Collective models of labor supply with nonconvex budget sets and nonparticipation: A calibration approach

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Abstract We suggest a methodology to calibrate a collective model with household specific bargaining rules and marriage specific preferences that incorporate leisure exter nalities. The empirical identification relies on the assumption that some aspects of indi vidual preferences remain the same after marriage, so that estimation on single individuals can be used. The procedure maps the complete Pareto frontier of each household in the dataset and we define alternative measures of a power index. The latter is then regressed on relevant bargaining factors, including a set of variables retracing the potential relative contributions of the spouses to household disposable income. In its capacity to handle complex budget sets and labor force participation decisions of both spouses, this frame work allows the comparison of unitary and collective predictions of labor supply reactions and welfare changes entailed by fiscal reforms in a realistic setting (see Michal Myck et al., 2006; Denis Beninger et al., 2006).

Keywords Collective model · Household labor supply · Intrahousehold allocations · Tax reform

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1. Introduction

The collective approach to household behavior is gradually gaining ground in the applied microeconomic literature. Introduced by Chiappori (1988, 1992), and Apps and Rees (1988), this approach can be seen as a substantial improvement over the standard, or unitary, model of household behavior. On the one hand, the model generalizes the unitary approach to a multi utility framework while nesting all other cooperative models leading to efficiency. On the other hand, it allows to derive original restrictions which are rarely rejected on observed behavior of individuals in couples, whereas tests of the unitary approach usually come to the opposite conclusion.

However, there is yet no simple way to identify and estimate a collective model of labor supply to perform satisfying simulations of tax benefit reform, i.e., in a setting which incorporates both participation decisions (in addition to the choice of work hours) and realistic nonconvex budget sets arising from means tested social and family benefits. These features are hard to handle by usual means.[1]

The present paper suggests a methodology to calibrate a collective model of labor supply with household specific bargaining rules and marriage specific preferences that incorporate leisure externalities. The empirical identification relies on the assumption that some aspects of individual preferences but not all remain the same after marriage, so that some parameters can be separately estimated on a sample of single individuals. The main novelty of the approach lies in a calibration procedure to map the complete Pareto frontier of each household in the dataset. Alternative measures of the final location on the frontier are interpreted as power indices. One of the latter is then regressed on relevant bargaining factors, including a set of variables retracing the potential relative contributions of the spouses to household disposable income (i.e., income after social contributions, taxes and benefits). In this way, tax reforms not only directly change the budget constraint, as in the unitary model, but also potentially alter the balance of power in the household by changing the potential relative contributions of each spouse.

In its capacity to handle nonconvex budget sets and labor force participation decisions of both spouses, this model offers a unique chance to compare behavioral responses to tax reform simulations by the two competing approaches, namely the unitary and the collective representations. This is the purpose of the following papers, Myck et al. (2006) and Beninger et al. (2006).

The paper is structured as follows. Section 2 presents a precise description of the Pareto allocations in the collective setting considered here. Section 3 describes the steps in the calibration procedure. These include the estimation of preference parameters for singles, as well as the calibration of leisure interaction coefficients, one for each spouse, and of a ‘‘power’’ index. Section 4 concludes, while the Appendix provides further details regarding the specification and estimation methods.

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[1] Within the collective approach, Beninger (2000) and Donni (2003) provide theoretical results for the case of convex budget sets. Moreau and Donni (2002) estimate a household labor supply model for France accordingly, by convexifying the budget sets and selecting participating couples. On the other hand, Blundell, Chiappori, Magnac and Meghir (2001) tackle nonparticipation with linear budget constraints. But the general case has not yet received proper treatment.

2. Pareto efficient household allocations

We focus on households consisting of two working age individuals (for simplicity they will be referred to as the spouses, although they need not be married), with their dependent children. If present, the latter are assumed to have no bargaining power in the household, and their preferences are supposed to be internalized by the parents.[2] Individual prefer ences are represented by the direct utility functions:

$$u^i \quad v^i\left(c^i, l^m, l^f, \mathbf{d}\right), \quad i \quad m, f, \tag{1}$$

defined over a Hicksian consumption aggregate c^i, and leisure amounts l^m and l^f as arguments, and parameterized by a vector **d** of demographic characteristics like number of children and education level. These utility functions are assumed to be increasing and concave in own consumption and leisure. They allow for an externality with respect to the other individual's leisure. Externalities with respect to consumption and household public goods (e.g., rent) are excluded because in the given setting we cannot identify which expenditures are for public consumption and which expenditures have external effects.[3]

To finance total consumption, c, the household has limited resources

$$c \quad c^m + c^f \leq g\left(\ell^m, \ell^f, w^m, w^f, y, \mathbf{d}\right), \tag{2}$$

where c^i, w^i and $\ell^i = T - l^i$ are, respectively, individual i's consumption, gross wage rate and labor supply (with T the individual's time endowment). Total nonlabor income, y, is composed of (possibly) individually assignable nonlabor incomes y^m and y^f and some other nonlabor income y^h. Finally, the function g represents the way in which the tax benefit system to which we henceforth refer simply as the tax system generates disposable income from these ingredients, including some household demographic characteristics.

A household allocation (c^m, c^f, l^m, l^f) is Pareto efficient if it is a solution to the following maximization problem:

$$\max_{c^m, c^f, l^m, l^f} v^m\left(c^m, l^m, l^f, \mathbf{d}\right)$$

subject to the budget constraint (2) and to

$$v^f\left(c^f, l^m, l^f, \mathbf{d}\right) \geq \overline{u}^f, \tag{3}$$

where $\overline{u}^f$ is some required utility level for the female. By varying $\overline{u}^f$, the set of Pareto efficient allocations can be traced out. The required utility level $\overline{u}^f$ captures the outcome of intrahousehold negotiation, and, hence, depends on relative wages, nonlabor incomes and distribution factors. This level of utility also depends on the overall productivity of the household.

Note that in the case of convex utility possibility sets (obtained when utility functions are strongly concave and budget sets are convex; see, e.g., Mas Colell, Whinston, & Green, 1995), the above maximization problem is equivalent to the maximization of a weighted

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[2] Dauphin, El Laga, Fortin, and Lacroix (2006) test for the number of decision makers in a study on consumption within the setting of Browning and Chiappori (1998).

[3] Examples of collective models with more general preferences can be found in Browning and Chiappori (1998) and Chiappori, Fortin, and Lacroix (2002).

mean of individual utilities, with weights that are functions of wages, nonlabor income and distribution factors. This dependence is the main distinction between the collective setting and the unitary model where preferences are independent from wages and nonlabor income (for a precise statement, see Browning, Chiappori, & Lechene, 2006).

3. Empirical specification and identification of the collective labor supply model

There are several applications of collective labor supply models in the literature. In Fortin and Lacroix (1997), Chiappori et al. (2002), Moreau and Donni (2002), and Vermeulen (2005), collective models of labor supply are estimated for two earner households. In all these papers both spouses' labor supplies are assumed to be flexible. This is a serious problem that has only been partially tackled for the case where the male's labor supply is rigid. Blundell et al. (2001) introduce nonparticipation and unobserved preference heterogeneity, under the assumption of constant marginal tax rates. Budget constraints resulting in convex budget sets are considered by Donni (2003). These are important improvements in the application of the collective setting, but they are not completely satisfactory for our purposes.

As already mentioned, we introduce both nonparticipation and nonlinear income tax ation in our setting. This may result in nonconvex budget sets and thus in nonconvex utility possibility sets, which rules out a weighted utilities approach (see above). A first char acteristic of our approach is that the labor supply of both spouses will be treated as discrete. This assumption eases econometric problems related to nonconvex budget sets and has proven useful in the unitary setting (see, for instance, Bingley & Walker, 1997; van Soest, 1995). Taking wages, nonlabor incomes, household characteristics, and the tax system as given, total household consumption can be calculated for each combination of the indi viduals' labor supplies.

However, the identification of a discrete choice collective model (i.e., of individual preferences and of the decision process) with nonconvex utility possibility sets and exter nalities, from observed couples' labor supplies alone, is, if achievable at all, beyond the scope of the present work. Therefore, a second characteristic of our approach is that the model is identified in a piecemeal way, making use of information obtained by econometric estima tions, as well as information coming from a calibration exercise. The crucial identifying assumption is the similarity of individuals' preferences before and after marriage. For the simpler case of two earner households and linearized budget restrictions, this approach is also followed by Barmby and Smith (2001). Manser and Brown (1980) even assume that pref erences do not change as a result of the formation of a new household.[4] Indeed, the (egoistic) utility functions for both spouses are taken to be independent of marital status. In this way, the authors can easily define threat points. As will be seen below, here both individual utility functions change after marriage with the addition of a leisure interaction term.

3.1. Specification of the individual utility functions

We now translate the Pareto optimality problem (3) in an empirically tractable form. First, the individual utility functions v^m and v^f are assumed to be of the following form (i=m, f and $i \neq j$):

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[4] An alternative approach can be found in Browning, Chiappori, and Lewbel (2004), who assume that singles and individuals in couples have the same preferences over a bundle of private good equivalents. For singles, private good equivalents equal observed quantities. For couples, a household consumption tech nology transforms observed quantities into private good equivalents.

$$
\begin{aligned}
v^i\left(c^i, l^i, l^j, \mathbf{d}\right) = \ & \beta_c^i(\mathbf{d}) \ln\left(c^i - \bar{c}(\mathbf{d})\right) + \beta_l^i(\mathbf{d}) \ln\left(l^i - \bar{l}(\mathbf{d})\right) \\
& + \delta^i(\mathbf{d}) \ln\left(l^i - \bar{l}(\mathbf{d})\right) \ln\left(l^j - \bar{l}(\mathbf{d})\right).
\end{aligned} \tag{4}
$$

The utility functions are seen to consist of two parts. First, a part corresponding to the familiar linear expenditure system (LES), where the preference parameters $\bar{c}(\mathbf{d})$ and $\bar{l}(\mathbf{d})$ capture ''subsistence'' or minimum consumption and leisure which are allowed to depend on household characteristics **d**. In particular, all time not spent in the market is interpreted as leisure, but the term $\bar{l}(\mathbf{d})$ is assumed to consist of time required for regeneration and for essential caring tasks when there are children in the household. Second, the utility functions include a leisure interaction term that accounts for potential complementarity in spouses' leisure or any other interaction between them. The introduction of this term relaxes the strong assumption of separability of individual preferences usually made in the empirical literature on collective models.[5] However, as already mentioned, this setting does not allow for externalities with respect to consumption. Finally, individual *i*'s utility function is strictly increasing and quasi concave in own consumption c^i and own leisure l^i, if the following conditions are satisfied ($i=m, f$ and $i \neq j$):

$$
\beta_c^i(\mathbf{d})>0, \quad \text{and} \quad \delta^i(\mathbf{d})> -\beta_l^i(\mathbf{d})/\ln\left(l^j - \bar{l}(\mathbf{d})\right). \tag{5}
$$

This setting basically interprets all time not spent on the labor market as leisure, although we do put some effort in deducting from non market time some time requirements related to the demographic structure of the household (see next section). Naturally, it would clearly be desirable to model shared leisure, the presence of children and household production in an explicit way.[6] We reach the usual limits of the literature here. In particular, the distinction between pure leisure and nonmarket time as an input in the household production process is difficult in the absence of adequate data (time use data containing also household characteristics and incomes).[7] Note, however, that recent findings in Donni (2004) show that (i) simple functional forms which are consistent with the traditional collective model of labor supply can sometimes be compatible with more sophisticated models incorporating domestic production, (ii) if the domestic good is marketable, these models can be tested and partially identified using traditional household surveys (i.e., without resort to time allocation surveys).

We now turn to the problem of identifying the parameters $\beta_c^i(\mathbf{d})$, $\beta_l^i(\mathbf{d})$ and $\delta^i(\mathbf{d})$ ($i=m$, f) from observed labor supplies and household aggregate consumption, given the mix of effects coming from individual preferences and an intrahousehold bargaining process that is reflected in $\bar{u}^f$ (see condition (3)).

3.2. Identification of the LES parameters of individual preferences

A crucial assumption made here is that the LES parameters $\beta_c^i(\mathbf{d})$ and $\beta_l^i(\mathbf{d})$ ($i=m, f$) can be identified on the basis of a sample of single males and females. This assumption, which

[5] Browning and Chiappori (1998) is an exception, but their paper is not primarily concerned with labor supply and focuses on linear budget constraits. See also Chiappori et al. (2002).

[6] Apps and Rees (2001) present a model with household production that also includes a calibration step. The theoretical distinction between individual and shared leisure in a collective framework is investigated by Fong and Zhang (2001). Apps (2003) discusses the limitations of time use surveys in this context.

[7] See Browning and Gørtz (2005) for an empirical study taking advantage of such data.

implies that, apart from a leisure interaction term, singles and individuals in couples have the same preferences, *ceteris paribus*, is of course questionable, but it is not totally unre alistic. Moreover, it should be stressed that, in so far as singles and married individuals can have different marginal rates of substitution between own leisure and consumption, they can have different reactions to tax reforms. Proceeding in a piecemeal way, as announced, we estimate the parameters $\beta_c^i(\mathbf{d})$ and $\beta_l^i(\mathbf{d})$ separately, using two samples of single males and single females. But ultimately, simultaneous estimation using data concerning single and married males and females would be desirable, as it would lead to efficiency gains.

Although the minimum consumption and leisure $\overline{c}(\mathbf{d})$ and $\overline{l}(\mathbf{d})$ can in principle be estimated, for simplicity we choose to calibrate them on the basis of published time use data and social minima for the different countries.[8] This information is typically available for different demographic compositions of the household.

For instance, for the UK, the National Statistics Omnibus Survey time use module indicates that the average regeneration time for childless singles under pension age is 10.1 h per day for women and 9.7 for men. For Germany, corresponding figures for childless singles (drawn from the German Time Budget Survey 1991/1992, see Beblo, 2001) are 10.2 h of daily physiological regeneration and almost 3.3 h for housework for women (10 and 2.4 for men).

In the presence of children, some additional time requirements need to be taken into account. For France, this information is obtained from the 1998 INSEE Time Allocation Survey. An additional weekly time requirement is set according to the age of the youngest child. For the wife and the husband this equals, respectively, 14 and 7 h if at least one child is below age 5, 6 and 3 h if at least one child is between 6 and 11, 5 and 2 h if at least one child older than 11. For German couples (see Beblo, 2001), the additional time requirement for a full time working man is 0.8 h per day, if he is living together with at least one child up to age 6. For a full time working woman this rise in household demands amounts to about 2 h. For older children, i.e., if all children are aged between 7 and 12, 0.4 h per day are added to the father's minimum ''leisure'' and 1 h per day to the mother's.

3.3. Identification of leisure interaction terms and bargaining power

The leisure interaction terms $\delta^m(\mathbf{d})$ and $\delta^f(\mathbf{d})$ and the female's required, or negotiated, level of utility $\overline{u}^f$ remain to be identified. If the Pareto frontier were concave to the origin (or the utility possibility set were convex), the optimal choice of hours could be obtained by maximizing a household social welfare function defined as a convex combination of individual utility functions. However, as a consequence of the nonconvexity of many budget sets, the utility possibility sets of many households turn out not to be convex (this concerns 42% of the households in Beninger et al., 2006). Thus, in order to capture all behavior possibilities, we must adopt another approach. The chosen procedure consists of determining each household's Pareto frontier and, for given preference parameters, searching for that point of the frontier which best corresponds to observed behavior.

3.3.1. Pareto frontier and identification of the bargaining power

Let us now describe the calibration procedure. Given the two observed labor supplies we can identify two parameters at the household level. Therefore, we first restrict attention to

[8] This is also what Barmby and Smith (2001) do.

the identification of the bargaining power and a common leisure interaction term. The idea is the following: for each household and for each element of a discrete set of values (δ, δ) for the leisure interaction terms (δ^m (**d**), δ^f (**d**)), chosen identical for both spouses, a discrete set of Pareto efficient allocations is calculated. An allocation from this set is singled out on the basis of the best fit with observed labor supplies. This serves to identify the corresponding bargaining power parameter. In a subsequent stage, individual leisure interaction terms δ^m and δ^f are identified on the basis of a parametric specification, common to all households, for the woman's bargaining power.

For each household, the algorithm used in practice is the following.

1. For each $\delta \in \{\delta_{\min},...,\delta_{\max}\}$, where $\delta=\delta^m$ (**d**)$=\delta^f$ (**d**), define the utility level $u^f_{\max}$ (δ) as the value of the woman's utility when she receives a maximum share of total household consumption (set, e.g., to 90% for Germany), and the leisure amounts l^m and l^f are chosen under condition (5) in such a way that her utility is maximal. Denote $u^m_{\min}$ (δ) the corresponding utility level achieved by the man. The other extreme [$u^m_{\max}$ (δ), $u^f_{\min}$ (δ)] is defined symmetrically. Thus, the points with coordinates [$u^m_{\min}$ (δ), $u^f_{\max}$ (δ)] and [$u^m_{\max}$ (δ), $u^f_{\min}$ (δ)] can be considered as the boundaries (dictatorial positions) of the Pareto frontier for each value of the common preference parameter δ.
2. Define K utility levels u^f_k $(\delta)=u^f_{\min}$ $(\delta)+(k/K)[u^f_{\max}$ $(\delta) - u^f_{\min}$ $(\delta)]$, where $k=0,...,K$. Note that the boundaries of the Pareto frontier are obtained for $k=0$ and $k=K$.
3. For each k, choose the labor supplies in their respective choice sets, D^i, $i=f$, m, so that the man maximizes his utility subject to the household budget constraint and her required utility level:

$$\max_{c^m,c^f,l^m,l^f} v^m\left(c^m, l^m, l^f, \mathbf{d}; \delta\right) \tag{6}$$

subject to

$$v^f\left(c^f, l^m, l^f, \mathbf{d}; \delta\right) \geq u^f_k(\delta),$$

$$c^m + c^f \leq g\left(\ell^m, \ell^f, w^m, w^f, y, \mathbf{d}\right),$$

$$\ell^i \in D^i, i \quad m, f.$$

This maximization procedure results in an optimal labor supply and consumption bundle [$\ell^m(\delta,k)$, $\ell^f(\delta,k)$, $c^m(\delta,k)$, $c^f(\delta,k)$] for each k.
4. Given δ, choose in the discrete set of Pareto efficient allocations the one that mini mizes the criterion

$$\lambda\left[\ell^m(\delta,k) \quad \ell^m_o\right]^2+\left[\ell^f(\delta,k) \quad \ell^f_o\right]^2 \tag{7}$$

over k, where $\ell^i_o(i=m,f)$ is individual i's observed labor supply, and λ is chosen in order to avoid dissymmetry in the fit for males and females (male labor supply is typically more concentrated).[9] This step results in an optimal labor supply and consumption

[9] In a full estimation approach, λ would be estimated in a two stage procedure like feasible generalized least squares. Here it is only roughly calibrated ex ante, with the main aim of taking into account different discretizations of labor supplies of men and women, as well as different hours distributions.

allocation $[\ell^m_*(\delta),\ell^f_*(\delta),c^m_*(\delta),c^f_*(\delta)]$ for each couple of individual utility functions $v^m(.;\delta)$ and $v^f(.;\delta)$. If there are multiple allocations that minimize the above criterion, then the allocation minimizing the difference between the consumption levels of the spouses is selected. This step results in the choice of an allocation indexed by $k_*(\delta)$.
5. Over all individual preferences characterized by δ, choose the value δ_* that minimizes criterion (7) with $\ell^i_*(\delta)$ in place of $\ell^i(\delta,k_*(\delta))$.

Given the above procedure, a measure for the female's bargaining power in the interval [0,1] can be defined as

$$\mu_f \quad k_*(\delta_*)/K. \tag{8}$$

This is simply a parameterization of the household's location on the Pareto frontier; the nearer this number is to one, the nearer the woman is to her dictatorial position $[u^m_{\min}(\delta),u^f_{\max}(\delta)]$, and thus the higher is her bargaining power. A symmetric parameteri zation can be based on the same algorithm with the genders permuted, and this leads to a measure μ_m for his bargaining power. If the Pareto frontier were a straight segment, μ_f and μ_m would add up to one, but if it were a portion of a circle, then their squares would add up to one instead. This motivates a second measure which depends on the local curvature of the Pareto frontier at the chosen point. Define the female's bargaining power as

$$\omega_f \quad \mu_f^\alpha, \tag{9}$$

where α is the solution to

$$\mu_f^\alpha + \mu_m^\alpha \quad 1. \tag{10}$$

This measure yields more meaningful comparisons across households than the first one because it takes account of the shape of the Pareto frontier which varies across households.

As an alternative, we could calibrate $u^f_k(\delta)$ directly, as Bargain, Beninger, Laisney, and Moreau (2002) do, and normalize the female utility scale by the multiplicative factor $\gamma=\sum u^m/\sum u^f$, so as to equalize wives' and husbands' utilities on average. Her utility becomes γu^f so that the sum of normalized female utilities over the population equals the sum of male utilities. Using calibrated values, we then suggest a third index for the female bargaining power whose interpretation is rather straightforward:

$$\varpi_f \quad \frac{\gamma u^f_*}{\gamma u^f_* + u^m_*}. \tag{11}$$

Note that the above proposed measures of an individual's bargaining power are not insensitive to the choice of the cardinalization. The index ϖ_f is clearly invariant to (identical) linear transformation (scaling) of the spouses' utilities, while some algebra shows that the index ω_f is invariant to affine transformations with identical slopes for both spouses. The characterization of measures of bargaining power that would be robust to the choice of cardinalization is still on our research agenda. An interesting avenue for future research may be the derivation of money metric measures of bargaining power. This approach, however, is not straightforward in a discrete setting and may become even more complicated within a collective model (see Bhattarai & Whalley, 1997; Browning et al., 2004; Small & Rosen, 1981, within a decentralized continuous collective model with a household consumption technology).

The next step in the calibration exercise consists of a logistic regression of the bargaining power index on a number of variables. Generally, the bargaining power is supposed to depend on wages, prices, nonlabor income and distribution factors. However, as mentioned by Browning and Lechene (2001), the theory of the collective model does not give precise guidance as to which variables should be included in the set of distribution factors. But the empirical literature has highlighted some exogenous factors which potentially influence bargaining power. Estimations usually concern the sharing rule in decentralized collective models.[10] We can think of socio demographic characteristics of the household such as (i) the allocation of child benefits among spouses, (ii) the age difference between the spouses, (iii) the difference in education level (under the assumption that human capital decisions are exogenous), and (iv) regional indicators that may pick up cultural differences or differences in sex ratios. The amount of unearned income brought independently by each spouse may also be crucial.[11]

Since we are primarily concerned with the effect of taxation in a collective setting, it seems appropriate to include a distribution factor related to the relative earnings of the spouses, or rather to their relative earning potential. Gross wage rates are clearly not attractive, because they do not change with the reforms at stake. Net wages are not attractive either because the theory of the collective model imposes the exogeneity of the distribution factors. Still, some measure summarizing the way in which the tax system potentially modifies the relative net earnings of the spouses can be included. In the empirical exercise, the female's and male's marginal contributions to the household's earnings when switching from the lowest labor supply choice to the highest are opted for. Specifically, we consider two such variables, y_d^{40} and y_d^{20} defined as follows. Let p_f^j and p_m^j denote the observed sample frequencies of (discretized) weekly labor supplies ℓ^j of wives and husbands, respectively. Denote $R_{mj}^{fj'}$ the household disposable income when the husband works ℓ^j hours and the wife works $\ell^{j'}$ hours. Variable y_f^{40} defined as:

$$y_f^{40} = \sum_{j=1}^{J} p_m^j \left(R_{mj}^{f40} - R_{mj}^{f0} \right) \tag{12}$$

measures the expected increase in the household disposable income if the wife switches from 0 to 40 h, the expectation being taken over the males' hours distribution. Defining y_f^{20} and y_m^{40} similarly, we then consider the two ratios $y_d^{40}=y_f^{40}/y_m^{40}$ and $y_d^{20}=y_f^{20}/y_m^{40}$ which we term ''relative earning power'' of the wife at 40 h (resp. 20).

In view of the above discussion, let **e** be a vector of explanatory variables that do not belong to the vector of household characteristics **d**, but that are assumed to affect spouses' bargaining power (relative earning power, assignable nonlabor incomes, etc.). The estimated bargaining power index is then denoted by $\omega_f(\mathbf{e}, \mathbf{d})$. The cross sectional variation in potential relative earnings identifies the effect of the tax system on the power index.[12] Note, however, that our indicator of relative earning power does not allow to distinguish

[10] For instance, using the PSID, Chiappori et al. (2002) find a significant impact on the sharing rule of the sex ratio and the divorce laws across states for US households.

[11] Indeed, the hypothesis of income pooling specific to the unitary model, but relaxed by the collective model, is rejected by most tests in the empirical literature on collective models already quoted.

[12] Clearly, a more reliable identification could be obtained by using data covering a tax reform along the lines of Blundell, Duncan, and Meghir (1998).

between different types of tax benefit instruments generating an identical change in earning capacity. The role of these tax benefit instruments may be different in the nego tiation and, hence, the nature of a reform can be important. A targeted reform with a ‘‘gender tag’’ as in Lundberg, Pollak, and Wales (1997) may have a stronger influence on household behavior than a reform of income taxation in a country with joint taxation (see Donni, 2003, for a more general discussion).

3.3.2. Identification of the leisure interaction terms

Once we have obtained an estimated bargaining power index for each household, the above algorithm is redefined to include this $\omega_f(\mathbf{e},\mathbf{d})$ and two different leisure interaction terms, one for each spouse.

1. First, for each pair (δ^m,δ^f) with $\delta^i \in \{\delta^i_{\min},\ldots,\delta^i_{\max}\}$, and $i=m,f$, calculate $u^f_{\max}(\delta^m,\delta^f)$ and $u^f_{\min}(\delta^m,\delta^f)$ in the same way as before.
2. Define K utility levels

$$u^f_k\left(\delta^m,\delta^f\right) \quad u^f_{\min}\left(\delta^m,\delta^f\right)+(k/K)\left[u^f_{\max}\left(\delta^m,\delta^f\right) \quad u^f_{\min}\left(\delta^m,\delta^f\right)\right], \tag{13}$$

   where k=0,...,K. Choose from this set the required utility level $u^f_{k_*}\left(\delta^m,\delta^f\right)$ such that k_* is the integer nearest to $\omega_f(\mathbf{e},\mathbf{d})^{1/\alpha}K$.[13]
3. Solve the following Pareto optimality problem:

$$\max_{c^m,c^f,l^m,l^f} v^m\left(c^m,l^m,l^f,\mathbf{d};\delta^m\right) \tag{14}$$

   subject to

$$v^f\left(c^f,l^f,l^m,\mathbf{d};\delta^f\right) \geq u^f_{k_*}\left(\delta^m,\delta^f\right),$$

$$c \quad c^m+c^f \leq g\left(\ell^m,\ell^f,w^m,w^f,y,\mathbf{d}\right),$$

$$\ell^i \in D^i,\ i \quad m,f.$$

   For each pair of utility functions, this results in an optimal labor supply and con sumption bundle

$$\left[\ell^m\left(\delta^m,\delta^f\right),\ell^f\left(\delta^m,\delta^f\right),c^m\left(\delta^m,\delta^f\right),c^f\left(\delta^m,\delta^f\right)\right]. \tag{15}$$

4. Choose the individual utility functions, defined by δ^m and δ^f, that minimize the cri terion

$$\lambda\left[\ell^m\left(\delta^m,\delta^f\right) \quad \ell^m_o\right]^2+\left[\ell^f\left(\delta^m,\delta^f\right) \quad \ell^f_o\right]^2. \tag{16}$$

   Again, in the case of multiple solutions select the solution entailing the smallest discrepancy between the consumption levels of the spouses.

[13] Recall the definition of μ_f in equation (8).

5. Finally, the calibrated δ^m and δ^f parameters are regressed on a set of household characteristics. Estimated parameters, say $\delta^m(\mathbf{d})$ and $\delta^f(\mathbf{d})$, close the model.[14]

At the end of the exercise we have at our disposal a set of ''collective'' parameters which allow to predict labor supplies by means of the identified collective model and conditional on observed and imputed gross wages, household characteristics and the tax system. In this way, we can generate a ''collective'' dataset which forms the baseline for (i) welfare analysis of tax reforms with a collective model, and (ii) estimation of a unitary model for quantification of the distortions in welfare analysis associated with the use of a unitary model in a collective world (see Myck et al., 2006; Beninger et al., 2006).

The Appendix gives further details on the estimation procedures used and empirical results are available for different countries in Bargain et al. (2002, France), Beninger, Laisney, and Beblo (2002, Germany), Blundell, Lechene, and Myck (2002, UK), Carrasco and Ruiz Castillo (2002, Spain), Chiuri and Longobardi (2002, Italy), and Vermeulen (2002, Belgium).

4. Conclusion

Despite very important theoretical advancements in the modeling of labor supply of individuals in couples in the recent decade, there are few empirical applications of the collective model. Accounting simultaneously for nonparticipation, nonconvex budget sets and nonegoistic preferences present difficulties which have not yet been overcome by usual identification and estimation techniques. This may explain why the unitary model has remained the favored model in practical applications for the purpose of fiscal reform analysis.

The aim of this methodological paper was to present an alternative approach to implement a collective model in the presence of nonconvex budget sets, nonparticipation and individual preferences accounting for possible complementarity in spouses' leisures. The methodology is original in that it provides a way to map the complete Pareto frontier of each household in the dataset using calibration. Identification relies on the estimation of preference parameters on samples of male and female singles, assuming some persistence in consumption leisure preferences, and on the calibration of the bargaining outcome and marriage specific preference parameters on observed labor supplies of individuals in couples.

In its capacity to handle nonconvex budget sets and labor force participation decisions of both spouses, this model allows to simulate realistic tax reforms and to account for their impact not only on budget constraints but also on the balance of power in the household. This is the purpose of the following paper of the current issue, Myck et al. In addition, the model allows to compare unitary and collective representations. More specifically, a unitary model can be estimated on collective behavior (labor supplies predicted by the collective model). Subsequently, behavioral responses of this unitary model can be com pared to those of the correct collective model, to gauge the discrepancies due to the

[14] One could think of iterating the procedure further, by re calibrating the bargaining power position given the estimated $\delta^m(\mathbf{d})$ and $\delta^f(\mathbf{d})$ until convergence, but this would be costly in terms of computer time.

wrongful unitary assumption. This is the topic of the last paper in the current issue, Beninger et al.[15]

In addition to familiar limitations (limited account of children, absence of domestic production, purely private consumption), the construction of the collective model presented here relies on a piecemeal approach that mixes estimation and calibration procedures. An obvious next step is to use the same identification assumptions with a pure estimation strategy.

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Appendix

For the sake of completeness we give here some details on the estimation procedures used.

A.1 Wage equations

In order to obtain wage rates for the whole population, including those not working, we estimate wage equations.

For singles we posit a linear normal selection model and use either the maximum likelihood method or the two steps Heckman procedure. We tried a number of different estimation methods, including also two step methods with other regressors than the predicted normal hazard used in the Heckman approach, but the choices mentioned above gave the most accurate predictions for working singles.

For couples we also estimate wage equations separately for wives and husbands. The following conceptual difficulty arises here due to selectivity: a participation model would need to be based on the collective framework, which is difficult. However, Lewbel (2000) proposes an estimation method for the selection model which does not require the specification of the selection mechanism.[16] The method relies on the existence of a variable which is monotonically related to the selection variable: in the case of participation, household unearned income is a plausible candidate though admittedly less so in the collective than in the unitary approach, because of the effect of household unearned income on intrahousehold allocation. We use the simplest of the estimators proposed by Lewbel for wives. For men we apply OLS, as the selectivity problem is much less severe for them, and the OLS predictions are more accurate than those based on the Lewbel estimator.

[15] Although we proceed in a fairly naive way by treating the collective model as deterministic in Beninger et al., there is scope for full scale simulation taking into account all types of unobserved heterogeneity considered in the estimation/calibration approach, as well as the uncertainty embodied in the estimated parameters. This concerns the estimation of wages for the nonparticipants, the estimation of preference parameters for singles, and the estimation/calibration of $\omega_f(\mathbf{e}, \mathbf{d})$, $\delta^m(\mathbf{d})$ and $\delta^f(\mathbf{d})$.

[16] We would like to thank Costas Meghir for pointing this out.

A.2 Preferences of single women and men

We estimate preferences separately for women and men, assuming LES type preferences:

$$v_i(c_i, l_i) \quad \beta_c^i \ln(c_i \quad \bar{c}_i) + \beta_l^i \ln\left(l_i \quad \bar{l}_i\right) \quad i \quad f, m, \tag{17}$$

where c_i represents consumption (i.e., disposable income in this static model) and l_i de mand for leisure. c_i and $\bar{l}_i$ are, respectively the ‘‘minimum’’ requirements in consumption and leisure. Instead of estimating these, which proved difficult, we chose to calibrate them.

The budget constraint is defined as:

$$c_i \quad g(l_i, w_i, y_i, \phi_i) \quad i \quad f, m, \tag{18}$$

where w_i and y_i are, respectively i’s gross wage rate and i’s unearned income, ϕ_i represents a vector of characteristics relevant to the tax system, and the function g expresses the tax benefit schedule.

For the estimation, we use a mixed multinomial logit model (MMNL) with mass points on the consumption and leisure coefficients in order to account for unobserved heteroge neity (see Heckman and Singer, 1984; Hoynes, 1996).[17] We suppose that each person has J alternative values ℓ^j for his/her weekly labor supply, leading to leisure choices l^j=T−ℓ^j, where T is the total time available in a week: 168 h a week. The contribution to the likelihood for person i choosing combination (c_i^j, l^j) is:

$$L \quad \sum_{r \quad 1}^{R} p_r \frac{\exp\left[\beta_{cr}^i \ln\left(c_i^j \quad \bar{c}_i\right) + \beta_{lr}^i \ln\left(l^j \quad \bar{l}_i\right)\right]}{\sum_{j' \quad 1}^{J} \exp\left[\beta_{cr}^i \ln\left(c_i^{j'} \quad \bar{c}_i\right) + \beta_{lr}^i \log\left(l^{j'} \quad \bar{l}_i\right)\right]} \tag{19}$$

where R denotes the number of mass points (or regimes), and p_r the probability associated with mass point r in the mixture. Three mass points appear sufficient, given the heavy dominance of one of the regimes for both preference estimations, single women as well as single men. We do not impose the constraint β_c^i+β_l^i 1 in the estimation, but check that the estimates are positive, which allows to rescale the utility function by β_c^i+β_l^i afterwards. An alternative specification with β_c^i $F(z_i\gamma)$, where F denotes the logistic cumulative distri bution function, and $z_i\gamma$ a linear index depending on characteristics for individual i, used for instance by Hoynes (1996), led to much lower likelihood values. The reason for the superiority of not imposing the restriction β_{cr}^i+β_{lr}^i 1 is that it amounts to fixing the scale of utility. The MMNL model results from adding iid error terms to (17) for each possible choice, with an extreme value distribution. This entails a fixed variance, which in turns identifies the scale of utility (and thus the sum of the marginal valuations of leisure and consumption).

In order to ensure that the probabilities p_r do lie between 0 and 1, we adopt the following logit parameterization:

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[17] We also estimated random parameter logit models (RPL, see, e.g., McFadden and Train, 2000) with a normal distribution of the constant terms in β_c^i and β_l^i. For several countries we obtained significant dis persion for the consumption term, but not for the leisure term, both for men and women. The specification with mass points on β_c^i alone strongly dominated the RPL specification on most cases, both in terms of likelihood and in terms of accuracy of predictions.

$$p_r = \exp(e_r) \Big/ \left[1 + \sum_{s=1}^{R-1} \exp(e_s)\right]; \quad r = 1, \ldots, R-1,$$
$$p_R = 1 - \sum_{s=1}^{R-1} p_s. \tag{20}$$

After estimation, we allocate each observation to the regime yielding the best hours prediction.

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