

Consumption Growth and the Real Interest Rate following a Monetary Policy Shock: Is the Habit Persistence Assumption Relevant? *

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Introduction

The SVAR literature that has studied the short-run effects of a monetary shock reports, among other facts, that following a contractionary monetary policy, (i) there is a persistent decline in real GDP; and (ii) the nominal and the real interest rates rise persistently. This is the so-called liquidity effect ¹ that a large strand of the theoretical literature has attempted to account for. Lucas (1990) and Christiano, Eichenbaum, and Evans (1996) proposed a limited participation model in which households decide the allocation of their money holdings between cash and deposit prior to observing the monetary shock. What is key in this model is that the intertemporal behavior of the agent is constrained by the timing of the model. The model then generates a liquidity effect which however lasts for one single period. Therefore, the model cannot explain the observed persistence of the liquidity effect. Christiano, Eichenbaum, and Evans (2005) develop a fully fledged DSGE model which features several intertemporal mechanisms enriching the ability of the model to generate persistence (adjustments costs on capital, habit persistence in consumption, sticky prices, sticky wages, etc.). This model

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¹ This result seems to be robust across different identification schemes and different sample periods (see Sims (1992), Leeper, Sims, and Zha (1996) and Christiano, Eichenbaum, and Evans (1999)).

therefore generates a persistent liquidity effect. All these models illustrate how the liquidity effect is fundamentally related to the intertemporal behavior of the agents, in particular the intertemporal substitution motive. This paper aims at understanding the exact role of these intertemporal mechanisms for the liquidity effect by focusing on the Euler equation. More precisely, we investigate the relationship between consumption growth and the real interest rate following a monetary shock as this relationship lies at the core of the liquidity effect.

This relationship is indeed theoretically and empirically intriguing and challenging: the permanent income hypothesis that lies at the core of most DSGE models implies that a high real interest rate is associated with high expected rate of growth of consumption. This is at odds with the data. For example, Hall and Mishkin (1982) have pointed out that there have been long periods of time in which average U.S. aggregate consumption growth was positive though real interest rates were very low (close to zero)². Over the period 1967:1–2003:4, results obtained from a SVAR model (to be described later) indicate that, following a monetary shock, the conditional correlation between the real interest rate and consumption growth is highly negative, -0.96. In other words, following a contractionary monetary policy, the real interest rate is found to rise whereas the economy experiences a (persistent) drop in consumption growth. This is at odds with the simplest permanent income model which predicts a Structural Vector Autoregressive model, a strong positive correlation between these two variables, for intertemporal substitution motives. Finally, while a direct result of the permanent income model is that consumption adjusts immediately to current “news” about lifetime resources (see Hall (1978)), the data suggest that consumption almost does not react immediately to a monetary shock. We argue that these implications of the permanent income hypothesis is a direct consequence of the intertemporal substitution effect that determines its overall behavior.

Since intertemporal substitution seems to play such a great role in the relationship linking the real interest rate and consumption growth, and since it seems to be one potential source of the inability of standard models to account for the joint behavior of these two variables, we need to weaken this mechanism. One way to break intertemporal substitution is to introduce habit persistence on consumption. Indeed, by introducing a time non-separability in consumption decisions and because the household cares about both future and past consumption decisions in determining her consumption/saving plans, habit persistence has two attractive features with regard to empirical findings. First, it weakens the intertemporal substitution mechanism. Second, it leads consumers to adjust slowly to non-anticipated shocks

² Instrumental variable regressions indicate that consumption growth is relatively weakly related to interest rates (see Campbell and Mankiw (1989) and (1991)). Chapman (1997) shows that real yields and consumption growth are (weakly) negatively correlated over the full sample of 1953-1991.

– among them monetary policy shocks. Hence, habit formation may theoretically help explaining the smoothness of consumption growth following a monetary policy shock. For these two reasons, the habit persistence assumption may be relevant in accounting for the joint behavior of the real interest rate and consumption growth.

In order to gauge the potential of habit persistence, we consider a single good economy where households' preferences are characterized by the presence of habit formation. We then assess the ability of this simple model to account for the behavior of the real interest rate and consumption growth following a monetary shock. Our methodology follows that of Beaudry and Guay (1996), Fuhrer (2000) or Christiano, Eichenbaum, and Evans (2005) in that it rests on an estimation/testing strategy based on conditional moments. For example, Fuhrer (2000) finds the deep parameters of his model by minimizing a distance between the theoretical moments generated by his model and the same set of moments obtained from an unconstrained SVAR. Fuhrer (2000) adopts a different specification of the utility function (involving the ratio, rather than the difference, of current consumption relative to some habit level) but considers the response of the model to different shocks including monetary shocks, and concludes that "one can reject the hypothesis of no habit formation with tremendous confidence". Fuhrer argues that the response of spending to monetary policy shocks are improved with habit persistence. In contrast to Fuhrer (2000), the present paper estimates the habit persistence parameter by matching the impulse response functions (IRF). Our approach also departs from Fuhrer (2000) as we focus on conditional moments – the comovements of the real interest rate and consumption growth following a monetary policy shock³. More precisely, we first estimate the conditional moments on consumption growth and the real interest rate by estimating the IRFs of these two variables to a monetary policy shock using a SVAR model. In a second step, this conditional information is used to estimate the habit persistence parameter, through method of moments estimation applied to the Euler equation characterizing the consumption behavior of our agent. Standard over-identification enables us to test the relevance of the model. Our approach is therefore closer to Christiano, Eichenbaum, and Evans (2005). They find that habit persistence of the form considered in the paper helps explaining the impulse response of consumption (or output) to monetary policy shocks. These authors apply limited information methods to estimate a fully-specified model, with the objective of estimating most of the deep parameters characterizing preferences and technology. In contrast, we focus on the consumption Euler equation and do not apply the full set of model restrictions to identify the habit persistence parameter. This enables to understand the role of habit formation in accounting for the joint behavior of the real interest rate and consumption growth following a monetary policy shock.

³ Beaudry and Guay (1996) focus on conditional moments following a technology shock.

Our results on quarterly US data suggest that habit persistence is pronounced and significant over the 1967:1–2003:4 period. The values of the estimated habit persistence parameter are similar to values obtained in previous studies with other estimation methods (see Constantinides and Ferson (1991) and Braun, Constantinides, and Ferson (1993) among others). Moreover, the steady state intertemporal elasticity of substitution (IES) implied by the estimated values of the habit persistence parameter are close to zero and match standard empirical estimates of the *IES*⁴. Further, the over-identification test never leads to rejection of the model. Otherwise stated, habit persistence seems to provide a relevant assumption to account for the joint dynamics of the real interest rate and consumption growth following a monetary shock. This suggests that weakening the intertemporal substitution mechanism at work in most models is a crucial step in developing a meaningful monetary model. Finally, we show that introducing more lags in consumption does not significantly improve our benchmark model.

The remaining of the paper is organized as follows. In the first Section we describe some monetary facts obtained from a standard SVAR approach. More precisely, we pay particular attention to the responses of the real interest rate and consumption growth to monetary policy shocks. In the second Section, we present our simple benchmark model – which includes internal habit with one lag. A third Section describes our evaluation methodology. A fourth Section describes the estimates results and discusses the role played by the habit formation on the intertemporal substitution mechanism. Finally, we check the robustness of these results to different specifications of the habit formation. A last section offers some concluding remarks.

1 The Monetary Facts

This section describes a set of stylized facts related to the behavior of the US economy in face of a monetary shock. More specifically, we report some empirical evidence about the real interest rate and consumption growth comovements following a monetary policy shock. In lines with Christiano, Eichenbaum, and Evans (1999), we identify this shock from restrictions imposed on a SVAR model estimated for the US economy.

1.1 The Structural Vector Autoregressive Model

Following Christiano, Eichenbaum, and Evans (1999), we assume that the Central Bank conducts its monetary policy relying on a simple reaction

⁴ For example, Campbell and Mankiw (1989) report estimates of the *IES* close to 0.2 on aggregate data and show a zero *IES* cannot be rejected by the data.

function. More precisely, in each period t , the policymaker sets its instrument – the short-term nominal interest rate S_t – in a systematic way using a simple rule which exploits the available information, Ω_t . Therefore, the monetary policy rule can be written as

$$S_t = f(\Omega_t) + \sigma^j \epsilon_t^j,$$

where we assume that $f(\cdot)$ is linear. The random variable ϵ_t^j , uncorrelated with any piece of information belonging to Ω_t , is the monetary shock. It is assumed to have zero mean and a constant standard deviation σ^j . Different interpretations may be given to this shock. For example, they can be viewed as measurement errors in the information set available to the central bank or as some exogenous shocks in the preferences of the central bank.

To identify those shocks, we consider a set of variables Y_t containing the instrument and the variables of the information set Ω_t of the central bank. We assume that the dynamic behavior of Y_t can be accurately represented by a SVAR of order q :

$$A(L)Y_t = \epsilon_t,$$

where L is the lag operator, and $A(L) = \sum_{i=0}^q A_i L^i$ is a polynomial of order q and $E(\epsilon_t \epsilon_t') = D$ where q is a diagonal matrix. Stated this way, this representation is assumed to be structural and ϵ_t is the vector of structural shocks that includes the monetary policy shock. This representation can then be used to analyze the effects of the monetary shock via the analysis of the Impulse Response Functions (IRFs) of the variables belonging to Y_t to a monetary shock. These IRFs can be obtained from the infinite Moving Average representation of the structural SVAR:

$$Y_t = H(L)\epsilon_t = \sum_{i=0}^{\infty} H_i \epsilon_{t-i},$$

with $H(L) \equiv A(L)^{-1}$. In particular, we can write the decomposition of the real interest rate and consumption growth⁵:

$$r_t = \sum_{i=0}^{\infty} h_i^r \epsilon_{t-i},$$

$$\Delta c_t = \sum_{i=0}^{\infty} h_i^c \epsilon_{t-i},$$

⁵ Initially the SVAR contains the following variables: the nominal interest rate i_t , prices P_t and consumption c_t . The IRF of the real interest rate and the consumption growth are actually respectively derived from the IRF of the nominal interest rate and inflation and the IRF of consumption *i.e.*

$$r_t = i_t - E_t[P_{t+1} - P_t] = \sum_{i=0}^{\infty} h_i^r \epsilon_{t-i} - \sum_{i=1}^{\infty} h_i^p \epsilon_{t-i+1} + \sum_{i=0}^{\infty} h_i^c \epsilon_{t-i},$$

$$\Delta c_t = c_t - c_{t-1} = \sum_{i=1}^{\infty} h_i^c \epsilon_{t-i+1} - \sum_{i=0}^{\infty} h_i^c \epsilon_{t-i}.$$

where the parameters $\{h_i\}_{i=0, \dots, \infty}$ correspond to the IRF to a shock on ϵ_t , where i is the horizon after the shock.

Estimation of the SVAR model is therefore a preliminary step to retrieve the IRF. However, the matrix A_0 exhibits some simultaneity problem, implying that the estimation has to be done in two steps. First, assuming that A_0 is invertible, we estimate the SVAR representation:

$$Y_t = B_1 Y_{t-1} + \dots + B_q Y_{t-q} + u_t,$$

using ordinary least squares. Note that $B_i = A_0^{-1} A_i$ for $i = 1, \dots, q$ and $u_t = A_0^{-1} \epsilon_t$ has covariance matrix V .

In order to recover the structural shocks, some identifying restrictions have to be placed on A_0 and D . Following Christiano, Eichenbaum, and Evans (1999), we use the recursiveness approach and assume that the matrix of contemporaneous impacts A_0 is lower triangular and that structural shocks are orthogonal with unit volatility – *i.e.* D is the identity matrix. Therefore, we have a recursive system which depends on the order of the variables in Y_t .

Consistent estimates of the IRF can then be derived and Monte-Carlo simulations can be used to obtain an estimate of their variance-covariance matrix, denoted M thereafter.

1.2 Monetary facts

We apply this methodology on US quarterly data⁶ over the period running from the first quarter of 1967 to the last quarter of 2003. Let GDP_t , $PGDP_t$, PPI_t , C_t , FF_t , NBR_t , TR_t and M_t denote the time t values of, respectively, the log of real GDP, the log of the implicit GDP deflator, the log of the producer price index (PPI, crude materials), the log of the real consumption of non-durable goods and services, the federal funds rate, the log of total reserves, the log of nonborrowed reserves and the log of M1. AIC and BIC information criteria led us to select a SVAR(3) representation for the vector $Y_t = \{GDP_t, PGDP_t, PPI_t, C_t, FF_t, NBR_t, TR_t, M_t\}$. As Christiano, Eichenbaum, and Evans (1999), the federal fund rate is taken to be the main instrument of monetary policy. The only departure from Christiano, Eichenbaum, and Evans (1999) is therefore the introduction of real consumption in our data set. We refer to the policy shock as a shock on the nominal interest rate, FF . As already stated, we use the recursiveness assumption to identify the shocks, which assumes, among other things, that the policymaker does observe current production, prices and consumption when it sets the federal funds rate (FF_t) but the private agents do not observe the current monetary policy shock. Another implication is that GDP, consumption and prices do not react to a monetary policy shock on impact. We further impose that monetary policy shocks are orthogonal to shocks to

⁶ We use Federal Reserve economic data's series (<http://www.stls.frb.org/fred/>).

non-borrowed reserves, total reserves and $M1_t$. Relative to Christiano, Eichenbaum, and Evans (1999), we add an identification assumption: consumption does not contemporaneously react to a monetary policy shock⁷.

Figure 1 reports the estimated IRF for all the variables after a contractionary monetary policy shock – that is a positive shock on the federal fund rate. The solid line reports the point estimates of the various dynamic response functions. The dashed lines correspond to the 95 per cent confidence interval obtained through Monte-Carlo simulations.

The main consequences of a contractionary monetary policy shock are similar to those obtained by previous studies. Following a contractionary monetary policy shock, there is a persistent decline in real GDP, the aggregate price level initially responds very little and positively and the federal fund rate rises. Finally, consumption decreases persistently. Focusing on the co-movements of consumption growth and the real interest rate leads to the following conclusion: a contractionary monetary policy shock leads to a persistent increase in the real interest rate and to a persistent decrease in consumption growth (see figure 2).

Let us now gauge the ability of the standard permanent income model to account for this fact. In fact, it turns out that this model is clearly unable to mimic the observed co-movement due to the intertemporal substitution mechanisms that lie at the core of the consumption behavior. Indeed, the log-linear version of the arbitrage condition defining the intertemporal allocation of consumption is given by:

$$r_t = \sigma E_t \Delta C_{t+1},$$

where r_t is the real interest rate, ΔC_{t+1} is consumption growth between t and $t+1$, and σ is the elasticity of substitution that enters implicitly in the utility function of the agents. This arbitrage condition clearly indicates that a high interest rate is associated with a high expected consumption growth. This just reflects the standard intertemporal substitution mechanism which determines the consumption/saving arbitrage in this type of model; a high interest rate creates an incentive to increase savings – *i.e.* to postpone consumption. Moreover this arbitrage condition makes it clear that agents adjust immediately their consumption levels to news about lifetime resources whereas Campbell and Deaton (1989) and Deaton (1992) show that consumption does not respond immediately to current “news” but that consumption exhibits “excess smoothness”. Hence, it should be clear that, due to the intertemporal substitution mechanism, the permanent income model is not able to explain qualitatively neither the co-movement of the variables, nor the persistence of the response. In other words, a standard permanent income model cannot generate co-movements in the real interest rate and consumption growth of the type observed in the data.

⁷ We checked the robustness of our SVAR model against different identification scheme and found some evidence in favor of robustness (the robustness results are available upon request).

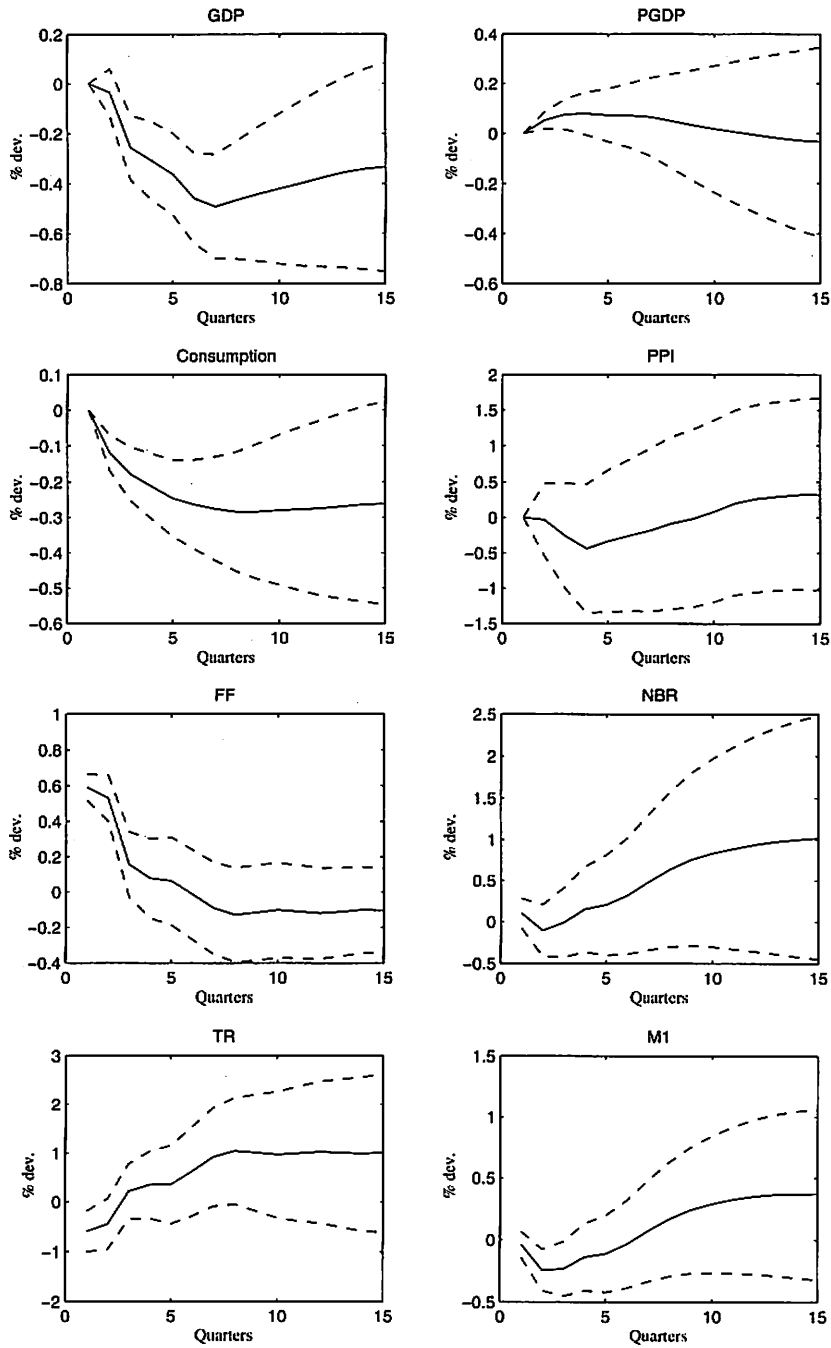


Figure 1: Responses to 1% interest rate shock

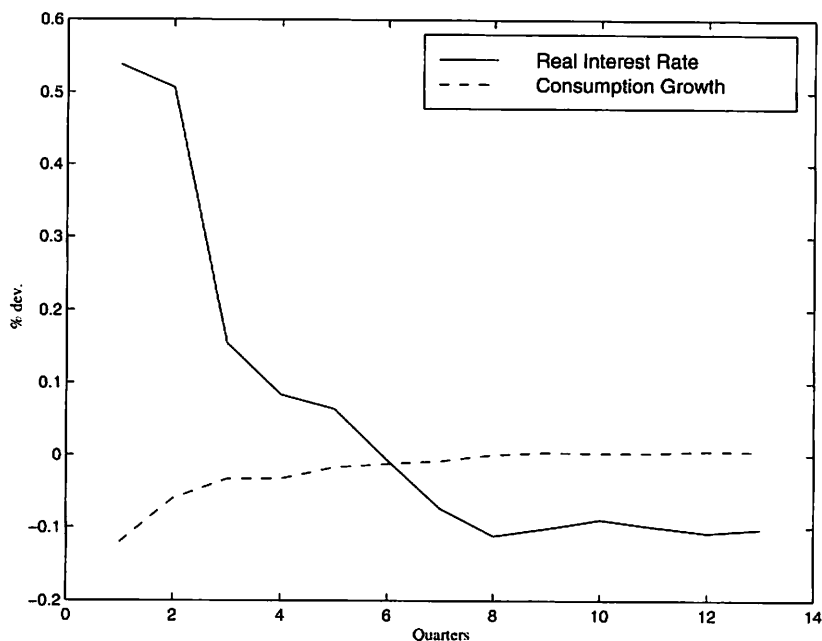


Figure 2: *Real Interest Rate and Consumption Growth*

2 The Model Economy

As aforementioned the permanent income model is not able to account for the co-movement of the real interest rate and consumption growth in face a monetary policy shock, mainly because of the intertemporal substitution mechanism. One way to reconcile the model and the data is then to weaken the intertemporal substitution mechanism. One possibility is to introduce habit persistence in consumption behavior. Under this assumption, the agents adjust their consumption levels only gradually to non-anticipated shocks, as they have to keep with their habits. We therefore expect the habit formation assumption to prove relevant in explaining the joint behavior of the real interest rate and consumption growth in face a monetary policy shock.

The theoretical model collapses to a standard consumption Euler equation with habit formation. Habit persistence actually raises three main modelling issues: *(i)* the speed with which habit reacts to consumption (habit depends on one lag of consumption vs. habit reacts only gradually to changes in consumption); *(ii)* whether it is internalized or not and *(iii)* the functional form (ratio vs. difference). As far as the first issue is concerned, we introduce

only one lag in our benchmark specification in order to avoid multicollinearity problems. Indeed, consumption growth rates are highly serially correlated and it has proven difficult to estimate accurately a specification which includes more lags (Constantinides and Ferson (1991))⁸. Since we do not want to introduce the distortion that comes from any externality, we consider internal habit persistence. Finally, as will become clear in a moment, we use a log-utility function. This implies, among other things, that the ratio specification suffers an identification problem. Thus we consider internal habit in difference with one lag⁹.

The economy is populated by an continuum of identical infinitely lived agents with mass one. We assume that there exists a representative household in the economy who has preferences over consumption represented by the following intertemporal utility function:

$$E_{t-1} \sum_{s=0}^{\infty} \beta^s [\log(C_{t+s} - \theta C_{t+s-1})] \text{ with } \theta \in (0, 1), \quad (1)$$

where θ is the habit persistence parameter, which may be taken as a measure of the time non-separability assumption in the model. $0 < \beta < 1$ is a discount factor. E_{t-1} denotes the mathematical expectation operator conditional to the information set available to the household at time $t-1$ ¹⁰.

The intertemporal Euler equation associated with the utility function (1) may be simply stated using a standard perturbation argument. Consider a reduction of the representative consumer's expenditures in period t from C_t to $C_t - \zeta$, $\zeta < 1$. The investment of ζ in a riskless asset with real return R_t yields an increase of the consumption expenditures in period $t+1$ from $E_{t-1}(C_{t+1})$ to $E_{t-1}(C_{t+1} + \zeta R_t)$. Optimality of consumption and investment plans requires that the expectation in period $t-1$ of the utility of the consumption flows is maximized at $\zeta = 0$, implying

$$E_{t-1} \left[\frac{1}{C_t - \theta C_{t-1}} - \frac{\beta \theta}{C_{t+1} - \theta C_t} \right] = \beta E_{t-1} R_t \left[\frac{1}{C_{t+1} - \theta C_t} - \frac{\beta \theta}{C_{t+2} - \theta C_{t+1}} \right]. \quad (2)$$

⁸ This point is examined in section 3.2.

⁹ For example (i) for one lag in habit of consumption see Abel (1990) and (1999), Dunn and Singleton (1986) and Constantinides and Ferson (1991) and for more gradual habit reaction to changes in consumption see Constantinides (1990), Sundaresan (1989), Campbell and Cochrane (1999) and Heaton (1995); (ii) for internal specification see Constantinides (1990) and Sundaresan (1989) and for external specification see Abel (1990) and (1999), Campbell and Cochrane (1999) (iii) for habit ratio specification see Abel (1990) and (1999) and for difference specification see Campbell and Cochrane (1999), Constantinides (1990) and Sundaresan (1989).

¹⁰ We assume that people make their consumptions decisions before the monetary policy shock, so that C_t is not affected by the monetary shock at t but at $t-1$ and earlier such that this timing is compatible with the identification scheme we imposed to identify the shocks (consumption does not respond on impact in the SVAR model). We thank a referee for pointing out an inconsistency between the timing of the theoretical model and of the SVAR model in the previous version of the paper.

This Euler equation relates real interest rate to past, current and future consumption expenditures and makes clear the role of the habit persistence parameter in this choice: the greater θ the greater the dependency of current consumption decision to past levels of consumption, therefore weakening the intertemporal substitution mechanism. Further, note that because habit persistence is internalized, not only does the household consider her past level of consumption to determine her current consumption, but also the impact of her decision on her future utility into account. Therefore, when habit persistence on consumption is introduced, a high interest rate may be associated with high current consumption relative to the future (low current consumption growth relative to the future).

In order to be able to exploit the estimated IRFs from the SVAR, the Euler equation is log-linearized to get

$$\alpha(\theta)E_{t-1}r_t = E_{t-1}[\alpha_1(\theta)\Delta c_{t+2} + \alpha_2(\theta)\Delta c_{t+1} + \alpha_3(\theta)\Delta c_t], \quad (3)$$

where

$$\begin{aligned} \alpha(\theta) &= (1 - \beta\theta)(1 - \theta), \\ \alpha_1(\theta) &= -\beta\theta, \\ \alpha_2(\theta) &= \beta\theta^2 + 1, \\ \alpha_3(\theta) &= \theta, \end{aligned}$$

where lowercase letters denote relative deviations of the variables from their steady state values.

This simple model furnishes an equilibrium condition summarizing the joint behavior of consumption growth and the real interest rate along a transition path. In particular, we focus on one single source of uncertainty: the unexpected monetary policy shock. If we assume that this condition should be true for all deviations of the variables from their equilibrium values, it should be also true for a deviation from their equilibrium values after a monetary policy shock. An attractive feature of this Euler equation is that it put simple restrictions on the data that can be tested using the methodology we describe in the next section.

3 Econometric Methodology

The ability of the simple model we propose in the previous section to account for the monetary facts we described in section 1 – and more precisely its ability to reproduce the IRFs – is tested using a method of moments. This section describes our estimation–testing strategy.

Intuitively our method consists in obtaining a value for the habit parameter θ that minimizes a metric between the implications of our theoretical model and the data. More precisely, we estimate θ from a set of

identifying restrictions derived from our theoretical model. The consistency between the timing of the theoretical model and of the SVAR model impose that we do not know r_t but only $E_{t-1}r_t$. Therefore, we can only start testing our simple model one period later. Let us start from rewriting the log-linear version of the Euler equation (3) one period later

$$\alpha(\theta)E_t r_{t+1} - \alpha_1(\theta)E_t \Delta c_{t+3} - \alpha_2(\theta)E_t \Delta c_{t+2} - \alpha_3(\theta)E_t \Delta c_{t+1} = 0. \quad (4)$$

Taking expectations based on the information available in period $t-1$, we get

$$\alpha E_{t-1} r_{t+1} - \alpha_1 E_{t-1} \Delta c_{t+3} - \alpha_2 E_{t-1} \Delta c_{t+2} - \alpha_3 E_{t-1} \Delta c_{t+1} = 0, \quad (5)$$

where the dependency of α_i to θ was dropped for exposition purposes. (4) together with (5) then implies

$$\begin{aligned} & \alpha(E_t r_{t+1} - E_{t-1} r_{t+1}) - \alpha_1(E_t \Delta c_{t+3} - E_{t-1} \Delta c_{t+3}) \\ & - \alpha_2(E_t \Delta c_{t+2} - E_{t-1} \Delta c_{t+2}) - \alpha_3(E_t \Delta c_{t+1} - E_{t-1} \Delta c_{t+1}) = 0. \end{aligned}$$

Note that $E_t r_{t+1} - E_{t-1} r_{t+1}$ and $E_t \Delta c_{t+i} - E_{t-1} \Delta c_{t+i}$ have their equivalent in the data. Further, if expectations are taken conditionally on the sole monetary shock, the IRFs obtained from the SVAR deliver

$$E_t r_{t+n} - E_{t-1} r_{t+n} = h_n^r,$$

$$E_t \Delta c_{t+n} - E_{t-1} \Delta c_{t+n} = h_n^c.$$

Plugging these empirical counterpart in the theoretical model, we should have, provided the model is true

$$\alpha h_1^r - \alpha_1 h_3^c - \alpha_2 h_2^c - \alpha_3 h_1^c = 0.$$

This gives us a condition to test the model.

Likewise, the Euler equation one period after the shock yields

$$\alpha E_t r_{t+2} - \alpha_1 E_t \Delta c_{t+4} - \alpha_2 E_t \Delta c_{t+3} - \alpha_3 E_t \Delta c_{t+2} = 0,$$

$$\alpha E_{t-1} r_{t+2} - \alpha_1 E_{t-1} \Delta c_{t+4} - \alpha_2 E_{t-1} \Delta c_{t+3} - \alpha_3 E_{t-1} \Delta c_{t+2} = 0,$$

implying

$$\begin{aligned} & \alpha(E_t r_{t+2} - E_{t-1} r_{t+2}) - \alpha_1(E_t \Delta c_{t+4} - E_{t-1} \Delta c_{t+4}) \\ & - \alpha_2(E_t \Delta c_{t+3} - E_{t-1} \Delta c_{t+3}) - \alpha_3(E_t \Delta c_{t+2} - E_{t-1} \Delta c_{t+2}) = 0, \end{aligned}$$

which furnishes another identifying restriction that exploits information carried by the dynamics of the real interest rate and consumption growth one period following the shock

$$\alpha h_2^r - \alpha_1 h_4^c - \alpha_2 h_3^c - \alpha_3 h_2^c = 0.$$

Applying the same methodology to different horizons, we obtain a system of identifying restrictions

$$\begin{cases} \alpha(\theta)h_1^r - \alpha_1(\theta)h_3^c - \alpha_2(\theta)h_2^c - \alpha_3(\theta)h_1^c & = 0 \\ \alpha(\theta)h_2^r - \alpha_1(\theta)h_4^c - \alpha_2(\theta)h_3^c - \alpha_3(\theta)h_2^c & = 0 \\ & \vdots \\ \alpha(\theta)h_{n+1}^r - \alpha_1(\theta)h_{n+3}^c - \alpha_2(\theta)h_{n+2}^c - \alpha_3(\theta)h_{n+1}^c & = 0 \\ & \vdots \end{cases}$$

Although an infinite number of identifying conditions can be derived from this system, we will only consider the N first conditions. Indeed, since the degrees of freedom is finite in the SVAR, IRFs do not carry any piece of information after a while. Otherwise stated, as any IRF results from a finite number of parameters from the SVAR model, there does not exist an infinite number of independent IRFs. Therefore, we have to choose the number of conditions, so as to extract as much information as possible from the conditional moments, without facing any collinearity problem. Therefore, when discussing our results, we will just report our estimates for θ obtained from different values for N . We will however impose that the number of conditions exceeds the number of parameters to be estimated (equals to one in our problem). Therefore, the model will be over-identified enabling us to test the model.

Let us turn to the estimation procedure. Let us denote by h the vector that collects the deviations of the variables r and Δc at different horizons. The system of identifying restrictions we obtained from our theoretical model may be simply summed up by the function g with arguments h and the structural parameter of interest θ

$$g(h; \theta) = 0.$$

Denoting by $\{\hat{h}_{Tn}\}_{n=1, \dots, N}$ the sequence of N impulse responses obtained from the SVAR using a data set of size T , the data will be said to support the model if:

$$g(\hat{h}_T; \theta) = 0,$$

where \hat{h}_T and then $g(\hat{h}_T; \theta)$ are $N \times 1$ vectors. A consistent estimator, $\hat{\theta}$, of θ can be obtained minimizing the quantity

$$J(\theta) \equiv g(\hat{h}_T, \theta)' \widehat{W}_T g(\hat{h}_T, \theta), \quad (6)$$

where \widehat{W}_T is a symmetric positive definite weighting matrix given by the inverse of the covariance matrix S of the N conditions $g(\hat{h}; \theta) = 0$.

Compared to Asymptotic Least Squares of Gouriéroux and Monfort (1996) or the Generalized Method of Moment of Hansen (1982), our estimates of h are derived from a first estimation. Therefore, we use a consistent estimate of S_t , which takes into account the uncertainty in the estimation of \hat{h}_T :

$$\widehat{S}_T = \left(\frac{\partial g(\widehat{h}, \theta)}{\partial h} \right)' \widehat{M}_T \left(\frac{\partial g(\widehat{h}, \theta)}{\partial \widehat{h}'} \right),$$

where \widehat{M}_T is the estimator of the variance-covariance matrix of \widehat{h}_T .

Since the weighting matrix depends on θ , we use an iterated approach. We construct the weighting matrix W using the parameter estimate from the n -stage, and use this matrix to find the parameter for the stage $n + 1$ which minimizes the quadratic form. The new parameter is used to update the weighting matrix. The iterations continue until the parameter estimation values are stable.

To test for the validity of our simple theoretical model, we apply the standard test of over-identifying restrictions (see Hansen (1982)). In standard cases, the test statistics is asymptotically distributed as a $\chi^2(n)$, with n being the degree of freedom of the model. But, since our estimation is done in two steps using a small sample, this usually fails to be the case. Indeed, in such an implementation of the method, the number of identifying restrictions is difficult to know exactly. Further, the distribution of the SVAR model estimators is degenerated because of multicollinearity problems arising in the model. Hence, to study inference, we rely on a Monte-Carlo approach which enables to draw inference both for the estimator of the structural parameter θ and the over-identifying test.

4 Econometric Results and Robustness

In this section we report estimated values of the habit persistence obtained in our model economy and discuss the role of habit formation in accounting for the joint behavior of the real interest rate and consumption growth. Finally, we check the robustness of our results against alternative specification for the habit persistence.

1 The benchmark model Economy

Table 1 reports the estimated value of the habit persistence parameter θ for different values of the time horizon, N , for the IRFs. Note that over the whole exercise, we will assume that the discount factor takes on the value 0.988, implying an annual real discount rate of 4%¹¹.

¹¹ We checked the robustness of our results against alternative values of β .

N	$\hat{\theta}$	s.e.	$\frac{J(\hat{\theta})}{\hat{\theta}}$		Critical Value	IES
			$\hat{\theta}$	$\theta = 0$		
1	0.5712	(0.0103)	-	-	-	0.1413
2	0.7396	(0.0083)	19.3793	85.2134	35.2633	0.0455
3	0.8139	(0.0077)	26.7594	86.9847	42.1061	0.0220
4	0.8646	(0.0082)	31.3024	93.5041	44.6166	0.0114
5	0.8888	(0.0087)	33.7882	94.7612	47.1413	0.0076
6	0.9025	(0.0090)	35.5487	95.2318	49.3064	0.0058
7	0.9096	(0.0091)	36.4360	96.7854	51.7203	0.0050
8	0.9140	(0.0093)	37.0592	97.6543	52.9016	0.0046
9	0.9170	(0.0099)	37.6872	98.1294	53.5580	0.0043
10	0.9190	(0.0101)	37.9001	98.9841	55.6956	0.0040
15	0.9216	(0.0117)	39.2585	101.1542	65.5919	0.0038

Table 1: One lag model

The reported estimates of the habit persistence parameter, $\hat{\theta}$, ranges from 0.57 to 0.92 depending on the time horizon we consider for IRFs. The estimated habit persistence values are similar to the values of habit obtained in previous studies. For example, Constantinides and Ferson (1991) and Braun, Constantinides, and Ferson (1993) obtained an estimated value for θ that lies within the interval [0.5; 0.9] on macro data. Using micro data on food consumption, Naik and Moore (1996) report a lower albeit significant estimates for habit persistence of 0.486. Further, note that the over-identification test (N greater than 1) never leads to reject the model as the J-statistics, $J(\hat{\theta})$, is always lower than the critical value at the standard 5% significance level, obtained from Monte-Carlo simulations. Conversely, a model where habit persistence is ruled out is systematically rejected by the data (column $\theta = 0$).

Beyond, note that the parameter is very precisely estimated no matter the selected time horizon. Nevertheless, it would be quite informative to restrict ourselves to a particular value for N . A simple selection criterion would be to choose the value for N that minimizes the mean squared error. But, as our estimations are, by construction, unbiased, the variance of the parameter is a sufficient statistics to measure the precision of the estimation. This leads us to select a time horizon $N = 3$, which implies that $\hat{\theta} = 0.8139$, which puts a high weights on consumption habits in the utility function and therefore weakens substantially the intertemporal substitution mechanism and therefore enhances the performances of the model (in terms of the stylized

fact we mainly focus on). Indeed, in the standard permanent income model, any increase in the real interest rate is associated with a higher consumption growth as individuals are led to save more and therefore postpone consumption. This intertemporal substitution effect is broken when the model takes habit persistence into account because the increase in future consumption is dampened by the habit it would create. Therefore, households adjust their consumption levels gradually to non-anticipated shock – which just reflects a weakening of the intertemporal substitution mechanism. In order to gauge this phenomenon, we report, in the last column of table 1, the implied Intertemporal Elasticity of Substitution (IES) computed in the steady state

$$IES = \frac{(1 - \theta)(1 - \beta\theta)}{(1 + \beta\theta^2)}.$$

The *IES* implied by all estimated values – for different N – are positive. Our results actually match standard empirical estimates of the *IES* (see Hall (1988), Campbell and Mankiw (1989), Attanasio and Weber (1993)) which are usually found to be close to zero. In other words, these results together with those on the overidentification test suggest that habit persistence, by weakening the intertemporal substitution mechanism, furnishes a potentially relevant propagation mechanism that can generate co-movements between the real interest rate and consumption growth similar of those found in the data¹².

4.2 A Two Lags Model

In this section, we assess the role of the length of habit persistence by introducing two lags in consumption. This enables us to gauge the role of consumption smoothing in the whole process.

We modify the speed with which habit reacts to individual consumption introducing two lags in habit formation. Valued consumption then rewrites as follows

$$C_t^* = C_t - \theta_1 C_{t-1} - \theta_2 C_{t-2} \text{ with } \theta_1, \theta_2 \in (0, 1). \quad (7)$$

We then estimate the two habit persistence parameters θ_1 and θ_2 for different values of N ¹³. Table 2 reports our estimation results. The estimated

¹² It should be noted that these results were obtained using a log-utility function. We also checked the robustness of the estimates against more general forms of preferences, in particular a general CRRA function with curvature parameter σ . However, identification problems arise if we estimate σ and θ simultaneously. Therefore, in order to gauge the robustness of our estimations against the specification of the utility function, we estimate θ conditional on several values for σ . Setting this parameter respectively to 0.1, 0.5, 2 and 5, we find that estimates of θ that lie between 0.8146 and 0.9693, 0.6723 and 0.9418, 0.4225 and 0.8901, 0.3117 and 0.8521 for different values of N (see tables 3, 4, 5 and 6 in appendix). The model is never globally rejected.

¹³ When $N = 2$, we are exactly identified.

N	$\hat{\theta}_1$	$\hat{\theta}_2$	<i>s.e.</i> $\hat{\theta}_1$	<i>s.e.</i> $\hat{\theta}_2$	$J(\hat{\theta}_1, \hat{\theta}_2)$	Critical Value
2	0.4016	0.2012	(0.0117)	(0.0106)	-	-
3	0.5014	0.1608	(0.0130)	(0.0132)	17.7241	31.6821
4	0.5321	0.1912	(0.0196)	(0.0201)	19.3104	35.4381
5	0.7021	0.1501	(0.0211)	(0.0221)	26.1411	41.2112
6	0.7214	0.1421	(0.0298)	(0.0272)	30.1412	43.2310
7	0.7318	0.1402	(0.0325)	(0.0298)	30.6212	44.5116
8	0.7302	0.1421	(0.0343)	(0.0312)	32.6147	45.6206
9	0.7218	0.1435	(0.0372)	(0.0360)	33.6702	48.3105
10	0.7182	0.1462	(0.0390)	(0.0372)	34.1409	49.7127
15	0.6321	0.1923	(0.0421)	(0.0394)	38.1782	55.3221

Table 2: *Two lags model*

value for θ_1 lies between 0.40 and 0.73 and seems consistent with our earlier estimates. The value for θ_2 lies between 0.14 and 0.20 and is always lower than the associated value of θ_1 . Both parameters are found to be always significant. However, the standard deviations in this two lags model are larger than the ones in the one lag model. This loss of precision comes from the collinearity between the two parameters (see Constantinides and Ferson (1991)). Moreover, the J-statistic shows us that the model is not rejected by the data¹⁴.

5 Concluding remarks

In this paper, we assess the ability of habit persistence to account for the joint dynamic behavior of the real interest rate and consumption growth following a monetary shock. We first fit a SVAR model on US quarterly data to obtain impulse response functions of consumption growth and the real interest rate to a monetary policy shock. Then, using these impulse response functions, we estimate the habit persistence parameter that minimizes the distance between our theoretical model – a single good monetary model with one asset – and the empirical IRFs. Our results are similar to those obtained using other estimation methods. Further, the model is not rejected by the

¹⁴ Very similar results are obtained when we introduce more lags in the model. These results are not reported but are available from the authors upon request.

data. Our results are of importance because they stress that the habit formation assumption is relevant to account for the joint behavior of consumption growth and the real interest rate.

We also show that habit formation allows weakening the intertemporal substitution mechanism. This assumption therefore proves relevant in accounting for the observed dynamic behavior of consumption growth and the real interest rate. Finally, introducing more lags in consumption specification does not improve our benchmark model even if these specifications are not rejected by the data.

Appendix

A The Algorithm

The simulation procedure is conducted as follows:

- Step 1: From the SVAR model, we obtain an estimate of the Data Generating Process (DGP) of the Y_t vector – that is the estimations of the parameters B and V . This DGP is used to simulate $I = 500$ realizations, $\{Y_t(i)\}_{i=1}^I$, of the initial vector of time series Y_t . Using these sets of simulated time series, we re-estimate the model, and obtain a set of new parameters $\{\hat{B}(i)\}_{i=1}^I$, from which we derive a set of simulated auxiliary parameters – *i.e.* the IRFs. We therefore end up with the set of parameters: $\{\hat{h}_T(i)\}_{i=1}^I$.
- Step 2: For each simulation i , we get an estimate, θ_i , of the deep parameter θ minimizing (6). We thus get a sequence $\{\hat{\theta}(i)\}_{i=1\dots I}$ of I estimates for θ .
- Step 3: From this sequence, $\{\hat{\theta}(i)\}_{i=1\dots I}$, we obtain an estimate of the covariance of the estimator of θ . We use this covariance to test for the significance of the parameter. For each $\hat{\theta}(i)$ we calculate the associated value of the over-identifying test statistic. We use this set of simulated statistics to derive the associated p-value.

B Robustness to the specification of the utility function

In this appendix, the robustness of our results is checked against the specification of the utility function, we estimate θ conditional on several values for σ . We set this parameter respectively to 0.1, 0.5, 2 and 5.

N	$\hat{\theta}$	<i>s.e.</i>	$J(\hat{\theta})$	Critical Value
1	0.8146	(0.0061)	-	-
2	0.8953	(0.0043)	17.3122	32.0553
3	0.9282	(0.0031)	24.8010	38.3688
4	0.9493	(0.0028)	29.7908	42.2608
5	0.9590	(0.0027)	32.5705	45.5767
6	0.9645	(0.0026)	34.5172	47.4428
7	0.9672	(0.0027)	35.5289	49.5470
8	0.9687	(0.0027)	36.2249	51.1034
9	0.9697	(0.0027)	36.9230	52.2193
10	0.9703	(0.0026)	37.1859	53.6582
15	0.9693	(0.0031)	38.6656	63.3767

Table 3: *One lag model* ($\sigma = 0.1$)

N	$\hat{\theta}$	<i>s.e.</i>	$J(\hat{\theta})$	Critical Value
1	0.6723	(0.0089)	-	-
2	0.8070	(0.0066)	18.5465	34.3280
3	0.8640	(0.0053)	25.9897	41.0255
4	0.9019	(0.0052)	30.7288	43.8135
5	0.9198	(0.0052)	33.3411	46.7589
6	0.9298	(0.0051)	35.1830	48.5306
7	0.9350	(0.0053)	36.1251	50.2910
8	0.9381	(0.0053)	36.7818	52.3733
9	0.9401	(0.0054)	37.4430	53.2471
10	0.9414	(0.0054)	37.6783	54.3762
15	0.9418	(0.0070)	39.0991	64.3643

Table 4: *One lag model* ($\sigma = 0.5$)

N	$\hat{\theta}$	<i>s.e.</i>	$J(\hat{\theta})$	Critical Value
1	0.4225	(0.0118)	..	–
2	0.6309	(0.0118)	20.3899	35.5603
3	0.7320	(0.0125)	27.6197	42.1924
4	0.8025	(0.0143)	31.8733	44.4905
5	0.8371	(0.0158)	34.1820	47.0805
6	0.8570	(0.0166)	35.8242	48.2441
7	0.8674	(0.0171)	36.6309	51.5004
8	0.8742	(0.0176)	37.2041	52.6864
9	0.8789	(0.0181)	37.7783	53.4342
10	0.8819	(0.0188)	37.9596	54.2129
15	0.8901	(0.0221)	39.2323	64.2059

Table 5: *One lag model* ($\sigma = 2$)

N	$\hat{\theta}$	<i>s.e.</i>	$J(\hat{\theta})$	Critical Value
1	0.3117	(0.0128)	..	–
2	0.5196	(0.0158)	20.7647	34.5307
3	0.6624	(0.0182)	27.8425	40.4167
4	0.7484	(0.0219)	31.9174	43.2902
5	0.7922	(0.0254)	34.1279	45.2619
6	0.8172	(0.0267)	35.6973	45.7677
7	0.8305	(0.0278)	36.4539	49.4716
8	0.8394	(0.0283)	36.9974	50.8233
9	0.8457	(0.0293)	37.5331	51.5184
10	0.8497	(0.0302)	37.6971	52.9690
15	0.8521	(0.0332)	38.9375	60.5984

Table 6: *One lag model* ($\sigma = 5$)

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