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Essays on Asset Pricing

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Introduction

My dissertation consists of three self-contained essays on asset pricing. The first one is my Job market paper called "The variance risk premium around the world". The second one is a joint work with with Lieven Baele, Joost Driessen and Oliver Spalt and is called "Cumulative Prospect Theory and the volatility premium". Finally, the third essay is a joint work with Lieven Baele and is called "Understanding Industry betas". Each essay is summarized in turn.

My first paper investigates the variance risk premium in an international setting. In this paper I first provide new evidence on the basic stylized facts traditionally documented for the US. I show that while the variance premiums in several countries are, on average, positive and display significant time variation, they do not predict local equity returns in countries other than the US. Then, I extend the domestic model in Bollerslev, Tauchen and Zhou (2009) to an international setting. In light of the qualitative implications of my model, I provide empirical evidence that the US variance outperforms all other countries' variance premiums in predicting local and foreign equity returns.

The second paper explores the ability of Cumulative Prospect Theory (CPT) to explain the observed negative volatility premium embedded in option prices. In this paper, we simulate equilibrium prices for zero-beta straddles when agents are endowed with CPT-type preferences. We find that overweighting the probability of extreme events, one of the components of CPT, plays a key role in increasing the implied price of straddles. In contrast, increasing the scale of the value function, the second component of CPT, yields minor changes in the equilibrium prices of these straddles unless agents display a very large degree of loss aversion. We also explore these implications in a time-varying framework where we find that the price agents are willing to pay to hedge the risk of extreme events depends on the previous performance of their portfolio.

Finally, the third paper models and explains the dynamics of market betas for 30 US industry portfolios between 1970 and 2009. We use a DCC-MIDAS and kernel regression technique as alternatives to the standard ex-post measures. In this paper, we find betas to exhibit substantial persistence, time variation, ranking variability, and heterogeneity in their business cycle exposure. While we find only a limited amount of structural breaks in the betas of individual industries, we do identify a common structural break in March 1998. Finally, we find the cross-sectional dispersion in industry betas to be countercyclical and negatively related to future market returns.

CHAPTER 1

The Variance Risk Premium around the World

Abstract

This paper investigates the variance risk premium in an international setting. First, I provide new evidence on the basic stylized facts traditionally documented for the US. I show that while the variance premiums in several countries are, on average, positive and display significant time variation, they do not predict local equity returns in countries other than the US. Then, I extend the domestic model in Bollerslev, Tauchen and Zhou (2009) to an international setting. In light of the qualitative implications of my model, I provide empirical evidence that the US variance outperforms all other countries' variance premiums in predicting local and foreign equity returns.

JEL Classification: E44, F36, G12, G13, G15.

Keywords: variance risk premium, economic uncertainty, interdependence, international integration, comovements, return predictability.

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1.1 Introduction

Traditional asset pricing models have mainly focused on characterizing the reward for equity risk. However, such models typically fail to capture the reward for bearing variance risk. The variance risk premium is formally defined as the difference between the risk neutral and the physical expectation of the total return variation. It can be estimated using model-free measures as the difference between the option implied variance and the expected realized variance. The observed variance premium in the US is large and varies significantly over time. In order to generate a time-varying variance premium, standard asset pricing models have been adjusted in different ways. One strand of the literature, and the one that will be followed in this paper, links the variance risk premium to macroeconomic uncertainty. This strand follows the intuition behind the long-run risk model in Bansal and Yaron (2004) (BY hereafter), and the idea that agents have a preference for an resolution of uncertainty in Bansal et. al. (2005). Extending BY's model, Bollerslev, Tauchen and Zhou (2009) (BTZ hereafter) show that the variance premium predicts equity returns; an implication for which they find empirical evidence for the US. An alternative strand of the literature relates the variance premium to agents' attitudes towards non-normalities in the distribution of returns. In Bakshi and Madam (2006), for example, the variance risk premium is explained by the desire of risk averse agents to buy protection against extreme events. In a similar vein, Bekaert and Engstrom (2010), Todorov (2010), and Gabaix (2009), using different methodologies, focus on the interplay between returns, risk aversion and extreme events to explain many asset pricing regularities, including the variance risk premium.

Existing work, both theoretical and empirical, has predominantly focused on the US market. This paper adds to the literature by extending the variance premium analysis to an international setting. The contribution is threefold. First, I provide new evidence on the basic stylized facts related to the variance premium for a total of eight countries. I show that while the variance premiums display significant time variation in all countries analyzed, the local return predictability does not hold internationally. Then, I extend the domestic model in Bollerslev, Tauchen and Zhou (2009) to an international setting. My model links the variance premium to local and aggregate macroeconomic uncertainty and yields a qualitative explanation for the local predictability puzzle. Finally, I provide new empirical evidence to investigate the main qualitative implications of my model. The empirical evidence suggests that the US variance premium predicts the equity returns in the US as well as in any other country in the sample. In addition, the evidence also suggest that the US variance premium plays a key role in predicting the variance premium correlations as well as the equity return correlations across countries.

I now discuss the different parts and contributions of the paper in more detail. In the first part, I investigate the main stylized facts related to the variance premium previously documented for the US in an international setting. In particular, I investigate whether the time-varying and positive nature of the variance premium as well as its capacity to predict returns holds internationally. In order to do so, I collect data for the US, Germany, UK, Japan, Switzerland, The Netherlands, Belgium, and France for the sample period 2000 to 2009. As it has become standard in the literature, the variance premiums for all countries are estimated using model-free measures of the expected variance of returns. Thus, the (squared of the) model-free implied volatility (IV) index for each equity market approximates the expectation of the total return variation under the risk neutral measure (Carr and Madan, 1998; and Britten-Jones and Neuberger, 2000) while the expectation under the physical measure is approximated by a conditional forecast of the actual realized variance.

The single-country evidence shows that the variance premiums display significant time variation and are, on average, positive for all countries in the sample. This international evidence is in line with previous findings for the US.[1] However, I show that the variance premium can predict local equity returns only in the US. For any other country analyzed, the evidence suggests that the local variance premiums cannot predict local equity returns. This finding suggests a puzzle that cannot be solved by the existing domestic models where the variance premium implicitly explains the variation in the local equity premium.[2]

The strictly domestic nature of the existing models motivates the theoretical contribution of this paper. In the second part, I propose a model to investigate the role of the variance premium in explaining the interactions across international equity and option markets. The model is a two-country extension of that in BTZ and extends the intuition that agents have a preference for an early resolution of uncertainty to an international setting. The macroeconomic uncertainty is characterized in my model by the volatility dynamics of the consumption growth of each country and is allowed to be transmitted across countries given a unique representative agent endowed with recursive preferences. In such a setting, the shocks to macroeconomic uncertainty in any country characterize the variance premium in all countries. In particular, the variance premiums of the two countries reveal the volatility of volatility of consumption generated in both countries. Now, given that changes in the volatility of volatility also explain a portion of the total risk premiums of any country, the model not only implies that variance risk is priced but also provides the intuition for the potential role of the variance premium of any country in predicting local and foreign equity returns. In other words, agents demand a reward for the existing local and foreign sources of risk (i.e., the volatility and the volatility of volatility of consumption). Although this uncertainty transmission mechanism is bidirectional, the model explicitly assumes a leader economy. The consumption process of this leader economy is entirely driven by local shocks. However, the shocks of the leader country consumption process can be partially transmitted to a second country, the follower.

My model yields several qualitative implications for the interactions across international equity and option markets that explain the inability of the variance premium to predict local equity returns in countries other than the US. The first main implication of my model is that the variance premium in each country is uniquely characterized by the volatility of volatility of consumption (VoV) of the two countries. The load of each country's VoV increases with the relative size of its economy and the degree of economic dependence among countries (leader-follower relation). As a consequence of having common components, the variance premiums are highly correlated across countries; and the cross-country variance premium correlation is mainly driven by the VoV generated in the leader country. Thus, the leader country variance premium plays the key role in predicting the variance premium correlations across countries. The second main implication of my model is that the VoV of the two countries also load on all countries' equity premiums. Similar to the implication for the variance premiums, the load of VoV increases with the relative size of each economy and the implied correlation of the consumption processes. This second implication links the variance premium to all countries' equity premiums. As a consequence, this implication explains the possibility that the variance premium of a leader economy predicts other countries' equity returns which in turn implies that the leader country variance premium plays the key role in predicting equity return correlations across countries.

The third contribution of this paper is that it provides new empirical evidence on the two main qualitative implications of my model. That is, I investigate the fundamental linkages between the variance premiums across countries as well as the interplay between the variance premiums and international equity returns. To do so, I first provide evidence that the

[1] See for instance Britten-Jones and Neuberger (2000), Jiang and Tian (2005), Bakshi and Madan (2006), Carr and Wu (2009), Bollerslev, Gibson and Zhou (2010), and BTZ, among others.

[2] BTZ, Zhou (2010), and Drechsler and Yaron (2010) find empirical evidence for their respective model-implied return predictability. However, Bekaert and Engstrom (2010) find weak evidence of return predictability.

variance premiums are highly correlated across countries as suggested by the common loads of volatility of volatility in the variance premiums suggested by my model. As a natural extension of the high variance premium correlation across countries, I also investigate the role of the variance premium in explaining unusual variance premium correlations across countries at the daily frequency. The analysis of unusual correlations closely follows the contagion literature (Bekaert, Harvey, and Ng, 2005) and suggests that the variance premiums are unusually correlated after extreme US variance premium episodes. The US variance premium contagion pattern only holds in the very short term, from 1 to 10 days.

Next, I investigate the second main implication of my model which suggests that the leader country variance premium plays the key role in predicting local and foreign equity returns. On the one hand, I confront the evidence on the poor performance of the local variance premiums in predicting local returns for countries other than the US. Thus, I provide new evidence that only the US variance premium predicts equity returns for all countries in the sample. The predictive power of the US variance premium over international equity returns holds for horizons between 1 to 6 months, and reaches its maximum at the quarterly horizon. In addition, I show that the US variance premium outperforms all other countries' variance premiums in predicting local and foreign equity returns. On the other hand, I provide evidence that international equity returns tend to comove more intensely following episodes of increasing US variance premium. The predictive power of the variance premium for both equity returns and cross-country return correlations holds for horizons between 3 and 6 months and is additional to that of traditional (local or US) variables such as the term spread and the dividend yield.

The remainder of the paper is organized as follows: Section 1.2 introduces the main definitions and data used throughout the paper. Section 1.3 provides international single-country evidence on the regularities related to the variance premium. Section 1.4 introduces the international consumption based general equilibrium model and analyzes its qualitative implications. Section 1.5 investigates the empirical evidence in light of the implications of my model. Finally, Section 1.6 concludes.

1.2 Data and Definitions

In this section, I introduce the data used to estimate the monthly variance premiums for the following countries: US, Germany, Japan, UK, Switzerland, The Netherlands, Belgium and France. The variance premium is defined as the difference between the risk neutral and the physical expectation of the market return variation between time t and one month forward $t+1$ for each market. It is estimated, as it has become standard in the related literature, using model-free measures for the expectations of the total return variation.

I approximate the risk neutral expectation of the market return variation as (the square of) the model-free options implied volatility (IV) index for each market. The methodology for the IV index was initially proposed by Carr and Madan (1998) and Britten-Jones and Neuberger (2000). The IV index has shown to provide a much better approximation to the expected risk neutral return variation than previously Black-Scholes based measures (Bollerslev, Gibson and Zhou, 2010). The IV indices are constructed from a portfolio of European calls where the underlying is a representative market index for each country as in

$$iv_{j,t} = 2\int_0^\infty \frac{C_{j,t}(t+1, \frac{K}{B_j(t,t+1)}) - C_{j,t}(t,K)}{K^2} dK,$$

where $C_{j,t}$ are the prices of calls with strikes from zero to infinity, and $B_j(t,t+1)$ are the local prices of zero-coupon bonds with one month ahead maturity.

The availability of the IV index for the countries analyzed is limited by the recent development of their option markets. The index was first reported for the US by the Chicago

Board Options Exchange (CBOE), the VIX, in 1993 (with data from 1990). The VIX was adapted to the model-free methodology in 2003, and was then called the New-VIX. An index for the German market, the VDAX, was released by the German Stock Exchange (Deutsche Beurse and Goldman Sachs) in 1994 (with data from 1992). The Swiss Exchange introduced the index for Switzerland, the VSMI, in 2005. Currently, Eurex estimates and reports both VDAX and VSMI following a unified New-VIX methodology. The Center for the Study of Finance and Insurance (CSFI) at Osaka University launched an index for Japan, the VXJ, with data from 1995. Finally, in 2007, Euronext announced IV indices for France (VCAC), Belgium (VBEL), the UK (VFTSE, in partnership with FTSE), and The Netherlands (VAEX) with data from 2000.[3] Considering the data restrictions for the European markets, the empirical analysis in this paper is centered on the sample period between 2000 and 2009.

Now, in order to construct the variance premiums, an expectation of the total return variation under the physical measure has to be estimated. I estimate a measure based on the first order autoregressive forecast of the total realized return variation or realized variance from the following equation:

$$rv_{j,t+1}= \gamma_o+\gamma_1 rv_{j,t}+\epsilon_t,$$

where the realized variance is calculated summing the squared daily equity returns for each market as in

$$rv_{j,t} = \sum_{t_i=1}^{N_t} (r_{j,t_i})^2,$$

where r_{j,t_i} are daily local returns within month t. I rely on daily returns since data at a higher frequency are not available for all countries in the sample.[4]

Now, in order to make the results comparable to those in the literature, and as a preventive solution to the possible underperformance of this benchmark measure, all results are checked using three alternative approximations of the expected realized variance. In the first measure, I use the martingale measure where the expected realized variance is approximated as the current realized variance ($E_t(rv_{t+1}) = rv_t$). In the second one, I estimate a forecast of the realized variance that includes the local IV index as in the following equation:

$$rv_{j,t+1}= \gamma_o+\gamma_1 rv_{j,t} + \gamma_2 iv_{j,t}+\epsilon_t.$$

Finally, in the third one, I estimate a forecast of the realized variance that includes the range-based variance for each country as in

$$rv_{j,t+1}= \gamma_o+\gamma_1 rv_{j,t} + \gamma_2 RangeV_{j,t}+\epsilon_t,$$

where $RangeV_{j,t}$ is the range-based variance calculated as

$$RangeV_{j,t} = \frac{1}{4\ln 2}\sum_{t_i=1}^{N_t} range_{t_i}^2,$$

where $range_{t_i}$ is the daily difference between the highest and the lowest price of the index.[5]

[3] Both, the UK (FTSE) and France (French March des Options Negociables de Paris) had previously introduced IV indices separately.

[4] It has been shown in the literature that the use of intradaily returns outperforms lower frequency data in the estimation of the realized variance (Andersen et. al., 2001, Barndorff-Nielsen and Shephard, 2002; and Meddahi, 2002).

[5] Martens and van Dijk (2007) provide a description of the range based estimation of volatility. Jacob and Vipul (2008) analyze the extension of the range based measure to forecast the variance.

In order to estimate the variance premiums, the monthly data (end of the month) for the IV indices as well as the daily returns for the underlying index returns for all countries are obtained from Datastream. All returns are expressed in local currencies.[6] Now, in order to obtain the local excess returns to investigate the return predictability, I consider the 3-months T-bill rates for each country. These T-bill rates are also obtained from Datastream. In order to save space, the discussion in this section is centered on the components of the variance premium. All other variables used in the paper are described in Appendix 1.B.

1.3 Variance Premium: Single-Country Evidence

In this section, I investigate whether the stylized facts observed for the variance premium in the US also hold internationally. In a first step, I analyze the positive and time-varying nature of the variance premium. Then, I investigate the ability of the local variance premium in predicting equity returns in each country separately.

In order to get an idea of the magnitude and the time-varying nature of the variance premiums, Figure 1.1 displays the (benchmark) time series for all countries considered. The main statistics of these series are summarized in Table 1.1. This table also displays the IV indices and their underlying equity market indices for each country. The volatility premiums $[volp_{j,t} = iv_t - \sqrt{\widehat{rv}_{j,t+1}}]$ are also included in the table in order to visualize the magnitude of the premiums in annual percentages. The average volatility premium ranges between 1.7% for Belgium to 3.8% for Japan. In order to get an intuitive idea of these magnitudes in terms of one month maturity at-the-money put options, the 3.8% volatility premium in Japan translates into a price difference of 18% in a Black Scholes world. That is, one month at-the-money put options priced at 26.75% implied volatility, which is the average IV index for Japan, are 18% more expensive than the same options priced at 22.87% implied volatility, which is the average realized volatility for this country in this sample.

The information in Table 1.1 and Figure 1.1 suggests that the variance premiums display significant time variation. In particular, the premiums show several episodes of high volatility and notorious spikes around the same periods of time which translate into large Kurtosis for all series. The first high-variance-premiums episode occurs around the end of the technological boom in 2000. A second episode occurs at the end of 2002. This second episode coincides with the high macroeconomic uncertainty reported in the second semester of 2002 in the US (first semester of 2003 for Germany. An episode also related to the corporate accounting scandals around those years). Finally, the most notorious variance premium spikes occur around the recent subprime crisis. Not surprisingly, the minimum and maximum values for all series, except for Germany, occur in the last quarter of 2008. For Japan, for example, the variance premium reached 3,398.2 (annual percentage squared) in October 2008.[7]

Now, in order to assess the positive nature of the average variance premiums, Figure 1.2 summarizes the results for a test on the significance of the mean variance premium for all countries. This figure displays the average variance premiums and their respective confidence intervals for the four alternative measures introduced in Section 1.2. The evidence suggests that the average variance premium is positive and significant for all countries analyzed and all alternative measures considered, except perhaps when the martingale measure is used. This evidence supports the idea that agents also price market volatility in countries other than the US. These results are new evidence that extends that found for the US by Britten-Jones and Neuberger (2000), Jiang and Tian (2005), Bakshi and Madan (2006), Carr and Wu (2009), Todorov (2010), Bollerslev, Gibson and Zhou (2010), Bekaert and Engstrom

[6] The results are checked for robustness when all returns are expressed in US dollars.

[7] See Bollerslev, Gibson and Zhou (2010), and Corradi, et. al. (2009) for a more detailed analysis of the relation between the variance premium and the business cycle in the US.

(2010), and BTZ, among others.[8] This paper is, to the best of my knowledge, the first to show that these stylized facts also hold in other developed markets.[9]

I now test another US-based stylized fact, namely that the local variance premium predicts local equity returns.[10] Given the new evidence presented above on the existence of a volatility premium in all countries analyzed, I investigate the role of the variance premium in predicting returns for all countries in the sample. To do so, Figure 1.3 reports the estimation results for the following regressions:

$$(r - r_f)_{j,t,t+h} = \gamma_{0,,j,h} + \gamma_{1,,j,h} vp_{j,t} + \gamma_{2,,j,h} dy_{j,t} + \gamma_{3,,j,h} ts_{j,t} + \epsilon_{j,h,t},$$

where $(r - r_f)_{j,t,t+h}$ represents future compounded annualized excess returns h-months ahead, $dy_{j,t}$ is the local dividend yield, and $ts_{j,t}$ is the local term spread.

The evidence in Figure 1.3 confirms most of the results previously found in the literature for the US. That is, the US variance premium predicts returns specially for horizons between 3 to 6 months. In fact, the evidence shows that the US variance premium explains up 15% of the total variation in future equity returns at the quarterly frequency. The predictive power, as well as the coefficient of the variance premium in these regressions, follows a hump-shaped pattern and becomes null for horizons around one year.

However, the evidence suggests that the local variance premium plays a modest or insignificant role in predicting returns in any other country analyzed except perhaps for Belgium. For example, the results show that for Germany, Japan, the UK and the Netherlands, the R^2 is modest and hardly ever above 1%. Not surprisingly, for these countries, the variance premium does not predict equity returns for any horizon considered. Now, for Belgium the R^2 is as high as 10% for the one-month horizon; and the predictive power of the variance premium follows a linearly decreasing as the horizon increases. Actually, the variance premium in Belgium plays a significant role in predicting returns for horizons up to 10 months.[11] Finally, for France, although the R^2 are also modest, the predictability follows a pattern similar to that found for the US. That is, both the R^2 and the variance premium coefficient is only significant at the 2-months horizon.

In sum, although this is, to the best of my knowledge, the first paper to present evidence on the role of the variance premium in predicting returns for countries other than the US, the single-country evidence is puzzling. My findings are on the one hand consistent with the existence of time-varying variance premiums for a large sample of countries. On the other hand, they suggests that the variance premium does not predict returns in countries other than the US. The concurrence of these two findings cannot be explained by the existing domestic models where the variance premium implicitly explains the variation in the local equity premium. This puzzling evidence is nonetheless the motivation for the international general equilibrium model introduced in the following section. The model proposed is able to qualitatively explain the poor evidence for the role of the local variance premium in predicting returns outside the US. This model suggests that the variance premium of a leader country plays a dominant role in predicting returns for all other countries; a key implication for which I provide empirical evidence in the subsequent section.

[8] A group of papers have also provided preliminary evidence of this regularity using Black–Scholes-based implied volatility. See, for instance, Bakshi, Cao and Chen (2000), Christoffersen, Heston and Jacobs (2006), and Bollerslev and Zhou (2006).

[9] This is certainly not the first one in analyzing the informational content of option markets internationally. Some preliminary evidence that volatiliy risk is priced in an international setting can be found in Mo and Wu (2007) and Driessen and Maenhout (2006). Implied volatility in international markets has also been analyzed in Konstantinidi, Skiadopoulos, and Tzagkaraki (2008), Siriopoulos and Fassas (2009), and Jiang, Konstantinidi and Skiadopoulos (2010).

[10] See for instance BTZ, Zhou (2010) and Drechsler and Yaron (2010).

[11] It is worth pointing out that the variance premium in Belgium shows the lowest Sharpe ratio (almost half that for the rest of the countries). This could preliminary suggest that the variance premium is particularly volatile in Belgium. This in turn implies a noisier measure in this country, potentially driven by the liquidity of the Belgian option market.

1.4 A Two-Country Model for the role of the Variance Premium in International Equity Markets

The domestic nature of the existing models in the literature restricts the analysis of the variance premium in an international setting. These models cannot provide an explanation for the poor role of the local variance premium in predicting returns in countries other than the US as shown in the previous section. Therefore, I propose an international consumption-based general equilibrium (GE) model where the variance risk is priced in the global as well as in the local portfolios. My model yields several new qualitative implications for the role of the variance premium in international markets. The most relevant implication of the model is that the variance premium of a leader economy plays a dominant role in predicting equity returns in all portfolios. In addition, the model implies that the leader country variance premium also plays a role in explaining equity and option markets correlations across countries.

In this section, I present the basic setup of the model as well as its main implications.[12] I do not attempt to estimate nor to test my model but rather to use its qualitative implications to investigate the inability of the variance premium to predict local equity returns in countries other than the US. Therefore, I propose a numerical simulation of the model in order to understand its implications and illustrate the mechanism behind it. These numerical simulations provide the link between the single-country evidence, the implications of the model and the empirical evidence presented in the following section.

1.4.1 Model Setup and Assumptions

The model presented here is a two-country extension of that in BTZ. It preserves two key ingredients in BTZ's model: the use of recursive preferences, and the time-varying nature of macroeconomic uncertainty characterized by the volatility of consumption. However, my model adds to the literature by extending the intuition that financial markets dislike macroeconomic uncertainty (BY and Bansal, et. al., 2005) to an international setting. Therefore, I include the additional sources of risk embedded in the consumption process of each country, namely the country-specific time-varying volatility and the volatility of volatility (VoV) of consumption.[13] The setup of the model requires several additional assumptions. First, the two countries are assumed to be of a "considerable" size. That is, they both play a role in determining the global consumption growth which is a weighted average of the two countries' consumption growth. Second, one of the countries is assumed to be "the leader". The consumption process for the leader country is assumed to be entirely driven by local shocks, while the consumption process for the second country, "the follower", is also affected by the shocks generated in the leader country. Finally, I assume fully integrated equity markets. That is, there exists a unique representative agent holding a global portfolio with positions in the two equity markets. The assumptions of fully integrated equity markets and potentially integrated economies seem adequate given the particular characteristics of the sample considered in this paper.

Formally, each country consumption process is modeled similar to BTZ. The log of the consumption growth $g_{j,t}$ for the leader country (labeled as 1) follows

$$g_{1,t+1} = \mu_{1,g} + \sigma_{1,t} z_{g_1,t+1}, \tag{1.1}$$

[12] In order to save space, the detailed solution of my model is presented in Appendix 1.A

[13] Bekaert, Engstrom and Xing (2009) survey the evidence on time-varying volatility of consumption for the US. Bansal, et. al. (2005) provide empirical evidence of time-varying macroeconomic uncertainty for the US, Germany, Japan, and the UK. Now, BTZ also find preliminary empirical evidence on the existence of time-varying VoV for the US.

$$\sigma^2_{1,t+1} = a_\sigma + \rho_\sigma \sigma^2_{1,t} + \sqrt{q_{1,t}} z_{\sigma_1,t+1},$$

$$q_{1,t+1} = a_q + \rho_q q_{1,t} + \varphi_q \sqrt{q_{1,t}} z_{q_1,t+1},$$

whereas the consumption process for the follower country (labeled as 2) follows

$$g_{2,t+1} = \mu_{2,g} + \phi_g \mu_{1,g} + \phi_\sigma \sigma_{1,t} z_{g_1,t+1} + \sigma_{2,t} z_{g_2,t+1}, \tag{1.2}$$

$$\sigma^2_{2,t+1} = a_\sigma + \rho_\sigma \sigma^2_{2,t} + \sqrt{q_{2,t}} z_{\sigma_2,t+1},$$

$$q_{2,t+1} = a_q + \rho_q q_{2,t} + \varphi_q \sqrt{q_{2,t}} z_{q_2,t+1}.$$

The global consumption growth is a weighted average of the two countries' consumption process as in

$$g_{w,t} = \omega g_{1,t} + (1-\omega) g_{2,t},$$

where ω is the weight of the leader country in the global economy.

In order to simplify the model, the parameters in the volatility and VoV processes in Eqs. (1.1) and (1.2) are assumed to be the same across countries. I also assume that there are neither within nor cross-country statistical correlations in the shocks. The only correlations assumed in my model are those implied by the parameters ϕ_g (level) and ϕ_σ (volatility) in Eq. (1.2). These two parameters control the extent to which the follower country is affected by the shocks generated in the leader country. In particular, ϕ_σ implies that the consumption process of the follower country is affected not only by the local macroeconomic uncertainty, but also by that generated in the leader economy. More importantly, the fact that both economies are exposed to the same sources of macroeconomic uncertainty yields the systematic component in both countries' variance premiums.[14]

Now, the unique world representative agent is endowed with Epstein Zin Weil preferences (Epstein and Zin, 1989; and Weil, 1989). That is, her life-time utility function is given by the following equation:

$$U_t = [(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \tag{1.3}$$

where $0 < \delta < 1$ is the time discount rate, $\gamma \geq 0$ is the risk aversion parameter, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 1$ is the intertemporal elasticity of substitution (IES).[15] These preferences have the property of assigning non-zero market prices to shocks not directly related to aggregate consumption. This property is crucial to investigate other risk factors such as news related to volatility which is the main objective of this paper.

[14] The parameters ϕ_g, and ϕ_σ can of course be set to 0; a case that I will also analyze in the numerical simulation of the model. Now, although ϕ_g turns out to have an insignifcant effect on the role of international variance premium, it is kept to maintain the possibility of a common level component in consumption. Alternative ways of characterizing the systematic component of the variance premiums outside the simplifications of a two-country model are being explored in my current research agenda.

[15] To be coherent with the idea of agents that fear an increase in macroeconomic uncertainty, ψ is assumed to be higher than 1. This assumption accomodates some empirical asset pricing regularities, among them: (i) a positive variance premium; (ii) the feedback effect between PD ratios and consumption volatility; and (iii) a low risk-free rate (BY and BTZ). See also Mehra and Prescott (1985) for reasonable values of γ.

1.4.2 Model-Implied Variance Premiums

Given the solution of the model in Appendix 1.A, it can be shown that the two countries' VoV isolate the variance premium in the global and the local portfolios. The expression for the global portfolio's variance premium is given by[16]

$$VP_{w,t} = E_t^Q[Var_{r_j,t+1}] - E_t^P[Var_{r_j,t+1}],$$

where $Var_{r_j,t}$ is the conditional variation of returns between time t and $t+1$ for portfolio j for $j = 1, 2, w$ (see appendix 1.A). The variance premium can be approximated as[17]

$$VP_{w,t} \approx (\theta - 1)\kappa_{w,1}(V_{w,1}q_{1,t} + V_{w,2}q_{2,t}), \tag{1.4}$$

where $(\theta - 1)\kappa_{w,1}V_{j,k}$ represents the load of $q_{k,t}$ on $VP_{j,t}$. For the global portfolio, these loads are characterized by the following expressions:

$$V_{w,1} = (\omega + (1-\omega)\phi_\sigma)^2 A_{w,1} + (A_{w,1}^2 + A_{w,2}^2\varphi_q^2)\kappa_1^2\varphi_q^2 A_{w,2},$$

$$V_{w,2} = (1-\omega)^2 A_{w,3} + (A_{w,3}^2 + A_{w,4}^2\varphi_q^2)\kappa_1^2\varphi_q^2 A_{w,4},$$

where $A_{j,1}$, $A_{j,2}$, $A_{j,3}$ and $A_{j,4}$ are respectively the loads of the risk factors $\sigma_{1,t+1}^2$, $q_{1,t+1}$, $\sigma_{2,t+1}^2$, $q_{2,t+1}$ on the wealth-consumption ratio of each portfolio. These loads are derived in detail in Appendix 1.A.

For the leader country, the variance premium is given by

$$\begin{aligned} VP_{1,t} &= E_t^Q[Var_{r_1,t+1}] - E_t^P[Var_{r_1,t+1}] \\ &\approx (\theta - 1)k_{w,1}(V_{1,1}q_{1,t} + V_{1,2}q_{2,t}), \end{aligned} \tag{1.5}$$

$$V_{1,1} = A_{w,1} + (A_{1,1}^2 + A_{1,2}^2\varphi_q^2)\kappa_{1,1}^2{\varphi_q}^2 A_2,$$

$$V_{1,2} = A_{1,3}^2 + (A_{1,4}^2\varphi_q^2)\kappa_{1,1}^2\varphi_q^2 A_{w,4},$$

while for the follower country

$$\begin{aligned} VP_{2,t} &= E_t^Q[Var_{r_2,t+1}] - E_t^P[Var_{r_2,t+1}] \\ &\approx (\theta - 1)\kappa_{w,1}(V_{2,1}q_{1,t} + V_{2,2}q_{2,t}), \end{aligned} \tag{1.6}$$

$$V_{2,1} = \phi_\sigma^2 A_{w,1} + (A_{2,1}^2 + A_{2,2}^2\varphi_q^2)\kappa_{2,1}^2\varphi_q^2 A_{w,2},$$

$$V_{2,2} = A_{w,3} + (A_{2,3}^2 + A_{2,4}^2\varphi_q^2)\kappa_{2,1}^2\varphi_q^2 A_{w,4}.$$

[16] It is important to keep in mind that this is actually the drift difference of the conditional variance between the two measures. In the case of Gaussian shocks, the level difference $(Var^Q(r_{t+1}) - Var^P(r_{t+1}))$ would be zero (see Drechsler and Yaron, 2010). I intentionally omit the use of models that generate a level difference in the variance premium to maintain the simplicity of the expressions and given that the main attention will be centered in the qualitative implications of my model and not its calibration.

[17] The risk neutral probability is replaced by its log-linear approximation:

$$E_t^Q(\sigma_{r,t+1}^2) \approx \log[e^{-rt,t}E_t(e^{m_{t=1}+\sigma_{r,t+1}^2})] - \frac{1}{2}Var_t(\sigma_{r,t+1}^2).$$

Bear in mind that a closed form solution to the risk neutral variance cannot be obtained in this setting.

Eqs. (1.4) to (1.6) imply that the VoV of both countries are the unique sources of the variance premiums in all portfolios. Actually, for $\theta < 1$, the two countries' VoV load positively on the variance premiums. That is, $V_{j,k} \leq 0$ for $j, k = 1, 2, w$ (see appendix 1.A). Consequently, the global and local variance premiums are positive if $\theta < 1$. While the load of foreign VoV in the leader country variance premium is explained by the recursive nature of the utility function given fully integrated equity markets, the leader country VoV load on the follower country variance premium has the following two sources: the recursive nature of the preferences, and the implied sensitivity to the leader country macroeconomic uncertainty (See Eq. (1.2)).

As an immediate consequence of the common components in the variance premium of all portfolios, the variance premium covariance across countries is uniquely characterized by the two countries' VoV. The expression for the variance premium covariance derived from Eqs. (1.5) and (1.6) can be written as follows:

$$Cov_t(VP^1_{t+1}, VP^2_{t+1}) = (\theta-1)^2 k^2_{w,1}\varphi^2_q(V_{1,1}V_{2,1}q_{1,t} + V_{1,2}V_{2,2}q_{2,t}) \tag{1.7}$$

where the VoV of both countries loads positively on the variance premium covariance across countries as long as $\theta < 1$.

1.4.3 Model-Implied Equity Premiums

In order to understand the relation between the variance premiums and the dynamics of returns, in this section, I find the expressions for the equity premiums.

The global equity premium is characterized by the following expression:

$$\begin{aligned} EP_{w,t} &= E_t(r_{w,t+1} - r_{f,t}) \qquad (1.8)\\ &= \gamma\sigma^2_{w,t} - \frac{1}{2}\sigma^2_{w,t} \\ &\quad +(1-\theta)k_{w,1}(P_{w,1}q_{1,t} + P_{w,2}q_{2,t}), \end{aligned}$$

where $r_{j,t+1}$ is the (log) gross return for portfolio j ($j = 1, 2, w$), $r_{f,t}$ is the global risk-free rate, $\sigma^2_{w,t} = \omega\sigma^2_{1,t} + (1-\omega)\sigma^2_{2,t}$ is the volatility of the world consumption, and $(-\frac{1}{2}\sigma^2_{r_w t})$ is the geometric adjustment term. The term $(1-\theta)k_{w,j}P_{j,k}$ represents the load of $q_{k,t}$ on $EP_{j,t}$. For the global portfolio these loads are given by

$$\begin{aligned} P_{w,1} &= k_{w,1}(A^2_{w,1} + A^2_{w,2}\varphi^2_q), \\ P_{w,2} &= k_{w,1}(A^2_{w,3} + A^2_{w,4}\varphi^2_q). \end{aligned}$$

Equation (1.8) shows the three model-implied components of the global risk premium. The first component is the classic risk-return trade-off $\gamma\sigma^2_{w,t}$. This first component is also present when the agents are endowed with CRRA preferences. Now, there are two additional components, one for the VoV generated in each country. The VoV components of the equity premium represent the true premium for variance risk since they are driven by the shocks to the volatility and the volatility of volatility of consumption in both countries. In the case of the global portfolio, the VoV of both countries load positively on the equity premium if $\theta < 1$. That is, $(1-\theta)k_{w,1}P_{w,j} \geq 0$, for $j = 1, 2$ (see Appendix 1.A). These positive loads are in line with the concept that, at least for the global portfolio, agents are positively compensated for the risk generated by the time-varying nature of the VoV.

The expressions for the equity premiums for each country are given by

$$\begin{aligned} EP_{1,t} &= E_t(r_{1,t+1} - r_{f,t}) \\ &= \gamma(\omega \ + (1-\omega)\phi_\sigma)\sigma^2_{1,t} - \frac{1}{2}\sigma^2_{r_1,t} \\ &\quad +(1-\theta)k_{w,1}(P_{1,1}q_{1,t} + P_{1,2}q_{2,t}), \end{aligned} \tag{1.9}$$

and

$$\begin{aligned} EP_{2,t} &= E_t(r_{2,t+1} - r_{f,t}) \\ &= \gamma\phi_\sigma(\omega + (1-\omega)\phi_\sigma)\sigma^2_{1,t} + \gamma(1-\omega)\sigma^2_{2,t} - \frac{1}{2}\sigma^2_{r_2,t} \\ &\quad +(1-\theta)k_{w,1}(P_{2,1}q_{1,t} + P_{2,2}q_{2,t}), \end{aligned} \tag{1.10}$$

where

$$\begin{aligned} P_{j,1} &= k_{j,1}(A_{w,1}A_{j,1} + A_{w,2}A_{j,2}\varphi^2_q), \\ P_{j,2} &= k_{j,1}(A_{w,3}A_{j,3} + A_{w,4}A_{j,4}\varphi^2_q), \text{ for } j = 1, 2. \end{aligned}$$

As for the global portfolio, the equity premium in each country is characterized by a volatility of consumption component, and two VoV components, one for each country. In particular, the VoV components in Eqs. (1.9) and (1.10) represent the true premium for local and foreign variance risk. Now, comparing the expressions for the Variance premium (Eqs. (1.5) and (1.6)) with those for the equity premiums (Eqs. (1.9) and (1.10)) yields the basic intuition for the role of local and foreign variance premium in predicting equity returns in any country. The intuition is as follows: the VPs reveal the VoV in both countries which in turn drives (in part) the time variation in the equity premiums. It is important to bear in mind that although the VoV is not a necessary condition to generate a variance risk premium, introducing the VoV isolates the risk premium on volatility and differentiate it from the consumption risk premium.

It seems natural from Eqs. (1.8) to (1.10) to expect that the VoV also explains the time variation in the covariance of returns across countries. The expression for the covariance of returns is given by

$$Cov_t(r_{1,t+1}, r_{2,t+1}) = \phi_\sigma\sigma^2_{1,t} + CO_1q_{1,t} + CO_2q_{2,t}, \tag{1.11}$$

where CO_j is the load of $q_{j,t}$ on the covariance of returns. These loads are given by

$$\begin{aligned} CO_1 &= \kappa_{1,1}\kappa_{2,1}(A_{1,1}A_{2,1} + A_{1,2}A_{2,2}\varphi^2_q), \\ CO_2 &= \kappa_{1,1}\kappa_{2,1}(A_{1,3}A_{2,3} + A_{1,4}A_{2,4}\varphi^2_q). \end{aligned}$$

1.4.4 Numerical Implications of the Two-Country Model

In this section, I present some numerical simulations of my model in order to investigate the mechanism of transmission of VoV shocks across countries. The purpose of these simulations is to analyze the qualitative implications of my model for the variance premiums and for the interaction between the variance premiums and the equity returns. I believe that understanding these qualitative implications provides a natural step between the model and the empirical evidence presented in the next section.

The base scenario for the numerical simulations is displayed in Table 1.2. In this scenario, the parameters in the preference function are calibrated as in BTZ. Now, in order to simplify the interpretation of results, I consider the hypothetical case where the world is composed of only two countries: the US, and Germany. Just for the purpose of illustrating the mechanism behind the model, the US is considered as the leader economy.[18] For these two countries, the parameters in Eqs. (1.1) and (1.2) are calibrated as follows: $\mu_{j,g}$ is estimated as the average IP growth in each country during the period 1973-2009; $\mu_{j,\sigma}$ is estimated as the IP growth unconditional variance for the same period; and the rest of the parameters are taken from BTZ (homogeneous parameters for the two countries). Now, the Campbell and Shiller constants k_o and k_1 are estimated using data for the Price-Dividend (PD) ratio for each country as well as for the Datastream world portfolio. The log-linearization constants are estimated as $k_1 = \frac{e^{E(PD)}}{1+e^{E(PD)}}$, where $E(PD)$ is the unconditional mean of the (log) PD ratio, and $k_0 = -k_1 \ln(1-k_1) - (1-k_1)\ln(1-k_1)$ (Campbell and Cochrane, 1999). Bear in mind that k_o and k_1 should actually be made dependent of the theoretical wealth-consumption ratio (see Appendix 1.A). However, I use the unconditionally expected PD ratio, to make these two parameters independent from the set of parameters considered in each case.[19]

Variance Premium Dynamics

According to the first main implication of my model, both countries' VoV load positively on all portfolios' variance premiums (see Eqs. (1.4) to (1.6)). In order to show this implication, Figure 1.4 displays the (unconditionally expected) VoV loads on the variance premium for all portfolios. The figure shows the components of the variance premiums for alternative values of the risk aversion (γ), the weight of the leader country (ω), and the correlation of consumption (ϕ_σ). The simulations show that the implied size of the US VoV load dominates that of Germany in all cases considered. The dominance of the US VoV increases with the relative size of the leader economy (ω), and with the relative dependence of the follower economy (ϕ_σ). The contribution of the follower economy VoV in the variance premiums, on the other hand, is almost insignificant no matter the size nor the independence of the consumption process in this economy.

The simulations also suggest that the magnitude of the expected variance premiums monotonously increases with the risk aversion, and decreases with the relative size of the riskiest market.[20] The riskiest market is assumed, for coherence, to be that in the follower country. However, for all cases considered, the average variance premium is quantitatively far from that empirically observed for these two countries (see Table 1.1). The limitation to quantitatively reflect the observed premium in models with recursive preferences has been previously documented by Drechsler and Yaron (2010) in a single-country setting.

In unreported results, I show that the model-implied variance premium correlation across the two countries is above 0.98 for all simulations. This results is to be expected given the high common component of the leader country VoV in all variance premiums. Actually, for all cases considered, the model implies that the leader country VoV accounts for more than 99% of the total cross-country variance premium covariance. Surprisingly, the result on the dominant role of the leader country's VoV holds no matter the relative size of the follower economy ($(1-\omega) < 0.5$) or its implied correlation with the leader economy.

In sum, the numerical simulations show that the VoV generated in the leader economy accounts for most of the systematic component of the variance premiums. Therefore, the VoV generated in the leader economy plays the key role in explaining the variance premium

[18] In the following section, the identification of the leader economy will be fully given by the empirical evidence.

[19] A full calibration of my model is out of the scope of this paper. This paper's attention is centered in the qualitative implications of my model. These implications explain in turn the main empirical findings of this paper such as the local predictability puzzle and the ability of the US variance premium to predict all other countries equity returns.

[20] It is easy to show the same monotonous relation for the IES ψ. Results for the relation between ψ and the model implications are available upon request.

for all portfolios. This in turn implies that the leader country VoV is also the key driver of the expected variance premium correlation across countries.

Return Dynamics

According to the second main implication of my model, the two countries' VoV that uniquely characterize the variance premiums also drive the time variation in equity premiums. Figure 1.5 displays the model-implied components of the equity premium for the global and the local portfolios for alternative sets of parameters. The leader country VoV load dominates that of the follower country in all portfolios' equity premiums for all cases considered. In some cases, the VoV of the leader country loads negatively on the follower country's equity premium. This case only occurs when economies are poorly correlated as can be seen in Panel J and K. However, the follower country VoV loads negatively on the leader country's equity premium for all cases considered, except of course for the extreme case where the size of the follower economy is insignificant (Panels C,F,I, and L).

The possibility of VoV loading negatively on the equity premiums can actually be explained by the mechanism of transmission of shocks to VoV implied by the model. According to this mechanism, a positive shock to VoV in the follower country has a negative impact on the leader country's equity premium. This effect can be interpreted as a macroeconomic uncertainty induced flight-to-safety from the follower to the leader economy. The possibility of an uncertainty flight-to-safety in this direction is actually generated by the fact that the leader country consumption process is, by construction, not sensitive to the shocks generated in the follower country (Eq. (1.1)). Investing in equities in the leader country is then expected to become a more attractive investment alternative with respect to this foreign source of risk. In contrast, an uncertainty flight-to-safety in the other direction (leader to follower) is not always possible. This is due to the fact that the follower country consumption process is affected by the shocks in the leader economy (Eq. (1.2)). Therefore, a flight-to-safety in this direction is only possible if the economies are assumed to be quite independent. For example, in the case of totally independent economies in Panel J, any equity market is free from the uncertainty risk generated in the foreign economy. Thus, in this extreme case, the VoV of one country will always load negatively on the other country's equity premium.

As a consequence of the second main implication of my model, both countries' VoV also play a role in explaining the covariance of equity returns across countries. As expected, even if the follower economy has a large (relative) size, the VoV of the leader country dominates. The dominance of the leader country VoV increases with the relative size of its economy (ω), and the degree of dependence across the two economies (ϕ_σ). In line with the simulations in Figure 1.5, the VoV generated in the follower country may even load negatively on the covariance of returns. Actually, in the case of totally independent or mildly correlated economies, the simulations confirm that even VoV generated in the leader economy might load negatively on the covariance of returns.[21]

Finally, the simulations in Figure 1.6 show the relation between the correlation across economies and the model-implied correlation across equity markets. The simulations reflect the documented disparity between the correlation of equity markets and the correlation of economies. They show that the equity return correlation is in some cases higher than the implied correlation of consumption. In particular, the simulations suggest that for moderately risk averse agents ($\gamma > 2$) and moderately correlated economies, the implied correlation across equity markets is larger than that implied by the correlation of consumption. This result arrives as a direct consequence of the recursive nature of the representative agent's preferences.

[21] These simulations are left unreported.

In sum, the numerical simulations show that the VoV generated in the leader economy plays the key role in explaining the time variation in the equity premium of all portfolios. As a consequence of this implication, the leader economy VoV also plays a dominant role in explaining the time variation in equity return correlations across countries. The simulations also show some consequences derived from the model setup. In particular, from the assumptions of integrated markets where the representative agent is endowed with recursive preferences and one economy behaves as a follower. For example, the model introduces the possibility of a macroeconomic uncertainty induced flight-to safety, which in turn introduces the possibility that the VoV of one country covaries negatively with the equity premium of another country.

1.5 The Variance Premium and International Equity and Option Markets: Empirical Evidence

In this section, I present the empirical evidence based on the qualitative implications of the GE model analyzed in Section 1.4. First, using the variance premiums for all countries in the sample, I investigate their role in (i) explaining the time variation in the variance premium for all other countries, (ii) predicting the variance premium correlations across countries, (iii) predicting not only the local equity returns, but also those in other countries, and (iv) predicting the correlation of equity returns across countries. Then, I propose an extension of the empirical evidence to analyze the potential role of the variance premium in explaining excessive comovements across equity and option markets. The excessive comovements analysis seems a natural extension to understand equity and option markets linkages around episodes of extreme macroeconomic uncertainty beyond the implications of the model.

1.5.1 Cross-country Variance Premium Correlations

A first implication of my model is that the variance premiums are highly correlated across countries. The high variance premium correlation is due to the common load of the leader country VoV in all variance premiums (leader, follower, and global portfolio). This in turn implies that the leader country variance premium plays a key role in predicting the variance premium correlations across countries. In order to analyze this implication, I first provide evidence for the variance premium correlations across countries. Then, I investigate the role of each all country's variance premium in predicting the variance premium correlations with any other country.

Table 1.3 displays the variance premium correlations across all countries in the sample. In line with the first implication of my model, all countries but Japan show correlations above 0.5. In particular, the US and the UK show a high correlation coefficient of 0.73. Among European markets, France and The Netherlands show the highest correlation coefficient in the sample: 0.89. However, Japan's variance premium shows a relatively low, or even negative, correlation with the variance premium of any other market excepts perhaps with Switzerland.[22] The evidence for Japan stands in sharp contrast to the implications of the model. In fact, my model can only accommodate positive variance premium correlations. This is in turn derived from the ability of my model to characterize only positive variance premiums.

The results on the high variance premium correlations has been previously documented in the literature for a shorter sample of countries. For example, Bekaert, Hoerova and Scheicher (2009) find evidence of high risk aversion and uncertainty correlation between

[22] The highly idiosyncratic dynamic of the variance premium in Japan has been previously documented in the literature (see, for instance, Driessen and Maenhout, 2006).

Germany and the US. Although their measures are not directly the variance premiums, their empirical methodology uncovers the risk aversion and uncertainty time series using the observed IV and realized volatilities for these two countries. Sugihara (2010) also finds evidence of strong linkages in volatility premiums between the US, Germany and Japan. He actually finds empirical evidence that the correlation between these three markets is stronger around certain episodes; in particular, after the subprime crisis. However, in this paper, I not only extend the evidence for a larger sample of countries but also provide a fundamental explanation for the dynamics of the variance premium correlation across countries. In particular, my model relates the high variance premium correlation across countries to a systematic component which is mainly driven by the leader country variance premium.

A direct consequence of the common component in all variance premiums is that the variance premium correlations are predicted by the leader country's variance premium. In order to test this consequence, Table 1.4 reports the estimated coefficients $\gamma_{1,jk}$ for the following regressions:

$$\rho_t(vp_{j,t,t+1}, vp_{k,t,t+1}) = \gamma_{0,jk} + \gamma_{1,jk} vp_{k,t} + \epsilon_{jk,t},$$

where the correlation coefficient for the period t to $t+1$ is calculated using daily data for the variance premiums of the two countries for the month starting immediately after the realization of $vp_{k,t-1}$.[23,24] The evidence suggests that the US variance premium predicts the one-month-ahead variance premium correlation between the US, Germany, and Japan (first horizontal block of results).[25] However, the results show that the US variance premium does not outperform all other countries' variance premium. For example, the first vertical block of results in the table suggests that the variance premiums in Germany, Japan, the UK, Switzerland and The Netherlands can also forecast the variance premium correlation between these countries and the US.[26]

In sum, the evidence in this section suggests that the variance premium correlations across countries increase following episodes of increasing variance premiums. It also suggests that the model-implied dominant role of the leader country variance premium restricts the potential ability of other countries in predicting one-month ahead variance premium correlations. In order to explore higher frequency correlation patterns, in section 1.5.3, I investigate the role of the US variance premium in explaining excessive variance premium comovements at the daily frequency.

1.5.2 Cross-Country Equity Return Correlations

The second main implication of my model is that the variance premiums covary with the equity premiums (Eqs. (1.9) and (1.10)). This is due to the fact that the VoV shocks that uniquely characterize the variance premiums also load on both countries' equity premiums. In particular, the model implies that the leader country's VoV dominates that of the follower country in all equity premiums. As a consequence, the variance premium of a leader country should outperform that of the follower country in predicting local and foreign returns. In this section, I provide evidence for the role of foreign variance premiums in predicting equity returns for all countries in the sample.

[23] The following month $(t, t+1)$ is assumed to be the period 22 days after the realization of $vp_{k,t}$.

[24] Equation (1.7) actually has an implication on the variance premium covariance. To avoid a potential scale problem, and make results easier to interpret, I only report cross-country correlations. An expression for the variance premium correlation from Eq. (1.7) is direct, although not necessarily linear in VoV.

[25] In unreported results, I actually show that, except for the variance premium measure based on the martingale assumption, the predictive role of the US variance premium over its correlation with Germany and Japan holds for all alternative variance premium specifications considered.

[26] Given the high correlation in vp_t across countries, it would be hard to disentangle the simultaneous role of $vp_{US,t}$ with any other $vp_{j,t}$ since multiple regressions will be highly affected by multicolinearity.

Table 1.5 reports the estimation results for the following regressions:

$$(r - r_f)_{j,t,t+3} = \gamma_{0,j,k} + \gamma_{1,j,k} vp_{k,t} + \gamma_{1,j,k} dy_{j,t} + \gamma_{1,j,k} ts_{j,t} + \epsilon_{j,k,t},$$

where $(r - r_f)_{j,t,t+3}$ represents future compounded annualized excess returns 3-months ahead, $dy_{j,t}$ is the local dividend yield, and $ts_{j,t}$ is the local term spread.[27] On top of the local predictability evidence discussed in Section 1.3, the evidence in Table 1.5 suggests that only the US variance premium plays a significant role in predicting equity returns for all other countries in the sample. Nevertheless, for other pairs of countries, the predictive power of the foreign variance premium over international equity returns holds. This is the case for the significant predictive power of the Japanese variance premium over the equity returns of Belgium and France. It is also the case for the (often borderline) predictive power of the variance premium of all countries, except for Switzerland and Japan, over the US equity returns.[28]

In order to investigate more in depth the predictive power of the US variance premium over international equity returns, Figure 1.7 reports the estimation results for the following regressions:

$$(r - r_f)_{j,t,t+h} = \gamma_{0,j,h} + \gamma_{1,j,h} vp_{US,t} + \gamma_{1,j,h} dy_{j,t} + \gamma_{1,j,h} ts_{j,t} + \epsilon_{j,h,t},$$

where $(r - r_f)_{j,t,t+h}$ represents future compounded annualized excess returns h-months ahead. The results suggest that he predictive power of the US variance premium for all countries except perhaps for Japan resembles the hump-shaped pattern found by BTZ for the US (local return predictability). This pattern reflects the fact that the variance premium should be a dominant predictor for horizons where the VoV is the main source of variation in equity returns. The extension of this evidence for other countries indicates that the US VoV is the dominant source of variation in all countries' equity returns for horizons between 3 and 6 months. The figure also suggests that the predictive power of the US variance premium is complementary to that of local term spreads and dividend yields.[29] Now, when compared to Figure 1.3, the evidence also suggest that the US variance premium outperforms the local variance premiums in predicting equity returns for all countries considered. In unreported results, I show that the ability of the US variance premium to predict one-quarter ahead foreign returns holds if a noise signal is added to the original variance premium. For all countries, except perhaps for the Netherlands and Japan, the standard deviation of the noise signal has to be at least 50% that of the original US variance premium before its predictive power disappears.[30] Moreover, the predictive power of the US variance premium holds for all alternative variance premium specifications considered, except perhaps for the range-based estimation.[31,32]

[27] The evidence suggests that the predictive power of the variance premium is stronger at the quarterly horizon. This result is in line with the findings in BTZ for the US and is discussed in detail in the international setting below.

[28] In fact, in unreported results, I show that not even a proxy for the world variance premium (with and without the US) is able to significantly predict equity returns for all other countries in the sample. The world variance premium exercise relates to concurrent independent evidence found by Bollerslev, Marrone, Xu and Zhou (2011).

[29] The hump-shaped predictability pattern, as well as the significance of the US variance premium in predicting foreign equity returns is robust to considering the US term spread and dividend yield. Results for these regressions are available upon request.

[30] For the Netherlands and Japan, adding any noise to the US variance premium almost immediately weakens its predictive power. In contrast, for the UK, the standard deviation of the noise signal has to be at least 70% that of the original variance premium before its predictive power disappears.

[31] When the range based forecast of realized volatility is used, the US variance premium predicts returns only for the UK, Belgium and France.

[32] Results for the robustness tests are left unreported in order to save space and center the discussion. The results for the noise stress tests, the alternative variance premium specifications, samples, currencies, as well as for alternative variance covariance matrix approximations (In particular, Hodrick, 1992) are available

As a consequence of the systematic component of equity premiums, the leader country variance premium should also be a useful predictor of equity return correlations across countries (Eq. (1.11)). In order to test this consequence, Figure 1.8 reports the estimation results for the following regressions:

$$\rho_t(r_{j,t,t+h}, r_{US,t,t+h}) = \gamma_{0,jk} + \gamma_{1,j,US} vp_{US,t} + \epsilon_{jk,t},$$

where $\rho_t(r_{j,t,t+h}, r_{US,t,t+h})$ is the h-months ahead equity return correlation between any country and the US. The results suggest that the US variance premium predicts equity return correlations between the US and any other country in the sample except for Japan and Belgium. As for the equity returns, the ability of the US variance premium to forecast return correlations holds for horizons between 3 and 6 months for most of the countries. Actually, for the equity correlation between the US and Germany, the US variance premium has predictive power for horizons up to 12 months. In unreported results, I also show that the US variance premium outperforms all other countries in the sample in predicting equity return correlations.

In sum, the evidence in this section supports the qualitative implications of my model for the role of the variance premium in predicting equity returns. It confirms the predominant role of the US variance premium in predicting foreign equity returns and return correlations across countries. Therefore, the evidence supports the theoretical solution implied by my model to the local return predictability puzzle in Section 1.3. That is, the local variance premium cannot predict returns in countries other than the US because the role of the variance premium in those countries is dominated by the variance premium in a leader country: the US.

1.5.3 Exploring the role of the Variance Premium in Explaining Excessive comovements

In this section, I extend the analysis of the model implications to understand the potential role of the variance premium in explaining excessive comovements across international equity and option markets. Although a natural extension, the excessive comovement analysis does not have an immediate linkage with the model in Section 1.4. Therefore, the evidence presented in this section is merely an investigation of the impact of unusual events from an empirical perspective. As in the previous sections, I investigate both the variance premium and the equity return correlations. The procedure to test for excessive comovements is done at the daily as well as at the monthly frequency. However, in order to save space and given the insignificant results at the monthly frequency, only the results for the daily frequency are reported.

The analysis of the excessive comovements in the variance premium dynamics requires the estimation of the following system of equations:

$$vp_{j,t+h} = \gamma_{0,,j,h} + \gamma_{1,,j,h,t} vp_{US,t} + \epsilon_{t,h}, \tag{1.12}$$

where h are the alternative horizons considered and the time varying coefficient $\gamma_{1,j,h,t}$ follows

$$\gamma_{1,j,h,t} = v_{o,,j,h} + v_{1,,j,h} D_{US,t},$$

where the dummy $D_{US,t}$ characterizes extreme values of $vp_{US,t}$. This variable takes the value 1 when $vp_{US,t}$ is above the 5^{th} percentile.[33] The estimated $v_{1,j,h}$ are displayed in Figure 1.9 for horizons between 1 and 22 days. The evidence suggests that episodes of unusually

upon request.

[33]The results for alternative percentiles as well as for two-sided extreme events are available upon request.

high US variance premium generate unusual variance premium correlations between the US and most of the countries in the sample. As can be seen from the figure, this effect is significant only in the very short term, usually up to 10 days. For all countries, except France, the significance of $v_{1,j,h}$ follows a decreasing pattern as the horizon increases, which in turn suggests that it could be hard to identify unusual variance premium comovements at the monthly frequency. In sum, the empirical evidence suggests that the variance premium correlations tend to intensify following episodes of extremely high US variance premium.

In order to investigate the excessive comovement of returns, I follow the literature on contagion. In particular, the procedures in Bekaert, Harvey and Ng (2005) and Baele and Inghelbrecht (2010). In this case, the test is performed on the residuals from a single-factor model. Therefore, excessive return comovements can be interpreted as cross-country equity return correlation beyond what is expected from the exposition of all returns to a common factor. To maintain the coherence with the rest of the paper, I assume that the common factor is the US equity returns. The procedure for this test is explained in detail in Appendix 1.B, and its results are displayed in Figure 1.10. The evidence shows that, in contrast to the evidence for the variance premium dynamics, there is no clear effect of extreme variance premium episodes over the dynamics of equity returns. Therefore, the evidence suggests that for the cross-country return correlations, only the fundamental relation implied by the model is supported by the empirical evidence.[34]

1.6 Conclusions

This paper presents several new findings related to the variance risk premium for a total of eight countries. First, I provide new evidence that the variance premiums display significant time variation and are, on average, positive for all countries analyzed. However, I also provide evidence that except for the US, the local variance premiums do not predict local equity returns. This evidence is in sharp contrast to the existing theoretical models where the variance premium explains the time variation in equity returns.

Motivated by the puzzling single-country evidence, I propose an international model to understand the role of the variance premium in explaining international equity returns. The model is a two-country general equilibrium model which extends that in Bollerslev, Tauchen and Zhou (2009). My model yields relevant qualitative implications that explain the inability of the variance premium in predicting local returns in countries other than the US. In particular, my model implies that the variance premium generated in a leader economy plays a key role in explaining the time variation in equity returns in the two countries. Therefore, the leader country variance premium outperforms the follower country variance premium in predicting not only equity returns, but also equity return correlations across countries. The dominant role of the leader country variance is a consequence of the common components in the variance premiums of all countries. In particular, a consequence of the dominant load of macroeconomic uncertainty shocks generated in the leader economy in the variance premiums of both countries.

Finally, I provide new empirical evidence for the qualitative implications of my model for the eight countries in the sample. I show that the US variance premium has predictive power over the equity returns for all countries in the sample. The predictive power of the US variance premium over international equity returns is (i) stronger for horizons between 3 and 6 months, (ii) additional to that of traditional local (or US) variables, and (iii) clearly outperforms the local variance premium themselves. Finally, I also show that the US variance premium predicts the correlation of equity returns between the US and all countries in the sample, except for Japan and Belgium.

[34] The insignificant effect of extreme variance premium episodes holds at the monthly frequency. These results are available upon request.

Given the new findings presented in this paper, exploring the dynamics of the variance premium in an international setting remains a very interesting topic in my research agenda. I am currently working in disentangling the systematic and country-specific components of the variance premiums. In future research, I will also investigate in depth the short-term dynamics of the variance premiums. In particular, the contagion patterns across countries.

APPENDIX

1.A Detailed Solution of the Two-country Model

This appendix explains in detail the solution to the model in Section 1.4.

Each country return process is assumed to be a claim on the local consumption growth, while the global portfolio return is a claim on the weighted global consumption $g_t^w = \omega g_t^1 + (1-\omega) g_t^2$, where ω is the weight of the leader country. Following Campbell and Shiller (1988), the returns are linearized as

$$r_{j,t+1} = \kappa_{j,0} + \kappa_{j,1} z_{j,t+1} - z_{j,t} + g_{j,t+1}, \text{ for } j = 1, 2, w, \tag{1.A-1}$$

where $z_{j,t}$ denotes the log of the wealth-consumption ratio of the asset that pays the consumption endowment $\{C_{j,t+i}\}_{i=1}^{\infty}$. As it is standard in the asset pricing literature, I conjecture a solution for $z_{j,t}$ as a function of the state variables of both countries as follows:

$$z_{j,t+1} = A_{j,0} + A_{j,1}\sigma^2_{1,t+1} + A_{j,2} q_{1,t+1} + A_{j,3}\sigma^2_{2,t+1} + A_{j,4} q_{2,t+1}. \tag{1.A-2}$$

Based on this solution, the basic asset pricing equation is imposed in order to determine the components of $z_{j,t+1}$. The basic asset pricing equation is the first order condition from the agent maximization problem given by

$$E_t[(\exp(m_{t+1} + r_{j,t+1})] = 1,$$

where m_{t+1} is the (log of) intertemporal marginal rate of substitution. For the case of Epstein-Zin-Weil preferences, and given that markets are assumed to be perfectly integrated, the unique marginal rate of substitution is given by

$$\begin{aligned} m_{t+1} &= \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1} \\ &= b_{mo} + b_{mg} g_{w,t+1} + b_{mr} r_{w,t+1}, \end{aligned}$$

where $0 < \delta < 1$ is the time discount rate, $\gamma \geq 0$ is the risk aversion parameter, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 1$ is the intertemporal elasticity of substitution (IES).

Solving for the world portfolio yields the following expressions for the components of $z_{j,t+1}$:

$$\begin{aligned} A_{w,0} &= \frac{\theta \log \delta + (1-\gamma)(\omega \mu_{1,g} + (1-\omega)(\mu_{2,g} + \phi_g \mu_{1,g}))}{\theta(1-\kappa_{w,1})} \\ &\quad + \frac{\kappa_{w,0} + \kappa_{w,1} A_{w,1} a_\sigma + \kappa_{w,1} A_{w,2} a_q + \kappa_{w,1} A_{w,3} a_\sigma + \kappa_{w,1} A_{w,4} a_q}{(1-\kappa_{w,1})}, \end{aligned}$$

$$A_1 = \frac{(1-\gamma)^2(\omega + (1-\omega)\phi_\sigma)^2}{2\theta(1-\kappa_{w,1}\rho_\sigma)},$$

$$A_{w,2}^{+,-} = \frac{(1-\kappa_{w,1}\rho_q) \pm \sqrt{(1-\kappa_{w,1}\rho_q)^2 - \theta^2 \kappa_{w,1}^2 \varphi_q^2 A_{w,1}^2}}{\theta \kappa_{w,1}^2 \varphi_q^2},$$

$$A_{w,3} = \frac{(1-\gamma)^2(1-\omega)^2}{2\theta(1-\kappa_{w,1}\rho_\sigma)},$$

and

$$A_{w,4}^{+,-} = \frac{(1-\kappa_{w,1}\rho_q) \pm \sqrt{(1-\kappa_{w,1}\rho_q)^2 - \theta^2\kappa_{w,1}^2\varphi_q^2 A_{w,3}^2}}{\theta\kappa_{w,1}^2\varphi_q^2}.$$

To avoid the load of time-varying volatilities $\sigma_{1,t}$, and $\sigma_{2,t}$ from growing without bounds, I only keep $A_{w,2}^-$ ($A_{w,4}^-$). The positive root discarded is explosive in φ_q, i.e., $\lim_{\varphi_q \to 0} A_{w,2}^+\varphi_q \neq 0$ ($\lim_{\varphi_q\to 0} A_{w,4}^+\varphi_q \neq 0$). Now $A_{w,2}^-$ ($A_{w,4}^-$) will be a solution to the model as long as $(1-\kappa_{w,1}\rho_q)^2 \geq \theta^2\kappa_{w,1}^2\varphi_q^2A_{w,1}^2$ ($(1-\kappa_{w,1}\rho_q)^2 \geq \theta^2\kappa_{w,1}^2\varphi_q^2A_{w,3}^2$). It is easy to show from these expressions that all state variables load negatively on the global wealth-consumption ratio. That is, $A_{w,1}$, $A_{w,2}$, $A_{w,3}$, $A_{w,4} \leq 0$ as long as $\theta < 1$.

Solving for the leader country 1 yields the following expressions:

$$\begin{aligned} A_{1,0} &= \frac{\kappa_{1,0} + \kappa_{1,1}A_{1,1}a_\sigma + \kappa_{1,1}A_{1,2}a_q + \kappa_{1,1}A_{1,3}a_\sigma^2 + \kappa_{1,1}A_{1,4}a_q + \mu_{1,g}}{(1-\kappa_{1,1})} \\ &- \frac{\begin{gathered}\kappa_{w,0} + (\kappa_{w,1}-1)A_{w,0} + \kappa_{w,1}A_{w,1}a_\sigma + \kappa_{w,1}A_{w,2}a_q + \\ +\kappa_{w,1}A_{w,3}a_\sigma + \kappa_{w,1}A_{w,4}a_q + \omega\mu_{1,g} + (1-\omega)(\mu_{2,g} + \phi_g\mu_{1,g})\end{gathered}}{(1-\kappa_{1,1})}, \end{aligned}$$

$$A_{1,1} = \frac{(1-\theta)(1-\gamma)^2(\omega + (1-\omega)\phi_\sigma)^2 + \theta(1-\gamma(\omega+(1-\omega)\phi_\sigma))^2}{2\theta(1-\kappa_{1,1}\rho_\sigma)},$$

$$\begin{aligned} A_{1,2}^{+,-} &= \frac{(1-\kappa_{1,1}\rho_q) + (1-\theta)\kappa_{1,1}\kappa_{w,1}A_{w,2}\varphi_q^2}{\kappa_{1,1}^2\varphi_q^2} \\ &\pm \frac{\sqrt{\begin{gathered}((1-\kappa_{1,1}\rho_q) + (1-\theta)\kappa_{1,1}\kappa_{w,1}A_{w,2}\varphi_q^2)^2 - \kappa_{1,1}^2\varphi_q^2((\theta-1)^2\kappa_1^2A_{w,2}^2\varphi_q^2 + \\ +2(\kappa_{w,1}\rho_q - 1)(\theta-1)A_{w,2} + (\kappa_{1,1}A_{1,1} + (\theta-1)\kappa_{w,1}A_{w,1})^2)\end{gathered}}}{\kappa_{1,1}^2\varphi_q^2}, \end{aligned}$$

$$A_{1,3} = \frac{(1-\theta)(1-\kappa_{w,1}\rho_\sigma)A_{w,3} + \frac{1}{2}\gamma^2(1-\omega)^2}{(1-\kappa_{1,1}\rho_\sigma)},$$

and

$$\begin{aligned} A_{1,4}^{+,-} &= \frac{(1-\kappa_{1,1}\rho_q) + (1-\theta)\kappa_{1,1}\kappa_{w,1}A_{w,4}\varphi_q^2}{\kappa_{1,1}^2\varphi_q^2} \\ &\pm \frac{\sqrt{\begin{gathered}((1-\kappa_{1,1}\rho_q) + (1-\theta)\kappa_{1,1}\kappa_{w,1}A_{w,4}\varphi_q^2)^2 - \kappa_{1,1}^2\varphi_q^2[(\theta-1)^2\kappa_{w,1}^2\varphi_q^2A_{w,4}^2 + \\ +2(\kappa_{w,1}\rho_q - 1)(\theta-1)A_{w,4} + (\kappa_{1,1}A_{1,3} + (\theta-1)\kappa_{w,1}A_{w,3})^2]\end{gathered}}}{\kappa_{1,1}^2\varphi_q^2}. \end{aligned}$$

Finally, for the follower country 2, solving the basic asset pricing equation yields

$$\begin{aligned} A_{2,0} &= \frac{\kappa_{2,0} + \kappa_{2,1}A_{2,1}a_\sigma + \kappa_{2,1}A_{2,2}a_q + \kappa_{2,1}A_{2,3}a_\sigma + \kappa_{2,1}A_{2,4}a_q + \mu_{2,g} + \phi_g\mu_{1,g}}{(1-\kappa_{2,1})} \\ &- \frac{\begin{gathered}\kappa_{w,0} + \kappa_{w,1}A_{w,0} + \kappa_{w,1}A_{w,1}a_\sigma + \kappa_{w,1}A_{w,2}a_q + \\ +\kappa_{w,1}A_{w,3}a_\sigma + \kappa_{w,1}A_{w,4}a_q - A_{w,0} + \omega\mu_{1g} + (1-\omega)(\mu_{2,g} + \phi_g\mu_{1,g})\end{gathered}}{(1-\kappa_{2,1})}, \end{aligned}$$

$$A_{2,1} = \frac{(1-\theta)(1-\gamma)^2(\omega+(1-\omega)\phi_\sigma)^2 + \theta(\phi_\sigma - \gamma(\omega+(1-\omega)\phi_\sigma))^2}{2\theta(1-\kappa_{2,1}\rho_\sigma)},$$

$$A_{2,2}^{+,-} = \frac{(1-\kappa_{2,1}\rho_q) + (1-\theta)\kappa_{w,1}\kappa_{2,1}A_{w,2}\varphi_q^2}{\kappa_{2,1}^2\varphi_q^2} \pm \frac{\sqrt{\begin{array}{c}((1-\kappa_{2,1}\rho_q) + (1-\theta)\kappa_{w,1}\kappa_{2,1}A_{w2}\varphi_q^2)^2 - 2\varphi_q^2\kappa_{2,1}^2((\theta-1)(\kappa_{w,1}\rho_q-1)A_{w,2}+ \\ +\frac{1}{2}((\theta-1)\kappa_{w,1}A_{w,1} + \kappa_{2,1}A_{2,1})^2 + \frac{1}{2}\varphi_q^2(\theta-1)^2\kappa_{w,1}^2A_{w,2}^2)\end{array}}}{(\kappa_1^2)^2\varphi_q^2},$$

$$A_{2,3} = \frac{(1-\theta)(1-\kappa_{w,1}\rho_\sigma^2)A_{w,3} + \frac{1}{2}(1-\gamma(1-\omega))^2}{(1-\kappa_{2,1}\rho_\sigma^2)},$$

and

$$A_{2,4}^{+,-} = \frac{(1-\kappa_{2,1}\rho_q) + (1-\theta)\kappa_{w,1}\kappa_{2,1}A_{w,4}\varphi_q^2}{\kappa_{2,1}^2\varphi_q^2} \pm \frac{\sqrt{\begin{array}{c}((1-\kappa_{2,1}\rho_q) + (1-\theta)\kappa_1\kappa_{2,1}A_{w,4}\varphi_q^2)^2 - 2\varphi_q^2\kappa_{2,1}^2[2(\kappa_{w,1}\rho_q-1)(\theta-1)A_{w,4}+ \\ +[(\theta-1)\kappa_{w,1}A_{w,3} + \kappa_{2,1}A_{w,3}]^2 + \varphi_q^2(\theta-1)^2\kappa_{w,1}^2A_{w,4}^2]\end{array}}}{\kappa_{2,1}^2\varphi_q^2}.$$

Again, following the same reasoning as for the world portfolio, it only makes sense to keep $A_{j,2}^-$ and $A_{j,4}^-$.

1.B Excessive Return Comovements

The test for the excessive return comovements in this paper is a simplified version of the contagion tests in Bekaert, Harvey and Ng (2005), and Baele and Inghelbrecht (2010). I assume that the only fundamental factor to which all countries are exposed is the US equity return. This assumption is coherent with the assumption of a leader economy throughout the paper. It is also consistent with the fact that except for the Euro-zone, not enough countries for any region are considered in the sample.[35]

The excess US returns (global factor) are modeled as follows:

$$r_{US,t} - r_{f,t} = \delta'_{US}Z_{US,t-1} + e_{US,t}, \tag{1.B-3}$$

where $e_{US,t} \mid Z_{US,t-1} \sim N(0, \sigma_{US,t}^2)$, where $Z_{US,t-1}$ contains the so-called global variables. The global variables considered are the world Datastream dividend yield, the Eurodollar spread, the 10 years US T-bill spread and the change in the 90 days US t-bill rate. Now, $\sigma_{US,t}^2$ follows a (potentially) asymmetrical GARCH process such as

$$\sigma_{US,t}^2 = a_{US} + b_{US}\sigma_{US,t-1}^2 + c_{US}e_{US,t-1}^2 + d_{US}\eta_{US,t-1}^2,$$

where $\eta_{US,t}$ is the negative return shock ($\eta_{US,t} = \min\{o, e_{US,t}\}$).

The local returns for all other countries are also assumed to follow GARCH (1,1) processes such as

[35] A single-factor model is considered in Tang (2001). However, it has been shown that models including a regional factor outperform those with only a global factor (see for instance Bekaert, Hodrick and Zhang, 2005).

$$r_{j,t} - r_{f,t} = \delta_j Z_{j,t-1} + \beta^{US}_{j,t-1}\mu_{US,t-1} + \beta^{US}_{j,t-1} e_{US,t} + e_{j,t}, \tag{1.B-4}$$

where $\mu_{us,t-1} = E[r_{US,t} - r_{f,t} \mid Z_{US,t-1}]$. The local information set $Z_{j,t-1}$ contains the local dividend yield. Now, the residuals in Eq. (1.B-4) follow $e_{j,t} \mid Z_{j,t-1} \sim N(0, \sigma^2_{j,t})$, where $\sigma^2_{j,t}$ follows a (potentially) asymmetrical GARCH process such as

$$\sigma^2_{j,t} = a_j + b_j \sigma^2_{j,t-1} + c_j e^2_{j,t-1} + d_j \eta^2_{j,t-1}.$$

Finally, the time varying sensitivity to the US (global) factor follows

$$\beta^{US}_{j,t-1} = \beta_{j,0} + \beta_{j,1} ts_{j,t-1} + \beta_j X^{US}_{j,t-1}, \tag{1.B-5}$$

where $X^{us}_{j,t-1}$ is the percentage of trade over GDP (sum of exports and imports over local GDP. Only for the monthly frequency), and $ts_{j,t-1}$ is the 1-year term spread for each country. Controlling by $X^{us}_{j,t-1}$ allows the betas to be impacted by trade as proposed by Bekaert, Harvey and Ng (2005), while the term spread is included to account for the possible cyclicality of betas as suggested by Baele and Inghelbrecht (2010).

Following the literature of contagion, in a first stage, I estimate the model in Eqs. (1.B-3) to (1.B-5). In a second stage, the residuals from this regression are used to test for unusual return comovements across countries. The excessive comovement test requires the estimation of the following regression:

$$\begin{aligned} \widehat{e}_{j,t} &= w_j + v_{j,t}\widehat{e}_{us,t} + u_{j,t}, \\ v_{j,t} &= v_o + v_1 D_{US,jt-1}, \end{aligned}$$

where $D_{US,j,t-1}$ are the dummy variables controlling for extreme realizations of the variance premium.[36] The estimated parameters for this tests are available upon request.

[36] All the attention is focused on the estimated parameter v_1. See Baele and Inghelbrecht (2010) for a discussion on the difference between the test for the correct specification of the model and the contagion test itself.

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Table 1.1: Summary Statistics. Variance (and Volatility) Premiums

The table reports the average volatility premiums (in annual percentages) calculated monthly as $volp_{j,t} = iv_{j,t} - E_t(rv_{j,t+1})$ for the eight countries considered for the sample period 2000 to 2009. It also reports the summary statistics for the variance premiums (in annual squared percentages). The variance premium in each country is estimated as $vp_{j,t} = iv_{j,t}^2 - (\widehat{rv}_{jt+1})^2$,where the benchmark specification for the expected realized variance is its first order autoregressive forecast. AR(1) is the estimated first-order autoregressive coefficient of the variance premium. *,** and *** represent significance at the standard 1, 5 and 10% confidence levels, both for the significance of the mean and the AR(1) coefficient. For the average volatility and variance premium, I perform a standard mean test and correct the standard deviations using Newey-West with 12 lags (given the evidence on the significance of the autoregressive coefficient). For the AR(1) coefficient, I correct the standard errors using the Newey-West HAC (Heteroskedasticity and autocorrelation correction) with 12 monthly lags (Newey and West, 1987).

			Volatility Premium	Variance Premium						
Country	IV Index	Index	Mean (%)	Mean (sq. %)	Min.	Max.	St. Dev.	Kurt.	Skew.	AR(1)
US	VIX	S&P 500	3.18***	124.32***	−1265.35	1155.62	257.55	11.90	−0.45	0.38***
GER	VDAX	DAX 30	2.42***	142.25***	−653.94	1612.70	288.28	10.56	2.10	0.28**
JAP	VXI	NIKKEI 225	3.87***	257.35***	−243.46	3398.24	485.43	21.64	3.91	0.74***
UK	VFTSE	FTSE 100	3.40***	152.68***	−810.06	1023.58	232.17	8.28	1.08	0.55***
SWI	VSMI	SMI	2.72***	140.48***	−284.18	1154.08	248.90	7.70	2.12	0.71***
NL	VAEX	AEX 25	3.24***	164.95***	−1357.79	1420.36	329.72	10.04	0.76	0.43***
BE	VBEL	BEL 20	1.71***	67.84**	−1491.42	1180.34	260.39	16.26	−0.72	0.22**
FR	VCAC	CAC 100	2.31***	115.87**	−1741.72	1805.45	322.28	18.31	0.36	0.27***

Table 1.2: Base Scenario for the Numerical Implications of the Two-Country Model

The table reports the values for the two-country model parameters considered as the base scenario to test its numerical implications. In this scenario, all parameters in the preference function (Eq. (1.3)) are taken from BTZ. The country-specific parameters in Eqs. (1.1) and (1.2) are estimated as follows: $\mu_{j,g}$ is estimated as the average IP growth for the sample 1973-2009; $\mu_{j,\sigma}$ is estimated as the IP growth unconditional variance for the sample 1973-2009. Finally, the parameters k_o and k_1 in the log-linearization of returns (Eq. (1.A-1)) are estimated using data for the Price-Dividend (PD) ratio for each country as well as for the Datastream world portfolio. The log-linearization constants are estimated as $k_1 = \frac{e^{E(PD)}}{1+e^{E(PD)}}$, where $E(PD)$ is the unconditional mean of the (log) PD ratio, and $k_0 = -k_1 \ln(1-k_1) - (1-k_1)\ln(1-k_1)$ (Campbell and Cochrane, 1999).

	Value			
Param.	Global	US	GER	Description
μ_g		1.6×10^{-3}	8.3×10^{-4}	Mean consumption growth
a_σ		1.2×10^{-6}	6.6×10^{-6}	Long-run consumption volatility
ρ_σ		0.98	0.98	Speed of reversion consumption volatility
a_q		2.0×10^{-7}	2.0×10^{-7}	Long-run VoV
ρ_q		0.80	0.80	Speed of reversion VoV
k_0	0.12	0.13	0.12	Campbell-Shiller k_0
k_1	0.97	0.97	0.97	Campbell-Shiller k_1
ψ	2.50			Intertemporal elasticity of substitution
$\log \delta$	1.00			Discount factor

Table 1.3: Variance Premium Correlations across countries

The table reports the correlation coefficients among the monthly variance premiums for all countries for the sample period 2000 to 2009.

	US	GER	JAP	UK	SWI	NL	BE	FR
US	1.00	0.56	−0.08	0.74	0.37	0.76	0.62	0.78
GER		1.00	0.24	0.74	0.79	0.82	0.42	0.78
JAP			1.00	0.18	0.61	0.07	0.07	−0.14
UK				1.00	0.70	0.85	0.64	0.77
SWI					1.00	0.67	0.48	0.51
NL						1.00	0.72	0.89
BE							1.00	0.57
FR								1.00

Table 1.4: Predicting Variance Premium Correlations across Countries

The table reports the estimated coefficients $\gamma_{1,j,k}$ in the following regression:

$$\rho_t(vp_{j,t,t+1}, vp_{k,t,t+1}) = \gamma_{0,jk} + \gamma_{1,j,k} vp_{k,t-1} + \epsilon_{j,k,t},$$

where j are the countries in the columns and k, the countries in the rows. For example, the information under the column JAP shows how the variance premium correlation between Japan (j) and any other country in the rows (k) is forecasted by the latter country variance premium. The correlation coefficient for the period t to $t+1$ is calculated using daily data for the variance premiums of the two countries for the month starting immediately after the realization of $vp_{k,t-1}$. The standard errors in all regressions are corrected by Newey-West with 12 lags. In order to make the coefficients easier to interpret, the variance premiums are taken in monthly squared percentages.

	$\rho(vp_{j,t,t+1}, vp_{k,t,t+1})$								
$vp_{k,t-1}$	US	GER	JAP	UK	SWI	NL	BE	FR	Avg. R^2
US		50.52**	45.53**	28.62	26.54	30.34	46.40	8.22	
		(2.05)	(2.03)	(1.58)	(1.42)	(1.61)	(1.48)	(0.41)	
R^2		4.57	2.50	1.88	1.16	2.04	2.30	0.14	2.08
GER	38.23***		22.16	31.31***	6.52	19.22*	24.32	10.32	
	(3.52)		(1.40)	(2.58)	(0.62)	(1.84)	(1.07)	(0.89)	
R^2	4.40		0.96	3.51	0.17	1.91	1.29	0.53	1.82
JAP	37.45**	30.90***		46.63***	55.76***	36.99***	47.99***	37.84***	
	(2.31)	(2.77)		(3.51)	(4.21)	(2.67)	(3.08)	(2.98)	
R^2	4.52	2.95		6.46	7.98	4.24	5.71	4.04	5.13
UK	39.45***	27.25*	66.80***		3.24	24.88*	−0.23	−8.33	
	(3.28)	(1.77)	(2.94)		(0.25)	(1.84)	(−0.01)	(−0.57)	
R^2	4.32	1.91	6.00		0.03	2.15	0.00	0.23	2.09
SWI	21.49**	16.95	42.14**	15.16*		18.70*	2.23	8.79	
	(2.31)	(1.60)	(2.09)	(1.78)		(1.74)	(0.07)	(0.93)	
R^2	1.12	1.02	2.52	0.83		1.22	0.01	0.26	1.00
NL	21.32**	21.08**	14.66	22.71**	15.80		29.42	5.20	
	(2.13)	(2.21)	(0.58)	(1.97)	(1.18)		(1.55)	(0.58)	
R^2	1.63	2.22	0.40	2.40	0.95		1.84	0.17	1.37
BE	−6.40	−6.08	−14.92	−13.84	−18.32	−34.07		−25.70	
	(−0.17)	(−0.13)	(−0.33)	(−0.36)	(−0.50)	(−0.64)		(−0.60)	
R^2	0.03	0.03	0.15	0.18	0.31	1.09		0.58	0.34
FR	−0.59	−1.99	19.24	−6.72	−21.20	−5.95	−2.48		
	(−0.04)	(−0.18)	(0.87)	(−0.48)	(−1.33)	(−0.52)	(−0.12)		
R^2	0.00	0.01	0.49	0.15	1.28	0.18	0.01		0.30

Table 1.5: The role of Foreign Variance Premium in Predicting International Equity Returns

The Table reports the estimated coefficient $\gamma_{1,j,k}$ in the following regressions:

$$(r-r_f)_{j,t,t+3} = \gamma_{0,j,k} + \gamma_{1,j,k} vp_{k,t} + \gamma_{1,j,k} dy_{j,t} + \gamma_{1,j,k} ts_{j,t} + \epsilon_{j,k,t},$$

where $(r-r_f)_{j,t,t+3}$ are 3-months (compounded annualized) excess returns, $dy_{j,t}$ are the (local) dividend yields and $ts_{j,t}$ the (local) term spreads calculated as the difference between the 1 year T-bill and the 3 months t-bill rate. The standard errors are corrected by Newey-West with a number of lags $l = 12$. The table also reports the average R^2 for the predictive power of each country's variance premium.

	$(r-r_f)_{j,t,t+h}$								
$vp_{k,t-1}$	US	GER	JAP	UK	SWI	NL	BE	FR	Avg. R^2
US	0.05***	0.04**	0.03*	0.03***	0.03***	0.03*	0.04**	0.04**	
	(4.94)	(2.50)	(1.67)	(3.85)	(2.61)	(1.96)	(2.50)	(2.53)	
R^2	22.87	10.94	7.93	15.16	11.39	6.84	9.11	12.77	12.13
GER	0.02*	0.00	−0.01	0.01	0.00	−0.01	0.00	0.00	
	(1.70)	(−0.12)	(−0.54)	(0.65)	(0.17)	(−0.95)	(0.25)	(0.29)	
R^2	13.05	5.24	4.30	7.66	6.13	4.22	2.86	5.87	6.17
JAP	−0.01	−0.03*	−0.01	0.00	−0.01	−0.02	−0.02**	−0.02***	
	(−1.15)	(−1.71)	(−0.82)	(−0.58)	(−1.01)	(−1.45)	(−2.00)	(−2.75)	
R^2	10.80	9.67	5.05	7.57	7.74	5.78	5.78	9.87	7.78
UK	0.03**	0.01	0.00	0.02	0.02	0.00	0.02	0.02	
	(2.37)	(0.26)	(0.09)	(0.95)	(0.96)	(−0.03)	(0.53)	(0.91)	
	14.14	5.39	4.03	8.63	7.30	3.57	3.42	7.30	6.72
SWI	0.00	−0.04	−0.02	0.00	−0.01	−0.03	−0.01	−0.01	
	(0.35)	(−1.45)	(−0.76)	(−0.31)	(−0.55)	(−1.39)	(−0.58)	(−0.99)	
R^2	9.84	7.92	4.75	7.28	6.35	5.73	3.22	6.43	6.44
NL	0.03***	0.02	0.01	0.02	0.01	0.00	0.02	0.02	
	(2.86)	(0.90)	(0.60)	(1.49)	(1.09)	(0.24)	(1.21)	(1.14)	
R^2	17.70	6.64	4.78	10.05	7.79	3.67	5.46	7.94	8.00
BE	0.01*	0.01	0.01	0.01	0.01	0.01	0.03***	0.01	
	(1.65)	(0.84)	(0.59)	(1.50)	(1.05)	(0.65)	(3.06)	(0.93)	
R^2	11.02	5.68	4.42	8.10	6.61	3.80	6.53	6.42	6.57
FR	0.03***	0.02	0.01	0.02	0.01	0.01	0.02	0.02	
	(2.84)	(0.99)	(0.77)	(1.64)	(1.29)	(0.66)	(1.27)	(1.57)	
R^2	19.14	7.27	5.16	10.47	8.07	4.17	5.55	9.86	8.71

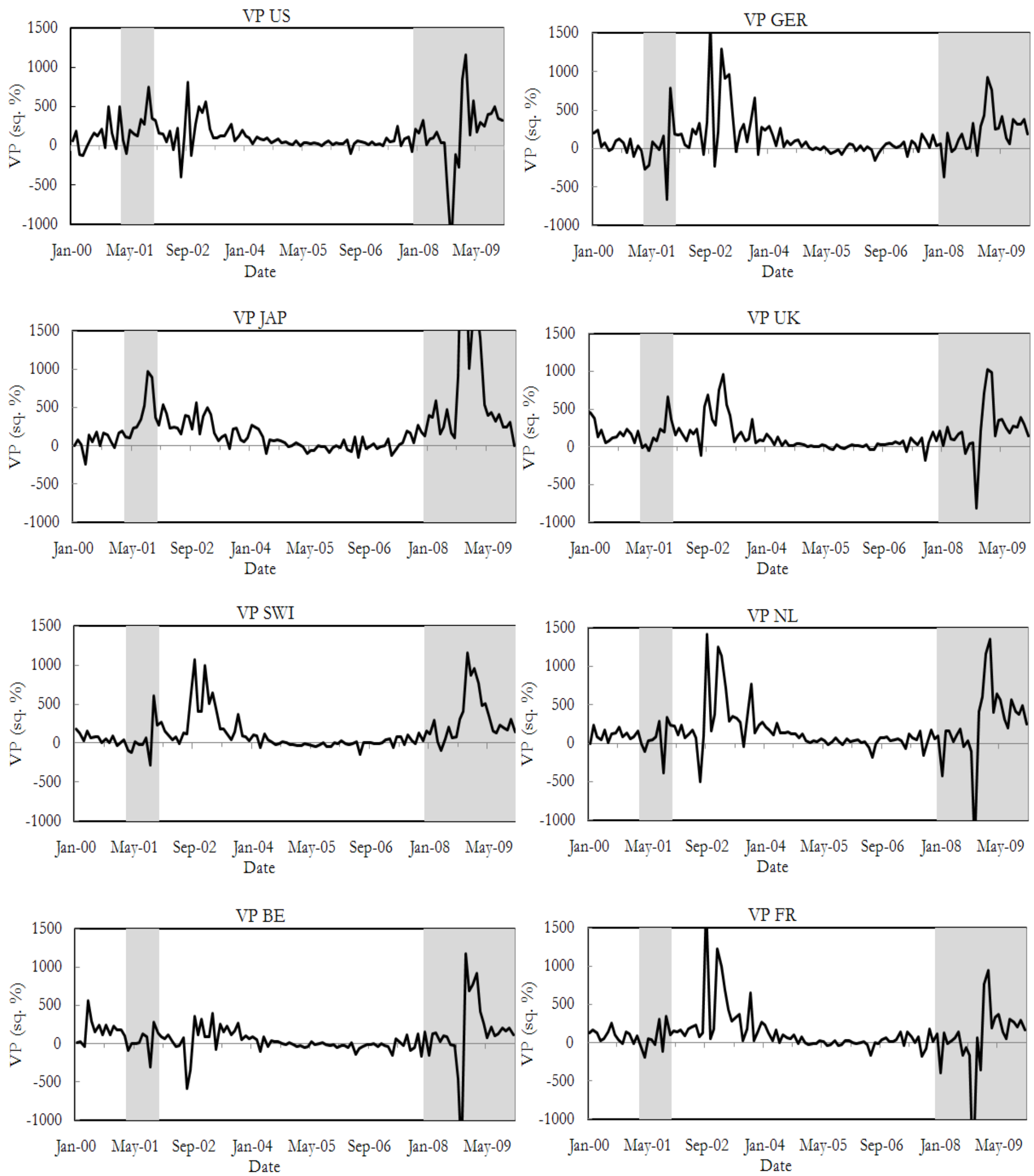

Figure 1.1: Estimated (model-free) Variance premiums

The figure shows the Variance Premiums vp_t in annual squared percentages for the eight countries the sample (see Table 1.1) for the sample period 2000 to 2009. The variance premium in each country is estimated as $vp_{j,t} = iv_{j,t}^2 - (\widehat{rv}_{jt+1})^2$, where the benchmark specification for the expected realized variance is its first order autoregressive forecast. The shaded areas represent NBER recession episodes for the US.

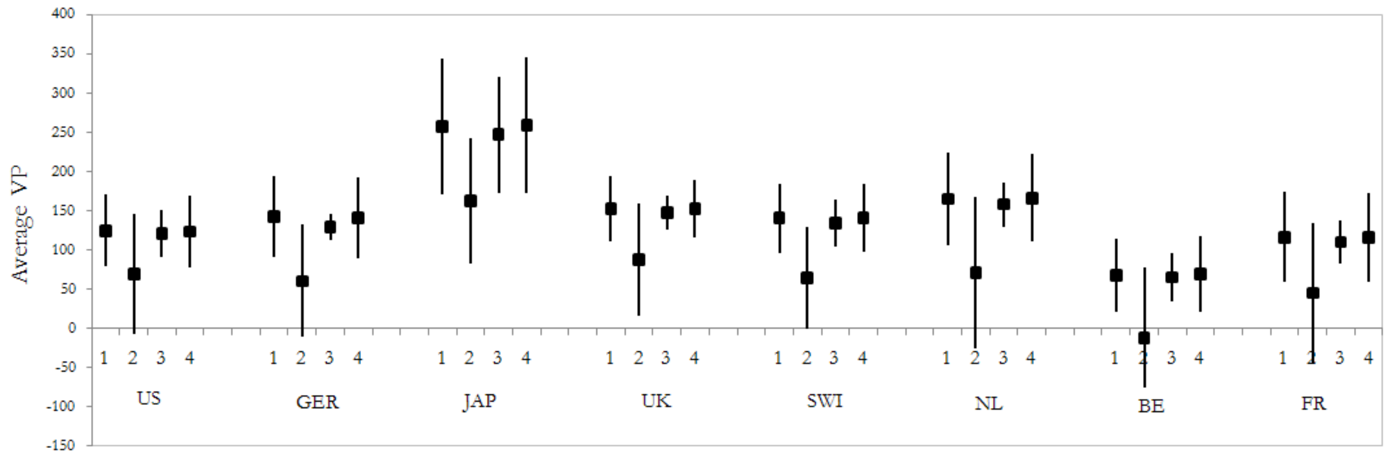

Figure 1.2: Significance of the average Variance Premiums. Alternative specifications

The figure reports the average variance premiums for each country for 4 alternative specifications (in bold squares) for the sample period 2000 to 2009. For each measure and each country, I also report the 95% confidence intervals for the significance of the average variance premium. Measure 1 is the benchmark measure (AR(1)) where the expected realized variance is estimated as its first order autoregressive forecast. Measure 2 assumes that the expectation of the volatility under the physical measure is well-proxied by rv_t or martingale assumption. In measure 3, the expected realized variance is estimated from a regression that includes the IV indices as in $rv_{j,t+1}= \gamma_o+\gamma_1 rv_{j,t} + \gamma_2 iv_{j,t}+\epsilon_t$. Finally, in measure 4, the expected realized variance is estimated from a regression that includes the monthly range-based variance for each country as in $rv_{j,t+1}= \gamma_o+\gamma_1 rv_{j,t} + \gamma_2 RangeV_{j,t}+\epsilon_t$, where $RangeV_{j,t}$ is the range based variance calculated as $RangeV_{j,t} = \frac{1}{4\ln 2}\sum_{t_i=1}^{N_t} range_{t_i}^2$, where $range_{t_i}$ is the daily difference between the highest and the lowest price of the index.

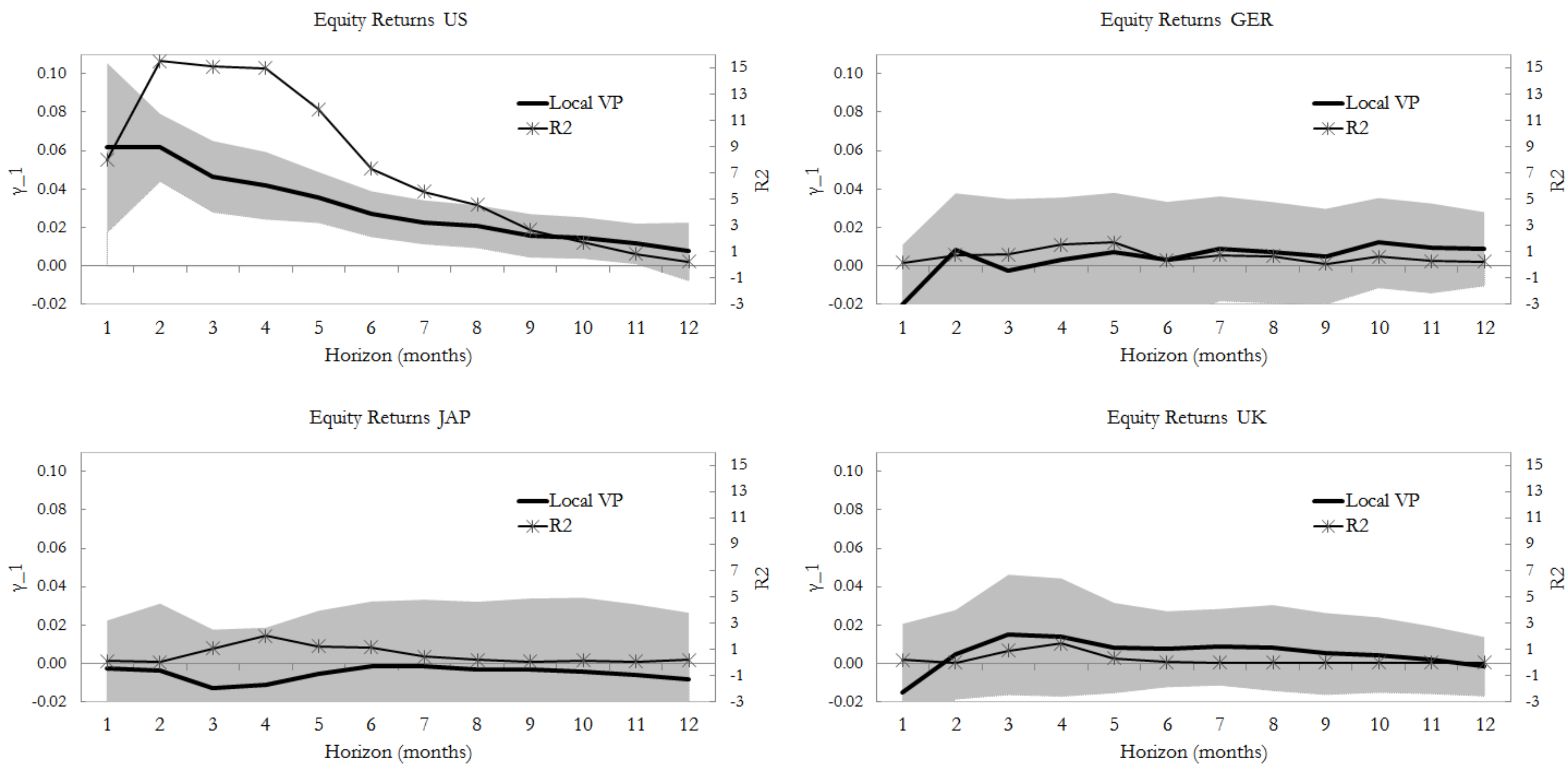

Figure 1.3: The role of the Local Variance Premium in Predicting Local Equity Returns

The Figure reports the estimated coefficients $\gamma_{1,j,h}$ in the following regressions:

$$(r - r_f)_{j,t,t+h} = \gamma_{0,,j,h} + \gamma_{1,,j,h} vp_{j,t} + \gamma_{2,,j,h} dy_{j,t} + \gamma_{3,,j,h} ts_{j,t} + \epsilon_{j,h,t},$$

where $(r - r_f)_{j,t,t+h}$ are h-months (compounded annualized) excess returns, $dy_{j,t}$ are the local dividend yields and $ts_{j,t}$ are the local term spreads calculated as the difference between the 1 year T-bill and the 3 months t-bill rate. I consider monthly forecasting horizons up to 12 months. The shaded areas represent the 95% confidence intervals for the Newey-West corrected standard errors with a number of lags $l = max\,\{2h, 12\}$. The figure also reports in the secondary axis the R^2 for each regression. In order to separately identify the predictive power of the variance premium, the R^2 are reported for regressions in which only the variance premiums are considered as in $(r - rf)_{j,t,t+h} = \gamma_{0,j,h} + \gamma_{1,j,h} vp_{j,t} + \epsilon_{j,h,t}$.

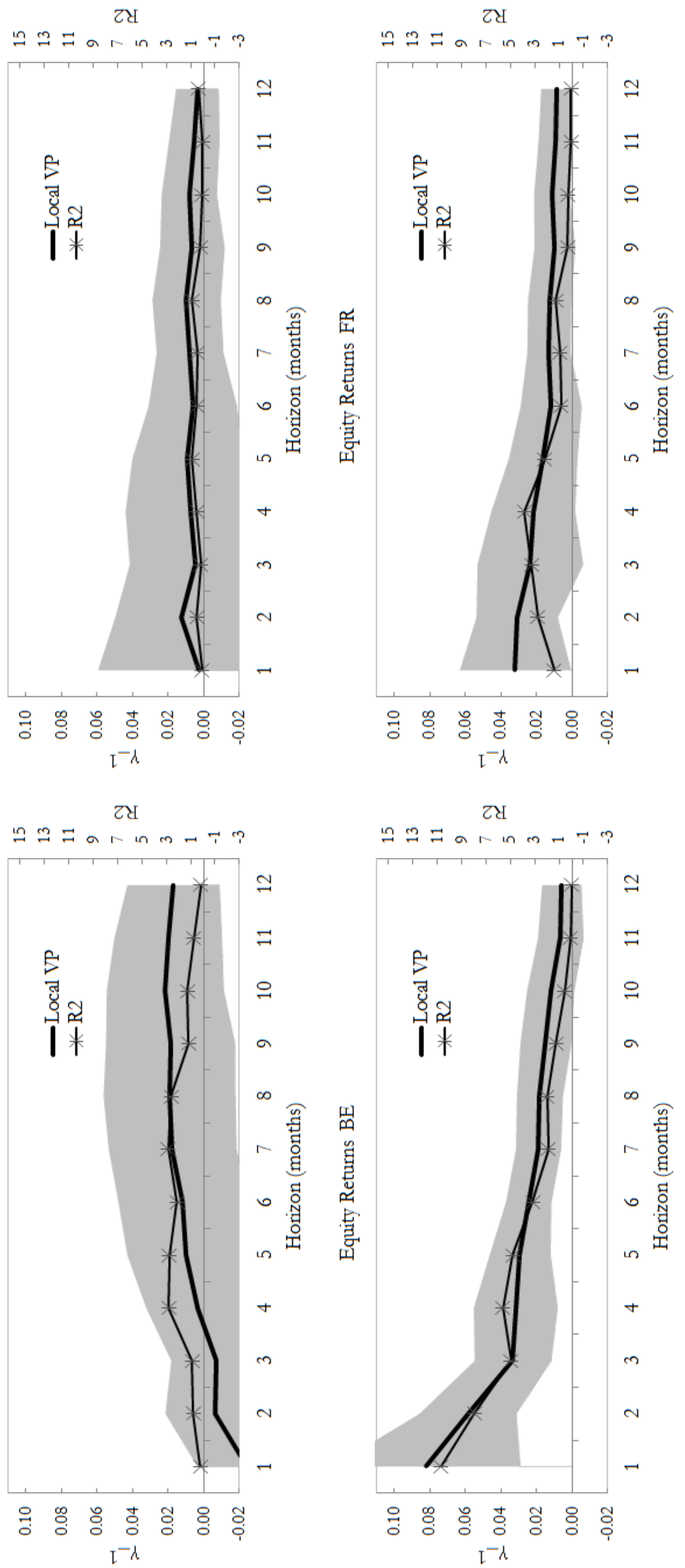

Figure 1.3: The role of the Local Variance Premium in Predicting Local Equity Returns. Continued

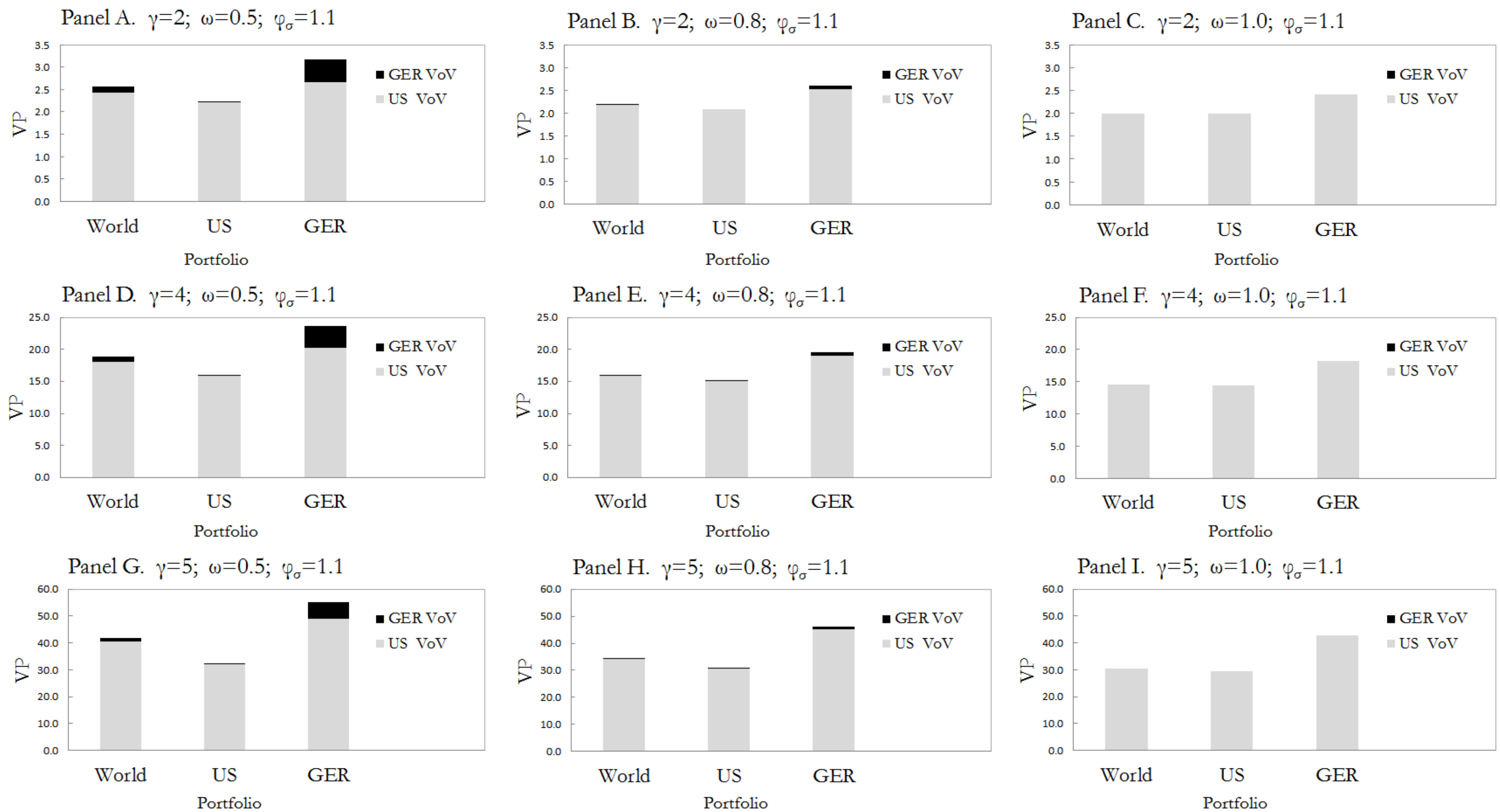

Figure 1.4: loads of VoV on the Variance Premiums. Alternative Parameters

The figure reports the total unconditional loads of the two countries' volatility of volatility (VoV) on the variance premium of each portfolio as implied by Eqs. (1.4) to (1.6) for alternative values of the parameters γ, ω, ϕ_σ. The unconditional loads are calculated as $(\theta-1)\kappa_{w,1}V_{j,k}E(q_{US,t})$ and $(\theta-1)\kappa_{w,1}V_{j,k}E(q_{GER,t})$ for $k = US, GER$, and j each one of the three possible portfolios: US, GER, and the global portfolio. Given the parameters in the base scenario (Table 1.2), the average VoV is given by $E(q_{US,t}) = E(q_{GER,t}) = \frac{a_q}{1-\rho_q} = 1.0 \times 10^{-6}$.

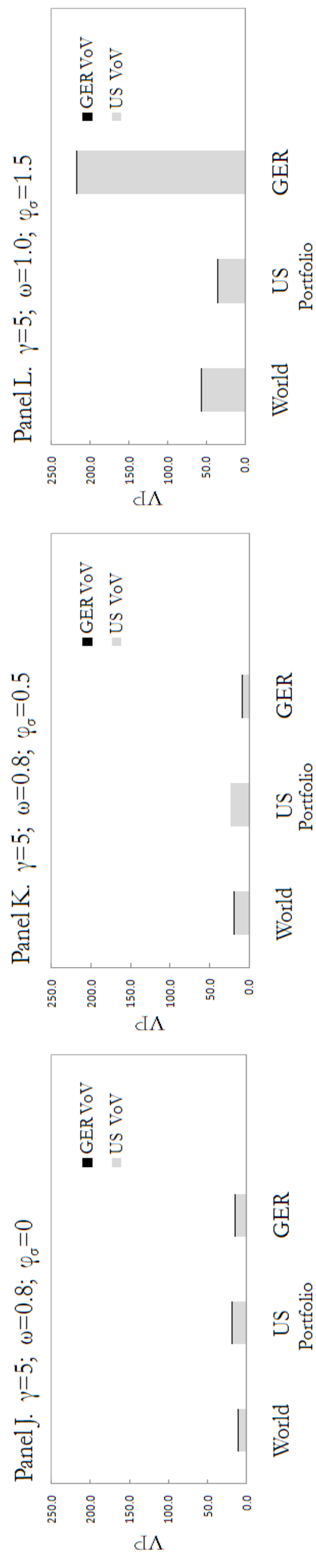

Figure 1.4: loads of VoV on the Variance Premiums. Alternative Parameters. Continued

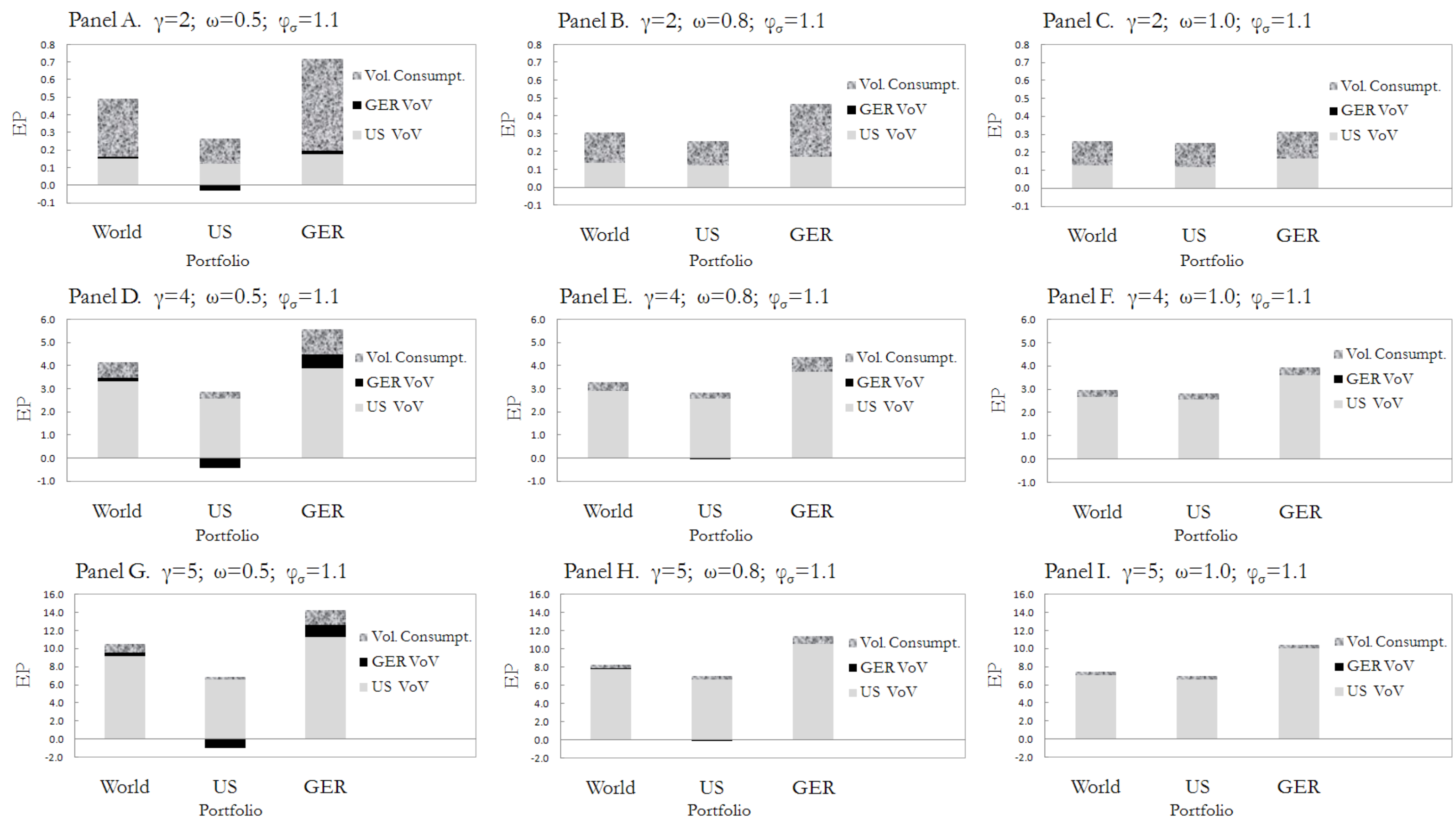

Figure 1.5: Consumption Volatility and VoV loads on Equity Premiums. Alternative Parameters

The figure reports the total unconditional components of all possible portfolio's equity premiums as implied by Eqs. (1.8) to (1.10) for alternative values of the parameters γ, ω, ϕ_σ. Only the exponential adjustment term $(-\frac{1}{2}\sigma^2_{r_1,t})$ is not considered in the figure. Given the parameters in the base scenario in Table 1.2, the loads of the volatility of consumption are calculated by assuming that (the average volatility of consumption) $E(\sigma_{US,t}) = E(\sigma_{GER,t}) = \frac{a_{\sigma,US}}{1-\rho_\sigma} = 6.0 \times 10^{-5}$. The loads of VoV are calculated by assuming that (the average VoV) $E(q_{US,t}) = E(q_{GER,t}) = \frac{a_q}{1-\rho_q} = 1.0 \times 10^{-6}$.

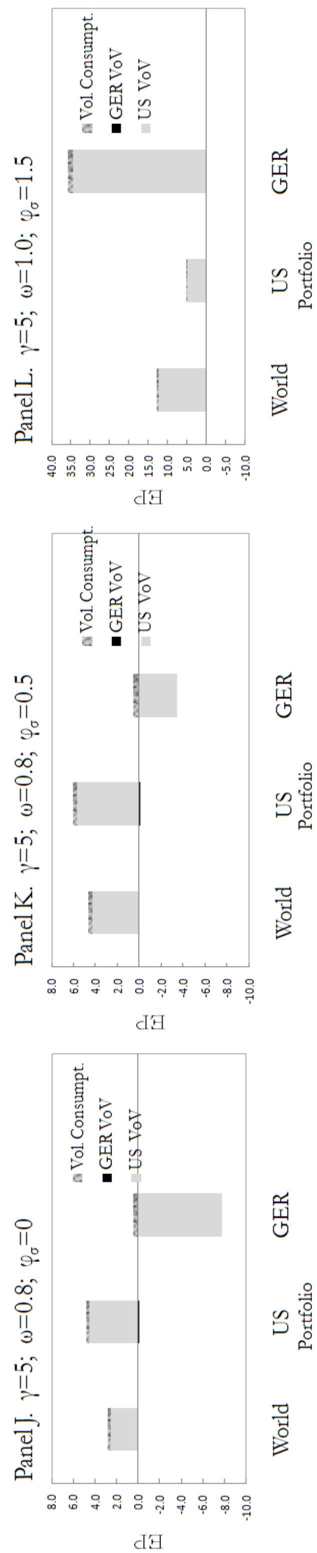

Figure 1.5: Consumption Volatility and VoV loads on Equity Premiums. Alternative Parameters. Continued

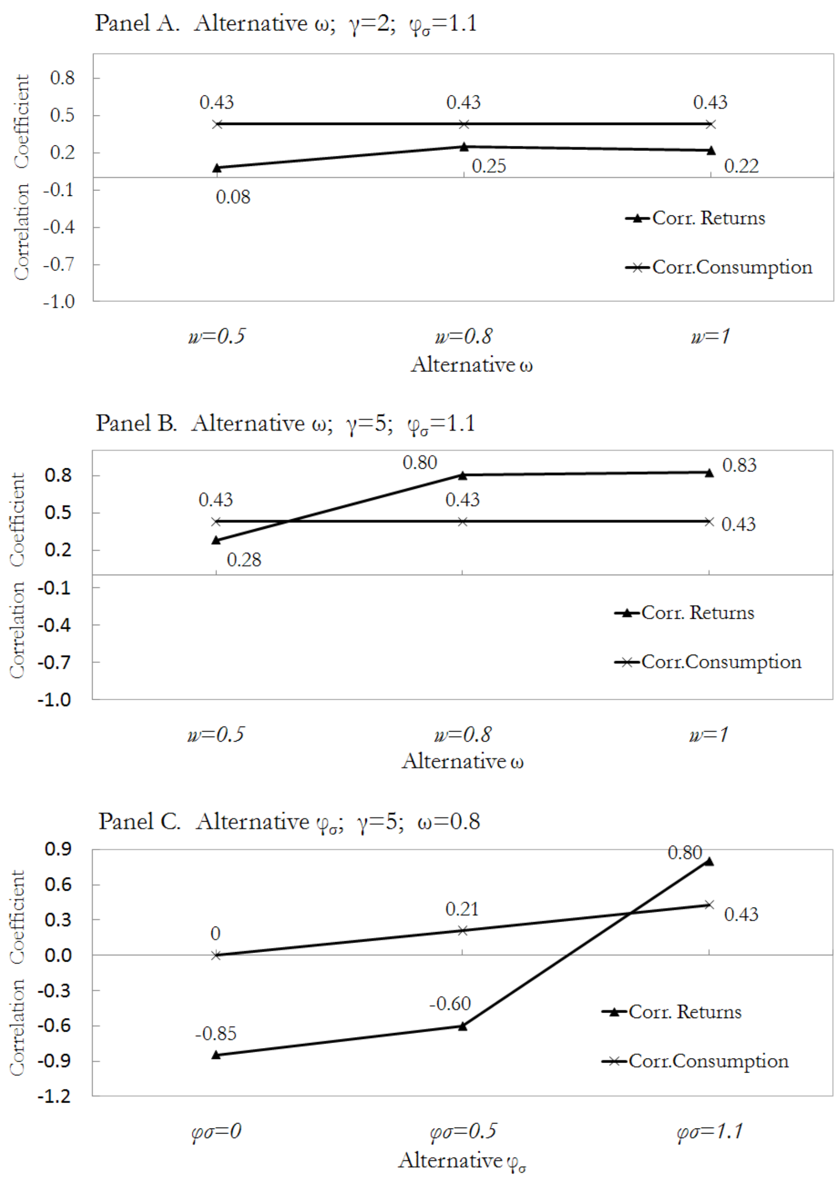

Figure 1.6: Cross-Country Return Correlations and Model-implied correlation of consumption

The figure shows the model-implied unconditional correlation of consumption ($\rho(g_{US,t}, g_{GER,t})$) and the model-implied equity return correlation ($\rho(r_{US,t}, r_{GER,t})$) between Germany and the US for several alternative values of the parameters γ, ω, ϕ_σ.

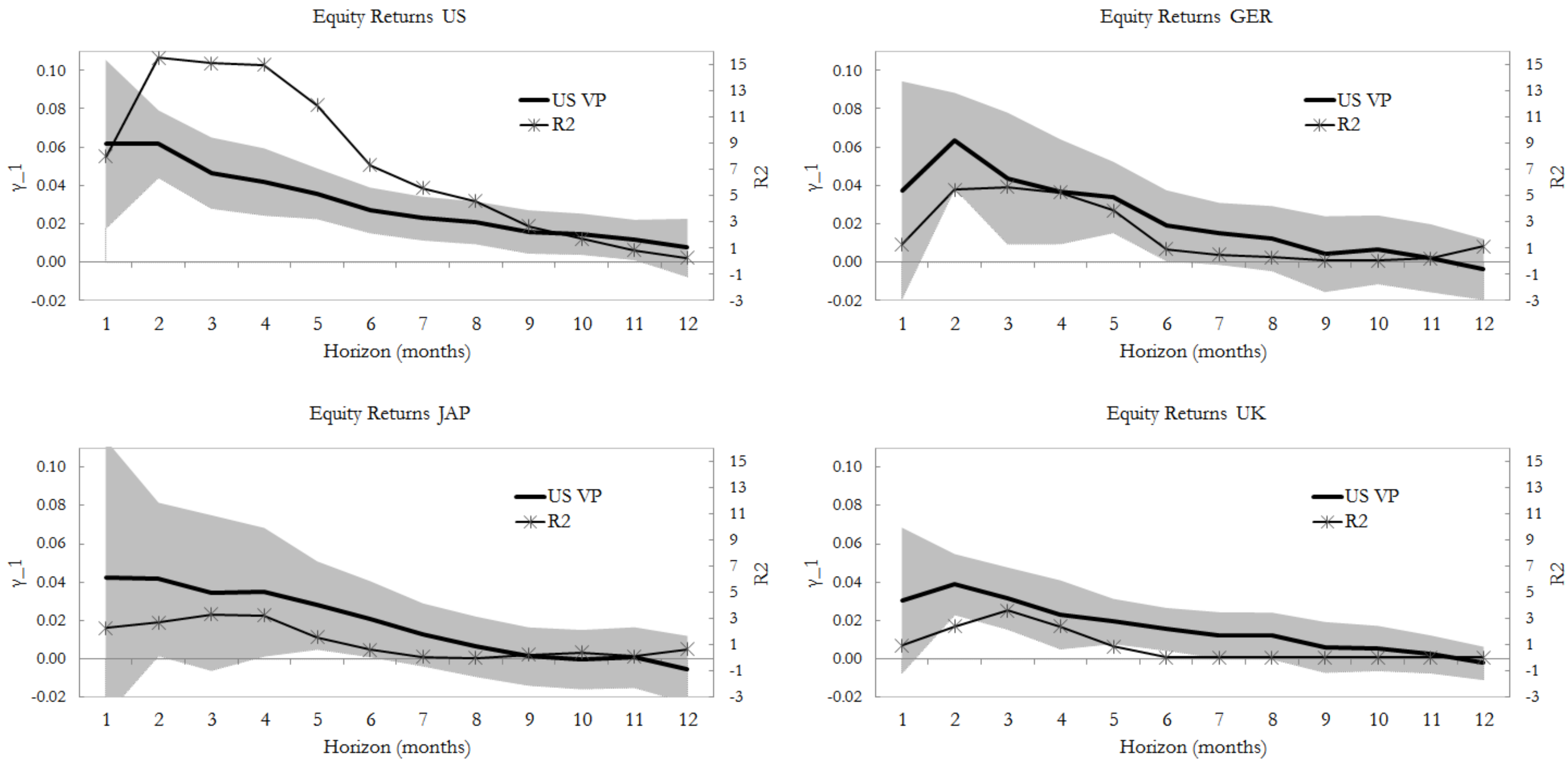

Figure 1.7: The role of the US Variance Premium in Predicting International Equity Returns

The Figure reports the estimated coefficient $\gamma_{1,j,h}$ in the following regressions:

$$(r - r_f)_{j,t,t+h} = \gamma_{0,j} + \gamma_{1,j,h} vp_{US,t} + \gamma_{1,j,h} dy_{j,t} + \gamma_{1,j,h} ts_{j,t} + \epsilon_{j,h,t},$$

where $(r - r_f)_{j,t,t+h}$ are h-months (compounded annualized) excess returns, $dy_{j,t}$ are the local dividend yields and $ts_{j,t}$ are the local term spreads calculated as the difference between the 1 year T-bill and the 3 months t-bill rate. I consider monthly forecasting horizons up to 12 months. The shaded areas represent the 95% confidence intervals for the Newey-West corrected standard errors with a number of lags $l = max\{2h, 12\}$. The figure also reports in the secondary axis the R^2 for each regression. In order to separately identify the predictive power of the variance premium, the R^2 are reported for regressions in which only the US variance premium is considered as in $(r - rf)_{j,t,t+h} = \gamma_{0,j,h} + \gamma_{1,j,h} vp_{j,US} + \epsilon_{j,h,t}$.

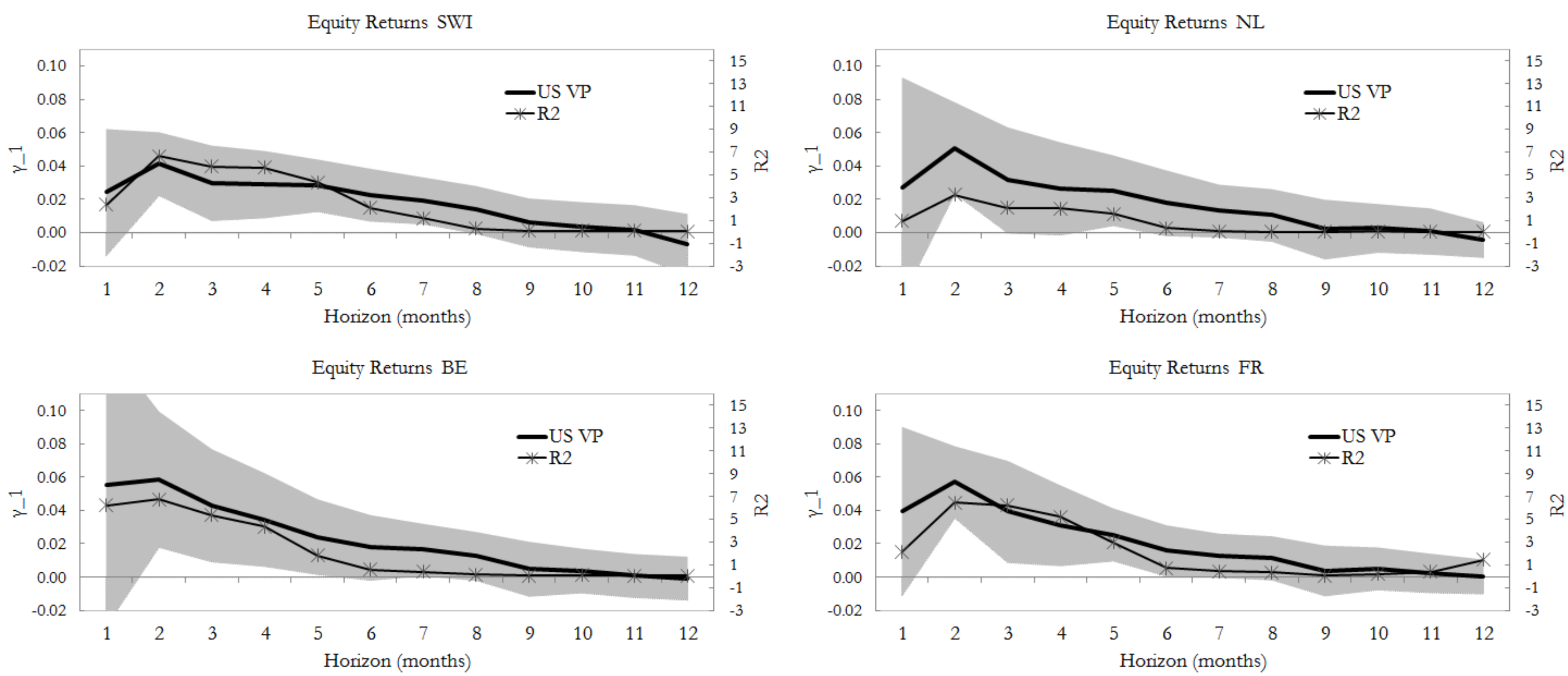

Figure 1.7: The role of the US Variance Premium in Predicting International Equity Returns. Continued

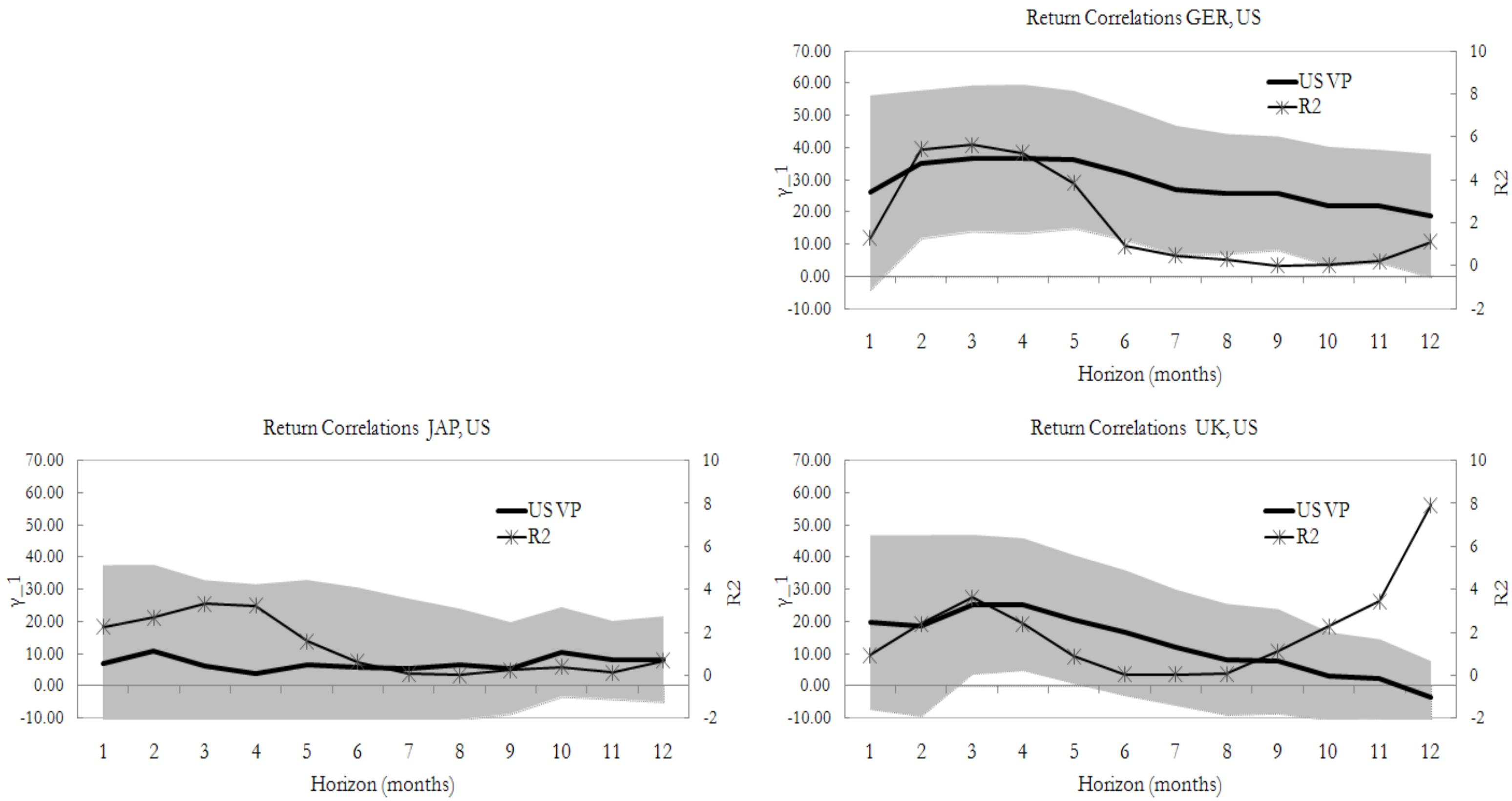

Figure 1.8: The role of the US Variance Premium in Predicting Equity Return Correlations across countries

The table shows the estimated coefficients $\gamma_{1,j,US}$ in the following regression:

$$\rho_t(r_{j,t,t+h}, r_{US,t,t+h}) = \gamma_{0,jk} + \gamma_{1,j,US} vp_{US,t-1} + \epsilon_{jk,t},$$

where $\rho_t(r_{j,t,t+h}, r_{US,t,t+h})$ is the h-months ahead equity return correlation between any country and the US. The correlation coefficient for the period t to $t+1$ is calculated using daily equity returns for the two countries for the month starting immediately after the realization of $vp_{k,t-1}$. The shaded areas represent the 95% confidence intervals for the Newey-West corrected standard errors with a number of lags $l = max\,\{2h, 12\}$. The figure also reports in the secondary axis the R^2 for each regression.

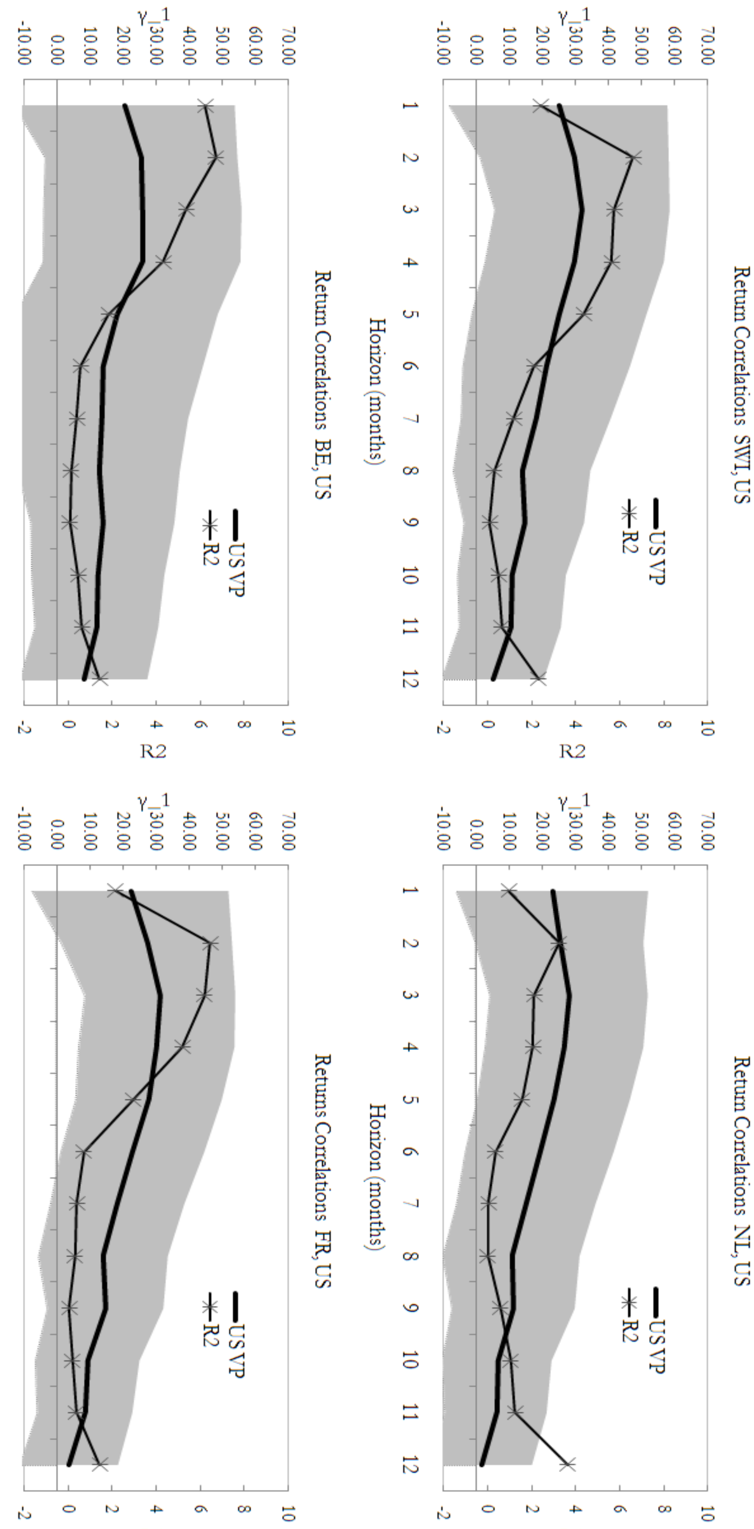

Figure 1.8: The role of the US Variance Premium in Predicting Equity Return Correlations across countries. Continued

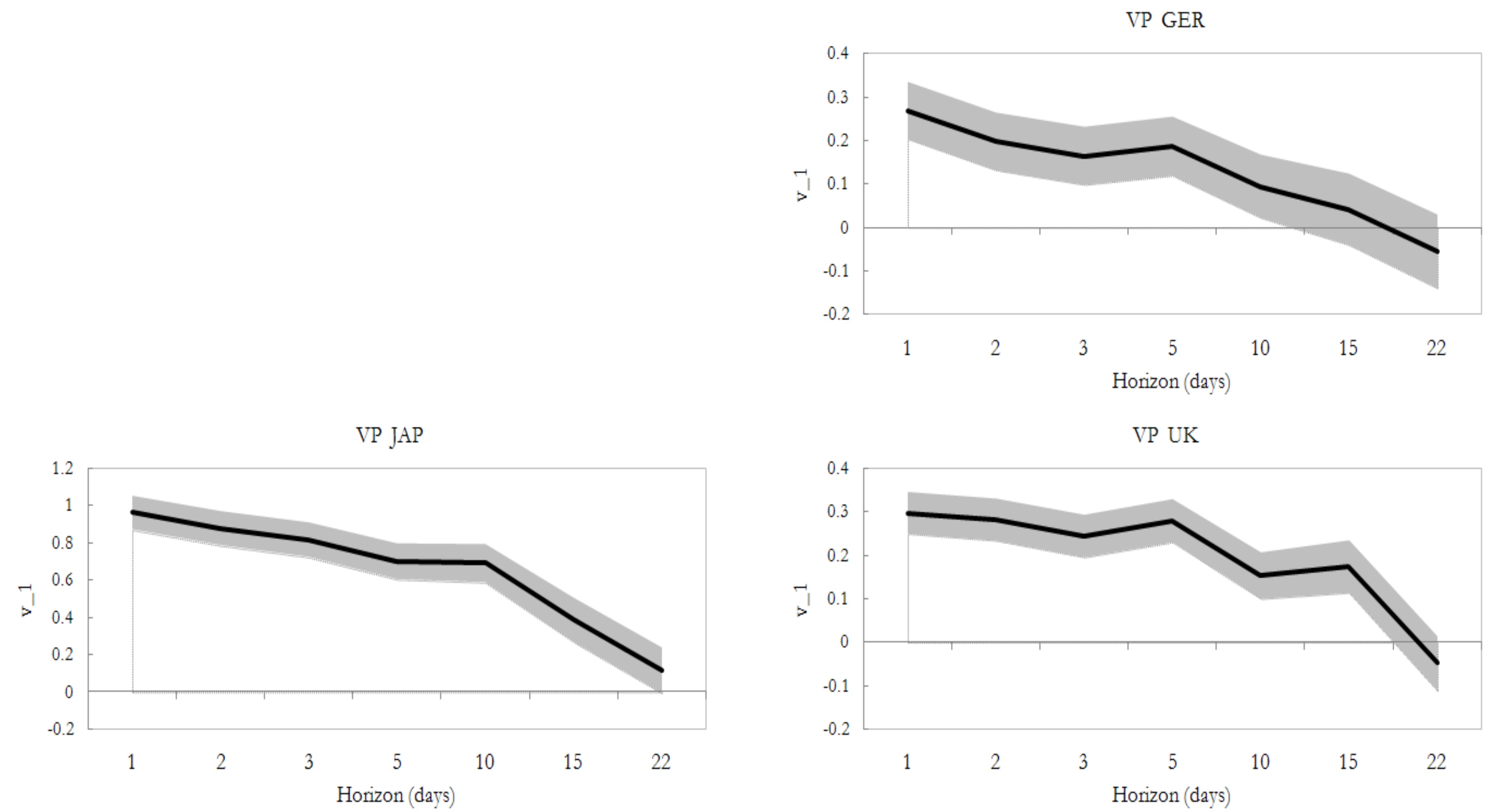

Figure 1.9: The role of the US Variance Premium in explaining Excessive Variance Premium Comovements
The figure reports the estimated coefficient $v_{1,j,h}$ in the following regressions:

$$vp_{j,t+h} = \gamma_{0,j,h} + \gamma_{1,,j,h,t} vp_{US,t} + \epsilon_{t,h}, \tag{1.2-6}$$

where $vp_{j,t+h}$ is the h-days ahead variance premium for country j. The time-varying coefficient $\gamma_{1,,j,h,t}$ follows the process $\gamma_{1,j,h,t} = v_{o,j,h} + v_{1,,j,h} D_{US,t}$, where the dummy $D_{US,t}$ characterizes the extreme values of the US variance premiums (5^{th} highest percentile). The shaded areas represent the 95% confidence intervals.

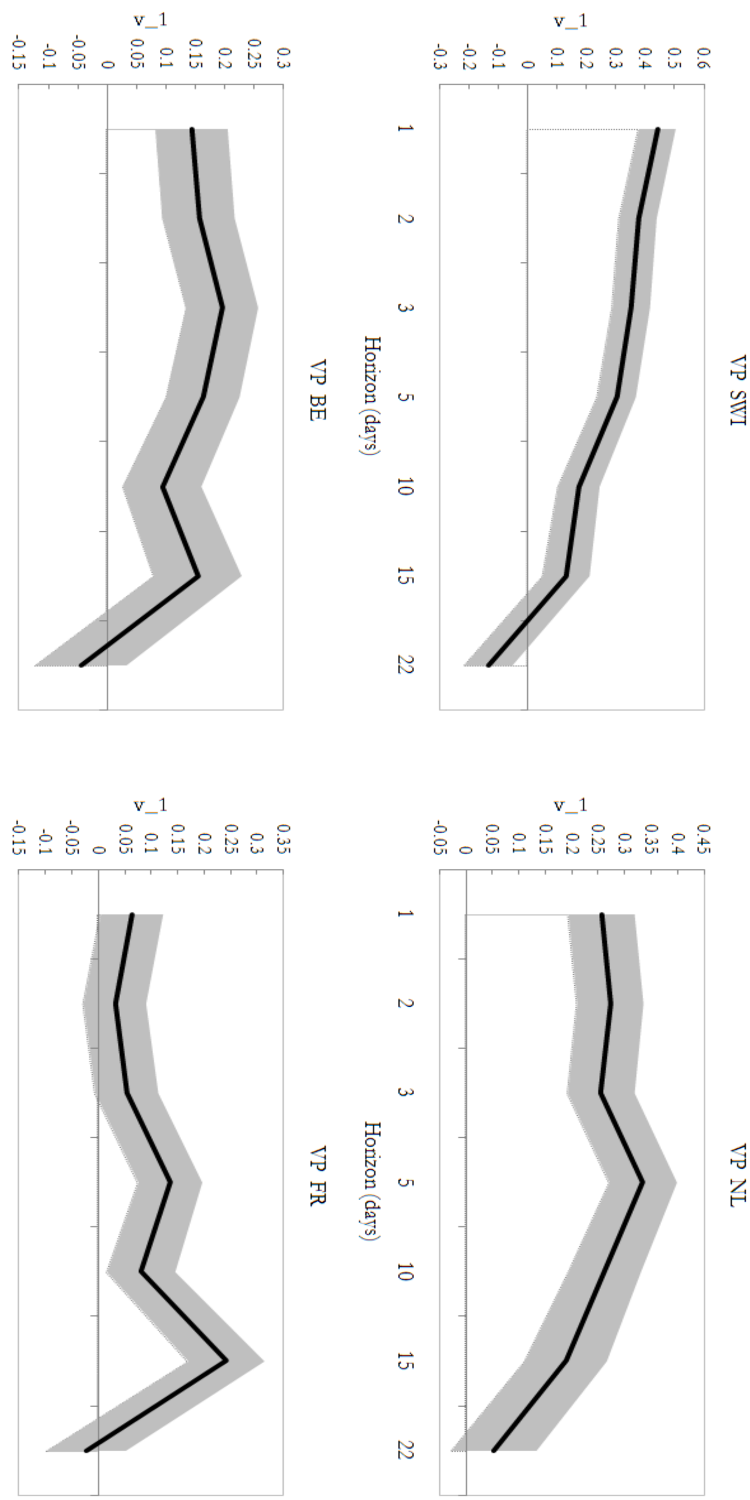

Figure 1.9: The role of the US Variance Premium in explaining Excessive Variance Premium Comovements. Continued

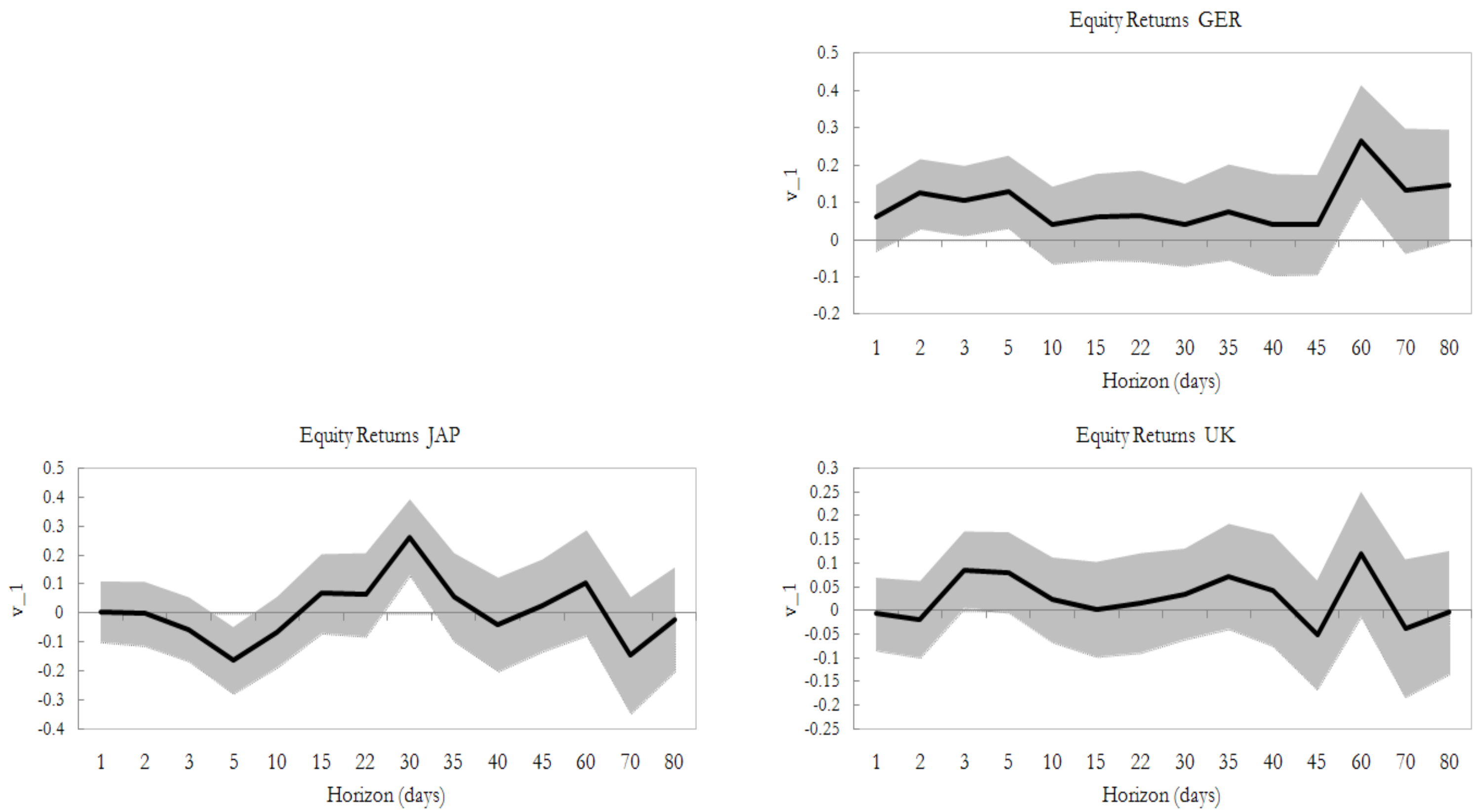

Figure 1.10: The role of the US Variance Premium in explaining Excessive Return Comovements
The figure reports the estimated coefficient $v_{1,j,h}$ in the following regressions:

$$e_{j,t+h} = \gamma_{0,j,h} + \gamma_{1,j,h} e_{US,t} + \epsilon_{,j,h,t},$$

where $e_{j,t+h}$, $e_{US,i}$ are the residuals from a standard contagion procedure as explained in Appendix 1.B, for alternative $h-$days ahead considered. The time-varying coefficient $\gamma_{1,,j,h,t}$ follows the process $\gamma_{1,j,h,t} = v_{o,,j,h} + v_{1,j,h} D_{US,t}$, where the dummy $D_{US,t}$ characterizes the extreme values of the US variance premiums. The shaded areas represent the 95% confidence intervals.

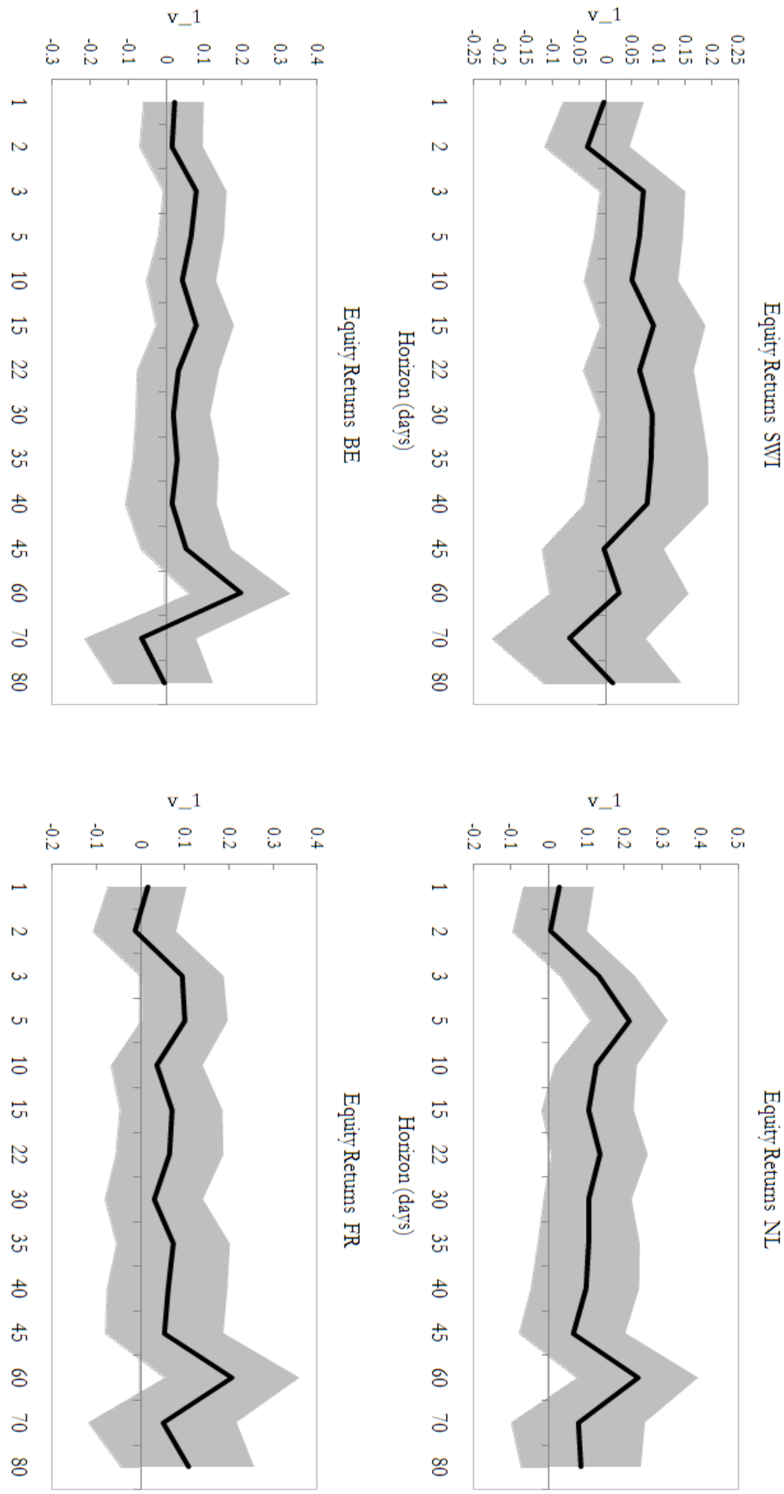

Figure 1.10: The role of the US Variance Premium in explaining Excessive Return Comovements. Continued

CHAPTER 2

Cumulative Prospect Theory and the Volatility Premium

Joint work with

Lieven Baele[1], Joost Driessen[1], and Oliver Spalt[2]

Abstract

This paper explores the ability of Cumulative Prospect Theory (CPT) to explain the observed negative volatility premium embedded in option prices. We simulate equilibrium prices for zero-beta straddles when agents are endowed with CPT-type preferences. We find that overweighting the probability of extreme events, one of the components of CPT, plays a key role in increasing the implied price of straddles. In contrast, increasing the scale of the value function, the second component of CPT, yields minor changes in the equilibrium prices of these straddles unless agents display a very large degree of loss aversion. We also explore these implications in a time-varying framework where we find that the price agents are willing to pay to hedge the risk of extreme events depends on the previous performance of their portfolio.

JEL Classification: C15, G11, G13

Keywords: Cumulative Prospect Theory, distorted probabilities, loss aversion, volatility premium.

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2.1 Introduction

There is ample evidence that agents demand a compensation for accepting the risk of extreme fluctuations in stock prices. This compensation is reflected in the substantial returns obtained by volatility sellers largely documented in the literature (Coval and Shumway (2001), Bakshi and Kapadia (2003) and Eraker (2009), among others). The observed volatility premium is, on average, negative and displays significant time variation. Nevertheless, our understanding of the sources of the volatility premium is still limited (Driessen and Maenhout (2007), and papers cited therein). In this paper, we explore the potential of Cumulative Prospect theory (CPT hereafter) to explain abnormal option returns and accommodate the observed stylized facts related to the volatility premium. We find the probability weighting scheme embedded in CPT to play the main role in explaining the observed high prices of derivatives exposed to extreme fluctuations in stock prices such as straddles. In contrast, the total effect of the value function, the second key component of CPT, is rather poor since the isolated effects of the degree of loss aversion and the curvature of the value function have similar magnitudes but opposite directions.

The intuition for the relation between the risk of extreme events, and in particular the volatility premium, and option returns is as follows. Volatility sellers usually take positions in straddles.[1] This strategy yields a positive return if equity prices display a moderate change at the maturity of the contract. In other words, if the implied volatility embedded in these options systematically exceeds the actual realized volatility within the duration of the contract. Therefore, the willingness of agents to pay (relatively) high prices for (holding long positions in) these options collects their desire to hedge against the risk of extreme movements in equity prices. Following this intuition, the literature has consistently found empirical evidence that the volatility premium is, on average, negative and significant. Coval and Shumway (2001) find for the US stock market that zero-beta straddles earn an average weekly return of -3% for the period between 1986 and 1996. They actually show that a volatility-selling strategy involving zero-beta straddles yields a Sharpe ratio considerably larger than that derived from an equity-index-investment strategy during their sample. Since zero-beta straddles are, by definition, exclusively exposed to market volatility, the evidence in Coval and Shumway (2001) suggests that systematic stochastic volatility is an important factor for pricing assets. In a similar vein, Bakshi and Kapadia (2003) use a derivative-hedging strategy to determine the sign and magnitude of the volatility premium. Using a volatility-exposed strategy, similar to that in Coval and Shumway (2001), they find an explicit relation between the return of ATM-hedged returns and the volatility premium. Finally, Driessen and Maenhout (2007) find evidence that the volatility risk is also priced in the UK and Japan.[2]

In order to understand the sources of the empirically observed volatility premium, theoretical models would at least require for agents not to be indifferent to the risk of extreme movements in equity markets. Therefore, in order to characterize the volatility premium, several adjustments to standard asset pricing models have been proposed in the literature. One strand of the literature, links the volatility risk premium to macroeconomic uncertainty. This strand follows the intuition behind the long-run risk model in Bansal and Yaron (2004) and the idea that agents have a preference for an early resolution of uncertainty in Bansal and Yaron (2004). These long-run risk models have the ability to endogenously generate a volatility risk premium by means of a representative agent who dislikes macroeconomic

[1] There are of course other alternatives to invest directly in volatility like taking position on variance swaps or betting on a volatility index.

[2] In order to maintain the coherence with our empirical analysis, we intentionally omit the related empirical evidence on the existence of the volatility premium measured as the difference between the option implied volatility index and the expected realized volatility. See, for instance, Britten-Jones and Neuberger (2000), Jiang and Tian (2005), Bakshi and Madam (2006), Carr and Wu (2009), Bollerslev, Gibson and Zhou (2011) and Bollerslev, Tauchen and Zhou (2009).

volatility. For instance, Bollerslev, Tauchen and Zhou (2009) show that the variance risk premium reveals the attitude of agents towards macroeconomic uncertainty. In a similar setting, Drechsler and Yaron (2011) find that the variance risk premium reveals the attitude of agents towards economic jumps. In particular, their attitude towards the probability of occurrence and intensity of these jumps. Also following a long-run-risk model strategy, Londono (2011) provides a model to understand the sources of the volatility premium in an international setting. An alternative strand of the literature relates the volatility premium to agents' attitudes towards the presence of non-normalities in the distribution of returns. In Bakshi and Madam (2006), the variance risk premium is explained by the desire of agents with time-varying risk aversion to buy protection against extreme events. In a similar vein, Bekaert and Engstrom (2010), Todorov (2010), and Gabaix (2009), using different methodologies, focus on the interplay between returns, risk aversion and extreme events to explain many asset pricing regularities, including the volatility premium.

In sum, the theoretical literature on the volatility premium suggests a joint role of agents' preferences and model-implied equity return dynamics in the characterization of the volatility premium. Nevertheless, theoretical attempts to understand the volatility premium have proven to be limited in generating its observed magnitude. Surprisingly, the potential role of CPT preferences has been, as far as we are aware, ignored until now in the volatility-premium literature.[3] The main contribution of this paper is to explore the ability of CPT preferences to explain the volatility premium in an equilibrium setting where equity returns follow a normal distribution with constant volatility. Thus, we explore the implications of the different components of CPT over the equilibrium price agents are willing to pay to hedge the risk of extreme events.

CPT is based on experimental evidence against traditional expected utility theory. It relies on two key features: a weighting function that distorts physical probabilities and a value function over gains and losses. The probability weighting scheme in CPT makes agents transform probabilities when forming expectations about their value function. In particular, the probabilities of extreme outcomes are distorted upwards by taking probability mass away from outcomes with moderate losses or gains. Now, the characteristics of the Prospect Theory (PT hereafter) value function, as originally proposed by Kahneman and Tversky (1979), are based on three experimentally observed deviations from expected-utility decision-making. First, individuals derive utility from losses and gains (relative to a reference level) rather than from a level of wealth. Second, marginal utility is larger for infinitesimal losses than for tiny gains so that investors are loss averse. Third, the utility function over gains and losses or value function exhibits risk-aversion in the domain of gains, but is convex in the domain of losses (Driessen and Maenhout (2007)).

Interestingly, some of the components of CPT have been previously related to agents' hedging strategies. Shiller (2000) suggests that the overweighting of probabilities might be useful to explain option pricing anomalies since it makes insurance and gambling attractive to agents. This mechanism is formally shown by Barberis and Huang (2008). However, Barberis and Huang (2008)'s setting is restricted to a model where the skewed asset is in small supply and not properly a derivative. In contrast to the intuition in Shiller (2000), Verslius, Lehnert and Wolff (2010) find a rather low and hardly interpretable effect of the probability weighting scheme over the price of ATM calls. As for the components of the PT-value function, loss aversion has been shown to increase the reward agents demand for holding risky-assets including those with skewness (Barberis and Huang (2008)). Moreover, Barberis, Huang and Santos (2001) introduce the possibility of a time-varying degree of loss aversion as a function of agents' recent portfolio performance. Their setting paves the way for the investigation of the potential of CPT to characterize the time variation of

[3] From an empirical perspective, in a non-equilibrium setting, an exception is riessen and Maenhout (2007) as we discuss below.

the volatility premium documented in the literature. Less attention has been paid to the potential role of the curvature of the value function. Barberis, Huang and Santos (2001) find that the effect of the curvature in the domain of gains and losses might offset its total effect over the price of skewed assets. In an attempt to isolate the effect of the curvature of the vale function in the domain of gains and losses, Verslius, Lehnert and Wolff (2010) find that more risk aversion in the domain of gains leads to higher call option prices while more risk seekingness in the domain of losses leads to lower call option prices. Finally, Driessen and Maenhout (2007) empirically investigate, in a non-equilibrium setting, the joint effect of probability weighting and loss aversion in explaining option pricing anomalies. They find evidence that the volatility premium implied by CPT-type preferences might be negative only for some parameter combinations. In particular, the volatility premium is negative for moderate probability weight parameters and low degrees of loss aversion.

This paper adds to the literature by investigating the ability of all CPT components to explain option pricing anomalies and potentially characterize the observed negative volatility premium in a general equilibrium setting. First, we propose an appropriate CPT setting and find equilibrium equity and option prices. Within this setting, we endow the representative agent with a utility function that combines a traditional constant-relative-risk-aversion utility function with a PT-type value function. Moreover, the representative agents' expectations might be distorted according to the probability weighting scheme embedded in CPT. Then, we simulate the equilibrium conditions for two parameter sets. The first one or benchmark parametrization includes several parameters previously used in the literature while the second one isolates the specific impact of each component by switching off the effect of all others. Using the results from our simulations, we disentangle the specific role of all CPT parameters on the equilibrium price of zero-beta straddles. This methodology allows us to gather information about the equilibrium-implied price agents are willing to pay to hedge the risk of extreme events depending on the characteristics of their preference function. Using the same methodology, we also investigate the impact of all CPT components over the price of at-the-market (ATM hereafter) straddles. Finally, we explore the impact of CPT over the price of straddles in a time-varying framework. In order to do so, and following Barberis and Huang (2001), we extend our one-period setting to allow for the degree of loss aversion as well as the reference level for gains and losses to vary as a function of the recent performance of the representative agent's portfolio.

Our methodology yields a number of interesting findings that, as far as we know, are new to the literature. First, we find that the parameter driving the probability distortion plays a key role in explaining the (relatively) high price of straddles. In particular, zero-beta straddle prices increase up to 51.2% when we compare our benchmark parametrization with a scenario where probabilities are not distorted. This price increase turns out to be as high as 215.6% for extremely distorted probabilities. Second, we find the effects of the components of the value function over the price of zero-beta straddles to have opposite directions. On the one hand, we find that increasing the degree of loss aversion increases the price zero-beta straddles. However, the price increase implied by changes in the degree of loss aversion is less relevant than that implied by changes in the probability distortion parameter. The price increase implied by changes in the degree of loss aversion turns out to be 2% if we compare our benchmark scenario with that where investors do not display any loss aversion. On the other hand, increasing the curvature of the value function decreases straddle prices by 2.9% for our benchmark scenario. The opposite direction of the effects of the value function components finally implies that, for our benchmark parametrization, increasing the scale of the PT-value function has a minor effect over the price of zero-beta straddles.

When we extend our one-period methodology to ATM straddles, the effects of the probability distortion and the curvature of the value function turn out to be similar in magnitude and direction to that for zero-beta straddles. In contrast, the effect of the degree of loss aversion over the price of ATM straddles has the opposite direction from that observed over the price of zero-beta straddles. For our benchmark parametrization, we obtain that ATM straddle prices are reduced by 5.6% compared to a scenario where investors display no loss aversion. The mechanism behind the reduction of ATM straddle prices is explained in turn by the increase in the expected return of equities implied by an increase in the degree of loss aversion or market beta effect.

Finally, when we extend our one-period results to a simplified time-varying setting, we find the dynamics of the implied return of straddles to depend on how agent's remember recent gains and losses. In particular, we find that increasing their memory horizon increases the persistence of the volatility premium. The average volatility premium implied by the return of zero-beta straddles is considerable (on average 34.58%), but displays a moderate variability (average standard deviation of 0.63).

The remainder of this paper is organized as follows. Section 2.2 reviews the basic concepts of CPT. Section 2.3 explains the main asset-pricing applications of CPT found in the literature. In Section 2.4, we derive the equilibrium prices for stocks and options implied by our CPT-setting. In Section 2.5, we simulate the equilibrium prices of zero-beta straddles in order to disentangle the specific impact of the CPT components on the volatility premium. Finally, Section 2.6 concludes.

2.2 Overview of Cumulative Prospect Theory

CPT is based on experimental evidence against expected utility. This theory has proven to be successful in explaining a variety of empirical regularities and phenomena that are puzzling from the point of view of expected utility.[4] There are two key components to CPT: a value function over gains and losses and a weighting function that distorts physical probabilities (Driessen and Maenhout (2007)). In this section, we introduce and explain in detail the components of CPT.

The original Prospect Theory (PT hereafter) value function was introduced by Kahneman and Tversky (1979) (KT hereafter) based on experimental evidence. This function collects several experimental violations of the expected utility theory. First, agents derive utility from losses and gains X (relative to a reference level) rather than from a level of wealth W. Second, agents' marginal utility is larger for infinitesimal losses than for small gains so that investors are loss averse. Note that loss aversion generates first-order risk aversion (Segal and Spivak (1990)). Third, while agents are risk averse in the domain of gains, they exhibit a risk-seeking behavior in the domain of losses. A typical representation of the value function $V(X)$ for a loss-averse investor with risk-averse and risk-seeking behavior is

$$V\left(X\right)=\left\{\begin{array}{c}\frac{X^{\widehat{\gamma}_1}}{\widehat{\gamma}_1}\\-\lambda\frac{(-X)^{\widehat{\gamma}_2}}{\widehat{\gamma}_2}\end{array}\right.\quad\text{for}\quad\begin{array}{c}X\geq0\\X\leq0\end{array},\tag{2.2-1}$$

where λ controls the degree of first-order risk aversion and makes the value function kinked at zero. The curvature parameters $\widehat{\gamma}_1$ and $\widehat{\gamma}_2$ are constrained to belong to the interval $[0, 1]$. These curvature parameters control the risk-averse and risk-seeking behavior in the domain of gains and losses respectively.

The second ingredient of (Cumulative) Prospect Theory makes decision-makers transform probabilities when taking expectations of their value function. In particular, the probabilities of extreme outcomes are distorted upwards by taking probability mass away from outcomes with moderate losses or gains. A weighting scheme that makes insurance and

[4] For an excellent survey, see Barberis and Thaler (2003).

gambling attractive to agents. The original weighting function in KT is a monotonic transformation of the individual outcome probabilities. The major drawback of such a transformation is that it does not always satisfy (first-order) stochastic dominance. In order to convey this drawback, Tversky and Kahneman (1992) introduced a weighting function in their Cumulative Prospect Theory (CPT hereafter). CPT adds probability distortions over cumulative rather than individual probabilities according to a nonlinear transformation of the loss-averse preferences. In particular, the transformed probabilities ('decision weights') overweight both extremely positive and extremely negative outcomes of the optimal portfolio.

The distorted probabilities or decision weights in Tversky and Kahneman (1992) are obtained from the objective probabilities as follows. First, the states or outcomes are ordered from worst to best according to the investors' endogenously chosen reference portfolio, R_R, and are labeled accordingly: $R_1 \leq ... \leq R_k \leq R_R \leq R_{k+1} \leq ... \leq R_N$. Denoting the objective probability of outcome n by p_n, the subjectively distorted probability of outcome n, π_n, is obtained as follows:

$$\begin{aligned} \pi_i &= w^-(p_1 + ... + p_i) - w^-(p_1 + ... + p_{i-1}) \text{ for } 2 \leq i \leq k \\ \pi_i &= w^+(p_i + ... + p_N) - w^+(p_{i+1} + ... + p_N) \text{ for } k+1 \leq i \leq N-1 \end{aligned} \quad (2.2\text{-}2)$$

where

$$w^-(p) = \frac{p^{c_1}}{\left[p^{c_1} + (1-p)^{c_1}\right]^{1/c_1}}, \quad (2.2\text{-}3)$$

$$w^+(p) = \frac{p^{c_2}}{\left[p^{c_2} + (1-p)^{c_2}\right]^{1/c_2}},$$

and $\pi_1 = w^-(p_1)\,, \pi_N = w^+(p_N)\,.$ Parameters c_1 and c_2 in Eq. (2.2-3) control the curvature of the weighting function for losses and gains respectively. Note that $c_1 = c_2$ takes us back to the case of rank-dependent expected utility introduced by Quiggin (1993). Moreover, $c_1 = c_2 = 1$ brings us back to a setting without distortions.

2.3 CPT and Asset Prices

Based on the ability of CPT to explain several experimentally observed behavioral puzzles, this theory has been used extensively to explain many asset-pricing related phenomena. Examples of which are the equity premium puzzle and low market participation (Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), BHS hereafter, and Ang, Bekaert and Liu (2005)), the house money effect (BHS), the disposition effect (Gomes (2005) and Barberis and Xiong (2009)), and the demand of stocks during booms and recessions (Berkelaar and Kouwenberg (2009)).[5] Nevertheless, most of the attention has been centered in the optimal portfolio choice problem when agents' investment opportunities are restricted to normally distributed assets. In contrast, little attention has been paid to the implications of CPT over the equilibrium prices of non-normally distributed assets and in particular over derivative prices. Nevertheless, a few exceptions can be found in the literature. BHS investigate the implications of the PT-value function (without probability distortion) over the price of a skewed asset that is in small supply and independent from the risky asset therefore not a

[5] A set of papers specifically deals with the cross-sectional implications of CPT. See, for instance, De Giorgi, Levy and Hens (2004), and Barberis and Huang (2001).

properly a derivative. Verslius, Lehnert and Wolff (2010) investigate the implied derivative prices given a marginal investor with CPT preferences.[6]

It is well known in this literature that in order to obtain realistic asset-pricing settings, several aspects of the original CPT need to be modified. In this section, we first investigate the convenience of using CPT for pricing equities and options as well as the modifications we will assume throughout the paper. Then, we discuss the consequences of using alternative parameter sets previously used in the literature as well as our benchmark parametrization.

2.3.1 Making CPT suitable for asset pricing

There are several inconveniences when applying the original CPT for pricing equities and options. These inconveniences are mainly related to the implications of CPT for the existence, finiteness and uniqueness of the market equilibrium. For instance, it is well known that the CPT setting might deliver non-existent market equilibria. Moreover, if one or several equilibria were to exist, portfolio weights might be non-finite. It has also been shown that CPT equilibria might not be unique. In this part of the section, we discuss in detail these drawbacks and describe the modified CPT setting used throughout the rest of the paper.

Original PT might deliver non-existent market equilibria due to the discontinuity in agents' demand function.[7] Berkelaar, Kouwenberg and Post (2004) provide a set of conditions under which a CPT-market equilibrium exists. These conditions include the non-negativity of wealth as well as the pseudo-concavity of the value function.[8] A related and very well known problem of CPT is the possibility of finding non-finite optimal portfolio weights. Pure CPT-preferences might cause an infinite short selling problem due to the fact that the value function in Eq. (2.2-1) is finite for negative levels of wealth. This problem is derived from the convexity of the function in the domain of losses. Nevertheless, this convexity is a key feature of CPT since it reflects the idea that agents display a risk-seeking behavior when facing losses. This behavioral aspect turns out to be a robust finding in experiments at least when losses are small.[9] However, there seems to be far less consensus among researchers about the behavior of agents when facing large losses as some evidence suggests concavity (Laughhunn, Payne and Crum (1980)). More specifically, in the finance literature, Gomes (2005) argues that having marginal utility decrease as wealth approaches zero is unappealing. This is especially relevant in our setting where investors have access to derivative-based returns with unusually asymmetric distributions. In such a setting, risk-seeking behavior becomes extreme and investors mainly take positions for which the non-negativity constraint on wealth becomes binding.

In order to guarantee a solution with finite portfolio weights in a CPT setting, several modifications have been proposed in the literature.[10] These modifications allow for the total utility function to be pseudo-concavified. The value function in Gomes (2005) is forced to be concave again for substantial losses. The agents' inflection point for determining 'substantial losses' needs to be decided by the researcher. As an alternative, Ait-Sahalia and Brandt (2001) impose portfolio constraints to rule out extreme positions due to the convexity

[6] Driessen and Maenhout (2007) find empirical portfolio weights when agents have access to options (including ATM straddles). They investigate the implications of several utility functions among them, the PT value function. However, their setting is not properly a general equilibrium one.

[7] De Giorgi, Hens and Rieger (2010) show that, if there is a continuum of agents with CPT preferences, and wealth is restricted to be non-negative, an equilibrium exists.

[8] The pseudo-concavity condition reduces to a strictly-increasing-function condition (Avriel (2005)) as long as parameter $\widehat{\gamma}_1$ belongs to the interval $(0, 1)$.

[9] See, for instance, Allais (1953), Williams (1966), KT, and papers cited therein.

[10] Ang, Bekaert and Liu (2005) argue that these modifications, although convey the problem of infinite leverage, change the nature of the original CPT specification.

of the value function.[11] Another alternative, and the one followed in this paper, is the utility function in BHS. Their total utility function differentiates two separate components. The first is a CRRA component that enforces a positive wealth constraint. The second component is precisely the value function in Eq. (2.2-1).

Our representative agent's problem reduces to the maximization of the following function:

$$E^{*}\left[U(W_T) + bV\left(X_T\right)\right], \tag{2.3-4}$$

where $U(.)$ is a traditional CRRA utility function, $V(.)$ is the PT-value function in Eq. (2.2-1), and b is the scaling parameter that controls the proportion of total utility that is derived from sources other than total wealth. See how, in order to incorporate the probability weighting scheme into BHS's utility function, expectations $E^{*}(.)$ are calculated using the "distorted" probabilities in Eq. (2.2-2).[12] Therefore, the main advantage of the utility function in Eq. (2.3-4) is that it considers simultaneously the two key ingredients of CPT: utility over gains and losses as well as probability distortion. In contrast, closely related papers usually center their attention exclusively on one of this components. For instance, Barberis and Huang (2001) and BHS do not consider the effects of the probability weighting function and center their attention on the asset pricing implications of the value function's parameters. Now, Barberis and Huang (2008) remove the CRRA component of the total utility function and center their discussion on the implications of the probability weighting function over equity prices. As it has been mentioned before, Barberis and Huang (2008) also explore the implications of probability weighting over the price of a skewed asset (in small supply and independent from equity).

Another advantage of the utility function in Eq. (2.3-4) is that it also allows to investigate different concepts related to the rationality of agents' decisions. BHS and Barberis and Huang (2001) argue that this function maintains the hypothesis of rationality since it is not irrational for agents to derive utility from sources other than wealth. In particular, agents might also derive utility from the value of their portfolios. In contrast, agents' rationality has been traditionally linked exclusively to the expected-utility setting with strictly risk averse agents.[13]

Finally, in order to make our CPT-setting suitable for asset pricing, we also need to decide the reference point with respect to which gains and losses are defined. It is important to point out that KT first formulated their theory in an atemporal setting and focused on experiments where subjects faced gambles with two possible non-zero outcomes (Barberis and Thaler (2003)). Bringing this theory to a temporal setting with gambles characterized by a richer support - a typical setting in financial economics - requires therefore that one imposes more structure on the dynamics of the reference point. Issues related to narrow framing or mental accounting and the updating of the reference point ('intertemporal framing') become crucial elements of the analysis.[14] The evolution of the reference point will prove particularly important when considering options in a time-varying framework. A reasonable assumption seems to be to have the reference level equal to initial wealth grown at the risk free rate: $X_t \equiv W_t - R_f W_0$.

[11] The leverage constraints in Ait-Sahalia and Brandt (2001), are often binding. However, Driessen and Maenhout (2007) argue that leverage constraints are less meaningful when derivatives are introduced since strategies involving derivatives allow for leverage by definition.

[12] Perhaps the most popular alternative to the weighting function in Eq. (2.2-2) is that introduced by Prelec (1998). Although both functions have nearly identical shapes, Prelec's specification is based on behavioral axioms rather than convenience of the functional form (Neilson and Stowe (2002)).

[13] See, for instance, Coval and Shumway (2005). They test the null hypothesis of standard rational investors' behavior against a number of potential alternative behavioral hypotheses including loss aversion. In a similar vein, Verslius, Lehnert and Wolff (2010) argue that breaking traditional expected utility theory means giving investors some sort of irrationality. Finally, Harrison and Rutström (2009) investigate the proportion of agents that are more expected-utility behaved from those who are CPT behaved.

[14] See, for instance Benartzi and Thaler (1995) and Barberis, Huang and Thaler (2006).

2.3.2 CPT parametrization

In this part of the section, we discuss the selection of our own benchmark parametrization. This benchmark parametrization as well as a summary of alternative sets for the parameters in Eqs. (2.2-1) and (2.2-3) previously used in the literature is displayed in Table 2.1.[15] Most of these parametrizations have been experimentally calibrated.[16] Their differences are explained by the particular CPT-setting or the alternative purpose of each investigation. This is particularly relevant, since it has been shown that most of the alternative calibrations cannot account simultaneously for all behavioral puzzles (Neilson and Stowe (2002)). Now, since a full calibration of these parameters is out of the scope of this paper, we briefly summarize the discussion on the alternative parametrizations previously used as well as their implications and restrictions.

Most papers find experimental evidence that the distortion of probabilities is homogeneous for positive and negative outcomes (Barberis and Huang (2008)). Probability distortion has proven to be very important to characterize the gambling, hedging and Allais-paradox-coherent behavior of agents. In particular, Neilson and Stowe (2002) find that all potential parametrizations that accommodate these behavioral facts require highly distorted probabilities (low c). However, Wakker and Tversky (1993) and Ingersoll (2008) warn how extremely high distortions ($c < 0.28$) generate negative weights. They show that c must be constrained to the interval $(0.28, 1]$ to ensure that $w(p)$ is strictly increasing for $P \in (0, 1)$. Our benchmark parametrization considers $c^+ = c^- = 0.65$.

As for the scale, we follow BHS and impose a benchmark value for parameter b that guarantees equal weights of the (CRRA) utility function and the (PT) value function in agents' decisions. That is, we make sure that, for the benchmark parametrization, the marginal utility obtained from the value of agents' portfolio is equal in magnitude to that derived from total wealth. Nevertheless, we also consider a wide range of scenarios where agents can be fully CRRA-behaved ($b = 0$) or mostly concerned about the return of their portfolio ($b = large$).

Our benchmark level for the first-order risk aversion is $\lambda = 2.25$ as in KT. However, in our simulations, we also investigate the possibility that the value function is not kinked ($\lambda = 0$) or that agents are very loss-averse in line with the findings of Hwang and Satchell (2010). Hwang and Satchell (2010) find that λ is as high as 3.25 for the US and increases by 1.5 during recession.

Finally, with respect to the curvature of the value function, most papers find experimental evidence that this parameter is homogeneous for gains and losses ($\widehat{\gamma}_1 = \widehat{\gamma}_2$). In particular, BHS argue that using $\widehat{\gamma}_1 = \widehat{\gamma}_2 = 1$ is convenient to evaluate decisions over stocks where the first-order loss aversion (λ) plays a more relevant role. However, a strictly linear value function over gains and losses can be problematic when trying to find non-finite optimal portfolio weights if the total utility function is not concave for extreme losses (as it is the case for extremely large values of b).[17] In contrast, Hwang and Satchell (2010) find empirical evidence that the value function is indeed curved. Moreover, they find that the curvature of losses is higher than that for gains (0.9 versus 0.7 for the US). In this paper, we also follow KT and assume a benchmark parametrization where $\widehat{\gamma}_1 = \widehat{\gamma}_2 = \widehat{\gamma} = 0.88$. However, we also investigate the specific impact of alternative values for $\widehat{\gamma}_1$ and $\widehat{\gamma}_2$ in the interval $[0.6, 1]$.

[15] An extensive summary of CPT-related parameters can be found in Neilson and Stowe (2002) and Stot (2006).

[16] An exception of experimental calibration can be found in Hwang and Satchell (2010). They argue that experiments can be problematic since agents' behavior may be different from the one you would observe in real financial markets.

[17] See Section 2.4 and Appendix 2.A.

2.4 Equilibrium Equity and Option Returns under CPT-Preferences

In this section, we find equilibrium equity and option prices in a simplified one-period (incomplete market) CPT setting. First, we explain the representative agent's optimization problem and provide some intuition on the impact of the key CPT parameters over equilibrium portfolio weights. Then, we formally introduce the equilibrium conditions for equity and derivative prices.

2.4.1 Optimal portfolio weights under CPT

The representative agent's problem reduces to finding a one-period optimal portfolio with positions in the risky asset, α_E, and the derivative,α_D, (and the risk free asset of course) such that she maximizes her utility over the next period. Her problem can be represented as follows:

$$\max_{\alpha_E,\alpha_D} E^*[U(W_T) + bV(X_T)], \tag{2.4-5}$$

$$s.t.\ W_T = [(1-\alpha_E-\alpha_D)R_f + \alpha_E R_E + \alpha_D R_D]W_o,$$

where expectations $E^*(.)$ are distorted according to the probability weighting function in Eq. (2.2-2).

The representative agent's total utility function, as introduced in Section 2.3, has two components: a traditional utility over total wealth and a PT-type value function defined over gains and losses. The contribution of the latter in the total utility is scaled by parameter b. We explain each component of the total utility function in turn.

The utility over wealth is defined as a traditional CRRA function such as

$$U(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{for} \quad \gamma \neq 1 \\ \ln W_T & \qquad\quad \gamma = 1 \end{cases},$$

where γ is the risk aversion coefficient.

The value function collects the agent's concern about her portfolio return. This function is introduced in Section 2.2 (Eq (2.2-1)) and is written here again for completeness.

$$V(X_T) = \begin{cases} \frac{X_T^{\widehat{\gamma}_1}}{\widehat{\gamma}_1} & \text{for} \quad X_T \geq 0 \\ -\lambda\frac{(-X_T)^{\widehat{\gamma}_2}}{\widehat{\gamma}_2} & \qquad\quad X_T \leq 0 \end{cases},$$

where parameter λ controls the degree of first-order risk aversion, and $\widehat{\gamma_1}$ and $\widehat{\gamma_2}$ drive the curvature of the function. The latter parameters characterize respectively the risk-averse and risk-seeking behavior of agents in the domain of gains and losses respectively. Gains and losses are defined with respect to a reference point. As it has been discussed before, a reasonable assumption is to have the reference level equal to initial wealth invested at the risk free rate. That is, $X_T \equiv W_T - R_f W_0$ (see Section 2.3).[18]

Before solving for the equilibrium, we briefly provide some intuition on the impact of the utility function assumed over the representative agent's investment decisions. Figure 2.1 illustrates the maximization problem in Eq. (2.4-5) when agents' investment opportunities are restricted to a (normally distributed) risky asset for alternative values of the key parameters c, λ, $\widehat{\gamma}$, and b in Panels A, B, C and D respectively. See how, the probability

[18] The conditions for the existence of finite optimal portfolio weights in problem (2.4-5) are discussed in Appendix 2.A.

weighting scheme makes extreme positions in stocks less attractive since agents overweight the tails of the return distribution as can be seen in Panel A. Now, increasing the first-order risk aversion (λ in Panel B) decreases the utility obtained by taking extreme positions in the risky asset. Therefore, to accept the risk of investing in this asset, loss averse investors will require a larger expected return. The information in Panel C reveals that increasing the curvature of the value function ($\widehat{\gamma_1} = \widehat{\gamma_2} = \widehat{\gamma}$) reduces the attractiveness of taking extreme (levered) positions in the risky asset. However, it is important to note that increasing the risk-averse and risk-seeking behavior has heterogeneous effects on the total utility function. Among other things, because the latter is magnified by the degree of loss aversion λ. Finally, the effect of increasing the scale of the PT-value function is displayed in Panel D. As expected, increasing the scale, b, makes the shape of the utility function converge to that in the original PT. Nevertheless, including even a small portion of utility over consumption guarantees that the function is again concavified for extreme losses. This in turn guarantees the existence of non-finite portfolio weights (infinite leverage problem. See Section 2.3).

Figure 2.2 extends the illustration of the problem in Figure 2.1 to the case where agents have access to the risky asset and a derivative (ATM straddle). The information in the figure reveals that including derivatives does not introduce the possibility of finding local optima as long as the scale is moderate. In particular, the utility surface in the domain of α_E and α_D turns out to be strictly concave for the case of 50% risk-averse/50% value-oriented investors as can be seen in Panel A. Nevertheless, in line with Barberis and Huang (2008), when the contribution of the value function over the total utility function increases, the possibility of finding local optima increases as can be seen in Panel B.[19] The information in this panel reveals that the utility surface for the particular case of 5% risk-averse/95% value-oriented investors is nos strictly concave and displays several peaks.

2.4.2 Equilibrium prices under CPT

Finding equilibrium equity (s_0) and derivative (d_0) prices is a two-steps procedure. First, we find optimal portfolio weights given the representative agent's maximization problem in Eq. (2.4-5)

$$[\alpha_E, \alpha_D] = \arg \max_{\alpha_E, \alpha_D} E^*[U(W_T) + bV(X_T)]. \tag{2.4-6}$$

The equilibrium prices are those for which the market clearing condition holds. This condition reduces, in our incomplete market setting, to $\alpha_E = 1$, $\alpha_D = 0$. Note that, since in equilibrium the representative agent's wealth is entirely invested in equities, the reference point for evaluating gains and losses depends exclusively on the distribution of equity returns. Therefore, we can find the distorted probabilities with respect to equity returns without having to solve for the price of the derivative.

The Euler conditions for the problem in Eq. (2.4-6) are given by

$$1.\ E^*[U' \frac{\delta W_T}{\delta \alpha_E} + bV' \frac{\delta X_T}{\delta \alpha_E}] = 0.$$

$$2.\ E^*[U' \frac{\delta W_T}{\delta \alpha_D} + bV' \frac{\delta X_T}{\delta \alpha_D}] = 0,$$

where $U' = W_T^{-\gamma}$, and

$$V' = \begin{cases} X_T^{\widehat{\gamma}_1 - 1} & \text{for} \quad X_T > 0 \\ \lambda(-X_T)^{\widehat{\gamma}_2 - 1} & \quad\quad\;\; X_T < 0 \end{cases}.$$

[19] In Section 2.5, we go again over the local optima problem. In particular, we investigate ex-post the uniqueness of the optimum for several problematic parametrizations.

In order to simplify the simulation of the equilibrium conditions, we discretize this problem to the case where there are N potential outcomes for the one-period ahead price of the risky asset. Therefore, the Euler equations can be rewritten as

$$1. \sum_{i=1}^{N} \pi_i (R_{E,i} - R_f)[U' + bV'] = 0. \tag{2.4-7}$$

$$2. \sum_{i=1}^{N} \pi_i (R_{D,i} - R_f)[U' + bV'] = 0, \tag{2.4-8}$$

where π_i are the distorted probabilities, $R_{E,i}$ and $R_{D,i}$ are respectively equity and derivative gross returns in each state of the nature, and R_f is the risk-free rate. For this simplified version of the problem, we obtain that in equilibrium $U' = (W_0 R_{E,i})^{-\gamma}$, and

$$V' = \begin{cases} (W_0(R_E - R_f))^{\widehat{\gamma_1}-1} & \text{for} \quad R_E > R_f \\ \lambda(W_0(R_f - R_E))^{\widehat{\gamma_2}-1} & \phantom{\text{for}} \quad R_f < R_E \end{cases}. \tag{2.4-9}$$

Again, note that, since the reference point only depends on the distribution of equity returns, we can solve first for the equilibrium price of the risky asset. Then, we can solve for the price of any derivative whose underlying is the risky asset.

2.5 CPT and the Volatility Premium: Numerical Simulations

In this section, we simulate the conditions in Eqs. (2.4-7) and (2.4-8) in order to find equilibrium prices for the risky asset as well as for zero-beta straddles for alternative parameter sets. See how, since zero-beta straddles are, by definition, exclusively exposed to the risk of extreme events, their simulated equilibrium prices reveal the impact of the CPT components on the ability of this theory to accommodate the observed negative volatility premium. In the first part of the section we investigate a one-period (incomplete market) setting. Within this setting, we explore the specific impact of each CPT parameter over the price of zero-beta straddles. In the second part, we also explore the implications of CPT over the price of ATM straddles and motivate the impact of the CPT parameters over the price of options at different degrees of moneyness. In a third part, we investigate the uniqueness of the one-period equilibrium solutions to problematic parametrizations. In the final part of this section, we explore a simple case where the volatility premium might be time-varying.

2.5.1 One-period setting

We first assume a one-period setting where there are $N = 1000$ potential outcomes for the return of the risky asset, $R_{E,i}$. These outcomes are normally distributed around an equilibrium expected return, μ_E, with volatility σ_E.[20] Given this distribution of equity returns, the payoffs of a straddle with strike K are given by $d_{1,i} = |K - s_{1,i}|$, where $s_{1,i}$ are one-period ahead prices for the risky asset. $s_{1,i} = s_0 R_{E,i}$, where the initial price of the risky asset, s_o, is normalized to 1 for convenience. For the particular case of a zero-beta straddle, its strike price would be the equilibrium expected return of the risky asset. That is, $K = \mu_E$. Finally, throughout the simulations, we assume a one-period constant risk free rate of 5% and an unconditional volatility of equity returns, $\sigma_E = 20\%$.

[20] Note that, μ_E is independent from the equilibrium price of the derivative as shown by the equilibrium conditions in Section 2.4.

For the setting described above, we quantify the specific impact of each CPT parameter over the price of one-period maturity zero-beta straddles. In order to do so, we investigate the impact of deviations of the probability distortion parameter as well as the components of the value function from two initial scenarios. On the one hand, we explore the isolated impact of each CPT component by switching off all others. For instance, in order to explore the specific ability of the probability distortion mechanism to characterize the volatility premium, we assume absence of loss aversion ($\lambda = 1$) and curvature ($\widehat{\gamma} = 1$) in the value function in Eq. (2.2-1). On the other hand, we investigate the impact of deviations of each parameter from the benchmark parametrization in Table 2.1. We describe the impact of each parameter in turn.

Probability distortion (c)

Figure 2.3 displays the physical and distorted distributions of excess equity returns (in standard deviations from the mean μ_E) for alternative values of parameter c. This figure illustrates the basic concepts introduced in Section 2.2 on the mechanism behind probability distortion. That is, as the probability distribution becomes more distorted (lower c), more probability mass is driven away from highly possible outcomes making extremes more likely to the eyes of agents.

Figure 2.4 displays the equilibrium expected return, μ_E, as well as the simulated equilibrium prices of a zero-beta straddle for the alternative values of parameter c considered. Overweighting the tails of the distribution not only increases the reward agents demand for investing in the risky asset, it also increases the price of zero-beta straddles. Thus, in the presence of distorted probabilities, the price agents are willing to pay to hedge against extreme movements in the price of the risky asset increases. This price increase is due to the fact that straddles provide the highest payoffs precisely for those states with larger distorted probabilities. Moreover, the magnitude of this price increase is economically meaningful. Our simulated results suggest that, in the absence of loss aversion and curvature of the PT-value function, straddle prices are driven up by 24.3% for $c = 0.65$ compared to a setting with non-distorted probabilities (up to 100% if we compare straddles under extremely distorted probabilities, $c = 0.30$). Our results also suggest that considering the benchmark levels of λ and $\widehat{\gamma}$ does not seem to have a significant effect over the changes in the expected return of straddles generated by increasing the probability distortion. Finally, our results reveal that the (one-period excess) expected return of long positions in zero-beta straddles is negative even if probabilities are moderately distorted ($c < 0.9$). This results is in line with the evidence in Driessen and Maenhout (2007).

Loss aversion (λ)

The effect of changes in the level of loss aversion over the price of zero-beta straddles is reported in Figure 2.5. Increasing λ systematically increases both the expected equity return and the price of zero-beta straddles. Thus, more loss averse agents are willing to pay a higher price to cover themselves against the (pure) risk of extreme events therefore decreasing the return of taking long positions in straddles. This evidence is in line with the intuition in Shiller (2000) and Barberis and Huang (2008). Our results suggest that, for $c = 1$ and $\gamma = 1$, straddle prices are driven up by 2.4% for moderately loss-averse investors ($\lambda = 2.25$) compared to a setting with non-loss averse investors. This price increase reaches 7.2% if we consider a setting with very loss-averse investors, $\lambda = 4$. The intuition behind this result is as follows. The presence of loss aversion increases the price of risk of the straddle payoffs corresponding to what the representative agent considers as negative outcomes (losses in the equity market). Our results reveal that the direction of the relation between loss aversion and the expected return of the straddle holds whether we consider the probability distortion and the curvature of the value function or not. However, considering these two characteristics significantly reduces the expected return of holding long positions in the straddle. Thus, we obtain a (gross) return for this position of -31.4% for the benchmark parametrization ($\lambda = 2.25$, $c = 0.65$, $\widehat{\gamma} = 0.88$) compared to -0.15% if only loss aversion is considered ($\lambda = 2.25$, $c = 1$, $\widehat{\gamma} = 1$).

Curvature of the value function ($\widehat{\gamma}$)

Figure 2.6 displays the effect of the curvature of the value function ($\widehat{\gamma}$) over the expected return of a zero-beta straddle. Increasing the curvature of the value function (simultaneously for gains and losses) systematically reduces the price of (long-positions in) zero-beta straddles since it reduces the price of risk of all straddle payoffs in proportion to their magnitude as suggested by Eq. (2.4-8). As for the case of loss aversion, we obtain that the expected return of a long position in zero-beta straddles is considerably lower if all components of CPT are considered together. This result suggests that loss aversion and, in particular, probability distortion play a key role in increasing the price agents are willing to pay to hold long positions in straddles. In particular, we obtain that considering the joint effects of all CPT components systematically yields negative excess return for holding a long position in straddles for all values of parameter $\widehat{\gamma}$. In contrast, the expected (excess) return of these positions is only negative for close-to-linear value functions ($\widehat{\gamma} > 0.9$) when probabilities are not distorted ($c = 1$) and agents are not loss averse ($\lambda = 1$).

The effect of the curvature of the value function over the price of straddles is expected to differ as agents become more risk-averse or more risk-seekers. Therefore, we also explore the specific effect of the curvature in the domain of gains and losses independently. Figure 2.7 displays the isolated effect of the curvature of gains (bold line) and the curvature of losses (dashed line) over the expected return of a zero-beta straddle. Our results suggest that increasing both the degree of risk aversion and risk-seekingness of agents reduces the price they are willing to pay for being exposed to the risk of extreme gains and losses respectively.[21] Moreover, these results suggest that decreasing the curvature of the value function in the domain of gains ($\widehat{\gamma}_1$), or alternatively increasing the degree of agents' risk aversion, implies a larger reduction in zero-beta straddle prices than decreasing the curvature in the domain of losses ($\widehat{\gamma}_2$).[22]

Scale (b)

In order to summarize our findings, we now explore the joint effect of all parameters embedded in CPT. Figure 2.8 reports the equilibrium expected return of a zero-beta straddle for alternative values of parameter b and alternative scenarios. This parameter controls up to what extent agents are concerned about changes in the value of their portfolio with respect to the utility they derive from consumption. We find a number of interesting results. First, even if agents do not incorporate any concern about the value of their portfolio ($b = 0$ or purely CRRA preferences), probability distortion is enough to characterize a negative excess return of holding long-positions in straddles. This result highlights the key role of the probability distortion mechanism in increasing the price agents are willing to pay to hedge their exposure to extreme events (equity return variations). Second, the isolated effect of loss aversion also yields higher straddle prices as the scale is increased. However, as pointed out above, the implied volatility premium implied only by changes in the level of loss aversion is hardly ever negative. In contrast, the isolated effect of the curvature of the

[21] In contrast to the evidence in Barberis and Huang (2008) who find that the effect of the curvature for gains and losses might offset its total effect over the price of skewed assets.

[22] This difference can be observed even in the absence of loss aversion and holds as long as $\mu_E \geq R_f$ in order to satisfy the Euler condition for the price of the derivative in Eq. (2.4-8).

PT-value function works in the opposite direction. That is, if only the curvature is considered, increasing the scale of the PT-value function yields lower prices of zero-beta straddles. In sum, we find that considering all CPT components together indeed generates negative expected returns for holding long positions in zero-beta straddles. This main result reflects the potential of CPT to accommodate the observed negative volatility premium. Nevertheless, and as a consequence of the opposite effect of the loss aversion and the curvature of the value function, our results also suggest that, after controling for the probability distortion mechanism, the total effect of CPT over the price of straddles hardly increases as we assume more PT-oriented investors (larger b).

2.5.2 Different degrees of Moneyness: ATM Straddles

We now extend the results in Section 2.5.1 to find equilibrium prices of ATM straddles. ATM straddles have strike prices equivalent to the initial price of the risky asset. That is, $K = s_o = 1$. See how, by construction, these straddles are not only exposed to the risk of extreme events, but also affected by changes in the expected return of the risky asset. In other words, the effect of the CPT components over their price will also be affected by their market beta.

Figure 2.9 displays the isolated effect of each CPT component over the price of ATM straddles. A comparisson between these results and those obtained for zero-beta straddles yields a number of interesting findings. First, although the magnitude of the implied return of ATM straddles is affected by their betas, the direction of the effect of the probability distortion mechanism and the curvature of the value function holds. That is, on the one hand, increasing the degree of probability distortion also increases the price of ATM straddles as can be seen in Panel A. Our simulations suggest that this price increase is as high as 180.1% if we compare a scenario with extremely distorted probabilities ($c = 0.30$) with one where probabilities are not distorted (46.8% if we compare a scenario with moderately distorted probabilities, $c = 0.65$, with a non-distorted one). Nevertheless, this price difference is lower than that obtained for zero-beta straddles. This is precisely because increasing c also increases the expected return of the risky asset. Therefore, the beta effect reduces part of the effect of the probability distortion over the price of ATM straddles. On the other hand, increasing the curvature of the value function (jointly for gains and losses) systematically decreases the price of ATM straddles as can be seen in Panel C. The magnitude of this effect is, in contrast to the effect of the probability distortion, comparable to that for zero-beta straddles. This in turn can be explained by the minor effect of $\widehat{\gamma}$ over the expected return of the risky asset. But perhaps the most interesting result in Figure 2.9 is the effect of changes in the degree of loss aversion over the price of ATM straddles. Panel C reveals that, in contrast to the results for zero-beta straddles, increasing the degree of loss aversion reduces the price agents are willing to pay for (holding long-positions in) ATM straddles. The mechanism behind this price reduction can be explained as follows. Since the expected equity return increases with λ, long positions in ATM straddles become more exposed to payoffs related to what the representative agent considers as gains. This in turn implies that agents holding long positions in ATM straddles are not compensated enough in states of the nature where they receive negative equity outcomes (losses). As a consequence, an increase in the degree of loss aversion systematically decreases the price of ATM straddles therefore increasing the return of taking long-positions in these derivatives.

2.5.3 Checking for the uniqueness of equilibria

We now investigate the uniqueness of equilibria for several parametrizations that might be problematic as explained in Section 2.4 and Appendix 2.A. Figure 2.10 displays the value of the expected total utility in Eq. (2.4-5) for derivative (ATM straddles) weights around the equilibrium condition $\alpha_D = 0$. It is important to note that, were the equilibrium unique, agents should not find it optimal to invest any proportion of wealth $\varepsilon > 0$ in the derivative at its equilibrium price. In other words, this figure checks the condition $E^*(.) \mid_{\alpha_{E=1};\alpha_{D=0}} > E^*(.) \mid_{\alpha_{E=1};\varepsilon>0}$ for several parametrizations. In particular, for parametrizations that (i) involve a large contribution of the value function (large b), (ii) strictly violate the pseudo-concavity condition ($\widehat{\gamma} = 1$), or (iii) have extremely distorted probabilities (low c). Our results suggest that the expected utility function $E^*(.) \mid_{\alpha_{E=1};\varepsilon>0}$ is not strictly concave for large values of b. In particular, we find that, when the contribution of the value function is large, agents with a linear value function ($\widehat{\gamma} = 1$) and extremely distorted probabilities might find negative position in straddles (locally) optimal. In contrast, agents whose preferences have curved value functions and moderately distorted probabilities find the opposite also (locally) optimal.

2.5.4 Exploring the time-varying nature of the volatility premium

In order to obtain a time-varying volatility premium, we extend our one-period setting as follows. Firts, we simulate L paths of size T for the return of the risky asset, $R_{E,t_l} \sim N(\mu_{E,T}, \sigma_{E,T})$. Then, for every simulated path, l, at each point in time, t_l, we solve the one-period equilibrium conditions following a procedure similar to that in Section 2.5.1. Now, in order to obtain time-varying expected returns for straddles, we follow BHS's methodology and assume that, at each point in time, the representative agent's first-order risk aversion as well as the reference level of her value function depend on the recent performance of her portfolio. In particular, we assume that the agent compares the current price of the risky asset with a benchmark level and calculates a measure of recent gains and losses, namely z_t.[23] If the representative agent has experienced recent gains, $z_t \leq 1$, her degree of loss aversion remains at a benchmark level λ. However, and given her recent positive experiences, she is now willing to accept more losses. As a consequence, her reference level for gains and losses becomes lower than R_f. Therefore, in this case, the value function in Eq (2.2-1) becomes

$$V_g(X_t) = \left\{ \begin{array}{c} X_t \\ R_f(z_t - 1) - \lambda(R_f(z_t - 1) - X_t) \end{array} \right. \text{ for } \begin{array}{c} X_t - R_f(z_t - 1) \geq 0 \\ X_t - R_f(z_t - 1) \leq 0 \end{array} . \tag{2.5-10}$$

In contrast, if she has experienced recent losses, $z_t > 1$, her reference level does not change but her degree of loss aversion increases as a response to the recent underperformance of her portfolio. The PT-value function after recent losses is then given by

$$V_l(X_t) = \left\{ \begin{array}{c} X_t \\ -\lambda_t(-X_t) \end{array} \right. \text{ for } \begin{array}{c} X_t \geq 0 \\ X_t \leq 0 \end{array} , \tag{2.5-11}$$

where

$$\lambda_t = \lambda + \kappa(z_t - 1), \tag{2.5-12}$$

where κ controls the agent's sensitivity to recently realized gains and losses as measured by z_t.

In order to introduce time variation in the benchmark level of gains and losses, BHS suggest that the price benchmark level should respond *sluggishly* to changes in the value of the stock.[24] In other words,

[23] BHS warn not to confuse the reference level in the value function with the price benchmark level used to judge recent gains and losses.

[24] The sluggishness is defined in BHS as follows: "when the stock price moves up by a lot, the benchmark level also moves up, but by less. Conversely, if the stock price falls sharply, the benchmark level does not adjust downwards by as much."

$$z_{t+1} = \eta(z_t \frac{\overline{R_E}}{R_{E,t+1}}) + (1-\eta), \tag{2.5-13}$$

where $\overline{R_E}$ is a fixed parameter calibrated to guarantee that half of the time the agent has prior gains ($R_{E,t} > \overline{R_E}$) and the rest of the time she has prior losses ($R_{E,t} < \overline{R_E}$), and η measures the degree of sluggishness.[25]

Figure 2.11 explains how the mechanism of time variation implied by Eqs. (2.5-10) to (2.5-13) works. The dynamics of z_t depends on the degree of sluggishness. For instance, for $\eta = 0.8$ -the value assumed in the figures-, z_t displays episodes where the agent accumulates recent gains ($z_t < 1$) or losses ($z_t > 1$) depending on the simulated performance of the stock. In turn, the level of z_t determines, at each point in time, the agent's degree of loss aversion. For instance, λ_t might display large peaks if the agent has accumulated serious losses as it is the case in several episodes of our simulated equity return path.

Figure 2.12 displays the effect of the implied dynamics of λ_t over the expected return of (long-positions in) a zero-beta straddle. In line with the results for the one-period setting, an increase in loss aversion increases the price of zero-beta straddles therefore decreasing the expected return of taking long-positions in these derivatives. In other words, agents become more willing to pay a higher price to hedge the risk of extreme events after experiencing losses. In contrast, when the agent has recently experienced gains, her reference level for valuing gains and losses becomes lower. Therefore, the payoffs of the zero-beta straddle corresponding to losses (relative to the performance of the risky asset) become less important in relation to those corresponding to gains. As a consequence, and despite the fact that the level of loss aversion remains constant, (long-positions in) zero-beta straddles become cheaper.

Finally, Table 2.2 reports a set of summary statistics for the return of a zero-beta for different values of parameter η.[26] As it is to be expected, a larger degree of sluggishness, η, implies that the return of (long-positions in) zero-beta straddles becomes more inertial. The first order autoregressive component ranges from 0.04 for $\eta = 0.2$ to 0.83 for $\eta = 1$. However, the average expected return of holding zero-beta straddles becomes more negative when the agent is assumed to have a longer-term memory. Thus, the average (one-period) volatility premium ranges from -33.99 for $\eta = 0.2$ to -36.05 for $\eta = 1$. In any case, the volatility of the implied return of zero-beta straddles is rather low and reaches a maximum of 2.24 for the case where agents have full-sample memory, $\eta = 1$.[27]

2.6 Conclusions

In this paper, we present a number of new findings on the potential of CPT to characterize several stylized facts extensively documented in the literature for the volatility premium. We first provide an appropriate setting to find equilibrium prices of equity and derivatives under CPT-preferences. Using the equilibrium prices of zero-beta straddles, we then disentangle the specific effect of the probability distortion mechanism as well as the value function embedded in CPT over the volatility premium.

[25] In other words, η measures the agent's memory (BHS). For instance, $\eta = 0$ implies that the investor has no memory and the benchmark level to compare gains and losses is always close to the current value of the stock; $\eta = 0.9$ is equivalent to assume that the "half-life" memory is around 6.6 years and $\eta = 1$ implies that the agent has a long-term memory (full-sample memory).

[26] See how, in general, simulating a very long path for equity returns would be equivalent to simulating many paths of a shorter length. However, this does not hold in this setting, since the representative agent's memory characterized by η also plays a role. This is particularly relevant for the case of full-sample memory agents ($\eta = 1$).

[27] To get an idea of this magnitude, the anualized volatility of the volatility premium reported by Londono (2011) between 2000 and 2009 is 13.861.

We show that several parameter combinations yield negative excess returns for assuming long-positions in zero-beta straddles. In particular, we show that the probability weighting mechanism plays a key role in explaining their (relatively) high prices. This effect is explained by the fact that zero-beta straddles provide the highest payoffs precisely for those states with larger distorted probabilities. In contrast, the total effect of the value function over the price of zero-beta straddles is rather poor since the isolated effects of the degree of loss aversion and the curvature of the value function have similar magnitudes but opposite directions.

When we investigate the effects of the CPT components over the price of ATM straddles, we find their market betas to play an important role in determining the magnitude and direction of these effects. In particular, we find that the effect of the degree of loss aversion switches sign when compared to that over zero-beta straddles. This evidence suggests in turn that CPT might have interesting implications for the volatility smile.

In the last part of the paper we also explore the ability of CPT to characterize the observed dynamics of the volatility premium. We find that after experiencing recent gains agents become more optimistic and zero-beta straddle prices are driven down. However, after having accumulated some losses, the degree of loss aversion increases thus increasing the price of zero-beta straddles. Nevertheless, this mechanism yields a moderate time variation in the price of straddles in contrast to what has been documented in the literature.

Our exploratory results pave the way for further research in several directions. First, we intend to make our findings comparable to equity and option market data. Our goal is to obtain parameter estimates that yield close approximations to the unconditionally expected volatility premium therefore conveying the limitations of existing models in the literature. Second, having understood the effect of the CPT components in the attitude of agents towards extreme events, we also aim to explain the observed dynamics of the volatility premium. In particular, the volatility premium peaks observed around high market uncertainty episodes.

APPENDIX

2.A Optimal non-finite portfolio weights

The conditions for the existence of finite optimal portfolio weights in problem (2.4-5) are[28]

$$\lim_{\alpha_j \to +\infty} E^*[\ln(\alpha_j(R_j - R_f)) - b\lambda \frac{(-\alpha_j^{\widehat{\gamma}}(R_j - R_f))^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) \leq 0\}} + b \frac{\alpha_j^{\widehat{\gamma}}(R_j - R_f)^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) > 0\}}] = -\infty, \tag{2.A-14}$$

and

$$\lim_{\alpha_j \to -\infty} E^*[\ln(\alpha_j(R_j - R_f)) + b \frac{\alpha_j^{\widehat{\gamma}}(R_j - R_f)^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) \leq 0\}} - b\lambda \frac{(-\alpha_j^{\widehat{\gamma}}(R_j - R_f))^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) > 0\}}] = -\infty, \tag{2.A-15}$$

for $j = E, D$ the proportion of the portfolio invested in the stock and the derivative respectively. In order to get some intuition on the existence of optimal finite portfolio weights, it is important to explore the three cases where Eqs. (2.A-14) and (2.A-15) hold. The first case is that where agents are only concerned about the utility they derive from total wealth (consumption). That is, when $b = 0$, the value function plays no role in agents' decisions which brings us back to the traditional CRRA setting.[29] In a second case, if b is large enough to make the utility over wealth contribution insignificant, a sufficient condition for Eqs. (2.A-14) and (2.A-15) to hold is any other parameter combination where $B_{1,j} < 0$ and $B_{2,j} < 0$, where[30]

$$B_{1,j} = -\lambda E^*[\frac{(R_j - R_f)^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) \leq 0\}}] + E^*[\frac{(R_j - R_f)^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) > 0\}}],$$

and

$$B_{2,j} = E^*[\frac{(R_j - R_f)^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) \leq 0\}}] - \lambda E^*[\frac{(R_j - R_f))^{\widehat{\gamma}}}{\widehat{\gamma}} \mathbf{1}_{\{(R_j - R_f) > 0\}}].$$

Finally, conditions (2.A-14) and (2.A-15) hold for any parameter combination as long as $U(W_T) = \ln W_T$ dominates the total utility function for extreme (levered) positions. That is, for any combination where the function is strictly concave in the extremes.

[28] In order to simplify the notation and center all the attention on CPT, we assume $\gamma = 1$ ($U(W_T) = \ln W_T$).

[29] See Hens and Pilgrim (2003) for a summary of the equilibrium conditions for the CRRA setting.

[30] These conditions converge to those in Ang, Bekaert and Liu (2005).

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Table 2.1: CPT parameters

Notes: this table reports alternative values for the parameters in Eqs. (2.2-1) and (2.2-3) previously used in the literature as well as our benchmark parametrization. The references are abbreviated as follows: TK1992 for Tversky and Kahneman (1992), CH1994 for Camerer and Ho (1994), BHS2001 for BHS, BN2002 for Burnes and Neilson (2002), ABL2005 for Ang, Bekaert and Liu (2005), DM2007 for Driessen and Maenhout (2007).

Parameter	Benchmark	TK1992	CH1994	WG1996	BHS2001, BN2002, ABL2005,	DM2007
$\widehat{\gamma}$	0.88	0.88	0.32	0.52	0.88	$0.8, 0.88, 0.9, 1$
λ	2.25	2.25			2.25	$1.25, 1.75, 2.25$
c^{+}	0.65	0.61	0.56	0.74		$0.65, 0.8$
c^{-}	0.65	0.69	0.56	0.74		$0.65, 0.8$

Table 2.2: Summary statistics. Time-varying volatility premium implied by CPT

Notes: this table reports the summary statistics for the expected return of a zero-beta straddle for alternative values of parameter η in Eq. (2.5-12). For these simulations, we assume $L = 100$ paths of size $T = 100$ periods each. We also assume a (one-period) expected equity return of $\mu_{ET} = 10\%$ and $\sigma_{ET} = 20\%$. All other parameters except for $\widehat{\gamma}$ are set according to our benchmark parametrization in Table 2.1 . Finally, as in BHS, we assume a linear value function ($\widehat{\gamma}_1 = \widehat{\gamma}_2 = 1$).

η	0.2	0.4	0.6	0.8	1
Mean	−33.99	−34.16	−34.31	−34.58	−36.05
Median	−33.96	−34.15	−34.33	−34.50	−35.14
Min.	−34.41	−34.88	−35.62	−37.26	−43.17
Max.	−33.78	−33.78	−33.78	−33.78	−33.78
St. Dev.	0.16	0.25	0.35	0.63	2.24
Kurtosis	2.68	2.72	5.08	7.68	5.32
Skew.	−0.73	−0.39	−0.87	−1.77	−1.54
AR(1)	0.04	0.12	0.23	0.52	0.83

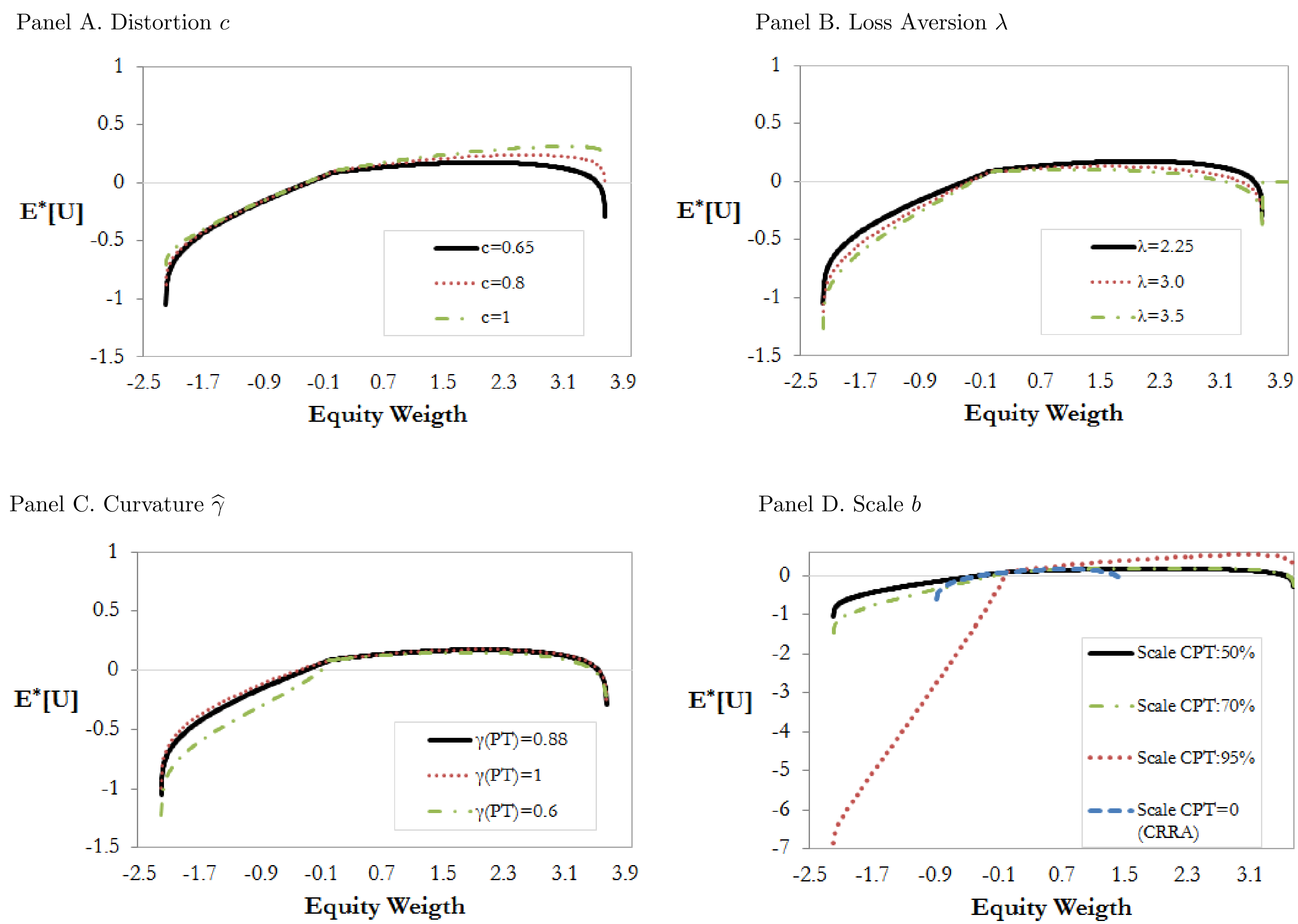
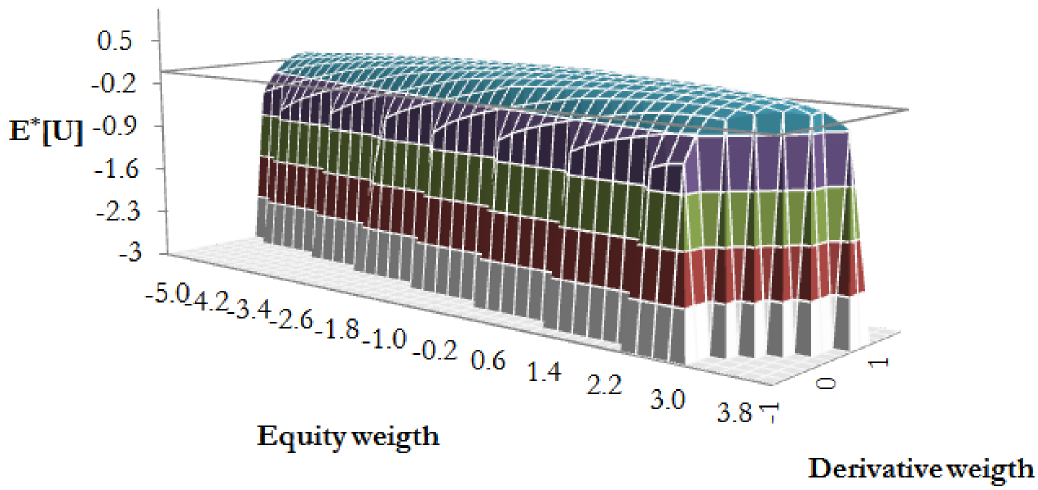


Figure 2.1: Maximization problem restricted to (normally distributed) risky assets. Alternative parametrizations

Notes: this figure illustrates the maximization problem in Eq. (2.4-5) when agents only have access to stocks. In the figure, equity returns are assumed to be normally distributed with mean $\mu_E = 10\%$ and volatility $\sigma_E = 20\%$. Unless otherwise stated, the parameters used in the figure are the following: $\gamma = 1$, $c = 0.65$, $\lambda = 2.25$, $\widehat{\gamma} = 0.88$, and $b = 0.5$ (benchmark parametrization).

Panel A. 50% CRRA, 50% CPT



Panel B. 5% CRRA, 95% CPT

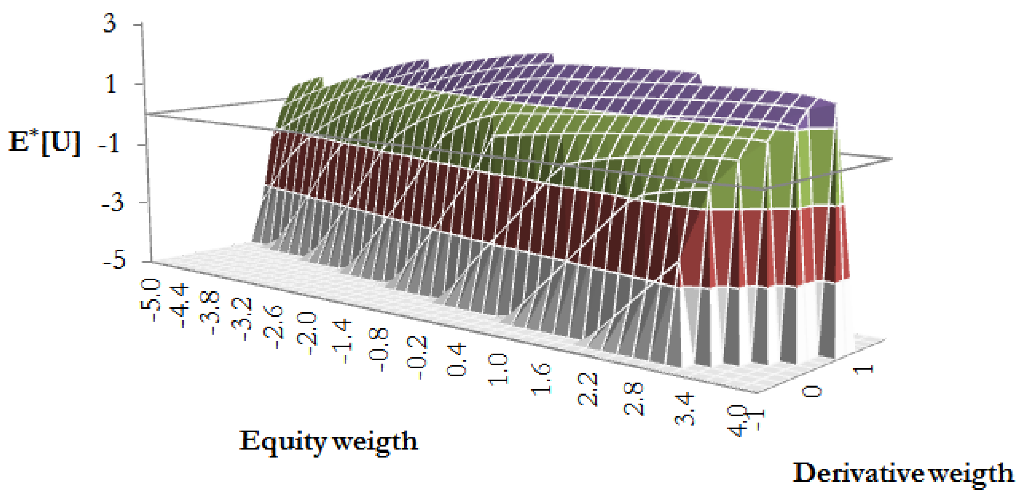


Figure 2.2: Maximization problem including a derivative (straddle). Alternative parametrizations

Notes: this figure extends the results in Figure 2.1 to the case where agents have access to stocks and derivatives. We report the expected value of the total utility function for the benchmark parametrization and alternative values of the scale (b). Panel A displays the benchmark case where the contribution of the value function is 50% while Panel B displays the case where its contribution is around 95%. The derivative is assumed to be an ATM straddle with maturity 1 period.

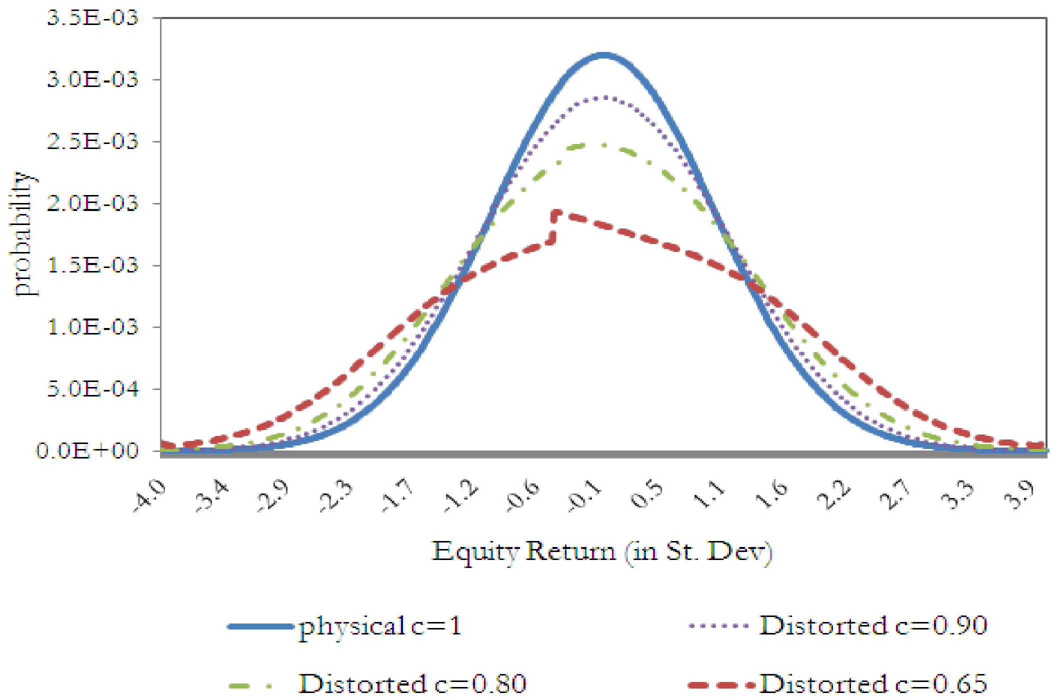


Figure 2.3: Distribution of equity returns under the physical and distorted probability measures

Notes: the figure displays the physical and distorted distribution of excess equity returns for several values of parameter c.

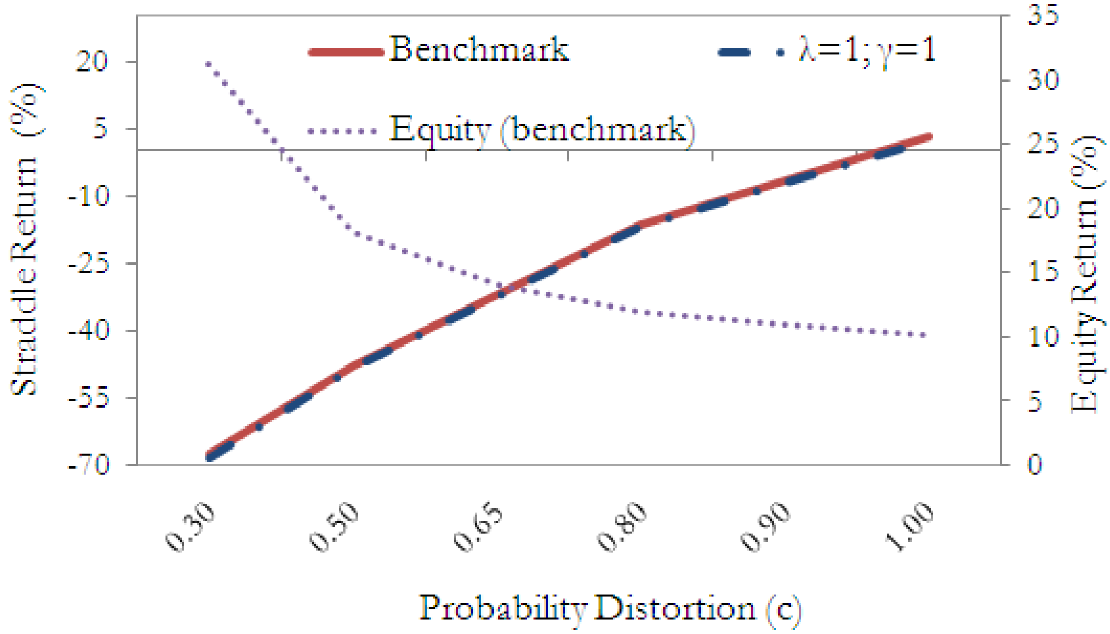


Figure 2.4: Numerical Simulations. Equilibrium expected return for stocks and zero-beta straddles for alternative values of parameter c

Notes: the figure displays the expected return of a zero-beta straddle ($E(d_1/d_0)$) with maturity 1 period for deviations of parameter c from two scenarios: the first one is the benchmark parametrization in Table 2.1 (bold line), and the second one is that where there is no loss aversion ($\lambda = 1$) and the value function is linear ($\gamma_1 = \gamma_2 = 1$) (dashed-dotted line). The figure also displays the equilibrium equity mean return, μ_E (dotted line), for alternative values of parameter c and the benchmark scenario.

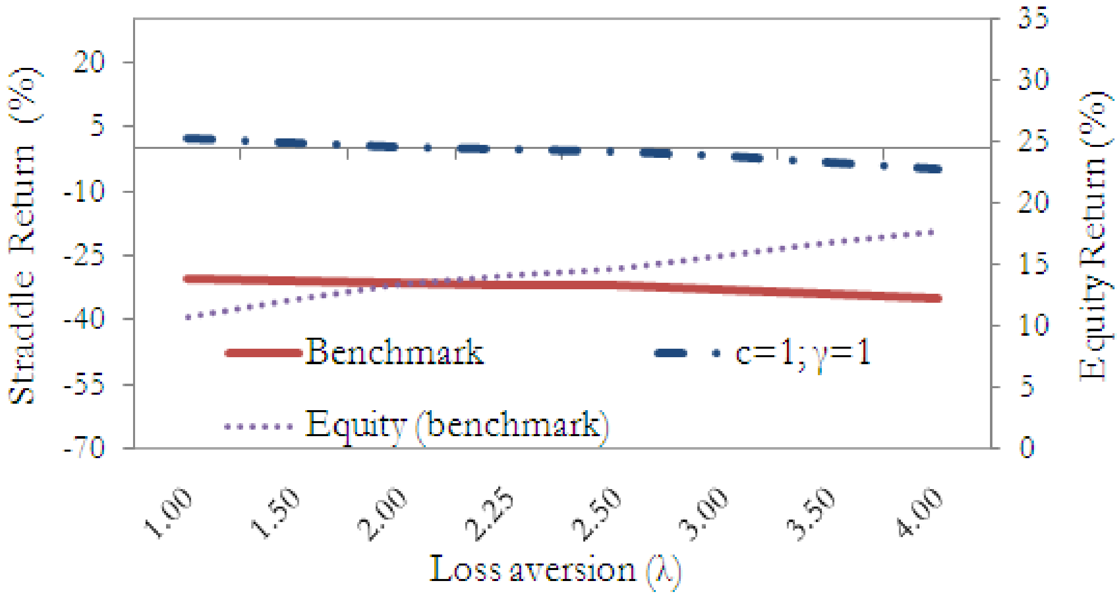


Figure 2.5: Numerical Simulations. Equilibrium expected return for zero-beta straddles for alternative degrees of loss aversion

Notes: similar to Figure 2.4, this figure displays equilibrium expected returns of a zero-beta straddle for deviations of parameter λ from the benchmark scenario (bold line) as well as from the scenario where probabilities are not distorted ($c = 1$) and the value function is linear ($\gamma_1 = \gamma_2 = 1$) (dashed-dotted line).

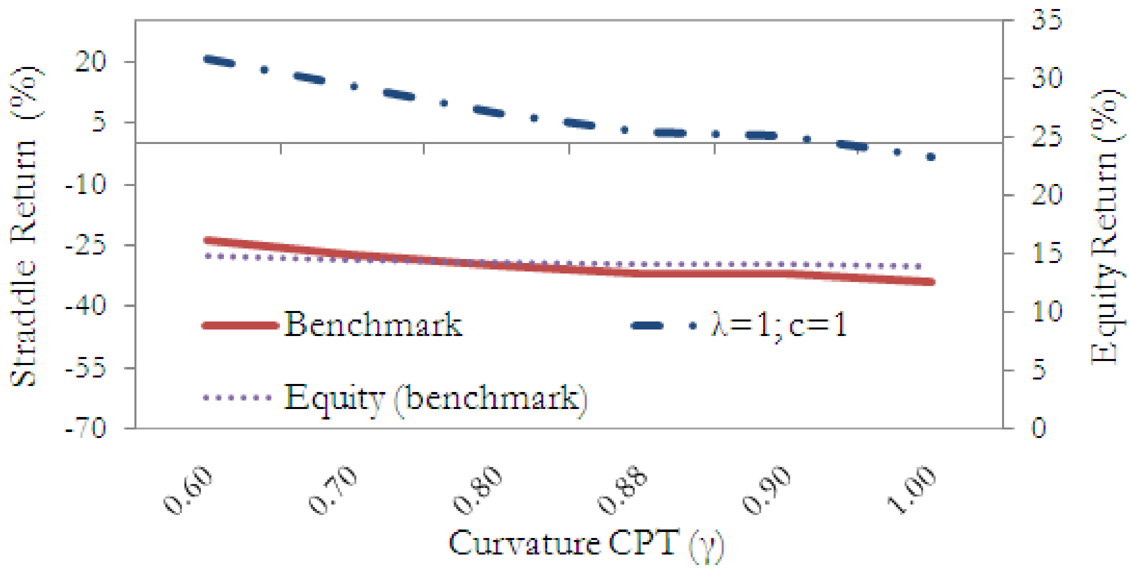


Figure 2.6: Numerical Simulations. Equilibrium expected return for zero-beta straddles for alternative values of the curvature of the value function

Notes: similar to Figure 2.4, this figure displays equilibrium expected returns of a zero-beta straddle for deviations of parameter γ ($\gamma = \gamma_1 = \gamma_2$) from the benchmark scenario (bold line) as well as from the scenario where probabilities are not distorted ($c = 1$) and there is no loss aversion ($\lambda = 1$) (dashed-dotted line).

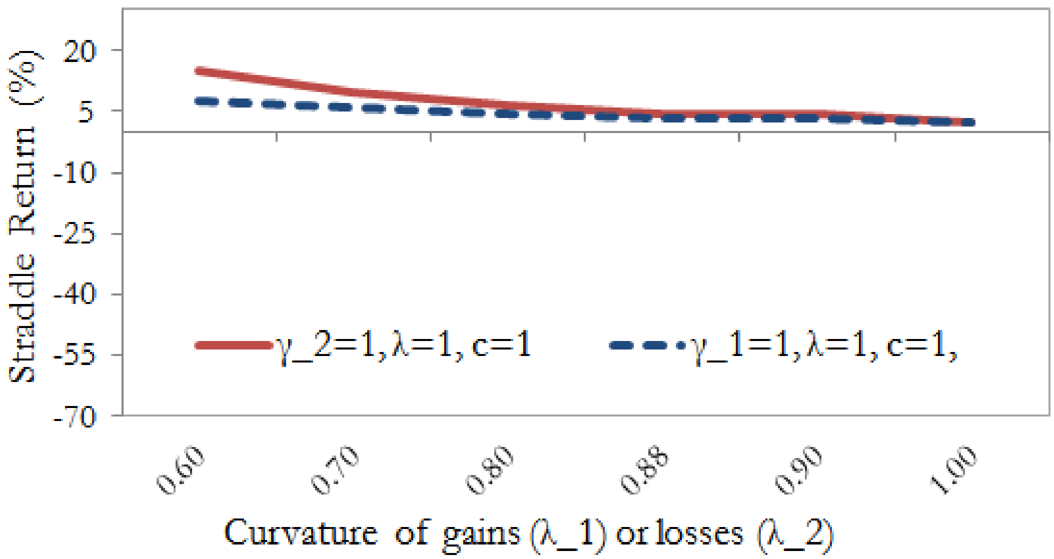


Figure 2.7: Numerical Simulations. Equilibrium expected return for zero-beta straddles for alternative values of the curvature in the domain of gains

Notes: Similar to Figure 2.6, this figure displays equilibrium expected returns of a zero-beta straddle for deviations of parameter $\widehat{\gamma}_1$ (bold line) and $\widehat{\gamma}_2$ (dashed line). In order to disentangle their specific effects, we only plot the scenario where there is no probability distortion, agents are not loss averse, and the value function is linear in the domain of losses and gains respectively.

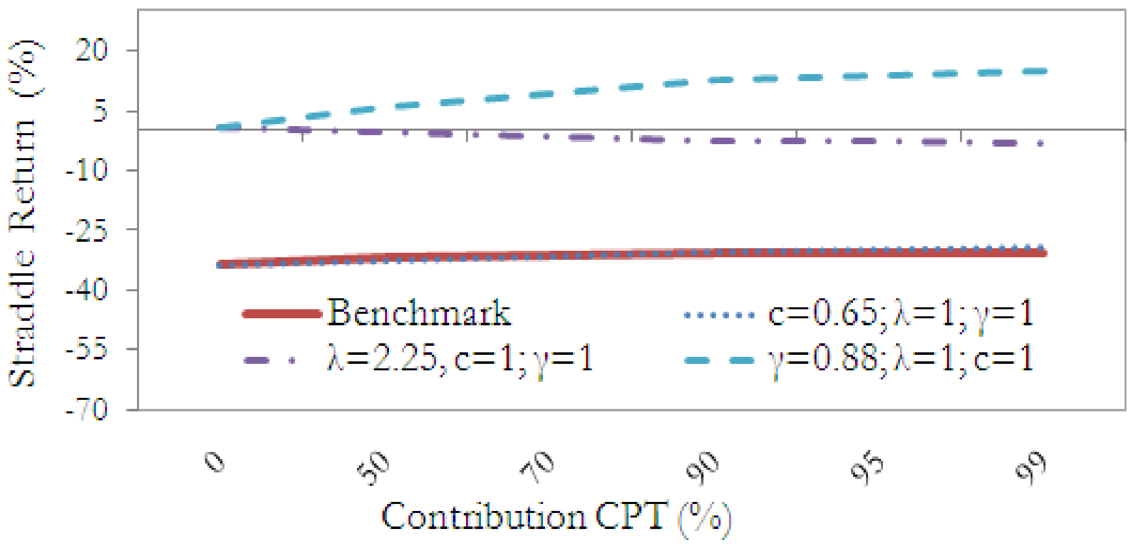
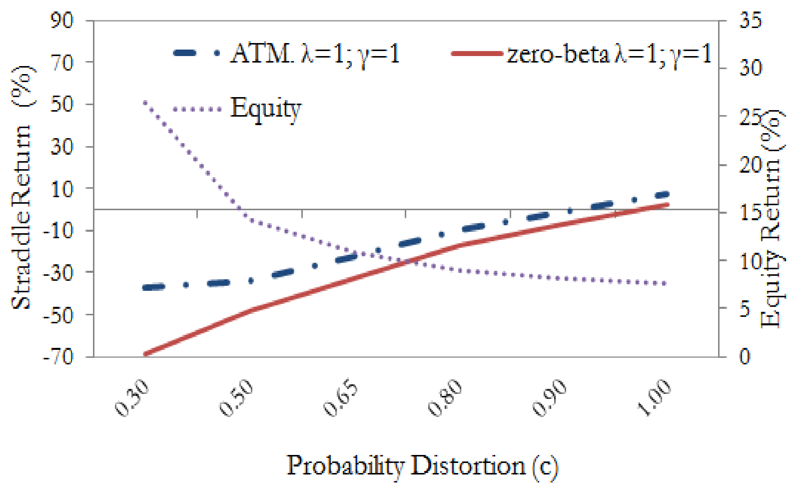


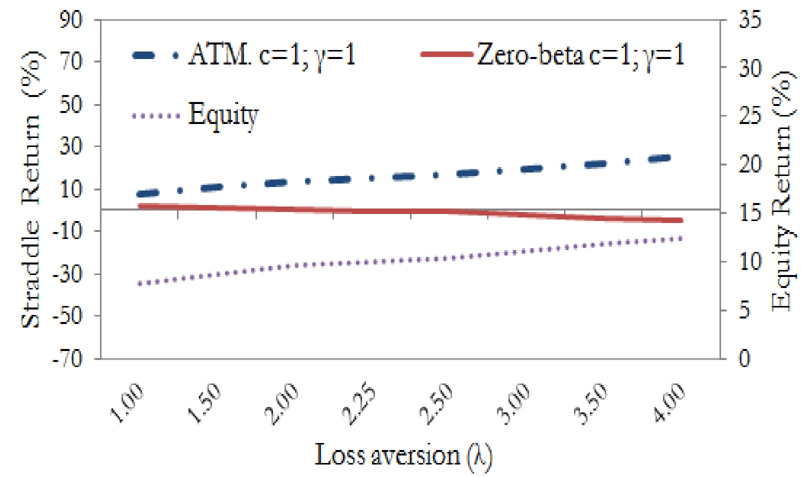
Figure 2.8: Numerical Simulations. Equilibrium expected return of zero-beta straddles for alternative values of parameter b

Notes: similar to Figure 2.4, this figure displays equilibrium expected returns of a zero-beta straddle for deviations of parameter *c* from four scenarios: (i) the benchmark scenario (bold line), (ii) a scenario where neither the loss aversion nor the curvature of the value function are considered (dotted line), (iii) a scenario where neither the probability distortion nor the curvature of the value function are considered (dashed-dotted line), and finally, (iv) a scenario where neither the loss aversion nor the probability distortion are considered (dashed line).

Panel A. c



Panel B. λ



Panel C. $\widehat{\gamma}$

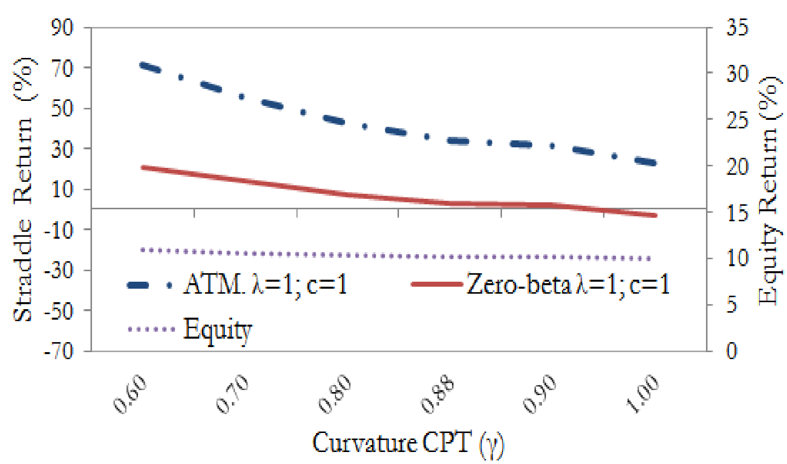


Figure 2.9: Numerical Simulations. Equilibrium expected return of ATM straddles for alternative parameter values

Notes: Similar to Figures 2.4 to 2.8, this figure displays the equilibrium expected returns of an ATM straddle (dashed-dotted line) for deviations of parameters c, λ, and $\widehat{\gamma}$ in Panels A, B, and C respectively. For the sake of completeness, the comparable prices of zero-beta straddles are also reported (bold line).

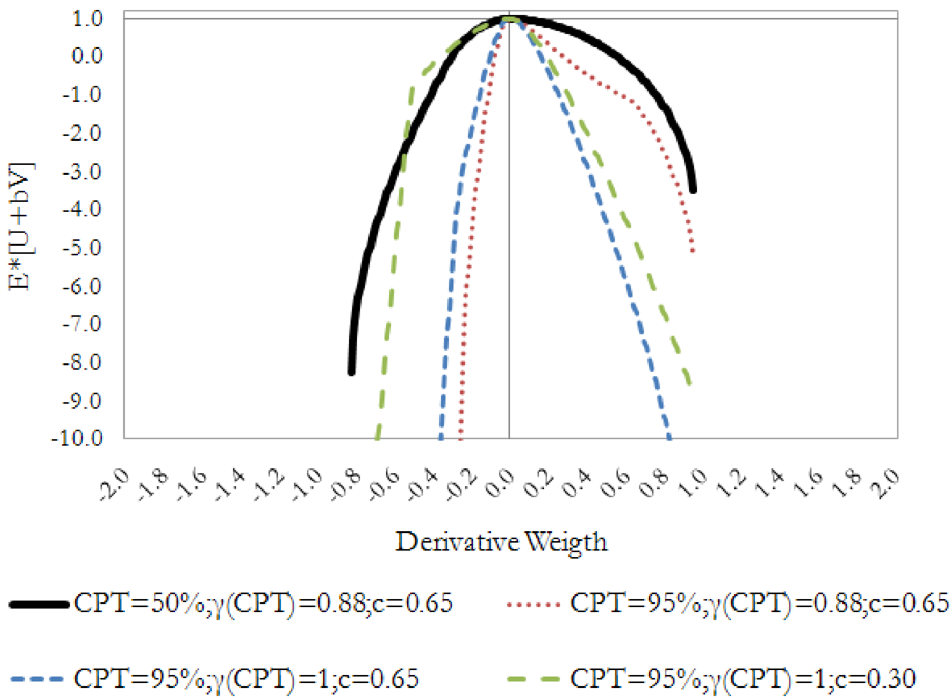


Figure 2.10: Numerical Simulations. Uniqueness of the optimum for alternative parametrizations

Notes: this figure displays the value of the expected total utility function ($E^*[U(W_T) + bV(X_T)]$) around the equilibrium condition ($\alpha_D = 0$). That is, we report the utility agents derive from investing an additional ε in the derivative at current equilibrium prices. For each parametrization, the value of the utility function is normalized to the maximum utility in order to make the figures comparable. We use four alternative parametrizations combining scales that imply contributions of the value function between 50% and 95%, $\widehat{\gamma}$ of 0.88 and 1.0, and c of 0.30 and 0.65.

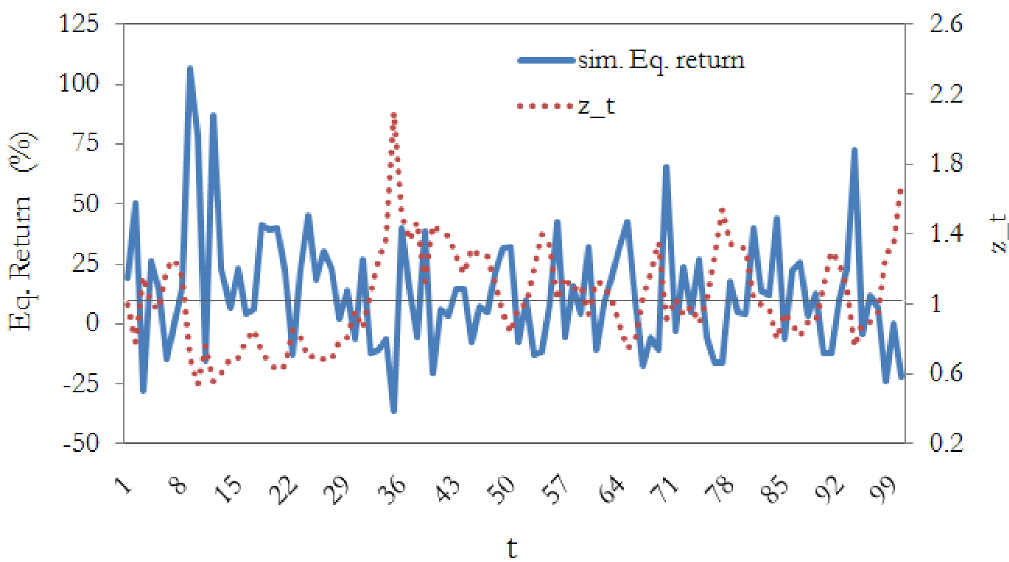


Figure 2.11: Dynamics of the measure of recent gains and losses for a simulated path for equity returns

Notes: This figure displays a simulated path for equity returns ($\mu_{ET} = 10\%$; $\sigma_{ET} = 20\%$) as well as the relative measure used by investors to track recent gains and losses, z_t. The sluggishness parameter (agent's memory) is assumed to be $\eta = 0.8$.

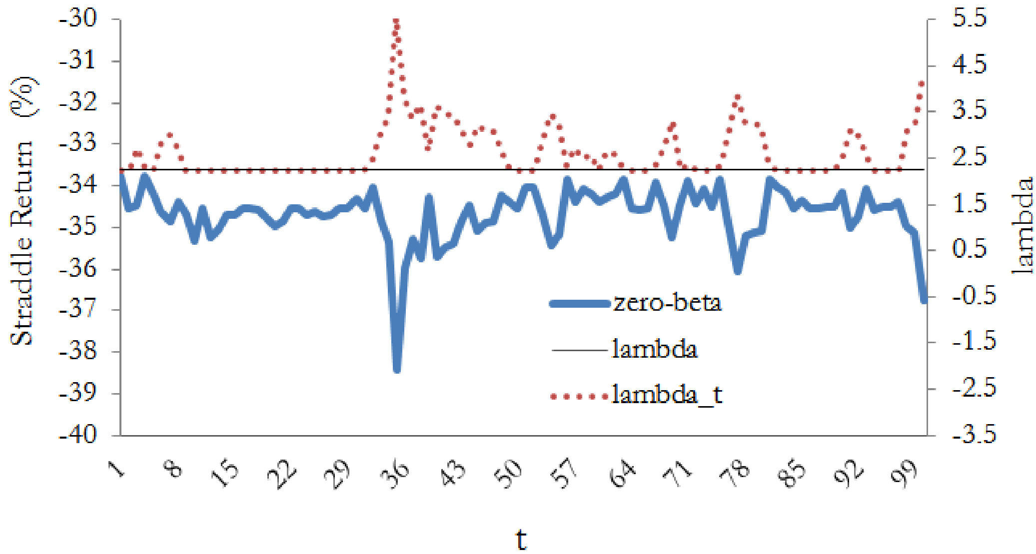


Figure 2.12: Dynamics of the expected return of zero-beta straddles for a simulated path for the return of the risky asset

Notes: This figure displays the implied (one-period) return for a zero-beta straddle (bold line) for the simulated return path in Figure 2.11. The figure also displays, in a secondary axis, the value of λ_t (dotted line) around its benchmark level, $\lambda = 2.25$.

CHAPTER 3

Understanding Industry Betas

Joint work with
Lieven Baele[1]

Abstract

This paper models and explains the dynamics of market betas for 30 US industry portfolios between 1970 and 2009. We use a DCC-MIDAS and kernel regression technique as alternatives to the standard ex-post measures. We find betas to exhibit substantial persistence, time variation, ranking variability, and heterogeneity in their business cycle exposure. While we find only a limited amount of structural breaks in the betas of individual industries, we do identify a common structural break in March 1998. Finally, we find the cross-sectional dispersion in industry betas to be countercyclical and negatively related to future market returns.

JEL Classification: C33, E32, G12

Keywords: Industry Betas, Component Models, Kernel, DCC-MIDAS, dispersion in betas.

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[1] Finance Department, CentER, and Netspar, Tilburg University.

3.1 Introduction

This paper investigates the dynamics and macroeconomic determinants of industry betas between 1970 and 2009. Improving our understanding on how to model industry betas is important for a number of reasons. First, industry portfolios are the base assets in many strategic and tactical asset allocation models and a popular investment strategy is based on the industry membership to stocks. These strategies are based on the intuition that industries have different fundamentals and thus react differently to market-wide changes. Second, while there is mounting evidence that industry betas are time-varying (Fama and French (1997), and Santos and Veronesi (2004), among others), there is still a lot of uncertainty both about the appropriate econometric techniques and the fundamental drivers of (industry) betas (see, e.g., Coseman, et. al. (2011)). Finally, existing models have problems explaining differences in industry cost of capital (see also Fama and French (1997)). Allowing betas to vary over time may at least partly help to solve that puzzle (see, e.g., Lettau and Ludvigson (2001) and Petkova and Zhang (2005)).

The first contribution of our paper is that we consider two alternatives to the traditional quarterly ex-post beta measures used in previous studies. While these ex-post measures have the advantage of being model-free and easy to calculate, they have, at least, two major disadvantages. First, they rely on the assumption that betas are constant within the window chosen. Second, all observations within this return window are equally weighted. As a first alternative to determine the length of the window as well as the weighting scheme for returns within the window, we consider the DCC-MIDAS model introduced by Colacito, Engle, and Ghysels (2009). This model not only allows for an optimal determination of the weighting scheme, but also decomposes each beta into a low and a high frequency component. This allows us to differentiate between instruments that are expected to predominantly affect the low frequency component, like business cycle indicators, from factors that should mainly have an influence on high frequency systematic risk. As a second alternative to ex-post betas, we consider the Kernel approach in Ang and Kristensen (2010). This econometric method, as the DCC-MIDAS, allows to estimate an optimal weighting scheme with more distant returns getting increasingly less weight. Moreover, this method allows to use efficiently the full sample information to estimate betas at each point in time by means of a two-sided Gaussian Kernel.

By using these alternative methods, we find a number of interesting results for the dynamics of industry betas. First, while the optimal window length implied by both methods differs between industries, it is always larger than the one quarter window typically used to estimate ex-post betas. While industry betas are highly persistent, we do find them to vary substantially over time. In particular, we find a substantial variability in beta ranking over time. In fact, nearly all industries had at some point the highest and the lowest beta. We find technology related industries like Business equipment, Games and Personal and business services to predominantly belong to the group of industries with the 30 percent highest betas, while the Mining industry as well as necessities related industries such as Utilities and Food belong mostly to the bottom 30 percentile.

Given that many industry characteristics may have fundamentally changed over our 40 year sample, we also test for the possibility of individual as well as common structural breaks. First, we identify industry-specific structural breaks in betas using the well-known Bai and Perron (2003) structural break test. We find the number of individual breaks to be small (only 9 out of 30 industries, and mostly just one break) and to be clustered in the period surrounding the Technology, Media and Telecom bubble. Then, we also identify common breaks in industry betas using the method in Qu and Perron (2007). We believe that a common break test makes more economic sense since the market-weighted sum of betas will still sum up to 1 and one would expect a break in the beta of one industry also to have an effect on the betas of all other industries. In line with the results from the individual break tests, when we impose the possibility of a common structural break, this break is identified in March 1998 around the beginning of the technology bubble.

A second contribution is that we identify for each industry the sensitivity of its market beta to the business cycle. We find a systematic increase during recessions in the betas of Chemicals, Steel, Fabricated Products, Mine, and Financial industries, and a systematic decrease in the betas of Smoke, Health, Electronic Equipment, and Retail Trade industries. However, for all other industries, and in particular for necessities related industries, we find betas not to be contemporaneously correlated with either an NBER dummy or the Chicago Fed National Activity Index. We show that this exposure holds when we correct for the common technology boom structural break while, at least, part of it disappears once we correct for individual structural breaks. Next, we show that industry betas are significantly related to a large set of lagged cyclical variables, but in a heterogeneous way. We find this relation to be stronger when DCC-MIDAS and Kernel betas are used as dependent variables precisely because these two approaches are less exposed to short-term noise than ex-post betas.

A final contribution of our paper is that it investigates the dynamics of the cross-sectional dispersion in industry betas and its predictive power for future returns. The existing theoretical literature has come up with different predictions about the cyclicality of beta dispersion. Gomes, Kogan and Zhang (2003) predict beta dispersion to be higher during recessions, while both the theoretical predictions and empirical findings of Santos and Veronesi (2004) suggest the opposite. We find cross-sectional dispersion in industry betas to be higher in recessions than in expansions, in line with the predictions of Gomes, Kogan and Zhang (2003). We also find that this relation becomes more clear when industry betas are corrected for structural breaks. Having shown that beta dispersion is negatively related with the business cycle, we next investigate whether it has predictive power for overall market returns. This relates to a recent study by Stivers and Sun (2010). They show that the cross-sectional dispersion in returns is positively related to the subsequent value premium, and negatively to the subsequent momentum premium. We find that industry beta dispersion is in fact a stronger predictor of future market returns than the non-systematic component of the cross-sectional dispersion in industry returns. However, the predictive power of both variables is centered in the last crisis-rich decade of our sample.

The remainder of the paper is structured as follows. Section 3.2 introduces the alternative econometric methods to measure industry betas. Section 3.3 discusses the differences in the dynamics of industry betas for the different methods considered. In this section, we also provide preliminary evidence for the dynamics of betas during recessions. Section 3.4 investigates the determinants of industry betas as well as their cross-sectional dispersion. In this section, we also investigate the ability of the cross-sectional dispersion to predict market excess returns. Finally, Section 3.5 concludes.

3.2 Alternative Measures of Industry Betas

In this section, we discuss two alternative beta measurement techniques for the standard ex-post betas estimated over rolling windows, namely the DCC-MIDAS model of Colacito, Engle, and Ghysels (2009) and a kernel regression technique recently refined by Ang and Kristensen (2010). As a starting point and benchmark, we introduce ex-post betas estimated over either the last quarter (**qbetas**) or the last year (**ybetas**) of daily returns. Given its simplicity and model-independence, this approach has been extensively used in the literature.[1] Nonetheless, this method has at least two major disadvantages. First, it relies on the assumption that betas are constant within the window chosen. It is far from clear,

[1] Specifically for the case of industry betas, see e.g., Fama and French (1997) and Ghysels and Jacquier (2006).

however, what the optimal window length would be. Too short windows will lead to noisy betas while betas estimated over too long windows will not react swiftly enough to new information. A second disadvantage is that all observations within the return window are equally weighted. This weighting scheme goes against the intuition and empirical evidence that recent observations contain more information about current market betas than past returns. The main advantage of the DCC-MIDAS and kernel method is precisely that both the window length and the weight assigned to distant returns are optimally determined.

3.2.1 DCC-MIDAS betas

As a first alternative to ex-post betas, we consider betas calculated from a bivariate DCC-MIDAS model for each industry and the market. The DCC-MIDAS method was introduced by Colacito, Engle, and Ghysels (2009). It combines the MIDAS-GARCH model in Engle, Ghysels, and Sohn (2009) with the DCC model in Engle (2002). The MIDAS-GARCH model combines in turn the Spline-GARCH model in Engle and Rangel (2008) with the mixed data sampling (MIDAS) in Ghysels, Santa-Clara, and Valkanov (2005, 2006). A key feature of the MIDAS model is that it allows to decompose each beta into a low (e.g., quarterly) and high (e.g., daily) frequency component. Distinguishing between different components may not only lead to more accurately measured betas[2], it will also allow us to differentiate between instruments that are expected to predominantly affect the low frequency component, like business cycle indicators, and factors that should mainly have an effect on high frequency changes in systematic risk.

In order to obtain the DCC-MIDAS betas, we model a bivariate system for the daily returns as follows:

$$\mathbf{r}_t \sim_{iid} N(\boldsymbol{\mu}, \mathbf{H}_t), \tag{3.2-1}$$

where $\mathbf{r}_t = [r_{i,t}, r_{m,t}]'$ contains daily returns for each industry (i) and the market (m). The market return is the value-weighted sum of returns over all industries. Given our focus on second moments, we assume the vector of expected returns $\boldsymbol{\mu}$ to be constant over time. The variance covariance matrix $\mathbf{H}_t$ follows

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \tag{3.2-2}$$

where $\mathbf{D}_t$ is a diagonal matrix containing standard deviations $D_{i,t}$ and $D_{m,t}$, and $\mathbf{R}_t$ is the time-varying correlation matrix.

The time-varying standard deviations are composed of a low and a high frequency component:

$$D_{k,t} = \sqrt{g_{k,t} \upsilon_{k,\tau}}, \qquad k = \forall i, m,$$

where $g_{k,t}$ is the short-term or high frequency component. This component is modeled as a standard mean-reverting GARCH(1,1) process:

$$g_{k,t} = (1 - \alpha_k - \beta_k) + \frac{\alpha_k (r_{k,t-1} - \mu_k)^2}{\upsilon_{k,\tau}} + \beta_k g_{k,t-1}, \qquad k = \forall i, m. \tag{3.2-3}$$

The long-term component,$\upsilon_{k,\tau}$, which is constant for all days t within quarter τ, follows the process

$$\upsilon_{k,\tau} = m_k + \gamma_k \sum_{l=1}^{L_k^{\upsilon}} \varphi_l(\omega_k^{\upsilon}) RV_{k,\tau-l}, \qquad k = \forall i, m, \tag{3.2-4}$$

[2] Chernov, et. al (2003) show that at least two components are needed to properly capture the dynamics of volatility.

where $RV_{k,\tau} = \sum_{j=\tau-N_\tau}^{\tau} r_{k,j}^2$ is the realized volatility for period τ calculated over a total of N_τ days, and the smoothing or weighting function $\varphi_l(.)$ is modeled as a beta function

$$\varphi_l(\omega_k^v) = (1 - l/L_k^v)^{\omega_k^v - 1} / \sum_{j=1}^{L_k^v} (1 - j/L_k^v)^{\omega_k^v - 1}). \quad (3.2\text{-}5)$$

The main advantage of this beta polynomial function is that its many possible shapes are determined by just two parameters, L_k^v and ω_k^v.[3] In general, for a given L_k^v, the higher ω_k^v, the lower the weight assigned to past returns and thus the lower the persistence of volatility. In contrast, for values of ω_k^v close to 1, the weight function decays very slowly, and volatility becomes highly persistent (as long as L_k^v is sufficiently large).

As for the components of the correlation matrix, the (short-term) correlation between the returns of industry i and the market m is calculated as:

$$\rho_{i,m,t} = \frac{q_{i,m,t}}{\sqrt{q_{i,i,t}}\sqrt{q_{m,m,t}}}, \quad (3.2\text{-}6)$$

with

$$q_{i,m,t} = \overline{\rho}_{i,m,\tau}(1 - a_i - b_i) + a_i \xi_{i,t-1}\xi_{m,t-1} + b_i q_{k,m,t-1}, \quad (3.2\text{-}7)$$

where $\xi_{i,t}$ and $\xi_{m,t}$ are the standardized residuals for industry i and the market respectively. These are calculated using the standard deviations obtained from the MIDAS-GARCH model (i.e., $\boldsymbol{\xi}_t = \mathbf{D}_t^{-1}(\mathbf{r}_t - \boldsymbol{\mu})$). The long-term correlation component $\overline{\rho}_{i,m,\tau}$ is specified as

$$\overline{\rho}_{i,m,\tau} = \sum_{l=1}^{L_i^c} \varphi_l(\omega_i^c) c_{i,m,\tau-l}, \quad (3.2\text{-}8)$$

where

$$c_{i,m,\tau} = \frac{\sum_{j=\tau-N_c}^{\tau} \xi_{i,,j}\xi_{m,,j}}{\sqrt{\sum_{j=\tau-N_c}^{\tau} \xi_{i,j}^2}\sqrt{\sum_{j=\tau-N_c}^{\tau} \xi_{m,j}^2}},$$

and the polynomial function $\varphi_l(\omega_k^c)$ is equivalent to that in Eq. (3.2-5).

In order to estimate the system of equations (3.2-1) to (3.2-8), we follow the two-step procedure introduced by Engle (2002) and applied for the DCC-MIDAS in Colacito, Engle, and Ghysels (2009). Thus, we estimate the parameters that maximize the following quasi-likelihood function:

$$\begin{aligned} QL(\Psi, \Xi) &= QL_1(\Psi) + QL_2(\Psi, \Xi), \\ &= -\sum_{t=1}^{T*N}(n \log(2\pi) + 2\log|\mathbf{D}_t| + \mathbf{r}_t'\mathbf{D}_t\mathbf{r}_t) + \sum_{t=1}^{T*N}(\log|\mathbf{R}_t| + \boldsymbol{\xi}_t'\mathbf{R}_t\boldsymbol{\xi}_t + \boldsymbol{\xi}_t'\boldsymbol{\xi}_t), \end{aligned}$$

where T is the total number of quarters in the sample and N the (average) number of days per quarter. In a first step, we estimate separately the parameters driving the dynamics of volatility for each industry and the market in Eqs. (3.2-3) to (3.2-5) and collect them in a vector $\widehat{\Psi} = [(\widehat{\alpha}_k, \widehat{\beta}_k, \widehat{\omega}_k^v \widehat{m}_k, \widehat{\theta}_k),\ k = \forall i, m]$. In a second step, we use the parameters in $\widehat{\Psi}$ to estimate the standardized residuals $\widehat{\xi}_{k,t-1}$ in Eq. (3.2-7). Then, we estimate the

[3] Ghysels, Santa-Clara and Valkanov (2005) and Ghysels, Sinko and Valkanov (2007) provide some interesting comparisons among alternative specifications for the weighting function.

parameters driving the dynamics of correlations (Eqs. (3.2-6) to (3.2-8)) and collect them in a vector $\widehat{\Xi} = [(\widehat{a}_i, \widehat{b}_i, \widehat{\omega}_i^c)]$. Finally, with the parameters in $\widehat{\Psi}$ and $\widehat{\Xi}$, the long-term (quarterly) DCC-MIDAS betas (**mbetas**) are calculated as follows:

$$\beta_{i,\tau} = \overline{\rho}_{k,m,\tau} \frac{\upsilon_{k,\tau}}{\upsilon_{m,\tau}}.$$

3.2.2 Kernel betas

In a recent paper, Ang and Kristensen (2010) use a kernel method to estimate time-varying alphas and factor exposures ("betas"). While the DCC-MIDAS method imposes a specific parametric model, this kernel method only imposes very weak restrictions on the dynamics of betas (see Ang and Kristensen (2010) for details). Similar to the DCC-MIDAS model, the kernel method estimates industry betas by applying a weighting scheme to daily returns with more distant (relative to the current observation) returns getting increasingly less weight. The shape of the kernel, K, determines how the different observations are weighted. While in principle the kernel could have many forms, we use the symmetric two-sided Gaussian kernel

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2}).$$

The quarterly kernel beta (**kbetas**) at any (normalized) point in time $\tau \in (0, 1)$ is given by:

$$\beta_{i,\tau} = [\sum_{t=1}^{n} K_{h_i}(t/n - \tau) r_{m,t}^2]^{-1} [\sum_{t=1}^{n} K_{h_i}(t/n - \tau) r_{m,t} r'_{i,t}],$$

where $K_{h_i}(z) = K(z/h_i)/h_i$ is a density kernel that controls the weights of the observations considered at every τ for a given bandwidth $h_i > 0$. Notice that because we use a two-sided kernel, the time τ beta is estimated using both past and future returns (relative to τ).[4] The bandwidth h_i controls the time window used in the estimation of the beta of the i'th industry. The bandwidth is estimated using a plug-in method that minimizes a mean squared error function. This function weighs the trade-off between the variance and the bias of the estimator, and should in general be different for each industry (for details on the bandwidth selection, see again Ang and Kristensen (2010)[5]).

3.3 The Dynamics of Industry Betas

In this section, we investigate the dynamics of industry betas for the alternative methods described in the previous section. After introducing the data, we report and discuss the estimated parameters for the DCC-MIDAS and kernel models. Then, we discuss the characteristics of estimated betas and highlight the main differences in beta dynamics across the three different econometric techniques. In the third part, we test for multiple structural breaks in the industry betas. Finally, we investigate the contemporaneous relation between these betas and the business cycle in order to motivate the investigation on their determinants.

[4] Our main motivation for this choice is that [1] show that using a two-sided instead of a one-sided kernel leads to significantly lower root mean squared errors. Of course, because of the inherent look-ahead bias, we will not use these kernel betas in the prediction exercise in Section 3.4.3.

[5] We would like to thank professor Kristensen for kindly providing the code for estimating the bandwidth.

3.3.1 Data

Our analysis of the dynamics and determinants of industry betas is based on the 30 industry portfolios available at Kenneth French's website. We download value-weighted returns at the daily frequency as well as industry market capitalizations at the monthly frequency. We use the yield on 3-month US treasury bills to calculate excess returns. Our sample period runs from January 1969 to December 2009 and includes several economic cycles and financial crisis such as the technology boom and subsequent bust as well as the recent subprime crisis.[6]

Table 3.1 reports the average number of firms for each industry portfolio, its average market weight and the summary statistics for their returns. The last columns also report (full sample) unconditional industry betas as well as the beta ranking for each industry (1 being the industry with the highest beta, 30 with the lowest). Unconditional industry betas range from 0.59 for Utilities to 1.31 for Business Equipment. This classification shows that, apart from the Utilities sector, the lowest unconditional betas are observed for the Beer, Food and Smoke industries. Next to the Business equipment industry, we observe the highest betas for the Steel, Mines and Games industries. In the following sections, we will analyze to what extent industries change their ranking over time, either temporarily or following a structural break.

3.3.2 Estimation results for DCC-MIDAS and kernel methods

Table 3.2 reports parameter estimates for the DCC-MIDAS model. Panel A shows the parameter estimates for the variance process, corresponding to Equations (3.2-3) to (3.2-5). The ARCH and GARCH parameters (α and β) are between $0.075-0.117$ and $0.851-0.910$ respectively. Interestingly, their sum is always safely below 1, suggesting that distinguishing between different volatility components alleviates the extreme persistency problem that plagues standard GARCH models estimated over daily data The lag length obtained through likelihood profiling is either short (3 quarters) or very long (16 quarters). A longer lag length, however, does not necessarily lead to higher volatility persistence, as long lag lengths seem to be associated with rapidly decreasing weighting functions (high values for ω^v). Similarly, industries with short lag lengths typically have a rather flat weighting function (ω^v relatively close to 1). Consider for instance the Health industry, which has an optimal lag length of 16 quarters and a $\omega^v = 10.14$. It is easy to show that observations within the first three previous quarters receive a total weight of 89% while observations lagged 6 quarters or more receive less than 2%.[7]

Panel B reports the estimated parameters for the covariance equation (Eqs. (3.2-6) to (3.2-8)). Since the likelihood functions turned out to be mostly flat for alternative lag lengths, we decided to fix the number of lags for all industries to 8. This implies in turn that the persistence of the long-term covariance component will be determined by the shape of the weighting function which is in turn exclusively determined by ω^c. Not surprisingly, we find substantial differences in the shape of the weighting function, ranging from nearly flat (Services industry, $\omega^c = 1.02$) to rapidly decreasing (Other industries, $\omega^c = 9.09$). Finally, the estimated parameters also suggest that the short-run covariance is highly persistent, with values for b between 0.85 and 0.95, and estimates for a typically below 0.05.

For the kernel method, the key parameter is the bandwidth which is determined by a plug-in method as explained in Section 3.2.2. Table 3.3 reports the bandwidths for all industries and compares them to the weighting parameters obtained for the DCC-MIDAS

[6] The market-wide variables used to investigate the determinants of industry betas are described in detail in Appendix 3.A.

[7] A full comparison of the weighting schemes across industries and methods is introduced later in Table 3.3.

approach. For the kernel method, we calculate the window length as the width of the 95% confidence interval from a normal distribution given the estimated bandwidth parameter h. For DCC-MIDAS, the window lengths (separately for the volatility and correlation) correspond to the 95% cutoff point of the cumulative weighting function (given L^v, ω^v for volatilities, and L^c, ω^c for correlations). We obtain an average kernel window length of 4.08. Notice that, while there are sizable differences between the different industries, optimal window lengths are never shorter than 2 quarters. This suggests that the common approach of estimating ex-post betas over just one quarter of daily returns is, at least from a statistical point of view, not optimal. Window length is among the highest for the Clothes, Games, and Electronic equipment industries, and among the lowest for the Mining and Beer industries. While we cannot directly compare the optimal window lengths for the kernel betas with those for the DCC-MIDAS model, it is nevertheless comforting that the window lengths for volatilities and correlations are roughly of the same magnitude (average of 3.49 for volatilities and 4.53 for correlations). Moreover, the lengths obtained for the components of the DCC-MIDAS model are on average positively correlated with the optimal window lengths for kernel betas.[8]

3.3.3 Beta dynamics

The summary statistics for ex-post betas (quarterly and annual windows), DCC-MIDAS betas, and kernel betas are reported in the left panel of Tables 3.4 to 3.7, respectively. We find industry betas, irrespective of the method being used, to vary substantially over time and to be highly persistent, consistent with the findings of Ferson and Harvey (1991), Fama and French (1997), Braun, Nelson, and Sunier (1995), Santos and Veronesi (2004), and Ghysels and Jacquier (2006). Except for the quarterly ex-post betas, which are substantially more variable and less persistent, we find similar first-order autocorrelations and standard deviations in betas for all other methods. Nevertheless, we find DCC-MIDAS betas to be slightly less persistent and variable than betas estimated using the kernel method (average persistence of 0.88 versus 0.95 and average standard deviation of 0.21 versus 0.23).

The fact that betas are time-varying does not necessarily imply that the relative ranking of industry betas also changes through time. The right panel of Tables 3.4 to 3.7 clearly show, however, that not only betas but also their relative ranking fluctuates significantly over time. In fact, almost all industries had at some point in time the highest and lowest betas among all industries. To investigate this further, we calculate for each industry the percentage of time that its beta belongs to the bottom 30, mid (between 30 and 70), and top 30 percentile. This classification suggests that, as for the unconditional betas in Table 3.1, the betas of industries like Business equipment and Games belong mostly to the top beta percentile (respectively 83% and 67% of the total sample among the highest betas for Kernel betas). In contrast, Utilities, Food, and Mining industries belong mostly to the bottom 30 percentile (respectively 92%, 74% and 68% among the lowest betas). Finally, other industries like Construction, Paper and Clothes can be classified into mid beta industries (respectively 77%, 73% and 62% among the mid betas).

[8] The window lengths for the two methods are not fully comparable for several reasons. First, in the DCC-MIDAS, the length of the window is estimated using likelihood profiling. Second, in the DCC-MIDAS approach, the window length is separately estimated for the volatility and the correlations. Third, the Kernel approach considers two-sided windows while DCC-MIDAS only considers lagged information. Finally, the DCC-MIDAS approach considers together the length of the window and the relative weight of the lagged information while in the Kernel approach, the shape of the weighting function is standard (Gaussian) and the bandwidth is the only parameter that needs to be estimated.

3.3.4 Structural breaks

Over the last 40 years (the length of our sample), the global economy has changed substantially, and so have industries. Ex ante, it seems likely that the risk characteristics of certain industries have structurally changed over time, for instance because of changes in technology, regulation, or (global) competition. To investigate this, we test for multiple structural breaks in the betas of the different industries using the well-known Bai and Perron (2003) test. Then, we test for the existence of common structural breaks using the method in Qu and Perron (2007). We shortly explain both methods as well as their respective results in turn.[9]

In order to identify the existence of industry-specific structural breaks, we estimate the following model for the dynamics of each industry's beta:

$$\beta_{i,\tau} = \mu_{i,l} + u_{i,\tau},$$

where $\mu_{i,l}$ for $l = 1, ..., m+1$ are regime-dependent levels of beta within break dates $\Gamma = (T_1, .., T_m)$, and m is the optimal number of breaks. The regime-dependent levels as well as the break dates are estimated by minimizing the sum of squared residuals $\sum_{l=1}^{m+1} \sum_{\tau=T_{l-1}+1}^{T_l} [\beta_{i,\tau} - \mu_{i,,l}]^2$. The optimal number of breaks, m, is obtained by following the sequential method in Bai and Perron (2003). The idea behind this method is to sequentially identify the statistical relevance of including an additional break. This relevance is assessed by comparing the minimal value of the sum of squared residuals over all segments including the additional break with that of a restricted model without this additional break.

The left panel of Table 3.8 reports the optimal number of breaks as well as the break dates for all industries' kernel betas.[10] The right panel reports the mean beta estimates, μ_l, as well as industry ranking between break dates. We observe a number of interesting findings. First, only 9 out of 30 industries exhibit a structural break. Second, for only 1 of these 9 industries, we observe more than one break (Mining, 3 breaks). Third, except for Meals (in September 1979), Telecommunications (March 1982) and Mines (September 1979 and September 1985), all structural breaks take place in the final part of our sample. The Smoke and Beer industries report significant drops in their betas in December 1997 and September 1998 respectively (from an average 1.02 to 0.50 and from 0.97 to 0.44 respectively). In 2000, the Steel (June) and Fabricated products and machinery (December) industries show substantial increases both in their betas and relative ranking. Finally, in 2001, the Mine (September), Coal (December) and Construction (December) industries increase their average betas from an average 0.18 to 1.45, 0.95 to 1.81, and from 0.95 to 1.39 respectively. Interestingly, the 2001 break has a particularly large impact for the Mines industry whose beta shifts from almost the lowest to among the highest betas around this period (average ranking goes from 29.35 to 6.40).

The fact that we find only a limited number of break dates for just 9 industries could be the result of the low power of the Bai and Perron (2003) break test. In order to increase the power, we also test for the presence of a common break test using the methodology in Qu and Perron (2007). We believe that a common break test also makes more economic sense because a structural break in the beta of one industry is expected to also lead to changes in the betas of other industries, simply because of the implicit identity that the market-weighed

[9] The detailed description as well as the distribution of the tests can be found in Bai and Perron (2003) and Qu and Perron (2007). The Gauss code used to identify structural breaks can be found in Professor Perron's webpage.

[10] In our analysis, we focus on the kernel betas since testing for structural breaks in DCC-MIDAS betas can be problematic as these betas are generated from a model that is assumed stationary in the first place. As for the ex-post betas, the number of breaks obtained turns out to be very large for almost all industries. Nevertheless, in almot every case, the quarterly betas breaks include those identified for the kernel betas.

sum of betas needs to be one. In order to identify common structural breaks, we estimate the following model for the dynamics of all industries' beta:

$$\boldsymbol{\beta}_\tau = x_\tau \boldsymbol{\mu}_l + \mathbf{u}_\tau, \tag{3.3-9}$$

where $\boldsymbol{\beta}_\tau = \beta_{i,\tau}$ contains the estimated betas for all industries, x_τ is an identity matrix which reflects the idea that the regime-dependent levels of beta, $\mu_{i,l}$, are allowed to change within each regime for each industry. The break dates as well as the regime-dependent levels of beta are obtained by following the restricted quasi-maximum likelihood method described in Qu and Perron (2007).

Similar to Table 3.8, Table 3.9 reports the mean beta estimates, μ_l, as well as industry ranking within regimes. Even when we chose Qu and Perron (2007)'s method specifically for its capacity to have multiple common structural breaks, we eventually had to restrict the number of breaks to one, as the common break algorithm failed to converge to a fixed number of breaks.[11] Again, we report results for the kernel betas even though results are robust to using the other measures. The algorithm identifies a common structural break in March 1998. That is, around the start of the dot-com bubble period. Not surprisingly, around this structural break, changes in the level of betas for those industries for which individual structural breaks are identified have the same magnitude. However, other industries such as Household, Health, and Meals also report significant changes in their betas around this structural break (from an average 1.09 to 0.61, 1.09 to 0.74 and from 1.19 to 0.77 respectively). For all other industries, changes in the level of betas are rather moderate.

3.3.5 Industry cyclicality

From an investor's perspective, it is important to understand how the market exposure of industries changes over the business cycle.[12] Therefore, in the last part of this section, we motivate the understanding of the determinants of industry betas and their cross-sectional dispersion by identifying those industries that either have significantly higher or lower market betas during economic recessions. We investigate this by regressing our different beta measures on a constant and two general business cycle indicators: a NBER dummy and the Chicago Fed National Activity Index ($cfnai$).[13] Apart from the quarterly ex-post, DCC-MIDAS, and kernel beta, we also consider kernel betas corrected for individual and common structural breaks and denote it by **dikbetas** and **dckbetas** respectively. More specifically, for the case of individual structural breaks, for those industries exhibiting structural breaks, we de-mean betas using the beta estimates between break dates (see Table 3.8). In a similar way, for the case of a common structural break, we de-mean all betas using the beta estimates before and after the structural break date (March 1998. See Table 3.9).

Table 3.10 reports estimates of the change in market beta during NBER recessions and with respect to $cfnai$ for all industries in Panel A and B respectively. First, we observe a (often only borderline) significant increase in market betas during recessions (or alternatively drops in the activity index) for the Chemicals, Steel, Fabricated Products, Mine, and Financial industries. In contrast, we observe a significant decrease during recessions in betas for the Smoke, Health, Electronic Equipment, and Retail Trade industries. In line with

[11] More specifically, we always find it optimal to add another break, and break dates are evenly spaced over the sample as soon as the number of breaks is equal or lager than 3.

[12] Other studies that have studied differences in cyclicality among industries include Bernanke and Parkinson (1991), Petersen and Strauss (1991), and especially Boudoukh, Richardson, and Whitelaw (1994). Gomes, Kogan and Zhang (2003), Gourio (2007), and Santos and Veronesi (2004), among others present theoretical models for the heterogeneous reaction of equity portfolios (not necessarily industries) to business cycle conditions.

[13] We do the same for the cross-sectional dispersion of industry betas. Although reported in this table, these results are explored in Section 3.4.2.

Boudoukh, Richardson, and Whitelaw (1994), we also find that all other industries, and in particular, those related to necessities appear to be less affected by the business cycle as it is the case for the Food, Clothing and Paper industries. Second, we find that accounting for individual structural breaks reduces the magnitude and in some cases the significance of the NBER dummy and $cfnai$. As it is to be expected, this effect is particularly strong for the mining industry, the only sector for which we found three break dates. In contrast, considering a common structural break does not have a significant impact on the direction or the significance of the relation between most industry betas and the two key business cycle indicators considered.

3.4 The Determinants of Industry Betas

In the previous section, we show that betas not only vary substantially over time, but also that there is substantial heterogeneity in their dynamics across industries. We also provide preliminary evidence that while the betas of some industries increase during recessions, those of others decrease, and some are just not significantly related to the business cycle. In this section, we investigate these findings further by linking industry betas to a set of variables that characterize the economic conditions. In the first part, we review the rather thin theoretical and empirical literature on the determinants of industry betas. The second part links the alternative estimates of industry betas to a set of market-wide variables. Following Fama and French (1997), Ferson and Harvey (1991, 1993 and 1999),Lewellen and Nagel (2006), Ghysels and Jacquier (2006), and Gourio (2007), we argue that (potentially time-varying) differences in industry characteristics are expected to lead heterogeneous reactions of industry market betas to market-wide events. Heterogeneity of characteristics and reactions will in turn explain the time variation in the cross-sectional dispersion in industry betas. Therefore, we also confront the contrasting model implications and empirical findings in Gomes, Kogan and Zhang (2003) and Santos and Veronesi (2004) and investigate the relation between the cross-sectional dispersion of industry betas and the business cycle. Finally, we test whether the cross-sectional dispersion in industry betas has predictive power for equity returns, and whether or not this relation explains the predictive power of return dispersion documented in Stivers and Sun (2010).

3.4.1 Existing models and previous empirical findings

Despite the CAPM being one of the cornerstones of modern finance, the theoretical literature on the determinants of time-varying market betas is surprisingly thin, and often makes conflicting predictions. Existing models propose alternative sources of heterogeneity across firms or portfolios (e.g., industries, size, book-to-market) to explain different reactions of betas to aggregate macroeconomic conditions. For instance, in Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004), the risk of assets in place as well as firms' investment decisions determine the time variation in systematic risk. In a similar vein, Gomes, Kogan and Zhang (2003) argue that differences in size and growth opportunities determine firm-specific reactions to aggregate productivity. In Gourio (2007), heterogeneous productivity and labor leverage levels as well as growth opportunities yield different reactions to the business cycle. Jacquier, Titman and Yalcin (2010) show that operating leverage is a more important determinant of systematic risk than financial leverage. In case of good news, the increase in the value of growth options is typically larger than the increase in the value of assets in place. Because growth options have larger betas, good (bad) news leads to increasing (decreasing) betas, which is opposite to what the financial leverage predicts. Finally, Santos and Veronesi (2004) model firms' cash flow risk and (asset's) duration to explain the relation between betas and aggregate variables.

Interestingly, some of these theoretical models explicitly yield (conflicting) predictions with respect to the dynamics of the cross-sectional dispersion in betas. On the one hand, in Gomes, Kogan and Zhang (2003), heterogeneity across firms increases during recessions, leading to increasing dispersion in betas. This prediction is in line with the findings of Chan and Chen (1988) for size-sorted portfolios. On the other hand, Santos and Veronesi (2004) argue that the relation between beta dispersion and the business cycle depends on the relative importance of cash flow and discount rate risk. If cash flow risk is negligible compared to discount rate risk, their model predicts the cross-sectional dispersion in conditional betas to be higher when the aggregate market risk premium is high (which is the case in recessions). Intuitively, when the market risk premium is high, differences in current expected dividend growth matter more in determining differences in asset valuation. This leads to a wide dispersion of price sensitivity to changes in discount rates, and hence more dispersed betas. In contrast, when cash flow risk is more important than discount rate risk, the model predicts beta dispersion to be lower (higher) when the market risk premium is high (low). Because an important part of the systematic volatility of assets with high cash flow risk is rather insensitive to changes in discount rates, market betas will be scaled up (down) by the low (high) market volatility in case of a low (high) market risk premium. Santos and Veronesi (2004) find empirical support for the latter, namely that the cross-sectional dispersion in betas is negatively related to the market risk premium, and hence also to the business cycle.

While there are several empirical papers linking conditional market betas to fundamentals, most of this work focuses on size and book-to-market sorted portfolios rather than industries (see, e.g., Jagannathan and Wang (1996), Zhang (2005), Lettau and Ludvigson (2001), Petkova and Zhang (2005), Lewellen and Nagel (2006), and Jacquier, Titman and Yalcin (2010)). There are, nonetheless, a few exceptions. For instance, as already mentioned, Santos and Veronesi (2004) focus on industry betas. However, their interest is mainly centered in testing the implications of their model for the cross-sectional dispersion in betas rather than investigating the determinants of time-varying industry betas. Also, Ghysels and Jacquier (2006) develop a new approach for estimating industry market betas by combining data-driven filters and parametric models that should be less vulnerable to measurement errors. While they find industry betas to be strongly autocorrelated, they do not find any evidence that quarterly betas are related to either aggregate (term and default spread, short rate, dividend yield) or firm-specific (market value of equity, debt-to-equity ratio, book-to-market ratio) variables.

3.4.2 Empirical evidence: industry betas and cross-sectional dispersion

As discussed above, the empirical evidence on the determinants of industry betas is limited, while theory often yields conflicting results, in particular with respect to the link between beta dispersion and the business cycle. Therefore, in this part of the section, we first investigate to what extent aggregate business cycle proxies explain the time variation in one-period ahead quarterly betas.[14] Then, we investigate the link between this set of aggregate variables and the cross-sectional dispersion in industry betas.

Table 3.11 reports estimation results from the following regression of conditional market betas on each of the lagged market-wide variables $x_{\tau-1}$:

$$\beta_{i,\tau} = \gamma_{0,i} + \gamma_{1,i} x_{\tau-1} + \epsilon_{i,\tau},$$

for four alternative market beta measures: **qbetas**, **mbetas**, **kbetas, dikbetas** and **dck-betas**. In order to save space, we only report for each instrument the average R-squared

[14] Details on our wide list of business cycle instruments can be found in Appendix 3.A.

(over all industries), an F-test ("F1") for the joint significance of a particular instrument across all industries, and, finally, a test for equal exposures ("F2"). We find strong evidence that industry betas are related to the different business cycle proxies, and that they react in a heterogeneous way. In fact, from the 18 proxies considered, only three (the liquidity measure *vol*, the corporate profits growth $cprofg$, and the return of the small-minus-big investment strategy *smb*) do not appear to be related to one-quarter ahead industry betas. We find similar results when we use the other beta measures. In contrast, the price-dividend ratio *pdratio* (18.55% for **mbetas**), the short-term risk free interest rate $3months$ (7.12%), two of the inflation measures $cpinf$ and *gdpd* (5.49% and 9.46% respectively) and an economic activity measure *empg* (4.94%) display the highest average R-squared among all variables. For other variables such as the monetary base growth *mbg* (1.88%), and two economic activity measures *ipg* and *gdpg* (2.22% and 2.28% respectively) the average R-squared is rather low. Interestingly, in all but two cases, the average R-squared is higher for DCC-MIDAS and Kernel Betas than for the ex-post quarterly betas, most likely because these two approaches lead to less noisy beta estimates.

With respect to the cross-sectional dispersion, different theoretical models lead to different predictions about the sensitivity of beta dispersion to the business cycle. Gomes, Kogan and Zhang (2003) predict beta dispersion to be higher during recessions, while both the theoretical predictions and empirical findings of Santos and Veronesi (2004) suggest the opposite. The last row of Table 3.10 shows the results from the *contemporaneous* regression between the cross-sectional dispersion in industry betas and the two general recession indicators *nber* and $cfnai$. The cross-sectional dispersion in betas is measured as

$$disp_\tau = \sqrt{\sum_i \omega_{i,\tau-1}(\beta_{i,\tau} - \sum_i \beta_{i,\tau})^2}, \tag{3.4-10}$$

where ω_i are industry market capitalizations (averaged over monthly data). Independent of the method used to measure betas, we find dispersion to be positively related to the NBER recession dummy and negatively to the $cfnai$ index. This finding is in line with the theoretical predictions from the model of Gomes, Kogan and Zhang (2003). That is, industry characteristics become more heterogeneous around recessions, thus leading to increasing dispersion in betas. However, we find that except for the betas corrected for structural breaks, the relation is, at best, borderline significant depending on the beta measure used.

Table 3.12 reports estimation results from regressing the cross-sectional dispersion in industry betas on the different lagged market-wide variables x_{t-1}. As for the contemporaneous effect, we find the one-period ahead dispersion in betas to mostly increase with negative information about the business cycle, even if estimates are mostly only borderline significant. We find, in particular, $cfnai$, *gdpd*, *empg*, *pd*, *divg*, and $3months$ to have a significant effect on the one-quarter ahead dispersion of **qbetas**. Among all regressions, the R-squared turns out to be particularly high for the *pd* ratio (13.85% for **qbetas**). However, we find an increase in the PD ratio to predict an increase rather than a decrease in the cross-sectional dispersion of betas. In unreported results, however, we find that both the positive sign of this relation as well as its significance disappear once we take out the period corresponding to the buildup and burst of the Technology, Media, and Telecom (TMT) bubble, a period during which the *pd* rapidly rose to and then dropped from unprecedented levels.

While the sign of the predictive relations is typically not affected by the choice of the beta proxy, we do find a marked increase in the number of significant relations when we calculate dispersion based on betas corrected for structural breaks (**dikbetas** and **dckbetas** for individual and common structural breaks respectively). In this case, the dispersion in betas increases following a decrease in economic activity characterized by $cfnai$, *empg*, *gdpg* and *ipg*. It also increases following an increase in inflation measured by *mbg* or a reduction in aggregate corporate profits growth $cprofg$, consumption growth *rcg* or the

1-year term spread $tsp1$. Notice that, apart from the improved statistical significance, we also observe larger R^2's and larger estimated coefficients. An interesting exception is the pd ratio: Once corrected for (individual or common) structural breaks, we find it to loose some of its significance and magnitude.

3.4.3 Dispersion in betas as a predictor of future equity returns

Having shown that beta dispersion is negatively related with the business cycle, we next investigate whether it has predictive power for overall market returns. This relates to a recent study by Stivers and Sun (2010). They show that the cross-sectional dispersion in equity returns is positively related to the subsequent value premium, and negatively to the subsequent momentum premium. Their dispersion measure differs from ours and is calculated from the 100 size and book-to-market portfolios. As a first step, we investigate to what extent dispersion in industry returns predicts returns on the value-weighted US market index. Second, we test whether its forecasting performance can be better understood by focusing precisely on the systematic component of return dispersion, namely beta dispersion. Third, given the structural breaks identified, we check whether or not our results and those of Stivers and Sun (2010) are driven by crisis periods (in particular the dot-com bubble and the subprime crisis[15]).

Table 3.13 reports the estimation results for the following regressions:

$$(r - r_f)_{\tau,\tau+h} = \gamma_{0,h} + \gamma_{1,h} disp_\tau + \gamma_{2,h} \widehat{u_\tau} + \epsilon_{j,\tau}, \tag{3.4-11}$$

where $(r-r_f)_{\tau,\tau+h}$ represents the return on the value-weighted annualized excess US market returns h-quarters ahead, $disp_\tau$ is the cross-sectional dispersion in industry betas for **qbetas** and **mbetas** in Panel A and B respectively, and $\widehat{u_\tau}$ is the component of cross-sectional dispersion in industry returns that is orthogonal to the dispersion in betas. $\widehat{u_\tau}$ is calculated as the error term from the following auxiliary regression:

$$disp\_ret_\tau = \kappa_0 + \kappa_1 disp_\tau + u_{j,\tau},$$

where $disp\_ret_\tau$ is the value weighted cross-sectional dispersion in industry returns calculated using an equation equivalent to (3.4-10).[16]

Panels A and B of Table 3.13 yield a number of interesting findings. First, both for the ex-post and MIDAS methods, we find the cross-sectional dispersion in betas to significantly predict a drop in the equity market up to one and a half years ahead. Second, we find the effect to be economically meaningful. For instance, again at the one and a half year horizon, a 1 (percentage) point increase in the dispersion in MIDAS betas leads to a 1.7 percent drop in the return of the index. Third, we do find the orthogonalized return dispersion measure to be significant at intermediate horizons. In contrast, for very short or long horizons, return dispersion looses its forecasting power once beta dispersion is introduced. Fourth, we find R-squareds to increase with the prediction horizon, and to be systematically higher for the MIDAS-based dispersion measure. At the one and a half year horizon, the R-squared reaches a respectable 8.3 percent (if only the cross-sectional dispersion in betas is included).

Table 3.14 runs the same forecasting regressions over two different subsamples, namely from 1969 until 1997, and from 1998 till the end of our sample in 2009. The latter sample includes both the TMT bubble period and the recent subprime crisis. Notice, also, that the common structural break as well as most of the individual breaks that we identify are located in the second subsample. We use the quarterly ex-post betas, as the MIDAS beta estimates

[15] The sample in Stivers and Sun (2010) runs up to 2005 only, and hence does not include the subprime crisis period.

[16] See how if the conditional CAPM with one factor holds the cross-sectional distribution of returns would be entirely determined by the distribution of betas.

may be vulnerable to look-ahead bias in subsamples. We find that the predictive power for both the return and beta dispersion measures is completely absent in the first subsample. In other words, all the predictive power is concentrated in the crisis-rich last decade. In fact, we even find this striking difference between the two subsamples after excluding the subprime crisis from the second subsample (which would make our sample more similar to the one in Stivers and Sun (2010)).

It is not clear whether the instability in the predictive power of both return and beta dispersion between different subsamples necessarily means both predictors are useless. It may just mean that the cross-sectional dispersion in industry characteristics and/or business cycle exposure have increased, and that this has made both dispersion measures more informative. We leave the investigation of this hypothesis for future research.

3.5 Conclusions

In this paper, we make an anatomy of industry betas. While it has proven difficult to link industry betas to industry expected returns (see, e.g., Fama and French (1997)), a good understanding of both the level and dynamics of industry betas is important in practice, for instance, as inputs in risk management and asset allocation models.

In a first part, we analyze two alternative beta measurement techniques for the standard ex-post moving window beta estimates, namely the DCC-MIDAS model of Colacito, Engle, and Ghysels (2009) and the kernel method recently refined by Ang and Kristensen (2010). While the optimal window length implied by both methods differs between industries, it is always larger than the one quarter window typically used to estimate ex-post betas. While industry betas are highly persistent, we do find them to vary substantially over time, both in absolute term and relative to each other. While nearly all industries had at some point the highest (lowest) beta, we find industries like Business equipment and Games to predominantly belong to the group of industries with the 30 percent highest betas, while Utilities, Food, and Mining industries belong mostly to the bottom 30 percentile.

Over our sample of more than 40 years, it is not unlikely that the very nature of some industries has changed over time, following changes in technology (e.g., the telecom sector) or regulation (e.g., the banking sector). We test for this possibility by performing an individual as well as a common structural break test on the industry betas. Overall, we find the individual number of breaks to be small (only 9 out of 30 industries, and mostly just one break) and to be clustered in the period surrounding the Technology, Media and Telecom bubble. In fact, when we impose a unique common structural break, where the break in the beta of one industry is expected to have an effect on the betas of all other industries, this break is identified in March 1998 around the beginning of the Technology bubble.

Consequently, we investigate whether the betas of different industries have different sensitivities to the business cycle. We find a systematic increase during recessions in the betas of Chemicals, Steel, Fabricated Products, Mine, and Financial industries, and a systematic decrease in the betas of Smoke, Health, Electronic Equipment, and Retail Trade industries. All other industry betas do not appear to be contemporaneously correlated with either an NBER dummy or the $cfnai$ leading indicator. Next, we show that industry betas are significantly related to a large set of lagged cyclical variables, but in a heterogeneous way. Interestingly, in all but two cases, the average R-squared is higher for DCC-MIDAS and Kernel Betas than for the ex-post quarterly betas, precisely because these two approaches lead to less noisy beta estimates.

The final set of results focuses on the dynamics of the cross-sectional dispersion in industry betas, and its predictive power for future returns. The existing theoretical literature has come up with different predictions about the cyclicality of beta dispersion. Gomes, Kogan and Zhang (2003) predict beta dispersion to be higher during recessions, while both the theoretical predictions and empirical findings of Santos and Veronesi (2004) suggest the opposite. Our results are in line with the predictions of Gomes, Kogan and Zhang (2003). That is, we find beta dispersion to be higher in recessions than in expansions. The natural next question is whether or not beta dispersion can also predict equity returns. This relates to a recent study by Stivers and Sun (2010), who show that the cross-sectional dispersion in *returns* is positively related to the subsequent value premium, and negatively to the subsequent momentum premium. In this paper, we assess the relative forecasting power of industry return dispersion and one of its components, industry beta dispersion. Our results suggest that industry beta dispersion is a stronger predictor of future market returns than the non-systematic component of industry return dispersion. However, we also find that the predictive power of both variables is centered in the last crisis-rich decade of our sample.

APPENDIX

3.A Market-wide Variables

Our dataset consists of several quarterly variables that characterize the business cycle. Our sample period runs from the fourth quarter of 1969 to the fourth quarter of 2009 (162 observations in total). We describe the exact data sources and how the variables are constructed in turn.

1. NBER recession indicator (*nber*): From the National Bureau of Economic Research. The indicator takes the value 1 if at least two months within that quarter are marked as recessions.

2. CFNAI MA3 (*cfnai*): National Activity Index from the Federal Reserve Bank of Chicago. This variables is constructed as the previous three months average of the weighted average of 85 monthly indicators of national economic activity.

3. Volume traded growth (*vol*): Log difference in the total volume traded for each quarter (sum of daily data) for the S&P 500, from Yahoo! Finance.

4. Monetary base growth (*mbg*): Log difference in the Federal Reserve Board of Governors monetary base adjusted for changes in reserve requirements, from the Federal Reserve Bank of St. Louis.

5. PPI growth (*ppinf*): Log difference in the Producer Price Index for All Urban Consumers (finished goods), from the Bureau of Labor Statistics.

6. CPI growth (*cpinf*): Log difference in the Consumer Price Index for All Urban Consumers (all goods), from the Bureau of Labor Statistics.

7. GDP deflator (*gdpd*): Log difference in the Gross Domestic Product implicit price deflator, from the Bureau of Economic Analysis.

8. Employment growth (*empg*): Log difference in the total employment (manemp), from the Bureau of Labor Statistics.

9. GDP growth (*gdpg*): Log difference in the Real Gross Domestic Product (3 decimals), from the Bureau of Economic Analysis.

10. IP growth (*ipg*): Log difference in the Industrial Production growth, from the Board of Governors of the Federal Reserve.

11. Price Dividend ratio (*pd*): three months average of the price dividend ratio, from Robert Schiller´s website. The PD ratio is calculated as $pd_t = p_t - d_{t-1}$, where p_t is the log of the (closing) price of the S&P 500 index and d_{t-1} is the log of the dividends accruing to the index paid out throughout period $t-1$.

12. Dividends growth (*divg*): Log difference in the real dividends accruing to the S&P 500 index, from Robert Schiller´s website.

13. Corporate Profits growth (*cprofg*): Log difference of the Corporate Profits with inventory Valuation adjustment (IVA) and capital consumption adjustment (CCAdj), from the Bureau of Economic Analysis.

14. Real Consumption growth (*rcg*): Log difference in the Real Consumption growth, from the Bureau of Economic Analysis.

15. 3-month Tbill (3*months*): 3-months Treasury bill (secondary market) rate, from the Federal Reserve.

16. Term Spread (*tsp*1): Difference between the 1 year Treasury bill (secondary market) rate and 3months, from the Federal Reserve.

17. Default Spread (*dsp*): Quarterly average of the credit spread expressed as the difference between the average rate for Moody's Baa-rated firms' bonds and AAA-rated firms' bonds.

18. Small minus Big strategy (*smb*): Annualized compounded quarterly return of the small minus big portfolios using monthly data, from Kenneth French´s website.

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Table 3.1: Fama and French's 30 industry portfolios. Summary statistics

The table reports the industry classification as well as the main characteristics for the 30 industry portfolios considered. It also reports summary statistics for the daily returns. Finally, the last two columns of the table report unconditional betas as well as their ranking. The ranking classifies industry betas from highest (1) to lowest (30). The sample period runs from January 1969 to December 2009.

	Short	(Avg) Characteristics		Daily excess return (annualized in %)				Unconditional Betas	
Industry	**Name**	**Num. Firms**	**Market Cap (%)**	**Mean**	**Stdev**	**Kurt**	**Skew**	**Beta**	**Rank**
Food	Food	109.99	3.42	12.22	17.19	21.54	−0.44	0.71	28
Beer	Beer	15.62	1.18	13.53	21.95	9.95	−0.04	0.70	29
Smoke	Smke	7.93	0.97	17.73	26.80	13.07	0.00	0.72	27
Games	Game	102.31	1.13	11.18	28.03	13.86	−0.38	1.16	4
Books	Book	60.25	1.11	9.16	21.13	25.31	0.08	0.91	19
Household	Hshl	101.88	4.61	9.98	20.97	26.00	−0.83	0.84	25
Clothes	Clth	74.16	0.53	10.41	22.73	15.31	−0.27	0.95	15
Health	Hlth	319.29	7.53	12.01	20.53	14.84	−0.38	0.89	21
Chemicals	Chem	83.76	3.54	9.48	22.52	15.84	−0.39	1.00	11
Textiles	Txtl	45.26	0.30	8.88	24.07	25.17	0.37	0.88	23
Construction	Cnst	182.26	2.01	9.08	22.43	16.90	−0.33	1.03	10
Steel	Stel	77.81	1.63	7.48	28.85	22.08	−0.35	1.24	2
Fabricated products	Fpro	183.35	2.42	9.60	23.37	17.30	−0.37	1.11	8
Electrical Equipment	Eeqp	68.53	1.39	14.47	25.68	13.22	−0.26	1.13	6
Autos	Auto	69.82	3.48	7.45	26.74	13.02	−0.15	1.11	7
Carry	Carr	36.03	1.22	12.46	24.09	11.65	−0.28	0.99	13
Mines	Mine	44.05	0.68	11.32	29.66	14.11	0.02	0.78	26
Coal	Coal	7.36	0.14	19.69	40.60	13.22	0.01	1.22	3
Oil	Oil	180.33	10.28	13.18	24.27	20.56	−0.15	0.92	18
Utilities	Utlt	158.23	6.57	7.97	15.66	30.32	0.09	0.59	30
Telecommunication	Tele	83.19	6.43	8.23	21.04	18.86	−0.06	0.90	20
Personal and Bus. services	Psrv	375.89	3.94	11.67	24.56	11.90	−0.17	1.14	5
Business Equipment	Beqp	462.18	9.38	10.50	29.19	12.24	0.12	1.31	1
Paper	Papr	78.08	2.47	9.37	20.03	23.16	−0.74	0.89	22
Transportation	Tran	104.03	1.86	9.24	22.94	12.25	−0.34	1.00	12
Wholesale trade	Wtrd	157.71	0.98	10.72	20.08	10.55	−0.31	0.87	24
Retail trade	Rtrd	234.29	5.46	11.40	21.84	14.17	−0.17	0.98	14
Meals	Meal	78.83	0.80	13.29	24.15	9.81	−0.17	0.94	16
Financial	Fina	771.95	12.35	10.36	23.02	26.62	0.09	1.09	9
Others	Othr	230.91	2.17	5.28	21.92	16.23	−0.35	0.92	17
Market		4505.43	100.00	7.21	18.58	20.72	−0.55		

Table 3.2: DCC-MIDAS betas. Estimated parameters

The table reports estimated parameters for the DCC-MIDAS method. Panel A reports the parameters for the first step of the estimation procedure in Eqs. (3.2-3) to (3.2-5). The maximum lag for the dynamics of the volatility is obtained by evaluating the value of the maximum-likelihood function for alternative horizons up to a maximum of 16 quarters. The (semi) optimal length is that for which the value of the likelihood function becomes flat. Panel B reports the parameters for the second step in Eqs. (3.2-6) to (3.2-8).

Panel A. MIDAS-GARCH (Step 1)

	Food	Beer	Smke	Game	Book	Hshl	Clth	Hlth	Chem	Txtl	Cnst	Stel	Fpro	Eeqp	Auto
L^v	3	16	16	3	16	3	16	16	3	16	3	3	3	16	3
β	0.880	0.867	0.844	0.833	0.851	0.895	0.861	0.880	0.879	0.797	0.847	0.852	0.836	0.910	0.880
	(0.010)	(0.015)	(0.030)	(0.017)	(0.022)	(0.011)	(0.018)	(0.012)	(0.013)	(0.028)	(0.014)	(0.021)	(0.018)	(0.010)	(0.013)
α	0.090	0.085	0.089	0.117	0.111	0.078	0.100	0.089	0.082	0.138	0.114	0.108	0.115	0.066	0.080
	(0.011)	(0.011)	(0.016)	(0.017)	(0.024)	(0.013)	(0.017)	(0.013)	(0.011)	(0.023)	(0.015)	(0.022)	(0.017)	(0.011)	(0.012)
μ	0.042	0.049	0.055	0.065	0.040	0.045	0.045	0.045	0.039	0.038	0.047	0.029	0.053	0.059	0.036
	(0.006)	(0.009)	(0.011)	(0.011)	(0.009)	(0.008)	(0.009)	(0.009)	(0.008)	(0.009)	(0.008)	(0.010)	(0.008)	(0.011)	(0.010)
m	0.269	0.369	0.554	0.869	0.469	0.724	0.552	0.655	0.327	0.450	0.334	0.694	0.414	0.839	0.618
	(0.054)	(0.085)	(0.144)	(0.143)	(0.095)	(0.136)	(0.115)	(0.118)	(0.068)	(0.107)	(0.083)	(0.156)	(0.076)	(0.174)	(0.109)
γ	0.011	0.011	0.012	0.009	0.010	0.005	0.009	0.006	0.011	0.011	0.012	0.010	0.010	0.007	0.010
	(0.002)	(0.001)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)
ω^v	1.594	12.491	18.807	2.904	11.940	1.154	9.699	10.140	1.064	14.391	1.055	1.058	1.738	6.106	1.354
	(0.590)	(5.418)	(7.770)	(0.956)	(5.704)	(0.139)	(4.060)	(7.028)	(0.064)	(5.871)	(0.050)	(0.055)	(0.667)	(4.087)	(0.557)

Panel B. DCC (Step 2)

	Food	Beer	Smke	Game	Book	Hshl	Clth	Hlth	Chem	Txtl	Cnst	Stel	Fpro	Eeqp	Auto
L^c	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
b	0.945	0.967	0.955	0.92	0.928	0.948	0.938	0.913	0.938	0.924	0.944	0.922	0.92	0.946	0.951
	(0.007)	(0.002)	(0.004)	(0.011)	(0.006)	(0.006)	(0.009)	(0.009)	(0.007)	(0.012)	(0.009)	(0.01)	(0.009)	(0.008)	(0.013)
a	0.036	0.023	0.026	0.039	0.045	0.033	0.031	0.056	0.039	0.03	0.033	0.047	0.048	0.03	0.024
	(0.006)	(0)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	(0.006)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.008)
ω^c	3.465	2.51	1.542	4.003	4.744	3.769	2.401	4.289	10.63	1.933	2.798	5.413	5.197	3.737	4.021
	(2.185)	(1.093)	(0.453)	(1.106)	(3.926)	(1.348)	(1.319)	(1.366)	(4.442)	(0.465)	(2.468)	(2.473)	(1.32)	(1.636)	(0.752)

Table 3.2: DCC-MIDAS betas. Estimated parameters. Continued

Panel A. MIDAS-GARCH (Step 1)

Ind	Carr	Mine	Coal	Oil	Utlt	Tele	Psrv	Beqp	Papr	Tran	Wtrd	Rtrd	Meal	Fina	Othr
L^v	16	16	3	16	3	3	3	16	3	3	3	3	16	3	16
β	0.875	0.880	0.863	0.922	0.760	0.852	0.838	0.896	0.896	0.857	0.850	0.883	0.884	0.862	0.844
	(0.017)	(0.011)	(0.017)	(0.018)	(0.030)	(0.018)	(0.013)	(0.012)	(0.013)	(0.020)	(0.013)	(0.010)	(0.011)	(0.016)	(0.020)
α	0.081	0.088	0.085	0.066	0.179	0.092	0.114	0.077	0.078	0.100	0.103	0.087	0.086	0.110	0.101
	(0.014)	(0.011)	(0.011)	(0.028)	(0.027)	(0.015)	(0.012)	(0.014)	(0.016)	(0.019)	(0.011)	(0.011)	(0.012)	(0.020)	(0.015)
μ	0.050	0.032	0.037	0.049	0.025	0.029	0.063	0.040	0.042	0.051	0.052	0.042	0.063	0.049	0.040
	(0.011)	(0.011)	(0.013)	(0.010)	(0.005)	(0.008)	(0.009)	(0.012)	(0.008)	(0.010)	(0.008)	(0.008)	(0.010)	(0.007)	(0.009)
m	0.585	0.319	0.499	0.585	0.075	0.297	0.650	0.963	0.486	0.633	0.399	0.502	0.896	0.328	0.328
	(0.118)	(0.116)	(0.142)	(0.411)	(0.019)	(0.057)	(0.102)	(0.205)	(0.100)	(0.117)	(0.073)	(0.093)	(0.166)	(0.088)	(0.084)
γ	0.010	0.015	0.014	0.011	0.017	0.011	0.008	0.008	0.008	0.009	0.009	0.009	0.007	0.012	0.012
	(0.002)	(0.002)	(0.001)	(0.009)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.003)	(0.002)
ω^v	13.299	6.867	1.260	1.465	2.228	2.351	1.514	11.926	1.063	1.033	1.088	1.054	10.980	1.101	14.108
	(4.845)	(2.399)	(0.556)	(0.957)	(0.533)	(0.730)	(0.617)	(5.199)	(0.079)	(0.030)	(0.211)	(0.053)	(5.574)	(0.186)	(4.269)

Panel B. DCC (Step 2)

	Carr	Mine	Coal	Oil	Utlt	Tele	Psrv	Beqp	Papr	Tran	Wtrd	Rtrd	Meal	Fina	Othr
L^c	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
b	0.926	0.941	0.95	0.89	0.86	0.943	0.953	0.942	0.943	0.939	0.946	0.924	0.916	0.903	0.902
	(0.011)	(0.009)	(0.007)	(0.019)	(0.02)	(0.008)	(0.006)	(0.004)	(0.008)	(0.009)	(0.011)	(0.01)	(0.008)	(0.01)	(0.024)
a	0.04	0.03	0.029	0.06	0.07	0.031	0.024	0.038	0.036	0.04	0.027	0.045	0.047	0.055	0.045
	(0.005)	(0.007)	(0.004)	(0.009)	(0.008)	(0.005)	(0.004)	(0.003)	(0.005)	(0.006)	(0.006)	(0.005)	(0.005)	(0.005)	(0.007)
ω^c	4.3	3.736	3.709	4.079	3.241	1.892	1.022	1.878	9.28	3.655	3.255	4.576	2.476	8.533	9.086
	(1.547)	(1.723)	(1.539)	(1.133)	(0.997)	(1.039)	(0.084)	(0.721)	(4.113)	(4.221)	(1.256)	(1.702)	(0.89)	(2.018)	(3.224)

Table 3.3: Estimated bandwidths

The table reports the horizon (in quarters) at which the value of the cumulative distribution function for the weighting schemes reaches 95%. For the Kernel, this value is obtained as

$$h_j \frac{1}{T} * 2 * \frac{1.96}{0.975},$$

where h_j is the optimal bandwidth for each industry and T the total number of quarters. The intervals $(-1.96, 1.96)$ and $(-0.975, 0.975)$ correspond to cumulative 95% probabilities for the normal and uniform kernel respectively (Ang and Kristensen (2010)). For DCC-MIDAS, we report the comparable bandwidth as the (95%) value for the weighting function (for volatility and correlation) in Eq. (3.2-5). For instance, for the volatility of the food industry, a maximum number of lags of $\widehat{L} = 16$ quarters and a weighting parameter of $\widehat{\omega} = 13.29$ implies that 95% of the observations required to estimate it are within the previous 2.53 quarters.

	Kernel	**DCC-MIDAS**	
Industry		**Volatility**	**Correlations**
Food	2.69	2.53	4.62
Beer	2.48	3.42	5.58
Smke	4.01	2.37	6.85
Game	7.13	1.92	4.21
Book	2.81	3.56	3.75
Hshl	4.20	2.76	4.39
Clth	9.05	4.26	5.70
Hlth	5.90	4.10	4.02
Chem	3.55	2.81	1.97
Txtl	3.66	3.02	6.29
Cnst	2.95	2.81	5.26
Stel	2.76	2.81	3.40
Fpro	5.84	2.45	3.51
Eeqp	7.44	6.21	4.42
Auto	4.17	2.67	4.19
Carr	3.75	3.23	4.02
Mine	2.29	5.66	4.42
Coal	4.67	2.70	4.43
Oil	4.57	13.95	4.16
Utlt	2.72	2.21	4.83
Tele	2.98	2.16	6.36
Psrv	4.62	2.57	7.56
Beqp	4.07	3.56	6.37
Papr	3.63	2.81	2.21
Tran	3.21	2.83	4.48
Wtrd	3.36	2.80	4.81
Rtrd	4.78	2.81	3.84
Meal	2.81	3.83	5.61
Fina	2.78	2.78	2.37
Othr	3.60	3.07	2.26

Table 3.4: Quarterly window ex-post betas. Summary statistics and ranking

The table reports summary statistics for quarterly betas obtained with the ex-post approach and a non-overlapping quarterly window. The ranking is obtained as follows: first, in every period, we rank betas from highest (1) to the lowest (30). Then, we classify each industry into highest (h.betas or 30^{th} highest percentile), mid (m.betas or between the 31^{st} and the 69^{th} percentile) or lowest (l.betas or lower 30^{th} percentile) beta. Finally, we estimate and report the percentage of periods in which each industry's beta belongs to each ranking classification.

						Ranking							
Ind.	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**h.betas(%)**	**m.betas(%)**	**l.betas(%)**
Food	0.77	−0.10	1.42	0.25	0.77	21.90	1	30	7.23	0.63	9	23	68
Beer	0.81	−0.27	1.59	0.35	0.77	19.50	1	30	9.72	0.75	22	23	54
Smke	0.87	−0.23	1.70	0.35	0.70	17.46	1	30	9.01	0.57	21	38	41
Game	1.15	0.47	1.68	0.26	0.65	8.81	1	29	6.52	0.41	62	33	6
Book	0.84	0.33	1.64	0.19	0.74	20.16	4	30	6.48	0.70	10	40	50
Hshl	0.95	−0.04	1.56	0.28	0.84	14.81	1	30	8.18	0.76	30	41	28
Clth	0.92	0.50	1.55	0.22	0.50	17.14	1	30	7.44	0.47	21	41	38
Hlth	0.98	0.21	1.56	0.25	0.74	13.8	2	30	7.72	0.74	35	44	21
Chem	1.00	0.07	1.56	0.24	0.69	13.46	2	28	6.21	0.52	31	58	11
Txtl	0.80	0.15	2.06	0.28	0.68	21.49	2	30	7.28	0.61	8	31	60
Cnst	1.00	0.39	1.99	0.24	0.76	14.54	2	26	5.75	0.52	21	67	12
Stel	1.10	−0.19	2.15	0.35	0.66	12.58	1	30	8.16	0.53	40	43	17
Fpro	1.04	0.31	1.69	0.20	0.65	13.01	1	26	6.58	0.58	36	49	15
Eeqp	1.11	0.49	1.65	0.20	0.46	10.17	1	29	6.64	0.48	54	38	8
Auto	1.12	0.42	1.90	0.25	0.55	9.87	1	26	6.16	0.37	56	40	5
Carr	1.04	0.20	1.82	0.25	0.67	12.63	1	29	6.98	0.61	38	51	10
Mine	0.62	−1.12	2.11	0.59	0.75	22.21	1	30	10.25	0.77	20	10	70
Coal	1.03	−0.31	2.40	0.56	0.65	14.99	1	30	11.40	0.69	41	17	42
Oil	0.90	−0.07	1.82	0.34	0.70	17.28	1	30	8.66	0.59	26	34	40
Utlt	0.56	0.02	1.28	0.21	0.59	26.64	6	30	4.56	0.43	2	9	89
Tele	0.84	0.13	1.43	0.23	0.67	19.41	1	30	8.44	0.73	17	33	50
Psrv	1.14	0.70	2.17	0.24	0.68	10.86	1	26	6.74	0.72	49	44	7
Beqp	1.34	0.91	2.71	0.31	0.66	5.77	1	21	5.17	0.62	78	22	0
Papr	0.91	−0.05	1.31	0.20	0.63	17.28	1	28	5.56	0.40	10	67	23
Tran	1.06	0.34	1.84	0.24	0.63	11.86	1	29	6.56	0.52	38	52	10
Wtrd	0.90	0.38	1.60	0.22	0.74	18.05	2	29	6.41	0.67	12	56	32
Rtrd	1.02	0.54	1.46	0.19	0.61	12.83	1	29	6.85	0.61	36	49	15
Meal	1.05	0.18	1.94	0.34	0.73	12.96	1	27	8.09	0.69	38	42	20
Fina	0.99	0.69	2.01	0.20	0.68	15.59	1	27	6.86	0.69	24	55	21
Othr	0.90	0.23	1.53	0.20	0.68	17.93	2	30	6.77	0.68	13	51	36
Disp.	0.29	0.13	0.76	0.11	0.64								

Table 3.5: Annual window ex-post betas. Summary statistics and ranking

Similar to Table 3.4, this table reports summary statistics for quarterly betas and their ranking obtained with the ex-post approach and an annual window.

						Ranking							
Ind.	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**h.betas(%)**	**m.betas(%)**	**l.betas(%)**
Food	0.77	0.01	1.28	0.22	0.96	22.66	1	30	6.44	0.91	6	24	70
Beer	0.82	−0.08	1.51	0.3	0.96	20.40	1	30	9.29	0.94	19	21	60
Smke	0.87	−0.06	1.59	0.31	0.95	18.57	1	30	8.76	0.89	17	38	45
Game	1.16	0.66	1.63	0.21	0.94	7.76	1	21	5.48	0.82	66	34	0
Book	0.84	0.37	1.33	0.15	0.98	20.73	5	30	5.95	0.94	9	31	60
Hshl	0.95	0.12	1.37	0.26	0.98	14.38	2	29	8.10	0.97	30	47	23
Clth	0.93	0.61	1.37	0.17	0.91	16.94	2	27	6.83	0.90	14	50	36
Hlth	0.99	0.29	1.44	0.22	0.96	13.44	1	28	7.48	0.94	38	41	20
Chem	0.99	0.24	1.28	0.19	0.95	13.53	4	27	5.93	0.9	34	52	14
Txtl	0.8	0.36	1.66	0.20	0.96	22.46	4	30	6.17	0.93	4	28	69
Cnst	1.00	0.45	1.70	0.20	0.97	14.46	2	24	4.79	0.87	14	83	3
Stel	1.10	0.49	1.93	0.28	0.95	12.10	1	29	7.77	0.87	41	43	16
Fpro	1.04	0.68	1.54	0.15	0.92	12.85	3	26	6.02	0.92	32	58	10
Eeqp	1.11	0.7	1.50	0.14	0.89	9.71	1	25	6.06	0.90	54	41	5
Auto	1.11	0.52	1.47	0.18	0.93	9.16	1	25	5.36	0.87	59	40	1
Carr	1.05	0.53	1.65	0.20	0.95	12.22	1	27	6.68	0.93	44	44	12
Mine	0.64	−0.47	1.80	0.53	0.96	22.03	1	30	10.59	0.96	22	11	67
Coal	1.03	0.03	2.06	0.46	0.95	14.78	1	30	11.66	0.96	48	12	41
Oil	0.90	−0.01	1.50	0.30	0.95	18.13	1	29	8.34	0.89	16	40	44
Utlt	0.56	0.03	1.02	0.17	0.91	27.65	12	30	3.18	0.77	0	6	94
Tele	0.83	0.48	1.27	0.20	0.96	19.96	2	30	8.41	0.95	15	38	47
Psrv	1.15	0.85	1.68	0.19	0.94	10.15	1	25	6.72	0.94	49	46	5
Beqp	1.33	0.93	2.15	0.24	0.94	5.12	1	19	4.35	0.93	85	15	0
Papr	0.91	0.28	1.20	0.15	0.92	17.90	3	27	4.76	0.81	6	73	21
Tran	1.06	0.48	1.56	0.20	0.94	11.56	1	24	5.92	0.91	42	51	7
Wtrd	0.92	0.39	1.60	0.21	0.93	18.15	1	27	6.67	0.92	14	51	36
Rtrd	1.03	0.63	1.40	0.16	0.95	12.08	1	27	6.65	0.92	41	46	14
Meal	1.07	0.26	1.73	0.31	0.96	12.30	1	25	8.03	0.95	43	41	15
Fina	0.99	0.68	1.78	0.17	0.99	15.63	1	27	6.88	0.94	25	51	24
Othr	0.91	0.44	1.31	0.17	0.93	18.17	3	28	6.91	0.92	13	46	41
Disp.	0.26	0.13	0.69	0.08	0.91								

Table 3.6: DCC-MIDAS betas. Summary statistics and ranking

Similar to Table 3.4, this table reports summary statistics for quarterly betas and their ranking obtained with the DCC-MIDAS approach.

						Ranking							
Ind.	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**h.betas(%)**	**m.betas(%)**	**l.betas(%)**
Food	0.76	0.03	1.21	0.2	0.94	24.27	2	30	5.04	0.75	3	14	83
Beer	0.79	0.02	1.44	0.27	0.94	21.57	1	30	8.86	0.92	16	16	68
Smke	0.85	0.10	1.45	0.29	0.91	18.66	1	30	8.68	0.82	15	41	44
Game	1.17	0.55	1.63	0.19	0.77	6.24	1	18	4.38	0.73	72	28	0
Book	0.88	0.51	1.29	0.13	0.81	20.19	5	29	5.68	0.84	7	39	54
Hshl	0.94	0.11	1.29	0.23	0.96	15.31	3	30	7.18	0.92	25	51	24
Clth	0.93	0.63	1.22	0.14	0.88	17.41	4	28	6.12	0.88	13	56	31
Hlth	0.98	0.32	1.29	0.19	0.89	14.38	3	29	6.38	0.84	29	57	14
Chem	0.97	0.22	1.27	0.20	0.89	14.98	2	28	6.34	0.81	22	57	20
Txtl	0.83	0.26	1.68	0.18	0.95	22.71	3	29	5.54	0.88	3	28	69
Cnst	1.01	0.51	1.57	0.18	0.98	14.02	1	25	5.23	0.73	19	77	5
Stel	1.11	0.31	1.66	0.23	0.86	10.27	1	28	7.00	0.75	56	35	9
Fpro	1.04	0.57	1.36	0.13	0.82	12.80	2	27	6.04	0.76	37	56	7
Eeqp	1.10	0.72	1.56	0.15	0.78	9.46	1	25	5.91	0.78	59	38	3
Auto	1.08	0.45	1.47	0.16	0.85	9.72	2	24	4.67	0.65	48	51	1
Carr	1.03	0.35	1.48	0.18	0.85	12.39	1	26	6.21	0.8	36	54	9
Mine	0.66	−0.22	2.04	0.49	0.98	22.18	1	30	10.36	0.97	20	13	67
Coal	1.00	0.05	2.17	0.47	0.94	15.78	1	30	11.74	0.93	40	16	44
Oil	0.93	0.01	1.62	0.31	0.90	16.77	1	30	9.35	0.87	30	27	44
Utlt	0.60	0.08	1.18	0.19	0.81	27.44	10	30	3.79	0.62	0	6	94
Tele	0.82	0.47	1.37	0.18	0.89	21.38	2	30	7.94	0.88	12	31	57
Psrv	1.15	0.73	1.57	0.12	0.90	8.46	1	23	5.43	0.85	60	38	2
Beqp	1.33	0.78	2.00	0.17	0.86	3.12	1	22	3.42	0.89	96	4	1
Papr	0.90	0.17	1.18	0.17	0.82	19.38	5	30	3.84	0.54	1	68	31
Tran	1.05	0.42	1.41	0.17	0.90	10.40	2	27	4.87	0.82	48	49	2
Wtrd	0.90	0.48	1.34	0.15	0.90	18.87	2	28	5.51	0.79	7	57	36
Rtrd	1.00	0.50	1.30	0.13	0.89	13.67	2	27	5.87	0.84	25	65	10
Meal	1.05	0.29	1.42	0.24	0.94	10.67	2	26	7.10	0.90	53	36	11
Fina	1.03	0.78	1.97	0.16	0.90	14.09	1	26	6.83	0.86	35	50	15
Othr	0.92	0.31	1.47	0.18	0.80	18.42	2	29	6.61	0.75	14	43	43
Disp.	0.23	0.11	0.59	0.08	0.91								

Table 3.7: Kernel betas. Summary statistics and ranking

Similar to Table 3.4, this table reports summary statistics for quarterly betas and their ranking obtained with the kernel approach. The sample considered for the kernel betas only runs to December 2008 to avoid end-point biases.

						Ranking							
Ind.	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**Mean**	**Min**	**Max**	**Stdev**	**AR(1)**	**h.betas(%)**	**m.betas(%)**	**l.betas(%)**
Food	0.77	−0.08	1.36	0.24	0.93	22.73	1	30	6.54	0.84	5	21	74
Beer	0.82	−0.17	1.59	0.34	0.92	20.13	1	30	9.53	0.89	21	21	58
Smke	0.87	−0.09	1.66	0.32	0.95	18.63	1	30	8.72	0.89	18	36	46
Game	1.18	0.70	1.58	0.20	0.97	7.33	1	20	4.84	0.93	67	33	0
Book	0.86	0.39	1.37	0.17	0.94	20.49	3	29	6.31	0.92	12	30	57
Hshl	0.95	0.10	1.41	0.27	0.98	14.96	2	30	8.17	0.97	30	46	25
Clth	0.95	0.67	1.35	0.16	0.98	16.98	3	26	6.10	0.95	7	62	30
Hlth	0.99	0.38	1.38	0.22	0.99	13.78	3	28	7.39	0.96	38	43	19
Chem	1.01	0.18	1.43	0.21	0.95	13.34	3	27	6.12	0.88	35	49	16
Txtl	0.83	0.34	1.73	0.24	0.97	22.09	3	30	6.59	0.95	5	30	65
Cnst	1.02	0.43	1.76	0.23	0.94	14.33	2	24	5.27	0.82	17	77	6
Stel	1.14	0.26	2.14	0.32	0.93	12.01	1	29	8.06	0.84	41	43	16
Fpro	1.07	0.73	1.54	0.15	0.98	12.36	3	25	5.78	0.95	35	57	7
Eeqp	1.12	0.83	1.44	0.13	0.97	9.63	2	22	5.46	0.94	55	43	2
Auto	1.13	0.50	1.57	0.20	0.94	9.33	1	24	5.45	0.89	58	40	2
Carr	1.06	0.42	1.63	0.21	0.93	12.41	1	26	6.88	0.91	43	44	12
Mine	0.67	−0.58	2.16	0.57	0.92	21.84	1	30	10.80	0.93	23	9	68
Coal	1.07	0.07	2.23	0.49	0.97	14.40	1	30	11.77	0.97	48	12	40
Oil	0.91	−0.02	1.52	0.30	0.97	17.96	1	29	8.41	0.94	18	38	44
Utlt	0.57	0.01	1.21	0.19	0.88	27.35	9	30	3.66	0.76	1	7	92
Tele	0.84	0.43	1.38	0.21	0.92	20.09	2	30	8.30	0.93	16	35	49
Psrv	1.16	0.80	1.77	0.21	0.97	10.43	1	26	6.87	0.95	49	46	5
Beqp	1.35	0.94	2.37	0.27	0.95	5.40	1	19	4.74	0.94	83	17	0
Papr	0.92	0.29	1.23	0.17	0.93	17.98	3	27	4.92	0.81	5	73	22
Tran	1.08	0.47	1.61	0.21	0.91	11.51	1	25	5.98	0.88	43	51	7
Wtrd	0.92	0.42	1.50	0.20	0.93	18.20	3	27	6.46	0.91	14	50	36
Rtrd	1.04	0.69	1.42	0.17	0.97	12.46	1	27	6.74	0.93	38	48	14
Meal	1.07	0.27	1.77	0.32	0.94	12.75	1	26	8.29	0.92	39	40	22
Fina	1.00	0.71	1.78	0.17	0.91	15.90	2	27	6.88	0.90	26	48	26
Othr	0.92	0.43	1.37	0.18	0.94	18.22	3	29	6.80	0.92	11	50	39
Disp.	0.27	0.13	0.79	0.10	0.92								

Table 3.8: Individual structural breaks in the level of betas

The table reports optimal number of breaks in the level of betas as well as break dates for each industry using the Bai and Perron (2003)'s method. For each regime (or period between breaks), we report the average beta and its ranking as well as their respective standard deviations (in parenthesis).

	Num.	**Break dates**			**State 0**		**State 1**		**State 2**		**State 3**	
Ind	**breaks**	**1**	**2**	**3**	**Beta**	**Rank**	**Beta**	**Rank**	**Beta**	**Rank**	**Beta**	**Rank**
Food	0	-	-	-	0.77	22.73						
					(0.02)	(0.51)						
Beer	1	$Sep-98$	-	-	0.97	17.10	0.44	28.00				
					(0.02)	(0.87)	(0.03)	(0.46)				
Smke	1	$Dec-97$	-	-	1.02	15.05	0.50	27.13				
					(0.02)	(0.72)	(0.03)	(0.50)				
Game	0	-	-	-	1.18	7.33						
					(0.02)	(0.38)						
Book	0	-	-	-	0.86	20.49						
					(0.01)	(0.50)						
Hshl	0	-	-	-	0.95	14.96						
					(0.02)	(0.64)						
Clth	0	-	-	-	0.95	16.98						
					(0.01)	(0.48)						
Hlth	0	-	-	-	0.99	13.78						
					(0.02)	(0.58)						
Chem	0	-	-	-	1.01	13.34						
					(0.02)	(0.48)						
Txtl	0	-	-	-	0.83	22.09						
					(0.02)	(0.52)						
Cnst	1	$Dec-01$	-	-	0.95	15.83	1.39	5.75				
					(0.01)	(0.33)	(0.04)	(0.73)				
Stel	1	$Jun-00$	-	-	1.00	14.89	1.59	2.63				
					(0.02)	(0.63)	(0.04)	(0.23)				
Fpro	1	$Dec-00$	-	-	1.01	14.23	1.29	5.83				
					(0.01)	(0.46)	(0.02)	(0.21)				
Eeqp	0	-	-	-	1.12	9.63						
					(0.01)	(0.43)						
Auto	0	-	-	-	1.13	9.33						
					(0.02)	(0.43)						

Table 3.8: Structural breaks in the level of betas. Continued

	Num.	Break dates			State 0		State 1		State 2		State 3	
Ind	**breaks**	**1**	**2**	**3**	**Beta**	**Rank**	**Beta**	**Rank**	**Beta**	**Rank**	**Beta**	**Rank**
Carr	0	-	-	-	1.06	12.41						
					(0.02)	(0.54)						
Mine	3	$Sep-79$	$Sep-85$	$Sep-01$	0.71	26.10	1.24	8.13	0.18	29.35	1.45	6.40
					(0.03)	(0.79)	(0.06)	(1.66)	(0.03)	(0.18)	(0.06)	(1.14)
Coal	1	$Dec-01$	-	-	0.95	16.64	1.81	1.54				
					(0.03)	(0.97)	(0.05)	(0.17)				
Oil	0	-	-	-	0.91	17.96						
					(0.02)	(0.66)						
Utlt	0	-	-	-	0.57	27.35						
					(0.01)	(0.29)						
Tele	1	$Mar-82$	-	-	0.59	28.76	0.96	16.10				
					(0.01)	(0.16)	(0.01)	(0.67)				
Psrv	0	-	-	-	1.16	10.43						
					(0.02)	(0.54)						
Beqp	0	-	-	-	1.35	5.40						
					(0.02)	(0.37)						
Papr	0	-	-	-	0.92	17.98						
					(0.01)	(0.39)						
Tran	0	-	-	-	1.08	11.51						
					(0.02)	(0.47)						
Wtrd	0	-	-	-	0.92	18.20						
					(0.02)	(0.51)						
Rtrd	0	-	-	-	1.04	12.46						
					(0.01)	(0.53)						
Meal	1	$Sep-79$	-	-	1.49	2.29	0.93	16.29				
					(0.03)	(0.29)	(0.02)	(0.58)				
Fina	0	-	-	-	1.00	15.90						
					(0.01)	(0.54)						
Othr	0	-	-	-	0.92	18.22						
					(0.01)	(0.53)						

Table 3.9: Common structural breaks in the level of betas

Similar to Table 3.8, this table reports the average beta and its ranking as well as their respective standard deviations (in parenthesis) for the episodes before and after the common structural break identified using the Qu and Perron (2003)'s method for the kernel betas. The structural break is identified in March 1998.

	Pre-March 1998		**Post-March 1998**	
Ind	**Beta**	**Rank**	**Beta**	**Rank**
Food	0.87	21.18	0.54	26.51
	(0.17)	(7.09)	(0.21)	(2.09)
Beer	0.97	17.16	0.46	27.40
	(0.25)	(9.51)	(0.23)	(4.17)
Smke	1.02	15.17	0.49	27.11
	(0.20)	(7.79)	(0.24)	(3.48)
Game	1.25	5.52	1.03	11.74
	(0.18)	(3.90)	(0.14)	(4.03)
Book	0.87	21.23	0.83	18.70
	(0.15)	(6.04)	(0.22)	(6.67)
Hshl	1.09	11.03	0.61	24.55
	(0.15)	(5.83)	(0.17)	(4.13)
Clth	0.94	18.37	0.96	13.55
	(0.17)	(6.61)	(0.13)	(2.20)
Hlth	1.09	10.90	0.74	20.85
	(0.14)	(5.67)	(0.17)	(6.30)
Chem	1.03	13.31	0.96	13.40
	(0.14)	(6.16)	(0.33)	(6.09)
Txtl	0.79	24.86	0.93	15.30
	(0.15)	(3.85)	(0.35)	(6.98)
Cnst	1.00	15.63	1.07	11.15
	(0.10)	(4.03)	(0.38)	(6.51)
Stel	1.02	14.85	1.43	5.06
	(0.16)	(6.86)	(0.42)	(6.42)
Fpro	1.02	14.77	1.21	6.49
	(0.09)	(5.00)	(0.18)	(2.32)
Eeqp	1.11	10.20	1.14	8.23
	(0.14)	(6.18)	(0.08)	(2.63)
Auto	1.13	10.06	1.14	7.53
	(0.17)	(5.99)	(0.27)	(3.26)

Table 3.9: Common structural breaks in the level of betas. Continued

	Pre-March 1998		**Post-March 1998**	
Ind	**Beta**	**Rank**	**Beta**	**Rank**
Auto	1.13	10.06	1.14	7.53
	(0.17)	(5.99)	(0.27)	(3.26)
Carr	1.12	11.23	0.92	15.32
	(0.20)	(7.36)	(0.19)	(4.39)
Mine	0.58	23.90	0.88	16.79
	(0.50)	(9.57)	(0.66)	(12.02)
Coal	0.92	17.48	1.44	6.87
	(0.37)	(11.46)	(0.55)	(8.80)
Oil	0.94	18.03	0.83	17.77
	(0.24)	(8.54)	(0.40)	(8.17)
Utlt	0.57	28.43	0.59	24.70
	(0.13)	(2.55)	(0.29)	(4.54)
Tele	0.80	22.11	0.95	15.13
	(0.22)	(7.64)	(0.16)	(7.82)
Psrv	1.15	10.49	1.19	10.30
	(0.19)	(6.44)	(0.27)	(7.90)
Beqp	1.31	5.12	1.46	6.09
	(0.19)	(4.03)	(0.39)	(6.15)
Papr	0.96	17.54	0.81	19.06
	(0.12)	(5.33)	(0.22)	(3.54)
Tran	1.12	10.84	0.97	13.13
	(0.18)	(6.53)	(0.24)	(3.99)
Wtrd	0.96	18.07	0.83	18.53
	(0.19)	(7.40)	(0.18)	(3.25)
Rtrd	1.07	11.93	0.96	13.74
	(0.17)	(7.12)	(0.15)	(5.57)
Meal	1.19	9.61	0.77	20.43
	(0.27)	(7.43)	(0.23)	(4.36)
Fina	0.96	17.86	1.09	11.09
	(0.12)	(6.21)	(0.24)	(6.08)
Othr	0.94	18.11	0.85	18.47
	(0.14)	(5.96)	(0.26)	(8.59)

Table 3.10: Industry cyclicality. Contemporaneous regressions
The table reports parameter estimates for a regression of industry betas on a NBER dummy (panel A) and the Chicago Fed National Activity index (panel B). *,** and *** represent significance at the standard 1, 5 and 10% confidence levels. **dikbetas** are industry betas corrected by individual structural breaks (see Table 3.8) and **dckbetas** are industry betas corrected for common structural breaks (see Table 3.9).

Panel A. *nber*

Industry	**qbeta**	**mbeta**	**kbeta**	**dikbeta**	**dckbeta**
Food	−0.08	−0.04	−0.07		−0.04
Beer	−0.12	−0.10	−0.12	−0.05	−0.06
Smke	−0.18*	−0.14	−0.16*	−0.11**	−0.11*
Game	0.15***	0.09*	0.07		0.09*
Book	0.03	−0.02	0.02		0.03
Hshl	−0.08	−0.12	−0.06		−0.01
Clth	0.05	−0.01	−0.01		−0.01
Hlth	−0.15*	−0.15**	−0.13*		−0.10*
Chem	0.09*	0.10**	0.08		0.09
Txtl	0.17	0.08	0.13		0.11
Cnst	0.10	0.10	0.08	0.01	0.07
Stel	0.14	0.16**	0.10	0.00	0.06
Fpro	0.09	0.07**	0.06	0.01	0.04
Eeqp	−0.05	−0.09***	−0.03		−0.03
Auto	0.03	0.02	0.01		0.01
Carr	0.07	0.03	0.04		0.06
Mine	0.39**	0.39**	0.37**	0.05	0.34*
Coal	0.20	0.27	0.23	0.11	0.18
Oil	0.11	−0.02	0.12		0.13
Utlt	0.03	0.09	0.02		0.02
Tele	−0.06	−0.04	−0.07	0.00	−0.09*
Psrv	−0.05	−0.05	−0.07		−0.07
Beqp	−0.05	−0.08	−0.07		−0.09
Papr	0.01	−0.01	0.00		0.02
Tran	0.07	0.02	0.04		0.06
Wtrd	0.04	0.01	0.03		0.05
Rtrd	−0.10**	−0.10***	−0.11**		−0.10**
Meal	0.06	−0.05	0.04	−0.01	0.08
Fina	0.14	0.14*	0.11		0.10
Othr	0.10*	0.11***	0.07		0.08
Dispersion	0.04	0.04	0.03	0.05*	0.04*

Table 3.10: Industry cyclicality. Contemporaneous regressions. Continued

Panel B. *cfnai*

Industry	**qbeta**	**mbeta**	**kbeta**	**dikbeta**	**dckbeta**
Food	0.03	0.02	0.02		-0.01
Beer	0.05	0.03	0.04	-0.03	-0.02
Smke	0.10^{**}	0.08^{**}	0.09^{**}	0.03	0.03
Game	0.02	0.02	0.03		0.01
Book	-0.01	0.00	-0.01		-0.02
Hshl	0.10^{**}	0.09^{***}	0.09^{**}		0.03
Clth	-0.02	0.02	-0.01		-0.01
Hlth	0.08^{**}	0.08^{***}	0.07^{**}		0.03
Chem	-0.02	-0.03	-0.01		-0.02
Txtl	-0.06	-0.02	-0.05		-0.04
Cnst	-0.02	-0.03	-0.03	0.01	-0.02
Stel	-0.06	$-0.05*$	-0.05	0.03^{*}	0.00
Fpro	-0.04^{*}	-0.03^{***}	-0.03	0.01	-0.01
Eeqp	-0.01	0.03^{***}	0.00		0.00
Auto	0.00	0.01	0.00		0.00
Carr	0.02	0.02	0.02		0.00
Mine	-0.09	-0.08	-0.09	-0.01	-0.06
Coal	-0.05	-0.10	-0.07	0.01	-0.01
Oil	-0.02	0.06	-0.02		-0.04
Utlt	-0.03	-0.06^{***}	-0.03		-0.03
Tele	-0.02	-0.04	-0.03	-0.01	-0.01
Psrv	0.02	0.02	0.02		0.02
Beqp	0.02	0.04	0.03		0.05
Papr	0.02	0.02	0.02		0.01
Tran	0.01	0.02	0.03		0.01
Wtrd	0.03	0.02	0.03		0.01
Rtrd	0.04^{*}	0.04^{***}	0.04		0.03
Meal	0.05	0.07^{**}	0.07	0.01	0.02
Fina	-0.07	-0.07^{*}	-0.06		-0.05
Othr	$-0.04*$	-0.04^{***}	-0.04		-0.05^{**}
Dispersion	-0.02^{*}	-0.01	-0.02^{*}	-0.04^{***}	-0.03^{***}

Table 3.11: Industry betas and their fundamentals

The table reports the average R-squared for the following regressions:

$$\beta_{i,\tau} = \gamma_{0,i} + \gamma_{1,i} x_{\tau-1} + \epsilon_{i,\tau}, \text{ for } i = 1, .., 30,$$

where $x_{\tau-1}$ is each one of the market wide variables defined in Appendix 3.A. "F1" is a test for the joint significance of each variable (the null hypothesis of this test is $\gamma_{1,1} = \gamma_{1,2} = .. = \gamma_{1,30} = 0$). "F2" is a test for equal exposures (the null hypothesis in this case is $\gamma_{1,1} = \gamma_{1,2} = .. = \gamma_{1,30} = C$). *,** and *** represent significance at the standard 1, 5 and 10% confidence levels.

	qbetas			**mbetas**			**kbetas**		
	avg. R2	**F1-test**	**F2-test**	**avg. R2**	**F1-test**	**F2-test**	**avg. R2**	**F1-test**	**F2-test**
nber	2.20	4.48***	4.33***	3.70	7.54***	7.61***	2.01	4.44***	4.31***
cfnai	2.64	4.37***	4.40***	4.51	7.39***	7.59***	2.66	4.53***	4.68***
vol	0.27	0.40	0.36	0.18	0.28	0.39	0.17	0.27	0.26
mbg	1.14	1.45*	1.35*	1.88	2.89***	2.82***	1.39	1.77***	1.73***
ppinf	2.83	4.70***	4.99***	2.84	5.09***	4.78***	3.25	5.16***	5.32***
cpinf	5.14	7.97***	8.27***	5.49	9.01***	7.70***	6.18	9.09***	9.34***
gdpd	8.52	14.65***	14.14***	9.46	17.64***	13.11***	10.46	17.45***	16.77***
empg	3.14	5.47***	5.59***	4.94	8.88***	8.99***	3.15	5.67***	5.87***
gdpg	1.58	2.63***	2.59***	2.28	3.53***	3.66***	1.37	2.45***	2.44***
ipg	1.43	2.82***	2.68***	2.22	3.94***	4.07***	1.26	2.64***	2.65***
pd	13.12	22.48***	17.86***	18.55	33.68***	20.83***	16.67	29.96***	24.96***
divg	1.59	2.02***	2.15***	2.74	3.82***	3.26***	2.14	2.33***	2.29***
cprofg	0.73	0.80	0.74	0.88	1.16	0.99	0.86	1.01	0.87
rcg	1.30	1.65**	1.56**	2.49	2.88***	2.85***	1.79	2.78***	2.63***
3months	6.12	9.71***	10.12***	7.12	11.72***	11.17***	7.97	11.89***	12.28***
tsp1	2.94	4.35***	4.02***	2.52	3.28***	3.18***	4.30	5.79***	5.04***
dsp	4.19	10.29***	9.29***	5.33	15.03***	12.89***	4.42	11.18***	10.69***
smb	0.75	1.66**	1.72**	0.58	1.12	1.13	0.64	1.02	1.05

Table 3.11: Industry betas and their fundamentals. Continued

	dikbetas			dckbetas		
	avg. R2	**F1-test**	**F1-test**	**avg. R2**	**F1-test**	**F2-Test**
nber	1.617	2.885***	2.905***	2.18	4.71***	4.39***
cfnai	2.120	3.371***	3.476***	1.44	2.21***	2.15***
vol	0.177	0.229	0.197	0.13	0.20	0.19
mbg	1.216	1.749***	1.761***	1.28	1.59**	1.44**
ppinf	2.445	4.885***	5.082***	2.74	5.62***	5.86***
cpinf	4.655	9.089***	9.298***	3.77	7.62***	7.96***
gdpd	8.151	17.437***	16.744***	6.75	16.50***	16.81***
empg	2.432	3.999***	4.101***	1.08	1.61**	1.52**
gdpg	1.108	1.818***	1.873***	1.16	2.10***	1.95***
ipg	1.003	1.611**	1.681**	0.83	1.77***	1.59***
pdratio	13.694	28.430***	21.640***	4.29	11.61***	11.00***
divg	2.617	4.333***	4.457***	3.19	3.50***	3.39***
cprofg	0.981	1.332	1.209	0.80	0.90	0.78
rcg	1.492	2.642***	2.618***	1.87	3.12***	2.83***
3*months*	5.588	9.787***	10.201***	4.54	7.97***	7.03***
*tsp*1	4.148	7.217***	6.176***	4.85	7.15***	6.26***
dsp	3.911	7.832***	7.689***	4.90	15.31***	14.84***
smb	0.591	0.837	0.841	0.60	0.85	0.88

Table 3.12: Cross-sectional dispersion and its fundamentals
The table reports the results for the regression of cross-sectional dispersion in industry betas with respect to the market wide variables as in

$$disp_\tau = \gamma_0 + \gamma_1 x_{\tau-1} + \epsilon_\tau,$$

where $x_{\tau-1}$ is each one of the (standardized) market wide variables defined in Appendix 3.A. $disp_\tau$ is measured in percentage terms to facilitate the interpretation of our results. The table reports the estimated coefficient γ_1 for each variable as well as their respective standard deviations (in parenthesis). *,** and *** represent significance at the standard 1, 5 and 10% confidence levels. Standard errors are corrected by Newey-West HAC with 4 (quarterly) lags ([27]). Finally, the table also reports the R-squared for each regression.

	qbetas	**mbetas**	**kbetas**	**dikbetas**	**dckbetas**
nber	1.30	1.23	0.88	1.31	1.00
	(1.15)	(1.22)	(1.02)	(1.08)	(1.01)
	1.38	2.57	0.81	2.41	1.65
cfnai	-2.02^{**}	-1.32	$-1.47*$	-2.65^{***}	-2.45^{***}
	(1.00)	(1.18)	(0.86)	(0.95)	(-2.73)
	3.38	3.06	2.33	10.03	10.17
vol	0.37	-0.12	0.46	0.68^{*}	0.67^{*}
	(0.44)	(0.37)	(0.34)	(0.39)	(1.74)
	0.11	0.02	0.23	0.65	0.74
mbg	0.53	1.05^{***}	0.70	1.29^{***}	1.04^{***}
	(0.67)	(0.23)	(0.48)	(0.42)	(2.75)
	0.22	1.89	0.51	2.32	1.80
ppinf	-1.16	-1.15^{*}	-0.78	-0.12	-0.18
	(0.75)	(0.66)	(0.66)	(0.90)	(-0.21)
	1.11	2.31	0.65	0.02	0.06
cpinf	-1.39	-1.52^{**}	-1.19	-0.47	-0.45
	(0.94)	(0.73)	(0.81)	(1.10)	(-0.41)
	1.60	4.06	1.52	0.31	0.35
gdpd	-1.88^{*}	-1.87^{**}	-1.75^{*}	-1.11	-1.17
	(1.10)	(0.89)	(1.06)	(1.20)	(-0.95)
	2.92	6.05	3.26	1.73	2.30
empg	-1.79^{*}	-1.43	-1.33^{*}	-2.08^{**}	-1.82^{**}
	(1.00)	(1.21)	(0.81)	(0.92)	(-2.04)
	2.65	3.57	1.89	6.17	5.58

Table 3.12: Cross-sectional dispersion and its fundamentals. Continued

	qbetas	**mbetas**	**kbetas**	**dikbetas**	**dckbetas**
$gdpg$	-1.05	-0.30	-0.51	-1.71^{**}	-1.68^{**}
	(0.82)	(0.82)	(0.73)	(0.83)	(-2.21)
	0.90	0.16	0.28	4.14	4.76
ipg	-1.35	-0.97	-0.95	-1.93^{**}	-1.71^{*}
	(1.03)	(1.13)	(0.89)	(0.95)	(-1.87)
	1.50	1.65	0.96	5.30	4.92
pd	4.09^{**}	2.86^{*}	4.33^{**}	3.06^{*}	2.62^{*}
	(1.98)	(1.52)	(2.13)	(1.82)	(1.70)
	13.85	14.30	20.10	13.39	11.67
$divg$	-2.61^{*}	-2.39^{**}	-2.46^{*}	-1.91^{*}	-1.35
	(1.36)	(1.01)	(1.38)	(1.11)	(-1.34)
	5.19	9.13	5.98	4.81	2.84
$cprofg$	-1.00	-0.24	-1.22	-2.07^{***}	-1.87^{***}
	(0.96)	(0.67)	(0.86)	(0.78)	(-2.80)
	0.83	0.10	1.60	6.14	5.97
rcg	-0.16	-0.03	0.00	-1.57^{*}	-1.59^{**}
	(0.67)	(0.78)	(0.73)	(0.83)	(-1.98)
	0.02	0.00	0.00	3.53	4.30
$3months$	-1.51^{*}	-1.81^{**}	-1.08^{*}	-0.35	0.07
	(0.78)	(0.83)	(0.60)	(1.12)	(0.07)
	1.86	5.62	1.23	0.17	0.01
$tsp1$	-1.11	0.30	-1.62	-2.32^{*}	-2.40^{**}
	(1.43)	(1.06)	(1.46)	(1.25)	(-2.08)
	1.01	0.16	2.76	7.59	9.62
dsp	-0.74	-0.04	-1.19	0.29	0.04
	(1.43)	(1.15)	(1.35)	(1.38)	(0.03)
	0.45	0.00	1.52	0.12	0.00
smb	0.12	0.96^{*}	0.59	0.40	0.23
	(0.56)	(0.53)	(0.52)	(0.48)	(0.46)
	0.01	1.59	0.37	0.23	0.09

Table 3.13: Predictive power of the cross-sectional dispersion in betas over equity returns

The table reports the estimation results for the following regression:

$$(r - r_f)_{\tau,\tau+h} = \gamma_{0,h} + \gamma_{1,h} disp_\tau + \gamma_{2,h} \widehat{u_\tau} + \epsilon_{j,\tau},$$

where $(r - r_f)_{\tau,\tau+h}$ represents future compounded annualized excess returns h-quarters ahead, $\widehat{u}_\tau$ is the component of return dispersion orthogonal to the dispersion in betas (measured in percentage terms) obtained from the auxiliary regression

$$disp\_ret_\tau = \kappa_0 + \kappa_1 disp_\tau + u_{j,\tau},$$

where $disp\_ret_\tau$ is the value weighted dispersion in industry returns. Standard errors are corrected by Newey-West HAC with max $\{2h, 4\}$ (quarterly) lags. *,** and *** represent significance at the standard 1, 5 and 10% confidence levels. Finally, the table reports the R-squared for the regression plus the R-squared for a regression where only dispersion in betas (and a constant) is considered ($R^2(disp_\tau)$).

Panel A. qbetas

h	1	2	3	4	5	6	9	12
$\gamma_{0,h}$	45.82**	50.50***	49.13***	44.12***	43.67***	44.15***	34.53***	29.34***
	(23.06)	(17.61)	(13.80)	(12.05)	(12.58)	(11.51)	(10.22)	(5.46)
$\gamma_{1,h}$	−1.02	−1.16**	−1.13***	−0.97***	−0.96***	−0.97***	−0.63*	−0.42***
	(0.76)	(0.56)	(0.39)	(0.35)	(0.37)	(0.34)	(0.36)	(0.13)
$\gamma_{2,h}$	−4.57	−3.95	−5.72***	−4.45**	−3.58**	−3.29*	−2.61	−2.81
	(4.16)	(2.84)	(2.09)	(1.88)	(1.81)	(1.93)	(2.39)	(2.45)
R^2	1.95	3.74	7.42	7.35	7.75	8.77	6.96	6.62
$R^2(disp_\tau)$	1.08	2.57	3.67	3.98	4.97	5.98	4.09	2.28

Panel B. mbetas

h	1	2	3	4	5	6	9	12
$\gamma_{0,h}$	63.23***	59.92***	50.84***	51.86***	56.43***	54.92***	42.01***	32.17***
	(22.91)	(20.87)	(18.93)	(17.62)	(15.21)	(14.80)	(12.08)	(11.81)
$\gamma_{1,h}$	−2.03**	−1.87**	−1.50**	−1.56**	−1.77***	−1.70***	−1.12**	−0.65
	(0.90)	(0.82)	(0.71)	(0.64)	(0.55)	(0.56)	(0.47)	(0.41)
$\gamma_{2,h}$	−3.84	−3.83	−5.96***	−4.29**	−3.21*	−3.01	−2.36	−2.72
	(4.10)	(3.11)	(2.25)	(1.76)	(1.69)	(1.89)	(2.11)	(2.12)
R^2	2.65	4.22	7.31	8.05	9.85	10.76	8.42	6.88
$R^2(disp_\tau)$	2.00	3.07	3.01	4.73	7.48	8.29	5.96	2.66

Table 3.14: Predictive power of the cross-sectional dispersion in betas over equity returns. Partitioned sample

Similar to Table 3.13, this table reports estimated results for the predictive power of the components of the cross-sectional dispersion in equity returns for a partition of the total sample.

Panel A. qbetas 1970-1997 sample

h	1	2	3	4	5	6	9	12
$\gamma_{0,h}$	−4.75	17.12	15.56	14.02	17.54*	21.12**	3.85	21.01*
	(31.66)	(23.36)	(16.23)	(11.86)	(9.26)	(9.84)	(7.62)	(10.99)
$\gamma_{1,h}$	0.96	0.14	0.20	0.27	0.12	−0.02	0.58	−0.09
	(1.01)	(0.75)	(0.52)	(0.42)	(0.33)	(0.44)	(0.44)	(0.64)
$\gamma_{2,h}$	−0.35	−1.20	−3.26	−2.12	−1.62	−1.97	0.28	0.05
	(5.95)	(4.65)	(3.80)	(3.24)	(2.97)	(3.12)	(2.23)	(1.85)
R^2	0.48	0.08	0.84	0.65	0.43	0.69	2.21	0.04
$R^2(disp_\tau)$	0.47	0.02	0.05	0.15	0.04	0.00	2.18	0.04

Panel B. qbetas 1998-2009 sample

h	1	2	3	4	5	6	9	12
$\gamma_{0,h}$	73.37**	65.57***	64.97**	49.77*	44.90	41.38	34.16*	16.87
	(30.18)	(24.84)	(29.45)	(29.02)	(32.11)	(30.50)	(19.75)	(12.55)
$\gamma_{1,h}$	−1.93**	−1.71***	−1.70***	−1.37***	−1.26**	−1.18**	−0.95***	−0.42***
	(0.76)	(0.56)	(0.55)	(0.52)	(0.58)	(0.54)	(0.30)	(0.16)
$\gamma_{2,h}$	−4.60	−4.87	−6.11***	−4.56**	−2.86	−2.65	−1.77	−2.57
	(6.41)	(3.84)	(1.87)	(1.83)	(1.91)	(1.74)	(2.30)	(1.76)
R^2	7.18	11.44	18.35	17.52	16.33	17.43	19.30	10.61
$R^2(disp_\tau)$	6.15	9.22	13.33	13.19	14.18	15.16	17.59	4.92