Subsidizing low-skilled jobs in a dual labor market

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Abstract

We introduce a matching model that allows for classical and frictional unemployment. The labor market is dual featuring low-skilled and high-skilled workers. Low-skilled jobs pay a minimum wage, while wages in the high-skilled jobs are determined by Nash bargaining. Opportunities for low-skilled workers are limited to low-skilled jobs; while high-skilled unemployed can apply for both types of jobs, and thereby can accept to be downgraded. We analyze the outcomes of low-skilled job subsidy policies assuming that government budget is balanced through taxes on occupied workers. We first give conditions for the existence and uniqueness of a steady-state equilibrium and we then analyze the effects of different fiscal instruments. We show that in this set-up, increasing low-skilled job subsidies does not necessarily reduce low-skilled unemployment or unemployment spells. We calibrate the model on French labor market data. It is found that for five low-skilled workers leaving classical unemployment, two high-skilled workers are downgraded.

Keywords: Matching, Taxation, unemployment policy

JEL Classifications: H21, H23, J41, J68

1 Introduction

Before the mid-1990s, European countries facing a large unemployment mainly used pure transfer policies to prevent income inequalities from rising. Following the policy initiative for Europe advocated by [Drèze and Malinvaud, 1994], some of them, like France or Belgium, switched to a mixed policy implementing low-skilled job subsidies. The financing of these low-skilled payroll tax cuts was supposed to go through a shift of the tax burden on skilled labor. Such a change modifies the relative demand for low-skilled labor. At the same time, in presence of a classical unemployment due to a minimum wage as observed in the above countries, the job subsidy strategy allows some low-skilled excluded workers to re-enter the labor market. Size and composition of the labor force are modified and the adjustments of the fiscal scheme, necessary to ensure a balanced budget, may affect the outcomes of the mixed policy. In this paper, we propose to analyze the changes in relative labor demand and supply, induced by the changes in the fiscal scheme, and to give some insights about the resulting effects on unemployment and welfare. A particular attention is put on the possibility for high-skilled unemployed to accept low-skilled jobs.

The baseline framework is a version of the matching model proposed by [Albrecht and Vroman, 2002] and [Gautier, 2002] with a dual labor market. There are two types of workers, low-skilled and high-skilled; two types of jobs, simple and complex. Jobs are characterized by the scope they offer for utilizing the worker capacities. The technology is such that a simple job can be done by either type of workers, but a complex job can only be done by a high-skilled worker. Facing unemployment risk, some high-skilled workers can accept a job that does not correspond to their level of professional qualification and become what we call hereafter *downgraded* (for empirical illustrations see for instance [Battu et al., 2000] or [Hartog et al., 1994]). In an economy where matches between high-skilled workers and simple vacancies are mutually beneficial, high skilled workers take jobs away from low-skilled workers. In other words, they crowd out low-skilled workers. [Teulings and Koopmanschap, 1989] and [Dolado et al., 2009] find evidence of crowding-out effect in UK and Europe. [Pierrard and Sneessens, 2004] show for Belgium that the phenomenon of low-skilled unemployment is jointly due to relative wage rigidities, an increase in the supply of skilled labor and job competition.

We follow [Gautier, 2002] and [Dolado et al., 2009] by considering directed search and assuming that high-skilled workers matched with simple jobs are allowed to search on the job for a complex position.[1] Nevertheless, in order to take account of classical unemployment, we depart from these

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[1][Blázquez and Jansen, 2008] also consider a matching model with heterogeneous jobs and workers, but search is

models in two directions.[2] First, there is a distribution of productivity among low-skilled workers. We focus on the changes induced by the job-subsidy policy on the low-skilled labor market and thus do not introduce a similar heterogeneity among high-skilled workers. Second, we assume that low-skilled jobs pay a minimum wage which is exogenously fixed. Employees in low-skilled positions are assumed to have very small negotiating power and be paid the minimum wage[3]. A simple job filled with a too low productivity worker would return negative profits because of the minimum wage constraint. Consequently, these workers are *excluded* in the sense that they do not search for a job any longer. They do not generate a congestion externality by searching. *Classical unemployment* consists of these excluded too low productivity workers.

Both types of workers pay a wage tax which depends on the type of the job they fill if they are working, but the government cannot discriminate between high-skilled and low-skilled whenever they fill a simple job.[4] Few papers discuss public policy within a dual market framework. Among them, [Kleven and Sorensen, 2004] and [Acemoglu, 2001] focus on the impact of labor market regulation on the composition of employment. The effects on the labor income distribution is explored by [Lommerud et al., 2004]. We, in contrast, consider an economy where workers have different skills and opportunities and in which the size of the labor force changes with the fiscal scheme. This relates our framework to papers by [Cardullo and Van der Linden, 2006], [Pierrard, 2005] and [Batyra and Sneessens, 2006]. This last paper for instance underlines that the presence of overqualified workers plays a significant role in the relative effectiveness of low-skilled worker unemployment policies, but does not consider the role of classical unemployment.

An important feature of our model is that, for simple jobs, employer tax/subsidy and employee tax/subsidy are not equivalent. In a simple model of equilibrium search with wage bargaining, [Pissarides, 1985] shows that lump-sum negative tax wages and employment subsidies have both the same effects on employment. Both fiscal instruments reduce wages and increase employment, by raising the surplus shared. However, this result does not apply when the minimum wage is exogenously determined. Job subsidies increase expected profit of low-skilled vacancies and thus

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not directed.

[2]Our set-up is different from [Albrecht and Vroman, 2002]'s one in which equilibrium multiplicity may arise because low and high skilled wages are negotiated and workers can coordinate on a segmented or a cross-matching equilibrium.

[3]In [Cahuc et al., 2006], bargaining power of French low-skilled workers is estimated to be equal to 0.

[4]These informational asymmetries are due to the fact that the government does not know the real opportunities of individuals. See also [Hungerbuhler et al., 2003] which characterize optimal non linear income taxation in an economy with a continuum of unobservable productivity level and endogenous involuntary unemployment due to frictions in labor market.

have a *direct effect* that always amounts to a reduction of classical unemployment and in some circumstances to a reduction of frictional unemployment. In this case, the rise in the arrival rate of low-skilled job offers increases the reservation wage of high-skilled unemployed and, in turn, reduces high-skilled employment. Then, an increased number of high-skilled workers compete with low-skilled ones on the simple job market. Consequently, the *indirect effect* of low-skilled job subsidy is an inflow of high-skilled unemployed on the simple job market, reinforcing the downgrading phenomenon. The fact that increased downgrading is beneficial or not for low-skilled employment depends on the relative importance of the two externalities, pointed by [Gautier, 2002]. The first externality comes from the productivity differential between high-skilled workers and low-skilled ones. We assume that, in simple positions, high-skilled workers are not less productive than low-skilled ones. Profitability of simple vacancies may then augment with the fraction of high-skilled people applying for simple jobs. In these circumstances, an increase in the number of downgraded high-skilled workers does not necessarily correspond to less numerous employment opportunities for low-skilled workers as it positively affects the value of simple job positions which may lead to open more simple vacancies. But on-the-job search introduces a second externality since downgraded high-skilled workers may quit a simple position for a better paid complex job. All in all, simple job subsidy first clearly reduces classical unemployment, whereas the net effect on frictional unemployment is ambiguous for low-skilled workers. Second, it induces an increase in the number of downgraded high-skilled workers. In addition, assuming that the government budget is balanced through taxes on high-skilled workers, the downgrading effect is reinforced and amplifies the two externalities discussed above. Consequently, subsidizing low skill jobs always diminishes classical unemployment, but at the cost of a possibly increasing average unemployment duration for all workers. Any additional classical unemployed re-entering the job market is accompanied by an increasing number of downgraded high-skilled workers. A simple calibration on French data allows us to illustrate the above mechanisms and the effects of various changes in the fiscal scheme. It appears that the level of subsidies in France induces an equilibrium in which any low-skilled worker reentering the job market is accompanied by 0.4 high-skilled worker accepting to be downgraded and that the creation of four low-skilled jobs is accompanied by the downgrading of one high-skilled worker.

The paper proceeds as follows. The setup of the model is described in the second Section. The equilibrium of the model is presented in the third one, where existence and uniqueness results are laid out. In the fourth Section we study the properties of the equilibrium and illustrate the

consequences of various changes in the fiscal scheme. The fifth one is devoted to a calibration exercise on the French economy. The last Section concludes.

2 The set-up

2.1 Basic assumptions

We consider an economy consisting of a fixed labor force, modeled by a continuum of agents that can be of two exclusive types of skill : low (l) and high (h). Populations of low-skilled and high-skilled people are respectively of mass ϕ_l and ϕ_h. Workers are infinitely lived and assumed to be risk-neutral. On the other side of the labor market, the number of jobs is endogenous. For simplicity, we assume single worker firms, with either a simple (type l) or a complex (type h) job.

All applicants whatever their type low or high (l or h) can get a simple job, which needs no special skill. There is a continuum of low-skilled worker types s distributed on the support $[0, \overline{s}]$; s is the output level of an s-type worker when matched with a simple job. Functions $x(s)$ and $X(s)$ are respectively the probability density function and the cumulative distribution function of s. In order to simplify the analysis, all high-skilled workers are supposed to be homogeneous in term of productivity and produce $\overline{s}$ when matching with a simple job[5]. By contrast, a complex job (type h) requires a minimum skill to be productive and employers observe worker's skill when the wage bargaining starts. The output of a complex job is s_h, when skill is h, and 0 otherwise. Consequently, low-skilled workers do not apply for complex jobs.

Job matchings result from a random process of search described by the following matching functions

$$M_i(z_i, v_i) = \lambda_i z_i^{1-\sigma_i} v_i^{\sigma_i}, \ \lambda_i > 0, \ 0 < \sigma_i < 1 \text{ for } i = l, h$$

where z_i denotes the mass of workers who applied, v_i, the mass of vacancies, λ_i, the usual efficiency parameter and σ_i, the elasticity of the matching function. For complex positions, high-skilled unemployed and high-skilled workers downgraded in simple positions compose the mass of applicants z_h. For simple jobs, z_l consists of high-skilled unemployed and of the fraction of low-skilled unemployed who are not classically unemployed as we shall see below.

The arrival rate of a job offer i ($i = h, l$) for a job-seeker is then given by $p_i = \lambda_i (v_i/z_i)^{\sigma_i}$, and the arrival rate of a worker for vacancy of type i is $q_i = \lambda_i (z_i/v_i)^{1-\sigma_i}$. The relation between those

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[5]The main results remain unchanged if we assume some productivity heterogeneity among high-skilled workers when matching with a simple job as long as the distribution first-order dominates the low-skilled one.

Figure 1: Flows chart
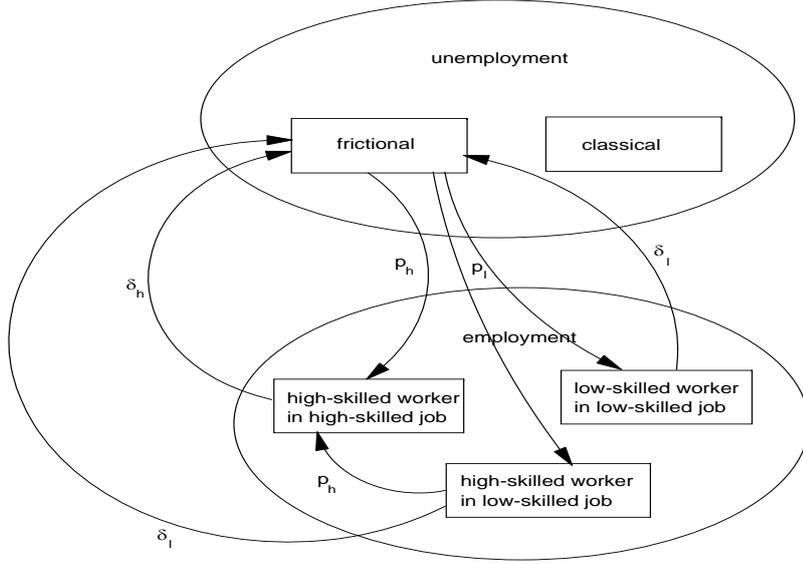
rates writes

$$q_i = Q_i(p_i) \equiv \lambda_i^{1/\sigma_i} p_i^{1-(1/\sigma_i)} \text{ for } i = l, h \tag{1}$$

Jobs end at the exogenous flow rate δ_j, $j = l, h$. In this case, the firm becomes an unfilled vacancy and the worker becomes unemployed. Figure 1 summarizes the flows of job creation and destruction involved in the modeling.

We focus on equilibria in which it is beneficial for high-skilled workers to match with simple vacancies and we refer to this type as equilibria with cross-skill matching. This requires some conditions on fiscal instruments as stated below.

2.2   Simple job market

Real wage in simple position is the minimum wage $\underline{w}$ set by the government. It is smaller than $\bar{s}$, so that the more productive workers in simple position produce output higher than labor cost. Nevertheless, since productivity s belongs to the interval $[0, \bar{s}]$, there is a mass of low-skilled workers for whom the potential output is lower than the minimum wage. These workers will never be hired and do not search any longer. They are excluded and constitute classical unemployment.

The value of a filled simple job depends on the productivity of the worker and on his opportunities. Values of a simple job filled with a high-skilled worker and a s-type low-skilled worker are respectively denoted by J_{hl} and $J_{ll}(s)$. They satisfy

$$rJ_{hl} = \bar{s} - \kappa_l - \underline{w} - (\delta_l + p_h)(J_{hl} - V_l) \tag{2}$$

$$rJ_{ll}(s) = s - \kappa_l - \underline{w} - \delta_l(J_{ll}(s) - V_l) \tag{3}$$

where V_l is the value of a simple job vacancy, and r is the instantaneous discount rate. We assume that employers pay a tax κ_l. The government cannot discriminate between low-skilled and high-skilled workers when they are matched with simple jobs. No information can be obtained with the observation of wages. Consequently, the government has to subsidy in the same way all simple jobs. In equation (2), separation occurs at rate $\delta_l + p_h$ since high-skilled workers in simple position continue to search for a complex job, and consequently, may quit at an additional rate p_h.

If a simple job is matched with a low-skilled worker, the value of the job depends on the worker's productivity s. A simple-job firm matched with a low-skilled worker accepts to fill the post when the match is advantageous, that is, when generating a non-negative surplus $J_{ll}(s) - V_l$. At equilibrium, the free-entry condition implies that the value of the vacancy is null. Therefore, $J_{ll}(s)$ must be positive. This defines a productivity threshold, $\underline{s} = \kappa_l + \underline{w}$, below which firms always break the match. In the following, we assume that low-skilled workers that have productivity lower than $\underline{s}$, internalize the fact that they will never be hired. They no longer apply for simple jobs and constitute classical unemployment.

At the moment the vacancy is opened, employers posting a simple job do not know the type of worker they will meet. But they know the aggregate composition of unemployment and therefore can calculate the probability of meeting each of the worker type, taking into account the fact that low-skilled workers with productivity lower than $\underline{s}$ will not search for a job. We denote by u_h the unemployment level of high-skilled workers and by $u_l(s)$ the unemployment level of low-skilled workers with productivity s. The value V_l of a simple-job vacancy satisfies

$$rV_l = -c_l + q_l\left(\frac{\int_{\underline{s}}^{\bar{s}} u_l(s)J_{ll}(s)\,ds + u_h J_{hl}}{\int_{\underline{s}}^{\bar{s}} u_l(s)\,ds + u_h} - V_l\right) \tag{4}$$

where c_l is a fixed cost paid by firms when posting a vacancy for a simple job. It can be viewed as an advertising cost. The free-entry condition implies that the expected cost of an advertised vacancy equates to the expected profit of a filled position

$$\frac{c_l}{q_l} = \frac{\int_{\underline{s}}^{\bar{s}} u_l(s)J_{ll}(s)\,ds + u_h J_{hl}}{\int_{\underline{s}}^{\bar{s}} u_l(s)\,ds + u_h} \tag{5}$$

The possibility of hiring a high-skill worker on a simple job introduces the two externalities pointed by [Gautier, 2002]. Both appear in the comparison of (2) and (3). On the one hand, production of a high-skilled worker $\bar{s}$ is never lower than the production of a low-skilled worker, that belongs to $[0, \bar{s}]$. This generates a positive externality on low-skilled labor market. The more high-skilled

workers search for simple jobs, the more attractive it is for employers to post simple vacancies. On the other hand, high-skilled workers search on-the-job and may quit at rate p_h. This imposes a negative externality on low-skilled labor market, since expected profit of simple filled job is reduced. Both externalities have opposite effects on the right-hand side of equation (5).

Equation (5) also gives some insights on the effect of low-skilled job subsidies that would correspond here to a reduction of the employer tax κ_l. On the one hand, a reduction of κ_l allows some low-productivity worker to enter the labor market and apply for simple jobs. The direct consequence on the right-hand side of (5) is to diminish the expected productivity and profitability of simple jobs. On the other hand, since the government cannot discriminate between low-skilled and high-skilled workers in simple positions, job subsidies will be accorded to all simple jobs whatever the type of the worker. This increases the expected profitability of simple jobs and induces employers to post more simple vacancies.

Equations (2) to (5) have been written under the assumptions that (i) all unemployed with productivity parameter higher that $\underline{s}$, search for a job, (ii) high-skilled unemployed search for simple jobs and accept to be downgraded, (iii) high-skilled workers matched with simple job continue to search for a complex job. Let U_i and N_{ij} be respectively the discounted values of being unemployed and employed, where $i = l, h$ denotes the skill of the worker, and $j = l, h$ the type of the job. The above assumptions formally write[6]

$$N_{ll} \geq U_l \text{ and } N_{hh} \geq N_{hl} \geq U_h \tag{6}$$

For a low-skilled worker, discounted values of a simple position N_{ll} and unemployment U_l satisfy[7]

$$rN_{ll} = \underline{w} - \tau_l - \delta_l (N_{ll} - U_l) \tag{7}$$

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[6]Assumptions (ii) and (iii) mean that we do not consider situations where fiscal incentives lead high-skilled workers to prefer to work in simple jobs rather than complex jobs, and continue to apply for simple jobs when they are matched with a complex one.

[7]In this model, the search cost is null. Then, people are indifferent between staying at home or searching without success. We consider in this case that, knowing the fact that they cannot be successful in their search on the job market, they do not search, and, then, do not increase job market frictions. With positive search cost, equation (8) should be rewritten as

$$rU_l = b - c_r + p_l(N_{ll} - U_l).$$

and a low-skilled unemployed which can potentially find a job may be incited to mimic an excluded one. p_l being endogenous, we have to face an indeterminacy of the equilibrium. The equilibrium depends on the belief of each agent on the behavior of the others and winds up with a coordination problem between low-skilled workers. Keeping to a simple analysis that enhances the consequences of interactions between the two sources of unemployment, we assume that $c_r = 0$.

$$rU_l = b + p_l(N_{ll} - U_l) \tag{8}$$

Since the minimum wage applies to all simple jobs, the discounted values N_{ll} and U_l do not depend on the productivity s of the worker under consideration. Unemployed people receive a grant b. A tax τ_l is paid by any worker who holds a low-skilled position. Straightforward manipulations of equations (7) and (8) show that $N_{ll} \geq U_l$ is equivalent to $\underline{w} \geq \tau_l + b$. Notice that the low-skilled workers which are below the threshold of productivity $\underline{s}$ and are consequently *excluded* from the job market, will receive the grant b. The corresponding value U_l^e satisfies

$$rU_l^e = b \tag{9}$$

For high-skilled workers, discounted values of unemployment (U_h), simple position (N_{hl}) and complex position (N_{hh}) satisfy

$$rU_h = b + p_l(N_{hl} - U_h) + p_h(N_{hh} - U_h) \tag{10}$$

$$rN_{hh} = w_h - \tau_h - \delta_h(N_{hh} - U_h) \tag{11}$$

$$rN_{hl} = \underline{w} - \tau_l - \delta_l(N_{hl} - U_h) + p_h(N_{hh} - N_{hl}) \tag{12}$$

where w_h is the wage in complex position, and τ_h is a tax. Since high-skilled wage is a Nash bargaining solution, an employer tax would be equivalent. From equations (10) and (12), we deduce that high-skilled workers accept to be downgraded $(N_{hl} \geq U_h)$ if and only if $\underline{w} \geq \tau_l + b$, which is the same condition on fiscal instruments and minimum wage as for low-skilled workers $(N_{ll} \geq U_l)$.[8] From equations (10), (11) and (12), the condition for on-the-job search, $N_{hh} \geq N_{hl}$, rewrites

$$w_h - \tau_h - b \geq \frac{r + \delta_h + p_l + p_h}{r + \delta_l + p_l + p_h}(\underline{w} - \tau_l - b). \tag{13}$$

If the destruction rates are equal in both types of jobs $(\delta_h = \delta_l)$, high-skilled workers search on-the-job if they expect a higher net wage in a complex position. We now turn to the determination of the high-skilled wage w_h.

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[8]A possible extension would be to introduce some disutility of being downgraded for a high-skilled worker, by adding a constant ρ_{hl} ($\rho_{hl} \geq 0$) in equation (12)

$$rN_{hl} = \underline{w} - \tau_l - \rho_{hl} - \delta_l(N_{hl} - U_h) + p_h(N_{hh} - N_{hl})$$

The parameter ρ_{hl} would represent the fact that a wage relation encompasses an economic and social relation, and that the social status procured by a job is not the least important aspect of the wage relation (see for instance on this subject [Solow, 1990]). Introducing disutility of being downgraded could sharply modify the nature of the equilibrium since high-skilled workers may be willing to only apply for complex jobs, whereas low-skilled workers apply to low-skilled jobs, leading to ex-post segmentation of the labor market.

2.3 Complex job market

Real wages in the high-skilled sector are determined by a generalized Nash bargaining process that takes place between the individual worker and the firm, after they meet. To keep the model tractable, we make the two following assumptions. (i) Search is not observable. [Wolinsky, 1987] and more recently [Abbring, 1999] show that this guarantees that workers and firms will not continue searching for similar partners during the bargaining. (ii) Following [Gautier, 2002], the wage is renegotiation proof, which implies that wages at complex jobs are independent of the workers previous labor market state.[9] Under these assumptions, firms and workers negotiate a real wage w_h which at the steady state remains constant until the job is broken up by the exogenous process assumed above.

The discounted value J_{hh} of a filled complex job and the discounted value V_h of a complex vacancy satisfy

$$r J_{hh} = s_h - w_h - \delta_h (J_{hh} - V_h) \tag{14}$$

$$r V_h = -c_h + q_h (J_{hh} - V_h) \tag{15}$$

where c_h is the fixed cost paid by firms which offer complex jobs. The wage rate w_h, solution of the Nash bargaining, maximizes $[N_{hh} - U_h]^\beta [J_{hh} - V_h]^{1-\beta}$, where β is associated to high-skilled worker negotiation power. Surplus is shared according to the following rule

$$\beta (J_{hh} - V_h) = (1 - \beta) (N_{hh} - U_h) \tag{16}$$

and the high-skilled wage rate is

$$w_h = \frac{\beta (r + \delta_h + p_h)}{r + \delta_h + \beta p_h} s_h + \frac{(1 - \beta)(r + \delta_h)}{r + \delta_h + \beta p_h} (b + \tau_h + p_l (N_{hl} - U_h)) \tag{17}$$

The opportunity for high-skilled unemployed people to fill a simple job increases their reservation wage in the bargaining. Similarly, the tax τ_h paid by the employee also puts up the reservation wage.

Equation(15) and the free-entry condition $V_h = 0$ imply that, at equilibrium, expected advertising cost is equal to expected profit

$$\frac{c_h}{q_h} = J_{hh} \tag{18}$$

An increase in p_l rises the reservation wage of the high-skilled workers and reduces profits in complex jobs, which become less attractive for employers. Therefore, a policy aimed at enhancing low-skilled

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[9]As [Gautier, 2002] explains, the intuition is the following. If an employed worker would use his simple-job wage as threat point in the bargaining, firms would initially agree but then re-open the bargaining at the moment the worker actually quits. In the new bargaining, the worker's outside option is similar to the outside option of an unemployed worker.

employment, like job subsidies, can reduce high-skilled employment in complex jobs, since it raises high-skilled reservation wage.

2.4 Government budget constraint

We focus on self-financing schemes, once taken into account a fixed public good spending (G). The equilibrium value of the benefit b then results from the government budget constraint

$$G + \left(\phi_l X \left(\underline{s} \right) + \int_{\underline{s}}^{\bar{s}} u_l \left(s \right) ds + u_h \right) b = \left(\int_{\underline{s}}^{\bar{s}} e_{ll} \left(s \right) ds + e_{hl} \right) \left(\tau_l + \kappa_l \right) + e_{hh} \tau_h$$

where G represents government spending, $\phi_l X \left(\underline{s} \right)$ is the mass of excluded workers, $u_l \left(s \right)$ and $e_{ll} \left(s \right)$ are unemployment and employment levels of type-s low-skilled workers, u_h is unemployment level of high-skilled workers and e_{hj} is their employment level in type-j jobs.

In order to avoid useless complexity, we assume that the government chooses the levels of κ_l, $\tau_l + b$ and $\tau_h + b$. As we shall see in the next section, this determines unique values of arrival rates, employment and unemployment levels. The value of the grant b is then obtained by the government budget constraint

$$\left(\phi_l + \phi_h \right) b = \left(\int_{\underline{s}}^{\bar{s}} e_{ll} \left(s \right) ds + e_{hl} \right) \left(\tau_l + b + \kappa_l \right) + e_{hh} \left(\tau_h + b \right) - G \tag{19}$$

Then, one obtains the tax levels τ_l and τ_h by subtracting b to the values of $\tau_l + b$ and $\tau_h + b$ chosen by the government. We add the following assumption.

Assumption 1 *The government announces instruments values $\left(\tau_l, \tau_h, b, \kappa_l \right)$ such that: (i) $\tau_l + b \leq \underline{w}$, (ii) $\tau_h + b < s_h$, (iii) $\kappa_l < \bar{s} - \underline{w}$ and satisfying the government budget constraint (19) for equilibrium values of unemployment and employment levels.*

Unemployed people apply for simple jobs if and only if condition (i) is satisfied. Condition (ii) implies that high-skilled unemployed search for complex jobs. Finally, condition (iii) guarantees that simple jobs are profitable when matched with a worker of highest productivity $\bar{s}$.

3   Definition and existence of equilibrium

We show existence and uniqueness of the steady-state cross-skill matching equilibrium for a collection of fiscal instruments $\left(\underline{w}, \tau_l + b, \tau_h + b, \kappa_l \right)$ satisfying Assumption 1. A steady state equilibrium consists of positive arrival rates p_l and p_h that satisfy the two free-entry conditions

$$\frac{c_l}{q_l} = \frac{\int_{\kappa_l + \underline{w}}^{\bar{s}} \frac{s - \kappa_l - \underline{w}}{r + \delta_l} u_l \left(s \right) ds + \frac{\bar{s} - \kappa_l - \underline{w}}{r + \delta_l + p_h} u_h}{\int_{\kappa_l + \underline{w}}^{\bar{s}} u_l \left(s \right) ds + u_h} \tag{20}$$

$$\frac{c_h}{q_h} = \frac{1-\beta}{r+\delta_h+\beta p_h} \left[(s_h - \tau_h - b) - \frac{p_l}{r+\delta_l+p_l+p_h} (\underline{w} - \tau_l - b) \right] \tag{21}$$

Equation (20) is obtained from (2), (3) and (5), while equation (21) is obtained from (14), (17) and (18). Unemployment levels are calculated from steady-state flow equilibrium (see Appendix B)

$$u_l(s) = \phi_l x(s) \frac{\delta_l}{\delta_l + p_l} \text{ and } u_h = \phi_h \frac{\delta_h}{\delta_h + p_h} \frac{\delta_l + p_h}{\delta_l + p_h + p_l} \tag{22}$$

An important characteristic of our setup is that the structure of unemployment matters for the determination of equilibrium arrival rates. Indeed, a high-skilled worker may leave a simple job at any time if he is matched with a complex one. Thus, the separation rate of a high-skilled worker is $\delta_l + p_h$, while the separation rate of a low skilled one is always δ_l. Then, a simple job filled with a high-skilled worker has a lower value than a simple job filled with a low-skilled worker with the same productivity $\bar{s}$. Moreover, the expected value of vacant simple jobs is also affected by the distribution of productivity among low-skilled unemployed. The more they are concentrated around $\bar{s}$, the larger this expected value will be.

Besides being positive, the two arrival rates p_l and p_h have to satisfy the on-the-job search condition $N_{hh} \geq N_{hl}$, that is, using (13) and (17),

$$\beta (r + \delta_l + p_l + p_h) \frac{s_h - \tau_h - b}{\underline{w} - \tau_l - b} \geq (r + \delta_h + \beta (p_l + p_h)) \tag{23}$$

We need the following assumption.

Assumption 2 *Instruments values (τ_l, τ_h) and parameters satisfy either (i) $s_h - \tau_h < \underline{w} - \tau_l$ and $r + \delta_h \leq \beta(r + \delta_l)$, or (ii) $s_h - \tau_h \geq \underline{w} - \tau_l$.*

Proposition 1 *Let $(\tau_l, \tau_h, b, \kappa_l)$ satisfy Assumptions 1 and 2. If a steady-state equilibrium exists, the equilibrium arrival rate p_h belongs to a non-empty interval $\left[\underline{p}_h, \bar{p}_h\right]$ defined by the following inequalities*

$$(s_h - \tau_h - b) - (\underline{w} - \tau_l - b) \leq \frac{(r + \delta_h + \beta p_h) c_h}{(1-\beta)Q_h(p_h)} \leq s_h - \tau_h - b \tag{24}$$

$$\frac{(r + \delta_h - \beta(r + \delta_l)) c_h}{(1-\beta)Q_h(p_h)} \leq (s_h - \tau_h - b) - (\underline{w} - \tau_l - b) \tag{25}$$

$$p_h \geq 0 \tag{26}$$

Proof. If $\underline{w} = \tau_l + b$, equation (21) determines p_h which satisfies inequalities (24), (25) and (26). If $\underline{w} > \tau_l + b$, equation (21) can be rewritten as

$$
\begin{aligned}
p_l &= P_l(p_h, \tau_l + b, \tau_h + b) \\
&\equiv (r + \delta_l + p_h) \frac{(s_h - \tau_h - b) - \frac{(r+\delta_h+\beta p_h)c_h}{(1-\beta)Q_h(p_h)}}{\frac{(r+\delta_h+\beta p_h)c_h}{(1-\beta)Q_h(p_h)} - [(s_h - \tau_h - b) - (\underline{w} - \tau_l - b)]}
\end{aligned}
\tag{27}
$$

For positive p_h, the arrival rate p_l is positive if the numerator and the denominator in P_l are both positive. This leads to inequality (24).

Replacing p_l in (23) by the expression of P_l, leads to inequality (25). To conclude the proof, one has to notice that the combination of the inequalities (24), (25) and (26) defines a convex interval of values of p_h.

Boundaries $\underline{p}_h$ and $\bar{p}_h$ are not always defined by the same inequality. This depends on the sign of $r + \delta_h - \beta(r + \delta_l)$ and $(s_h - \tau_h) - (\underline{w} - \tau_l)$. The case $\{s_h - \tau_h < \underline{w} - \tau_l$ and $r + \delta_h - \beta(r + \delta_l) > 0\}$ leads to an empty interval, since inequality (25) would never be satisfied. In any other case, the interval $[\underline{p}_h, \bar{p}_h]$ is non-empty (details are given in appendix B). ∎

Using (22), the free-entry condition for simple jobs (20) can be rewritten

$$\frac{Q_l(p_l)}{r + \delta_l + p_h} \frac{\left(1 + \frac{p_h}{r + \delta_l}\right) B\left(\kappa_l\right) + \gamma}{1 + \gamma} - \frac{c_l}{\bar{s} - \kappa_l - \underline{w}} = 0 \tag{28}$$

where

$$\gamma = \frac{\phi_h}{(1 - X\left(\kappa_l + \underline{w}\right))\phi_l} \frac{\delta_h}{\delta_l} \frac{\delta_l + p_h}{\delta_h + p_h} \frac{\delta_l + p_l}{\delta_l + p_l + p_h},$$

$$B\left(\kappa_l\right) = \int_{\kappa_l + \underline{w}}^{\bar{s}} \frac{s - \kappa_l - \underline{w}}{\bar{s} - \kappa_l - \underline{w}} \frac{x\left(s\right) ds}{1 - X\left(\kappa_l + \underline{w}\right)} < 1.$$

Notice that $B\left(\kappa_l\right) = 1$, if all low-skilled workers have productivity $\bar{s}$.

If $\tau_l + b = \underline{w}$, then the arrival rate p_h is the solution of equation (21) and p_l is the unique positive solution of equation (28) (since the LHS is decreasing from $+\infty$ to $-c_l/\left(s_l - \kappa_l - \underline{w}\right)$ when p_l increases from 0 to $+\infty$).

Now, if $\tau_l + b < \underline{w}$, equation (28) rewrites, using (27),

$$H\left(p_h, \kappa_l, \tau_l + b, \tau_h + b\right) = 0.$$

In general, there may exist zero or more than one equilibrium. Nevertheless, it is possible to state conditions for existence and uniqueness. We proceed in two steps. First, we state existence and uniqueness when the productivity parameter is $\bar{s}$ for all low-skilled workers. Second, we release this assumption and give conditions for existence and uniqueness when s is distributed on $[0, \bar{s}]$ according to the density function $x\left(s\right)$. In the first case, we have $B\left(\kappa_l\right) = 1$, while, in the second case $B\left(\kappa_l\right) < 1$.

Proposition 2 *Let $(\tau_l, \tau_h, b, \kappa_l)$ satisfy Assumptions 1 and 2. Suppose that the productivity parameter is $\bar{s}$ for all low-skilled workers $(B\left(\kappa_l\right) = 1)$. If*

$$\sigma_l + \sigma_h \leq 1 \ and \ \delta_l \geq \delta_h, \tag{29}$$

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the function H is increasing with respect to p_h on the interval $\left[\underline{p}_h, \bar{p}_h\right]$. If, in addition,

$$H\left(\underline{p}_h, \kappa_l, \tau_l + b, \tau_h + b\right) < 0 \tag{30}$$

then there exists a unique cross-skill matching equilibrium in which the arrival rate for high-skilled workers is denoted p_h^*.

Proof. Existence directly results from (30) and uniqueness is a consequence of Lemma 2 in Appendix B. ∎

Proposition 3 *Let $(\tau_l, \tau_h, b, \kappa_l)$ satisfy Assumptions 1 and 2. Suppose that conditions (29) and (30) in Proposition 2 hold. If*

$$B\left(\kappa_l\right) > \frac{r + \delta_l}{r + \delta_l + p_h^*}, \text{ for all } \kappa_l \in [-\underline{w}, \bar{s} - \underline{w}], \tag{31}$$

then there exists a unique cross-skill matching equilibrium. Moreover, p_h increases with respect to $\tau_l + b$ and κ_l and decreases with respect to $\tau_h + b$.

Proof. Note that $H\left(p_h, \kappa_l, \tau_l + b, \tau_h + b\right)$ shifts downward when $B\left(\kappa_l\right)$ diminishes. By consequence, there exists an equilibrium. It remains to show that the condition $B\left(\kappa_l\right) > \frac{r + \delta_l}{r + \delta_l + p_h^*}$ implies uniqueness. We proceed by stating that, under this condition,

$$B\left(\kappa_l\right) > \frac{r + \delta_l}{r + \delta_l + p_h}$$

which implies $\frac{\partial H}{\partial p_h} > 0$ and thus that the equilibrium is unique. Since H is shifted downward, the new equilibrium arrival rate p_h (with $B < 1$) is higher than p_h^*. Thus

$$B\left(\kappa_l\right) > \frac{r + \delta_l}{r + \delta_l + p_h^*} > \frac{r + \delta_l}{r + \delta_l + p_h}$$

which states uniqueness.

Moreover $P_l\left(p_h, \tau_l + b, \tau_h + b\right)$ is increasing (decreasing) with respect to its second (third) argument. Since, an increase in p_l has a negative effect on the LHS in (28), the function H is increasing (decreasing) with respect to $\tau_l + b$ (resp. $\tau_h + b$). Finally, H is decreasing with respect to κ_l. This concludes the proof. ∎

4 Equilibrium properties

In order to graphically illustrate equilibrium properties, we assume $\sigma_l = \sigma_h = 1/2$ and $\delta_l \geq \delta_h$. Rearranging equation (28), we obtain:

$$\left(\frac{p_l}{p_l^s(\kappa_l)} - \frac{1}{B(\kappa_l)} \frac{r + \delta_l}{r + \delta_l + p_h} \right) \left[1 + \frac{\phi_h}{(1 - X(\kappa_l + \underline{w}))\phi_l} \frac{\delta_h}{\delta_l} \frac{\delta_l + p_h}{\delta_h + p_h} \frac{\delta_l + p_l}{\delta_l + p_l + p_h} \right]$$

$$= 1 - \frac{1}{B(\kappa_l)} \frac{r + \delta_l}{r + \delta_l + p_h} \qquad (32)$$

with

$$p_l^s(\kappa_l) \equiv B(\kappa_l) \frac{\lambda_l^2}{c_l} \frac{(\bar{s} - \kappa_l - \underline{w})}{r + \delta_l}$$

The level $p_l^s(\kappa_l)$ corresponds to the arrival rate of simple jobs if no high-skilled worker were posting for a low-skilled job. For a given arrival rate p_h, we obtain a polynomial of degree two in p_l whose solutions are given in Appendix B. We select the solution $p_{l,l}^c(p_h)$ that satisfies the condition

$$p_{l,l}^c\left(\left(\frac{1}{B(\kappa_l)} - 1 \right)(r + \delta_l) \right) = p_l^s(\kappa_l)$$

and is positive. It is a continuous function such that $\lim_{p_h \to \infty} p_{l,l}^c(p_h) = p_l^s(\kappa_l)$. We plot this relation between p_l and p_h in Figure 2 (low-skilled job market curve). On the other hand from (27), we have

$$p_{l,h}^c(p_h) = \frac{(r + \delta_l + p_h)\left(s_h - \tau_h - b - \frac{c_h(r + \delta_h + \beta p_h)p_h}{(1-\beta)\lambda_h^2} \right)}{\frac{c_h(r + \delta_h + \beta p_h)p_h}{(1-\beta)\lambda_h^2} - [(s_h - \tau_h - b) - (\underline{w} - \tau_l - b)]} \qquad (33)$$

which is a decreasing function of p_h that is positive for $\underline{p}_h < p_h < \overline{p}_h$. We also plot this relation in Figure 2 (high-skilled job market curve). The steady-state equilibrium corresponds in Figure 2 to the intersection of the curves (1) and (2) that respectively represent the free-entry conditions in the high-skilled sector and in the low-skilled one.

Interactions between high-skilled and low-skilled workers on the low-skilled job market explain the U-shape of the free-entry condition for simple positions. As stated before, the two externalities pointed by [Gautier, 2002] play a important role in our setting: (i) downgraded high-skilled workers have a higher productivity mean, but (ii) continue to apply for complex jobs, shortening the expected duration of simple job. An increase in p_h has therefore two confluent effects. First, it reduces the share of simple jobs filled with workers who have productivity $\bar{s}$, reducing the expected value of filled simple job. This effect, that we call the *productivity effect*, reduces p_l. Second, an increase in p_h enhances the probability for high-skilled workers to find more suitable job for them and shortens the expected duration of simple job. This *turnover effect* also reduces p_l.

Figure 2: Change in equilibrium related to a change in τ_h
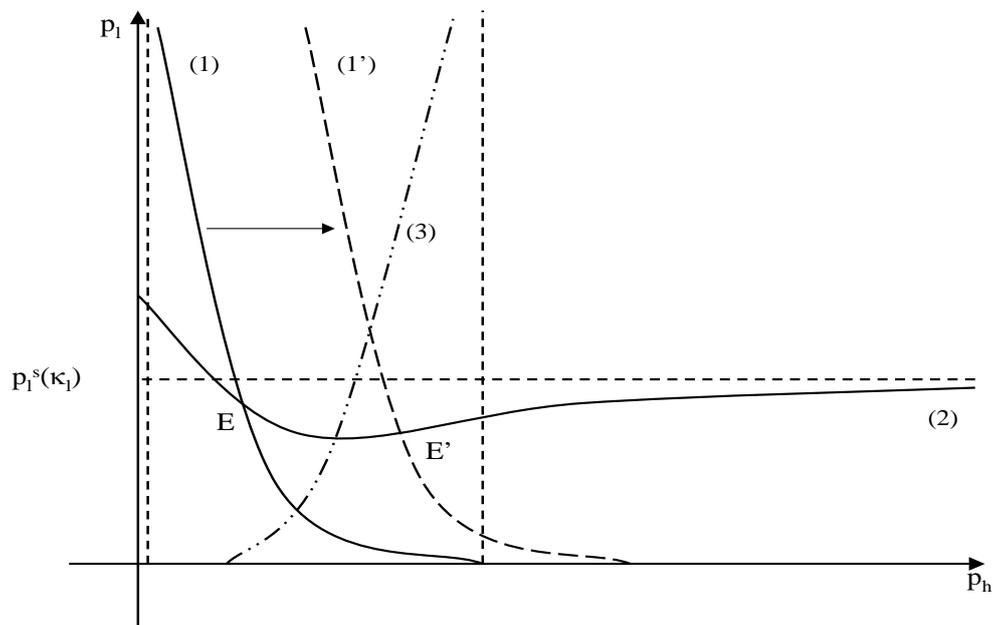
Both effects, *turnover* and *productivity*, explain the decreasing part of the low-skilled curve in Figure 2. For low values of p_h, the *turnover* effect vanishes. Since high-skilled workers are more productive and may quit at a very low rate, the expected value of simple jobs is higher than in a segmented simple job market, i.e. $p_l > p_l^s(\kappa_l)$. Nevertheless, the higher p_h, the more important the *turnover* effect. Therefore, in Figure 2, the free-entry curve for low-skilled jobs passes below the segmented market value $p_l^s(\kappa_l)$. But, an increase in p_h also reduces the number of high-skilled unemployed. As p_h increases, the simple job market moves closer to the segmented equilibrium, where only low-skilled workers look for simple jobs. The turnover and productivity effects are progressively attenuated and, when p_h goes to infinity, the arrival rate p_l tends to the segmented market value $p_l^s(\kappa_l)$.

In Figure 2, the two curves intersect at E in the decreasing part of the low-skilled curve. In this sense, p_h and p_l can be viewed as "substitutes". A small increase in p_h can be associated to a decrease in p_l. Conversely, in an economy where the two curves intersect in E' on the increasing part of the low-skilled curve ((1') and (2)), p_h and p_l can be viewed as "complementary".

We now turn to the effect of the fiscal instruments ($\kappa_l, \tau_l, \tau_h, b$). They work differently on the two free-entry conditions. The low-skilled job tax κ_l affects only the position of the low-skilled curve, whereas other instruments affect the high-skilled curve.

Figure 3: Example of a change in equilibrium related to a change in κ_l
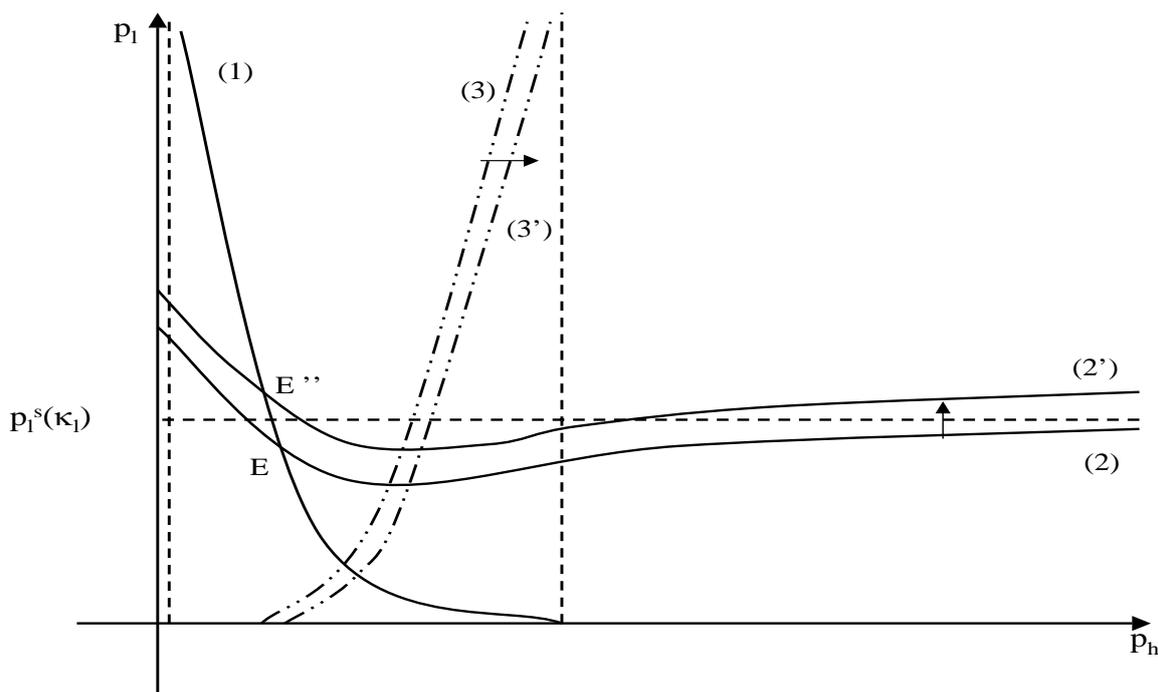
A marginal increase in b enhances the outside opportunity of high-skilled workers in the wage bargaining. This leads to a rise in w_h and, consequently, a fall in the mass of complex jobs through the free-entry condition. We then observe a decrease in p_h. Depending on whether the initial level of p_h belongs to the complementarity or substitutability regime, the resulting effect on p_l may be respectively negative or positive. *An increase in the tax* τ_h is shared between the firm and the workers because of the wage bargaining. Here again, the wage w_h increases and one observes the same qualitative changes in p_h and p_l. *An increase in* τ_l has also no effect on the low-skilled curve, since there is no bargaining on simple jobs. Nevertheless, a higher tax τ_l means that low-skilled jobs become less attractive for high-skilled workers, weakening their outside opportunity in the wage bargaining on w_h. This results in a higher arrival rate p_h.

Let us now turn to *a decrease in* κ_l, the policy that we focus on. For a given p_h, this may have contradictory effects on low-skilled unemployment. Job subsidy lowers the productivity threshold for positive profit and reduces classical unemployment. But the widening of the interval of productivity that allows for positive profits has also a double effect on frictional unemployment: an increase in the competition between workers in order to find a job and a reduction in the expected productivity of low-skilled jobs. Both increase low-skilled unemployment. Simultaneously, the fall in κ_l reduces the cost of simple jobs and contributes positively to profits generated by these jobs.

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This tends to increase the number of simple vacancies and affects positively p_l. Consequently, some additional high-skilled unemployed will find downgraded positions. Summarizing these results, a decrease in κ_l allows for a reduction in classical unemployment and has an ambiguous effect on p_l, i.e. the average duration of low-skilled vacancies and of low-skilled frictional unemployment. Graphically, the ambiguous effect on p_l means that the low-skilled curve may shift upward or downward depending on the relative importance of the contradictory effects.

From the above discussion, when the change in p_l is positive, we wind up with a decrease in low-skilled unemployment. When negative, the average decrease in low-skilled job productivity may induce an increase in the frictional low-skilled unemployment ($u_{l,f}$) that may dominate the reduction of classical unemployment ($u_{l,c}$). But if the fall in average low-skilled job productivity remains moderate, the decrease in κ_l would result in an decrease in p_l, allowing nevertheless for a lower low-skilled unemployment $u_{l,c} + u_{l,f}$. This would be associated to longer frictional unemployment spells.

The consequences of the above mechanisms on p_h, when for instance p_l increases, are quite simple: change in p_h is negative since high-skilled workers have now a better outside opportunity and can claim for higher wages w_h. Notice that the fall in p_h, in turn, increases the number of downgraded high-skilled workers[10] and contributes to a higher average productivity of low-skilled jobs.

We are now able to describe the consequences of balanced-budget fiscal policies that consist in subsidizing low-skilled jobs. We only focus on a financing scheme that amounts to an increase in taxes on high-skilled jobs τ_h [11]. Such policies imply shifts of the two free-entry conditions. Let us assume that a fall in κ_l shifts upward the free-entry condition for low-skilled jobs and increases the arrival rate p_l for any given value of p_h, as plotted in Figure 3. The direct consequence of the upward shift of the low-skilled free-entry condition is to increase p_l and reduce p_h. Therefore, the number of filled simple jobs $e_{ll} + e_{hl}$ rises, while the number of filled complex jobs e_{hh} falls (see equations (35) in Appendix B). The overall decreasing number of unemployed people reduces the burden of unemployment benefits but this may not offset the cost of the subsidy policy that applied to every low-skilled position. This may lead to some budget deficit that can be compensated by an increase in τ_h. Combining Figures 2 and 3, we can picture the situation in which resources that finance the subsidy are levied through an increase in the tax on high-skilled jobs τ_h. The change in τ_h

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[10]Indeed, from equation (35) in Appendix B, we have $e_{hl} = \frac{\phi_h \delta_h}{p_h + \delta_h} \frac{p_l}{p_h + p_l + \delta_l}$, which is decreasing with respect to p_h.

[11]Reducing unemployment benefits b is an alternative scheme we do not consider because it can affect voluntary unemployment we do not model.

shifts the high-skilled free-entry condition to the left, leading to a lower arrival rate on high-skilled job p_h. Increasing high-skilled job taxes affects negatively the number of high-skilled jobs. Due to high-skilled worker bargaining power, tax increase is shared between the firms and the workers. This increases the frictional high-skilled worker unemployment and the burden of unemployment benefits. This effect is nevertheless limited by an increase in the number of downgraded high-skilled workers. In some circumstances, these additional downgraded high-skilled workers may even outnumber the high-skilled job destructions leading to a decrease in high-skilled unemployment. This change in the probability of finding a high-skilled job can induce a negative change in the number of low-skilled jobs when current equilibrium position is in the complementarity zone or a positive change in the alternative zone. The indirect effect on p_l may then be positive or negative. We end up with three schematic situations that are presented in Figures 4, 5 and 6. In Figure 4, the substitutability effect reinforces the consequences of the subsidy policy on the probability of finding a low-skilled position. On the contrary, in Figures 5 and 6, the complementary effect plays against the subsidy policy. It reduces its effects in Figure 5 and can offset them in Figure 6 for some well-chosen parameter values. Unemployed high-skilled workers and re-entered low-skilled workers swell the ranks of people looking for a low-skilled position. This increases frictional unemployment on the low-skilled job market and affects negatively the probability of finding a low-skilled position.

5 An example: The French economy

5.1 Calibration

As illustrated in the preceding section, low-skilled job subsidies can have a positive or negative effect on low-skilled unemployment. Moreover, a reduction in low-skilled unemployment may be obtained with longer unemployment spells. In practice, it is of interest to determine in which situation we stand. We propose to illustrate this issue with a simple calibration in the case of the French economy.

In the sequel, we only consider jobs in market activities. The population under study is therefore composed of the working population minus civil servants plus those out of the working population who benefit from the minimum welfare payment given to those who are not entitled to unemployment benefit ("R.M.I"). We improperly use the expression "labor force" to refer to this population of 19.3 millions, 3.9 millions of unemployed people included[12].

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[12]Our definition of unemployed people does not match the ILO's definition, in order to take into account classical unemployment, that comprises people who could be transitorily discouraged.
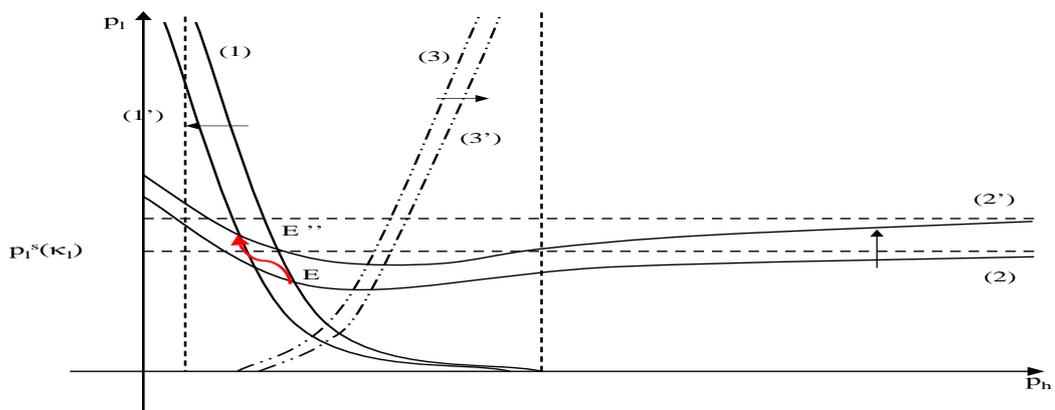
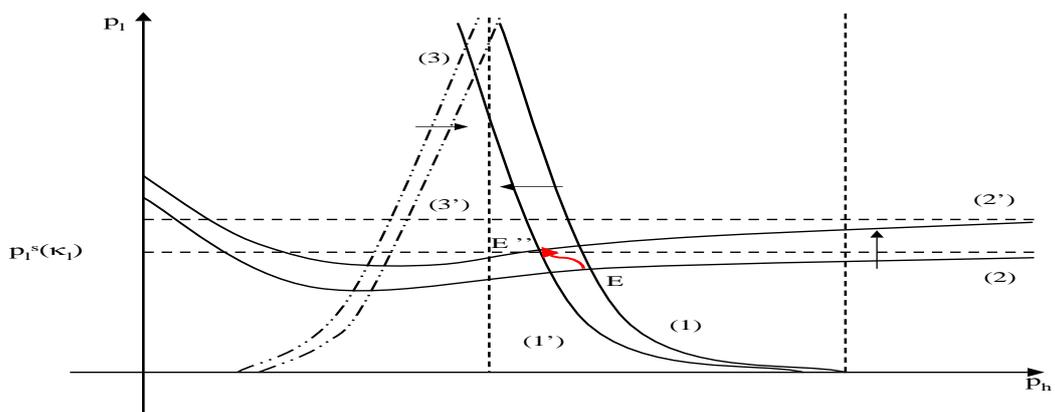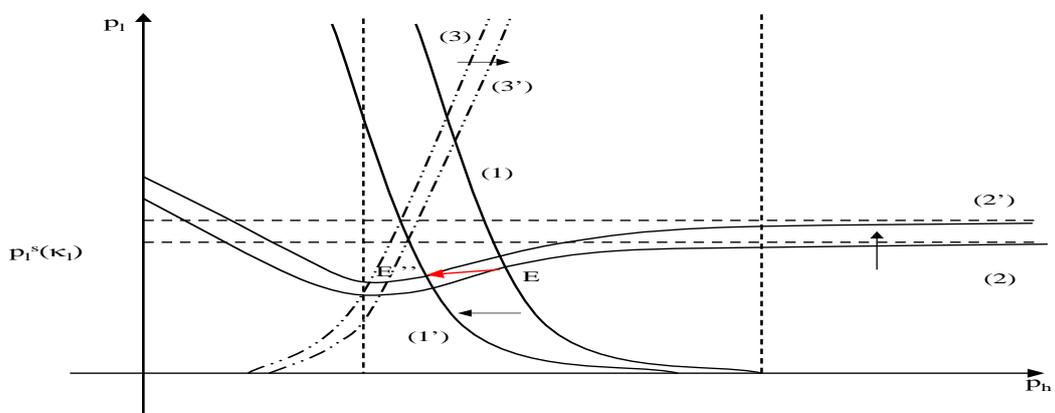Figure 4: Situation 1

Figure 5: Situation 2

Figure 6: Situation 3

We are first led to raise the issue of the definition of skilled and unskilled persons. We cannot uniquely define these classes from the last academic degree (particularly for the older persons), there exists a gap between the last degree and the skills required to hold the position declared by the firms or the employees. Following [Burnod and Chenu, 2001], we consider as skilled employees, persons that hold a position in which is used his/her professional know-how acquired by educational or professional training. We thus construct our two classes on the basis of the professional position (or the last position for the unemployed ones) as described in Burnod and Chenu (2001) and the last degree, both declared in INSEE Employment Survey. If their last degree is a A-level or corresponds to a college training and they hold a high-skilled professional position, they are considered as high-skilled person. For those who are unemployed and have never worked, we only use the last academic degree. Unskilled persons are mainly unskilled workers and clerks (office clerks, sales clerks, janitors, night watchmen,...) without any degree or a degree less than a A-level. In 1998 according to INSEE Employment survey, they accounted for 36.2 of the labor force ($\phi_l = 0.362$). Based on this classification, the survey allows us to compute the rate of unemployment for each class and the share-out between high-skilled and low-skilled employment and unemployment, we get : $e_{hh} = 0.547$, $e_{hl} = 0.029$ and $e_{ll} = 0.222$, $u_h = 0.062$ and $u_l = 0.139$. For sake of simplicity, we assume that voluntary unemployment is only composed of low-skilled persons, so that low-skilled unemployment has to be broken down into voluntary $(u_{l,v})$, classical $(u_{l,c})$ and frictional unemployment $(u_{l,f})$. We assume that low-skilled productivities are distributed in $[0, \overline{s}]$ according to a $Beta\,(a_1, a_2)$ distribution, where a_1 and a_2 are two real positive numbers. Depending on the values of these parameters, this distribution can have no mode or a mode between 0 and $\overline{s}$.

We then use administrative data sets (DADS) to measure the median monthly wage cost of low-skilled and high-skilled workers declared by firms. The above model focuses on a limited number of questions. In particular, pension, health and family contributions and benefits are overlooked and there are not retirees and children. We keep to this simplified approach and consider that these contributions are equal to the benefits over a life-cycle. We present all the details of the calibration in Appendix C. Table 1 gives the values we end up with. For such a set of parameters, Figure 7 illustrates that there exists a unique cross-skill equilibrium. The distribution of low-skilled worker productivity is given in Figure 8. It is characterized by a mode in $\overline{s}$ and suggests a continuous distribution of productivities over low- and high-skilled workers.

From the calibration exercise, we first can notice that the probability of finding a job for low-

Table 1: Calibrated parameters

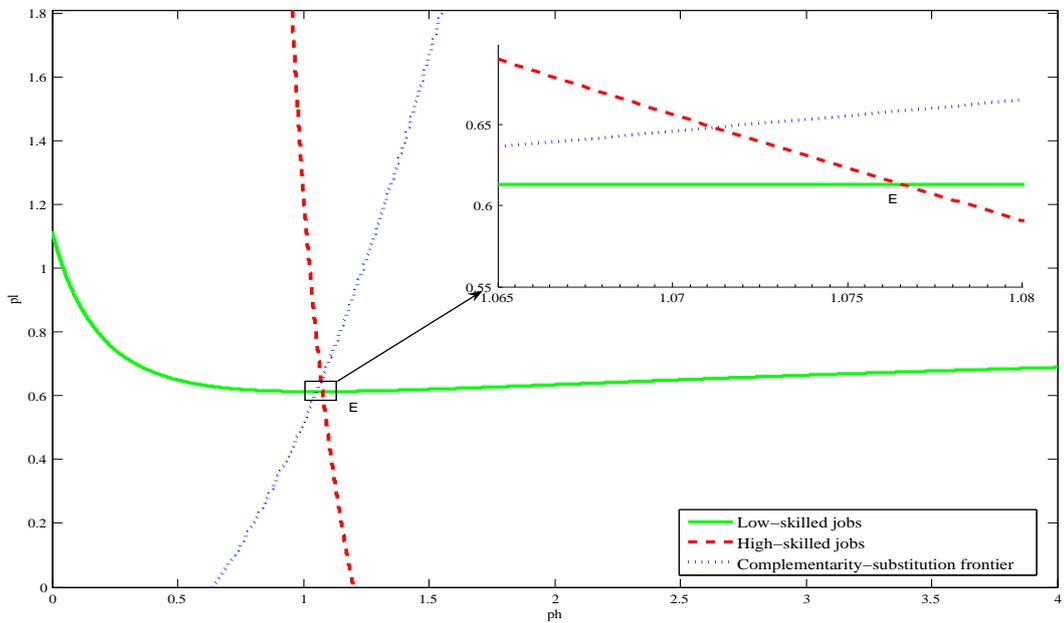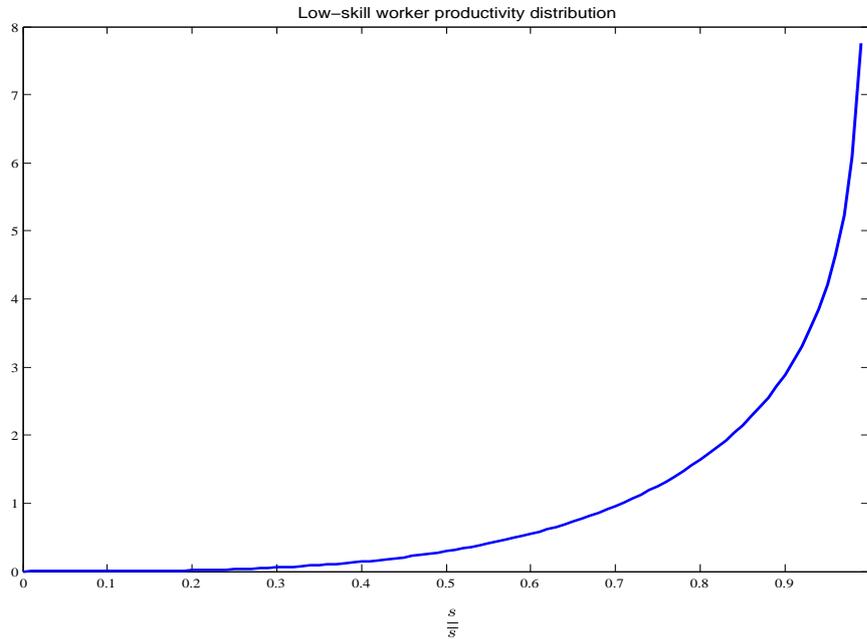| parameters	ϕ_l	β	r	σ	δ_h	δ_l
calibrated values	0.362	0.25	0.05	0.5	0.1791	0.2341
parameters	b	τ_l	τ_h	κ_l	G	$u_{l,v}$
calibrated values	0.34	0.0227	0.143	-0.0218	0.0098	0.0281
parameters	s_h	$\overline{s}$	a_1	a_2	$\frac{c_l}{\lambda_l^2}$	$\frac{c_h}{\lambda_h^2}$
calibrated values	1.7632	1.608	4.048	0.696	1.793	1.5133

Figure 7: Equilibrium

Figure 8: Low-skilled worker productivity distribution

skilled workers is close to the lowest value. The frictional unemployment and consequently the waiting time in unemployment are very large for low-skilled people. There is then a large room for maneuver for fiscal policies in order to decrease the delay for finding a job. Moreover, the equilibrium is closed to the frontier between the substitution and complementarity zones, which affects the design of optimal labor market policies. When limiting our attention to equilibrium positions, a change in a fiscal instrument can have non monotonous effect on p_l depending on the amplitude of the shift of the curves it implies. A small increase in τ_h may induce a decrease in the equilibrium value of p_l but a larger one may have the opposite effect. Second, the simple-job-firm free-entry curve is flat around the equilibrium position. Large changes in the fiscal schemes are necessary to get a significant change in p_l. We therefore conclude that we are in situation 2 (see Figure 5). A balanced low-skilled tax cut policy leads to a proportional small decrease of high-skilled job rate, and a more important increase of low-skilled job rate.

5.2 Study of low-skilled payroll tax cuts financed by an increase of tax burden on high-skilled labor

5.2.1 Empirical illustration

In Figure 9, we illustrate the change in the equilibrium values when the low-skilled job subsidy is modified and the government budget is balanced by an appropriate increase in the high-skilled wage taxes, unemployment benefits are unchanged to not affect voluntary unemployment. In the first plot, we observe that the high-skilled wage tax rate decreases with κ_l, but additional computations show that for larger values of κ_l, it increases. When low-skilled jobs are significantly taxed, the implied increase in unemployment and associated unemployment benefits necessitates larger net taxes on high-skilled workers. This echoes a "Laffer"-type property.

In presence of classical and frictional unemployment, the effects of low-skilled job subsidy policy get smaller as the subsidies increase. Classical unemployment is reduced and in our calibration exercise, frictional one decreases, but average productivity tends to decrease. The decreasing amplitude of consequences of low-skilled job subsidy policy is illustrated in Figures 10, 11 and 12.

Figure 10 illustrates that a 1% increase in subsidy is never self-financed by the implied decrease in the total amount of unemployment benefits, but requires a larger and larger increase in high-skilled job tax to balance the budget. We can notice that this result is obtained as we observe a simultaneous reduction of high-skilled employment and high-skilled unemployment (Fig. 11). When κ gets smaller, low-skilled unemployment (classical and frictional) decreases and the number of downgraded high-skilled workers increases as due to higher net taxes on high-skilled jobs. This change is nevertheless accompanied by a decreasing number of high-skilled unemployed, downgraded high-skilled workers outnumbering the high-skilled job destructions.

As subsidies get larger, a marginal increase in subsidies induces a smaller decrease in classical unemployment and a larger increase in the number of downgraded high-skilled workers. This is illustrated in Figure 12 by the ratios of the marginal change in the number of downgraded high-skilled workers over, on the one hand, that in the number of low-skilled workers who leave classical unemployment and on the other hand, that in the number of low-skilled job creations. From our calibration, the level of subsidies in France induces an equilibrium in which any low-skilled worker reentering the job market is accompanied by 0.4 high-skilled worker accepting to be downgraded (Fig. 12). Similarly, we observe that the creation of four low-skilled jobs is accompanied by the downgrading of one high-skilled worker.

Shift of high-skilled workers to low-skilled jobs implies that equilibrium with minimum unem-

Figure 9: Changes in equilibrium values with κ when b is given and τ_h ensures government budget balance
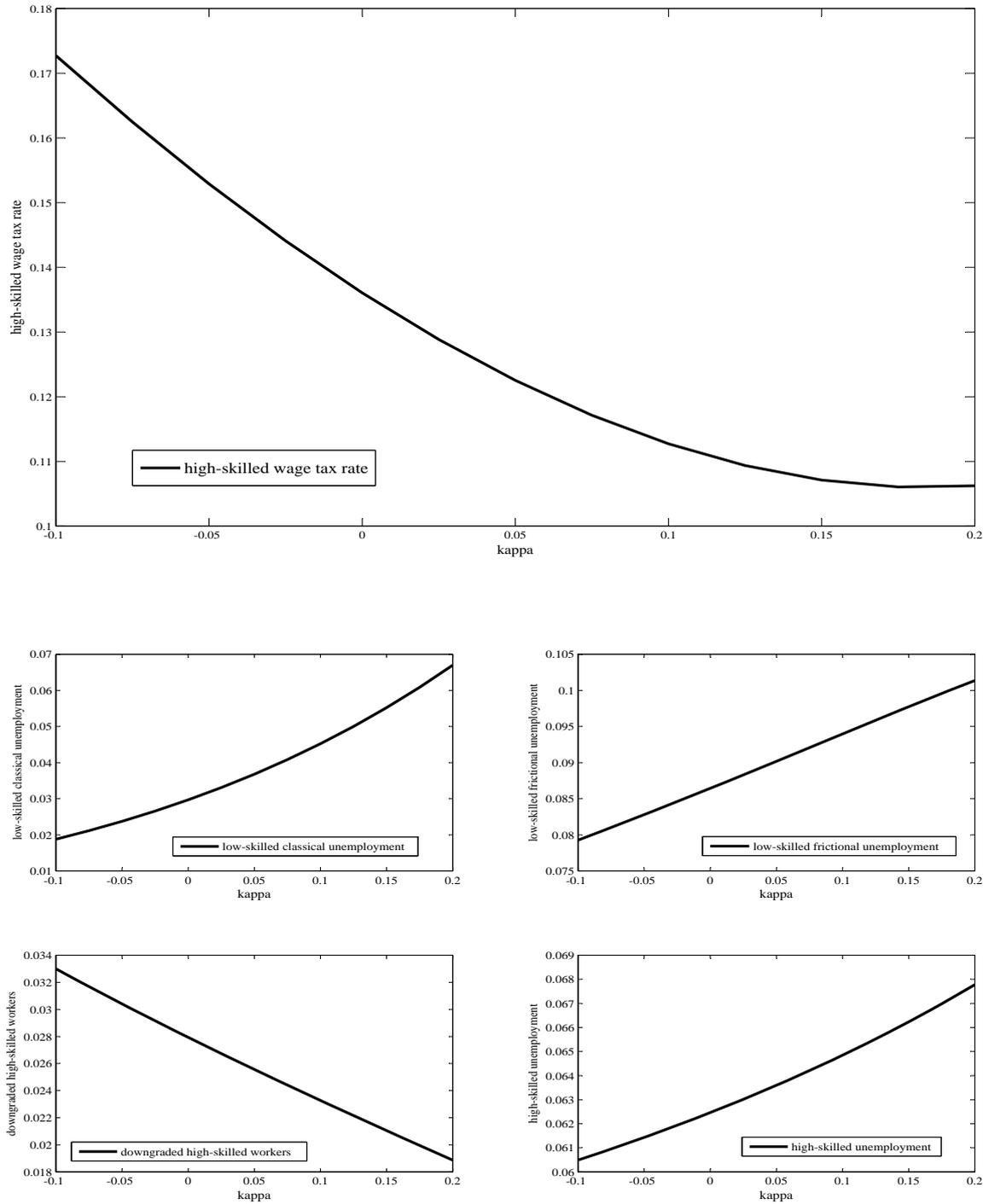
Figure 10: A marginal change in low-skilled job subsidies induces...
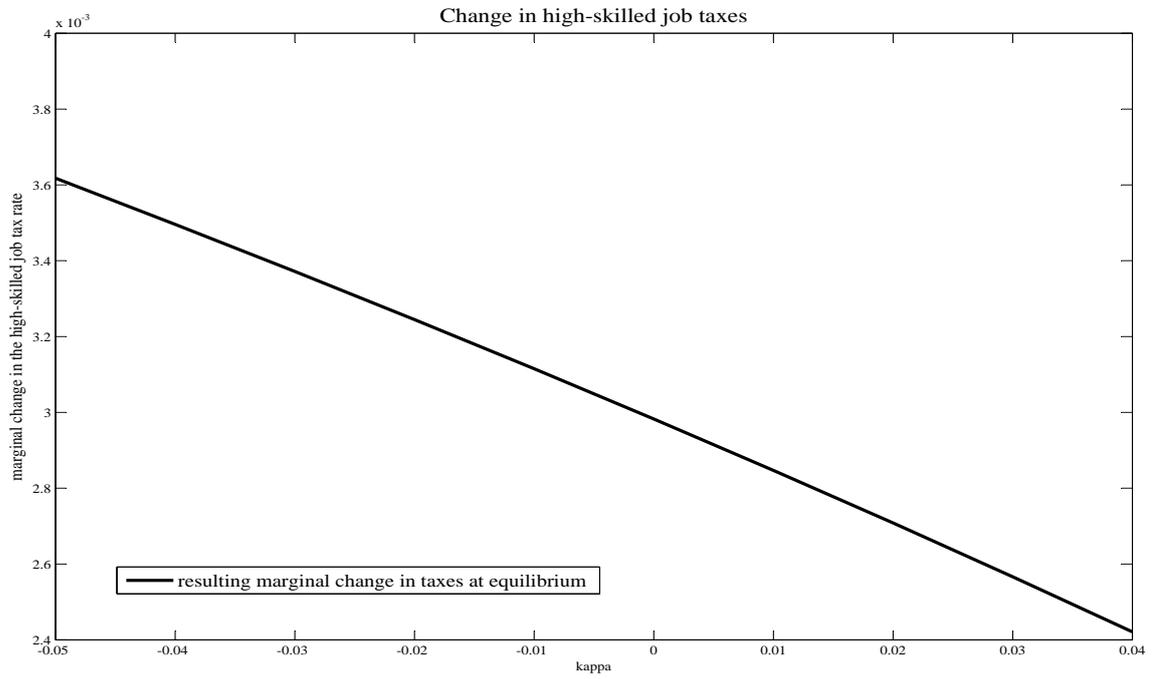

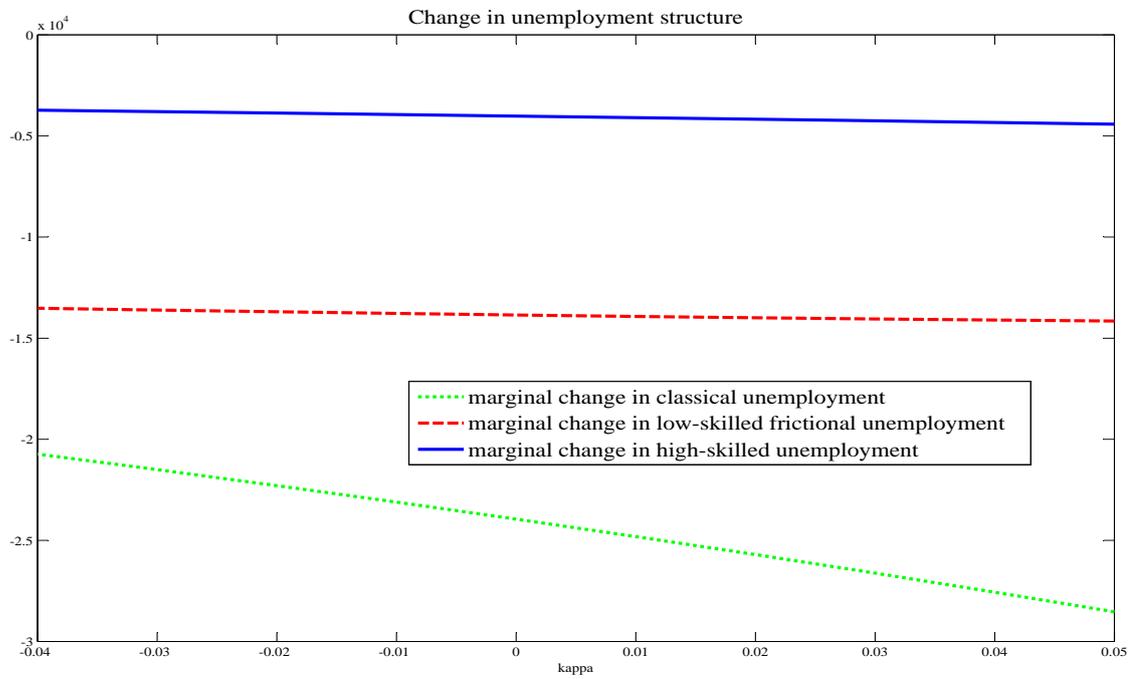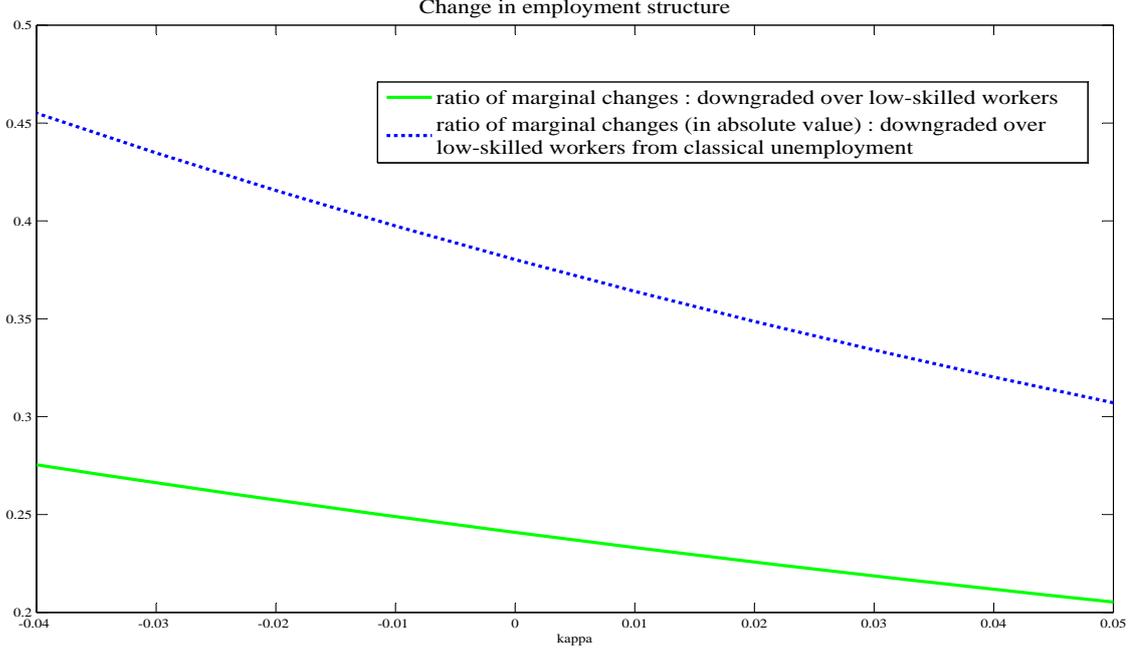Figure 11: A marginal change in low-skilled job subsidies induces...

Figure 12: A marginal change in low-skilled job subsidies induces...

ployment does not correspond to that induced by the maximization of a utilitarian criterion.

5.2.2 Normative properties

In order to evaluate simple-jobs subsidy policy, we consider the traditional utilitarian criterion, defined as the sum of individual utilities. Recalling that steady state intertemporal utility of an excluded worker is b/r, the utilitarian criterion writes

$$
\begin{aligned}
W &= \left(\phi_l X \left(\underline{w} + \kappa_l\right)\right) \frac{b}{r} + \left(\int_{\underline{w}+\kappa_l}^{\bar{s}} u_l\left(s\right) ds\right) U_l + u_h U_h \\
&\quad + \left(\int_{\underline{w}+\kappa_l}^{\bar{s}} e_{ll}\left(s\right) ds\right) N_{ll} + e_{hl} N_{hl} + e_{hh} N_{hh}
\end{aligned}
$$

Let us rewrite this equation in terms of flows of income:

$$
\begin{aligned}
rW &= \left(\phi_l + \phi_h\right) b + \phi_l \left(1 - X\left(\underline{w} + \kappa_l\right)\right) \left(rU_l - b\right) + \phi_h \left(rU_h - b\right) \\
&\quad + \left(\int_{\underline{w}+\kappa_l}^{\bar{s}} e_{ll}\left(s\right) ds\right) r\left(N_{ll} - U_l\right) + e_{hl} r\left(N_{hl} - U_h\right) + e_{hh} r\left(N_{hh} - U_h\right)
\end{aligned}
$$

and, from equations (8) and (10), we deduce

$$
\begin{aligned}
rW &= \left(\phi_l + \phi_h\right) b + \left[\phi_l \left(1 - X\left(\underline{w} + \kappa_l\right)\right) p_l + r \int_{\underline{w}+\kappa_l}^{\bar{s}} e_{ll}\left(s\right) ds\right] \left(N_{ll} - U_l\right) \\
&\quad + \left[\phi_h \left(p_l + p_h\right) + r \left(e_{hl} + e_{hh}\right)\right] \left(N_{hl} - U_h\right) + \left(\phi_h p_h + e_{hh} r\right) \left(N_{hh} - N_{hl}\right) \quad (34)
\end{aligned}
$$

At steady-state, rW is the total flow of income distributed among workers. Equation (34) presents a decomposition of this total flow. The first term $(\phi_l + \phi_h)\, b$ represents the fact that anyone can expect to receive at least the minimum income b. The three other terms represent the expected flows of income that workers will receive as far as they are not excluded. First, there is the expected flow of income for an unskilled worker to have a productivity parameter above $\underline{w} + \kappa_l$ and thus to have the opportunity to apply for a simple job. Of course this term is proportional to the difference $N_{ll} - U_l$ and will be positive as soon as $\tau_l + b < \underline{w}$. Second, we have isolated the expected flow of income that a skilled worker enjoys at a cross-skill matching equilibrium since he can apply for simple jobs. This term is proportional to $N_{hl} - U_h$ and, here again, will be positive as soon as $\tau_l + b < \underline{w}$. Finally, skilled workers have also the opportunity to apply for complex jobs. This gives them an excepted flow of income represented by the term proportional to $N_{hh} - N_{hl}$ in equation (34). From the wage bargaining equation, one may deduce that

$$N_{hh} - U_h = \frac{\beta}{1 - \beta} J_{hh} = \frac{\beta}{1 - \beta} \frac{c_h}{q_h} > 0$$

Thus, the presence of search frictions in the labor market forces the utility of high-skilled workers to be higher than high-skilled unemployed $(N_{hh} > U_h)$. This excludes that both equalities $N_{hh} = N_{hl}$ and $N_{hl} = U_h$ can be jointly satisfied. This implies that the maximum of social surplus is associated to particular constraints on income distribution that limit the effects of redistributive policies.

In Figure 13, we have plotted the variation of the total flow of income rW with respect to the subsidy rate, keeping the unemployment benefit b constant. The net lump-sum wage tax for high-skilled job is changed accordingly to balance the government budget. We have also drawn the relative importance of the different expected flow of income that a worker may expect according to his skill. We observe that the minimum of unemployment under incentive constraints (6) corresponds to the situation $N_{hl} = N_{hh}$, which is very different from the Utilitarian criterion. Implementing low-skilled job subsidy policy aims at reducing unemployment but it may induce some costs which move the equilibrium far away from the Utilitarian criterion. This last one is at its maximum for a moderate low-skilled job subsidy, slightly larger in absolute value than the current one in France.

The comparison of the welfare levels for each type of situations between the calibrated situation and the minimum unemployment equilibrium in Table 2 shows that the second situation corresponds to a deterioration of unemployed high-skilled worker situations and an improvement of low-skilled persons who are out of classical unemployment. From a fiscal point of view, this corresponds to a large subsidy of low-skill jobs balanced by a large increase in net high-skilled wage taxes that reduces their return and their number. On the labor market, this corresponds to a reduction of job
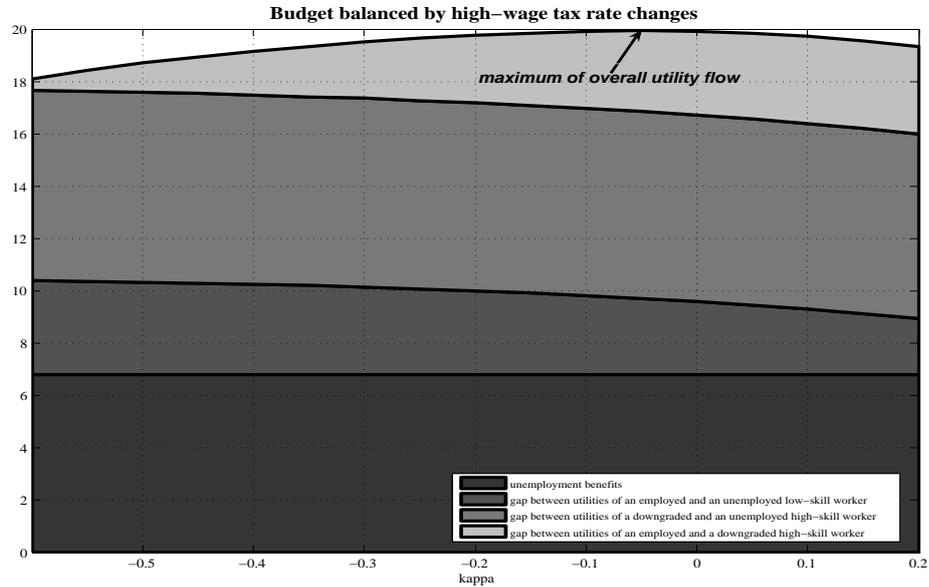
Figure 13: Expected flows of income

Table 2: Comparison of welfare levels

	N_{ll}	N_{hl}	N_{hh}	$U_{l,c}$	$U_{l,f}$	U_h	W
Calibrated values	16.22	22.775	22.995	6.8	15.51	22.452	19.924
Unemployed minimization	17.771	18.216	18.216	6.8	17.392	17.932	17.71

opportunities for high-skilled people in low-skilled positions and a decrease in low-skilled person unemployment. Reducing their job opportunities implies a larger number of downgraded high-skilled workers that stay longer in these positions. This may affect the return of high-skilled degrees and create some disincentives to invest in higher education.

6 Conclusion

In this paper, we provide a formal set-up to analyze the impact of low-skilled job subsidies policies on labor productivity. By introducing heterogeneous skills and possible downgrading of the high-skilled workers, we show that the effectiveness of such policies in reducing the classical unemployment are decreasing. In fact, any additional classical unemployed re-entering the job market is accompanied

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by an increasing number of downgraded high-skilled workers.

When the model is calibrated on French data, it is found that for five low-skilled workers leaving classical unemployment, two high-skilled workers are downgraded (although they might have been previously unemployed).

In our model, differences in productivity among workers are due to differences in their initial ability, but they can also be influenced by the unemployment duration, the type of job occupied and their career. Still, our model does not allow us to analyze the impact of unemployment and downgrading on workers' skill accumulation. We provide groundwork for future research comparing the effectiveness of training policies along the career with the ones based on low-skilled job subsidies to reduce classical unemployment. Importantly, as underlines also by [Oskamp and Snower, 2006], these two policies may affect initial educational choices since returns of schooling may change.

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7 Appendices

7.1 Appendix A : List of symbols

r	instantaneous discount rate
G	fixed public good spending
β	parameter of the negotiation power of high-skilled workers in the Nash bargaining
$\phi_i, i = l, h$	percentage of i-type workers
$x(s)$	probability density of low-skilled worker output
$X(s)$	cumulative density of low-skilled worker output
$\overline{s}$	upper bound of the support of low-skilled worker output
s_h	high-skilled worker output
$\delta_i, i = l, h$	exogenous flow rate of termination of employment
$\lambda_i, i = l, h$	constant of the i-type matching function
$c_i, i = l, h$	cost of posting a i-type vacancy
c_r	search cost (=0 in the text)
$p_i, i = l, h$	probability for an unemployed to move into a i-type job
$q_i, i = l, h$	probability to fill a i-type vacancy
τ_h	net lump-sum wage tax for high-skilled jobs
τ_l	lump-sum wage tax for low-skilled jobs
κ_l	low-skilled job tax
b	unemployment benefit
$\underline{w}$	minimum wage
w_h	high-skilled worker wage
$V_i, i = l, h$	discounted value of a vacancy of type i
$U_i, i = l, h$	discounted value of being unemployed (frictional unemployment)
U_l^e	discounted value of being excluded from the job market
$N_{i,j}, i, j = l, h$	discounted value of being employed for a i-type worker in a j-type firm
$J_{i,j}, i, j = l, h$	discounted value of a j job filled by i-type worker
$e_{i,j}, i, j = l, h$	percentage of i-type workers employed in j-type jobs
$u_i, i = l, h$	percentage of i-type unemployed
$u_{l,c}, u_{l,v}, u_{l,f}$	percentage of classical, voluntary and frictional unemployed low-skilled people (calibration section)

7.2 Appendix B : Proofs

7.2.1 Unemployment and employment levels.

At the cross-skill matching equilibrium satisfying inequalities (6), the flow equilibrium conditions for each state become:

$$p_l u_h = (p_h + \delta_l) e_{hl}$$

$$p_l u_l (s) = \delta_l e_{ll} (s)$$

$$p_h (u_h + e_{hl}) = \delta_h e_{hh}$$

Then, we obtain

$$e_{ll} (s) = \frac{\phi_l x (s) p_l}{p_l + \delta_l}, \quad e_{hl} = \frac{\phi_h \delta_h}{p_h + \delta_h} \frac{p_l}{p_l + p_h + \delta_l}, \quad e_{hh} = \frac{\phi_h p_h}{p_h + \delta_h}, \tag{35}$$

$$u_l (s) = \frac{\phi_l x (s) \delta_l}{p_l + \delta_l}, \quad u_h = \frac{\phi_h \delta_h}{p_h + \delta_h} \frac{p_h + \delta_l}{p_l + p_h + \delta_l} \tag{36}$$

7.2.2 Interval $\left[\underline{p}_h, \bar{p}_h \right]$

Proposition 1 states the range of values of the arrival rate p_h that allow for positive arrival rates p_h and p_l, and that satisfy incentives constraints $N_{hh} \geq N_{hl} \geq U_h$.

Let introduce the following notations

$$S_l = \underline{w} - (\tau_l + b)$$

$$S_h = s_h - (\tau_h + b)$$

Three cases have to be considered

- $S_h - S_l \geq 0 \geq r + \delta_h - \beta(r + \delta_l)$: the interval $\left[\underline{p}_h, \bar{p}_h \right]$ is defined by

$$S_h - S_l \leq \frac{(r + \delta_h + \beta p_h) c_h}{(1 - \beta) Q_h(p_h)} \leq S_h$$

- $S_h - S_l < 0$ and $r + \delta_h - \beta(r + \delta_l) \leq 0$: then $\underline{p}_h$ and $\bar{p}_h$ are defined by

$$\frac{(S_h - S_l) (r + \delta_h + \beta p_h)}{r + \delta_h - \beta (r + \delta_l)} \leq \frac{(r + \delta_h + \beta p_h) c_h}{(1 - \beta) Q_h(p_h)} \leq S_h$$

- $S_h - S_l \geq 0$ and $r + \delta_h - \beta(r + \delta_l) > 0$: then $\underline{p}_h$ and $\bar{p}_h$ are defined by

$$S_h - S_l \leq \frac{(r + \delta_h + \beta p_h) c_h}{(1 - \beta) Q_h(p_h)} \leq \min \left\{ S_h, \frac{(S_h - S_l) (r + \delta_h + \beta p_h)}{r + \delta_h - \beta (r + \delta_l)} \right\}$$

7.2.3 Uniqueness of the steady-state equilibrium.

Note first that uniqueness is trivial if $S_l = 0$. Therefore, we focus on the case $S_l > 0$. The free-entry condition for high-skilled jobs allows us to define function P_l

$$p_l = P_l(p_h) \equiv (r + \delta_l + p_h) \frac{S_h - (r + \delta_h + \beta p_h)\frac{c_h}{(1-\beta)Q_h(p_h)}}{(r + \delta_h + \beta p_h)\frac{c_h}{(1-\beta)Q_h(p_h)} - (S_h - S_l)}$$

The free-entry condition for simple jobs writes

$$q_l \frac{B(\kappa_l) + \frac{r+\delta_l}{r+\delta_l+p_h}\gamma}{1+\gamma} = \frac{(r+\delta_l)c_l}{\bar{s} - \underline{w} - \kappa_l}$$

where

$$\gamma = \Gamma(p_h) \equiv \frac{\phi_h}{\phi_l(1 - X(\underline{w} + \kappa_l))} \frac{\delta_h}{\delta_l} \frac{\delta_l + p_h}{\delta_h + p_h} \frac{\delta_l + P_l(p_h)}{\delta_l + P_l(p_h) + p_h}.$$

Let us define

$$\tilde{H}(p_h) \equiv \frac{Q_l(P_l(p_h))}{r + \delta_l + p_h} \frac{B\left(1 + \frac{p_h}{r+\delta_l}\right) + \Gamma(p_h)}{1 + \Gamma(p_h)} - \frac{c_l}{\bar{s} - \underline{w} - \kappa_l}$$

Lemma 1 *Let $B(\kappa_l) = 1$. Assume there exists a cross-skill matching equilibrium. If*

$$\frac{d}{dp_h}\left(\frac{Q_l(P_l(p_h))}{r + \delta_l + p_h}\right) \geq 0, \text{ for } p_h \in \left[\underline{p}_h, \bar{p}_h\right],$$

then this equilibrium is unique. A sufficient condition for the previous inequality to be satisfied is

$$\left(\frac{-p_l Q_l'(P_l(p_h))}{Q_l(P_l(p_h))}\right)\left(\frac{-p_h Q_h'(p_h)}{Q_h(p_h)}\right)\left(1 + \frac{r + \delta_h}{p_h}\right) \geq 1 \tag{37}$$

Proof. Since $\frac{d}{dp_h}\left(\frac{Q_l(P_l(p_h))}{r+\delta_l+p_h}\right) \geq 0$ for $p_h \in I$, then P_l' is also negative on this interval. This implies that Γ' is also negative at the equilibrium value of the arrival rate p_h and the result immediately follows.

Let now prove that (37) is a sufficient condition for uniqueness. The first-order derivative of $\frac{Q_l(P_l(p_h))}{r+\delta_l+p_h}$ has the same sign as

$$\frac{-1}{r + \delta_l + p_h} + \frac{Q_l' P_l'}{Q_l} = \frac{-1}{r + \delta_l + p_h}\left[1 - \left(\frac{-p_l Q_l'}{Q_l}\right)\left(\frac{-(r + \delta_l + p_h)P_l'}{P_l}\right)\right]$$

where

$$\frac{-(r + \delta_l + p_h)P_l'}{P_l} = -1 + \frac{(r + \delta_l + p_h)\left(1 + \left(\frac{-p_h Q_h'}{Q_h}\right)\left(1 + \frac{r+\delta_h}{\beta p_h}\right)\right)\beta c_h}{(1 - \beta)Q_h(p_h)}$$
$$\times \left[\frac{1}{g(p_h) - (S_h - S_l)} + \frac{1}{S_h - g(p_h)}\right]$$

and

$$g(p_h) \equiv (r + \delta_h + \beta p_h)\frac{c_h}{(1 - \beta)Q_h(p_h)}.$$

Thus, the inequality $\frac{d}{dp_h}\left(\frac{Q_l(P_l(p_h))}{r+\delta_l+p_h}\right) \geq 0$ is equivalent to

$$\frac{(r + \delta_l + p_h)\beta c_h}{(1 - \beta)Q_h(p_h)}\left[\frac{1}{g(p_h) - (S_h - S_l)} + \frac{1}{S_h - g(p_h)}\right] \geq \frac{1 + \left(\frac{-p_l Q_l'}{Q_l}\right)^{-1}}{1 + \left(\frac{-p_h Q_h'}{Q_h}\right)\left(1 + \frac{r+\delta_h}{\beta p_h}\right)} \tag{38}$$

From the inequality (25) in Proposition 1, we deduce

$$g\left(p_h\right) - \left(S_h - S_l\right) \le \frac{\left(r + \delta_l + p_h\right)\beta c_h}{\left(1 - \beta\right)Q_h\left(p_h\right)}$$

which implies that LHS in (38) is higher than

$$\frac{S_l}{S_h - g\left(p_h\right)}$$

which is greater than 1. Thus, (37) is a sufficient condition for $\frac{d}{dp_h}\left(\frac{Q_l(P_l(p_h))}{r+\delta_l+p_h}\right) \ge 0$ to be satisfied. ∎

7.2.4 Analysis of $p_{l,l}^c\left(p_h\right)$:

Solutions of (32) are the roots of the following second order polynomial

$$\left[1 + b\left(p_h\right)\right]p_l^2 + c\left(p_h\right)p_l + d\left(p_h\right) = 0$$

with

$$
\begin{aligned}
a\left(p_h\right) &= \frac{p_l^s\left(\kappa_l\right)}{B\left(\kappa_l\right)}\frac{r + \delta_l}{r + \delta_l + p_h} \\
b\left(p_h\right) &= \frac{\phi_h\delta_h\left(\delta_l + p_h\right)}{\left(1 - X\left(\kappa_l + \underline{w}\right)\right)\phi_l\delta_l\left(\delta_h + p_h\right)} \\
c\left(p_h\right) &= \left(\delta_l + p_h\right) + \left[\delta_l - a\left(p_h\right)\right]b\left(p_h\right) - p_l^s\left(\kappa_l\right) \\
d\left(p_h\right) &= -a\left(p_h\right)b\left(p_h\right)\delta_l - p_l^s\left(\kappa_l\right)\left(\delta_l + p_h\right)
\end{aligned}
$$

For positive values of p_h, since $d\left(p_h\right) < 0$, the discriminant $\Delta = c\left(p_h\right)^2 - 4\left[1 + b\left(p_h\right)\right]d\left(p_h\right)$ is positive. The solutions have the following form

$$p_{l,\pm} = \frac{-c\left(p_h\right) \pm \sqrt{c\left(p_h\right)^2 - 4d\left(p_h\right)\left(1 + b\left(p_h\right)\right)}}{2\left(1 + b\left(p_h\right)\right)}$$

Simple algebraic manipulations show that $p_{l,-}$ is negative, so we set $p_{l,l}^c\left(p_h\right) = p_{l,+}$.

7.3 Appendix C : Details of the calibration

We detail the calibration procedure. We first present some parameter values we derived from already available statistics we picked in various works and then list the set of equations we used to calibrate the remaining parameters.

We decompose the wage cost into a labor contribution paid by the firm and by the employee and a remainder. In case of low-skilled worker, this remainder corresponds to w_l that we set equal to 1 by convention ($w_l = \underline{w} = 1$). The labor market contribution paid by the employee is the unemployment benefit contribution that amounts to 0.0227. For sake of simplicity, we assume that low-skilled workers do not paid direct income tax [13], so that $\tau_l = 0.0227$. The labor market contribution paid by employers is equal to the net sum of the unemployment benefit contribution and the subsidies on low-skilled jobs. We average this

---

[13] About one half of French households do not pay income taxes.

subsidy available for job paid between the minimum wage and 1.3 minimum wage we assume to be low-skilled position. This should lead to an over-evaluation as some high-skilled workers earn less than 1.3 minimum wage. We obtain that $\kappa_l = -0.0218$. Turning to high-skilled workers, a similar definition of the wage cost gives $w_h = 1.39$. We then compute the amount of income tax paid on average by the 70% richest households that we add to the unemployment benefit contribution. This gives $\tau_h = 0.143$. We then set b equal to the monthly amount per capita paid by Social Security and the government by way of unemployment benefits and "R.M.I." benefits : $b = 0.34$ and compute the "government budget" which gives us $G = 0.0098$.

[Cahuc et al., 2006] estimate bargaining power parameters for various type of workers from a large French employer-employee dataset. They obtain for high-skilled workers in various sectors values between 0.16 and 0.38 and no bargaining power for low-skilled workers. We reasonably pick the value $\beta = 0.25$ [14].

We set $r = 0.05$ on a yearly basis, consistently with the estimation framework used by [Cahuc et al., 2006]. From flow equilibrium conditions, we get :

$$e_{hh} = \frac{p_h \phi_h}{\delta_h + p_h} \tag{39}$$

and

$$e_{hl} = \frac{\phi_h \delta_h}{p_h + \delta_h} \frac{p_l}{p_l + p_h + \delta_l} \tag{40}$$

From [Leclair and Roux, 2004], we get the share ($P_{ls} = 0.269$) of low-skilled jobs and ($P_{hs} = 0.164$) of high-skilled jobs that last less than one year. In the steady state of our model, we have

$$P_{hs} = 1 - e^{-\delta_h} \tag{41}$$

$$P_{ls} = \left(1 - e^{-\delta_l}\right) \frac{e_{ll}}{e_{ll} + e_{hl}} + \left(1 - e^{-(\delta_l + p_h)}\right) \frac{e_{hl}}{e_{ll} + e_{hl}} \tag{42}$$

Equation (41) gives $\delta_h = 0.1791$ and then (39) $p_h = 1.077$, it follows from (42) that $\delta_l = 0.2341$ and from (40) that $p_l = 0.6136$ which implies that frictional low-skilled unemployment is $u_{l,f} = (\delta_l/p_l)e_{ll} = 0.085$ and the sum of voluntary unemployment and classical unemployment is $u_{l,v} + u_{l,c} = \phi_l - e_{ll} - u_{l,f} = 0.055$.

We consider the same Cobb-Douglas matching function as in section 4 :

$$m_i(\theta_i) = \lambda_i(\theta_i)^{1/2} \quad \text{for} \quad i = l, h$$

Notice that based on the set of available equations (free-entry and flow equilibrium conditions), we do not need to identify c_h, c_l, λ_h and λ_l, but only the ratios c_h/λ_h^2 and c_l/λ_l^2. Thus, it remains seven deep parameters to calibrate: $u_{l,v}$, c_h/λ_h^2, c_l/λ_l^2, s_h, $\overline{s}$, a_1, a_2.

With the free-entry condition $V_h = 0$, the wage bargaining equation $\beta J_{hh} = (1 - \beta)(N_{hh} - Uh)$ rewrites

$$\beta \frac{s_h - w_h}{r + \delta_h} = \frac{1 - \beta}{r + \delta_l + p_h} \left[w_h - \tau_h - b - \frac{p_l(\underline{w} - \tau_l - b)}{r + \delta_l + p_l + p_h} \right]$$

and allows us to compute $s_h = 1.7632$. The free-entry condition $V_h = 0$ also implies that $q_h J_{hh} = c_h$ or equivalently

$$\frac{s_h - w_h}{r + \delta_h} = \frac{c_h p_h}{\lambda_h^2}$$

---

[14]These values appear to be country dependent. For the US, [Flinn, 2006] got estimates between 0.37 and 0.43 for young workers.

We deduce $c_h/\lambda_h^2 = 1.5133$.

For the last five parameters $(u_{l,v}, c_l/\lambda_l^2, \overline{s}, a_1, a_2)$, we have at our disposal

- the flow equilibrium equation for low-skilled jobs

$$
\begin{aligned}
e_{ll} &= \frac{p_l}{p_l + \delta_l}\left(\phi_l - u_{l,v}\right)\left(1 - X\left(\underline{w} + \kappa_l\right)\right) \\
&= \frac{p_l}{p_l + \delta_l}\left(\phi_l - u_{l,v}\right)\left(1 - betacdf\left(\frac{\underline{w} + \kappa_l}{\overline{s}}, a_1, a_2\right)\right)
\end{aligned}
$$

- the free-entry condition (32) that we rewrite as

$$
\frac{1}{r + \delta_l}\left[\int_{\kappa_l + \underline{w}}^{\overline{s}} s\frac{betapdf\left(\frac{s}{\overline{s}}, a_1, a_2\right)ds}{1 - betacdf\left(\frac{\kappa_l + \underline{w}}{\overline{s}}, a_1, a_2\right)} - \underline{w} - \kappa_l\right] + \frac{\gamma\left(\overline{s} - \underline{w} - \kappa_l\right)}{r + \delta_l + p_h} = (1 + \gamma)p_l\frac{c_l}{\lambda_l^2}
$$

where $\gamma = u_h/u_{l,f}$.

- Drawing on [Crépon et al., 2002], we compute an estimate of the average ratio of productivity between a low-skilled job and a high-skilled job. This gives the ratio

$$
\frac{1}{e_{ll} + e_{hl}}\left[e_{ll}\int_{\kappa_l + \underline{w}}^{\overline{s}} s\frac{betapdf\left(\frac{s}{\overline{s}}, a_1, a_2\right)ds}{1 - betacdf\left(\frac{\kappa_l + \underline{w}}{\overline{s}}, a_1, a_2\right)} + e_{hl}\overline{s}\right] = 0.819\ s_h
$$

- Moreover, drawing on the various studies ([Crépon and Deplatz, 2001], [Laroque and Salanié, 2000]) on the job creations induced by the subsidies on low-skilled jobs implemented between 1992 and 1997 ($\Delta\kappa_l = -0.0756$), we set that this subsidy contributes to the creation of 450 000 jobs, i.e. $\frac{\Delta(e_{hh} + e_{hl} + e_{ll})}{e_{hh} + e_{hl} + e_{ll}} = 0.0227$, with

$$
e_{hh} + e_{hl} + e_{ll} = \frac{\phi_h}{\delta_h + p_h}\left(p_h + \frac{\delta_h p_l}{p_l + p_h + \delta_l}\right) + \frac{p_l\left(\phi_l - u_{l,v}\right)}{p_l + \delta_l}\left(1 - betacdf\left(\frac{\underline{w} + \kappa_l}{\overline{s}}, a_1, a_2\right)\right)
$$

where p_l and p_h vary with κ_l.

We then have 4 non-linear equations to determine the five parameters c_l/λ_l^2, $\overline{s}$, $u_{l,v}$, a_1 and a_2. All these parameters must be positive and $s_h > \overline{s} > \underline{w} + \kappa_l$. Technically speaking, we still need additional information but at this stage the solution set might already be empty. The natural additional information might be given by vacancy statistics, but we did not find any usable statistic in this area. We then proceed as follows. Determining the sets of parameter values that satisfy the set of inequality constraints and the above equations, we observe that some of these sets are very small intervals. We then select the middle of the smallest interval as the value of the related parameter.