Animal spirits in cash-in-advance economies*

Stefano Bosi [†]
EPEE, University of Evry
Frédéric Dufourt
BETA-theme, University Louis Pasteur
Francesco Magris
EPEE, University of Evry

1 Introduction

In this paper, we present a one-sector infinite horizon economy with capital accumulation, endogenous labor supply and liquidity constraint on consumption expenditures. We depart from similar model economies (see Stockman, 1981; Abel, 1985; Svensson, 1985; Lucas and Stokey, 1987; Coleman, 1987; Cooley and Hansen, 1989; Woodford, 1994) by assuming that only a share between zero and one of current consumption purchases must be paid by cash holdings accumulated from the previous periods, in the spirit of Grandmont and Younès (1972). We study the stability properties of this model, and assess the conditions under which it may become indeterminate. When it is the case, we analyze the cyclical properties of the model when animal spirits act as a driving force of the business cycle. Notice that our fractional cash-in-advance constraint can be viewed as the inverse of the velocity of money, according to the Cambridge Cash Balance Approach. We will refer to it in the sequel equivalently as to the amplitude of the financial constraint or to the inverse of the velocity of money.

By deriving analytically the conditions under which the model is indeterminate, we establish two results which we believe are quite surprising with regard to the literature on endogenous fluctuations. First, we show that a

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† Corresponding author. EPEE, Département d'économie, Université d'Evry - Val d'Essonne. 4, Bd F. Mitterrand. 91025 Evry Cedex, France. Tel: + 33 1 69 47 70 52. Fax: + 33 1 69 47 70 50.
E-mail: stefano.bosi@univ-evry.fr.

small departure from the traditional Ramsey-Cass-Koopmans model – the requirement that an *arbitrarily small* amount of consumption purchases must be paid by cash in the hands of the representative consumer – is sufficient to make the equilibrium indeterminate and to allow for the existence of sunspots fluctuations. Next, we show that the *occurrence* of such fluctuations becomes more and more likely as long as the velocity of money is continuously increased from one (as in Cooley and Hansen, 1989) to infinite (as in the standard Ramsey-Cass-Koopmans model). These two findings are rather surprising, since it is often believed that indeterminacy can occur only in the presence of *strong* market imperfections and that the occurrence of such fluctuations is made more and more likely as long as the degree of market imperfection is *increased*, and not *decreased*.

In particular, we demonstrate that when the velocity of money is close to one, indeterminacy arises only when consumption is not very substitutable across periods (i.e. strong enough income effects). However, as long as the velocity of money is increased, the range of admissible values for an indeterminate equilibrium becomes larger and larger. Above a certain threshold, it includes the logarithmic case (unitary elasticity of substitution) and above a higher one, it coincides with the whole domain of definition of the elasticity of intertemporal substitution in consumption. Finally, when the velocity of money reaches infinite, the model is not continuous since it looses one dimension: Actually, it collapses into the standard Ramsey-Cass-Koopmans model which, as is well known, exhibits the saddle-path stability. The interpretation of this feature consists in the fact that the nominal indeterminacy, prevailing when there are no financial constraints, becomes real as soon as an (arbitrarily small) liquidity constraint is introduced.

Bosi and Magris (2003) find analogous results in a deterministic model in which labor is supplied inelastically. Of course, these results will appear to be consistent, as a particular case, with those found in the more general framework considered in this paper. Although more analytically demanding, considering an elastic labor supply is important for two reasons. First, it influences strongly the stability properties of the model. We show in particular that the range of values for the elasticity of intertemporal substitution in consumption compatible with indeterminacy decreases when labor supply becomes more elastic, and that the threshold for the velocity of money above which indeterminacy occurs for whatever parameters configuration shrinks (and eventually tends to zero) as the labor supply elasticity tends to infinity. Second, the introduction of an elastic labor supply allows us to perform simulations experiments and to study the cyclical properties of the model in the spirit of the Real Business Cycle literature.

A possible criticism which can be addressed toward our framework is that, in a one-sector model, agents could avoid a financial constraint applying exclusively on consumption by buying only the investment good and consuming *ex-post* a part of it according to their optimal plans. But a one-sector model can be easily viewed as a special case of a two-sector model in which the capital intensities of the two production functions are the same and the relative price is constant and equal to unity. This is proved in, e.g., Becker and Tsyganov (2002; Proposition 2.2, page 194). It follows that one can interpret our aggregate one-sector model as a degenerate two-sector economy, the first sector producing only the investment good, the second being devoted to the production of the consumption good. This will then prevent agents to "transform" investment into consumption and avoid the cost represented by the nominal interest rate. In addition, our result could appear at first sight in contradiction with Abel's (1985) finding, which shows that a model with a full cash-in-advance constraint applying on total expenditure is always determinate. However, as shown in Bosi and Dufourt (2005), Abel's results can be reverted if one considers a *partial* liquidity constraint applying on consumption and investment purchases, in a similar fashion to the present paper. In addition to being much more tractable analytically, our choice to focus on consumption (in this general case with an elastic labor supply) comes from the observation that capital can be more easily used as a collateral asset, and is therefore more likely to be bought using credit.

The fact that indeterminacy and sunspots fluctuations may occur for an arbitrarily high velocity of money is important, since it tends to suggest that such phenomena, far from being "exotic" or simple theoretical curiosities, are by contrast quite pervasive. The recent literature on endogenous fluctuations had already considerably alleviated the conditions under which such phenomena may arise. However, even the most recent papers continue to rely upon rather discussed features such as mild increasing returns to scale [1], complementariness in production factors [2], unconventional specifications of the utility function [3], or other controversial calibrations of the fundamentals [4]. Our paper can thus be seen as a new step toward this increase in realism, since it shows that indeterminacy may occur even for an *arbitrarily small* departure from the conventional Ramsey-Cass-Koopmans model. In our model, the only imperfection – if we adopt this point of view rather than making reference to the velocity of money – is that a certain amount of cash must be accumulated to buy the consumption good. All the other elements are perfectly standard. This simple form of imperfection is sufficient to make the model indeterminate, and to be consistent with self-fulfilling revisions in expectations to act as an independent source of the business cycle. Our model also completes some results obtained in a recent paper by Carlstrom and Fuerst (2003) in which the cash-in-advance constraint is viewed as a limit case of a Money-in-Utility formulation and preferences are linear in

[1] See, e.g., Benhabib and Farmer (1996), Perli (1998), Wen (1998), Dos Santos Ferreira and Dufourt (2006).
[2] E.g., Grandmont *et al.* (1998).
[3] E.g., Farmer (1997).
[4] E.g., Bennett and Farmer (2000), Barinci and Chéron (2001). We provide a brief discussion of all these models in the second part of this paper. See also the very complete survey by Benhabib and Farmer (1999) for a more exhaustive presentation.

labor. For example, our assumption of an elastic labor supply entails the existence of a whole interval of low shares of consumption to be paid cash generating indeterminacy for whatever fundamentals specification, and not only for some, as it is the case in Carlstrom and Fuerst (2003). At the same time, focusing directly on a partial cash-in-advance constraint makes it easier to define the degree of market imperfection of the economy and study its impact on the emergence of indeterminacy.

Of course, one could still remain skeptical that such a sunspots driven model may be able to account for the main features of actual fluctuations. In the second step of the paper, we thus perform an empirical assessment of our model by verifying under which circumstances it can correctly describe the main features of the US business cycle. We also discuss the performance of this model in regard to some recent results established in the related literature.

We show in that respect that when it is driven by sunspots shocks only, our model suffers from the same difficulties as companion models which do not rely on large increasing returns to scale to generate indeterminacy. In particular, our model does not generate sufficient persistence in output movements, and it tends to predict counter-cyclical movements of investment. This implies that animal-spirit shocks alone are not sufficient in our model to explain the most salient features of the US business cycle. However, when animal spirits are instead given the interpretation of an *overreaction* to fundamental events such as technological innovations, like it is the case in the recent literature, our model becomes able to account for most empirical observations.

Overall, we find that the model performs as well as comparable models in the literature. This is true even though we only require a modest degree of market imperfection, and that we do not rely on any unconventional features regarding the production structure or the utility function.

The remainder of the paper is organized as follows. In Section 2 we present the model and derive the intertemporal equilibrium. Section 3 is devoted to the study of the local stability of the stationary solution while Section 4 provides an interpretation of the indeterminacy mechanism. In Section 5, we assess the cyclical properties of the model and Section 6 concludes the paper.

2 The model

2.1 The environment

We consider a discrete-time one-sector economy populated by identical long-lived agents and identical firms distributed uniformly along the unitary interval. The representative consumer is endowed with rational expectations and seek to maximize the expected stream of utility functions

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-1/\sigma} - 1}{1-1/\sigma} - \frac{l_t^{1+\chi}}{1+\chi} \right) \tag{1}$$

In (1), E denotes the rational expectation operator, $0<\beta<1$ the discount factor, c is consumption, $\sigma>0$ is the elasticity of intertemporal substitution in consumption, l is labor supply and $\chi>0$ is the inverse of the elasticity of labor supply. In each period $t\geq 0$, households must respect the dynamic budget constraint

$$p_t c_t + p_t[k_{t+1} - (1-\delta)k_t] + M_{t+1} = p_t r_t k_t + p_t w_t l_t + M_t + \tau_t \tag{2}$$

where p is the price of the good, k physical equipment, M money balances, r the real rental price of capital, w the real wage, $\delta \in [0, 1]$ the depreciation rate of capital and τ nominal lump-sum transfers issued by the government. We assume in addition that an amount $q \in (0, 1]$ of consumption purchases must be paid by "cash-in-hands" previously accumulated by the representative consumer. In other words agents, when maximizing, must also respect the liquidity constraint

$$q p_t c_t \leq M_t \tag{3}$$

Straightforward computations show that the FOC's of the household maximization problem write

$$c_t^{-\frac{1}{\sigma}} = p_t(\lambda_t + q v_t) \tag{4}$$

$$\lambda_t p_t = \beta E_t[\lambda_{t+1} p_{t+1} R_{t+1}] \tag{5}$$

$$\lambda_t^{\chi} = \lambda_t p_t w_t \tag{6}$$

$$\lambda_t = \beta E_t[\lambda_{t+1} + v_{t+1}] \tag{7}$$

where λ and v are non-negative Lagrange multipliers associated to, respectively, the budget and cash-in-advance constraints, and $R \equiv 1-\delta+r$ is the gross real interest rate on physical equipment. According to the arbitrage condition (7), the price of money at time t, λ_t, is equal to its expected value in the following period plus the expected value of the implicit dividends ξ_{t+1} it will pay off. At the same time, $p_t\lambda_t$ can be viewed as the marginal indirect utility of real income in period t: However, as (4) establishes, at the optimum, it does not equalize the marginal utility of consumption, since the individual cannot transform income into consumption unless part of the former is first used to purchase money balances. Condition (5) says that no intertemporal transfer of real income is still possible to increase total utility. Finally, equation (6) states that the marginal disutility of labor must equalize the induced increase in utility.

Constraint (3) binds when the nominal interest factor $i_t \equiv (1-\delta+r_t)\pi_t$ is greater than one, $\pi_t \equiv p_t/p_{t-1}$ being the inflation factor between periods

$t-1$ and t. Under this condition and by manipulating conditions (4)-(7), we obtain the stochastic Euler equation for the consumer

$$c_t^{-\frac{1}{\sigma}} = \beta E_t\left[c_{t+1}^{-\frac{1}{\sigma}} \frac{q\pi_t R_t + 1 - q}{q\pi_{t+1} + (1-q)R_{t+1}^{-1}}\right] \tag{8}$$

Again from the FOC's (4)-(7), we get the consumption-labor arbitrage condition

$$l_t^{\chi} = \frac{c_t^{-\frac{1}{\sigma}} w_t}{q\pi_t R_t + 1 - q} \tag{9}$$

Optimal plans for the single household must also satisfy the transversality condition

$$\lim_{t \to +\infty} E_t\left[\beta^t c_t^{-\frac{1}{\sigma}}(k_{t+1} + \pi_{t+1} m_{t+1})\right] = 0 \tag{10}$$

where $m_t \equiv M_t/p_t$ are the real balances held by the representative agent at the outset of period t.

The good is produced by mean of a Cobb-Douglas aggregate production function

$$zK^{\alpha}L^{1-\alpha}, \ \alpha \in (0,1) \tag{11}$$

where K and L are, respectively, aggregate capital and labor and z is a exogenous technological process. Profit maximization implies that in each period the real interest rate and the real wage equalize, respectively, the marginal productivity of capital and the marginal productivity of labor:

$$r_t = z_t \alpha K_t^{\alpha-1} L_t^{1-\alpha} \tag{12}$$

$$w_t = z_t(1-\alpha) K_t^{\alpha} L_t^{-\alpha} \tag{13}$$

Government follows a very simple monetary policy: It issues in each period lump-sum transfers of money balances at the constant rate $\mu - 1 > 0$, so that in period t the supply of money, M_t^s, satisfies $M_t^s = \mu^t M_0^s$, where M_0^s is the initial amount of nominal balances. Thus nominal transfers are given by $\tau_t = (\mu - 1)M_t^s$.

2.2 Equilibrium

Equilibrium in factors market is obtained by setting $K_t = k_t$, $L_t = l_t$ for every $t \geq 0$. When constraint (3) binds, money market equilibrium requires $\pi_{t+1} = \mu m_t/m_{t+1}$ for every $t \geq 0$. Finally, Walras law ensures good market clearing in each period. By manipulating the first order conditions and by substituting in them the equilibrium conditions, intertemporal competitive equilibria can be described by a dynamic system in terms of (k_t, l_t, c_t) explicitly formulated in the following definition.

Definition 1. *An interior rational expectations intertemporal equilibrium is a strictly positive sequence* $\{k_t, l_t, c_t\}_{t=0}^{\infty}$ *satisfying, for every* $t \geq 0$ *and for a given sequence of technological process* $\{z_t\}_{t=0}^{\infty}$, *equations*

$$(1-\delta)k_t + z_t k_t^\alpha l_t^{1-\alpha} - c_t = k_{t+1} \tag{14}$$

$$z_t k_t^\alpha l_t^{-(\alpha+\chi)} = E_t\left[\frac{z_{t+1}k_{t+1}^\alpha l_{t+1}^{-(\alpha+\chi)}}{\beta(1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha})}\right] \tag{15}$$

$$q\mu c_t = E_t\left[\frac{(1-\alpha)z_{t+1}k_{t+1}^\alpha l_{t+1}^{-(\alpha+\chi)}c_{t+1}^{-1/\sigma} - (1-q)}{1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}}c_{t+1}\right] \tag{16}$$

subject to the initial endowment of capital $k_0 > 0$ *and the transversality condition (10).*

2.3 Steady state

A deterministic steady state of the dynamic system defined by equations (14)-(16) is obtained by solving for (k, l, c) system

$$\begin{cases} c = k^\alpha l^{1-\alpha} - \delta k \\ \beta(1-\delta+\alpha k^{\alpha-1}l^{1-\alpha}) = 1 \\ (1-\alpha)k^\alpha l^{-(\alpha+\chi)}c^{-1/\sigma} = 1-q+q\mu\beta^{-1} \end{cases}$$

from which we get the unique solution

$$\bar{k} = \left[\frac{1}{1-\alpha}\left(1-q+q\frac{\mu}{\beta}\right)\left(\frac{\theta}{\alpha}\right)^{\frac{\alpha+\chi}{1-\alpha}}\left(\frac{\theta}{\alpha}-\delta\right)^{1/\sigma}\right]^{-\frac{1}{\chi+1/\sigma}} \tag{17}$$

$$\bar{l} = \left(\frac{\theta}{\alpha}\right)^{\frac{1}{1-\alpha}}\bar{k} \tag{18}$$

$$\bar{c} = \left(\frac{\theta}{\alpha}-\delta\right)\bar{k} \tag{19}$$

where $\theta \equiv \beta^{-1} - 1 + \delta$. It is easy to verify that constraint (3) binds at the steady state if and only if $\beta < \mu$. Under this condition, which we will assume to be satisfied throughout the paper, system (20) is consistent with intertemporal equilibria remaining in a small neighborhood of the steady state.[5] By direct inspection of expressions (17)-(19), one immediately sees that $\bar{k}$, $\bar{l}$ and $\bar{c}$ are decreasing in both the factor of money growth μ and the amplitude of the liquidity constraint q: Not surprisingly, both an inflationary monetary policy and a higher imperfection in the financial market tend to reduce economic activity.

[5] Notice that if the cash-in-advance constraint is not binding, we have two possibilities. According to the first, money dominates capital in terms of returns. But in such a case agents will shift investment from capital to money until the marginal productivity of capital will be at least equal to the return on the liquid asset. Second, if the returns of the two assets are the same, money has to be viewed as a bubble, which in not sustainable in the long term within an infinite-horizon model.

3 Stability analysis

In order to study the occurrence of (local) indeterminacy, and without any loss of generality, we study the stability of the deterministic dynamics around the steady state. Dropping the expectation operator from (14)-(16) and identifying x with (k, l, c), we can define $G_0(x_t)$ and $G_1(x_{t+1})$ as the left-hand side and right-hand side, respectively, of the deterministic counterpart of equations (14)-(16). Therefore a deterministic intertemporal equilibrium of the economy can be now written in the more compact form as a sequence $\{x_t\}_{t=0}^{\infty}$ satisfying

$$G_0(x_t) = G_1(x_{t+1}) \tag{20}$$

The steady state of the economy is said to be locally indeterminate if there exists a continuum of sequences $\{x_t\}_{t=0}^{\infty}$ satisfying system (20) for all $t \geq 0$, subject to the initial stock of capital, k_0, all of which converging to the steady state $\bar{x}$. Following the usual procedure, the study of (local) indeterminacy requires an exam of the linear operator

$$A = [DG_1(\bar{x})]^{-1} DG_0(\bar{x}) \tag{21}$$

which regulates the linear tangent motion to (20) near the steady state. [6] System (20) contains a unique pre-determined variable, the initial capital stock k_0. It follows that the stationary solution will be indeterminate if and only A possesses at least two eigenvalues lying inside the unit circle. In this case, there will exist different possible choices for placing the remaining initial conditions (l_0, c_0) in such a way that the equilibrium dynamics converges and, therefore, respects the transversality condition (10). In the opposite case, the steady state of system (20) is determinate. This means that there is only one pair (l_0, c_0) ensuring convergence of the system towards this steady-state. The characteristic polynomial of A is

$$P(\xi) \equiv \xi^3 - T\xi^2 + \Sigma\xi - D \tag{22}$$

where T, Σ, D are, respectively, the trace, the sum of the principal minors of order two and the determinant of A. Straightforward but tedious computations give the following expressions for T, Σ and D: [7]

$$T = \psi - \zeta(1 - \varphi\upsilon)$$
$$\Sigma = 1/\beta - \zeta(\varphi + \psi)$$
$$D = -\zeta/\beta$$

where $\upsilon \equiv \frac{1-q}{q}\frac{\beta}{\mu}$, $\zeta \equiv \frac{\sigma}{1-\sigma+\upsilon}$; $\varphi \equiv \chi\frac{\vartheta-\alpha}{\vartheta+\chi}\left(\frac{\theta}{\alpha} - \delta\right)$, $\psi \equiv \gamma + \frac{\vartheta-\alpha}{\vartheta+\chi}\left(\frac{\vartheta}{\alpha\beta} - 1\right)$, $\vartheta \equiv \alpha + (1-\alpha)\beta\theta$ and $\gamma \equiv (1+\beta)/\beta$. The next proposition characterizes the modulus of the eigenvalues of A. It is shown that when the amplitude

6 $DG_i(x)$, with $i = 0,1$, denotes the matrix of the derivatives of G_i with respect to x. A is an isomorphism provided $\sigma \neq 1 + \frac{\beta}{\mu}\frac{1-q}{q}$.

7 A document explaining how these expressions are derived can be obtained upon request from the authors.

of the liquidity constraint q is close to one, indeterminacy comes about only for low elasticities of intertemporal substitution in consumption σ. This result is analogous to those found in the cash-in-advance economies ($q = 1$) studied in Bloise *et al.* (2000) and Barinci and Chéron (2001). Conversely, as soon as q is continuously relaxed, the range of values for σ generating indeterminacy becomes larger and larger and includes eventually, for q low enough, the whole domain of definition of σ.

Proposition 2. *The linear operator A possesses a real eigenvalue $\xi_1 \in (0,1)$ and an eigenvalue ξ_2 with modulus greater than one. Moreover, defining*

$$\hat{q} \equiv \frac{1}{1+\frac{\mu}{\beta}\left(1+\frac{2}{\eta}\right)} \tag{23}$$

and

$$\hat{\sigma} \equiv \frac{1+\frac{1-q}{q}\frac{\beta}{\mu}}{2+\eta\left(1-\frac{1-q}{q}\frac{\beta}{\mu}\right)} \tag{24}$$

where

$$\eta \equiv \frac{\beta\left(\frac{\theta}{\alpha}-\delta\right)(1-\alpha)r\chi}{2\frac{1+\beta}{\beta}(\alpha+\chi)+(1-\alpha)\theta\left(3+\beta+\frac{1-\alpha}{\alpha}\beta\theta\right)} \tag{25}$$

we have:

(i) $0 < q < \hat{q}$. Then the third root ξ_3 belongs to $(-1, 0)$ and the steady state is locally indeterminate.

(ii) $\hat{q} \leq q \leq 1$. Then for $\sigma < \hat{\sigma}$ the third root ξ_3 belongs to $(-1, 0)$ and the steady state is locally indeterminate, meanwhile for $\sigma > \hat{\sigma}$, it has modulus greater than one and the steady state is locally determinate. In addition, when σ goes through $\hat{\sigma}$, the steady state undergoes a flip bifurcation.

Proof. The eigenvalues of A correspond to the roots of the characteristic polynomial (22). Performing simple computations we obtain

$P(0) = \zeta/\beta$

$P(-1) = (\zeta-1)(\gamma+\psi)-(\upsilon-1)\zeta\varphi$

$P(1) = (\zeta+1)(\gamma-\psi)-(\upsilon+1)\zeta\varphi$

Observe that $\lim_{\xi\to+\infty} P(\xi) = -\infty$, $\lim_{\xi\to-\infty} P(\xi) = -\infty$ and that the polynomial is a continuous function and its domain is connected. One can easily verify that $P(1)P(0) < 0$. This implies that there is always a real root, say ξ_1, in $(0,1)$. At the same time straightforward computations show that $P(-1)P(1) > 0$ either when $0 < q < \hat{q}$ or, for $\hat{q} < q \leq 1$, when $\sigma < \hat{\sigma}$. Therefore two main regimes are possible.

(i) $0 < q < \hat{q}$. Then $P(-1)P(1) > 0$ for all σ. It follows that there is a root belonging to $(-1, 0)$ and by continuity of the polynomial, a third real root with modulus greater than one.

(ii) $\hat{q} < q \leq 1$. Then $P(-1)P(1) > 0$ if and only if $\sigma < \hat{\sigma}$. In such a case there is a root belonging to $(-1, 0)$ and a third one with modulus greater than one. The steady state is thus locally indeterminate. When $\sigma > \hat{\sigma}$ observe in addition that $D = -P(0) > 1$ and that the determinant corresponds to the product of the eigenvalues. It follows that there exists at least one root with modulus greater than one. If such a root is real and positive, by the continuity of the polynomial, one has $\xi_i > 1$ for $i = 2, 3$. If it is real and negative, one has $\xi_i < -1$ for $i = 2, 3$. If such a root is complex, there are two conjugate eigenvalues ξ_2, ξ_3 such that $|\xi_2| = |\xi_3| > 1$. In either cases, the steady state is locally determinate.

Finally, since -1 is a root of the characteristic polynomial when $\sigma = \hat{\sigma}$, the steady state undergoes a flip bifurcation.

In view of Proposition 2, for $0 < q < \hat{q}$, indeterminacy comes about for any value for σ, meanwhile for $\hat{q} < q \leq 1$, indeterminacy prevails for all $\sigma < \hat{\sigma}$. It is easy to verify that $\hat{\sigma}$ is decreasing in $q \in [\hat{q}, 1]$. This implies that indeterminacy is more and more likely to emerge as soon as q is *decreased,* as it is illustrated in Fig. 1, before including eventually the whole domain of definition for σ $(0 < q < \hat{q})$. [8]

Remark 1. *It is worthwhile to notice that the model displays a discontinuity in $q = 0$: Indeed, in such a case, the dynamic system looses one dimension and boils down to the standard Ramsey-Cass-Koopmans model, which is always determinate. It is therefore this discontinuity – and the associated increase in the dimension of the system – which is at the source of the strong differences in the stability conditions of each model.*

Proposition 2 also shows that the change in the stability properties of the dynamic system always occurs through a flip bifurcation. Indeed, for a given $q > \hat{q}$, when $\sigma = \hat{\sigma}$, there is one characteristic root going through -1. Given the non-linearity of the system, this in turn implies (see, e.g., Grandmont, 1988) that when σ is set arbitrarily close to $\hat{\sigma}$, there will generically emerge, according to the direction of the bifurcation, a stable or unstable two-period cycle. [9]

Fig. 1 summarizes the results emphasized in proposition 1 by displaying the stability and instability regions for different q and σ. In particular, the shaded area corresponds to the indeterminacy region, and its boundary coincides with the values of $\hat{\sigma}$ undergoing a flip bifurcation. We have assumed the period to be short, of about one quarter. As a consequence, we focus on the following calibration for the structural parameters: $\beta = 0.99$,

[8] It is worthwhile to emphasize that such a critical share $\hat{q}$ below which the steady-state is *always* indeterminate also exists in the case of a constraint applying simultaneously on consumption and investment purchases. See Bosi and Dufourt (2005) for more details, as well as for figures comparing the values of $\hat{q}$ in the two cases.

[9] In an analogous way one may fix σ and then study the family of maps indexed by the amplitude of the liquidity constraint q. A flip bifurcation will possibly emerge in correspondence to those q such that $\sigma = \hat{\sigma}(q)$.

$\delta = 0.025$, $\mu = 1.01$, $\alpha = 0.3$. In addition, we assume utility to be linear in labor, namely $\chi = 0$.

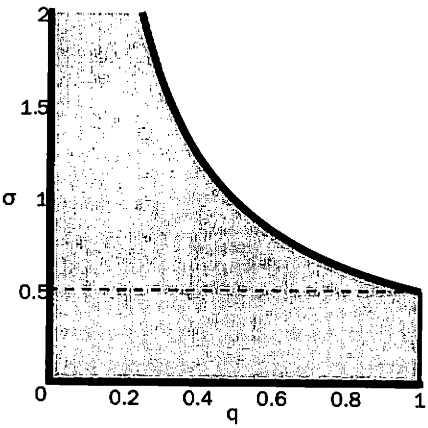

Fig. 1: *Indeterminacy region*

As it emerges in Fig. 1, for $q = 1$, indeterminacy requires $\sigma < 1/2$. As soon as q is relaxed, the interval of values for σ consistent with indeterminacy enlarges: It is, for example, $(0, 1)$ when $q = 0.48$ (including in this case the Cobb-Douglas production function), and $(0, 2)$ when $q = 0.22$.

4 The indeterminacy mechanism

Proposition 2 includes a result which may appear at first sight surprising. In our model, indeterminacy becomes more or more likely to emerge as long as the amplitude of the liquidity constraint – as measured by q, the share of consumption purchases to be paid cash – decreases continuously. In short, indeterminacy is more likely to occur for low degrees of credit market imperfections. Because such a result contrasts widely with most related models in the literature, and in particular in models of increasing returns to scale for which the required degree of IRS must be sufficiently high, it is important to understand the mechanisms which lead to an indeterminate equilibrium in our model.

In fact, a simple analysis of the arbitrage equations (4)-(7) and the induced equation (8) is sufficient to provide an intuitive interpretation of why indeterminacy occurs for low degrees of market imperfection. In order to illustrate this, let us rewrite the deterministic counterpart of (8) in the following way:

$$\left(\frac{1}{\pi_{t+1}}\right)^{1/\sigma} = \beta \frac{q i_t + 1 - q}{q \pi_{t+1} + \pi_{t+1} i_{t+1}^{-1} (1 - q)} \tag{26}$$

which incorporates the equilibrium condition in money market $c_{t+1}/c_t = \pi_{t+1}^{-1}$ and takes into account the definition of the nominal interest factor $i \equiv R\pi$. At the same time, it is useful to manipulate appropriately (4)-(7) and to assume utility to be linear in labor [10] (i.e. $\chi = 0$) in order to obtain the following consumption-labor static arbitrage condition:

$$w_t = [1 + q(i_t - 1)]c_t^{1/\sigma} \tag{27}$$

Now, let us suppose that the system is in period t at its steady state equilibrium and verify under which conditions it may be possible to construct an alternative equilibrium where agents anticipate, say, a fall in next period's price level, i.e. a decrease in the inflation rate π_{t+1}. Under this conjecture and the assumption that the liquidity constraint is still binding, both foregoing investment and tomorrow consumption will be driven up. But, in view of (27) evaluated in period $t+1$, this in turn will require a depreciation in tomorrow's nominal interest factor i_{t+1} (assuming that the real wage does not move significantly, an hypothesis which is not too strong in view of the fact that capital is a predetermined variable and that the labor supply elasticity is infinite).

Now suppose that the degree of the liquidity constraint is high, i.e. q is rather close to one: If the intertemporal elasticity of substitution σ is not very low (and, so, the expected contraction in π_{t+1} does not translate into an arbitrarily large variation in $\pi_{t+1}^{-1/\sigma}$), the re-establishment of (26) requires only a relatively mild increase in foregoing nominal interest factor i_t, namely an increase lower, in absolute terms, than the corresponding contraction of i_{t+1}, the weight of the former, q, being larger than that of the latter, $1-q$. But this implies a contracting backwards dynamics, and therefore an explosive forward dynamics violating the transversality condition.

If, conversely, q is rather small, the right-hand side of (26) is more sensitive to i_{t+1} than to i_t. Therefore, in order to restore (26), i_t must increase considerably more than how i_{t+1} has fallen. As a consequence, this will make the system to move back towards its steady state, although following an oscillatory path. The sensitivity of i_t and i_{t+1} with respect to q has to be interpreted in light of equation (27): For a given variation of consumption in period t, the associated variation of i_t depends negatively on q (the same holds in period $t+1$). Therefore, the lower q the higher the required adjustment for i.

Following the logic of this experiment, one immediately verifies that the higher σ, the lower the increase of the left-hand side of (26) induced by a given fall of π_{t+1}, and therefore, for any given q, the softer the required adjustment in its right-hand one. Thus, the mechanism above described will

[10] In the indeterminacy mechanism, the elasticity of labor supply plays a role relatively less important than the curvature of the utility function in consumption. Thus, the assumption of an infinite elastic labor supply – which we also make in our simulations – does not affect considerably the generality of our explanation.

lead to convergent dynamics only in correspondence to lower q's; This explains the negative trade-off occurring between σ and q in the indeterminacy conditions, as described in Fig. 1.

5 Model properties

In this final section, we assess the cyclical properties of the model and study the conditions under which it is able to account for the main features of the US business cycle.

Since the seminal papers by Benhabib and Farmer (1994), Farmer and Guo (1994) and Galí (1994), it is widely recognized that *some* business cycle models driven by sunspots only may account for most empirical stylized facts in a way which is at least as satisfactory as standard *RBC* models. However, the *conditions* under which these successful models may generate an indeterminate equilibrium have been criticized for being implausible and at odds with the data. For example, in Benhabib and Farmer (1994), externalities and increasing returns to scale must be high enough to imply that the labor demand curve is upward sloping, and slopes even steeper than the labor supply curve. The required degree of increasing returns to scale is around 60%, which is considerably higher than what is suggested by most empirical studies, which find - if any - much smaller markups and increasing returns to scale of the order of 5% (see Basu and Fernald, 1997, and Burnside, Eichenbaum and Rebelo, 1995). [11]

For these reasons, a more recent generation of papers have been devoted to show that indeterminacy could be obtained under much weaker restrictions regarding markups and increasing returns to scale. In Benhabib and Farmer (1996) and Perli (1998), two-sector versions of the original Benhabib and Farmer's 1994 model are shown to become indeterminate for increasing returns to scale around 7% and 16%, respectively. This is closer to the empirical estimates reported in Basu and Fernald (1997). Similarly, Wen (1998) shows that adding a variable capital utilization rate to a standard one-sector model may make indeterminacy occurring for a degree of increasing returns to scale of the order of 10%.

These models have made a crucial step toward more realism by proving that the most counter-factual assumptions required to obtain indeterminacy could be considerably alleviated as long as more sophisticated versions of the standard one-sector models were considered. But it is fair to recognize

[11] In Galí (1994), returns to scale are constant, but markup margins above 100% are assumed to generate an indeterminate equilibrium. No empirical studies tend to confirm such high levels for the markups, most empirical estimates lying instead in the range 10-40% (see, e.g., Morrison, 1988, Rotemberg and Woodford, 1991).

that this gain toward more realism has raised some new difficulties, however. [12] For example, simulations experiments show that if increasing returns to scale are set at their minimal value, multi-sector models are generally unsuccessful to match several moments of the US data, especially when business cycles are driven by sunspot shocks only. More precisely, several counterfactual features tend to emerge in the context of indeterminate models with low increasing returns to scale: (i) the tendency of these models to generate counter-cyclical movements of consumption or investment, (ii) the tendency to overestimate or underestimate the volatility of consumption or investment, and (iii) the lack of sufficient persistency in output movements, as measured by the first-order auto-correlation coefficient in the production series. The main causes for these failures are well documented in Wen's mentioned paper and in the recent survey by Benhabib and Farmer (1999), so we refer the interested reader to these papers for a more detailed discussion.

Two complementary solutions have been advocated to overcome these difficulties. The first one is to finally increase, when performing simulations, the degree of returns to scale to a value which is above the minimal value that is required to obtain indeterminacy. Because increasing returns to scale act as a major propagation mechanism and allow for a more pro-cyclical pattern of consumption's movements, this may prove sufficient in itself to make the model more consistent with the data. The disadvantage of this practice is that the new degree of IRS is generally inconsistent with the empirical evidence reported in Basu and Fernald (1997) and Burnside *et al.* (1995). [13] As a consequence, a second solution which has been introduced in the literature is to consider that animal spirits, rather that being purely exogenous shifts in expectations disconnected from any real event, are in fact *overreaction* to technological innovations. Models of this type have been generally found to be successful in matching the data as long as the degree of increasing returns to scale is in the range 10-20% (see e.g. Benhabib and Farmer, 1996, Perli, 1998, and Weder, 2000).

In this section, we wish to test the performance of our model in accounting for the main features of the US business cycle, allowing for both possibilities regarding the role of animal spirits. In order to do so, we first assume that technology follows the simple first order process

$$\ln z_t = \rho \ln z_{t-1} + \gamma \varepsilon_{z,t} \tag{28}$$

where $\rho \in [0,1]$, $\varepsilon_{z,t}$ is an *i.i.d.* technological innovation with zero mean and variance σ_z^2, and γ is a parameter taking values 0 or 1. Under the conditions generating indeterminacy, animal spirits shocks η_t - as measured by unforecastable increases in the consumption level, $\eta_t \equiv \{C_t - E_{t-1} C_t\}$ - act

[12] For a detailed discussion, see the survey by Benhabib and Farmer (1999) and Schmitt-Grohé (2000).

[13] In Benhabib and Farmer (1996), for example, 20% of increasing returns to scale are required for the model with sunspots only to generate empirically realistic business cycles.

as a second source of disturbance. We assume that these shocks evolve according to

$$\eta_t = \gamma \varepsilon_{z,t} + (1-\gamma)\varepsilon_{x,t} \quad (29)$$

where $\varepsilon_{x,t}$ is an *i.i.d.* shock with zero mean and variance σ_x^2. The advantage of equations (28)-(29) is that they can handle as special cases the two different interpretations of animal spirits that we emphasized earlier. In particular, for $\gamma = 0$, animal spirits are uncorrelated to any real events and act in this case as the unique source of disturbance. By contrast, for $\gamma = 1$, animal spirits are perfectly correlated to technological innovations and act in this case as an amplification mechanism of these shocks.

We show that, when driven by sunspots shocks only, our model suffers from the same difficulties as other models which do not rely on large increasing returns to scale or large markups to generate indeterminacy. Hence, animal spirits alone are not sufficient in our model to explain the most salient features of the US business cycle. However, when coupled with technological shocks, our model performs at a similar level as these formerly mentioned papers. This is true even though we do not rely on increasing returns to scale or on other non-standard features regarding the production structure or the utility function. In our model, the only source of imperfection is the requirement that some cash holdings must be accumulated from the preceding period to pay a fraction of total consumption purchases. All other elements are perfectly standard : Preferences are separable, technology is Cobb-Douglas, and the intertemporal elasticity of substitution is unitary. [14] Only the presence of this arbitrarily small degree of market imperfection, and the assumption that technology shocks and animal spirit innovations are positively correlated, are required for our model to generate appropriate statistical properties, as we illustrate now.

5.1 Calibration

We first briefly discuss the calibration of our main parameters (see Table 1). Since we want to remain as close as possible to comparable business cycle models with indeterminacy, we follow Benhabib and Farmer (1996) by assuming a share of capital in total income of $\alpha = 0.3$, a quarterly depreciation rate of capital of $\delta = 0.025$ and a discount factor of $\beta = 0.99$. Following Hansen (1985) and Rogerson (1988), we also assume an infinitely elastic labor supply with a unitary coefficient and set $\chi = 0$. Finally, we assume that the growth rate of money is $\mu = 1.01$, based on the $M0$ series for the US economy.

[14] As shown in Barinci and Chéron (2001), a strong complementarity in the intertemporal baskets of consumption may be a cause of indeterminacy.

In our simulations, we proceeded with two experimentations, $\gamma = 0$ and $\gamma = 1$. When technological shocks are present, we set an auto-correlation coefficient in the technological process of $\rho = 0.95$ and assume a standard deviation of technological innovation of $\sigma_z = 0.007$.

β	α	δ	χ	σ	q	p	σ_z
0.99	0.3	0.025	0	1	0.33	0.95	0.007

Table 1: *Structural parameters*

Two important parameters remain to be calibrated at this stage. The first one is σ, the value of the intertemporal elasticity of substitution. This value is highly controversial, since it influences directly some important properties of the models, and since the existing empirical evidence does not allow a clear statement about it. Some papers, like Bennett and Farmer (2000), assume a rather strong intertemporal substitutability ($\sigma > 3$) in order to get an indeterminate equilibrium. Others papers, like Barinci and Chéron (2001), assume instead a strong intertemporal complementariness ($\sigma < 0.5$) to get indeterminacy. By contrast to these papers, indeterminacy may occur in our model for a large range of values for σ including the complementariness and substitutability cases (see Fig. 1 above). We thus follow the most conventional *RBC* practice by simply setting $\sigma = 1$, i.e. we assume that the utility function is logarithmic in consumption.

Finally, the last parameter that remains to be calibrated is q, the overall degree of market imperfection we are willing to introduce in our model. Note that a simple interpretation of q is that of the (inverse) long run velocity of money, $q = 1/v$. Using the monetary base as their definition of money, Carlstrom and Fuerst (2003) estimate an average velocity of money (at a quarterly frequency) for the US economy which is around $v = 3$. We thus follow Carlstrom and Fuerst (2003) and set correspondingly $q = 1/3$.

5.2 Simulation results

For each version of the model, we simulated 100 series of 150 innovations each, and we used them to derive from our dynamic system below a set of series for the main macroeconomic variables. The model-implied moments reported in Table 2 are then the average moments for all these series, after each series was filtered through the HP filter. These theoretical moments are then compared to their empirical counterparts in the US economy, which are taken from the survey of RBC models by Cooley and Prescott (1995).

We first discuss the results for the version of the model which introduces animal spirit shocks only ($\gamma = 0$). Results from this experiment can be seen in the second column of Table 2. As we stressed earlier, our model

a. Relative standard deviations std(x)/std(output)					
	Data	autonomous sunspots ($\gamma = 0$)	correlated sunspots ($\gamma = 1$)	RBC	BF (correlated sunspots)
Y	1	1	1	1	1
C	0.74	13.79	0.74	0.24	0.74
I	4.79	44.64	3.73	4.41	3.45
H	0.98	1.26	0.78	0.57	0.89
Q	0.42	0.49	0.29	0.45	0.74
b. Autocorrelation coefficient on output: AR(1)					
	Data	$\gamma = 0$	$\gamma = 1$	RBC	BF ($\gamma = 1$)
	0.85	-0.44	0.67	0.70	0.66
c. Contemporaneous correlation with output					
	Data	$\gamma = 0$	$\gamma = 1$	RBC	BF $\gamma = 1$
Y	1	1	1	1	1
C	0.83	0.98	0.61	0.84	0.51
I	0.91	-0.98	0.81	0.99	0.83
H	0.92	0.93	0.98	0.98	0.70
Q	0.34	-0.35	0.86	0.98	0.50

Table 2: *Cyclical properties*

suffers from the same difficulties as former models which do not rely on large increasing returns to scale to generate indeterminacy. In particular, our model strongly overestimates the relative volatilities of consumption and investment with respect to output, and predicts counter-cyclical movements of investment. Furthermore, the auto-correlation coefficient on output is negative in the model, when it is strongly positive in the US data. This conforms to the Benhabib and Farmer's findings that most endogenous persistence in traditional sunspots models comes from the assumption of large increasing returns to scale.

We now turn to the simulation results when we let technological and animal spirit shocks coexist, and that we assume a perfect correlation between them ($\gamma = 1$). In this case, animal spirits act as an important amplification mechanism of technological innovation. Results from this experiment are reported in the third column of table 2. From this table, we see that the match with actual US fluctuations is pretty good for all the dimensions that have been considered.

First, the model now predicts the correct pattern of fluctuations for the relative volatilities between all variables. Consumption is about three-quarter as volatile as output, while investment is about four times as volatile as output. Productivity also remains significantly less volatile than output, which is consistent with what is found in the data. Finally, hours worked appears slightly less volatile in the model than they are in the US economy. It remains that overall, these numbers are in a much closer accordance with the data than what could be inferred from the version of the model in which animal spirits were the only source of disturbance.

The same kind of results holds for the contemporaneous correlation between all variables. As can be seen from Table 2, consumption, investment and hours worked are now strongly pro-cyclical, and productivity is no longer counter-cyclical: The contemporaneous correlation between productivity and output is even slightly higher now in the model than it is the data.

Finally, Table 2 reports the first-order auto-correlation coefficient on the production series. The model-implied coefficient of 0.67 compares to a value of 0.85 in the data. Thus, there remains a slight deficit of persistence in our theoretical model with liquidity constraints. However, the performance of the model remains rather good, especially if one reminds that returns to scale are assumed to be constant in our simulation experiments. The rather high first-order auto-correlation coefficient reported in Table 2 is thus not obtained at the price of a counter-factually strong degree of increasing returns to scale.

Overall, our model does therefore pretty well in accounting for the main features of actual fluctuations. The remainder of Table 2 displays for comparison purpose the corresponding theoretical moments which are implied by the standard RBC model of Cooley and Prescott (1995), and the multi-sector model of Benhabib and Farmer (1996). As is apparent, our model performs at a similar level as these former models, even though we do not rely on increasing returns to scale or on non-standard features concerning the production structure or the utility function.

6 Conclusion

In this paper we have presented a one-sector productive economy with partial cash-in-advance constraint on consumption purchases and studied the occurrence of indeterminacy and endogenous fluctuations as well as the cyclical properties of the model. We have shown that the relaxation of the liquidity constraint makes indeterminacy more and more likely to occur. For an amplitude of the liquidity constraint sufficiently low, the unique steady state is even bound to be indeterminate for whatever calibration of fundamentals.

These findings seem to suggest that indeterminacy and sunspot fluctuations – very far from being some kind of "exotic" or "pathological" feature in macroeconomic models – are very pervasive. The paper shows that it is sufficient to perturb slightly the canonical Real Business Cycle model in order to obtain multiple, deterministic and stochastic (expectations driven) equilibria. When performing simulations, we have shown that this model is able to correctly account for the main features of US economic fluctuations, as long as animal spirits and technological innovations coexist and are correlated. In this aspect, the model performs as well as other indeterminate models without relying upon strongly increasing returns (at odds with empirical estimates) or any other specific – if not even unconventional – calibration of fundamentals or specification of utility and production functions.

Références

Abel A. B. (1985). Dynamic Behavior of Capital Accumulation in a Cash-in-Advance Model. *Journal of Monetary Economics* **16**, 55-71.

Barinci, J.-P., and A. Chéron (2001). Real Business Cycles and the Animal Spirits Hypothesis in a Cash-in-Advance Economy. Document de Recherche EPEE 01-13, University of Evry.

Basu, S., and J. G. Fernald (1997). Returns to Scale in U.S. Production: Estimates and Implications. *Journal of Political Economy* **105**, 249-83.

Becker R. A., and E. N. Tsyganov (2002). Ramsey Equilibrium in a Two-Sector Model with Heterogeneous Households. *Journal of Economic Theory* **105**, 188-225.

Benhabib, J., and R. E. A. Farmer (1994). Indeterminacy and Increasing Returns. *Journal of Economic Theory* **63**, 19-41.

Benhabib, J., and R. E. A. Farmer (1996). Indeterminacy and Sector Specific Externalities. *Journal of Monetary Economics* **37**, 397-419.

Benhabib, J., and R. E. A. Farmer (1999). Indeterminacy and Sunspots in Macroeconomics. Handbook of Macroeconomics. Taylor and Woodford Eds., Elsevier.

Bennet, R. L., and R. E. A. Farmer (2000). Indeterminacy with Non-Separable Utility. *Journal of Economic Theory* **93**, 118-43.

Bloise G., S. Bosi, and F. Magris (2000). Indeterminacy and Cycles in a Cash-in-Advance Economy with Production. *Rivista Internazionale di Scienze Sociali* **CVIII 3**.

Bosi S., and F. Dufourt (2005). Indeterminacy with Constant Money Growth Rules and Income-Based Liquidity Constraints. Document de recherche EPEE 05-22, Université d'Evry.

Bosi S., and F. Magris (2003). Indeterminacy and Endogenous Fluctuations with Arbitrarily Small Liquidity Constraint. *Research in Economics* **57**, 39-51.

Burnside, C., Eichenbaum, M., and S. T. Rebelo (1995). Capacity Utilization and Returns to Scale. In: Bernanke, S., and J. J. Rotemberg Eds., NBER Macroeconomics Annual 10, 67-110.

Carlstrom, C. T., and T. S. Fuerst (2003). Money Growth Rules and Price Level Determinacy. *Review of Economic Dynamics* **6**, 263-75.

Coleman W. J. II (1987). Money, Interest, and Capital. Doctoral Dissertation, University of Chicago.

Cooley, T. F., and G. D. Hansen (1989). The Inflation Tax in a Real Business Cycle Model. *American Economic Review* **79**, 492-511.

Cooley, T. F,. and E. C. Prescott (1995). Economic Growth and Business Cycles, In: Cooley ed., Frontiers of Business Cycles Research, Princeton University Press, chapter 1.

Dos Santos Ferreira, R., and F. Dufourt (2006). The Influence of Animal Spirits over Free Entry in the Business Cycle. *Journal of Monetary Economics* **53**, 311-328.

Farmer, R. E. A. (1997). Money in a Real Business Cycle Model. *Journal of Money Credit and Banking* **29**, 568-611.

Farmer, R. E. A., and J. T. Guo (1994). Real Business Cycles and the Animal Spirit Hypothesis. *Journal of Economic Theory* **63**, 42-73.

Galí, J. (1994). Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand. *Journal of Economic Theory* **63**, 73-96.

Grandmont, J-M., (1988). Nonlinear Difference Equations, Bifurcations and Chaos. CEPREMAP Working Paper, Paris.

Grandmont J.-M., Pintus, P., and R. de Vilder (1998). Capital-Labor Substitution and Competitive Nonlinear Endogenous Business Cycles. *Journal of Economic Theory* **80**, 14-59.

Grandmont, J.-M., and Y. Younès (1972). On the Role of Money and the Existence of a Monetary Equilibrium. *Review of Economic Studies* **39**, 355-72.

Hansen, G., (1985). Indivisible Labor and the Business Cycle. *Journal of Monetary Economics* **16**, 309-27.

King, R. G., Rebelo, S., 1999. Resuscitating Real Business Cycles. In: Taylor, J.B., and M. Woodford Eds., Handbook of Macroeconomics; North-Holland, Amsterdam; Vol. 1.

Lucas, R. E., and N. Stokey (1987). Money and Interest in a Cash-in-Advance Economy. *Econometrica* **55**, 491-513.

Morrison, C. J., (1988). Markups in U.S. and Japanese Manufacturing: A Short Run Econometric Analysis. National Bureau of Economic Research Working Paper: 2799.

Perli, R., (1998). Indeterminacy, Home Production and the Business Cycle: A Calibrated Analysis. *Journal of Monetary Economics* **41**, 105-25.

Rogerson, R., 1988. Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics* **21**, 3-16.

Rotemberg, J., and M. Woodford (1991). Markups and the Business Cycle. NBER macroeconomics annual, 63-129.

Schmitt-Grohé, S. (2000). Endogenous Business Cycles and the Dynamics of Output, Hours, and Consumption. *American Economic Review* **90**, 1136-1159.

Stockman, A. C., (1981). Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy. *Journal of Monetary Economics* **8**, 387-93.

Svensson, L. E. O., (1985). Money and Asset Prices in a Cash-in-Advance Economy. *Journal of Political Economy* **93**, 919-44.

Weder, M., (2000). Animal Spirits, Technology Shocks and the Business Cycle. *Journal of Economic Dynamics and Control* **24**, 273-95.

Wen, Y., (1998). Capacity Utilization Under Increasing Returns to Scale. *Journal of Economic Theory* **81**, 7-36.

Woodford, M., (1994). Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. *Economic Theory* **4**, 345-80.