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Is the matching function Cobb-Douglas?

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Abstract

We study the theoretical properties that particular matching functions must satisfy to represent a frictional labor market within a general equilibrium random matching model. We analyze Cobb-Douglas, CES and other functional forms of the matching function. Our findings establish restrictions on the parameters of these matching functions to ensure that the equilibrium is interior. These restrictions provide both theoretical reasons to choose among several functional forms and model misspecification tests for empirical work.

Estudiem les propietats teòriques que una funció d'emparellament ha de satisfer per tal de representar un mercat laboral amb friccions dins d'un model d'equilibri general amb emparellament aleatori. Analitzem el cas Cobb-Douglas, CES i altres formes funcionals per a la funció d'emparellament. Els nostres resultats estableixen restriccions sobre els paràmetres d'aquests formes funcionals per assegurar que l'equilibri és interior. Aquestes restriccions aporten raons teòriques per escollir entre diverses formes funcionals i permeten dissenyar tests d'error d'especificació de model en els treballs empírics.

Keywords: matching function, random matching, interior equilibrium, Cobb-Douglas

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1 Introduction

The use of matching functions in economic models allows for the introduction of market frictions in a tractable fashion. This has proven important when studying unemployment and its relationship with other phenomena. The extensive body of literature on matching models shows its many applications (see Petrongolo and Pissarides, 2001 and Rogerson et al., 2005 for surveys).

While labor market frictions might account for the coexistence of unemployment and empty vacancies observed in reality, not all matching functions are consistent with this result. For some specifications, truncation is necessary to avoid job finding probabilities above unity, which would imply that labor market frictions are not active. In this paper, we study theoretical properties that particular matching functions must satisfy to be compatible with labor market frictions. We analyze the Cobb-Douglas functional form under different returns to scale, and the constant elasticity of substitution (CES) form to allow for different degrees of substitution between vacancies and unemployment. We also analyze the matching function that den Haan et al. (HRW, 2000) propose to avoid corner equilibria in their numerical exercises. Our results have implications for the empirical and theoretical literature on matching.

The paper is organized as follows. Next section specifies the basic framework of our analysis. Sections 3 to 5 show the analysis of a Cobb-Douglas, two CES and the HRW matching functions, respectively. We discuss the implications of our results in the concluding section.

2 Basic framework

Let us denote by M the number of matches in the economy, which is a function of the number of job seekers (U) and the number of open vacancies (V), $M = m(U, V)$. We assume, as it is common in this literature, that the matching function is increasing in its arguments and concave. In this set-up, the average probability of finding a job by a worker is the number of matches per unemployed, $p = m(U, V)/U$, and the average probability of filling in a vacancy by a firm is the number of matches per vacancy, $q = m(U, V)/V$.

We assume that the number of vacancies is endogenously determined. This requires a positive cost of opening a vacancy and free entry in the market of vacancies. The free-entry condition is the following:

$$qB = \kappa, \tag{1}$$

where q is the probability to fill in a vacancy, B are the expected future profits of a filled vacancy for the firm and $\kappa > 0$ is the cost of posting a vacancy. This condition applies to a large variety of random matching models, from those with any type of separation rate to those with match-specific productivity. These models will differ in their specification of B (Rogerson et al., 2005).

Definition 1 *An* ***interior equilibrium*** *occurs when the number of matches is bounded from above by the number of job seekers and the number of vacancies in the economy (* $M < \min\{U, V\}$ *).*

Definition 1 implies that the probability of finding a job (p) and the probability of filling in a vacancy (q) are both below unity. Notice from (1) that, in equilibrium, the probability of filling in a vacancy is lower than 1 if and only if $B > \kappa$. This result holds for any functional form of the matching function. However, the matching function will be important in determining whether the probability of finding a job is lower than 1. In the following sections, we study the conditions under which several matching functions lead to an interior equilibrium. Throughout the paper we assume that $B > \kappa$ so that, in equilibrium, the condition $q < 1$ is automatically satisfied. In the following sections it is only left to check whether $p < 1$ holds in equilibrium.

3 Cobb-Douglas matching function

In this section, we consider the Cobb-Douglas functional form, which allows us to investigate the parameter restrictions that lead to an interior equilibrium under different returns to scale on the matching technology.

Proposition 1 *In a general equilibrium random matching model with a Cobb-Douglas matching function,* $m = AU^{\alpha}V^{\beta}$*, where* $A > 0$ *is a scale parameter,* $\alpha \in (0, 1)$ *and* $\beta \in (0, 1)$*, an interior equilibrium exists if and only if* $B > \kappa$ *and:*

a) $\alpha + \beta = 1$ *and* $A < (\kappa/B)^{\beta}$.

b) $\alpha + \beta > 1$ *and* $U < A^{1/(1-\alpha-\beta)}(B/\kappa)^{\beta/(1-\alpha-\beta)}$.

c) $\alpha + \beta < 1$ *and* $U > A^{1/(1-\alpha-\beta)}(B/\kappa)^{\beta/(1-\alpha-\beta)}$.

Proof. Let us define the matching function to be Cobb-Douglas, $m = AU^{\alpha}V^{\beta}$, where $A > 0$, $\alpha \in (0, 1)$ and $\beta \in (0, 1)$. An interior equilibrium exists if $M < U$.[1]

[1] Recall that since we assume $B > \kappa$ the condition $M < V$ is satisfied in any equilibrium.

To prove that $M < U$, we substitute q in (1) for its expression, m for its assumed functional form and solve for V. Then we substitute V for this expression in the condition $M < U$, where M is defined by the Cobb-Douglas matching function. Re-arranging we obtain the following inequality:

$$U^{1-\alpha-\beta} > A\left(\frac{B}{\kappa}\right)^{\beta}. \tag{2}$$

When there are constant returns to scale (CRS), the left-hand side (LHS) of equation (2) is equal to 1 ($U^0 = 1$). Then the inequality is satisfied if and only if $A < (\kappa/B)^\beta$.

In the case of increasing returns to scale (IRS), inequality (2) determines an upper bound to unemployment ($U < A^{1/(1-\alpha-\beta)}(B/\kappa)^{\beta/(1-\alpha-\beta)}$) and consequently to the number of matches.

In the case of decreasing returns to scale (DRS), inequality (2) determines a lower bound to unemployment, $U > A^{1/(1-\alpha-\beta)}(B/\kappa)^{\beta/(1-\alpha-\beta)}$. ■

Proposition 1 states the necessary conditions for any Cobb-Douglas matching function to lead to an interior equilibrium. Results depend on the returns to scale. In the case of CRS, a Cobb-Douglas matching function with scale parameter (A) larger or equal to 1 cannot be a good representation of labor market frictions, since it would imply that any job seeker fills a vacancy with probability one.[2]

In the case of IRS, proposition 1 specifies an upper bound to the number of unemployed individuals, which at the same time, sets an upper bound to the number of matches created. This represents an arbitrary restriction on endogenous variables that can only be checked for a posteriori. When the number of job seekers is above this threshold, increasing returns ensure that everybody finds a job. Note that this result holds for any population size. The larger the population size (L), the lower the upper bound on unemployment rate (U/L) that ensures an interior equilibrium. This result raises questions on the adequacy of a Cobb-Douglas form for a matching function with IRS.

The case of DRS also implies an a priori restriction on the value of equilibrium unemployment. However, this is less problematic in the sense that it does not bind the number of matches. Moreover, there is little empirical evidence supporting this case.

[2]Recall that since $B > \kappa$ and $\beta > 0$, then $(\kappa/B)^\beta < 1$.

4 CES matching function

Proposition 2 *In a general equilibrium random matching model with a matching function represented by a CES function* $m = A(\eta U^{\sigma} + (1-\eta)V^{\sigma})^{1/\sigma}$*, where* $A > 0$ *is a scale parameter,* $\eta \in (0,1)$*,* $\sigma < 1$ *and* $1/(1-\sigma)$ *is the elasticity of substitution between unemployed people and vacancies, an interior equilibrium exists if and only if* $B > \kappa$ *and:*

a) $\sigma < 0$ *and* $(1-\eta)^{-1/\sigma}(\kappa/B) < A < ((\kappa/B)^{\sigma}\eta + 1 - \eta)^{-1/\sigma}(\kappa/B)$ *or,*

b) $\sigma > 0$ *and* $A < ((\kappa/B)^{\sigma}\eta + 1 - \eta)^{-1/\sigma}(\kappa/B)$*.*

Proof. Let us define the matching function to be CES such that

$$m = A(\eta U^{\sigma} + (1-\eta)V^{\sigma})^{\frac{1}{\sigma}},$$

where $A > 0$, $\eta \in (0,1)$ and $\sigma < 1$. To prove that $M < U$, we substitute q in (1) for its expression, m for its assumed functional form and derive the following equation:

$$\eta\left(\frac{U}{V}\right)^{\sigma} = \left(\frac{\kappa}{AB}\right)^{\sigma} - (1-\eta). \tag{3}$$

Notice that for the market tightness (V/U) to be positive, the RHS of the previous equation must be positive. Therefore, in any equilibrium $(\kappa/AB)^{\sigma} > (1-\eta)$. This translates into $A < (1-\eta)^{-1/\sigma}(\kappa/B)$ if $\sigma > 0$ and $A > (1-\eta)^{-1/\sigma}(\kappa/B)$ if $\sigma < 0$.

Using equation (3) we solve for V and use this expression to substitute V in the condition $U > M$, where M is defined by the CES matching function above. We obtain the following inequality:

$$\left(\left(\frac{\kappa}{AB}\right)^{\sigma} - (1-\eta)\right)^{1/\sigma} > \frac{\kappa}{B}\eta^{1/\sigma}. \tag{4}$$

This inequality is satisfied if and only if $A < ((\kappa/B)^{\sigma}\eta + 1 - \eta)^{-1/\sigma}(\kappa/B)$ for any $\sigma < 1$. Putting all the conditions together, we find the result in proposition 2. ■

Proposition 3 *In a general equilibrium random matching model with a matching function represented by the following CES function,* $m = (U^{\sigma} + V^{\sigma})^{1/\sigma}$*, where* $\sigma < 1$ *and* $1/(1-\sigma)$ *is the elasticity of substitution between unemployed people and vacancies, the equilibrium is interior if and only if* $B > \kappa$ *and* $\sigma < 0$*.*

Proof. Let us define the matching function to be CES such that

$$m = (U^\sigma + V^\sigma)^{\frac{1}{\sigma}}$$

where $\sigma < 1$. In order to prove that $M < U$, we substitute q in (1) for its expression, m for its assumed functional form and solve for V. Rearranging we obtain the following:

$$\left(\frac{\kappa}{B}\right)^\sigma - 1 = \left(\frac{U}{V}\right)^\sigma.$$

Notice that, given $B > \kappa$, the market tightness (V/U) is positive if and only if $\sigma < 0$. Therefore, we need $\sigma < 0$ in any equilibrium.

To continue the proof, we solve for V and substitute it in the condition $U > M$, where M is defined by the CES matching function above. We obtain the following inequality:

$$1 > \frac{\kappa}{B} \frac{1}{((\kappa/B)^\sigma - 1)^{1/\sigma}},$$

which, given $B > \kappa$, is satisfied for any $\sigma < 0$. ■

Propositions 2 and 3 report the necessary conditions for two different CES matching functions to lead to an interior equilibrium. The CES function specified in proposition 2 requires restrictions on the scale parameter, as in the Cobb-Douglas case with CRS. In contrast, the only requirement for the CES matching function studied in proposition 3 is to have an elasticity of substitution between vacancies and unemployed below unity.

5 The HRW matching function

Den Haan et al. (2000) study the propagation of aggregate shocks in a dynamic general equilibrium with labor frictions represented by the following matching function:

$$m(U, V) = \frac{UV}{(U^\rho + V^\rho)^{1/\rho}}.$$

They recognize that with a Cobb-Douglas matching function "*truncation is necessary to rule out matching probabilities greater than unity*" (den Haan et al. 2000, p. 485). In section 2, we showed under which conditions this is true. In this section, we analyze the conditions under which the matching function they propose leads to an interior equilibrium.

Proposition 4 *In a general equilibrium random matching model with a matching function represented by* $m(U,V) = UV/\left(U^\rho + V^\rho\right)^{1/\rho}$*, the equilibrium is interior if and only if* $B > \kappa$ *and* $\rho > 0$*.*

Proof. Let us define the matching function to be such that

$$m(U,V) = \frac{UV}{\left(U^\rho + V^\rho\right)^{1/\rho}}.$$

To prove that $M < U$, we substitute q in (1) for its expression, m for its assumed functional form and solve for V. Then we substitute V for this expression in the condition $M < U$, where M is defined by the HRW matching function. Re-arranging we obtain the following inequality:

$$B > \left(B^\rho - \kappa^\rho\right)^{1/\rho}.$$

Given $\kappa > 0$, this inequality is only satisfied when $\rho > 0$. ■

Proposition 4 states that the parameter in the HRW matching function must be positive in order to obtain an interior equilibrium. Although den Haan et al. (2000) do not explicitly state this condition in their paper, their calibration is consistent with our results.

6 Conclusions

We show theoretical properties that particular matching functions must satisfy in a general equilibrium random matching framework to allow for labor market frictions in equilibrium. Our results show that a Cobb-Douglas function with increasing returns to scale is not a good representation of the matching process, since it implies an upper bound to the number of matches independent of the population size. In contrast, we find that all the matching functions analyzed with constant returns to scale (Cobb-Douglas, CES, HRM) lead to an interior equilibrium as long as they satisfy some restrictions on their parameters.

These results have several implications. First, most studies that estimate aggregate matching functions use a Cobb-Douglas functional form (see Petrongolo and Pissarides 2001 for a review). Our results provide tests to check whether the assumed functional form is miss-specified in a model with endogenous vacancies. Similarly, calibration exercises using a matching function can avoid truncation problems by taking into account our results. Generally, our results provide restrictions on Cobb-Douglas and other functional forms that any aggregate matching function should satisfy.

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