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Testing habits in an asset pricing model

Melisso Boschi* Stefano d'Addona[†] Aditya Goenka[‡]

Abstract

We develop a model of asset pricing assuming that investor's behavior is habit forming. The model predicts that the effect of consumption growth shocks on the risk premium depends on the business cycle phase of the economy. This empirical implication is tested with a Markov-switching VAR model on the US postwar economy.

The results show that the response of the risk premium to shocks to consumption is not significantly different over the business cycle phases of the economy. We interpret this as evidence against the habit formation hypothesis of the investor's behavior.

Keywords: Habit formation, Equity premium, Business cycle, Markov-switching VAR models.

JEL Classification: E21, E32, E44, G11, G12.

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1 Introduction

In the past two decades there has been growing interest in the implications of preferences that are not time separable, with a focus on those exhibiting habit formation. When consumers have preferences with habit formation, current utility depends not only on current expenditure, but also on a “habit stock” formed by past expenditures. For a given level of current expenditure, a larger habit stock lowers utility. This implies that habit formation causes consumers to adjust slowly to shocks to permanent income and it can explain the “excess” smoothness of aggregate consumption documented by Campbell and Deaton (1989) as well as by Carroll and Weil (1994). In addition, habits have provided a partial solution to the equity premium puzzle, since they increase the disutility associated with large declines in consumption (Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999)). Habit formation has also been pushed forward as a solution to a number of other empirical anomalies associated with permanent income hypothesis models of inter-temporal consumption behavior. Carroll et al. (2000) use habits to explain the direction of causality from national growth rates to aggregate saving. Fuhrer (2000) shows that habits allow the delayed response of consumption and inflation to monetary shocks that is observed empirically. Gruber (2004) proposes a solution to the “excess volatility” problem of inter-temporal current account models by incorporating consumption habits in the standard model.

Notwithstanding the wide use of habit formation as a device to solve empirical puzzles, tests of this hypothesis based on aggregate consumption data yield only mixed conclusions. Dunn and Singleton (1986), Eichenbaum et al. (1988), and Heaton (1993) find very little evidence of habit formation in

U.S. aggregate monthly consumption data, and Muellbauer (1988) produces similar results with U.S. quarterly consumption data. In contrast, Ferson and Constantinides (1991) find large and statistically significant amounts of habit formation in monthly, quarterly, and annual U.S. consumption data, while Braun et al. (1993) find some habit formation in aggregate Japanese consumption. These widely varying conclusions stem from differences in the estimated first-order conditions, data, and instruments. Moreover, all studies of habit formation based on aggregate data face a common problem: their conclusions hinge on the serial correlation of aggregate consumption growth, which is appreciably influenced by a number of factors unrelated to preferences, such as time averaging, aggregation across individuals, data construction methods among others. Dynan (2000) uses data on food expenditures from the *Panel Study on Income Dynamics*(PSID) to estimate the first-order condition of a life-cycle consumption model with habit formation. She argues that such data are far less influenced by the factors that distort the serial correlation of aggregate data. Her results yield no evidence of habit formation at the annual frequency. However, these results as well suffer from problems related to data since any durability in consumption figures could partially or even completely obscure habit formation, while habit formation and durability parameters cannot be estimated separately with PSID data.

Besides problems related to data aggregation and measurement errors, as discussed by Dynan (2000), all tests reviewed above rely crucially on heavy assumptions required to estimate the Euler equation. In addition, a number of special problems arise when estimating consumption Euler equations with households data, and the solutions provided by the existing literature are designed for linear equations, while the first-order condition of a consumption

model with habits is highly non-linear.

In this paper we test the hypothesis of habit formation by assessing the empirical implications of a simple model of portfolio decision in which the investor's behavior is affected by (a slow moving) external habit based on past aggregate consumption (Abel (1990), Campbell and Cochrane (1999)). One of the key insights of habit models is that as investors' wealth and consumption fall to the habit level, the effective curvature of the utility function increases making investors act as if they were more risk averse. This increases the risk premium that investors need in order to hold risky assets. The latter point links closely to the empirical analysis provided here. In fact, we test a set up where the relevant variables follow a two-state Markov-switching process. Because models of this class are able to capture non-linearities that are missed by more traditional ones, they are suitable to model the different effect on excess returns of a shock to consumption growth that depends on the relative position of consumption with respect to habits. Depending on the state of economic activity, whether in expansion or contraction, the effect of a shock to consumption growth on excess returns will either be smaller or greater, respectively.

Notice that this does not happen with the usual specification of utility functions. In fact, when we consider the standard CRRA utility the effect of consumption shocks on the risk premium is independent of the business cycle phase. At the same time, considering a general non-linear specification of the utility function generates a non-linear relationship between consumption shocks and the risk premium (see Boschi and Goenka (2007)) which, however, does not necessarily depend on the business cycle regime. In this sense, the empirical methodology applied below is a valid test of the habit formation hypothesis in financial markets.

We test the model on the US postwar economy. The estimation results show that, as expected, the risk premium reacts negatively to a positive shock in consumption, but the response seems to be linear with respect to different regimes of the economy. This empirical evidence is in contrast with the habit models prediction is that, as investors' consumption fall to the habit level, they become more risk averse, increasing the required premium.

The remainder of the paper is structured as follows: section 2 outlines the asset pricing implications of an open economy model with habit formation. Section 3 discusses the empirical methodology while section 4 presents the data used in the estimation. Finally, section 5 discusses the results and section 6 concludes.

2 The model

This section derives the risk premium from an asset pricing model with habit formation.

There are N identical price-taking investors. The time horizon is infinite. The representative investor maximizes the period utility flow which depends on current consumption and is affected by external habit formation.

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s - X_s) \right\} \quad (1)$$

where E_t is the expectation operator conditional on information available at date t , $\beta \in \{0, 1\}$ is the constant subjective time-preference factor, $u(\cdot)$ is the period utility function which is assumed to be twice-continuously differentiable, strictly increasing, and strictly concave, i.e. $u'(\cdot) > 0$, $u''(\cdot) < 0$, C_s is real consumption on period s , X_s is the habit level and depends on

the history of aggregate consumption. The price of the consumption good is normalized to 1.

The period-by-period budget constraint is given by:

$$B_{s+1}^f + x_{s+1}B_s = (1 + r_s^f)B_s^f + x_s(I_s + B_s) - C_s \quad (2)$$

where B_s^f is the real net risk-free bond purchase at time $s - 1$, x_s is the fractional share of the risky equity purchased by the agent in period $s - 1$, B_s denotes the date s real market value of equity, r_s^f is the net real interest rate on the risk-free bond B_s^f between period $s - 1$ and s , I_s is the dividend paid on equity at time s . Equation (2) expresses the link between period s 's saving and period $s + 1$ financial wealth. One can think of B_s^f as the net purchase of a United States Treasury bill.

Maximizing the utility function (1) subject to the constraints (2) with respect to B_{s+1}^f and x_{s+1} gives the following Euler equations:

$$u'(C_s - X_s)B_s = \beta E_s \{u'(C_{s+1} - X_{s+1})(I_{s+1} + B_{s+1})\} \quad (3)$$

and

$$u'(C_s - X_s) = (1 + r_{s+1}^f)\beta E_s [u'(C_{s+1} - X_{s+1})] \quad (4)$$

Define the *ex post* net real rates of return on the risky equity as:

$$r_{t+1} \equiv \frac{I_{t+1}}{B_t} + \frac{B_{t+1} - B_t}{B_t}$$

Therefore, from (3), recalling that $E(XY) = Cov(X, Y) + E(X)E(Y)$, we obtain:

$$\begin{aligned}
u'(C_s - X_s) &= \beta Cov \{u'(C_{s+1} - X_{s+1}), (1 + r_{s+1})\} \\
&\quad + \beta E_s [u'(C_{s+1} - X_{s+1})] E_s(1 + r_{s+1}) \quad (5)
\end{aligned}$$

Dividing both sides by $u'(C_s - X_s)$, using (4) to substitute out $\beta E_s u'(C_{s+1} - X_{s+1})/u'(C_s - X_s)$, and rearranging, we obtain, for $s = t$:

$$E_t(1 + r_{t+1}) - (1 + r_{t+1}^f) = -(1 + r_{t+1}^f) Cov \left\{ \frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r_{t+1} \right\} \quad (6)$$

Equation (6) is the crucial expression of the consumption-based CAPM with habit formation. It says that, given the assumptions on the period utility function, the risk premium on the risky asset depends positively on the covariance of the asset's return with the surplus consumption growth. If the covariance term is negative, the risk premium will be positive, meaning that the asset yields unexpectedly high returns in states of nature when the level of surplus consumption is unexpectedly high. Therefore, the asset does not provide a hedge against consumption fluctuations and the investor will require an excess return with respect to the risk-free bond's return to be persuaded to hold the asset.

2.1 Model predictions

In this subsection we analyze some of the theoretical predictions of the model outlined above. The first analysis can be done on the relation between risk aversion and consumption level.

Proposition 2.1. *The risk premium is negatively related to the surplus*

consumption ratio:

$$E_t(r_{t+1}) - r_{t+1}^f \approx (1 + r_{t+1}^f)\beta \frac{\gamma}{S_t} Cov \left\{ \frac{(C_{t+1} - X_{t+1})}{C_t}, r_{t+1} \right\}$$

Proof. We take a second order Taylor expansion at the points $C_{t+1} - X_{t+1} = C_t - X_t$ and $r_{t+1} = E_t(r_{t+1})$ of the function

$$G(C_{t+1} - X_{t+1}, r_{t+1}) \equiv \frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)} [r_{t+1} - E_t(r_{t+1})]$$

the expected value of which equals the covariance entering the risk premium in equation (6).¹

$$G[C_t - X_t, E_t(r_{t+1})] = \frac{\beta u'(C_t - X_t)}{u'(C_t - X_t)} [E_t(r_{t+1}) - E_t(r_{t+1})] = 0;$$

$$\frac{\partial G[C_t - X_t, E_t(r_{t+1})]}{\partial (C_{t+1} - X_{t+1})} = \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} [E_t(r_{t+1}) - E_t(r_{t+1})] = 0;$$

$$\frac{\partial G[C_t - X_t, E_t(r_{t+1})]}{\partial r_{t+1}} = \frac{\beta u'(C_t - X_t)}{u'(C_t - X_t)} = \beta;$$

¹Recall that the second-order approximation to $G(X, Y)$ near $X = \bar{X}$ and $Y = \bar{Y}$ is:

$$\begin{aligned} G(X, Y) \simeq & G(\bar{X}, \bar{Y}) + G_X(\bar{X}, \bar{Y})(X - \bar{X}) + G_Y(\bar{X}, \bar{Y})(Y - \bar{Y}) \\ & + \frac{1}{2}G_{XX}(\bar{X}, \bar{Y})(X - \bar{X})^2 + \frac{1}{2}G_{YY}(\bar{X}, \bar{Y})(Y - \bar{Y})^2 \\ & + G_{XY}(\bar{X}, \bar{Y})(X - \bar{X})(Y - \bar{Y}). \end{aligned}$$

$$\frac{\partial^2 G [C_t - X_t, E_t (r_{t+1})]}{\partial (C_{t+1} - X_{t+1})^2} = \frac{\beta u'''(C_t^* - X_t^*)}{u'(C_t - X_t)} [E_t (r_{t+1}) - E_t (r_{t+1})] = 0;$$

$$\frac{\partial^2 G [C_t - X_t, E_t (r_{t+1})]}{\partial (r_{t+1})^2} = 0;$$

$$\frac{\partial^2 G [C_t - X_t, E_t (r_{t+1})]}{\partial (C_{t+1} - X_{t+1}) \partial r_{t+1}} = \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)};$$

Therefore:

$$\begin{aligned} G(C_{t+1} - X_{t+1}, r_{t+1}) &\approx \beta [r_{t+1} - E_t (r_{t+1})] + \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} \\ &\quad [C_{t+1} - X_{t+1} - (C_t - X_t)] [r_{t+1} - E_t (r_{t+1})] \end{aligned} \quad (7)$$

Taking conditional expectations of both sides of equation (7), yields:

$$\begin{aligned} E_t [G(C_{t+1} - X_{t+1}, r_{t+1})] &= Cov \left\{ \frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r_{t+1} \right\} \\ &\approx \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} \cdot \\ &\quad E_t \{ [(C_{t+1} - X_{t+1}) - (C_t - X_t)] \cdot \\ &\quad [r_{t+1} - E_t (r_{t+1})] \} \\ &= \beta \frac{C_t u''(C_t - X_t)}{u'(C_t - X_t)} \cdot \\ &\quad Cov \left\{ \frac{(C_{t+1} - X_{t+1})}{C_t}, r_{t+1} \right\}. \end{aligned}$$

Hence equation (6) becomes:

$$E_t(r_{t+1}) - r_{t+1} \approx (1 + r_{t+1})\beta \frac{\gamma}{S_t} Cov \left\{ \frac{(C_{t+1} - X_{t+1})}{C_t}, r_{t+1} \right\} \quad (8)$$

where

$$\frac{\gamma}{S_t} = \frac{-C_t u''(C_t - X_t)}{u'(C_t - X_t)}$$

denotes the local curvature of the utility function with habits. \square

We will now show how expectations about future consumption growth will affect current consumption level. In order to do this, we use the log-linear approximation to the budget constraint proposed by Campbell (1993)

Consider that the representative investor's dynamic budget constraint (equation (2)) can be alternatively written as:

$$W_t = (W_{t-1} - C_{t-1})(1 + r_t^w) \quad (9)$$

where W_t denotes total real wealth and $(1 + r_t^w)$ is defined to be the gross real return on wealth invested from period $t - 1$ to period t . Given portfolio diversification, the *ex post* gross return can be decomposed as follows:

$$(1 + r_t^w) = q_t^f(1 + r_t^f) + q_t(1 + r_t) \quad (10)$$

where q_t^f is the proportion of wealth invested in the risk-free bond and q_t is the proportion of wealth invested in the risky asset at time $t - 1$, implying that $q_t^f + q_t = 1$.

Taking logarithms of expectations of both sides of (10) gives:

$$E_{t-1}(r_t^w) \approx \log\{q_t^f(1+r_t^f) + q_t \exp[E_{t-1}(r_t)]\} \quad (11)$$

Proposition 2.2. *An unexpected decrease in wealth through an unexpected fall in current consumption and a decrease in the surplus consumption ratio, leads to an increase in the risk premium.*

Proof. Dividing (9) by W_{t-1} and taking logarithms, we obtain:

$$\Delta w_t \approx r_t^w + \log[1 - \exp(c_{t-1} - w_{t-1})] \quad (12)$$

where $r_t^w \approx \ln(1 + r_t^w)$.

Taking a first-order Taylor expansion around the mean $(\bar{c} - \bar{w})$ of the second term on the right hand side of equation (12) we get the following approximation to the budget constraint (9):

$$\Delta w_t \approx r_t^w + k + \left(1 - \frac{1}{\eta}\right)(c_{t-1} - w_{t-1}) \quad (13)$$

where $k = \log(1 - \exp(\bar{c} - \bar{w})) - \left(1 - \frac{1}{\eta}\right)(\bar{c} - \bar{w})$, $\left(1 - \frac{1}{\eta}\right) = -\frac{\exp(\bar{c} - \bar{w})}{1 - \exp(\bar{c} - \bar{w})}$, and $\eta \equiv 1 - \exp(\bar{c} - \bar{w})$.

Next, consider the equality:

$$\Delta w_t = \Delta c_t + (c_{t-1} - w_{t-1}) - (c_t - w_t) \quad (14)$$

Equating the left hand sides of (13) and (14), solving forward the resulting difference equation in $c_{t-1} - w_{t-1}$, assuming that $\lim_{j \rightarrow \infty} \eta^j (c_{t+j} - w_{t+j}) = 0$, and taking expectations at time $t - 1$ we obtain:

$$c_{t-1} - w_{t-1} = E_{t-1} \sum_{j=1}^{\infty} \eta^j (r_{t-1+j}^w - \Delta c_{t-1+j}) + \frac{\eta k}{1 - \eta} \quad (15)$$

Finally, substitute out equation (15) into (13) and (14) to obtain:

$$c_t - E_{t-1}c_t = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \eta^j r_{t+j}^w - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \eta^j \Delta c_{t+j} \quad (16)$$

Paraphrasing Campbell (1993), equation (16) indicates that an unexpected decrease in consumption today must be determined by an unexpected reduction of return on wealth today, or by news that future returns will be lower, or, finally and most importantly to the present analysis, by news that future consumption growth will be higher.

The decrease in current consumption implies a decrease in the surplus consumption ratio through equation (8), and an increase in the risk premium by Proposition 1. \square

3 Testable hypothesis and empirical methodology

As shown in proposition 2.1, an implication of the habit formation model of Campbell and Cochrane (1999) is that during economic expansions consumption increases above habits leading to a decline in risk aversion. This decline in risk aversion, in turn, leads to a greater demand for risky assets and a decrease in expected excess returns, or risk premia. However, from equation (8) we see that the effect of a shock to consumption growth or excess returns is different depending on the relative position of consumption with respect to habits, i.e. depending on the level of S_t . Given the level of the shock, the effect on S_t is different according to the state of the economy.

It will be greater over economic contractions while it will be smaller over economic booms.

This leads to a straightforward empirical methodology suitable to test for the implications of the habit model outlined above. A Markov-switching VAR model will allow us to model the dynamic relationship among consumption, domestic and foreign expected excess returns over different state of the economy that we may interpret as different phases of the business cycle. By simulating a shock to the different variables of the model, we expect to find significantly different responses of the risk premia over different states of the economy. To derive the empirical specification of the model, we follow the approach proposed by Campbell (1991) in modeling excess stock returns and consumption as elements of a vector auto-regression. First we define a vector \mathbf{y}_{t+1} which has k elements, i.e. the risk free real interest rate, the excess return, the consumption level, and the consumption growth. Then I assume that the vector \mathbf{y}_{t+1} follows a first order VAR whose parameters are subject to regime shifts, as detailed in the next section. The assumption that the VAR is first-order is not restrictive, since a higher-order VAR can always be stacked into first-order (companion) form in the manner discussed by Campbell and Shiller (1988).

3.1 Markov-switching VAR models

Since Hamilton Hamilton (1988), Hamilton (1989) and Hamilton (1994) popularized this approach through his seminal research, the Markov-switching model has been widely applied in the analysis of various economic phenomena. Its multivariate version extends a standard linear VAR model by allowing its parameters to be subject to regime shifts. In such situation, rather

than time-varying, the VAR process can be modeled as time-invariant conditional on an unobservable regime variable s_t which indicates the regime prevailing at time t . Therefore, Markov-switching vector auto-regressions are generalizations of the basic VAR model of order p :

$$\mathbf{y}_t = \nu + \sum_{i=1}^p \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{u}_t \quad (17)$$

where $\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$ is a K -dimensional vector, ν is an intercept term, \mathbf{A}_i , for $i = 1, \dots, p$, are $K \times K$ matrices of coefficients, and \mathbf{u}_t is a vector of residuals. Denoting $\mathbf{A}(L) = \mathbf{I}_K - \mathbf{A}_1 L + \dots + \mathbf{A}_p L^p$ as the lag polynomial of dimension $K \times K$, we assume that there are no roots on or inside the unit circle $|\mathbf{A}(z)| \neq 0$ for $|z| \leq 1$, where L is the lag operator. Under the additional assumption that $\mathbf{u}_t \sim NID(\mathbf{0}, \mathbf{\Sigma})$, equation (17) is the intercept form of a stable *Gaussian* VAR model of order p .

Since we assume that the parameters of the *observed* time series vector \mathbf{y}_t depend on the *unobservable* regime variable s_t , a model for the regime generating process is required. In the Markov-switching VAR model the regime $s_t \in \{1, \dots, M\}$ is assumed to be governed by a discrete time, discrete state Markov stochastic process characterized by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}. \quad (18)$$

The transition probabilities can be represented by the following transition

matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \quad (19)$$

where $p_{iM} = 1 - p_{i1} - \dots - p_{i,M-1}$ for $i = 1, \dots, M$. A crucial assumption for the theoretical properties of MS-VAR models is that s_t follows an irreducible ergodic M state Markov process with transition matrix given by (19).

Therefore, if time series are subject to shifts in regime, the M regimes Markov-switching form of the VAR(p) model of equation (17) is given by:

$$\mathbf{y}_t = \nu(s_t) + \sum_{i=1}^p \mathbf{A}_i(s_t) \mathbf{y}_{t-i} + \mathbf{u}_t. \quad (20)$$

where $\mathbf{u}_t \sim NID(\mathbf{0}, \mathbf{\Sigma}(s_t))$ and $\mathbf{A}_1(s_t), \dots, \mathbf{A}_p(s_t), \mathbf{\Sigma}(s_t)$ are shifts functions describing the dependence of the parameters $\mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{\Sigma}$ on the realized regime s_t .

A simple way to backup the choice of the variables in the empirical specification of the VAR is based on a straightforward manipulation of equation (16). Let us remind t that $q_t^f + q_t = 1$, we can rewrite equation (10) as follows:

$$(1 + r_t^w) = (1 + r_t^f) + q_t[(1 + r_t) - (1 + r_t^f)] \quad (21)$$

which can transformed in an approximate equation for log returns (see Lettau and Ludvigson (2001)) taking the form:

$$r_t^w \approx r_t^f + q_t(r_t - r_t^f) \quad (22)$$

This means that the equation (16) can be rewritten as follows:

$$\begin{aligned}
c_t - E_{t-1}c_t &= r_t^f - E_{t-1}r_t^f + r_t - E_{t-1}r_t + (E_t - E_{t-1}) \sum_{j=1}^{\infty} \eta^j r_{t+j}^f \\
&\quad + (E_t - E_{t-1}) \sum_{j=1}^{\infty} \eta^j q_t r_{t+j} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \eta^j \Delta c_{t+j} \quad (23)
\end{aligned}$$

Equation (23) indicates that we can include four variables in the VAR specification of the empirical model: the cyclical component of the consumption, the detrended risk free rate, the market return, and consumption growth rate. At the same time it allow us to identify the main shocks of the VAR by imposing some structure on the covariance matrix. In fact, equation (23) implies that consumption unexpected movements are influenced contemporaneously by shocks to the risk-free rate and by shocks to the excess return $(r_t - r_t^f)$.

The parameters of the model are estimated with the maximum likelihood method (see Hamilton (1989) and Hamilton (1994)). The maximization of the likelihood function of an MS-VAR requires an iterative estimation of the parameters of the auto-regression and the transition probabilities governing the Markov chain of the unobserved states. This is usually obtained through the implementation of the *Expectation Maximization* (EM) algorithm introduced by Dempster et al. (1977) and proposed by Hamilton (1990) for this class of models.

In order to compute impulse response functions, which are necessary to gauge the effect of a consumption growth shock on the risk premium, identifying structural shocks is required. It is popular to identify the system

for contemporaneous relationship among endogenous variables. To this end some authors, such as Christiano et al. (1999), make use of the Cholesky decomposition, which assumes that the system is recursive and hence allows identification. This identification scheme is also employed in this paper.

We employ the procedure developed by Ehrmann et al. (2003) to obtain regime-dependent impulse response functions to depict the relationship between endogenous variables and fundamental disturbances within a regime. As is standard for impulse responses, these illustrate expected changes in the endogenous variables after one standard deviation shock to one of the fundamental disturbances. However, regime-dependent impulse response functions are conditional on the regime prevailing at the time of the disturbance continuing to prevail throughout the duration of the responses. Therefore, as mentioned earlier, this concept is valid only when each regime is persistent. To obtain confidence intervals for the impulse response functions we bootstrap the system resorting once again to the procedure advocated by Ehrmann et al. (2003).

4 Data and regimes estimation

We test our empirical prediction on the US postwar economy. The dataset is quarterly over the sample period 1952:1 to 2008:4. All variables are in real terms and are obtained using the CPI index provided by Robert Shiller through his website. We take log of per capita consumption from the dataset provided by Martin Lettau and Sidney Ludvigson on their web site, and used in Lettau and Ludvigson (2001). As a risk free, we employ The 30-day US Treasury bill rate taken from CRSP while as a market return we use the rate of return on the S&P composite index taken from Robert

Table 1: Estimation of Regime-Switching VAR

This table reports the estimated parameters of the regime switching model for the US postwar data.

State	Transition Matrix	
	p_{ij}	Average Duration
Regime 1	0.0272	23.80
Regime 2	0.0420	36.83

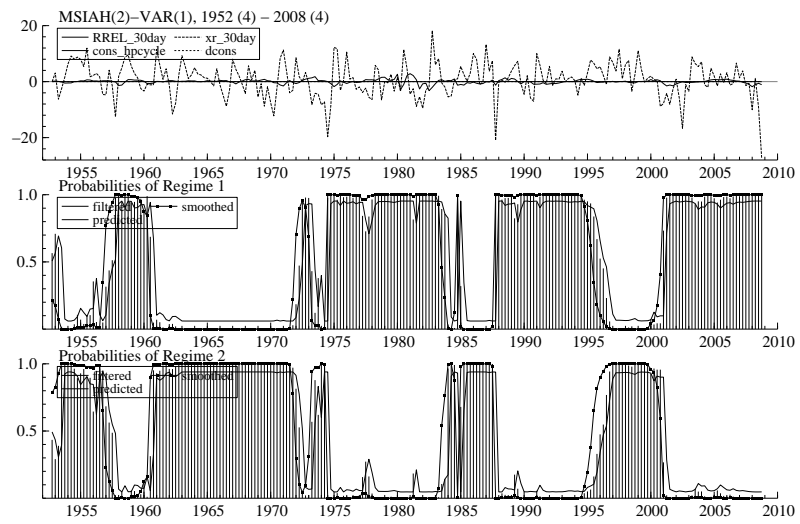
Shiller web site. The cyclical component of the consumption level (c_t) is obtained with a Hodrick and Prescott (1997) filter. The excess return (xr_t^i) is calculated as the excess return of the S&P composite index over the 30-day US Treasury bill rate. As a measure of the risk free we use the difference between the real US Treasury bill rate and its 12-months backward moving average. Computing this difference is a crude way to obtain a stochastically detrended measure of the risk free rate, denoted with RREL, and it has been used by Campbell (1991) and Hodrick (1992).

We estimate a MS-VAR, where the vector of the endogenous variables \mathbf{y}_t includes the RREL, the excess return, xr_t^{US} , the HP filtered consumption level, c_t , as well as the consumption growth rate calculated as the log first difference of real per capita consumption. The model specification allows for regime changes in the intercept, the autoregressive coefficient matrix \mathbf{A} , and the variance-covariance matrix. That is, we estimate a MSIAH-VAR model, in the terminology of Krolzig (1997).

The first panel of Figure 1 shows the time plot of the four variables of the system, while the second and third panel of show the regime probabilities of the same model.

The transition matrix, reported in table 1, shows a regime that is more

Figure 1: Regime probabilities



persistent than the other. In fact the regime labelled “Regime 1” has an average duration of 9 years while the second regime displays a duration of less than 6 years. Some inference can be drawn from a simple association of regime periods to business cycle phases in the last twenty years. The plots of regime probabilities (panel 2 and 3 of figure 1) suggest that the first regime can be identified with a low consumption growth state of the economy, while the second corresponds to a high consumption growth state, thus implying that regime 1 identifies a recession or stagnation state, while regime 2 corresponds to an expansion state. Therefore, the economy is predicted to be in the recession state in the beginning and the middle of the eighties, at the beginning of the nineties and at the beginning and the end of the last decade.

The graphical analysis depicted above is confirmed when we turn to the VAR estimates. Table 2 reports the unconditional means of the four variables in the VAR estimation. The values for both consumption measures confirm that we can interpret the first regime with a low consumption period and the second regime with a high consumption period. In fact the cyclical component displays a negative unconditional mean in the first regime and a positive value in the second regime, while the growth rate of consumption has a higher unconditional mean in the second regime.

5 Results

The natural way to look at the response of the risk premium to a shock to consumption is via the impulse response functions. Figure 2 reports these impulse response functions along with bootstrapped confidence intervals. To obtain them we employ a Cholesky factorization of the variance covariance

Table 2: Unconditional means of the VAR variables

This table reports the estimated unconditional means of the MSVAR in the two regimes.

State	Unconditional Means			
	RREL	$r_t - r_f$	c_t	Δc_t
Regime 1	-0.957	0.581	-0.002	0.004
Regime 2	-0.800	0.740	0.001	0.006

matrix to recover structural shocks, along the lines of Ehrmann et al. (2003).

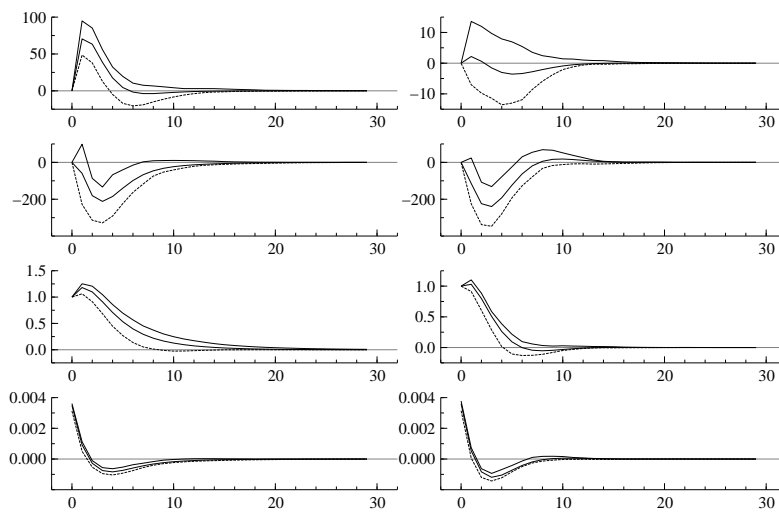
Specifically, figure 2 shows the responses of $RREL_t$ (top row), xr_t (second row), c_t (third row) and Δc_t (fourth row) in regime 1 (left-hand column) and regime 2 (right-hand column) to a one s.d. shock to c_t .

from the second row of figure 2 we can draw two main conclusions. First the direction of the risk premium response is as expected: a positive shock to the cyclical component of the consumption reduces the risk premium. Second, it is clear that the response of the risk premium to the shocks to the cyclical component of consumption does not differ in regime 1, i.e. in the low consumption growth state of the economy, from the response in regime 2, i.e. in the expansion state of the economy.

The linearity between regimes displayed by the risk premium response to the consumption shock is in contrast with the implication of equation (8), that predicts a higher required equity premium when the consumption level falls towards the habit level (i.e. when the economy is in a low consumption state).

Figure 2: Impulse response functions

This figure shows the responses of $RREL_t$ (top row), xr_t (second row), c_t (third row) and Δc_t (fourth row) in regime 1 (left-hand column) and regime 2 (right-hand column) to a one s.d. shock to c_t .



6 Conclusion

Habit formation has been widely used in the literature on asset pricing implications of macroeconomic models as a possible solution to several empirical anomalies in finance. We develop a model of asset pricing which has clear implications about the empirical relationship between the current level of consumption and the risk premium over the business cycle. In order to test these implications we specify a multivariate econometric model where all parameters are subject to Markovian regime switching. In order to simulate regime-dependent impulse responses of endogenous variables we recover structural shocks through a Cholesky factorization of the variance-covariance matrix of residuals in line with the procedure proposed by Ehrmann et al. (2003). Estimation results show that following a one standard deviation shock to consumption, the risk premium in the US is not affected much differently in economic recessions with respect to economic expansions. We interpret this result as evidence against habit formation in investors behavior.

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