

## **PORTUGUESE YIELD CURVE Volatility and Correlations**

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### **Summary**

*Stronger competition in Europe triggered by the launch of the Euro and increasing regulatory demands upon the capitalisation of banks and other financial intermediaries in the continent have raised the importance of a sound and accurate measurement of the risks brought to them by the positions they take in different assets. In Portugal banks still invest predominantly in debt instruments due to the small size of the domestic equity market, and this makes their profitability and solvency more sensitive to interest rate volatility than to the uncertainty of equity indices. But some VaR models require statistic information about the form of the local yield curve as well as how it tends to evolve along the time. However, it is important to note that there are reasons to believe that a common currency within the EU does not eliminate differences at this level between, on one hand, small and peripheral countries and, on the other hand, central and larger ones. This paper makes a first approach into this direction suggesting models to estimate the Portuguese yield curve, the spreads against German spot rates, and some values of volatility and correlation for our domestic interest rates.*

A word of recognition is due to Prof. Luís Catela Nunes for his criticism of an earlier but somewhat different version of this paper. Of course, full responsibility remains with the author.

## 1. Objectives of this study

Since 1988, with the first Capital Accord, the Basel Committee of Bank Supervision has been issuing a number of recommendations relative to the level of capital that banks and other financial companies should maintain in their balance sheets in order to cope with the different types of risks that springs out of their normal commercial activities. In many countries around the world some of these recommendations have been transformed into regulatory requirements by local national supervisory authorities. But the European Union (EU) went one step further and turned most of them into European law through a number of Directives that have been issued. And the future seems to point to even tighter demands in this field to be imposed to banks in Europe.

One of those capital requirements stems from the so-called Market Risk. Here, by the first time, VaR methodologies are nowadays a possibility to quantify that minimum bank capital.

In Portugal, the relatively underdeveloped equity market led local banks and other intermediaries to be exposed mainly to the interest rate segment of this Market Risk, especially through their large positions taken in domestic Treasury Bonds and Notes. Neither derivatives nor corporate debt are significantly represented on their balance sheets, including off balance sheet items.

At first sight, the introduction of the single currency, in 1999, would suggest that those investments in debt instruments and in other interest sensitive contracts should have their prices dependent upon some common Euroland interest rates. Under that hypothesis, our intermediaries could simply borrow the relevant statistical data from some central European countries (like Germany) and calculate their VaR exposures accordingly.

However, the loss of monetary sovereignty did not eliminate the credit risk premium demanded to take debt from each individual member state, although it might have determined some improvement in that field due to the Stability and Growth Pact. And the different levels of liquidity of the sovereign debt issues might also contribute to some market segmentation. These two effects should therefore contribute to an additive spread to be considered on top of some form of "a Central European yield curve".

In fact, empirical evidence accumulated since the launch of the Euro shows that Portugal, - like other members of the EU as Italy and Belgium -, has been required to pay some tens of basis points above the German equivalent interest rates, a spread that has fluctuated along the years and that also has showed some dependence upon the maturity of the loan.

Unfortunately the Portuguese yield curve has not been frequently studied, most likely due to the yet short story of reliable market data to base such empirical studies. Escalda (1992), Ferreira (1994) and Cassola & Luís (1996) have conducted some studies in this area, but all of them concentrated their attention in the domestic yield curve without any concern for the stochastic behaviour of it. Also these studies refer to a time frame when the Portuguese currency, in spite of integrating the European Monetary System, still fluctuated against the other European currencies. The Euro was still a currency to be borne. In addition, our secondary market of debt securities underwent great changes in the year 2000 aiming at a deeper and more European integrated market for the Treasury issues.

All in all, the recent changes in our Treasury secondary market, the introduction of the Euro and the increasing demands placed upon the banking industry by the Basel Committee led us to

write this paper. So, in parallel with a daily estimate of the domestic “risk free” yield curve, we have tried a first approach to some of its statistic characteristics. Also a comparison was made between the estimated interest rates for Portugal (for a number of constant maturities) and the correspondent values for Germany, as published everyday by the Bundesbank.

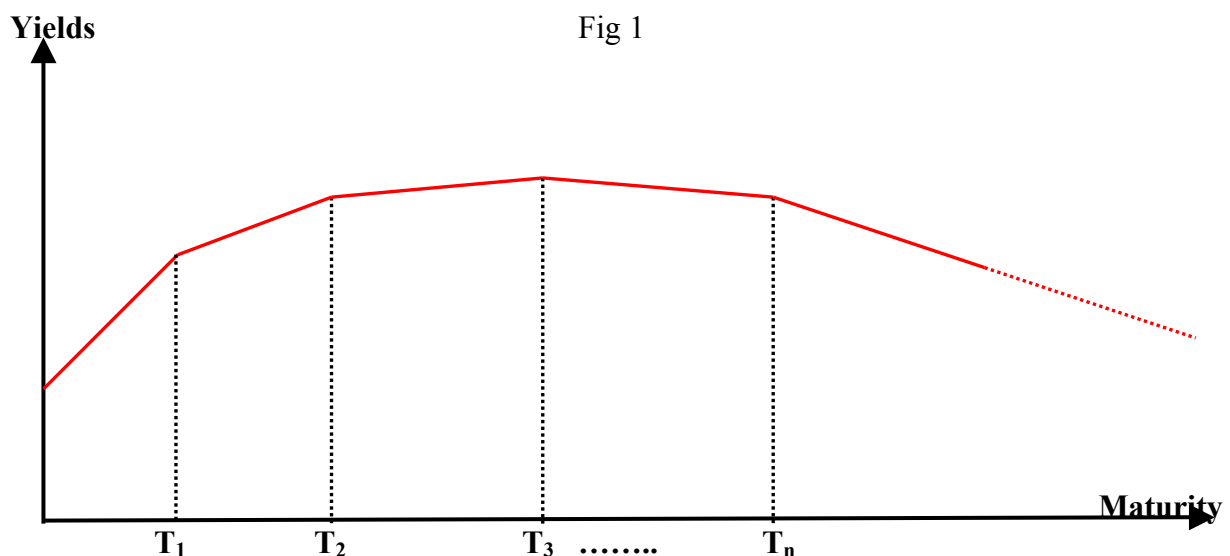
## 2. Yield Curve Models

The economic science has produced a number of different interest rate theories in an effort to explain the shape of the yield curve at any moment. In the beginning, only human behaviour interpretations were introduced, and a relevant role was frequently attributed to the current expectations predominant in the market about the future level of short-term interest rates. That is, whether market is anticipating an increase or a decrease of those future short-term rates is central to explain a positive or negative slope of the curve. More recently, a number of proposals have been popping up from a completely different origin: typically, one or tow interest rates are considered as random variables, but forbidding profitable arbitrage operations between similar debt instruments produces an equilibrium form of the yield curve.

In this paper none of these two sets of theories is the focus of our analysis. We simply took some mathematical models proposed by various authors – the so called Econometric Models - to represent the actual shape of the yield curve without much consideration to any explanatory reasons behind the profile of the curve. And out of them, we selected essentially two models, since our purpose was simply to adjust the most practical ones to our domestic empirical data.

### Selection of Models

The simplest mathematical description of a yield curve is a set of straight lines interconnected at both ends and best adjusted to the actual curve by some criterion. Basically, this approach substitutes a round curve by a broken but continue line, made up of a set of short segments of straight lines. Of course, the quality of this representation will be low, unless one uses a large number of splines (at the cost of increasing the number of parameters to be estimated).



This type of spline approach was first developed for the discount factor  $\mathbf{d}(T)$  and not for the spot interest rates  $\mathbf{R}(T)$ , and one of the most referred models is McCulloch's (1971). He began by using second order polynomials in place of those segments of straight lines, but later on, he improved his approach through third order polynomials (his 1975 version) of the following type

$$d(T) = 1 + a_{21} \cdot T + a_{31} \cdot T^2 + a_{41} \cdot T^3 + \sum_{i=1}^{k-3} a_{4i+1} \cdot (T - T_i)^3 \cdot h_i(T)$$

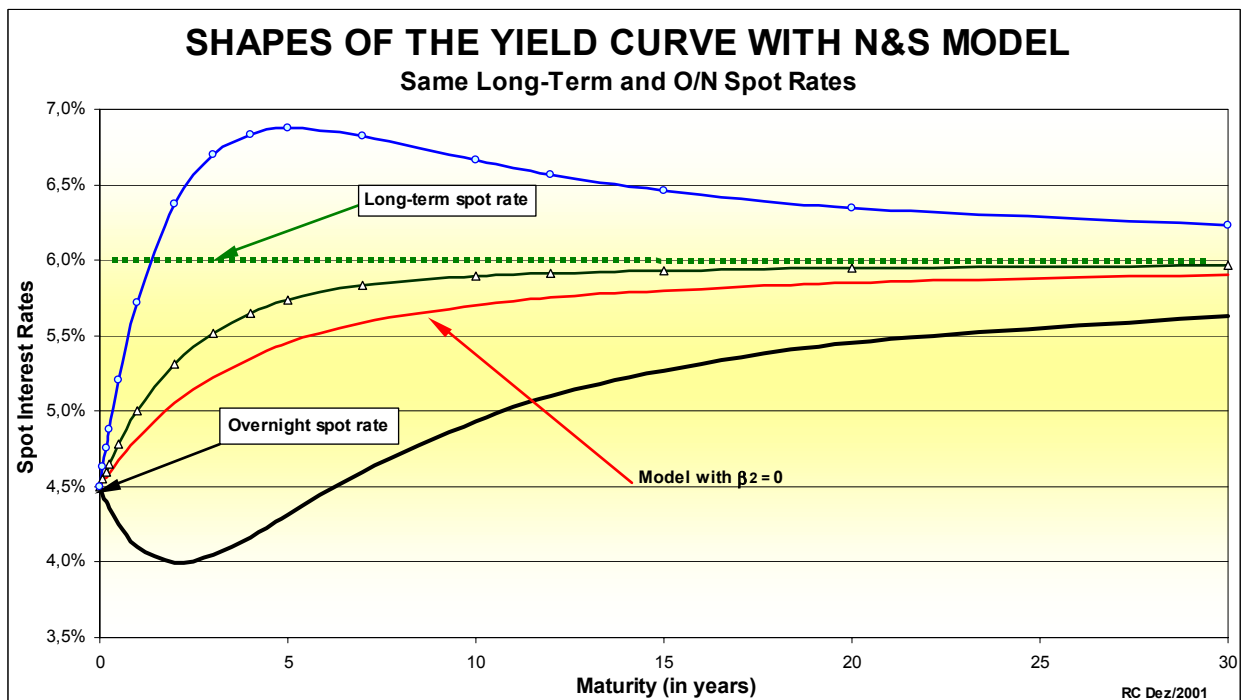
In this last version,  $\mathbf{k}$  is the number of parameters  $\mathbf{a}_{ij}$  to be estimated from market data, and  $\mathbf{h}_i(T)$  is a Heaviside step function that starts at  $T = T_i$ . McCulloch proposes to select  $\mathbf{k}$  according to the number of observations available: the integer nearest to  $\sqrt{n^o}$  of observations .

However, any of these models – linear or not – inevitably suffer from a common drawback: when extending the rightmost spline beyond the most distant cash flow, one is led either to too large or too small interest rates, or even to negative rates. That is, out of sample estimates lose their economic meaning and that makes those models less proper to extrapolate estimates of rates for (much) out of the sample maturities.

For that reason, Nelson and Siegel (1987) suggested a more elaborated mathematical model based on a few exponentials, with the advantage that their model is characterised by only four parameters:

$$R(T) = \beta_0 + \beta_1 \cdot \left( \frac{1 - e^{-\frac{T}{\tau}}}{\frac{T}{\tau}} \right) + \beta_2 \cdot \left( \frac{1 - e^{-\frac{T}{\tau}}}{\frac{T}{\tau}} - e^{-\frac{T}{\tau}} \right)$$

Fig 2

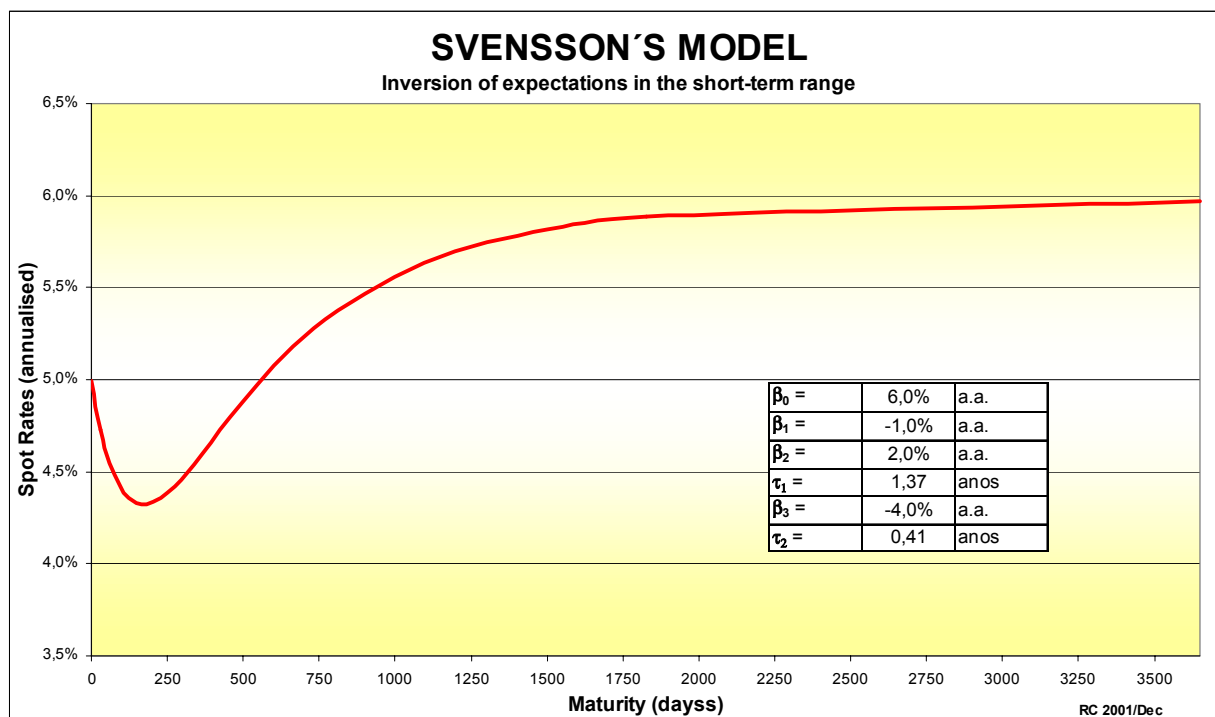


Those four parameters have the following economic interpretation:

- $T$  is the maturity of the estimated spot rate
- $\beta_0$  is the spot interest rate for very long-term maturities (when  $T \rightarrow +\infty$ )
- $\beta_0 + \beta_1$  is the interest rate for very short-term operations (in practice, 24 hours)
- $\tau$  measures the transition time from that very short-term rate to the long-term level
- $\beta_2$  is a correction term for mid-term rates that is necessary to accommodate deviations from the simple exponential shape<sup>1</sup> (that is, the function that multiplies  $\beta_1$ ).

If this N&S model is used when there is a known and explicitly disclosed overnight (O/N) market rate - the value  $\beta_0 + \beta_1$  - the number of parameters to be estimated is reduced to 3, a fact that brings important practical consequences in case of frequent adjustments to empirical data.

Fig 3



Later on, Svensson (1994) suggested an improvement to this N&S model in order to describe better the occasional "abnormal" shape of the yield curve at the short-term end of the spectrum. In fact, sometimes it is found that the yield curve begins with a negative (positive) slope that, subsequently, shifts to a positive (negative) slope - a horizontal S-shaped curve. Specifically, he added a fourth term to the N&S expression with two more parameters to be estimated<sup>2</sup>:

<sup>1</sup> In Fig 2, the difference between the two central curves is that one has  $\beta_2 > 0$  and the other has  $\beta_2 = 0$ . That is, the  $\beta_2$  term is intended to increase or to decrease the curvature of the yield curve for mid-term maturities.

<sup>2</sup> Notice that, mathematically, the two terms in  $\beta_2$  and  $\beta_3$  are very similar, although with different time constants  $\tau_1$  and  $\tau_2$ . These two similar terms allow a better fit to the shorter part of the yield curve without disturbing the adjustment to the mid-term portion of it.

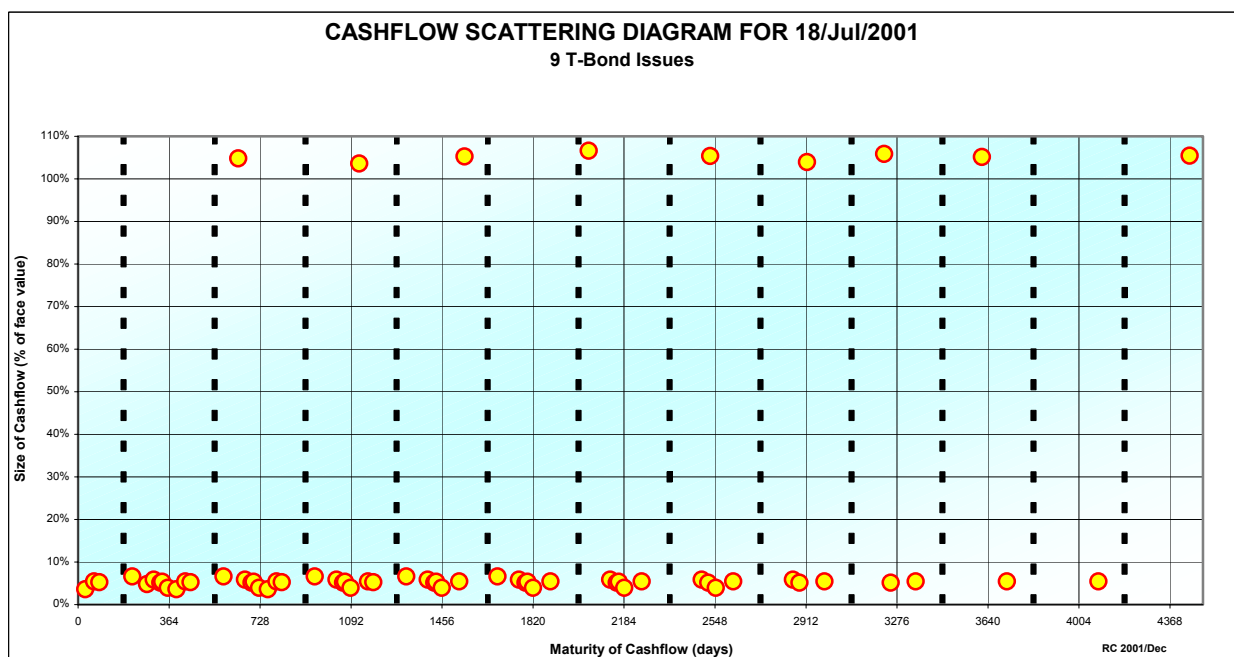
$$R(T) = \beta_0 + \beta_1 \left( \frac{1 - e^{-\frac{T}{\tau_1}}}{\frac{T}{\tau_1}} \right) + \beta_2 \left( \frac{1 - e^{-\frac{T}{\tau_1}}}{\frac{T}{\tau_1}} - e^{-\frac{T}{\tau_1}} \right) + \beta_3 \left( \frac{1 - e^{-\frac{T}{\tau_2}}}{\frac{T}{\tau_2}} - e^{-\frac{T}{\tau_2}} \right)$$

Of course, with six parameters this model brings an improved capacity to adjust to any shape of the curve, but this at the cost of more demanding and elaborated procedures<sup>3</sup> to adjust the model to a concrete set of market data.

However, because the term in  $\beta_3$  simply repeats the term in  $\beta_2$  with a different time constant, unless one really needs to adjust to very “unusual” short-term shapes – pronounced hump or deep trough – in parallel with significant changes in the “normal” curvature in the middle of the yield curve, it is frequently acceptable to stick to the N&S model and to its lower number of parameters (3 or 4).

In the particular conditions of the Portuguese case - small number of issues available (9 at most), low market liquidity of most of those issues, and lack of price data for the very low maturities<sup>4</sup> - the model from Svensson becomes even less desirable for a pragmatic usage. It brings a lot of complications without adding much accuracy to the estimates, due to the quality and quantity of the data available.

Fig 4



<sup>3</sup> Minimising the sum of square deviations requires a number of iterations, and that number tends to increase with the number of parameters involved. Also, it becomes more likely to stop in a relative minimum instead of in an absolute minimum.

<sup>4</sup> This is due to the fact that Floating Rate Notes issued by our Treasury and paying coupons twice a year do not trade frequently (if any) in both organised markets that exist in Portugal – Stock Exchange and MEDIP. See Annex 5.

Finally, a recent model was proposed by Costa (2000) which, in its simplest version (RC1, for single regime model), requires only 3 parameters to be estimated. In fact, assuming that there is no liquidity risk premium<sup>5</sup> and also taking for granted that the yield evolves monotonically upwards or downwards - no “abnormal” behaviour of short-term rates -, this model is expressed by the following simple formula:

$$R_n = \bar{r} + (r_1 - \bar{r}) \cdot \frac{\tau}{n} \left( 1 - \left( 1 - \frac{1}{\tau} \right)^n \right)$$

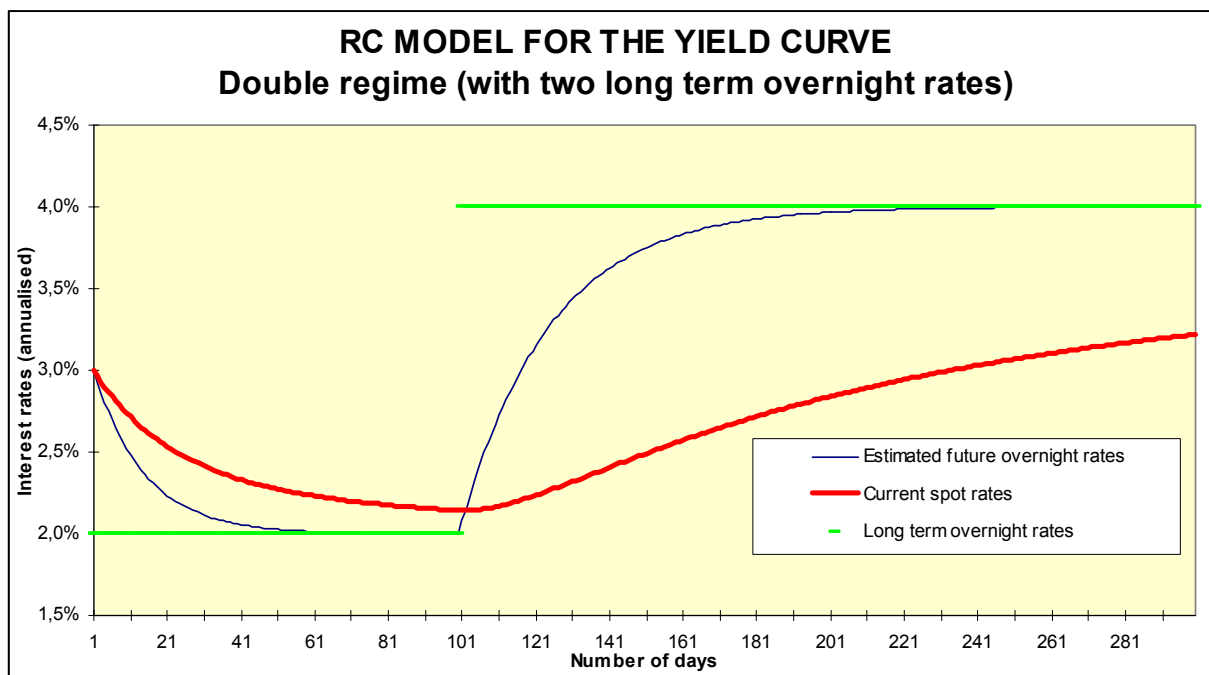
where:

- $n$  is the maturity (expressed in number of days)
- $r_1$  is the spot rate for immediate O/N operations
- $\bar{r}$  is the long term asymptotic rate for future O/N operations
- and  $\tau$  is a measure of the transition time from  $r_1$  to  $\bar{r}$  (in days)

Since, in the Portuguese market, the O/N rate is calculated and disclosed every business day, the daily work with this version boils down to estimating only 2 parameters. Therefore, it would be a very convenient alternative if it could be applicable to our empirical data.

But, as the very construction of this simplest RC1 version makes it less well adjustable to curves with more the one curvature (S-shaped), it was crucial to check the capacity of this simpler version to describe our market. That was one of the aims of this study.

Fig 5



<sup>5</sup> Besides RC1, there is a more flexible but elaborated version – RC2 – that considers a double transition regime to forecast future interest rates. And both those versions can be complemented with an added liquidity premium term.

Nevertheless, for more "detailed" days, one can extend the basic idea of Costa's model to a cascade of two regimes (RC2, double regime version), where:

- an initial regime is applicable from day **1** until day **n**, and is determined by parameters  $r_1$ ,  $\bar{r}_1$  and  $\tau_1$ ; **n** is the termination day of the first transition regime
- and a subsequent regime is applicable for any maturity  $N > n$ , which starts on day **n+1** and is characterised by  $r_{n+1}$  (the forward O/N rate for day **n+1**), plus the parameters  $\bar{r}_2$  and  $\tau_2$ .

$$R_N = \bar{r}_2 + \left\{ \left[ \underbrace{\bar{r}_1 + (r_1 - \bar{r}_1) \left(1 - \frac{1}{\tau_1}\right)^n}_{r_{n+1}} \right] - \bar{r}_2 \right\} \cdot \frac{\tau_2}{N-n} \left( 1 - \left(1 - \frac{1}{\tau_2}\right)^{N-n} \right)$$

Unfortunately, this extended version adds 3 more parameters to the daily estimation workload, and that reduces the appeal of this version in comparison to the N&S model (only 3 parameters needed in total).

In summary, our analysis used only two basic models: Nelson & Siegel's and Costa's RC1 version, and the criteria to select these two models were:

- exclude all spline based models because they do not allow to estimate rates for maturities that fall much out of the sample
- substitute the Svensson model by N&S one due to the reduced number of market prices for a significant number of days in our domestic market (some days with only 4 issues with market prices), even knowing that the Bundesbank estimates German yields with this model and we would like to compare these foreign estimates to our Portuguese counterparts
- use RC1 version on the grounds of its simplicity and the low number of parameters required; the idea is precisely to check the actual quality of adjustment to our reality.

As a complement to this analysis and to gauge the potential gains (and subsequent costs) of moving one step further and make use of the same model as Bundesbank, some sparse comparisons (only five) were also made between Svensson's and N&S's models. Also for these same days along the sample, we used the double regime RC2 version to confirm its much larger capacity to fit to market prices.

### 3. Source of data and sampled window

In the recent past, Portuguese T-Bonds and T-Notes were traded both OTC and in the Lisbon Stock Exchange, with a clear majority of the volumes being agreed in the Exchange. However, since 24/Jun/2000, a new centralised market - MEDIP - was launched in the domestic market following the European trend of integrating all sovereign debt into a much larger, deeper and harmonised EU market.



Although our T-Bonds are still listed in the Exchange, their liquidity has moved almost completely away of the Exchange and into the new MEDIP system. Therefore, no prices from the Stock Exchange integrate the sample that was analysed. Nevertheless, that transition took a rather long time in terms of liquidity building, and that fact recommended some postponement to the beginning of the sampling period, after Jun/2000.

A similar liquidity problem also applies to the Floating Rate Bond and Notes issued by the Portuguese Treasury and listed in the Stock Exchange. No price information from these issues could meaningfully be included in the sample due to the frequent number of trading days with no single deal agreed on the Exchange. This brought important consequences to the different yield curve estimates: all those FRN pay coupons twice a year, and that means that those large volumes of debt in the hands of investors could not provide market prices for maturities below 6 months - their repricing frequency. This is precisely the range of horizons where the different debt issues included in the sample - fixed coupon bonds - did not offer large cash flows. That is, the information available is more meaningful for long-term maturities than for short-term ones.

The sample we used runs, uninterrupted<sup>6</sup>, from 14/May/2001 until 26/Sep/2001, for a total of 97 days. This is a sufficiently large sample to provide a first approach to the instability characteristics of the statistical parameters of our domestic yield curve, yet short enough to supply that first hand information without further delays.

<b>SAMPLE 14-May-2001 to 26-Sep 2001</b>	
Number of days in sample	97
Maximum number of T-Bond issues in a day	9
Minimum number of T-Bond issues in a day	4
Average number of T-Bond issues in a day	7,89
Maximum daily traded volume (€ million)	867
Minimum daily traded volume (€ million)	115
Average daily traded volume (€ million)	391
Maximum maturity of a cash flow (years)	12,4
Minimum maturity of a cash flow (years)	0,32

No information was used from the inter-bank market (MMI<sup>7</sup>) due to the lack of liquidity and homogeneity<sup>8</sup> of all its maturities above O/N. Additionally, MEDIP and MMI are different markets, in particular, with different levels of credit risk (spreads to Germany may be different).

Fortunately, the sample we used includes the unusual events on the 11<sup>th</sup> of September 2001. This gives some precious indications as to the potential effects of such a stressful situation in our domestic market. At least in terms of liquidity, that experience is rather meaningful: only two debt issues were traded on the day after that event (on 12/Sep)!

<sup>6</sup> The session on 12/Sep/2001 was excluded because only two bond issues were traded at MEDIP. But the session on the 11<sup>th</sup> was included.

<sup>7</sup> MMI = "Mercado Monetário Interbancário".

<sup>8</sup> For example, all operations between 5 and 9 days are reported as a single lump figure. And frequently there are no trades in many of these segments.

#### 4. Adjustment to the Yield Curve

Due to the non-linear character of the different models, non-linear regressions were applied to estimate the correspondent parameters from the available market data.

In this paper that adjustment was done through minimising the sum of squares of the errors between market prices<sup>9</sup> and the correspondent prices produced by each set of parameters. As the table above shows, the "best" days provided only 9 prices to the optimisation process as that was the maximum number of debt issues traded on those days in the new MEDIP market.

Because the different debt issues show significant asymmetric volumes along the sample, each individual residual was weighted by the traded volume of the day for the correspondent issue. That is, those issues less traded contributed less to the final value of the parameters than the most heavily transacted ones.

#### The role of the O/N rate

Every business day, the Portuguese domestic market provides an overnight rate which is calculated as a weighted average of all 24 hours operations executed in the MMI market and maturing on the following day. Fortunately, this O/N rate seems to be representative enough of our market due to the relatively large volumes and number of operations that are executed everyday.

Because the issues used in the adjustment process have their most relevant cash flows – the principal - falling above the 12 months frontier, we decided to force all the estimates to pass through the O/N rate as a means to compensate for that lack of data in the monetary zone of the yield curve. Also, the basic concept of the RC model so recommended<sup>10</sup>.

#### Day-count convention. Constant maturities selected

Nowadays in Europe, overnight rates and other money market rates adopt the Act/360 day count convention. On the opposite, for instruments placed with maturities longer than 12 months, Europe has standardised the Act/Act day count convention. To harmonise these different interest rates along one single yield curve, we adopted the Act/Act convention for all estimated interest rates irrespective of the associated maturity, with the following deviations:

- estimated rates consider a year of 365,25 days for maturities above 1 year
- O/N market rates were linearly converted from the 360-day basis to a 365-day year.

On the other hand, since the number of days included in the traditional constant maturity horizons is fully arbitrary, a specific number of days was adopted for each selected maturity as indicated in the table below:

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<sup>9</sup> Some authors have suggested minimising Yield to Maturity (YTM) differences instead of market prices deviations. But the larger number of calculations required by the YTM alternative to minimise the sum of the square of the residuals led us to opt for the price comparison.

<sup>10</sup> The RC model is based on the forecast of future short-term *forward* rates from the current O/N value. It is the average of these successive forecasts that produces the *spot* rate forecasted for any maturity (**n** days).

<b>Maturity</b>	<b>O/N</b>	<b>1 Month</b>	<b>3 Months</b>	<b>1 Year</b>	<b>3 Years</b>	<b>5 Years</b>	<b>10 Years</b>
<b>N° of Days</b>	1	31	91	365	1 095	1 826	3 652

The choice of these 6 + 1 maturities was the result of:

- the purpose of this paper (statistic data for VaR calculations)
- some Basel papers in that field have recommend, at least, six distinct maturities to represent the stochastic characteristics of any domestic yield curve
- the longest domestic T-Bonds were initially placed with 15 years maturities, but, along the sampled period, the largest maturity available was in the order of 12 years; this did not recommend to adopt any vertices much above 10 years
- the most common maturities internationally used.

### **Settlement of MEDIP transactions**

Because Bond and Note agreed prices are understood as clean of the running coupon, the MEDIP disclosed prices were subsequently complemented with the interests accrued since the last coupon payment day.

On the other hand, since MEDIP transactions are settled 3 days after the trading day **T**, all calculations were duly corrected for this **T+3** rule. In particular, daily interests were accrued until the settlement day **T+3**, not until the trading day **T**.

Also, because payment of the agreed price and delivery of the correspondent securities - a DvP<sup>11</sup> process - occurs, on **T+3**, comparison between market prices and estimated prices was executed on **T+3**. This means that all future cash flows were discounted from their occurrence day until **T+3**, that is, 3 days less than until trading day.

### **Income Tax. Transaction costs**

Under "normal" rules, coupon income and capital gains are liable to income tax. However there is a great deal of segmentation among investors in these Portuguese T-Bonds and T-Notes in terms of this tax impact. Beyond individual investors (who shall pay this tax), some non-residents are free of that tax, and others, like resident banks, can postpone somewhat their tax liabilities (and so decreasing their impac).

For this reason calculations were performed without consideration of any tax effects: coupons were taken gross and capital gains were considered in full. The consequences are that the estimated interest rates are gross and therefore do not translate the actual return obtained from an actual investment in these securities.

Also transaction costs were ignored because, normally, these costs represent a tiny fraction of the return obtained from the investment in debt securities. Here too, occurs a similar segmentation because different parties have specific conditions to access this market.

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<sup>11</sup> DvP = "Delivery versus Payment", meaning that payment from the buyer occurs simultaneously with his reception of the traded securities.

Our belief is that by excluding these two costs we followed the most harmonised approach to estimate interest rates in our market.

## 5. Results from the 97 days Sample

### 5.1. Goodness of fit

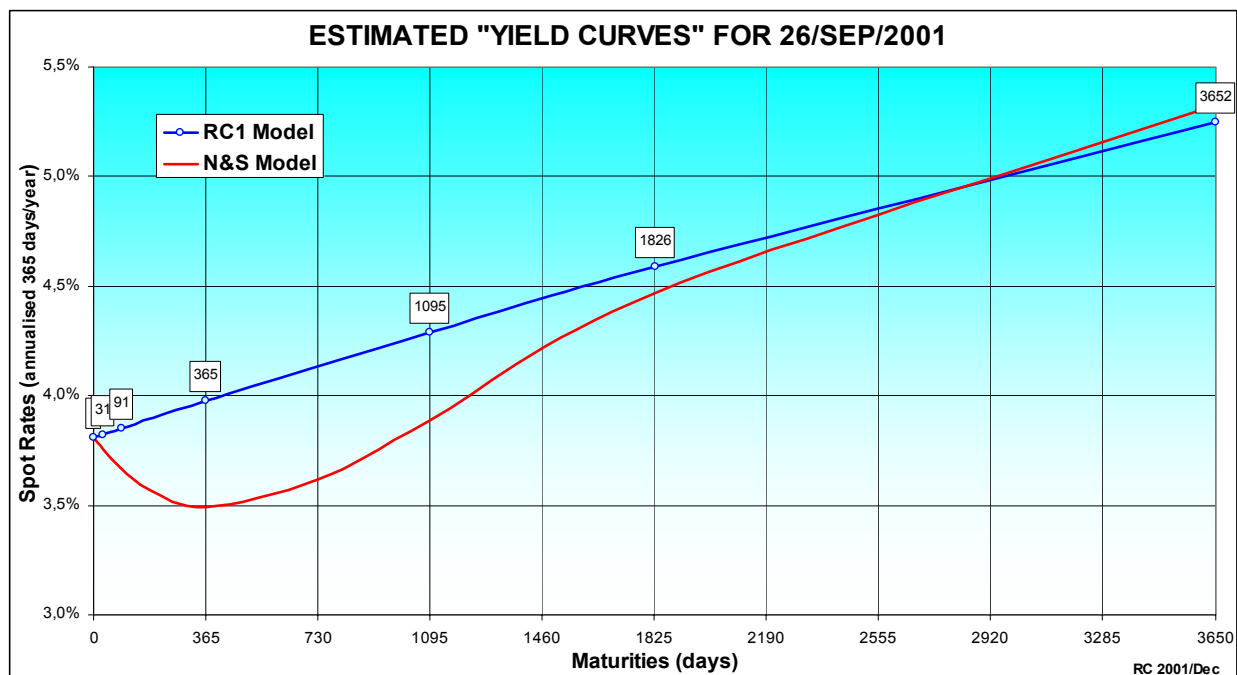
Judging from the 97 days sample analysed, both models - N&S and RC1 - adjust rather well to the public prices discovered in the MEDIP market: always the non-linear indicator  $R^2$  remained above 0,9 threshold<sup>12</sup>. On the other hand, and as it could be expected from the extended flexibility of N&S in comparison to RC1, this last model always produced a lower  $R^2$ .

Part of this "high quality" is due to the small number of points available per day - between a maximum of 9 and a minimum of 4 issues per day. The curve could almost everyday be designed to cross all those market points.

It is noticeable that, in spite of this limitation, the RC1 version maintained that high quality, even when a negative sentiment developed in the world economy at the end of the sampled period - particularly after the sad events of Sep/11 in USA - and triggered a significant fall of the expected short-term interest rates for the near future (see the graph below).

For the N&S model this high quality could be expected from the larger number of parameters adopted in relation to RC1, and in particular from the fact that the  $\beta_2$  term can sometimes perform the same role as  $\beta_3$  term in the Svensson's model.

Fig 6



<sup>12</sup> See Annex 1 for details.

Due to the similarities between the results of the two models in terms of  $R^2$  all along the sample, Fig 6 complements this analysis of the goodness of fit for one specific day (around the middle of the sample) when the abnormal short-term behaviour of the curve was already visible.

The greatest difference between the two models appeared at the level of the asymptotic rates  $\bar{r}$  and  $\beta_0$ , and for the transition times  $\tau$ . Above all, RC1 always estimated long-term asymptotic rates much above reasonable levels, wherever N&S produced counterpart values much more economically acceptable.

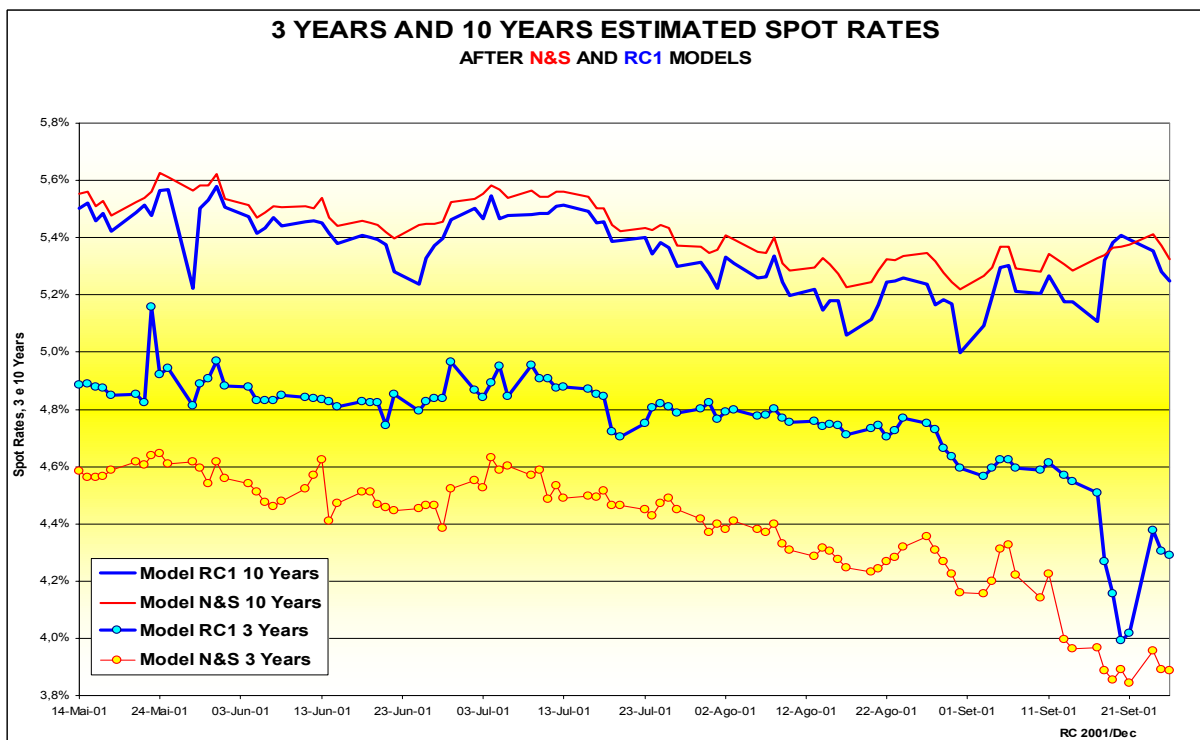
RC1 VERSION - 7/Sep/2001			N&S MODEL - 7/Sep/2001		
$r_1 =$	4,3258%	p.a.	$\beta_1 =$	-1,8424%	p.a.
$\bar{r} =$	43,4848%	p.a.	$\beta_0 =$	6,1682%	p.a.
$\tau =$	79 501	days	$\beta_2 =$	-3,7672%	p.a.
			$\tau =$	577	days

This fact restricts the use of RC1 for out of sample interest rates estimates. In the Portuguese case, this means maturities above 10 years

## 5.2. Comparison between estimated interest rates

For 10 years maturities, RC1 (almost) always produced estimated interest rates slightly below N&S, but for 3 years, the opposite is valid. Nevertheless, the spread between the two estimates is always larger for 3 years than for 10 years. This difference in spreads is due to the incapacity of RC1 to accommodate double curvature shapes as it lacks an additional parameter like  $\beta_2$ .

Fig 7



## 6. Improvements from RC2 Version and from Svensson's Model

In spite of the good quality of adjustments obtained with both RC1 and N&S, we know that:

- by construction, RC1 may produce estimates with large errors in case of mid-term maturities
- and any comparison of spot rates with Germany recommends the use of the same model in the two countries - Svensson's model.

So we decided to apply the RC2 version and the Svensson's model to a shorter sub-sample made of only 5 days and distributed almost evenly along the 97 days sample. This provided a first insight into the level of improvements that one could expect from such two sophistications.

### 6.1. RC2 Version

RC2 version improved the quality of the adjustment to the market data, in terms of the  $R^2$  statistic, in terms of the asymptotic interest rate  $\bar{r}_2$ , and of the estimated 1 year and 3 years spot rates, as shown in the next table:

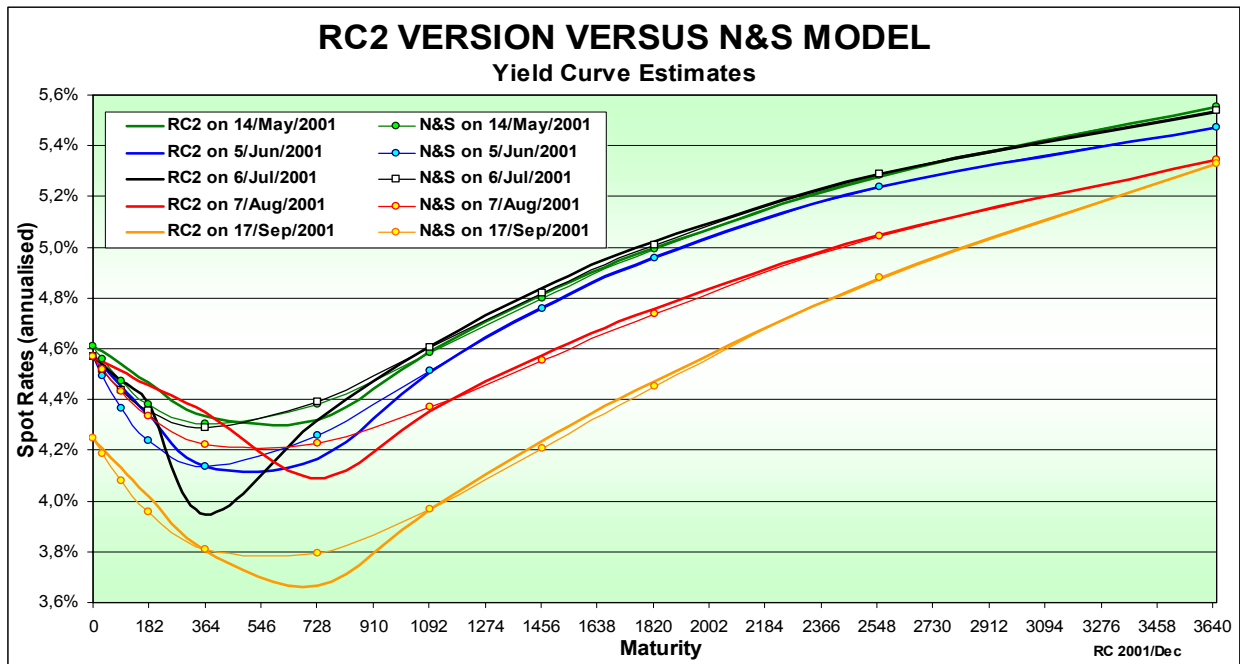
Day	Model	$R^2$	1Y rate	3Y rate	Asymptotic rate
14-May-2001	RC1	0,9876	4,7032%	4,8842%	28,2071%
	RC2	0,9986	4,3342%	4,5909%	6,3324%
	N&S	0,9987	4,3022%	4,5822%	6,2238%
05-06-2001	RC1	0,9499	4,6602%	4,8306%	27,7725%
	RC2	0,9964	4,1351%	4,5078%	6,0245%
	N&S	0,9968	4,1355%	4,5126%	6,0341%
06-07-2001	RC1	0,9714	4,6621%	4,8459%	32,1231%
	RC2	0,9963	3,9486%	4,6098%	6,1670%
	N&S	0,9974	4,2891%	4,6028%	6,1555%
07-Aug-2001	RC1	0,9929	4,6405%	4,7814%	20,1967%
	RC2	0,9996	4,3517%	4,3530%	6,2210%
	N&S	0,9997	4,2236%	4,3700%	6,1662%
17-Sep-2001	RC1	0,9450	4,3354%	4,5084%	55,6889%
	RC2	0,9991	3,8019%	3,9680%	7,2359%
	N&S	0,9994	3,8104%	3,9678%	6,5986%

Additionally and in comparison to N&S, Fig 8 shows in amplified form that:

- for these five selected days, the RC2 version produces essentially the same type of double curvature yield curves as the N&S Model
- and, above 3 years, both models produce almost the same estimates for all maturities.

It is in the range of maturities around 1 year that the diagram shows more visible differences between these two models. Part of those discrepancies might be attributed to the lack of debt issues with maturities below 1 year. Without price information for the low end of the yield curve, the margin of error of any interest rates estimated for that region tends to increase. Further details can be found in Annexes 2 and 5.

Fig 8



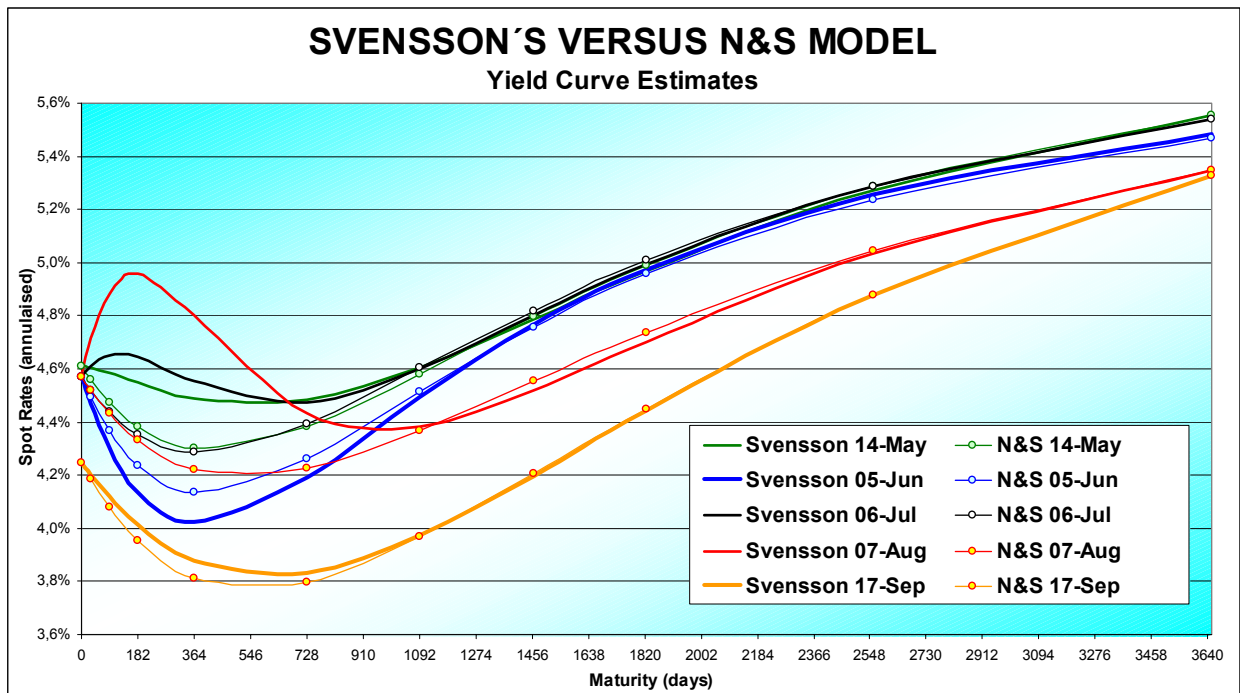
## 6.2. Svensson's Model

Most of comments above can be repeated here where N&S model estimates are compared to Svensson's.

Day	Model	R <sup>2</sup>	1Y rate	3Y rate	Asymptotic rate
14-May-2001	RC1	0,9876	4,7032%	4,8842%	28,2071%
	Svensson	0,9989	4,4899%	4,6071%	6,2655%
	N&S	0,9987	4,3022%	4,5822%	6,2238%
05-06-2001	RC1	0,9499	4,6602%	4,8306%	27,7725%
	Svensson	0,9892	4,0219%	4,4944%	6,0186%
	N&S	0,9968	4,1355%	4,5126%	6,0341%
06-07-2001	RC1	0,9714	4,6621%	4,8459%	32,1231%
	Svensson	0,9979	4,5540%	4,6008%	6,1437%
	N&S	0,9974	4,2891%	4,6028%	6,1555%
07-Aug-2001	RC1	0,9929	4,6405%	4,7814%	20,1967%
	Svensson	0,9998	4,8015%	4,3827%	6,1243%
	N&S	0,9997	4,2230%	4,3700%	6,1662%
17-Sep-2001	RC1	0,9450	4,3354%	4,5084%	55,6889%
	Svensson	0,9994	3,8785%	3,9736%	6,5645%
	N&S	0,9994	3,8104%	3,9678%	6,5986%

Specific is only the apparent opposite behaviour of the short-term part of the curves for 6/Jul and 7/Aug, as shown in Fig 9. The two humps in the curves designed after Svensson's model seem out of the general shape of the others. Once again, that contradiction can be attributed to the adjustment process performed without a significant number of cash flows occurring in that region of the yield curve.

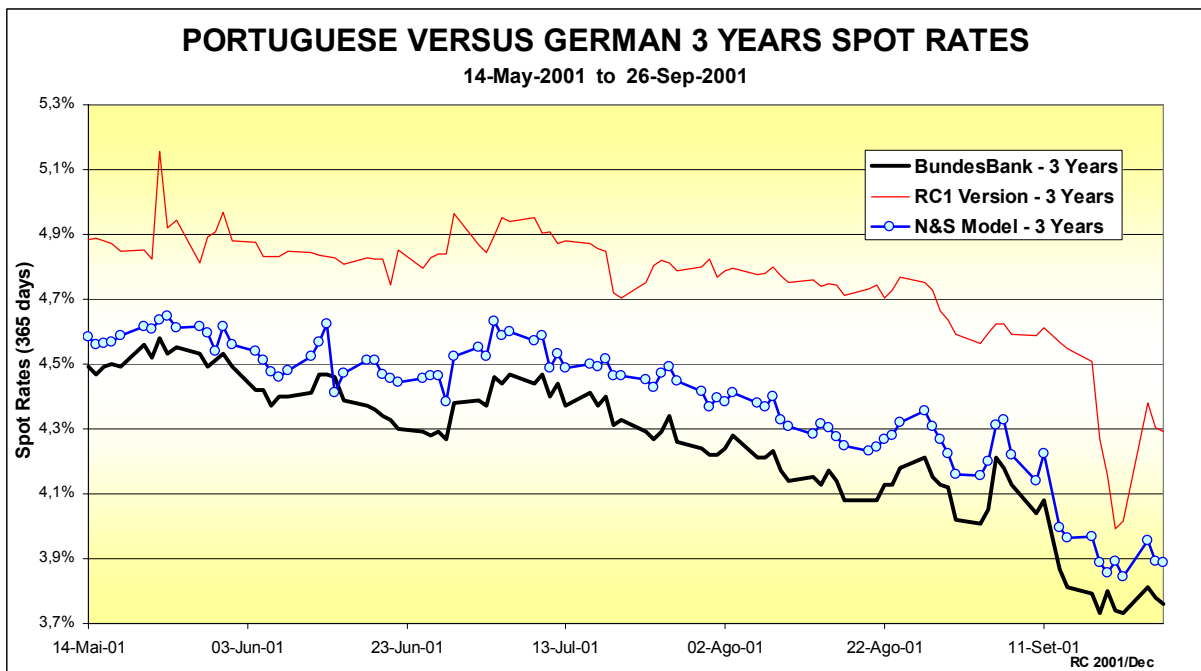
Fig 9



**7. Comparison to the Bundesbank Estimates**

Since 1997 the German Central Bank has adopted Svensson's Model to estimate daily the local yield curve. Therefore, any comparison between German and Portuguese yield estimates should be based on the same underlying econometric model. However, the number of limitations

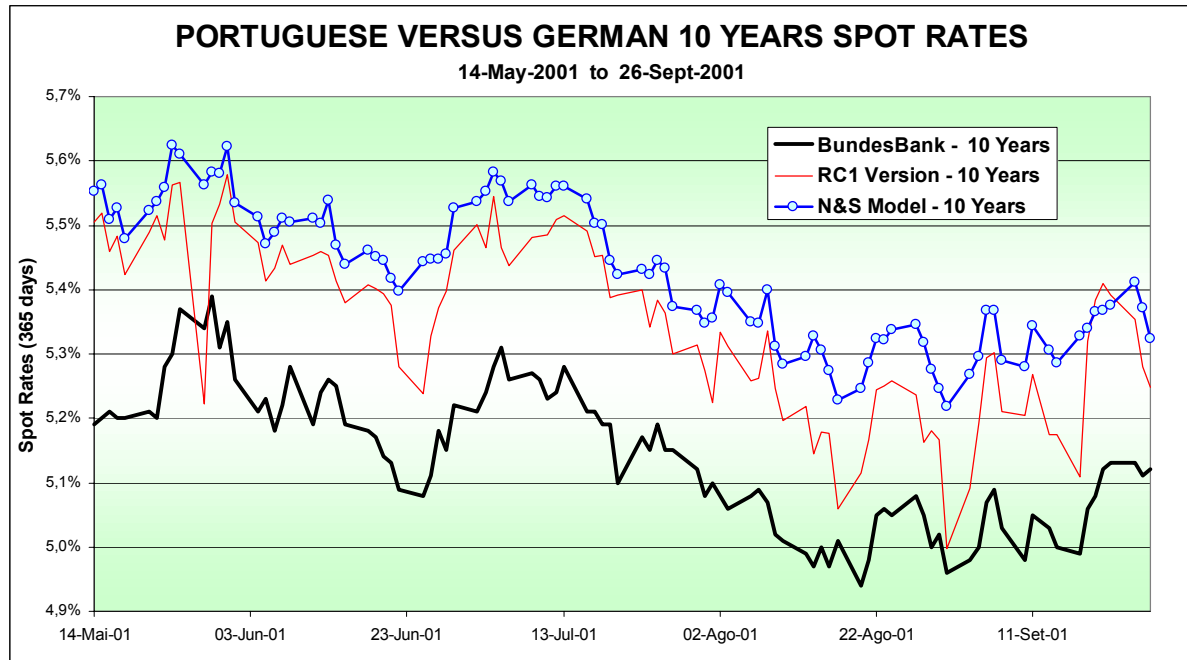
Fig 10





of the Portuguese data-base and their relevance - reduced number of debt issues, low liquidity of those issues, and lack of information for the low end of the yield curve - do not recommend the use of this more elaborated alternative to N&S.

Fig 11



Nevertheless, it is wise to have an idea of what part of that difference of estimates for the two countries comes from this model selection dilemma. With that information the remaining part of that spread might be attributed to the rating differences and to the liquidity characteristics of the respective T-Bond issues. Judging from this 97 days sample, Figs 10 and 11 show that:

- Portuguese rates are always - and with both models - above German counterparts
- at the 3 years maturity, it is N&S that replicates better the German yields - lower spread than RC1 estimates
- at the 10 years maturity, RC1 approaches closer the German spot interest rates
- however, all RC1 estimates seem more volatile than N&S counterparts, in particular, for 10 years (frequent and large negative "spikes")

The conclusion is that part of the Portuguese excess cost of debt (yet unclear from this small sample) may radicate in a model selection problem.

## 8. Volatility

Either VaR models based on variances-covariances or those based on Monte Carlo simulations require the use of some statistic parameters of the stochastic behaviour of some selected interest rates. The sample with 97 days selected along 2001 provided a first opportunity to estimate the historical values of some of these parameters.

The aim of this paper was precisely to estimate statistic those figures for both interest rates and discount factors<sup>13</sup> and for 7 constant maturities along the yield curve: one O/N plus six arbitrarily selected maturities. For that purpose, the following definitions are due:

- interest rates: 7 spot rates  $R_T$  estimated using one of the two yield curve models
- volatility of interest rates: standard deviation of the 7 spot rates along the sample
- correlation between interest rates: along the sample and for the same 7 maturities
- discount factors: calculated as  $1/(1+R_T)^T$
- volatility of discount factors: standard deviation along the sample and for the correspondent 7 factors
- correlation between discount factors: along the sample and for the 7 discount factors.

For the whole sample, the two tables below summarise the volatilities estimated for the 6+1 time horizons and for both interest rates and the correspondent discount factors.

<b>SPOT RATES</b>	<b>O/N</b>	<b>1M</b>	<b>3M</b>	<b>1Y</b>	<b>3Y</b>	<b>5Y</b>	<b>10Y</b>
14/May to 26/Sep/2001	<i>1</i>	<i>31</i>	<i>91</i>	<i>365</i>	<i>1 095</i>	<i>1 826</i>	<i>3 652</i>
<b>MODEL RC1</b>							
<b>Average rate</b>	4,4724%	4,4804%	4,4961%	4,5675%	4,7524%	4,9307%	5,3505%
<b>Volatility</b>	0,2818%	0,2788%	0,2727%	0,2468%	0,1904%	0,1522%	0,1329%
<b>MODEL N&amp;S</b>							
<b>Average rate</b>	4,4704%	4,4127%	4,3168%	4,1192%	4,3869%	4,8107%	5,4277%
<b>Volatility</b>	0,2816%	0,2766%	0,2716%	0,2578%	0,2046%	0,1716%	0,1063%

For interest rates, volatility decreased with time  $T$ , a fact that fits well into the basic idea of the RC model<sup>14</sup>. For the discount factors, longer maturities determine a larger volatility due to the amplification effect of the duration of each cash flow.

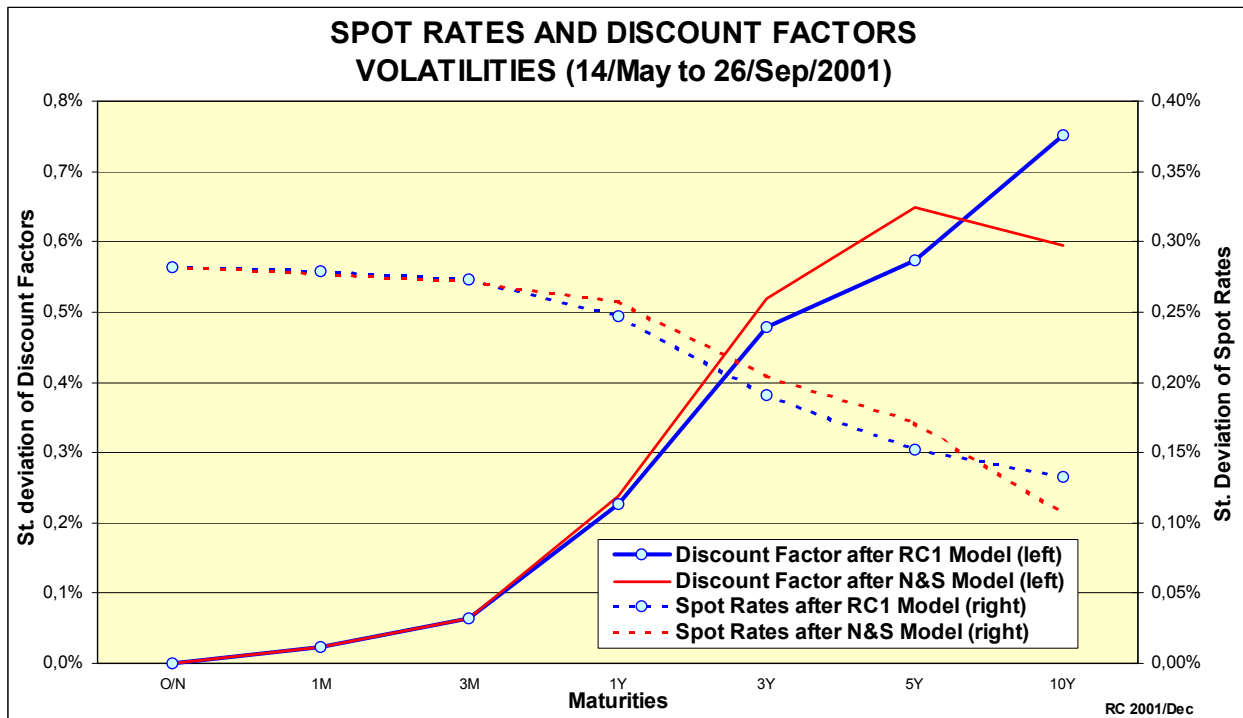
<b>DISCOUNT FACTORS</b>	<b>O/N</b>	<b>1M</b>	<b>3M</b>	<b>1Y</b>	<b>3Y</b>	<b>5Y</b>	<b>10Y</b>
14/May to 26/Sep/2001	<i>1</i>	<i>31</i>	<i>91</i>	<i>365</i>	<i>1 095</i>	<i>1 826</i>	<i>3 652</i>
<b>MODEL RC1</b>							
<b>Average factor</b>	0,9999	0,9963	0,9891	0,9563	0,8700	0,7860	0,5937
<b>Volatility</b>	0,0007%	0,0227%	0,0647%	0,2272%	0,4779%	0,5727%	0,7516%
<b>MODEL N&amp;S</b>							
<b>Average factor</b>	0,9999	0,9963	0,9895	0,9604	0,8792	0,7906	0,5893
<b>Volatility</b>	0,0007%	0,0225%	0,0645%	0,2389%	0,5192%	0,6489%	0,5946%

In spite of the oversimplification of RC1, the two models produced quite similar estimates for the 7 volatilities, either of spot rates or of discount factors. The only significant difference occurred for 10 years where N&S produced a lower volatility for the discount factor than RC1 - even below the 5 years level - a fact that is contradictory to a larger duration. In this respect, RC1 behaved better than N&S.

<sup>13</sup> Mind that, every business day, RiskMetrics supplies estimated volatilities of the discount factors for 14 maturities from one month to 30 years.

<sup>14</sup> RC model calculates the "noise" of a long-term spot rate as a kind of an average of the independent "perturbations" that affect each individual forward rate included in that long rate. As an average, that "noise" decreases with the number of components included in that computation, that is, with maturity  $T$ .

Fig 12



Mid range maturities - around 1 to 5 years - deserves a closer look. The fact that N&S model has a special term to improve the quality of adjustments in this range of maturities makes  $\beta_2$  determined by a sub-sample of the entire sample of cash-flows. That is, estimates of mid-term spot rates (or discount factors) under N&S suffer from an enlarged uncertainty if the number of debt issues used is not large (as is the domestic case).

On the contrary, the RC1 model does not suffer from this mid-range drawback and that might explain part of the larger volatilities estimated with the N&S model.

Combining that larger uncertainty of N&S estimates with the incapacity of RC1 to accommodate S-shaped functions, the conclusion to be taken is that volatilities (and correlations) estimated for those central range maturities - with any of the two models - are less certain than identical statistics for shorter (because O/N rate is a certain information) and longer (because of the larger number of cash flows) maturities.

This is relevant for our domestic market since the clear majority of debt securities “parked” in our domestic bank assets (beyond T-Bonds and T-Notes) have remaining maturities falling in a central range running from 1 to 5 years. That is, more uncertain volatilities (and correlations) of interest rates in this “middle” range of maturities bring an additional risk to bond and note portfolios in our banks than can be concluded from the formal figures produced by VaR models based on these statistic estimates.

## 9. Correlations

VaR of a portfolio of equities is much more dependent upon the mutual covariances among those securities than upon the volatilities of each individual component. The same applies to the impact of the different spot rates on the variance of a portfolio of debt instruments.

So, we complemented the above estimates of volatilities with the correlations between the same 6+1 constant maturity interest rates and between the correspondent discount factors.

### CORRELATIONS BETWEEN SPOT RATES. 14/May to 26/Sep/2001

#### MODEL RC1

O/N	1M	3M	1Y	3Y	5Y	10Y
1,0000	1,0000	0,9998	0,9962	0,9548	0,8353	0,1722
	1,0000	0,9999	0,9968	0,9567	0,8389	0,1779
		1,0000	0,9978	0,9606	0,8461	0,1896
			1,0000	0,9769	0,8790	0,2468
				1,0000	0,9601	0,4338
					1,0000	0,6616
						1,0000

#### MODEL N&S

O/N	1M	3M	1Y	3Y	5Y	10Y
1,0000	0,9962	0,9742	0,8858	0,8006	0,6498	0,3411
	1,0000	0,9902	0,9206	0,8106	0,6419	0,3369
		1,0000	0,9631	0,8196	0,6260	0,3322
			1,0000	0,8616	0,6531	0,4002
				1,0000	0,9421	0,7493
					1,0000	0,8899
						1,0000

The relevant results are:

- both models indicate that correlation decreases with the “distance” between the two considered horizons;
- in the particular sample analysed – 14/May to 26/Sep - all correlations were positive; this is partly explained by casual reasons, as the results from the different sub-samples show some negative correlations for some distant maturities (see Annex 4)
- nevertheless, the two models show different evolutions: correlation falls faster in the N&S model than in RC1 version, but this last model shows a steeper fall of the correlation statistic from 5 years to 10 years; this difference may be attributed to the better capacity of N&S to translate the reality of the mid-term rates
- and this, once again, alerts to the importance of model selection on the estimates of these correlations.

## CORRELATIONS BETWEEN DISCOUNT FACTORS. 14/May to 26/Sep/2001

## MODEL RC1

O/N	1M	3M	1Y	3Y	5Y	10Y
1,0000	1,0000	0,9998	0,9963	0,9558	0,8372	0,1699
	1,0000	0,9999	0,9969	0,9577	0,8408	0,1755
		1,0000	0,9978	0,9615	0,8478	0,1870
			1,0000	0,9774	0,8801	0,2434
				1,0000	0,9601	0,4287
					1,0000	0,6572
						1,0000

## MODEL N&amp;S

O/N	1M	3M	1Y	3Y	5Y	10Y
1,0000	0,9962	0,9744	0,8866	0,8017	0,6509	0,3394
	1,0000	0,9903	0,9212	0,8118	0,6432	0,3352
		1,0000	0,9633	0,8208	0,6276	0,3306
			1,0000	0,8625	0,6550	0,3985
				1,0000	0,9423	0,7465
					1,0000	0,8884
						1,0000

## 10. Instability of Volatility and Correlations

## 10.1. Volatilities

Although the sample used is not very large (only 97 days), it is worth dividing it into 5 non-overlapping sub-samples in order to gauge the degree of instability of the different volatilities.

VOLATILY (St Dev) Interest Rates	MODEL RC1						
	O/N	1M	3M	1Y	3Y	5Y	10Y
<b>SAMPLES</b>							
14/5 to 7/6	0,1018%	0,1009%	0,0992%	0,0920%	0,0769%	0,0686%	0,0776%
8/6 to 4/7	0,0540%	0,0534%	0,0523%	0,0481%	0,0427%	0,0460%	0,0732%
5/7 to 31/7	0,0663%	0,0658%	0,0650%	0,0634%	0,0685%	0,0740%	0,0750%
1/8 to 27/8	0,0275%	0,0272%	0,0266%	0,0249%	0,0272%	0,0370%	0,0711%
28/8 to 26/9 (w/ 11/Sep)	0,4159%	0,4096%	0,3970%	0,3422%	0,2154%	0,1145%	0,1063%
28/8 to 26/9 (w/o 11/Sep)	0,4233%	0,4168%	0,4040%	0,3481%	0,2184%	0,1151%	0,1088%
14/5 to 26/9	0,2818%	0,2788%	0,2727%	0,2468%	0,1904%	0,1522%	0,1329%

VOLATILY (St Dev) Interest Rates	MODEL N&S						
	O/N	1M	3M	1Y	3Y	5Y	10Y
<b>SAMPLES</b>							
14/5 to 7/6	0,1016%	0,0995%	0,1086%	0,1368%	0,0509%	0,0492%	0,0458%
8/6 to 4/7	0,0538%	0,0650%	0,1142%	0,1830%	0,0648%	0,0651%	0,0504%
5/7 to 31/7	0,0661%	0,0604%	0,0542%	0,0513%	0,0619%	0,0839%	0,0740%
1/8 to 27/8	0,0276%	0,0308%	0,0390%	0,0630%	0,0568%	0,0479%	0,0486%
28/8 to 26/9 (w/ 11/Sep)	0,4156%	0,4058%	0,3879%	0,3196%	0,1728%	0,0839%	0,0496%
28/8 to 26/9 (w/o 11/Sep)	0,4230%	0,4132%	0,3952%	0,3257%	0,1740%	0,0815%	0,0506%
14/5 to 26/9	0,2816%	0,2766%	0,2716%	0,2578%	0,2046%	0,1716%	0,1063%

Each of these sub-sets counts only 19 days, except the one for September which counts 21 days (or 20, if excluding Sep/11<sup>th</sup>).

The table above indicates the estimates obtained for the volatility estimates of interest rates for the five sub-periods analysed and for the entire sample. These results suggest that:

- short-term rate volatilities are much more sensible to stressful situations than long-term rates
- even excluding those less frequent events, short-term volatilities seem to be more unstable than long-term ones
- once more, the two models produced similar estimates of volatility; but similarities are larger for the larger sample than for each of the smaller sub-samples
- therefore, credible estimates require a minimum dimension of samples (and, consequently, the use of the most recent data).

For the discount factors, the equivalent tables of volatilities are summarised below where similar results can be observed.

VOLATILY (St Dev) Discount Factors	MODEL RC1						
	O/N	1M	3M	1Y	3Y	5Y	10Y
<b>SAMPLES</b>							
14/5 to 7/6	0,0003%	0,0081%	0,0233%	0,0836%	0,1900%	0,2549%	0,4350%
8/6 to 4/7	0,0001%	0,0043%	0,0123%	0,0439%	0,1060%	0,1716%	0,4112%
5/7 to 31/7	0,0002%	0,0053%	0,0154%	0,0594%	0,1796%	0,2900%	0,4143%
1/8 to 27/8	0,0001%	0,0022%	0,0063%	0,0227%	0,0677%	0,1388%	0,4068%
28/8 to 26/9 (w/ 11/Sep)	0,0011%	0,0334%	0,0944%	0,3163%	0,5447%	0,4357%	0,6074%
28/8 to 26/9 (w/o 11/Sep)	0,0011%	0,0340%	0,0960%	0,3217%	0,5524%	0,4381%	0,6215%
14/5 to 26/9	0,0007%	0,0227%	0,0647%	0,2272%	0,4779%	0,5727%	0,7516%

VOLATILY (St Dev) Discount Factors	MODEL N&S						
	O/N	1M	3M	1Y	3Y	5Y	10Y
<b>SAMPLES</b>							
14/5 to 7/6	0,0003%	0,0080%	0,0256%	0,1258%	0,1279%	0,1836%	0,2528%
8/6 to 4/7	0,0001%	0,0053%	0,0270%	0,1692%	0,1628%	0,2443%	0,2801%
5/7 to 31/7	0,0002%	0,0049%	0,0127%	0,0457%	0,1548%	0,3167%	0,4118%
1/8 to 27/8	0,0001%	0,0025%	0,0092%	0,0581%	0,1439%	0,1819%	0,2750%
28/8 to 26/9 (w/ 11/Sep)	0,0011%	0,0331%	0,0924%	0,2972%	0,4418%	0,3208%	0,2807%
28/8 to 26/9 (w/o 11/Sep)	0,0011%	0,0337%	0,0941%	0,3029%	0,4448%	0,3116%	0,2864%
14/5 to 26/9	0,0007%	0,0225%	0,0645%	0,2389%	0,5192%	0,6489%	0,5946%

## 10.2. Correlations

Annex 4 provides details for the behaviour of the correlations between interest rates and between discount factors.

The most striking aspect is the much larger instability of correlation figures in comparison to volatility values: even the sign of the correlations changed from one sub-sample to the next, particularly, for long-term maturities.

Also the two models produced larger differences (comparing to volatilities) between correlation estimates among the five sub-samples. One might here be faced with another component of what is commonly called Model Risk.

The impact of the events on the 11<sup>th</sup> of September is not clear. On one hand, September figures were different from previous months equivalents, but, on the other hand, excluding that day from the sub-sample does not seem to affect much the final results.

## 11. Conclusions

### a) The Yield Curve

- In spite of a common currency, there is room to speak of a domestic yield curve in Portugal somewhat different from the German one.
- Generally speaking, Portugal pays a premium above German interest rates, but that premium is unstable from one day to the next, and tends to increase with time to maturity of the loan (more visible if estimated with N&S).
- There is some Model Risk when estimating these spreads, that is the same market data produces significantly different spreads in relation to German rates, depending upon the model used to estimate the shape of the Portuguese risk-free yield curve.
- The Portuguese market does not supply much information to allow sound daily estimates of interest rates; interest rates, their volatilities and their correlations can only be estimated without much confidence.

### b) Usable Models

- The empirical evidence from this study suggests that two alternative models can be adopted for these daily estimates: the traditional Nigel & Siegel model and the simplest RC1 version of Costa's Model.
- Svensson's model would be a viable alternative if our market supplied a much larger set of prices (of debt instruments), in particular, in the low end of the spectrum of maturities (where the  $\beta_3$  term is specially adapted to).
- A decision between RC1 and N&S models should only occur after both are tested against market data for the short-term part of the yield curve.
- Anyway, one may already suspect that N&S will be selected in the future: closer similarity of this model to Svensson's (adopted by the German central bank to estimate their daily yield curve), and the much better capacity of N&S, in comparison to RC1, to accommodate double curvature yield shapes are two strong arguments in that direction.

### c) Volatility and Correlation

- Volatility of interest rates seems to decrease with time to maturity but the small size of our domestic Treasury market makes those volatility estimates very dependent upon the particular sample used and its size.
- In this respect the two models seem relatively similar as both lead to similar volatilities.
- Correlations are even more unstable and dependent upon the samples utilised. Also there is a much clear dependence upon the model selected to produce the estimates.
- Due to the low number of Treasury issues available and the low liquidity of all of them, we might, most likely, be forced to use longer samples of daily trades to improve the quality of

all these estimates. This means that VaR estimates may be more vulnerable to rapid changes in these parameters due to the inherent inertia brought by these longer samples.

d) Extreme Events

- A clear impact of stressful events upon our market (in the case, MEDIP) is the tremendous reduction in liquidity, both in terms of number of issues traded (2 issues, against the maximum of 9) and in terms of volumes transacted (€ 50 million, against an average of € 391 million in the sample).

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## ANNEX 2

### Improvement from the RC2 Version

Out of the entire sample from May to September 2001, five days were selected to compare the quality of adjustment of the RC2 version of Costa's model to the N&S alternative. The selection criterion was to have one day per month of the sample.

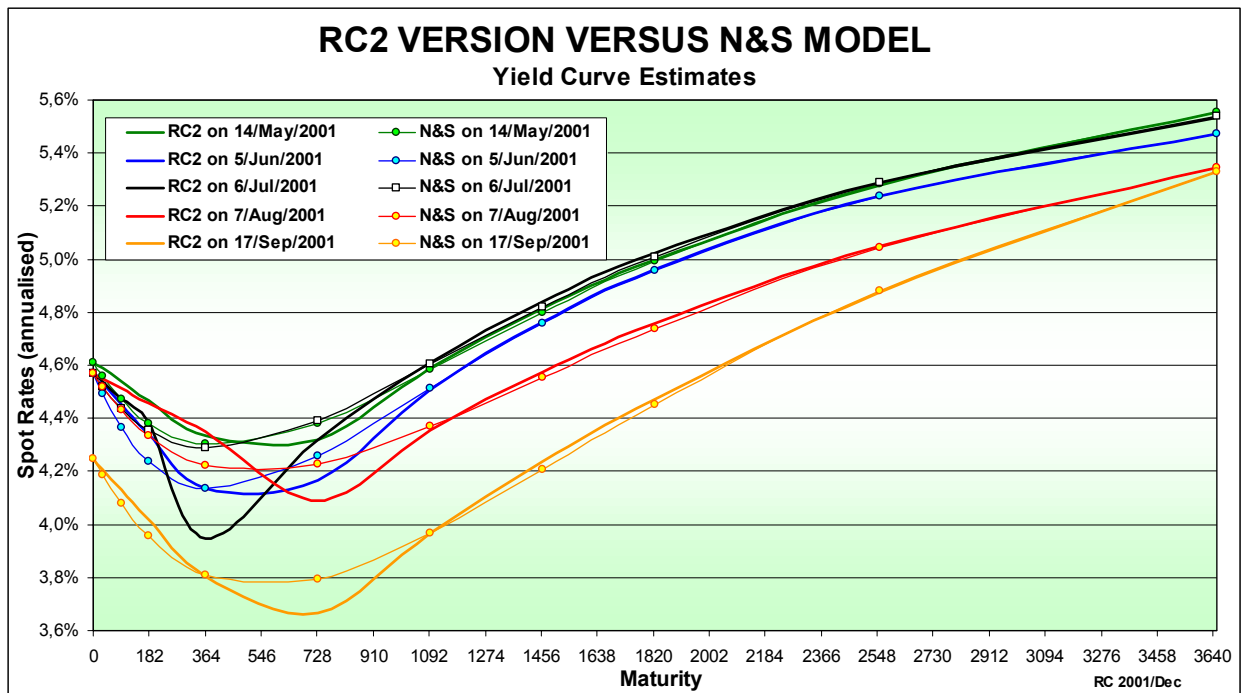
INTEREST RATE ESTIMATES													
Comparison Between RC2 and N&S Models on 5 Different Days Along the Sample													
	O/N	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y	Asymptotic	
Days	1	31	91	182	365	730	1095	1461	1826	2557	3652	Rate	
14-May-2001	RC2	4,6122%	4,5879%	4,5400%	4,4694%	4,3342%	4,3200%	4,5909%	4,8139%	4,9974%	5,2778%	5,5529%	6,3324%
	N&S	4,6103%	4,5587%	4,4730%	4,3814%	4,3022%	4,3817%	4,5822%	4,7980%	4,9909%	5,2852%	5,5524%	6,2238%
05-06-2001	RC2	4,5747%	4,5328%	4,4524%	4,3384%	4,1351%	4,1674%	4,5078%	4,7657%	4,9626%	5,2359%	5,4720%	6,0245%
	N&S	4,5719%	4,4934%	4,3661%	4,2358%	4,1355%	4,2602%	4,5126%	4,7561%	4,9563%	5,2363%	5,4708%	6,0341%
06-07-2001	RC2	4,5698%	4,5375%	4,4747%	4,3841%	3,9486%	4,3193%	4,6098%	4,8399%	5,0227%	5,2894%	5,5355%	6,1670%
	N&S	4,5680%	4,5184%	4,4371%	4,3530%	4,2891%	4,3918%	4,6028%	4,8197%	5,0083%	5,2891%	5,5382%	6,1555%
07-Aug-2001	RC2	4,5696%	4,5505%	4,5130%	4,4576%	4,3517%	4,0898%	4,3530%	4,5739%	4,7591%	5,0489%	5,3431%	6,2210%
	N&S	4,5679%	4,5180%	4,4323%	4,3334%	4,2236%	4,2271%	4,3700%	4,5546%	4,7370%	5,0428%	5,3471%	6,1662%
17-Sep-2001	RC2	4,2489%	4,2093%	4,1316%	4,0177%	3,8019%	3,6675%	3,9680%	4,2355%	4,4728%	4,8736%	5,3306%	7,2359%
	N&S	4,2468%	4,1858%	4,0798%	3,9552%	3,8104%	3,7964%	3,9678%	4,2051%	4,4497%	4,8787%	5,3280%	6,5986%

Note the slight differences between O/N rates due to the different ways the two models treat the shortest spot rate (n = 1 or T = 0).

The table above compares identical interest rate estimates along the entire spectrum of maturities, as well as the asymptotic rates. The graph below provides the same information in visual form, showing clearly that:

- there is almost a coincidence of results from the two models for maturities above (around) 3 years
- but there is a clear divergence of estimates between O/N and that 3 years borderline.

The larger number of parameters of RC2 in relation to N&S justifies this divergence as there is almost no price information from the domestic market in that mid range of maturities. FRN were not included in the sample. So, where N&S follows a "middle path" determined by O/N and long-term data, RC2 is almost "free" to wander between those two points.



### ANNEX 3

#### Improvement from Svensson's Model

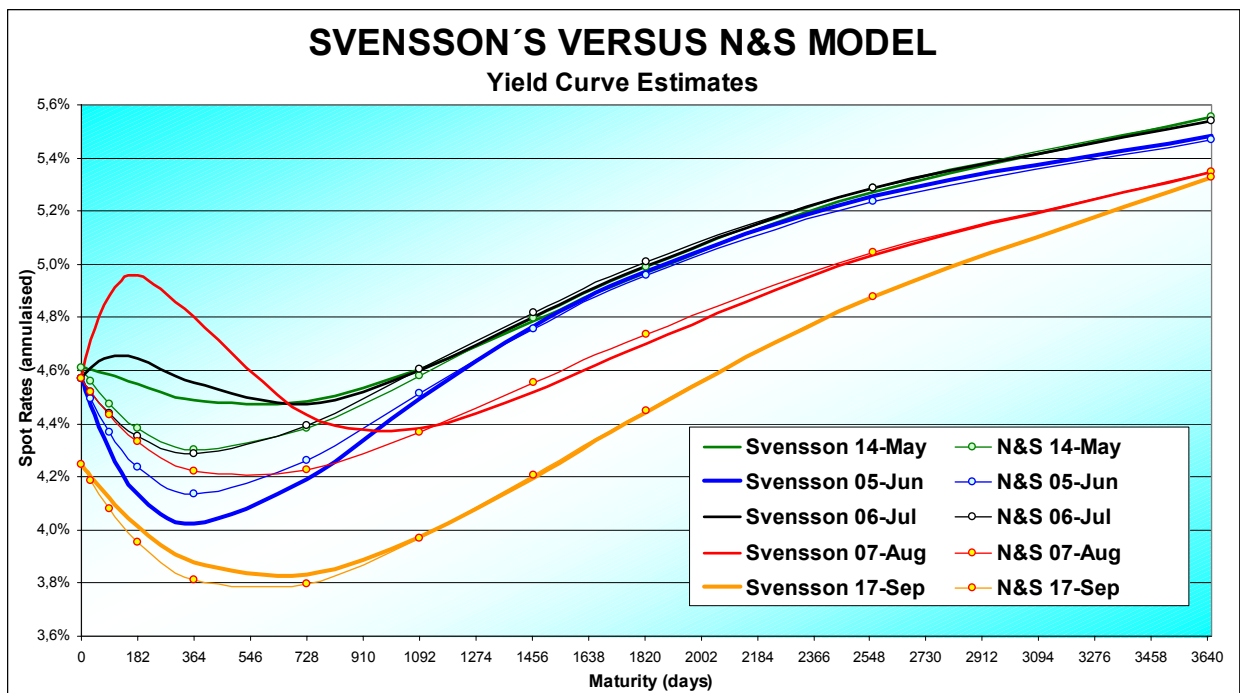
For the same five days used to test RC2 version, a comparison was made between N&S results and the output from Svensson's model.

INTEREST RATE ESTIMATES													
Comparison Between Svensson's and N&S Models for 5 Different Days Along the Sample													
Days		O/N	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y	Asymptotic
		1	31	91	182	365	730	1095	1461	1826	2557	3652	Rate
14-May-2001	Svensson	4,6120%	4,6054%	4,5866%	4,5517%	4,4899%	4,4824%	4,6071%	4,7874%	4,9701%	5,2710%	5,5538%	6,2655%
	N&S	4,6103%	4,5587%	4,4730%	4,3814%	4,3022%	4,3817%	4,5822%	4,7980%	4,9909%	5,2852%	5,5524%	6,2238%
05-Jun-2001	Svensson	4,5710%	4,4674%	4,3020%	4,1374%	4,0219%	4,1932%	4,4944%	4,7651%	4,9762%	5,2574%	5,4835%	6,0186%
	N&S	4,5719%	4,4934%	4,3661%	4,2358%	4,1355%	4,2602%	4,5126%	4,7561%	4,9563%	5,2363%	5,4708%	6,0341%
06-Jul-2001	Svensson	4,5714%	4,6111%	4,6501%	4,6448%	4,5540%	4,4760%	4,6008%	4,8004%	4,9938%	5,2867%	5,5385%	6,1437%
	N&S	4,5680%	4,5184%	4,4371%	4,3530%	4,2891%	4,3918%	4,6028%	4,8197%	5,0083%	5,2891%	5,5382%	6,1555%
07-Aug-2001	Svensson	4,5750%	4,7144%	4,8845%	4,9564%	4,8015%	4,4331%	4,3827%	4,5173%	4,7024%	5,0324%	5,3451%	6,1243%
	N&S	4,5679%	4,5180%	4,4323%	4,3334%	4,2236%	4,2271%	4,3700%	4,5546%	4,7370%	5,0428%	5,3471%	6,1662%
17-Sep-2001	Svensson	4,2473%	4,2019%	4,1196%	4,0157%	3,8785%	3,8321%	3,9736%	4,1980%	4,4409%	4,8766%	5,3297%	6,5645%
	N&S	4,2468%	4,1858%	4,0798%	3,9552%	3,8104%	3,7964%	3,9678%	4,2051%	4,4497%	4,8787%	5,3280%	6,5986%

Note the slight differences between O/N rates due to the different ways the two models treat the shortest spot rate (n = 1 or T = 0).

The graph below almost replicates the same pattern found in Annex 2 with RC2 versus N&S. And the reasons are very much the same: lack of mid-term information to determine a specific form to the Svensson curves in that mid range.

However, here the divergence seems to be even larger for the mid-term maturities. For 6/Jul and for 7/Aug the Svensson curves took rather "unexpected" forms which might be due to the minimization procedure where no certainty exists as to having reached an absolute minimum instead of a most probable local minimum.







## ANNEX 5

### Lack of Data for Mid-term Maturities

The graph below shows the set of cash flows that were available to the adjustment procedure in a "good" day - that is with 9 issues traded - somewhere in the middle of the full sample. It is clear that there were many more "small" cash-flows - correspondent to the coupons of the 9 debt issues - than "large" cash-flows - correspondent to the 9 principals.

The shortest maturity of a "large" cash flow was more than 1,5 years ahead. The impact on the adjustment procedure of any change of a spot interest rate below 1,5 years could only result from those "small" short-term cash-flows and, therefore, had to be much smaller than a similar effect from any change in the spot rate applicable to that "first" large cash-flow.

Therefore, without proper data, all models with specific terms included to adjust to this range of maturities will benefit from a great deal of freedom to let those terms fluctuate widely, as those fluctuations will have negligible impacts within the minimization procedure.

However, because Floating Rate Notes and Bonds (FRN) are frequently repriced once every semester, it means that a portfolio with such instruments would supply "large" cash flows maturing at 6 months or less horizons. The conclusion is that some FRN's should be included in the sample to supply such missing data and improve the quality of those mid range estimates.

