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Asset pricing and the role of macroeconomic volatility*

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Abstract

Standard Real Business Cycle (RBC) models are well known to generate counter-factual asset pricing implications. This paper provides a simple extension to the prior literature where we study an economy that follows a regimes switching process both in the mean and the volatility, in conjunction with Epstein-Zin preferences for the consumers. We provide a detailed theoretical and numerical analysis of the model's predictions. We also show that a reasonable parameterization of our model conveys reasonable financial figures. Furthermore, we provide evidence in support of the necessity to model the decline of macroeconomic risk in this particular class of models.

Keywords: Asset Pricing, Real Business Cycle Models, Recursive Preferences, Markov Switching Models

JEL: G12, E32, E23

1 Introduction

We study an economy that switches between booms and busts where technological shocks follow a hidden two state Markov chain, in conjunction with recursive preferences for the consumers. Our work is closely related with the main literature on asset pricing with a non-trivial production side¹, and it contributes a novel theoretical framework where a sizeable equity premium can be obtained without imposing any kind of rigidity on the production side of the model (e.g. Boldrin et al., 2001 and Jermann, 1998) and without the need of an implausibly high value for the risk aversion of the agents (e.g. Danthine et al., 1992, Rouwenhorst, 1995, and Tallarini, 2000).

We show that, when the role of macroeconomic risk is taken into account, the model can replicate the US postwar financial figures. This provides support in favor of modelling the macroeconomic volatility as regime dependent, in order to capture the “great moderation” of the last twenty years. Moreover we show how the model, in line with the empirical evidence, is able to produce a higher, and sizeable, required premia during a downturn of the economy.

Building on Lettau et al. (2008), where the role of macroeconomic risk in an endowment economy is studied, we adopt a recursive utility specification for the consumption side, referred to as Epstein-Zin preferences (Epstein and Zin, 1989, Epstein and Zin, 1991, and Weil, 1989). Two reasons drive this

¹for an exhaustive analysis on the role of asset prices in RBC models see Lettau (2003).

choice. First, this form of utility function is widely used in the latest asset-pricing research (see Bansal and Yaron, 2004; Campbell and Viceira, 2001; Campbell et al., 2003; Brandt et al., 2004; Guvenen, 2006 among others). Second, since Epstein-Zin preferences nest the power utility function as a special case, these are particularly useful to provide a closer comparison with the standard models based on power utility specification.

In the RBC literature, the first analysis on asset prices, (Danthine et al., 1992, Rouwenhorst, 1995) while unsuccessful in explaining the behavior of returns over the business cycle, provided useful insights on what would be a necessary ingredient of a successful model. Along this line, to improve the capability in explaining financial figures, the main literature on asset pricing with a non-trivial production side (Jermann, 1998, Boldrin et al., 2001), introduced some form of rigidity in the model. While both Boldrin et al. (2001) and Jermann (1998) specify a habit utility for consumers, the former relies on a limited mobility of production's factors and the latter introduces capital adjustment costs².

The assumption of a recursive but time non-separable felicity function is not novel in this literature. Tallarini (2000), or more recently, Gomes and Michaelides (2008), assume Epstein-Zin utility function, and both document the inability to generate reasonable return's figures without introducing some production frictions in the model. Kaltenbrunner and Lochstoer (2010) study

²A different approach can be found in Cochrane (1991) who evaluates asset pricing implications from the producer's first order conditions.

the role of long run risk in a production setting, while Ai (2010) is additionally concerned with the equity premium implications of this class of models.

This paper takes a different approach on the production side of the model and instead of imposing any kind of restriction, provides a simple extension of a standard RBC model where the economy switches between booms and busts. This is accomplished by letting the economy follow a hidden markov chain. Most of the literature studies the implication of a Markov switching process in the conditional mean and in the volatility of the endowment process³. Differently, here the regimes are introduced via the production side by allowing the technology shocks to follow a latent two states process both in the mean and the volatility⁴.

The role played by the volatility of the underlying state of the economy in determining the returns is the key to understand our results. Intuitively, if we shut down the regime on the volatility, we are preventing the agent from entering a “high risk” regime (i.e. the high volatility regime), but we

³Regime switching is widely used in economics since the seminal contribution by Hamilton (1989). In particular, in the asset pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti et al. (1990); Kandel and Stambaugh (1991); Cecchetti et al. (1993); Abel (1994); Abel (1999). Recently, the time series properties of the second moments gained popularity in this framework: by setting up an equilibrium economy where the endowment process follows a latent two state regime switching process, Veronesi (1999) shows a better explanatory power of volatility clustering than a model without regimes. In the same setting, Whitelaw (2000) introduces time-varying transition probabilities between regimes, finding a complex nonlinear relation between expected returns and stock market volatility. A recent contribution that studies the impact of regime switches in the volatility of the endowment process is in Lettau et al. (2008).

⁴In a different setting Cagetti et al. (2002) model the technology shocks as a Markov switching model in the first moment.

are also depriving her of the possibility of entering a “low risk” regime. This creates an economy with a smoother path for the consumption claim. Such a path is appealing for agents, and this would push the price of the risk free asset down increasing its return and reducing the equity premium. So, an economy with a volatility regime gives a higher incentive to agents to use the risk free asset to transfer consumption overtime, pushing its prices up, lowering the risk free return and thus increasing the equity premium.

The remainder of the paper is organized as follows: Section 2 introduces the general model, derives equilibrium asset prices, and analyzes the determinants of the equity premium predicted by the model. Section 3 discusses model calibration and provides numerical analysis of asset returns’ properties over the business cycle, while section 4 provides a sensitivity analysis of the results’ determinants. Section 5 concludes. Proofs, algebraic derivations, and additional results are provided in Appendix C.

2 Model

A standard production economy with two actors is considered. Consumers are modeled via a representative, risk averse, agent which derives utility from consumption, while the production side is modeled through a standard representative firm that maximizes its shareholders’ value. There are two securities in the economy: a riskless bond that agents can use for transferring their wealth to the future, and equity, which provides a claim on firm’s profits.

2.1 Consumers

The representative agent has preferences defined over current consumption and future utility. Following Epstein and Zin (1989, 1991), the utility function is defined recursively by:

$$U(C_t, \mathbb{E}_t(U_{t+1})) = \left[(1 - \beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(\mathbb{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\alpha}} \right]^{\frac{\alpha}{1-\gamma}}, \quad (1)$$

where C_t indicates aggregate consumption, β is the time preference parameter, and $\alpha \equiv (1 - \gamma)/(1 - 1/\psi)$, with $\gamma > 0$.

The parameter γ is the coefficient of relative risk aversion (RA), while the elasticity of intertemporal substitution (EIS) is given by ψ ⁵.

2.2 Firms and technology

Each period the firm has to decide how much human capital to employ and how much capital to invest in physical assets.

In particular, there is only one traded good which is produced through a constant return to scale technology. Analytically, production can be described using a Cobb-Douglas production function with human and physical capital as factors:

$$Y_t = A_t K_t^\theta H_t^{1-\theta}, \quad (2)$$

⁵An interesting feature of this utility function specification is that it nests the power utility. In fact, when $\gamma = \frac{1}{\psi}$ equation 1 can be solved forward to get the standard power utility function.

where θ is the share of physical capital.

The human capital H evolves according to:

$$H_{t+1} = (1 - \delta_H)H_t + E_t, \quad (3)$$

where δ_H is its depreciation rate, and E_t is the investment in education.⁶

The capital stock's K evolution is governed by:

$$K_{t+1} = (1 - \delta_K)K_t + I_t, \quad (4)$$

where δ_K is its depreciation rate, and I_t indicates capital investment.

The resource constraint for this economy is written as

$$C_t + I_t + E_t \leq Y_t. \quad (5)$$

If we consider the productivity shock (A) in a regime switching model, we can express its law of motion as a process with stochastic parameters depending on the state of the economy:

$$\Delta \log A_t = \mu(s_t) + \sigma(s_t)\varepsilon_t, \quad (6)$$

where μ and σ define the mean and the volatility of the process, and s_t indicates the state of the economy. We assume that s_t follows a hidden

⁶As in Barro and Sala-I-Martin (2004) we can think of human capital as the number of workers multiplied by the human capital of a typical worker.

Markov chain with transition probabilities matrix P (see Hamilton (1989)). The evolution of the state of the economy in terms of state beliefs (ξ_{t+1}) can be expressed as realizations of the equation:

$$\xi_{t+1} = P\xi_t + \epsilon_t. \quad (7)$$

The agents cannot directly observe the state of the economy, s_t , and they have to rely on interpreting external signals. The agents update their belief according to the posterior probabilities computed as

$$\hat{\xi}_{t+1|t} = P \frac{\hat{\xi}_{t|t-1} \odot \zeta_t}{\mathbf{1}' (\hat{\xi}_{t|t-1} \odot \zeta_t)}, \quad (8)$$

where \odot denotes the Hadamard product, ζ_t is a vector that stacks the conditional densities of the technological shocks' growth rates:

$$\zeta_t = \begin{bmatrix} f(\Delta \log A_t | s_t = 1, \Omega_{t-1}) \\ \vdots \\ f(\Delta \log A_t | s_t = n, \Omega_{t-1}) \end{bmatrix} \quad (9)$$

with the density of $\Delta \log A_t$ conditional on state s_t is defined as:

$$f(\Delta \log A_t | s_t = i, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma(s_t)} \exp \left\{ -\frac{(\Delta \log A_t - \mu(s_t))^2}{2\sigma(s_t)^2} \right\}, \quad (10)$$

where Ω denotes the information set.

2.3 The firm's problem

The management of the representative firm maximizes firm value through optimal investment given the current capital stock, the level of human capital hired, the current level of technology, and the stochastic discount factor.

In this economy the firm's optimality condition for investment is summarized by the existence of a pricing kernel (Q) for pricing the investment return. That is, the equality $\mathbb{E}_t [Q_{t+1} R_{t+1}^I] = 1$ holds true for the investment return (R^I) defined as $\theta \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_K)$ (see Appendix A).

2.3.1 Asset prices

Next, we derive the equilibrium asset prices implied by the model. We examine two different types of assets: a one period asset that yields one unit of consumption (i.e. a "risk-free" asset), and a claim to the physical capital (i.e. an "equity asset").

First, we know from Hansen and Richard (1987) that optimality of the solution implies the existence of a *unique* pricing kernel for pricing *all* the available assets (i.e. the Euler equation holds true for any asset return (R) as $\mathbb{E}_t [Q_{t+1} R_{t+1}] = 1$).

Epstein and Zin (1989) show that the stochastic discount factor for the case of utility over consumption is given by

$$Q_{t+1} = \beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}} (R_{c,t+1})^{\alpha-1}, \quad (11)$$

where $R_{c,t+1}$ is the equilibrium gross return to consumption claim between t and $t + 1$.

Second, with the assumed production technology (see Restoy and Rockinger, 1994), if firms only use retained earnings to finance their capital the equity holder gets a dividend $D_t = \frac{\partial Y_t}{\partial K_t} K_t - I_t = \theta Y_t - I_t$. This implies that the unlevered stock market of this economy represents the value of the capital stock. Given the above dividends' expression, it is easy to show the following:

Remark 2.1. *When the “equity asset” is considered,*

a) the gross dividends' growth rate can be expressed as:

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}}{A_t} \left[\frac{K_{t+1}}{K_t} \right]^\theta \left[\frac{H_{t+1}}{H_t} \right]^{1-\theta} \frac{\theta - I_{t+1}/Y_{t+1}}{\theta - I_t/Y_t} = \frac{A_{t+1}}{A_t} \lambda_t^\theta \eta_t^{1-\theta \Sigma_{t+1}}, \quad (12)$$

where $\eta_t = \left(1 - \delta_H + \frac{E_t}{H_t} \right)$, $\lambda_t = \left(1 - \delta_K + \frac{I_t}{K_t} \right)$ and $\Sigma_{t+1} = \frac{\theta - I_{t+1}/Y_{t+1}}{\theta - I_t/Y_t}$;

b) consumption and dividends share the same gross growth rate.

Proof. The expression in a) is obtained by simple algebra, starting from the expression of dividends given above (see appendix B). The statement in b) implies that education expenses are part of the consumption. \square

Given remark 2.1 we can express the price to consumption ratio (PC) of the consumption claim $R_{c,t+1} = \left(\frac{P_{t+1}^c + C_{t+1}}{P_t^c} \right)$ as:

$$PC_t^\alpha = \mathbb{E}_t \left[\beta^\alpha \left(\frac{A_{t+1}}{A_t} \lambda_t^\theta \eta_t^{1-\theta} \Sigma_{t+1} \right)^{1-\gamma} (PC_{t+1} + 1)^\alpha \right]. \quad (13)$$

In the same fashion we can express the price to dividend ratio (PD) of the dividend claim⁷ $R_{e,t+1} = \left(\frac{P_{t+1}^e + D_{t+1}}{P_t^e} \right)$ as

$$PD_t = \mathbb{E}_t \left[\beta^\alpha \left(\frac{A_{t+1}}{A_t} \lambda_t^\theta \eta_t^{1-\theta} \right)^{1-\gamma} \Sigma_{t+1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1} + 1) \right] \quad (14)$$

Following the approach of Lettau et al. (2008), we solve Equations 13 and 14 numerically⁸. Thus we can get the equity return from:

$$R_{e,t+1} = \frac{1 + PD_{t+1} \frac{D_{t+1}}{D_t}}{PD_t}, \quad (15)$$

and calculate its second moment as

$$\mathbb{E}_t [R_{e,t+1}^2] - \mathbb{E}_t [R_{e,t+1}]^2. \quad (16)$$

Finally, the risk free rate can be expressed as: $R_{f,t+1} = (\mathbb{E}_t [Q_{t+1}])^{-1}$,

from which we can calculate both its first and second moment.

⁷Appendix C provides the derivation for both equation (13) and (14).

⁸The problem is solved by fixed point iteration over the price-consumption ratio. Given the price-consumption ratio as a function of the state beliefs, we start from a first guess value and we calculate the value the price-consumption ratio at each point of the state beliefs grid. Next, we update the guess of the function using these new values and start over the iteration process.

2.4 The equity premium

In this section we provide a first grasp on the determinants of the equity premium implied by the model. We do that by log-linearizing the Euler equations for the equity asset and the risk free asset respectively, and solving them for the expected excess return. Thus, we can study the role played by the interplay of the utility function parameters, namely the risk aversion and the elasticity of intertemporal substitution.

2.4.1 The role of utility parameters

Following the analysis in Campbell et al. (1997), if consumption growth rates and asset returns are homoskedastic and jointly lognormal, the equity premium can be expressed as⁹

$$EP = \gamma\sigma_g^2 + \left(1 - \frac{1-\gamma}{1-1/\psi}\right)\sigma_\omega^2 + \left(1 - \frac{1-\gamma}{1-1/\psi} + \gamma\right)\sigma_{\omega,g} \quad (17)$$

where σ_g^2 is the variance of the log consumption growth, σ_ω^2 is the variance of $\log \frac{1+PD_{t+1}}{PD_t}$, and $\sigma_{\omega,g}$ is their covariance¹⁰.

The first component of equation 17, $\gamma\sigma_g^2$, is the determinant of the equity premium when an agent has a power utility function. Clearly, as has already been established in the literature, the only way to increase the equity premium through this term is to increase the coefficient of risk aversion, leading

⁹Even if the log-linearization does not strictly hold for our non-linear economy, the implications that follow are valuable.

¹⁰For a detailed loglinear analysis in a similar framework see Brevik and d'Addona (2010).

to the well known finance puzzles.

The second component links the variance of increases in the price-dividend ratio with the utility function parameters. The variance is always positive so, we can focus on the coefficient $\left(1 - \frac{1-\gamma}{1-1/\psi}\right)$ to analyze the contribution of this term to the equity premium. To have a positive contribution of the above coefficient we need the term $\left(\frac{1-\gamma}{1-1/\psi}\right)$ to be less than 1. When the elasticity of intertemporal substitution (EIS) is larger than 1, this implies a higher RA parameter relative to the inverse of the EIS, or a preference for early resolution of uncertainty, in the language of Kocherlakota, 1990.

It is important to note that an EIS larger than 1, implies procyclical prices (see Brevik and d'Addona (2010)). Thus, in order to have a positive contribution by the variance of prices to the equity premium we need an agent with a preference for the early resolution of uncertainty coupled with procyclical prices.

In the same fashion we can analyze the third component of equation 17. Focusing on the case when both RA and EIS are greater than 1, the procyclicality of prices leads to a positive covariance between consumption growth rates and prices themselves. So a positive contribution of this third term is assured if the numerator of its coefficient, rewritten as $\frac{\gamma(2-1/\psi)-1/\psi}{1-1/\psi}$ is positive. This turns out to be always true for the case we are focusing on.

The above analysis can be used to define the restrictions we can impose on the model to expect a better performance in explaining financial figures. That is: in order to have a positive contribution by all the terms in equation

17 to the equity premium, it is sufficient to assume preference for the early resolution of uncertainty for the representative agent and a EIS parameter strictly greater than 1.

Interestingly enough, both the preference for the early resolution of uncertainty and procyclical prices are not necessary requirements for a positive contribution by the two second moments to the equity premium. In fact it is straightforward to see that, when ψ is less than 1, we still can have a positive value of the second component in equation 17, provided that agents have a preference for the late resolution of uncertainty (that is $\gamma < 1/\psi$). Moreover, the covariance of prices and consumption growth rates would be negative in this case. So a sufficient condition for having a positive contribution from the last term in equation 17 is to have a RA parameter greater than 1. Summarizing, if prices are countercyclical, then the equity premium is monotonically increasing in both the variance of prices and the (negative) covariance between consumption growth rates and prices themselves, if agents have preference for late resolution of uncertainty and have a RA parameter greater than 1.

3 Empirical analysis

3.1 Estimation

Having provided the theoretical implications given by our framework, we can now turn to the numerical analysis of the model.

The data used for the calibration span from the beginning of 1952 to the end of 2004. The dataset is expressed in real terms with a quarterly frequency. The financial series (prices and dividends) are on the S&P 500 composite, while the risk-free rate is the yield on 1 year treasury bills. These series are from Robert J. Shiller's webpage¹¹. The main economic series are downloaded from the Bureau of Economic Analysis' website¹². Consumption is quarterly real total personal consumption expenditures (NIPA table 2.3.6, line 1), GDP is quarterly real gross domestic income (NIPA table 1.1.6, line 1), investment in physical capital is quarterly non residential fixed investment (NIPA table 5.3.5, line 2), and education expenditures are personal education and research expenditures (NIPA table 2.5.5, line 104). Both the human capital and the physical capital series are constructed using the perpetual inventory method. Finally, we use the official recession dates as reported on the website of the National Bureau of Economic Research¹³.

To estimate the technology shocks, a standard technique based on the growth accounting framework is applied. In particular, with constant return to scale, it is possible to decompose the output growth in two parts, and thus the change in technology shocks can be estimated as:

$$\Delta \log A = \Delta \log Y - \theta \Delta \log K - (1 - \theta) \Delta \log H. \quad (18)$$

[Figure 1 about here.]

¹¹<http://www.econ.yale.edu/~shiller/data.htm>.

¹²<http://www.bea.doc.gov/>.

¹³<http://www.nber.org/cycles.html/>.

Figure 1 shows the time series of the estimated technological shocks. A procyclical behavior in the series is clearly evident.

The regime switching specification for the US economy with two possible states both for the mean and the volatility of the productivity shocks is also estimated. Parameter estimates for the model were computed using a Markov-Chain Monte-Carlo (MCMC) procedure following Kim and Nelson (1999).

The results from this analysis are given in Table 1. An important finding from our estimation is the high persistence of the states associated with the mean. In fact, the probabilities for the first moment of switching from the two states are 5.67% and 19.72%, respectively. This implies an average duration of more than four years (17.7 quarters) for high mean states (associated with booms), and more than one year (5.3 quarters) for low mean states (associated with busts). Hence, if we find ourselves in either of the two states, we expect to stay there for several periods. The results are more striking for the second moment. Looking at its switching probabilities, it is clear how the volatility state is extremely persistent, so if we find ourselves in either one of the two states for the volatility, it is very well the case that we will face that state for the majority of a sample period.

[Table 1 about here.]

To get a final grasp on the estimation of the regime switching economy, we investigate the capability of the model in picking up the historical busi-

ness cycles of the US postwar economy. Figure 2 reports this analysis by plotting the estimated posterior probability, associated with the mean of the productivity shock process, of being in the recession state. It is evident how the Markov switching model is able to capture fairly well the US recessions as chronicled by the official NBER business cycle dates (the gray areas in the graph).

[Figure 2 about here.]

Figure 3 reports a similar analysis by plotting the estimated posterior probabilities, associated with the volatility of the productivity shock process, of being in the low state. The obtained graph is consistent with the declining macroeconomic volatility starting in the mid eighties and documented widely in the literature (see Blanchard and Simon, 2001, and Lettau et al. (2008) among others), also named as “the great moderation” by Stock and Watson (2002).

[Figure 3 about here.]

3.2 Calibration

The basic calibration sets the model’s parameters as follows: the share of physical capital θ , is fixed to 0.36 which is the standard approach taken in the literature on business cycles. The depreciation rate for physical capital is fixed at 0.021, which is also standard in the literature. For human capital, we follow Heckman (1976) by choosing a value of 0.009.

The ratio of investment to physical capital and the ratio of education expenses to human capital are set to replicate the long run mean of the Σ_{t+1} ratio obtained by simulation¹⁴. This gives investment to capital ratios of 0.031 and 0.002 during booms and during busts respectively. Similarly, education expenses to human capital ratios are set to 0.0145 and 0.002 respectively.

3.3 Results

In this subsection we present the results from solving the model numerically. As a first comparison, we set the utility function parameters as in Lettau et al. (2008): $\gamma = 30$, $\psi = 1.5$, and we fix $\beta = 0.9925$.

The basic results from this calibration are presented in table 2. It reports the set of estimates discarding the first five years of data, in order to address the well known critique to the perpetual inventory methodology used in the capital estimations.

[Table 2 about here.]

As shown in this first set of results, the model overshoots in estimating the mean equity premium. This is mainly due to the poor performance in matching the risk free rate. In fact, with the proposed parametrization, we obtain a real risk free rate that is negative and big (-4.5%).

¹⁴To check its behavior we simulate the model for 10,000 periods obtaining economic figures that are in line with the empirics. Simulation results are available upon request.

Again, we can use a loglinear approximation as in Campbell et al. (1997), to interpret this quite odd result. Loglinearizing the Euler equation, and solving it for the risk free return, gives us a simplified framework to analyze it. The expected risk free rate can be expressed as:

$$r_{f,t+1} = -\log \beta + \frac{1}{\psi} E_t [g_{t+1}] - \frac{\alpha}{2} \left[\frac{1}{\psi^2} \sigma_g^2 + \left(\frac{1}{\alpha} - 1 \right) \sigma_{pc}^2 \right] \quad (19)$$

where $E_t [g_{t+1}]$ is the expected consumption growth rate, and σ_g and σ_{pc} denote the volatility of the log consumption growth rate and the volatility of the log return to the consumption claim, respectively.

By inspection, we can see that the coefficient on the volatility of the consumption claim, which is $(.5(\alpha - 1))$ is negative with the parametrization adopted above. So, this is the term that is pulling down the required risk free in the model, given the positivity of volatilities.

Given the unsatisfactory performance of the previous calibration, especially regarding the risk free rate, we perform a simple exercise: we fix the risk aversion parameter to 10, and we let the elasticity of intertemporal substitution vary to match the mean risk free rate of the postwar US economy¹⁵. The results for this calibration, obtained discarding the first five years of data, are reported in table 3.

¹⁵It is well known that Mehra and Prescott (1985) indicate 10 as an upper bound for an acceptable RA parameter in their setting. But it is important to point out that even if a risk aversion parameter higher than 10 can be perceived as implausible in a standard power utility setting, the parameter's implications in a Epstein-Zin utility framework change dramatically with respect to the power utility case. For a detailed theoretical characterization of these implications see Campanale et al. (2009).

[Table 3 about here.]

As shown in this table, the model performs fairly well in matching the mean equity premium and the first moment of the equity asset. In fact the model predicts a yearly equity return of 6.6%, and we match the real risk free return to a level of 1.3%, with a EIS parameter of 1.391. This leads to a predicted yearly equity premium of 5.3%. The matching of second moments is also satisfactory. In particular, both the standard deviations of the risk free asset and the equity asset are of the same magnitude of their empirical counterparts. Regarding the correlation and the autocorrelation of the assets, the model performs well in matching the autocorrelation of the risk free asset as well as the correlation between equity and the risk free asset. Less satisfaction is derived from the performance in matching the equities' autocorrelation. This is probably due to the nature of the model in which prices have a strong persistence with respect to the states of the economy.

The drastic improvement in the model performance is due to the decrease of both the risk aversion and the elasticity of intertemporal substitution assumed for the representative agent. By lowering the agent risk aversion we directly influence the equity claim: a lower value of γ makes equities more appealing; this leads to higher prices and so decreases the equity return. Instead, the risk free claim is directly influenced by the elasticity of substitution: the lower is the EIS the lower is the willingness of an agent to transfer her wealth overtime; this leads to lower prices for a risk free asset and so to a higher risk free return.

Before moving ahead in our empirical analysis, it is worth analyzing the value obtained for the EIS parameter to match the risk free return, given that the empirical estimates in the literature vary considerably. One approach of the empirical research focuses on a representative agent setup and uses aggregate consumption data. This leads to estimates of the EIS coefficient below 1, and even close to 0 (see e.g. Hall, 1988; Campbell and Mankiw, 1989, 1991; Hahm, 1998; Yogo, 2004; and Zhang, 2006). Another strand of research relies on microeconomic survey data to avoid potential biases in the aggregate data. If a stockholder is considered, these studies find EIS parameters around or above 1. (See Beaudry and van Wincoop, 1996; Vissing-Jørgensen, 2002; Vissing-Jørgensen and Attanasio, 2003; and Guvenen, 2006). Even if there is this lack of consensus in the economic literature, the recent asset-pricing literature relies on the higher EIS estimates of the latter literature, in fact both Bansal and Yaron (2004) or Lettau et al. (2008) calibrate their models with an EIS greater than one. As discussed above (cf. section 2.4.1) this choice is mainly linked to the capability of generating procyclical prices when agents have a recursive utility function. Consequently, we can consider the value of 1.391 for the EIS, obtained by matching the risk free rate, in line with the latter literature, and theoretically well grounded.

3.4 The role of macroeconomic risk

Loosely speaking, the insight we can gather from Lettau et al. (2008) is that the reduction in macroeconomic volatility in the last twenty years can account

for a good portion of assets' valuations in the recent past. So, it is natural to ask ourselves what is the role of the volatility regimes in the economy we are studying. We then re-estimate our regime switching economy, imposing the restriction of a single state for the volatility of the productivity shocks.

The parameters for the restricted regime switching model are again obtained using the same Markov-Chain Monte-Carlo (MCMC) algorithm used for the unrestricted model. The resulting estimates are given in table 4.

[Table 4 about here.]

The estimated value for the volatility is in between the values we obtained for the two states version of the regime switching process. The main departure from the unrestricted estimation is that the difference between the means in the two states is sharper. Furthermore, the estimated probabilities of switching from the two mean states are 6.30% and 22.08%, respectively. These probabilities confirm the persistence of each state also in the restricted setting.

We can now move to analyze the performance of the model, when the decline in the macroeconomic risk is not considered. We re-estimate the main financial figures, implied by our model with a restricted regime switching economy, fixing the utility parameters at $\gamma = 10$ and $\psi = 1.391$. The results for this calibration, obtained by discarding the first five years of data, are presented in table 5. The equity returns are not greatly effected by disregarding the decline in the macroeconomic risk, given that we lose 40 basis

points over the year estimate of 6.6% obtained by modelling the macroeconomic volatility. Instead, the implied risk free rate is more effected and it moves in the expected direction. In fact we obtain an estimate of 2.2% on an annual basis, that is about 90 basis point higher than the observed risk free rate on which we calibrated the EIS parameter in the unrestricted version of the model (cf. table 3).

This result can be better interpreted if we focus on the role played by the volatility of the underlying state of the economy in determining the prices and thus the returns. When we shut down the regime on the volatility, we are preventing the agent from entering a persistent “high risk” regime associated with a high volatility of the economy, but we are also depriving her of the possibility of entering a persistent “low risk” regime. This creates an economy with a smoother path for the consumption claim. Such a path is appealing for the representative agent of this economy, and this would push the price of the risk free asset down increasing its return. So, an economy with a volatility regime gives a higher incentive to the representative agent to use the risk free asset to transfer consumption overtime, pushing its prices up and lowering the risk free return.

A mathematical interpretation follows from equation 19: in the parametrization adopted above, the risk free rate is positively related with the volatility of the consumption growth rate (σ_g) and negatively related with the volatility of the consumption claim (σ_{pc}). When we move from an economy with no regimes for the volatility of the underlying state to an economy that models

such possibility, we don't expect any change to the (unconditional) volatility of consumption growth rate. To the contrary, the volatility of the consumption claim is expected to increase in the latter economy, given the higher volatility of prices. This, given the above mentioned negative relation of σ_{pc} and the risk free, pulls down the risk free rate.

[Table 5 about here.]

3.5 Time varying properties of the equity premium

Another notable feature of the model is the ability of replicating the behavior of the equity premium over the business cycle. The model delivers a higher risk premium in recessions than in booms: the implied annualized equity premium during recession varies from 3.19% under the high volatility state to 3.06% under the low volatility state. During booms the model predicts a much lower level of equity premium on an annual basis: 2.07% in the high volatility state and 1.94% in the low volatility state.

As a thoughtful exercise, we can calculate the expected equity premium by an investor that only observes consumption realizations. In practice, at each point in time, we let investors only have access to consumption growth rates. They know the structure of the economy, but they can use only consumption data in order to infer the current state of the economy. The probabilities the investors attach to each of the two states correspond to the Hamilton's filtered state probabilities.

Figure 4 shows the expected equity premium for the US postwar period. The plotted line clearly picks up in recessions, confirming the expected countercyclical behavior of the required premium an investor would ask in order to hold a risky asset.

[Figure 4 about here.]

4 Sensitivity analysis

In order to get a better grasp on the forces driving our results, we provide a sensitivity analysis of the main financial figures implied by the model to the three most relevant parameters: risk aversion (γ) and elasticity of intertemporal substitution (ψ).

Figure 5 provides a general overview of how the equity premium is affected by different values of the parameters. The figure shows the mesh of the equity premium by changing RAs and EISs. As expected the equity premium is monotonically increasing in the risk aversion. The same increasing relation is displayed for the intertemporal substitution and the equity premium itself, but somewhat less pronounced.

[Figure 5 about here.]

After having discussed the general behavior of the implied equity premium with respect to risk aversion and intertemporal elasticity of substitution, it could be interesting to focus on the relevance of the utility function's

parameters. Figure 6 delivers some insights on this. Panel A plots the obtained equity return by letting the risk aversion vary, keeping the remaining parameters fixed to the benchmark case. As expected the equity return is increasing in γ with a higher rate for lower values of γ . The same exercise, but on the risk free return, is reported in Panel B by letting the elasticity of intertemporal substitution vary. Again the relation is as expected: The higher the EIS the higher the willingness of an agent to transfer her wealth overtime; this leads to higher prices for a risk free asset and so to a lower risk free return.

[Figure 6 about here.]

5 Conclusion

This paper deals with asset pricing implications of production economies. In particular, we propose a simple extension to a standard Real Business Cycle (RBC) model where the economy switches between booms and busts and consumers have a recursive utility.

Our first contribution is a detailed theoretical analysis of the equity premium's determinants in the proposed framework. Secondly, we show that a plausible parametrization of the model conveys financial figures that are in line with the empirical observations on the postwar U.S. data. A detailed analysis on the relative contribution of prominent parameters of the model is also provided. This allows us to clarify the role of different choices on

the utility function. In particular, we investigate the role of risk aversion and elasticity of intertemporal substitution in determining asset prices and thus, in determining the equity premium. Furthermore, we study the role of macroeconomic risk in the proposed economy, providing evidence in favor of modelling the “great moderation” observed in the last two decades.

The model, presented herein, is also shown to be able to replicate the empirical evidence of higher, and sizeable, required premia during a downturn of the economy, by simply letting the agent infer the state of the economy from consumption realizations.

A The firm's problem

The firm maximizes firm value, by choosing the optimal investment decision, given the current capital stock, the level of human capital hired, and the technology level. Let Q_t denote the stochastic discount factor. The firm's problem can be written as

$$f(\cdot) = \max_{\{I_t, K_{t+1}, H_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \{ (Y_t - W_t H_t - I_t) - z_t (K_{t+1} - (1 - \delta_K) K_t - I_t) \} \right]. \quad (20)$$

where z_t is the shadow price of the capital accumulation constraint and W_t is the price paid for human capital.

The first order conditions are:

$$\frac{\partial f(\cdot)}{\partial H_t} = (1 - \theta) \frac{Y_t}{H_t} - W_t; \quad (21)$$

$$\frac{\partial f(\cdot)}{\partial I_t} = -1 + z_t; \quad (22)$$

$$\frac{\partial f(\cdot)}{\partial K_{t+1}} = \mathbb{E}_t \left[Q_{t+1} \left\{ \theta \frac{Y_{t+1}}{K_{t+1}} + z_{t+1} (1 - \delta_K) \right\} \right] - z_t. \quad (23)$$

Equating the focs to 0 and substituting out z in equation (23), gives:

$$1 = \mathbb{E}_t [Q_{t+1} R_{t+1}^I], \quad (24)$$

where R_{t+1}^I indicates the return on investment and is given by $\left(\theta \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_K)\right)$

B Deriving Dividend growth rate

We can start from the dividend equation:

$$D_t = \theta Y_t - I_t,$$

divide both sides by Y_t :

$$\frac{D_t}{Y_t} = \theta - \frac{I_t}{Y_t},$$

and divide D_{t+1}/Y_{t+1} by D_t/Y_t :

$$\frac{D_{t+1}}{D_t} = \frac{Y_{t+1} \theta - I_{t+1}/Y_{t+1}}{Y_t \theta - I_t/Y_t}.$$

C Deriving Asset Prices

Starting from Equation 11, we can rewrite it as

$$\begin{aligned} Q_{t+1} &= \beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P_{t+1}^c + C_{t+1}}{P_t}\right)^{\alpha-1} = \\ &= \beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P_{t+1}^c}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P_t^c}{C_t}\right)^{1-\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1} = \\ &= \beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi} + \alpha - 1} \left(\frac{P_{t+1}^c}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P_t^c}{C_t}\right)^{1-\alpha} \end{aligned} \quad (25)$$

giving us an expression for the stochastic discount factor as a function of consumption and price of its claim.¹⁶

First we price the consumption claim $R_{c,t+1} = \left(\frac{P_{t+1}^c + C_{t+1}}{P_t^c} \right)$

$$\mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi} + \alpha - 1} \left(\frac{P_{t+1}^c}{C_{t+1}} + 1 \right)^{\alpha - 1} \left(\frac{P_t^c}{C_t} \right)^{1 - \alpha} \left(\frac{P_{t+1}^c + C_{t+1}}{P_t^c} \right) \right] = 1 \quad (26)$$

Define $PC_t = \frac{P_t^c}{C_t}$

$$PC_t^\alpha = \mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi} + \alpha} (PC_{t+1} + 1)^\alpha \right]. \quad (27)$$

Then we solve for the dividend claim $R_{e,t+1} = \left(\frac{P_{t+1}^e + D_{t+1}}{P_t^e} \right)$

$$\mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi} + \alpha - 1} \left(\frac{P_{t+1}^c}{C_{t+1}} + 1 \right)^{\alpha - 1} \left(\frac{P_t^c}{C_t} \right)^{1 - \alpha} \left(\frac{P_{t+1}^e + D_{t+1}}{P_t^e} \right) \right] = 1 \quad (28)$$

Define $PD_t = \frac{P_t^e}{D_t}$

¹⁶It is worth noting that Q_{t+1} can be further simplified:

$$\begin{aligned} Q_{t+1} &= \beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{\alpha(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} \\ &= \beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} = \\ &\quad \beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} \end{aligned}$$

where PC indicates the Price Consumption ratio on the consumption claim

$$PD_t = \mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi} + \alpha - 1} (PC_{t+1} + 1)^{\alpha - 1} (PC_t)^{1 - \alpha} (PD_{t+1} + 1) \left(\frac{D_{t+1}}{D_t} \right) \right]. \quad (29)$$

Finally, using remark 2.1, we can plug $\frac{A_{t+1}}{A_t} \lambda_t^\theta \eta_t^{1-\theta} \Sigma_{t+1}$ in place of the growth rates of consumption and dividends in Equations 27 and 29 for getting Equations 13 and 14 respectively.

We can solve this set of equations using the same technique as Lettau et al. (2008) and the estimation on the $\Delta \log A_{t+1}$ process (i.e. solve for the fix point and then compute expected PC and PD using posterior probabilities from the Hamilton (1989) filter.)

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Figure 1: Estimated technological shocks

This figure plots the empirical estimate of technological shocks. Data, transformed with logarithms, are quarterly starting from I-1952 to IV-2004. Panel A plots the estimated series of technological shocks, coupled with the recession periods according to NBER (shadow areas in the graph). Panel B shows a scatter-plot of GDP cyclical component, estimated with a HP filter, versus the shocks.

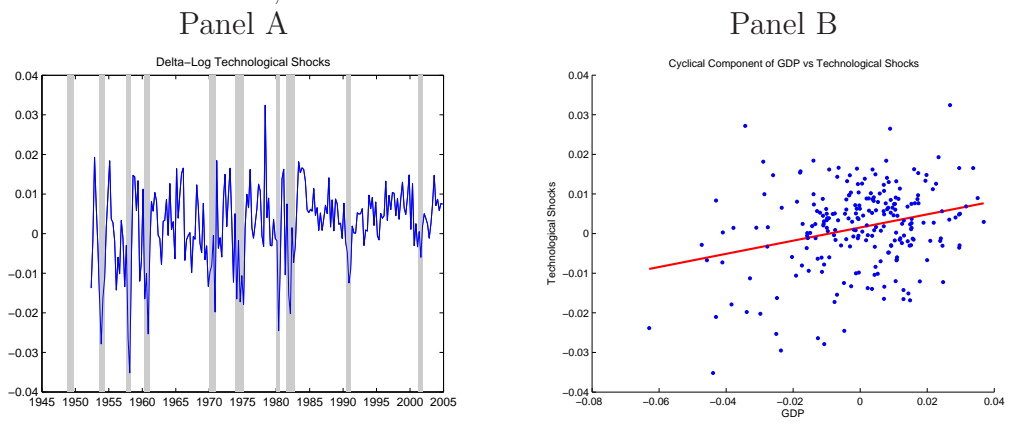


Figure 2: Posterior probabilities of a recession

This figure shows the estimated posterior probabilities of being in a recession coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.

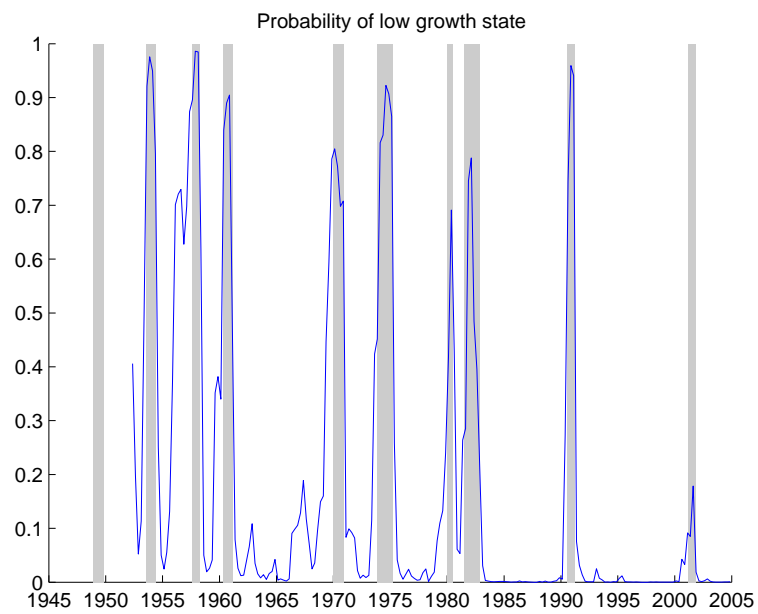


Figure 3: Posterior probabilities of the low volatility state

This figure shows the estimated posterior probabilities of being in a low volatility state coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.

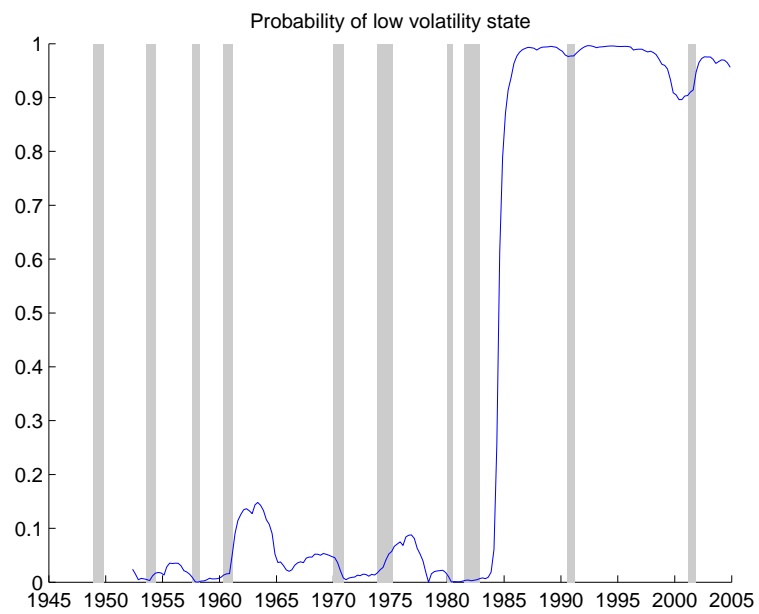


Figure 4: The time varying properties of the equity premium

This figure shows the time-series of the expected equity premium by an agent that only observes consumption realizations. The state probabilities are those inferred using the Hamilton's 1989 filter. The equity premium is calculated fixing the utility parameters to the benchmark values (i.e. $\gamma = 10$, $\psi = 1.391$, and $\beta = 0.9925$).

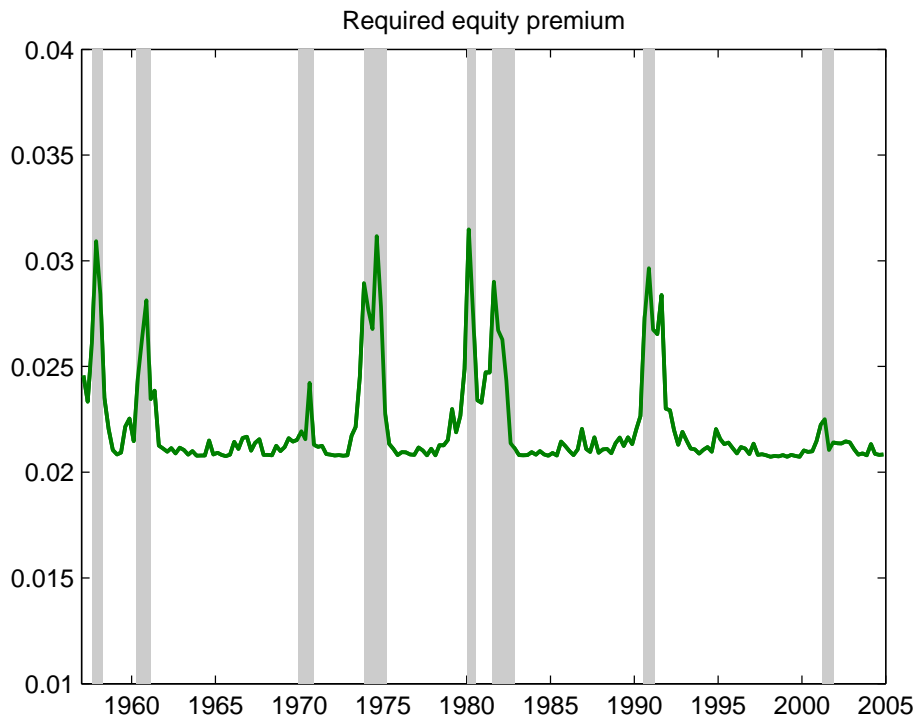


Figure 5: Equity premium's sensitivity

This figure shows the sensitivity of the implied equity premium to the relevant parameters used in the model. It plots the role of both Risk Aversion and Elasticity of Intertemporal Substitution in determining the equity premium.

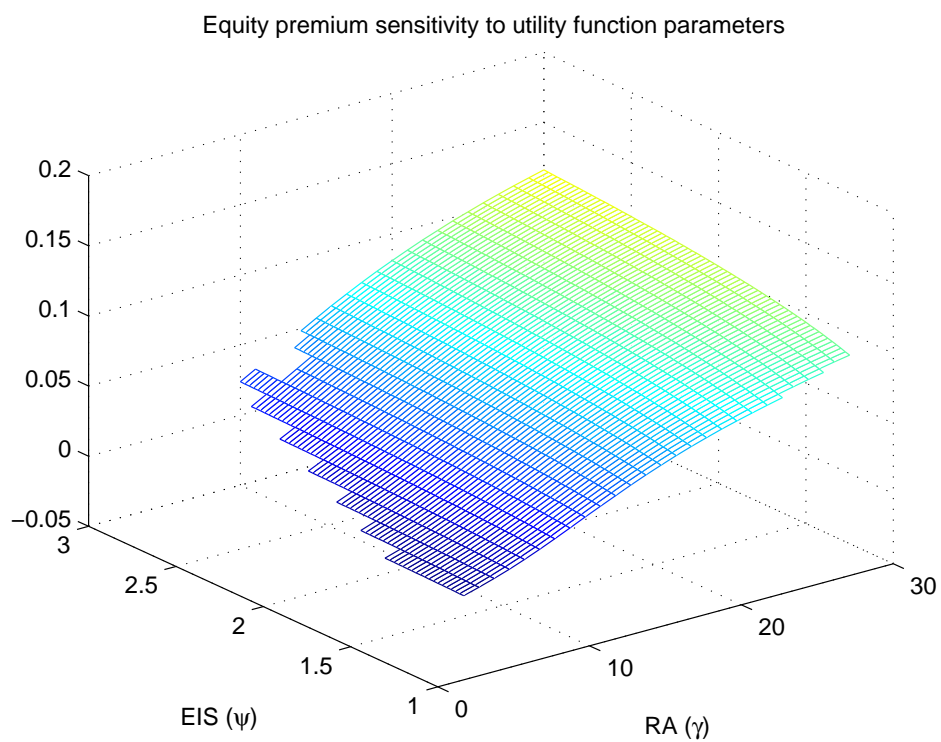


Figure 6: Returns' sensitivity to risk aversion and intertemporal substitution
This figure shows the sensitivity of assets' returns to utility function's parameters. Panel A plots the role of Risk Aversion in determining the required return on a risky asset. Panel B analyzes the influence of Elasticity of Intertemporal Substitution in determining the required risk free return.

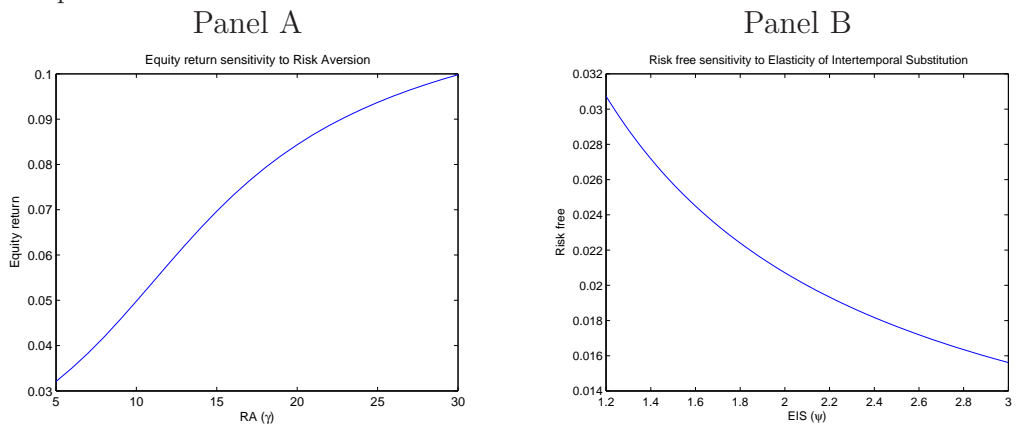


Table 1: Estimation of regime switching economy

This table reports the estimated parameters of a two state Markov switching model for the US postwar economy. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with both the mean and the volatility of technological shocks being different in the two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

State	Technological shocks' process estimation			
	$\mu(s)$	$\sigma(s)$	p_{ij}^μ	p_{ij}^σ
High (s=high)	0.0047 (0.0010)	0.0101 (0.0009)	0.0567 (0.0227)	0.0191 (0.0164)
Low (s=low)	-0.0109 (0.0069)	0.0046 (0.0005)	0.1972 (0.0680)	0.0209 (0.0207)

Table 2: Financial series of the US economy using Lettau et al. (2008) utility parameters

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (<http://www.econ.yale.edu/shiller/data.htm>). The coefficient of risk aversion is set to 30, the EIS is set to 1.5.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.080	0.074	0.031
Risk Free	0.013	-0.045	0.006	0.000
Equity premium	0.058	0.125	0.074	0.044
$\rho(r^f; r^e)$	0.047	-0.078	-	-
$\rho(r_{t+1}^e; r_t^e)$	0.105	0.540	-	-
$\rho(r_{t+1}^f; r_t^f)$	0.890	0.540	-	-

Table 3: Empirical series of US financial markets matching the risk free rate
This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (<http://www.econ.yale.edu/~shiller/data.htm>). The coefficient of risk aversion is set to 10. The obtained EIS by matching the empirical mean of the risk free rate is 1.391

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.066	0.074	0.036
Risk Free	0.013	0.013	0.006	0.009
Equity premium	0.058	0.053	0.074	0.051
$\rho(r^f; r^e)$	0.047	0.059	-	-
$\rho(r_{t+1}^e; r_t^e)$	0.105	0.598	-	-
$\rho(r_{t+1}^f; r_t^f)$	0.890	0.610	-	-

Table 4: Estimated parameters for the restricted regime switching model
 Reported are the estimated parameters for the restricted version of the model. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with the mean of technology shocks being different in two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

State	$\mu(s)$	σ	p_{ij}^μ
High (s=high)	0.0051 (0.0007)	0.0074	0.0630 (0.0219)
Low (l=low)	-0.0112 (0.0019)	(0.0004)	0.2244 (0.0692)

Table 5: Empirical series of US financial markets: restricted regime switching model

This table shows the asset returns implied by the restricted model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (<http://www.econ.yale.edu/shiller/data.htm>). The coefficient of risk aversion is set to 10, the EIS is set to 1.391.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.062	0.074	0.035
Risk Free	0.013	0.022	0.006	0.010
Equity premium	0.058	0.040	0.074	0.050
$\rho(r^f; r^e)$	0.047	0.062	-	-
$\rho(r_{t+1}^e; r_t^e)$	0.105	0.540	-	-
$\rho(r_{t+1}^f; r_t^f)$	0.890	0.540	-	-