it is materially impossible to cover all aspects of his theory, I have decided to focus on just one of the most important and basic characteristics of his work: the role of a stock of money as the external financing form of capital investment. From a methodological standpoint, there is great distance between Minsky's own presentation of his ideas—which was mainly rhetorical, lacked formalization, and therefore allowed him to connect many multidisciplinary financial issues—and the rather narrow mathematical way developed here to deal with the specific role of the stock of money. The ideas and concepts of the model here presented are constrained by chosen mathematical specifications, which cannot help but isolate the main elements of Minsky's theory. However, I want to stress that while writing this article I have kept in mind the entirety of Minsky's work, which constitutes a complete financial fragility theory: one that stresses the fundamental role played in real dynamics by the financial side of the economy; one in which finance could in due time take any economy to a fragile state.

The mathematical approach used here aims to formalize my ideas, but I am aware of the limitations of dressing up a vision of financial fragility in mathematical clothing, limitations due not only to financial fragility's dynamic nature but also to many institutional variables: e.g., evolutionary changes in financial institutions and money-market usages (Minsky, 1957a). Putting them into mathematical terms is difficult if not impossible. Nevertheless, mathematics remains a powerful instrument for the dissection of Minsky's ideas and is used here to try to simplify his original and complex theory.

Aside from Minsky's original contribution to a mathematical version of his financial theory, I have taken into account the more recent contributions of several authors who deal with systemic fragility. Since the mid-eighties, with Taylor's pioneering article (1985), there have been a handful of articles dealing with this issue methodically. Gatti and Gallegatti (1990) proposed different mathematical models within an IS-LM framework, which Minsky himself approved even though this approach constituted a very conventional framework of analysis. Sethi (1995) set out the theoretical question in terms of modern game theory, including the much-needed microeconomic foundations of the theory, and with instruments that capture the dynamic behavior of the population, like the so-called replicators. Finally, Keen (2000), using chaos theory, proposed an expression of Minky's ideas in terms of non-linear dynamic models, while closely relating these models to well-known models of economic growth, like those proposed by Goodwin (1967: "A Growth Cycle"), as well as to Marx's thoughts about the two price levels and money (Keen, 1996).

My principal task has been to elaborate a concept of money whose financial implications for the growth of capital are suited to Minsky's financial fragility theory. I have relied in particular on a definition of money that, on the one hand, stresses money's role as a form of private credit and, on the other, can be considered a minimalist synthesis of a great

many unorthodox perspectives on that role (*i.e.*, the non-monetarist one). Nevertheless, this is a rather common concept of money, many authors having considered private credit its most important feature. In chronological order, they are: Innes (1919), who gave us an excellent foundation for money as pure credit; Pedersen (1946), who proposed to measure money as income, a notion that follows straightforwardly from the notion of money as credit; Gurley (1960), who made the fundamental distinction between inside and outside money, an opposition that derives from the social accounting system described in his book, where money flows and the total cash deficits at the end of different periods connect agents' debt and credit financial decisions; and Kaldor (1982), with his criticism of the monetarism theory, which dismisses the quantitative notion of money. Finally, our notion of money as credit is also linked to the workings of some modern authors within a Keynesian and circulation approach: Randall (1990), Rochon (1999), and Delaplace and Nell (1996), among others.

Consequently, I focus on the concept of money that best adapts to Minsky's financial fragility theory: that of money as credit. That Minky embraced it in his writings is apparent in his clear distinction between ordinary and extraordinary flows of money. For him, ordinary flows allow a debt repayment with the normal income of an economic unit (*i.e.*, with flow money), while extraordinary flows have to be obtained to cover contractual payments with the selling of assets (*i.e.*, with the use of stock money):

> This division of money flows into three transaction types—those due to balance sheet account, income account, and portfolio account—is useful analytically because it sets out quite precisely some of the financial constraints within which any economic unit functions and defines the circumstances which make a sharp drop in asset values possible. It is also useful because it divides money flows such that behavioral relations for the various flows classified by accounts can be postulated. If the data are available, these behavioral relations can be estimated, and once estimated they can be integrated into aggregate models of income determination (Minsky, 1964: p. 234–235).

The next section provides a more precise definition of money, one that allows an exploration of the financial relation between the flow of money and the stock of money. Section II explains a collection of basic equations. There, variables and parameters are connected to generate a macroeconomic model, which will be a mere mathematical expression of the concept of money proposed above. Such a model considers the external finance of investment as either constant in time or exogenous, so it is possible to establish stock money or debt as a function of the external finance of investment. Section III, finally, shows a more complex, dynamic model of debt: a model in which the external finance will be endogenous. The development of that model produces a monetary and financial theory of the economic cycle. Section III also studies the model's features, particularly the sensitivity to initial conditions and the stability of its structure when parameters change; this will reveal the non-linear nature (the chaos) of the stated model.

I.2 Money in Space and Time

The notion of money adopted for this paper requires first that money not be considered an entity independent from the specific economy in which it is created and circulates. This is common sense, as money cannot exist by itself. Money is, rather, an immaterial symbol of a social nature. To grasp the basic nature of money while making the

methodological assumptions of this inquiry, it is necessary to posit a very simple representation of an economy. We'll assume that it consists of two sectors, families and firms, and therefore will not allow for banks or any public administration. Besides this spatial dimension, the definition of money requires a second dimension: time, as all transactions take place at a given moment in time: *e.g.*, the present. The definition of time is necessary, as it makes intelligible the notion of money required in this paper. Without a proper representation of time, the concept of money as credit would be senseless. Here we adopt a conventional representation of time, which is understood as an ordinate succession of arbitrarily discrete intervals or periods: *i.e.*, months or years

These periods have become standardized, and each period of a given type can be thought of as identical with every other period of that given type. Each minute, for example, can be thought of as identical to every other minute. However, our notion of chronology, of time's advancing in a line, intervenes to distinguish each individual period from all the others. We conceive of a past, a present, and a future, and we order our otherwise equivalent periods accordingly. Once we begin to consider our periods as part of a non-arbitrary series, the first minute after three o'clock is no longer the same as the fifth minute after three o'clock.

With this conceptual background in mind, let us assume that in the present time both the families and the firms have free access to emit or create money to satisfy their consumption and production needs. That is, new money is created whenever they make any kind of purchase: *e.g.*, of goods, of factors of production, etc . Therefore, at present, families create money when they buy goods from the firms and firms simultaneously create money when paying family wages. This assumption is consistent with a strict concept of money as account money and money for transactions. More importantly, it demonstrates the notion of money that interests us: money as flow of primary credit.

Flow money is credit insofar as it is a relation of trust between two parties making an exchange. It is an IOU. It is a relationship established at a certain moment. Money emitted by any agent as part of a normal expenditure is credit that must be confirmed before the present time expires. Because it represents a commitment of debtors, money makes their actions the opposite of a creditor's, for a buyer must also be a seller .

In this simple model of two agents, the families validate their emitted money to buy goods and services with the money they have accepted in wages from the firms. In this sense, money is related to the production and consumption process—a social income account—and to the way both activities are coordinated in the market through bargaining processes This positive circulation of money makes clear that there is no need for any stock of money (despite conventional wisdom to the contrary), because money is spontaneously created. From the perspective of a social income account, it is just a pure flow of money that happens in the present, being born and dying at the same moment. It is not necessary that money go from the past to the present. Nor is money intended to be saved for the future. We can say, then, that an economy presenting nothing but flows of money is an economy without past or future. However, it is clear that there might be many circumstances in which the money cannot be validated by the issuing agents. In such a case, money has to negate itself.

Since credit flows constitute the only money that circulates in the economy, a given agent might see, at the end of the present, imbalances between the flows of money that he cashes and the flows of money that he pays. Consequently, an agent can incur a cash deficit

after checking his net position at the end of the period. However, this situation represents a contradiction, since the promise to pay inherent in the money he has just created in the present is not fulfilled. This is the negation of money itself, which is responsible for money's transformation into another form: *the stock of money*. It is also the origin of the process of time in its three classical periods: past, present and future.

That there is stock money does not mean that it could be directly exchanged for consumption goods or production factors. In this context, the function of the stock money is to pay for or validate future total cash deficits of the agent who owns it. It is an asset whose value is known with certainty and that therefore exhibits perfect liquidity. As a result, the case of a basic economy can be thought of as a case that has only one asset with perfect liquidity. From the perspective of the firms, stock money is employed as external funds for current investment. And, as proposed in the next section, it represents the only source of money that validates past debts: *i.e.*, that can be used to validate contractual payments of debt. Finally, with regard to the critique of monetarism, it is important to note that no relationship can be established between the only quantifyable form of money (the stock of money) and prices.

II. Primitive Financial Anatomy: Basic Equations. External Financing and the Stock of Money.

2.1 Basic Equations

It follows from the previous section that money created in the present might, by the end of the present period, be added to the total cash deficit of the agent who has created it, thus constituting stock money. It is normally said that if money flows become money stock, then those goods or production factors that generate it are externally financed. The basic equations of this section formalize the transformation process from primary flow money to stock money. That is, we propose a comprehensive account of a wide range of concrete questions, like: what are the goods and production factors that, in being purchased, give birth to stock money? why do these purchases produce an imbalanced situation generating a cash deficit? what, then, is the net position of the economy with regard to the stock money that has been created? and, finally, what are the consequences of this financial situation in the future?

In our economy, where only two types of agents exist—firms (denoted by f = 1,...,F) and families—and where wages paid by the firms are equal to the families' consumption, the investment of the firms sector matches its total profits. The following equation depicts this balanced situation for the firms sector:

$$\sum_{f=1}^{F} I_t^f = \sum_{f=1}^{F} B_t^f = I_t = B_t \ , \qquad (1)$$

where t =1..., T corresponds to a given moment in time: *e.g.*, the present. According to this equation, any investment level I_t can be paid with the profits obtained by the firms, B_t. However, it is quite likely that this accounting identity, which holds for the sector as a whole, will not be hold for individual firms. Because even if we know that a firm's profit is a share of the overall profit (no matter how it is distributed) and that it is the result of the firm's

investment (yet another share of overall investment), nothing guarantees that these two shares will be equal for a given firm: $I_t^f \neq B_t^f$, f =1,...F. Therefore, it is likely that a firm that invests and contributes to the generation of overall profit would not make enough individual profit to cover its investment level: *i.e.*, would not get back enough money to cover the amount of money it has previously created.

Since profit (calculated at the end of the present time t) is the only source of money available in our model to face the investment bought with primary money (issued at the beginning of the present time t), we can say that the part of the firm's investment that exceeds its profits must be externally financed: $EF_t^f = \left|I_t^f - B_t^f\right|, I_t^f > B_t^f$. Therefore, a firm's profit represents its only source of internal financing: $IF_t^f = B_t^f$. By aggregating the individual financial situations at the end of the period while taking into account the internal and external financing sources, we obtain the equation:

$$I_t = EF_t + IF_t, \tag{2}$$

which reflects how just one part of overall investment is financed by way of individual profits.

In this context, a firm with external financing is a firm with access to the stock money, which becomes the financial means to acquire productive capital. This runs contrary to primary money, which all firms have free access to and, moreover, which all firms are forced to use if they want to produce. Firms with access to stock money to finance investment enjoy a privilege: a privilege denied to families.

In accordance with how it materializes in the stock of money over a given period, we divide the effects of external finance into two categories: (a) those that originate a direct increase in the quantity of stock money and, therefore, that create new stock money (without replacing the stock previously inherited: Δ^+S) and (b) those that cause any of the following: (1) destruction or a direct diminishing of the stock of money, denoted in contrast to (a) by Δ^-S, or (2) a substitution or shift in the stock of money $\Delta\pm S$, such as when an old debt is replaced by a new one within the same portfolio, or when an old debt is transferred from one agent's portfolio to another's. Considering this classification of the effects of external finance on the stock of money, we can represent the former by way of the following expression:

$$\begin{aligned} &EF_t = \Delta^+S + \left[\Delta^-S + \Delta \pm S\right] = (1-d_t)EF_t + \left[d_t EF_t\right] = \\ &= (1-d_t)EF + \left[(\gamma_t + \mu_t + \left[1-(\gamma_t + \mu_t)\right])d_t EF_t\right]; \quad (\gamma_t + \mu_t) \leq 1,\ d_t \leq 1 \end{aligned} \tag{3}$$

where in the third equation the coefficient d_t represents the financing of investment by the old inherited stock of money. Thus $d_t EF_t$ represents the external investment amount being financed by the existing stock of money. As a result, it also indicates to what extent aggregate profit has implications for the future of the economy, because it is the source of money necessary to fulfill the contractual payments on debt, both repayment and interest. This is introduced in our model by the variables γ_t and μ_t, which jointly represent a percentage over external financing $d_t EF_t$. In particular, γ_t represents the destruction of the stock of money

as a percentage of $d_t EF_t$. Two criteria must be met for this destruction to happen: a debtor's appropriation of part of the aggregate profit, which is later used to pay off some of his debts, and the existence of a cash deficit with regard to the creditor: *i.e.*, the owner of stock money resulting from the debtor's investment. However, if just the first criterion is met, there will be a simple shift in the nature of the stock of money that would not affect its quantity; there will be simply a change in the name of the debtor inside the same portfolio, which is represented by the coefficient μ_t. Therefore, a shift in the inherited stock of money not affecting its quantity (represented by a simple debt shift from one portfolio to another), as expressed by the coefficient $[1-(\gamma_t + \mu_t)]$, would not be useful to the validation of the contractual debt payments due in the period t.

We have assumed that stock money is debt or obligation and, accordingly, that its payment must be contractual. But it is necessary to explain in more detail the exact form in which this debt —and the contractual payments it carries along— is created. We already know that the quantitative relation between the generation of new debt (new stock money) and the quantity of stock of money that existed at the beginning of the present period *t* can be ascertained at the end of this period. This new debt (new stock money) can be found in the total cash deficits of those firms that do not have the necessary stock money in their portfolio to eliminate their deficits. While a part of new debt (new stock money) replaces the old stock money or inherited money —in particular, that represented by the coefficient μ_t— not all new money can be considered as a partial positive increase in the stock of money, as shown in equation (3). To stress money creation, we introduce the following representation of this new debt (stock money):

$$C_t = [(1-d_t) + \mu_t d_t] EF_t, \tag{4}$$

where C_t is accordingly derived from equation (3) to specify this particular notion. All new debt is a commitment to forward payment, therefore originating future payments —which will be repaid in future periods under different conditions: contingent, dated, or on demand (Minsky, 1964). Because of the institutional nature of these future payments, we call them contractual and we denote their volume period *t* as:

$$P_t = P_t(C_{t-1}, C_{t-2}, ..., C_{t-N}), \tag{5}$$

reflecting how total contractual payments depend on the past flows of credit. A simple relation between contractual payments and the past flows of credit can be shown if we consider debt repaid on date —for example, during N periods from the moment it was born and for which we do not consider the payment of interest. In this case we assume that all credit is distributed uniformly in the future, as it is represented in the following equation —where the stock of money in the first period $t = 0$ is zero:

$$P_t = \begin{cases} 0 & t = 0 \\ \frac{1}{N} C_0 & t = 1 \\ \sum_{t-N}^{t-1} \frac{1}{N} C_t & t < N \rightarrow t - N = 0 \end{cases}$$

Here, the amount of money necessary to cover these payments does not correspond to future aggregate profits, but only to part of them. The variable representing the volume of fulfillment and effective payment of due debt in any period of time is given by:

$$R_t = (\gamma_t + \mu_t) d_t EF_t \qquad \gamma + \mu_t \leq 1 \tag{6}$$

This equation establishes the mechanism of validation of contractual payments that fall due in period *t*. The total repayment of expected payments due to mature debt. Here the first additive part represents the net destruction of stock money, $\gamma_t d_t EF_t$ —where γ indicates the ability of debtors to generate profits that originate in creditors' total spending on investment.

2.2 External Financing and the Stock of Money

In this subsection we will establish the mathematical relation between the stock of money and external financing, and develop its particular dynamics. This relationship is obtained from the basic equations already introduced. Particularly, we depart from the d_t, γ_t, and μ_t variables that determine how investment is financed by the stock of money. In this sense, if $\mu_t = 0$, the partial increase and reduction of the stock of money, Δ^+S and Δ^-S, will coincide respectively with the increase in debt and with the repayment of debt: C_t and R_t. If this relationship holds, then our model becomes much simpler, since $C_t = \Delta^+S$ and $R_t = \Delta^-S$. With regard to d_t, this variable can be expressed through a percentage of the stock of money —denoted by λ_t. In this case, by multiplying this last variable by the stock of money: $\lambda_t \cdot S_t$, we obtain the part of the stock of money employed in the external financing of investment: $d_t EF_t$. Finally, for this situation both λ_t and γ_t will adopt constant values, denoted by $\lambda_t = L$ and $\gamma_t = G$ respectively, that remain constant in time.

For these L and G values, we want to determine what amount of the stock of money would be required to ensure the complete fulfillment of the debt's contractual payments, given a certain value of external finance that remains constant during a large enough periods of time (presents). Let us consider that for this quantity of the stock of money the economy is in equilibrium: S^E. Given the dynamics of the economy, if this quantity of the stock money is reached then any the new debt will be equal to the amount of the contractual payments. Because once the economy reaches this steady state, the stock of money will not change —nor will the new debt; that is, eventually $C_t = C$. This situation shows that:

$$P_t = \frac{1}{N} \sum_{t-N}^{t-1} C_t = \frac{1}{N} NC \Rightarrow t > T, P_t = C$$

From this relationship it is clear that if $t > T \rightarrow P_t = C_t = C$, and if it is given that $P_t = R_t$ in equilibrium, then in this situation $R_t = C_t$. It is precisely this last equation that reflects the relationship between external finance and the stock of money in equilibrium. This implies that $Gd_tEF_t = (1\text{-}d_t)EF_t$, and since $L^*S_t = d_tEF_t$, then there is a relation between different levels of external finance and different values of the quantity of the stock of money that fulfill all the conditions for equilibrium already mentioned; that is, those values for which a particular value for the stock is an equilibrium quantity. The specific relationship between the equilibrium stock of money and external finance is given by:

$$S^E = \frac{EF}{(1+G)L}, \tag{7}$$

Let us now focus on the stock of money dynamics from the moment when the stock of money of the economy is $S_0 = 0$ until it reaches its equilibrium quantity in a final period. The value of this financial variable is given in each time period *t* by the following two equations —which can be obtained from the basic equations introduced in the previous section:

$$S_t = \sum_{t=0}^{t-1} C_t - \sum_{t=1}^{t-1} L*G*S_t, \tag{8}$$

$$S_t = \frac{FE_t - C_t}{L} \tag{9}$$

These two equations can be summarized in a single one, replacing (9) in (8), and therefore determining the dynamics of the new debt C_t from the initial value $C_0 = EF$. Given the dynamics of the new debt, we can derive the dynamics corresponding to the stock of money from the initial value $S_0 = 0$. The solutions of these dynamical equations are given in the following equations, which are developed as potential series by way of the Newton's binomial formula:

$$C_t = \left(1 - \sum_{n=1}^{t} (-1)^n \binom{t}{n} L^n (1+G)^{n-1}\right) * EF \tag{10}$$

$$S_t = \left(\sum_{n=1}^{t} (-1)^{n-1} \binom{t}{n} L^{n-1} (1+G)^{n-1}\right) * EF \tag{11}$$

Since $0 \leq L \leq 1$ and $0 \leq G \leq 1$, these solutions generate a series that converges to an equilibrium point, as we expected, when *t* is large enough. Additionally, as previously stated, in this equilibrium the contractual payments are seen to be equal to new debt.

To illustrate the process just described, we will show the equilibrium values as dynamics of the financial variables S_t and C_t, which depend on external finance, EF; on the value of two parameters, L and G; and on the number of periods for the repayment of debt, N. Assuming for example that EF = 555, L = 0.2, G = 0.55, and N = 3, at the end of the period new debt reaches an equilibrium value of 197 units from the initial 555. On the other hand, departing from zero, the stock of money reaches a final equilibrium value of 1790 units. The nature of these dynamics is linear, no matter what initial value for external finance, parameters, or time periods is selected. This is illustrated in Figs. 1A and 1B, where alternative values for L and G are selected.

Please introduce figure 1A and 1B

As shown in the Fig. 1A, changes in L do not have an effect on the final equilibrium value of new debt, and only affect the stock's equilibrium values, which decrease as L increases. This implies that changes in the stock of money, together with inverse changes in L, do not affect the contractual payments that are due to mature in each period. In relation to changes in G (Fig. 1B) this variable keeps an inverse relation to the equilibrium value of the stock of money —from equation (7)— and a positive relation to the generation of new debt in each period of time: *i.e.*, to external financing. Therefore, greater or smaller liquidity preferences from creditors affect the quantity of stock money in their portfolio, but do not affect the financing of investment through greater quantities of new debt or, equivalently, do not bring new partial increments of the inherited stock of money. Therefore, they do not have an effect on the structure of contractual payments or their volume. An increase in G reduces the stock, because money destruction carried out by debtors is high. As a result of that high destruction, contractual payments could also be higher, which explains the existence of more new debt —used to finance new investment.

Within this mathematical framework it is possible to imagine the consequences of a fast fall in liquidity preference, a greater L, on the equilibrium values of new debt and the stock of money —as usually happens in the euphoric phase of the economic cycle (as agents' expectations hold an ever growing economy). This increase would cause a reduction in the stock of money, but would keep constant the contractual payments, while it could raise investment levels. If in some advanced moment of this euphoric time, the value of G were to fall —*i.e.*, if the debtors' ability to destroy the stock money were reduced— then there would have to be higher levels of stock of money to face the situation, as well as reduced levels of new debt. Additionally, if in this economic situation there were a sharp increase in liquidity preference, L would fall, and debtors could have serious difficulty repaying the debts incurred during the euphoric phase. This situation could get even worse if there were a concurrent delay the stock of money increase. This could describe the archetypal financial crises taking place in more complex institutional environments.

Finally, we illustrate what would happen if the external finance of investment changed in time —a detailed discussion of this issue is postponed until the next section. In Fig. 3 we present two different simulations for alternative evolutions of external finance in time, EF_1 and EF_2, and their different effects on the evolution of the stock of money —Figs. 2A and 2B.

Please introduce figure 2A

Please introduce figure 2B

The evolution of EF_1 shows a continuous rise in this variable. In this case the economy will never reach equilibrium levels for the stock and the new debt. On the contrary, as it is shown for EF_2, if this variable eventually reaches the specific value of 555 presented in the previous example, then the economy will reach the equilibrium values already introduced for both variables —Figs. 2A and 2B. However, the paths followed by these variables are completely different if compared with those presented in Figs. 1A and 1B. These last are linear, while Fig. 2A shows cycles.

III. The Dynamics of Investment and the Stock Money: a Model, Cycles, Initial Conditions and Changes in Parameters.

3.1 The Model

The mathematical evolution of the stock of money and new debt as described by equations (10) and (11) necessarily takes these magnitudes to an equilibrium. The final values of the stock and the new debt that is added depend on the values of the two parameters L and G; on the level of external financing, EF; and on the initial quantity of the stock of money, S_0. Given the equilibrium levels for the stock of money, all contractual payments of debt are fulfilled by construction —that is, the expected value of such payments coincides with the effective repayment of debt. However, it is reasonable to conceive investment and therefore external finance as endogenous variables in the system. As such, it is shown in equation (12) how the investment of the firms sector depends on the main financial variables of the economy: profits, internal funds, and the repayment of debts.

$$I_t = I_{t-1}\left(1+\left(U\left(\frac{B_{t-1}}{B_{t-2}}-1\right)+V\left(\frac{IF_{t-1}}{IF_{t-2}}-1\right)+W\left(\frac{R_{t-1}}{P_{t-1}}-1\right)\right)\right) \quad 0 \le I_t \le 2J\,, \tag{12}$$

where an increment in profits and internal funds of one hundred percent is transformed at the investment level into an increment of U and of V respectively; while a default of one hundred percent, for example, is transformed, according to this equation, into a decline of the investment level by W percent.

To further discuss this analytic proposal for the investment equation, it is necessary to address several important questions:

1) How the internal and external finance values for each level of aggregate investment are obtained. One possibility is to calculate the percentage of internally financed investment as a function of the overall volume of the firm sector's investment —equation (13).

2) What percentage of the investment is externally financed with stock money: *i.e.*, d_t —a magnitude that is positively related to λ_t as described in equation (14).

3) What percentage of total external finance is done with the stock of money which can be used by debtors to repay their debts is shown in equation (15).

$$\frac{IF_t}{I_t} = f(B_t) = H * \sin\left(\frac{\pi}{2J} B_t\right) \tag{13}$$

$$d_t = \frac{D\left(S_t / EF_t\right)}{1 + K\left(S_t / EF_t\right)} \tag{14}$$

$$\gamma_t = A * \sin\left(\frac{\pi}{2J} B_t\right) \tag{15}$$

In equations (13) and (15), it is assumed that there is a maximum value for the percentage of internally financed investment, H, and also for the percentage of externally financed investment by stock money used to fulfill the contractual payments, A: $0 \leq H < 1$, $0 \leq A < 1$. In relation to these maximum values A and H —because the mathematical notation proposed in equations (13) and (15) to determine the values of two basic variables in our model involves trigonometric relations— investment changes are restricted to the range $0 \leq I_t \leq 2J$, where J is a middle value of the investment, an arbitrary number, such that when $I_t = J$ these maximum values H and A are reached. Otherwise, if $I_t \neq J$ both values in (13) and (15) are lower than H and A respectively. The percentage of externally financed investment by stock money, d_t, is determined by equation (14), which shows how it corresponds to the ratio of stock of money to external finance in each period. In this case, $D \leq 1$ and $K \geq 1$. When the ratio of stock of money to external finance is great enough, and $K = 1$, then the value of d_t approaches to D.

These last three equations, along with those already introduced in the second section, allow us to generate a system of two difference non-linear equations that describes the behavior of our economy. Assuming that $N = 3$ and $\mu_t = 0$, the system is described by the following two equations:

$$\frac{B_t - B_{t-1}}{B_{t-1}} = u\frac{B_{t-1} - B_{t-2}}{B_{t-2}} + v\frac{f(B_{t-1})B_{t-1} - f(B_{t-2})B_{t-2}}{f(B_{t-2})B_{t-2}} + \\ + w\frac{3d_{t-1}\gamma_{t-1}[1-f(B_{t-1})]B_{t-1}}{[1-d_{t-2}(1+\gamma_{t-2})][1-f(B_{t-2})]B_{t-2} + [1-d_{t-3}(1+\gamma_{t-3})][1-f(B_{t-3})]B_{t-3} + [1-d_{t-4}(1+\gamma_{t-4})][1-f(B_{t-4})]B_{t-4}} \quad (16)$$

and

$$S_t - S_{t-1} = [1-d_{t-1}(1+\gamma_{t-1})][(1-f(B_{t-1}))B_{t-1}] \quad (17)$$

Here, the equilibrium values for the stock of money and the investment flow can be found once the initial conditions and values for the parameters are given. However, nothing ensures that the dynamics corresponding to the stock take it to a significant equilibrium level other than zero or to explosive values that exceed the boundaries imposed by the investment flows. The reason is that all those variables that define the dynamics for these flow and stock variables —*i.e.*, the initial conditions or the particular parameters values— present an interdependent behavior when contributing to a final solution.

Since the result to the system is not linearly determined, the uncertainty regarding its final outcome increases and the description of the system dynamics becomes more complex. In the case of equations (16) and (17), eight parameters are needed: H, A, J, D, K, U, V, and W, as well as the two initial conditions I_0 and I_1, —while in the previous case (equations 7 to 11) only two parameters and one initial condition were necessary.

3.2 Cycles

Being more complex than the model presented in the second section, the new system can produce a cyclic behavior of the flow variables corresponding to the firms sector's investment and profits. This behavior can be easily related to the general activity of the economy —to national income and, therefore, to the economic cycle. The dynamics of the system can be related in particular to depression and recovery phases. That is, the growth of investment may end once the economy enters a recovery phase, but it may end also while the economy remains depressed. In what follows we will describe the process that defines a cycle, from drops in investment to subsequent recovery. We will focus on two main factors that explain investment behavior: namely, internal funds and the financial equilibrium between contractual payments and the amount of money available to fulfill them.

Given the initial conditions and parameters values, the stock of money and the flow of investment grow to levels that make an induced fall in aggregate investment probable. This behavior, which is numerically and graphically illustrated in the next section —Fig. 3— proceeds as follows. Initially, a reduction in investment concurrent with a increase in the stock variable takes place. Later follows a simultaneous reduction in investment and in the stock of debt. Let us examine the principal elements of this investment declining process for the firms sector. As previously noted, one main factor of these variables' behavior is the internal funds. The previous equations have taken into account the existence of a threshold for internal finance (and for its corresponding investment). If for a given investment rate internal finance surpasses that threshold, its value begins to decrease as investment exceeds

the limit mentioned above. Symmetrically, if investment falls short of the value corresponding to such threshold, then the quantity of internal finance available to the firms rises.

When depression starts, it is important to have in mind the particular increase in internal funds. Because if the investment values reached are superior to the threshold, then a deterioration in the levels of internal funds will follow and, as assumed in our model, these opposing forces will break the necessary expansion process of capital. But this financial situation does not justify by itself the existence of a the long period of investment decline that could follow. On the contrary, since investment falls to a value that generates an improvement in internal funds, as soon as the economy reduces the levels of investment it generates an incentive to increase investment in the following period. And this does not happen in the example analyzed. Something else has to take place: *i.e.*, the balance between contractual payments and the amount of money necessary to fulfill them.

We must consider, then, how the relation between contractual payments and the money source with which firms cover them evolves in time. The increasing imbalance between these two flows of money is the second main reason for the decline in investment. On the one hand, contractual payments depend on the behavior of credit —new debt— in the previous three periods. On the other, the capacity of repayment results from a more complicated mechanism. Given A, the level of profits determines the capacity of indebted firms to obtain money to repay their debts, γ_t . And given D, the level of profits determines total external finance and, therefore, the ratio S/EF, which eventually determines what part of total external finance of investment is finance by the existing stock of money.

When investment begins to fall, the increments in investment that precede this decline generate greater contractual payments in the present. Although the ratio S/EF increases, the value of γ_t diminishes, because of the high values of investment; that is, the capacity of debtors to obtain money from the externally financed investment in order to repay their debts reaches a very low level. The net effect of this opposing trend between d_t and γ_t represents a serious problem when debts need to be repaid. In this context, lower investment values bring a reduction in the level of external finance, while the large amount of stock money will eventually reduce the necessity of creating new debt. This situation causes a reduction in the volume of contractual payments in successive periods of time. Another effect of the decline in investment is the increased ability of debtors to obtain money to pay their debts. Both of these effects, the lower new created debt and the higher capacity of repayment by debtors, reduce the stock of money that will be inherited in the future from the present. And this situation drives the economy into a second phase of the recession, when signals of a recovery start to appear.

As we know, whether a recovery in investment takes place or not depends on the initial conditions of the system as well as on the investment sensibility to changes in the financial conditions. When a recovery of investment happens, it derives from the conditions existing in the preceding period, when the stock of money was falling. However, it is necessary that the corresponding reduction in profits and internal funds —which take place at the end of the recession— not be high enough to counterbalance the positive effect of the previously stated improvement in the repayment capacity of debtors and of the reduction in the creation of new debt.

The recovery of investment from values below a particular threshold —as is probably the case after a recession— produces an improvement in the internal funds of the firms' sector. This situation results in the conjunction of three favorable forces that boost the expansive phase of the cycle and that are represented in the model by U, V, and W. But once investment growth exceeds the aforementioned threshold, the financial situation of the firms worsens and the increments in the stock of money accelerate. Eventually, the use of new debt increases, bringing about an increase in the stock of money. In this context it is quite likely that a general deterioration in the debt repayment capacity of firms will appear; so we conclude that during the rising phase of the cycle their financial structure enters a fragile state —which will eventually come to an end with the continuous increase in the level of investment and the rising phase of the economic cycle.

3.3 The Initial Conditions

In this section we will illustrate the economic cycle as it has already been described, proposing two possible solutions to the general system defined by equations (16) and (17) —which differ only in their initial conditions. To calculate the trends in all the variables describing the economy, it has been written and executed a program in visual basic, available upon request. This program generates trajectories for the investment and stock of money variables with these equations.

The solutions to the system we are offering are quite representative, as they exhibit the desirable features of nonlinear dynamical systems. That is, despite the closeness of the initial values, the two trajectories follow a completely different paths. Departing for both solutions from the parameters H = 0.7, A = 0.7, J = 1000, D = 0.85, K = 1, U = 0.8, V = 2, and W = 0.07, let us assume that the initial conditions for the investment levels are $I_0 = 1000$ and $I_1 = 1061.3$. In this case the system exhibits a cycle similar to the one described in the preceding section, resulting in a stable trajectory that finishes with an equilibrium value for the stock of money and the investment flow: $S^E = 1441$ and $I^E = 1335$. However, if a slight change to the initial values is made, then the results are quite different. If we assume now that the initial conditions are $I_0 = 1000$ and $I_1 = 1061.2$ (one decimal point smaller than in the previous case), then an unstable solution for the system is observed after a period of coincidence, and an equilibrium value for the stock of money is never reached, because there is no recovery from the production and investment decline once it happens —as described in the previous section. We can visualize both solutions in the phase space of Fig. 3. We can observe how a bifurcation in the two trajectories is observed once a critical situation in the system is reached —I = 1301, S = 1377— even if they are almost identical to that point.

Please Insert Figure 3

3.4 The Parameters of the Investment Function

We now deal with the sensitivity of our nonlinear dynamical system to different parameter values. Particularly, we focus on the effects that a slight change in the parameters of the investment function —U, V, and W, in equation (12)— have on a solution that represents a cyclic but stable behavior that ends up in a equilibrium value for the stock of money. Recalling the initial values for these three parameters, U = 0.8, V = 2, and W = 0.07,

which yield the trajectories depicted in Fig. 3, we now alter each individually and analyze how the system behavior changes.

In relation to U and V, it can be seen in Figs. 4A and 4B that when successively reduced they yield quite different trajectories from the one corresponding to the base solution. In both cases, the characteristics of the base solution —that is, cycles and equilibrium— disappear at the end. This shows the role that each these forces U and V (that is, the way in which changes in profits and internal funds affect investment flow) plays in the system as well as their relationship —complementary or substitutable— when yielding these alternative solutions. This despite the fact that both forces, whose effects on investment are represented by way of the parameters U and V, are similarly defined and interpreted from an economic point of view. However, if investment is not sensitive to the joint values of these two financial parameters —and for particular values of U and V— then a stable behavior of the economic dynamics cannot be observed.

Please insert figures 4 A and 4 B

It is also necessary to mention the sensitivity of investment to the level of debt repayment in relation to what could be expected from the arithmetic rule that generates the contractual payments: *i.e.*, in accordance with different values of W. If W has a base value of 0.07, and if the ratio of debt reimbursement to due contractual payments R/P tends towards zero, then investment must decline by 7% from its previous value, as shown in Fig. 5. This is a remarkable decline, but, as we can observe, a slight increase in this parameter to 0.08 causes a greater change in the initial trajectory. The reason is that even if this value can reach quite low levels in the future, it can be quite high in the initial periods, when the economy exhibits strong increases in contractual payments and debt cannot be repaid. On the contrary, for values smaller than 0.07, but not as low as 0.04, the system still exhibits the stability of the base trajectory, allowing it to reach higher level of investment and, therefore, of economic activity.

Please insert figure 5

IV. Conclusions

The financial theory of primitive economics has two levels of content, which correspond exactly —except in the boundaries of the levels— to the organization of this essay's parts. Section I where we define the terms and section II where we state the preliminary relations between them. There remains section III, the highest level, which breathes life into those concepts and the relationships between them, so as to give movement to a system that begins with an inert base.

From the purely semantic question of the inquiry, let us remember in the conclusion the fundamental questions considered. Money, as primary flow for transactions and as a unit of account, was defined as credit circumscribed within a present time, without past or future, and representing a promise that is always affirmative and that never and in any way supposes a negation. Neither for families nor for firms could it be created or used freely; rather, it was obligatory for both. The negation of flow money and its anterior primacy, which does not cease to exist simply for not being announced at its origin, was said to cede its place to a transformation of primary money into stock, which thereby gained a past and a future. Access to stock money was neither free nor obligatory for families or firms. Stock money served neither for transactions nor for accounting but was freely available to whoever was priviledged enough to possess it. In the example given, only the firms were so priviledged.

From semantic questions we reach the logic of stock money. We can indicate three successive moments. First, the moment occupied by the obvious asymmetry between the *how* or the *who* allowing the firms (as a group) to affirm their primary money (*i.e.*, how profits for the group of firms are created) and the distribution of the group's profits between the members of the group. The second moment deals with how a certain volume of stock money is created for that asymmetry at the end of the present, how it could not happen in any other way, and how two factors that affect the future of the economy result from this: one factor having to do with the composition of a stock of money (with what is measurable) and its various movements —renewal, growth, and destruction— and the other having to do with the mathematical structure of certain rules of payment or contracts. The third moment identifies the mechanism of the payment of past stock money in the present, envisaged as movements of stock money already inherited.

The method for giving stock money an initial dynamic in time was to establish with all possible precision the basic relationship between external investment financing and the stock of money. Because of the semantics of money, this relationship is possible insofar as external finance is a complex but discernable sum of created units of stock money and of units of stock money inherited from the past. And, with that logic, each implies the other in the process of the creation of contractual payments derived from the existence of stock money and from the same mechanism permitting said payments. In this way it is possible to assign to each given level of external financing a corresponding volume of stock money that ensures the perfect functioning of the mechanism for generation and validation of contractual payments. It is what we call equilibrium stock of money. We determined for this first dynamic that, beginning with any initial situation in which the stock of money is zero and given external financing, the economy reaches a state of equilibrium. We determined that there were Newtonian series of powers that describe it. We determined, too, that cases in which the external financing is subject to movement but is able in the end to maintain a certain constant value.

In the last part of the inquiry, external financing is treated as a function of changes in the model's financial variables, of those changes that we have chosen: profits, internal funds,

and the validation of contractual payments. The model thus defined becomes more complex, but it requires more and more simplifications of and assumptions about certain variables in the system, or the relationships between them. These become ever more demanding. In this new conception of primitive economics, the trajectory followed by stock money and investment (and hence by external financing) towards their equilibrium values can reveal a cyclical behavior. But the system presents the characteristics proper to dynamic nonlinear systems: that is, stable solutions are very sensitive to every small variation in the initial conditions and to changes in the system's parameters. Unlike the more elementary, anterior dynamics, financial dynamics is therefore not certain to end up in a state of equilibrium. Nor are there exact equations to define the trajectory from initial conditions to final equilibrium.

Since this inquiry has roots expressly in Minsky's hypothesis of financial fragility, it is logical to ask, as we have managed it, what role an idea of equilibrium could play in a heterodox theoretical context. First, we could modify the definitions of the states of equilibrium, specifying the firms' normal income in the repayment of debt and increasing the margin that separates between those income and contractual payments. This would allow a study of the evolution and reduction of this margin in the course of time. And, what seems most important, how the economy breaks free of that equilibrium in reaching it. That break could be presented as the exit from a state of equilibrium into a situation of instability. Also, the the structure of debt must be made more complex, first by incoporating the kinds of interest and by widening the institutions: banks and state. Finally, the study of the semantics of money, in which we have reached a specific understanding of money's nature an function, could be extended to other the study of other monetary institutions, particularly banks, and advance towards practical (or utopic, depending on one's point of view) intervention for the structure of communitary banks providing access to the market for unpriviledged sectors, not only within a country but also internationally.

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Figure 1A

Liquidity

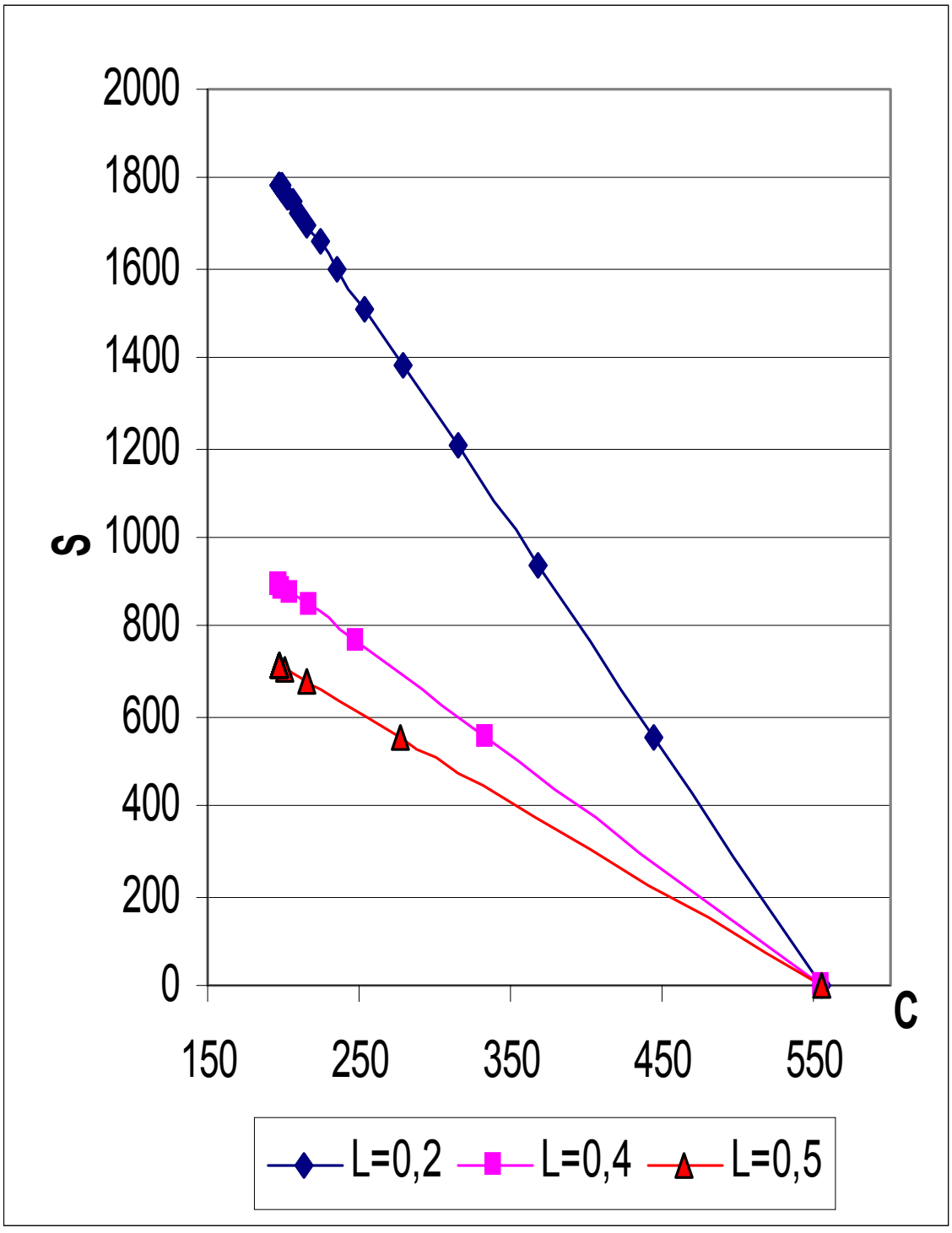
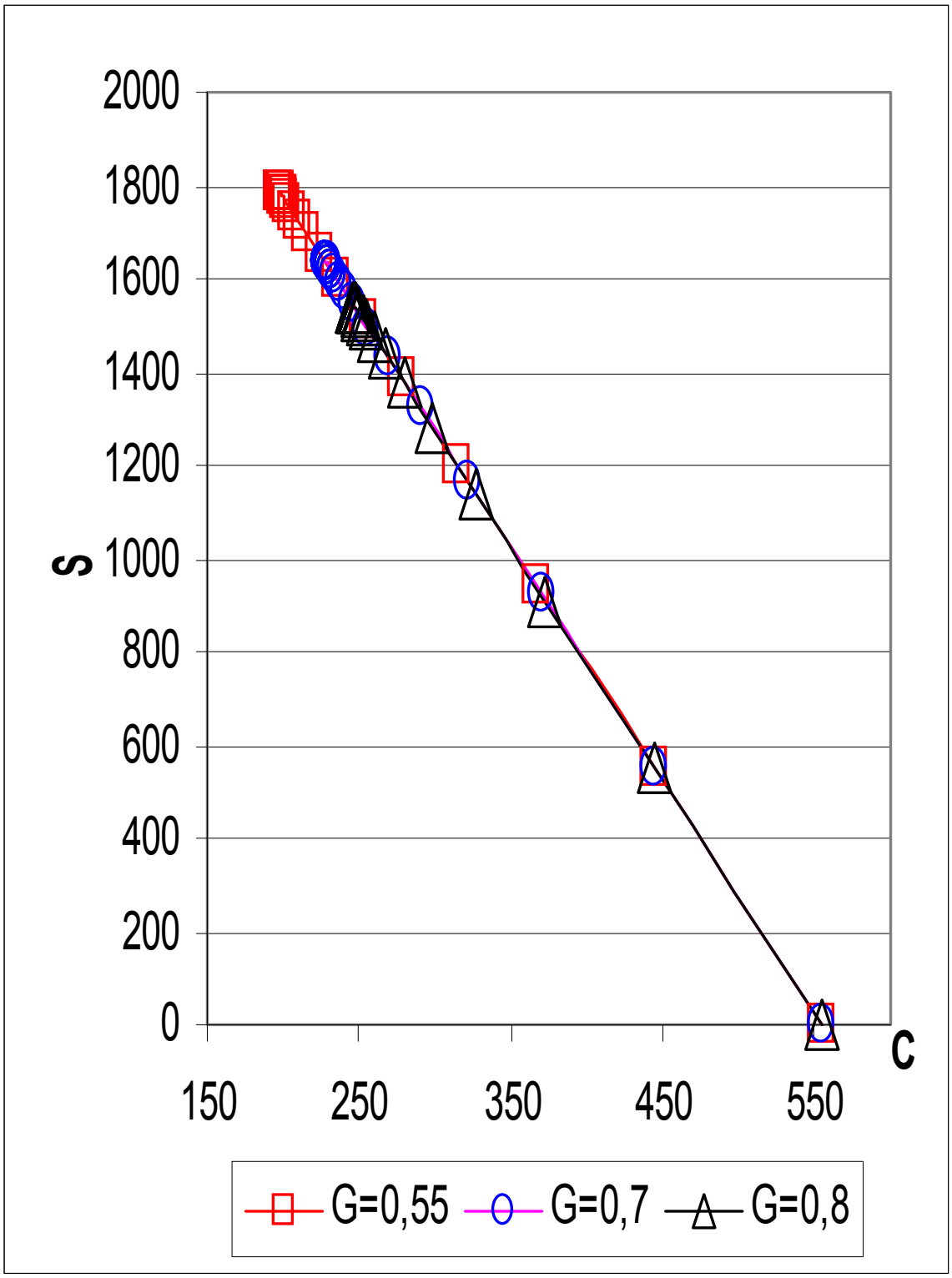


Figure 1 B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1200
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450
550
C
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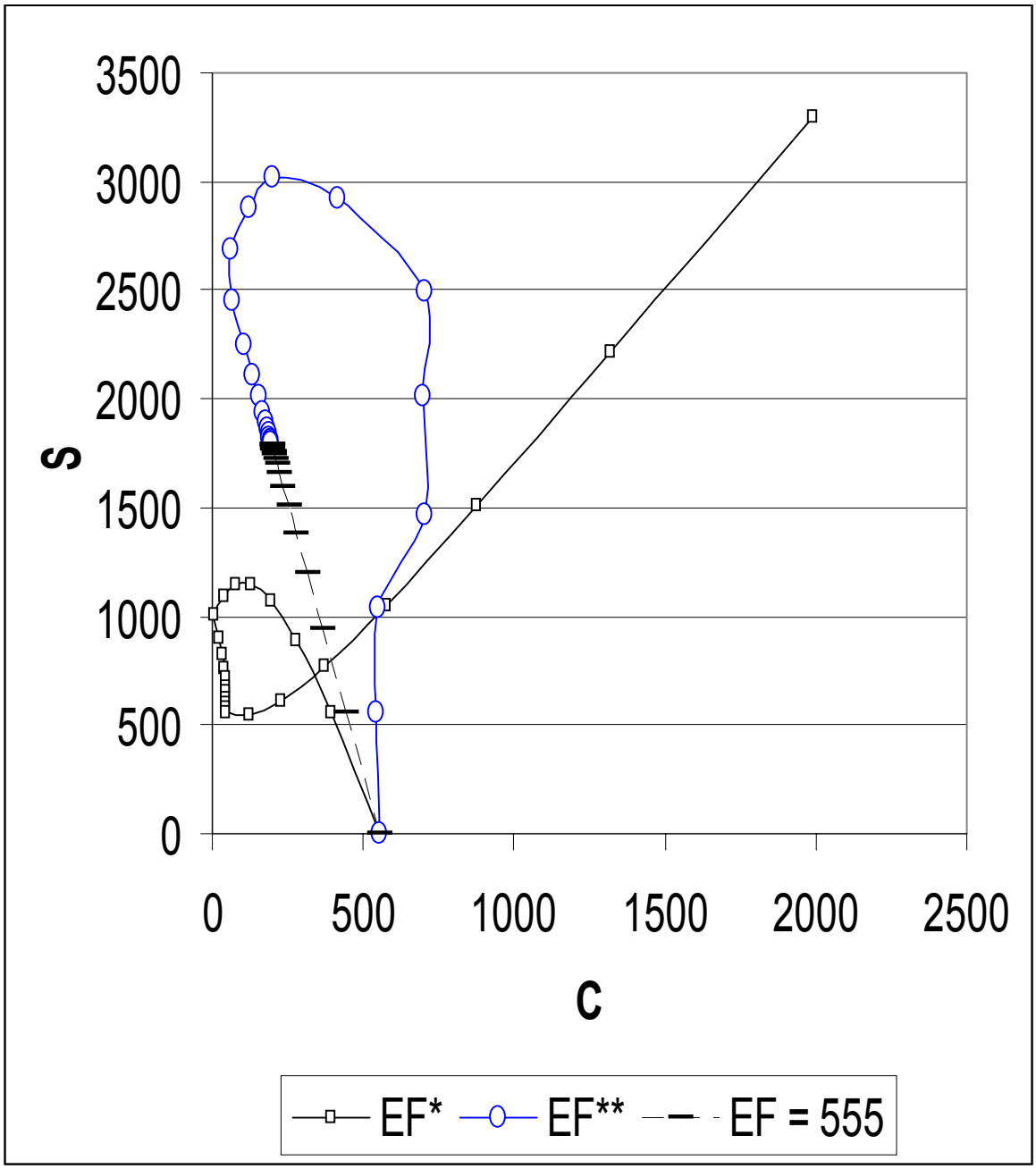
G=0,7
G=0,8

Figure 2A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EF**
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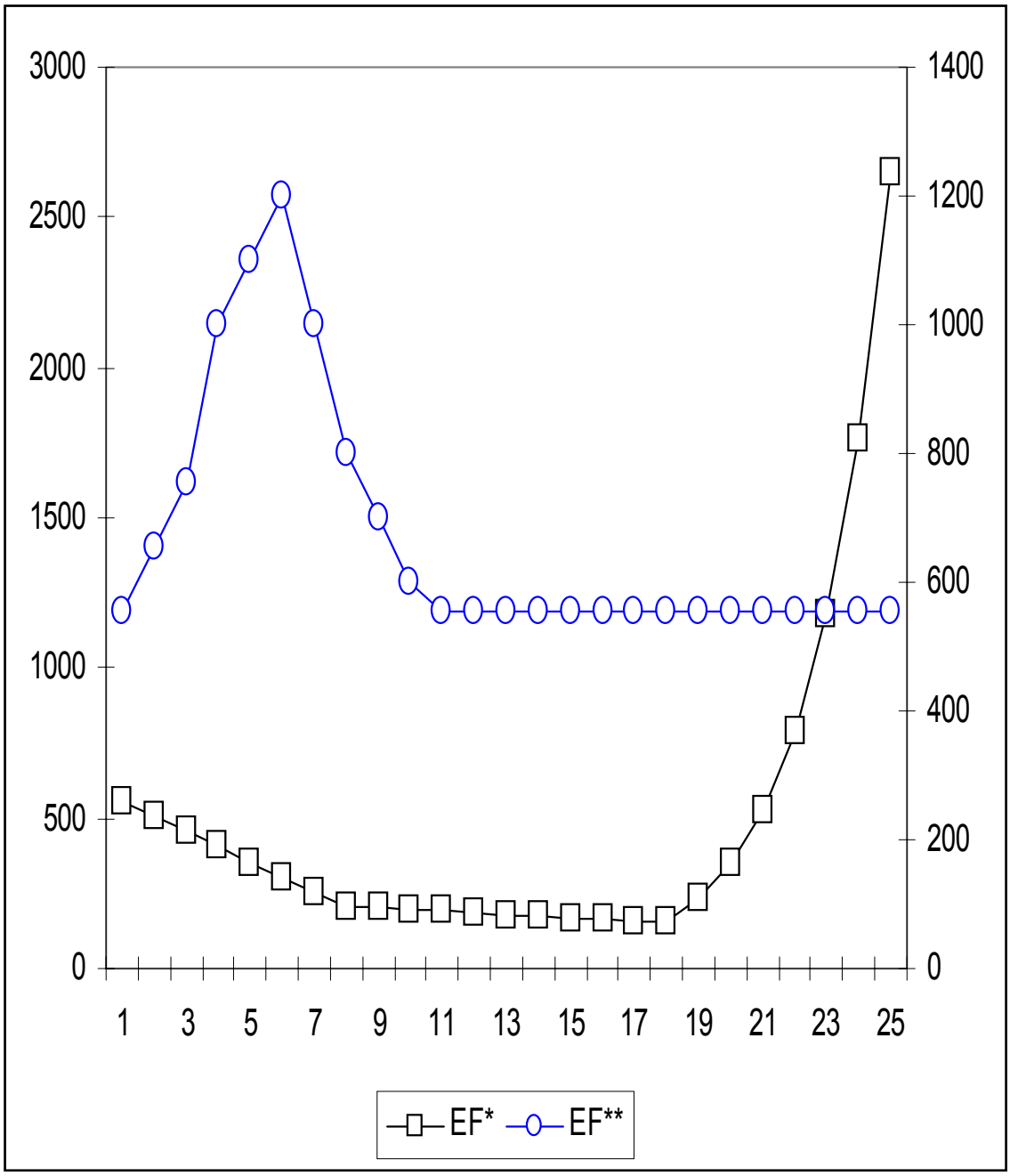
Figure 2B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EF*
EF**

Figure 3

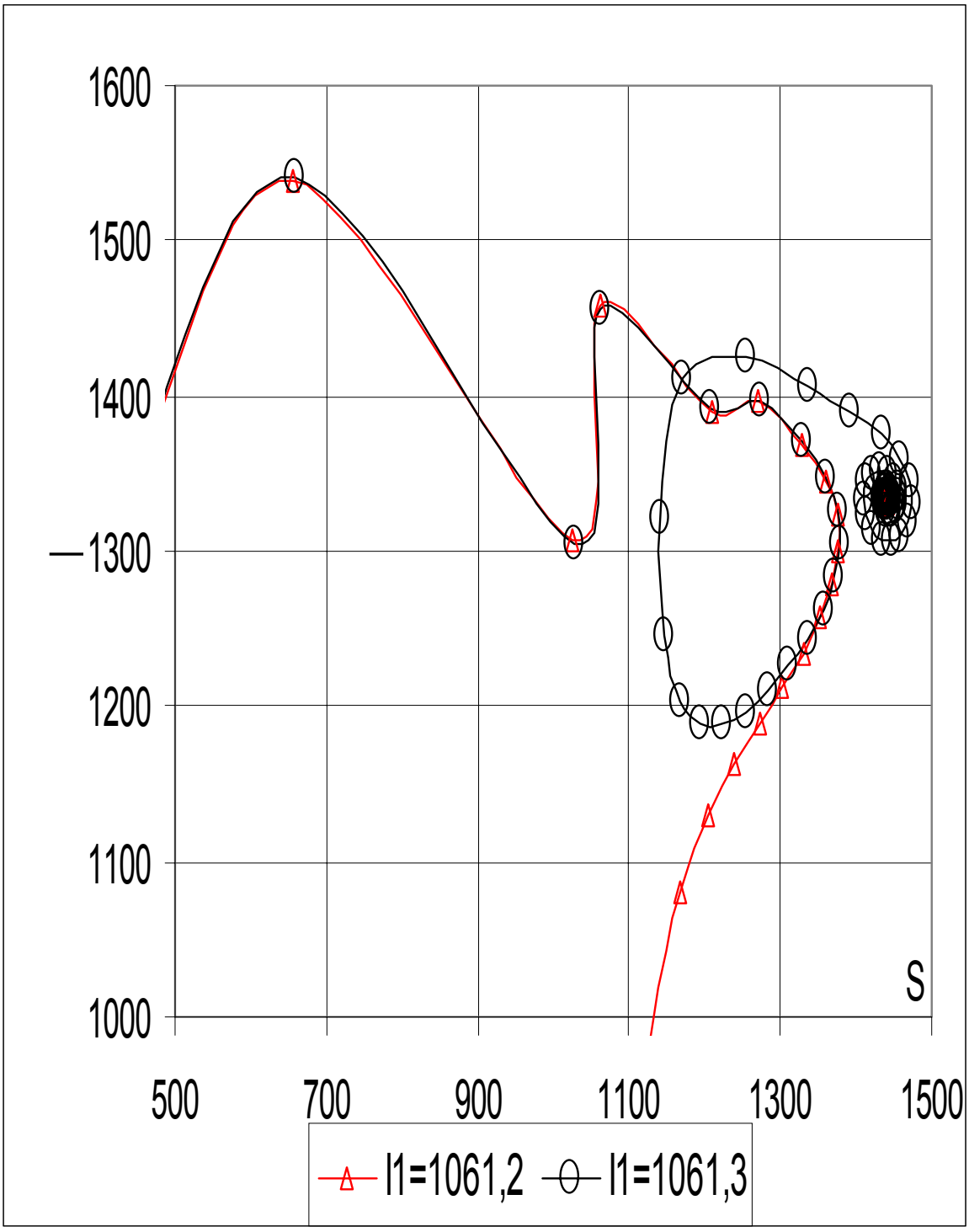
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Figure 4A

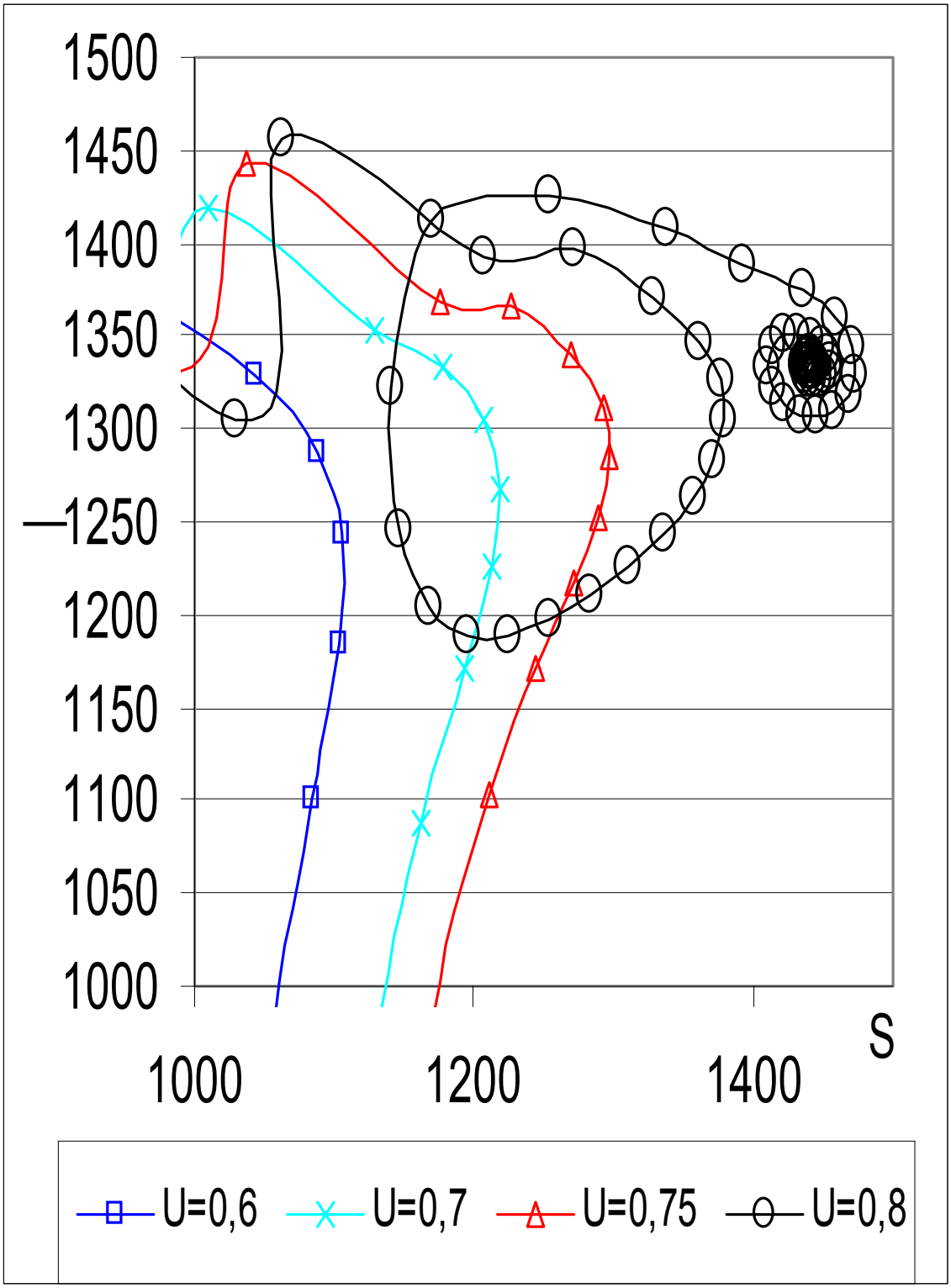
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Figure 4B

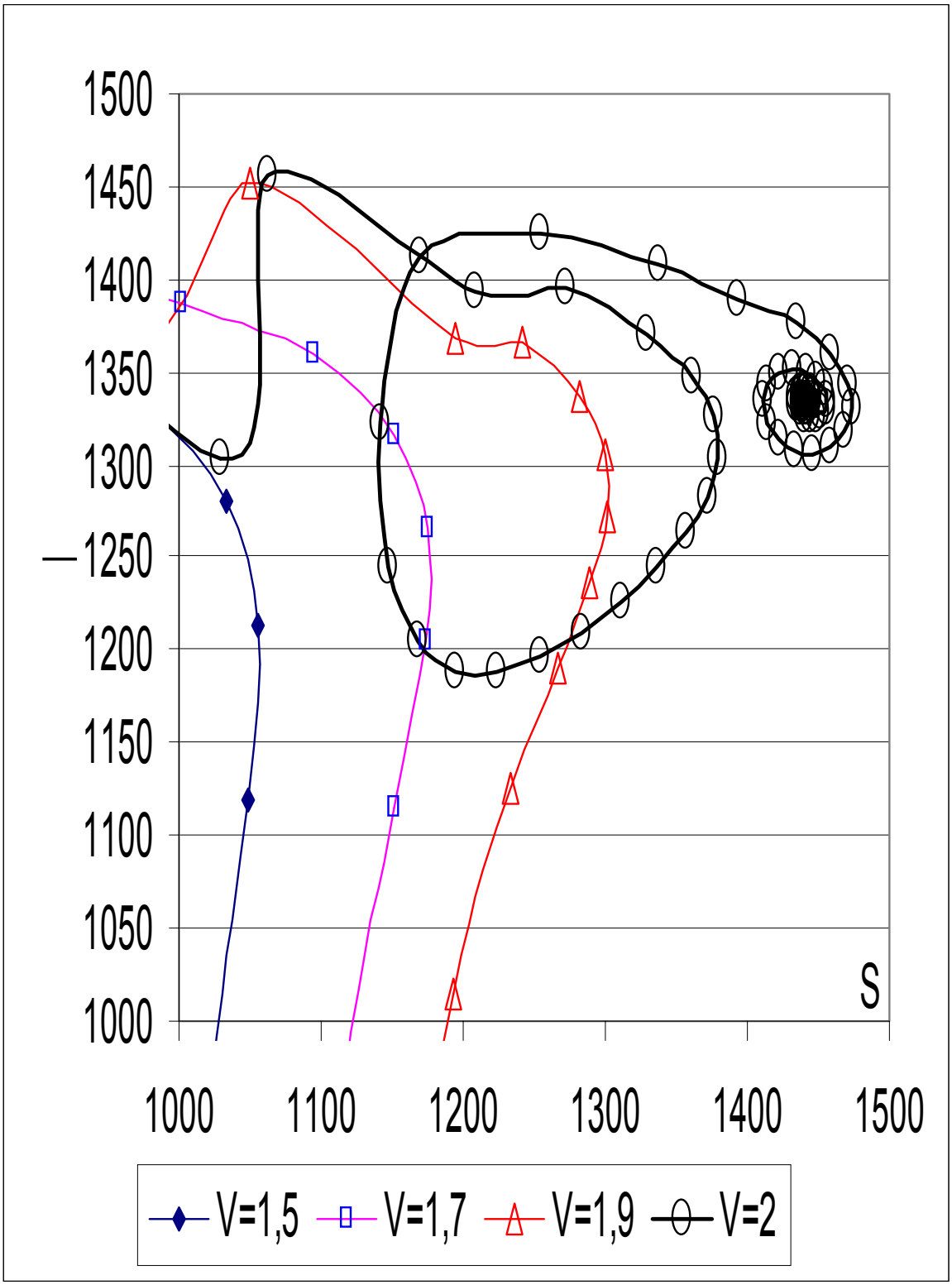
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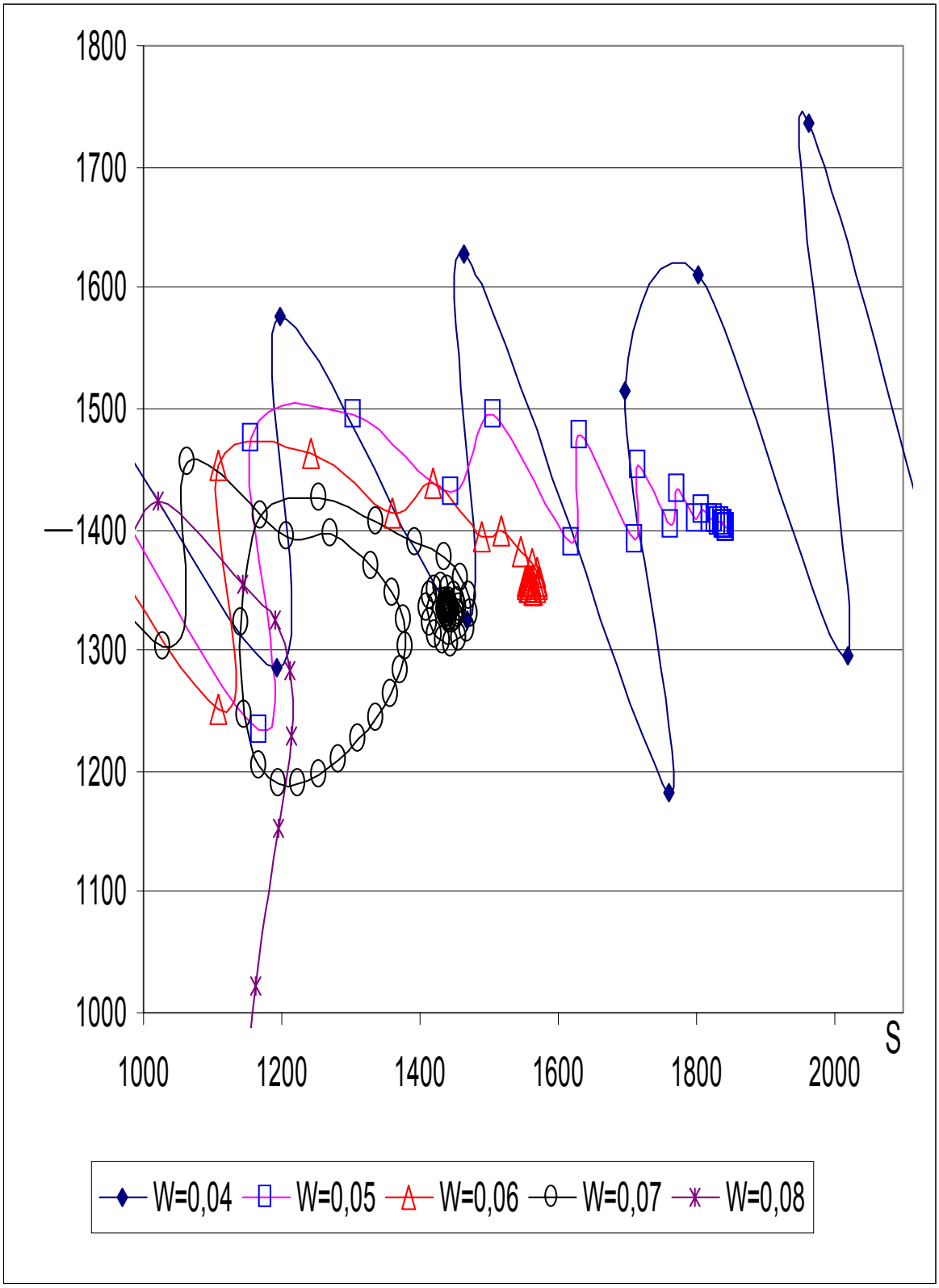
Figure 5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