Time−Varying Parameters in the Almost Ideal Demand System and the Rotterdam Model: Will the Best Specification Please Stand Up?

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Abstract

This paper assesses the ability of the Rotterdam model and of three versions of the almost ideal demand system (AIDS) to recover the time-varying elasticities of a true demand system and to satisfy theoretical regularity. Using Monte Carlo simulations, we find that the Rotterdam model performs better than the linear-approximate AIDS at recovering the signs of all the time-varying elasticities. More importantly, the Rotterdam model has the ability to track the paths of time-varying income elasticities, even when the true values are very high. The linear-approximate AIDS, not only performs poorly at recovering the time-varying elasticities but also badly approximates the nonlinear AIDS.

Key Words: AIDS, Rotterdam model, structural time series models, Monte Carlo experiment, theoretical regularity.

JEL Codes: D12, C51, C52

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1 Introduction

The almost ideal demand system of Deaton and Muellbauer (1980a,b) and the Rotterdam model (Barten, 1964, 1968, 1977; Theil, 1965, 1975a,b) have been widely adopted in applied demand research. Their attractiveness is explained by the fact that both demand specifications share desirable properties that are not possessed by other local flexible functional forms such as the Generalized Leontief (Diewert, 1971) and the Translog (Christensen et al., 1975): local flexibility, consistency with demand theory, linearity and parsimony with respect to the parameters. They also have identical data requirement so that no additional variable is needed in order to estimate one specification whenever the estimation of the other is possible.

However, the two specifications lead to different results in some applications (Alston and Chalfant, 1991), prompting the question of the appropriateness of either specification for a given dataset. Nevertheless, the adoption of one of the models for empirical demand analysis has been purely arbitrary and possibly motivated by the personal acquaintance of the researcher with each of them. This is understandable since economic theory does not provide a basis for ex ante discriminating among the flexible functional forms in general, and between the AIDS and the Rotterdam model (RM) in particular.

The observed discrepancies between the outcomes of the two specifications require adopting a research strategy that allows to discriminate between them not only based on the demand properties contained in the specific dataset, but also on their consistency with the particular maximization problem that has produced or that is believed to have produced the data. Thus, choosing the best approximating structure for the true underlying model should be the result of a well-defined methodology that establishes the true properties contained in the data as a benchmark. This applies whether consumer preferences are postulated to be fixed as in the neoclassical demand theory, or otherwise subject to shifts of a specific nature.

Alston and Chalfant (1993) developed a statistical test of the linear-approximate AIDS against the RM and then applied it to the meat demand in the United States. The test concluded in favor of the acceptance of the RM, rejecting the AIDS. The same conclusion obtained with Barten (1993)'s test. However, the authors warned that their finding could not be interpreted as an evidence of the superiority of the RM over the AIDS in a general way. Furthermore, their test may lead to a different conclusion if applied to a different dataset. Although this does not clearly appear from Alston and

Chalfant's conclusion, the difference in the performance of the AIDS and the RM from one dataset to another may mainly result from the fact that the datasets are produced by different data generating processes.

On the other hand, Barnett and Seck (2008) conducted a Monte Carlo comparison of the nonlinear AIDS, the linear-approximate AIDS and the RM. They sought to determine which of the three specifications could perform better in terms of the ability to recover the elasticities of the true demand system. Their finding was that both the nonlinear AIDS and the RM performed well when substitution among goods was low or moderately high. However, the nonlinear AIDS model performed better when the substitution among goods was very high. Finally, the RM performed better at recovering the true elasticities within separable branches of a utility function. In this experiment, the linear-approximate AIDS performed badly and was found to be a poor approximation to the nonlinear AIDS.

It is noteworthy that both papers postulated constant parameters in the demand functions and the underlying utility functions. However, when using real data, the consistency of the estimated coefficients of the demand system can be compromised if one wrongly assumes the constancy of the parameters while they are actually random or varying over time. In this case the constant-coefficient model will not only fail to capture the possible long-run dynamics in the data but also will produce a poor approximation to the underlying data generating process (Leybourne, 1993). In addition, it is important that further investigation be conducted in order to determine whether or not the advantages of one demand specification on the other can be preserved when the constant-parameters assumption is abandoned in a Monte Carlo study.

This paper evaluates the performance of the nonlinear AIDS, the linear-approximate AIDS and the Rotterdam model when the parameters of the model of consumer preferences and that of the resulting demand system are permitted to vary over time. To the best of our knowledge, such an assessment has not been attempted yet. The present paper shall contribute to the literature by filling this gap.

The motivation for undertaking this study can be put forth into a threefold argument. First, the real world economic system is constantly subject to shocks that translate into technological and institutional changes as well as shifts in consumer preferences. The interaction of these shocks leads to more or less permanent changes in economic behavioral relationships. Therefore, assuming time-varying parameters helps to capture the dynamics of specific nature in these economic relationships. Second, accounting for shifting consumer preferences allows to deepen our understanding of consumer behavior outside the neoclassical framework of fixed tastes. Moreover, such an approach helps break with the old tradition of considering the subject as pertaining to social disciplines other than economics. Third, both the RM and the AIDS are local first-order Taylor series approximations that are intended to approximate a true demand system derived from any utility maximization problem. When fitting the data to any of these flexible functional forms, an implicit assumption is that there exists an unknown true function of the variables of interest that has generated the observed data given a set of parameters. Since the approximation provided by each functional form is only locally valid, assuming a single value for the parameter vector is more unlikely to provide an adequate approximation of the true demand system that underlies the observed data. This idea has been expressed for the RM by Barnett (1979) and Bryon (1984), and for the AIDS by Leybourne (1993).

It is customary to assume that consumer preferences are affected by taste-changing factors. These factors can be captured in the consumer's behavioral model by postulating, on the one hand, the interdependence of consumer preferences in terms of myopic habit formation (Gaertner, 1974; Pollak, 1976, 1978; Alessie and Kapteyn, 1991; Kapteyn et al., 1997). On the other hand, one can make the assumption of simultaneous consumption decisions (Karni and Schmeidler, 1990) or of intrinsic reciprocity or consumer altruism (Sobel, 2005). Finally, the parameters in the functional form of the consumer model may be assumed as functions of the exogenous taste changing factors or depending on stochastic variables (Ichimura, 1950; Tintner, 1952; Basmann, 1955, 1956, 1972; Barnett, 1979; Basmann et al., 2009; Barten, 1977; Brown and Lee, 2002). In this paper's analytical framework, stochastic factors are considered to affect the marginal utilities and to induce preference changes over time through the parameters of the utility function.

The treatment of varying marginal utilities in this paper differs from Basmann (1985) in that we will not consider multiplicative functional forms for the marginal utilities. In contrast, we shall assume that the stochastic shocks to consumer preferences affect parameters of the marginal rates of substitution over time. In addition, we shall explicitly specify the time-varying process for the stochastic chocks to consumer preferences and estimate the implied time-varying parameters in the demand functions.

The analysis shall be conducted in the framework of Harvey (1989)'s structural time series models. We first assume a pure random walk process for the parameters in the demand systems and compute the time-varying elasticities accordingly. Second, we assume a local trend model specification where the time-varying intercept in each demand equation is specified as a random walk with drift, with the drift itself being a random walk. The two approaches have been respectively used by Leybourne (1993) and Mazzocchi (2003) to estimate time-varying parameters in the linear-approximate AIDS. However, none of the papers attempted to compare the performance of the linear-approximate AIDS neither to that of the nonlinear AIDS nor to that of the RM.

The scope of the results in this paper will be limited to the approximating time-varying elasticities (elasticities of substitution, income and compensated price elasticities) that have a counterpart in the set of relevant elasticities derived from the true model. The approximating time-varying elasticities will be calculated using the estimated time-varying coefficients in each demand specification. Time-varying parameters shall be estimated in each demand system by the Kalman filter and passed through the Kalman smoother for their revision, after appropriately representing each demand specification in a state space form.

The paper is organized in 8 sections, including this introduction. The true model is described in section 2 and the time-varying parameter versions of the AIDS and the Rotterdam model are specified in section 3. Section 4 provides the state space representation of the time-varying parameter AIDS and RM. The Monte Carlo experiment and the data generation procedure are described in section 5, while the estimation method and results are presented in sections 6 and 7 respectively. Section 8 summarizes the findings and concludes with their empirical implications.

2 The true model

The consumer's problem is specified as that of maximizing the time-varying parameter utility function

$$\begin{aligned} u_t &= u\left(\mathbf{x_t}; \mathbf{\Theta}_t\right), \\ \text{subject to} \qquad & \\ \mathbf{p_t'}\mathbf{x_t} &= m_t \\ \mathbf{\Theta}_t &= \mathbf{\Theta}_{t-1} + \varepsilon_{\mathbf{\Theta},\mathbf{t}}. \end{aligned} \tag{2.1}$$

where $\Theta_t = (\theta_{1t}, \theta_{2t}, \cdots, \theta_{nt})$ is the vector of the parameters that describe the form of the ordinal utility function at each time period $t = 1, 2, \ldots, T$; $\mathbf{p}_t = (p_{1t}, p_{2t}, \cdots, p_{nt})$ is the price vector and m_t is the consumer's expenditure. The specification in equation (2.1) implies that only the parameters of the utility function are time-varying and that the functional form of the utility function is time-invariant.

It is assumed that the specification of the time-varying structure of the parameter vector is such that the utility function u_t possesses nice properties at each time period t, that is u_t is assumed to be a well-behaved function that satisfies all the regularity conditions of consumer demand theory(increasingness, quasiconcavity, continuity, etc.). In addition, the shocks to the parameter vector affect the marginal rates of substitution and hence translate into demand functions with time-varying parameters. An important assumption that underlies the model in equation (2.1) is that the parameters of the utility function are affected only by the stochastic process that govern the preference shifting factors. More specifically, the parameters of the utility function and the shocks to consumers' preferences follow exactly the same stochastic process (Kalonda-Kanyama, 2012).

2.1 Illustration: The WS-Branch Utility Tree

To illustrate the above considerations, we shall use a known functional form of the utility function that will serve as the benchmark. The weak separable(WS-) branch utility function shall be used to serve this purpose. It was first introduced by Barnett (1977) and subsequently used by Barnett and Choi (1989) as the underlying true utility function in testing weak separability in four demand specifications. This utility function, which is a macroutility function over quantity aggregator functions, is a flexible blockwise weakly separable utility function when defined over no more than two blocks with a total of two goods in each block. The constant-parameter homothetic form of the WS-branch utility function with two blocks q_1 and q_2 is defined as follows:

$$U = U(q_1(x_1, x_2), q_2(x_3)) = A\left[A_{11}q_1^{2\rho} + 2A_{12}q_1^{\rho}q_2^{\rho} + A_{22}q_2^{2\rho}\right]^{(1/2\rho)} \tag{2.2}$$

where $\rho < 0.5$, the constants $A_{ij} > 0$ are elements of a symmetric matrix such that $A_{ij} = A_{ji}$ and $\sum_i \sum_j A_{ij} = 1$. The constant $A > 0$ produces a monotonic transformation of the utility function and thus can be normalized to 1 without loss of generality. Assume that there are only three goods and that the first block consists of the two first goods x_1 and x_2 while the second block consists only of the third good, x_3. Then the sub-utility functions q_1 and q_2 are defined as follows in terms of the vector of supernumerary quantities $\mathbf{y} = \mathbf{x} - \alpha$, where $\mathbf{x} = (x_1, x_2, x_3)$, and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is a vector of translation parameters:

$$q_1 = q_1(x_1, x_2) = B\left[B_{11}y_1^{2\delta} + 2B_{12}y_1^{\delta}y_2^{\delta} + B_{22}y_2^{2\delta}\right]^{(1/2\delta)} \tag{2.3}$$

$$q_2 = q_2(x_3) = y_3 + \alpha_3 \tag{2.4}$$

where $\delta < 0.5$, $B_{kl} > 0$ for $k,l = 1,2$; $B_{kl} = B_{lk}$ for $k \neq l$ and $\sum_k \sum_l B_{kl} = 1$. Notice that the specification of the aggregator function q_1 in equation (2.3) is the same as the specification of the macroutility function (2.2). Therefore, both functions share the same properties. For example, both functions are monotone and quasi-concave as a result of the restrictions on their parameters. These restrictions insure their theoretical regularity as well.

2.2 True time-varying elasticities

Barnett and Choi (1989) have derived the properties of the WS-branch utility function(income elasticities and elasticities of substitution). When the parameters of the WS-branch utility function are assumed to vary over time as in problem (2.1), the income elasticity of the elementary good x_j *(j = 1,2,3)* is, for every time period t, given by

$$\eta_{jt} = \left(\frac{1}{1 - \overline{\mathbf{p}}_t^{'}\alpha_y}\right)\frac{x_{jt} - \alpha_{jt}}{x_{jt}}. \tag{2.5}$$

On the other hand, the elasticity of substitution between two elementary quantities x_i and x_j is given by

$$\sigma_{ij,t} = \xi_{ij,t}\left(\frac{1}{1 - \overline{\mathbf{p}}'\alpha}\right)\frac{(x_{it} - \alpha_{it})(x_{jt} - \alpha_{jt})}{x_{jt}x_{it}} \tag{2.6}$$

where $\overline{\mathbf{p}}_t = (\overline{p}_{1t}, \overline{p}_{2t}, \overline{p}_{3t})$ is the income normalized price vector, $\mathbf{p}_t/m_t$, with $m_t = \mathbf{p}_t^{'}\mathbf{x}_t$ being the total consumer expenditure at time t. In equation (2.6), $\xi_{ij,t}$ represents the elasticity of substitution between the i^{th} and the j^{th} *(j=1,2)* aggregator function in the WS-branch utility function, and is defined as follows, $\forall t$:

$$\xi_{ij,t} = \frac{1}{(1-\rho_t + R_t)} \tag{2.7}$$

where

$$R_t = -\rho_t \frac{A_{11,t}A_{22,t} - A_{12,t}^2}{(A_{11,t}(\frac{q_{2t}}{q_{1t}})^{-\rho_t} + A_{12,t})(A_{12,t} + A_{22,t}(\frac{q_{2t}}{q_{1t}})^{\rho_t})} \tag{2.8}$$

However this formula applies only when $\alpha_1 = \alpha_2 = 0$ or when the aggregate function is defined in terms of the supernumerary quantities as in equations (2.3) and (2.4)[See Theorem 2.2 in Barnett and Choi (1989)].

The time-varying compensated elasticity of the demand for the elementary good x_i with respect to the price, p_j, of the elementary good x_j obtains from the relation between the Allen-Uzawa elasticity of substitution and the compensated price elasticity, that is

$$\eta^*_{ij,t} = \sigma_{ij,t} w_{jt} \tag{2.9}$$

where $w_{jt} = p_{jt}x_{jt} / \sum_k p_{kt}x_{kt}$ is the expenditure share for the elementary good x_{jt}.

3 Structural time-varying coefficients AIDS and RM

This section introduces the AIDS and the Rotterdam model in the framework of Harvey (1989)'s structural time series models. The resulting demand specifications are respectively referred to as the structural time-varying coefficients (TVC-) AIDS and RM. This framework allows the time-varying specification of the parameters in each demand function and their estimation by means of the Kalman filter, after appropriately representing the demand systems in a state space form.

3.1 The structural TVC-AIDS

In the n-goods unrestricted model, the demand equation for the ith good in the TVC linear-approximate AIDS is specified as follows (see for example Mazzocchi (2003)):

$$w_{it} = \mu_{it} + \sum_{j=1}^{n} \gamma_{ijt} \log p_{jt} + \beta_{it} \log \left(\frac{x_t}{P_t^*} \right) + \phi_{it} + u_{it} \tag{3.1}$$

where w_{it} is the budget share of good i at time t, x_t is the aggregate consumer expenditure on the n goods and P_t^* is the Stone price index defined as $P^* = \prod_{i=1}^{n} p_i^{w_i}$; μ_{it} and the ϕ_{it} are respectively the time-varying intercept and the seasonal components. Finally, u_{it} is an error term that is assumed to be a random noise process. Following Harvey (1989), the time-varying intercept is specified as a random walk with drift, with the drift itself following a pure random walk process. On the other hand, the seasonal dummies ϕ_{it} are constrained to sum to zero over a year. All the price and income coefficients in equation (3.1) are assumed to follow a pure random walk process.

Given the similarity between the nonlinear AIDS and the linear-approximate AIDS, the structural TVC specification for the nonlinear AIDS obtains by using the appropriate price index in equation (3.1) to obtain:

$$w_{it} = \alpha_{it} + \sum_{j=1}^{n} \gamma_{ijt}^* \log p_{jt} + \beta_{it} \log \left(\frac{x_t}{P_t} \right) + \phi_{it} + u_{it}, \tag{3.2}$$

where P_t is the translog price aggregator defined by

$$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j. \tag{3.3}$$

The following constraints are imposed on the parameters of both the nonlinear and the linear-approximate AIDS models to respectively satisfy linear homogeneity, adding-up and Slutsky symmetry at every time period t:

$$\sum_{i=1}^{n} \gamma_{ij,t}^* = 0 = \sum_{i=1}^{n} \beta_{it} \tag{3.4}$$

$$\sum_{i=1}^{n} \alpha_{it} = 1 \tag{3.5}$$

$$\gamma_{ij,t}^* = \gamma_{ji,t}^* \tag{3.6}$$

3.2 The Structural TVC-RM

One important feature of the Rotterdam model is that the constancy of its parameters obtains by assuming constant mean functions involved in the formulas of its marocoefficients. However, Barnett (1979) has shown that the macrocoefficients in the Rotterdam model are not necessarily constant. In contrast they vary over time and are income-proportional-weighted theoretical population averages of microcoefficients. By admitting time-varying microparameters and macroparameters in the Rotterdam model, the implicit assumption is that the coefficients of the utility function that the Rotterdam is approximating are also time-varying. However, the neoclassical theory leaves open the question of how consumer preferences are affected by exogenous factors over time.

We assume that shocks to preferences reflect into the utility function in the form of time-varying parameters. Hence the Rotterdam model is theoretically well suited to incorporate the analysis of change in preferences over time. The specification of the ith equation in the structural TVC-RM is given in equation (3.7) as follows:

$$\overline{w}_{it} Dq_{it} = \varpi_{it} + \theta_{it} DQ_t + \sum_{j=1}^{n} \pi_{ij,t} DP_{jt} + \psi_{it} + \nu_{it} \tag{3.7}$$

where $\overline{w}_{it} = (1/2)(w_{i,t-1} + w_{i,t})$ is an arithmetic average of the ith good income share over two successive time periods t and $t-1$; $\pi_{ij,t}$ is the Slustky coefficient that gives the total substitution effect of the change in the price of good j on the demand for good i; ν_{it} is the error term; DQ_t and DP_t are the finite change versions of the Divisia quantity and price indexes[1]. The income effect of the n price changes on the demand for good i at time t is given by θ_{it}. The time-varying coefficients ϖ_{it} and ψ_{it}'s have the same meaning and follow the same stochastic processes as μ_{it} and the ϕ_{it}'s in equation (3.1). Each of the time-varying coefficients θ_{it} and π_{ijt}'s follows a pure random walk process. For more details on the derivation of the Rotterdam model in its constant-parameters version, see Barten (1964), Theil (1965, 1971, 1975a,b, 1980a,b), Barnett (1979), and Barnett and Serlertis (2008).

The following restrictions are imposed on the coefficients in order for

[1] The formulas for the Divisia quantity and price indexes are respectively $\text{dlog}Q = \text{dlog}m - \text{dlog}P = \sum_{j=1}^{n} w_j \text{dlog}x_j$ and $\text{dlog}P = \sum_{j=1}^{n} w_j \text{dlog}p_j$, where m is total consumer expenditure.

the Rotterdam model to satisfy Engel aggregation, linear homogeneity and symmetry respectively, at each time period:

$$\sum_{i=1}^{n} \theta_{it} = 1; \ \sum_{i} \pi_{ij,t} = 0 \tag{3.8}$$

$$\sum_{i=1}^{n} \pi_{ij,t} = 0 \tag{3.9}$$

$$\pi_{ij,t} = \pi_{ji,t} \tag{3.10}$$

The next section discusses the state space representation of the AIDS and the Rotterdam model, a framework that allows estimating the time-varying parameters, using the Kalman filter. We shall consider two specifications of the time-varying parameters in the demand system: the random walk model (RWM) where all the parameters are assumed to follow a random walk process, and the local trend model (LTM) where the intercept in each demand equation is assumed to follow a random walk with drift while all the other parameters follow a pure random walk process.

4 State space Representation of the AIDS and the RM

Consider the following state space representation of the demand system:

$$\begin{aligned} y_t &= Z_t \alpha_t + w_t \\ \alpha_{t+1} &= S_t \alpha_t + v_t \end{aligned} \tag{4.1}$$

For an n-goods demand system, the $n \times 1$ vector y_t is the vector of the dependent variables in the demand system, the m vector α_t is the state vector of the m unknown parameters for t =*1, . . . , T*. The above state space representation has two matrices. The $n \times m$ matrix Z_t contains all the exogenous variables of the system while the $m \times m$ matrix S_t is the transition matrix that links the state vector at time period *t+1* to its current value, and the entries of which are supposed to be known. Finally, the $n \times 1$ vector $\mathbf{w}_t$ and the $m \times 1$ vector $\mathbf{v}_t$ are the serially uncorrelated and independent error vectors in the measurement equation and the transition equation respectively, with zero means and respective nonnegative definite covariance matrices H_t and Q_t, that is

$$E(\mathbf{w}_t) = 0 \text{ and } Var(\mathbf{w}_t) = H_t; \quad E(\mathbf{v}_t) = \mathbf{0} \text{ and } Var(\mathbf{v}_t) = Q_t; \quad t = 1, \ldots, T, \tag{4.2}$$

where H_t and Q_t are respectively of order $n \times n$ and $m \times m$. In addition, the error vectors in the state space model are assumed to be independent of each other at all time points, that is

$$E(\mathbf{w}_t \mathbf{v}_t') = \mathbf{0}, \forall t \tag{4.3}$$

An explicit formulation of different matrices in the state space model of the demand system, as they relate to the AIDS and the Rotterdam model[2], is provided in the next subsection. The homogeneity and symmetry restriction are imposed, following Mazzocchi (2003), by modifying the measurement equation and the transition equation accordingly rather than by augmenting the measurement equation prior to estimation as suggested by Doran (1992) and Doran and Rambaldi (1997). We shall underline the fact that the restriction on the coefficients in each demand specification are assumed to hold at every time point.

4.1 The Random Walk Model

The state-space representation matrices for each demand specification incorporate the restrictions that are imposed on its parameters. However, When linear homogeneity is imposed the disturbances become linearly dependent and their covariance matrix becomes singular. In order to circumvent this problem, one equation must be deleted from the demand system prior to estimation as advocated by Barten (1969). The parameters of the deleted equation will then be recovered by using the imposed restrictions or by estimating the system with a different equation dropped.

[2]Although we shall only consider two specifications of the parameters' time varying structure, other stochastic processes can be specified for the time-varying coefficients as well, such as the autoregressive structure suggested by Chavas (1983).

4.1.1 State-space representation of the structural TVC-AIDS

In the 3-goods case, the measurement equation, with homogeneity and symmetry imposed on the coefficients and the third equation deleted is as follows, for every $t = 1, 2, \ldots, T$:

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} = \begin{bmatrix} 1 & log\left(\frac{p_{1t}}{p_{3t}}\right) & log\left(\frac{p_{2t}}{p_{3t}}\right) & log\left(\frac{m_t}{P_t}\right) & 0 & 0 & 0 \\ 0 & 0 & log\left(\frac{p_{1t}}{p_{3t}}\right) & 0 & 1 & log\left(\frac{p_{2t}}{p_{3t}}\right) & log\left(\frac{m_t}{P_t}\right) \end{bmatrix} \times \begin{bmatrix} \alpha_{1,t} \\ \gamma_{11,t} \\ \gamma_{12,t} \\ \beta_{1,t} \\ \alpha_{2,t} \\ \gamma_{22,t} \\ \beta_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

When the state vector is assumed to follow a pure random walk process, the transition equation at every time period is given by

$$\begin{bmatrix} \alpha_{1,t} \\ \gamma_{11,t} \\ \gamma_{12,t} \\ \beta_{1,t} \\ \alpha_{2,t} \\ \gamma_{22,t} \\ \beta_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{1,t-1} \\ \gamma_{11,t-1} \\ \gamma_{12,t-1} \\ \beta_{1,t-1} \\ \alpha_{2,t-1} \\ \gamma_{22,t-1} \\ \beta_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_t^{\alpha_1} \\ e_t^{\gamma_{11}} \\ e_t^{\gamma_{12}} \\ e_t^{\beta_1} \\ e_t^{\alpha_2} \\ e_t^{\gamma_{22}} \\ e_t^{\beta_2} \end{bmatrix}$$

4.1.2 State-space representation of the structural TVC-RM

When linear homogeneity is imposed the ith equation in the n-goods Rotterdam model (3.7) becomes:

$$\overline{w}_{it} Dq_{it} = \varpi_{it} + \theta_{it} DQ_t + \sum_{j=1}^{n-1} \pi_{ijt}(Dp_{jt} - Dp_{n,t}) + \psi_{it} + \nu_{it} \tag{4.4}$$

With the constant and the seasonal dummies dropped from equation (4.4), the measurement equation of the state space representation of the Rotterdam model can be expressed explicitly as follows, in the 3-goods case when symmetry is imposed and the third equation deleted:

$$\begin{bmatrix} \overline{w}_{1,t}Dq_{1,t} \\ \overline{w}_{2,t}Dq_{2,t} \end{bmatrix} = \begin{bmatrix} DQ_t & (Dp_1 - Dp_3) & (Dp_2 - Dp_3) & 0 & 0 \\ 0 & 0 & (Dp_1 - Dp_3) & DQ_t & (Dp_3 - Dp_3) \end{bmatrix} \times$$

$$\begin{bmatrix} \theta_{1,t} \\ \pi_{11,t} \\ \pi_{12,t} \\ \theta_{2,t} \\ \pi_{22,t} \end{bmatrix} + \begin{bmatrix} \nu_{1,t} \\ \nu_{2,t} \end{bmatrix}$$

The transition equation in matrix form is given $\forall t$ by

$$\begin{bmatrix} \theta_{1,t} \\ \pi_{11,t} \\ \pi_{12,t} \\ \theta_{2,t} \\ \pi_{22,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{1,t-1} \\ \pi_{11,t-1} \\ \pi_{12,t-1} \\ \theta_{2,t-1} \\ \pi_{22,t-1} \end{bmatrix} + \begin{bmatrix} e_t^{\theta_1} \\ e_t^{\pi_{11}} \\ e_t^{\pi_{12}} \\ e_t^{\theta_2} \\ e_t^{\pi_{22}} \end{bmatrix}$$

4.2 The Local Trend Model

The local trend model assumes that the intercept in each equation of both the AIDS and the Rotterdam model follows a random walk process with a drift, that is

$$\begin{aligned} \mu_{it} &= \mu_{i,t-1} + \lambda_{i,t-1} + e_{it}^{\mu} \\ \lambda_{it} &= \lambda_{i,t-1} + e_{it}^{\lambda} \end{aligned} \tag{4.5}$$

for the ith equation in the AIDS, and

$$\begin{aligned} \varpi_{it} &= \varpi_{i,t-1} + \omega_{i,t-1} + e_{it}^{\varpi} \\ \omega_{it} &= \omega_{i,t-1} + e_{it}^{\omega} \end{aligned} \tag{4.6}$$

for the ith equation in the Rotterdam model. All the other parameters of the demand systems follow the random walk process as in the random walk model. The measurement and transition equations are modified accordingly.

5 Data generation procedure

This section explains the steps used to generate the data for the Monte Carlo simulations. In this process, all the parameters in the utility functions in equations (2.2) and (2.3), except ρ and δ, are assumed to be time varying. The constancy of δ and ρ is assumed for convenience, since these parameters can be considered as time-varying as well. The data generation procedure proceeds as follows:

Step 1: Set the value of the elasticity of substitution between the supernumerary quantities y_1 and y_2 in the microutility function in equation (2.3) for each time period, $t = 1, 2, \ldots, T$, where $T = 60$.

Step 2: Generate the stochastic process for the time-varying parameters in the microutility function q_1. The parameters $B_{11,t}$,$B_{12,t}$,$B_{21,t}$ and $B_{22,t}$ are assumed to follow a random walk process and are constrained so that they satisfy the condition $\sum_k \sum_l B_{kl,t} = 1$, with $B_{12,t} = B_{21,t}$, $\forall t$.

Step 3: Obtain the ratio between y_{1t} and y_{2t} from the formula of the elasticity of substitution between the two supernumerary quantities, using the values set in *Step 1*.

Step 4: Generate the first order autoregressive time series for the two supernumerary quantities y_{1t} and y_{2t} and the supernumerary income m_{1t}[3]; then adjust the time series of the two supernumerary quantities so that the ratio y_{2t}/y_{1t} corresponds to the one obtained in *Step 3*.

Step 5: Use the first order conditions for maximizing q_1[4] and the supernumerary budget constraint to solve for the price system (p_{1t}, p_{2t}) at every time period.

Step 6: Calculate the aggregate quantity q_{1t} and the corresponding price index using the Fisher factor reversal test.

[3]The autoregressive models for the supernumerary quantities and income are the following: $y_{1t} = 2 + 0.75y_{1,t-1} + e_{1t}$; $y_{2t} = 1 + 0.739y_{2,t-1} + e_{2t}$; $m_{1t} = 125 + 0.98m_{1,t-1} + e_{3t}$ where e_{1t}, e_{2t} and e_{3t} are zero mean and serially uncorrelated normal error terms with variance 1.

[4]See Barnett and Choi (1989)

Step 7: Set the value of the elasticity of substitution between the two aggregate quantities q_{1t} and q_{2t} in the macroutility function (2.2) and solve for the ratio q_{2t}/q_{1t} from equation (2.7) for each time period $t = 1,2,\ldots,T$.

Step 8: Generate the time path of the time-varying parameters in the macroutility function, such that $\sum_i \sum_j A_{ijt} = 1$ and $A_{12t} = A_{21t}$. The parameter vector in the macroutility function is assumed to follow a random walk process. The only constant parameter in the macroutility function is ρ.

Step 9: Generate the supernumerary quantity $y_{3t} = q_{2t}$ according a first order autoregressive process[5] and adjust the resulting time series so that the ratio q_{2t}/q_{1t} corresponds to the ratio obtained in *Step 7*.

Step 10: Solve for p_{3t} from the first order conditions for the maximization of the macroutility function[6].

Step 11: Set the value of α_1,α_2 and α_3 and obtain the elementary quantities x_1, x_2 and x_3 from their relationships with the supernumerary quantities, that is $x_i = y_i + \alpha_i$[7], i=*1,2,3* and calculate total expenditure on the elementary quantities.

Step 12: Add noises to the elementary quantities x_{1t}, x_{2t} and x_{3t} that constitute the reference dataset and estimate the time varying parameters of the resulting demand system, bootstrapping the model 2000 times while recalculating the total expenditure on x_{1t}, x_{2t} and x_{3t}.

For the bootstrap procedure we have generated three vectors of 2000 seeds each, to use in generating the normally distributed random numbers that are added as shocks to the reference dataset. Relevant elasticities are calculated and stored at each replication from the estimated time-varying parameters. Finally, the income and compensated price elasticities as well as the elasticities of substitution at each time period are calculated as the

[5] $y_{3t} = 3 + 0.69 y_{3,t-1} + e_{4t}$

[6] See Barnett and Choi (1989) for the specification of this utility maximization problem.

[7] The values used to generate the data are: $\alpha_1 = 1$, $\alpha_2 = 10$ and $\alpha_3 = 4$. This specification is used for the random walk model. For the local trend model, each of the α_i's is specified as a random walk plus a shift, where the shift itself follows a random walk process.

averages of the values stored during the bootstrap procedure. The true time-varying cross-price elasticities are obtained from the WS-branch utility model by using the relationship between the Allen-Uzawa elasticity of substitution, the income shares, and the Hicksian demand elasticities.

6 Estimation method

The time-varying parameters in the AIDS and RM are estimated by Kalman filtering. The exact Kalman filter (Koopman, 1997) is used for initial states and variances and implemented in the RATS software (Doan, 2010b,a, 2011; Estima, 2007a,b). Under the normality assumption for the disturbance vectors $\mathbf{w}_t$ and $\mathbf{v}_t$ in equations (4.1), the distribution generated by the Kalman filter is given by

$$y_t|y_1, y_2, \ldots, y_{t-1} \sim N(Z_t^{'}\alpha_t, \Lambda_t) \tag{6.1}$$

where $\Lambda_t = Z_t^{'}P_{t|t-1}Z_t + Q_t$. The essential part of the likelihood function for the full sample, which is the objective function of the Kalman filter(smoother) is therefore

$$-\frac{1}{2}\sum_t log|\Lambda_t| - \frac{1}{2}\sum_t (y_t - Z_t^{'}\alpha_{t|t-1})^{'}\Lambda_t^{-1}(y_t - Z_t^{'}\alpha_t). \tag{6.2}$$

The AIDS models have been estimated in first-differenced form by assuming time-varying coefficients rather than constant coefficients like, for example, in Deaton and Muellbauer (1980a), Eales and Unnevehr (1988), Moschini and Meilke (1989), Brester and Wohlgenant (1991) and Alston and Chalfant (1993). An intercept is included in each demand equation. Leybourne (1993) and Mazzocchi (2003) have estimated time-varying parameters in the AIDS model. However, we have found no journal article that has attempted to estimate time-varying parameters in the Rotterdam model.

6.1 Calculation of the time-varying elasticities

The Kalman filtered and Kalman smoothed time-varying parameters in the AIDS and RM are used to calculate the demand elasticities using the formulas in Table 1. The elasticity formulas in the linear-approximate AIDS are the corrected elasticity formulas from Green and Alston (1990, 1991).

Table 1: Time-varying demand elasticities in the AIDS the RM

Model	η_{it}	η_{ijt}	η^*_{ijt}
Rotterdam	$\frac{\theta_{it}}{wit}$	$\frac{\pi_{ijt}-\theta_{it}w_{jt}}{w_{it}}$	$\frac{\pi_{ijt}}{w_{it}}$
AIDS	$1+\frac{\beta_{it}}{w_{it}}$	$-\delta_{ijt}+\frac{\gamma_{ijt}}{w_{it}}-\frac{\beta_{it}\alpha_{jt}}{w_{it}}$ $-\frac{\beta_{it}}{w_{it}}\sum_k \gamma_{kjt} lnp_{kt}$	$\eta_{ijt}+w_{jt}\left(1+\frac{\beta_{it}}{w_{it}}\right)$
LA-AIDS	$1+$ $\frac{\beta_{it}}{w_{it}}\left[1-\sum_{jt} w_{jt} lnp_{jt}(\eta_{jt}-1)\right]$	$-\delta_{ijt}+\frac{\gamma_{ijt}}{w_{it}}-\beta_i \frac{w_{jt}}{w_{it}}$ $-\frac{\beta_{it}}{w_{it}}\left[\sum_k w_{kt} lnp_{kt}(\eta_{kjt}+\delta_{kjt})\right]$	$\eta_{ijt}+w_{jt}\eta_{it}$

However, Alston et al. (1994) have shown, in a Monte Carlo study, that if the nonlinear AIDS is viewed as the underlying demand system and that the linear-approximate AIDS is indeed an approximation of it, the simple formulas of elasticities can be used. we shall consider both versions of the formulas in calculating the income and price elasticities in the linear approximate AIDS.

On the other hand, we shall use the Morishima formulas (Morishima, 1967; Blackorby and Russell, 1975) in calculating the elasticities of substitution. In contrast to the Allen-Uzawa elasticity of substitution (AUES), this measure of the elasticity of substitution is both quantitatively meaningful and qualitatively informative. Moreover, it is a measure of curvature or ease of substitution and a logarithmic derivative of a quantity ratio with respect to marginal rate of substitution (Blackorby and Russell, 1981, 1989; Blackorby et al., 2007).

The Morishma elasticity of substitution (MES) between goods i and j is calculated as follows:

$$\sigma^M_{ij} = \frac{p_i C_{ij}(\mathbf{p}, u)}{C_j(\mathbf{p}, u}) - \frac{p_i C_{ii}(\mathbf{p}, u)}{C_i(\mathbf{p}, u)} = \epsilon_{ij}(\mathbf{p}, u) - \epsilon_{ii}(\mathbf{p}, u), \tag{6.3}$$

where $C(\mathbf{p}, u)$ is the cost function and the subscripts on $C(\mathbf{p}, u)$ are the partial derivatives with respect to the relevant prices; $\epsilon_{ij}(\mathbf{p}, u)$ is the Hicksian compensated elasticity of good i with respect to the price of good j. The cost function in equation (6.3) depends on the price vector $\mathbf{p}$ and the utility level u; and it is assumed to satisfy all the regularity conditions [8].

[8] A regular cost function is continuous, nondecreasing, linearly homogeneous and concave in $\mathbf{p}$, increasing in u and twice continuously differentiable.

It is important to mention that both the MES and the AUES are used to classify inputs/goods as substitutes or complements. However, they yield different stratification sets in general (Barnett and Serlertis, 2008). In fact, two Allen substitutes goods must be Morishima substitutes while two Allen complements may be Morishima substitutes. The goods that we have constructed in our experiments are all substitutes to each other so that the AUES and the MES will produce an identical stratification.

7 Results

In introducing the results of this paper, we shall underline the fact that the linear-approximate AIDS with corrected elasticity formulas (LA-AIDS/CF) and the Rotterdam model are the most used demand specifications in empirical demand analysis, among all the local flexible functional forms. Therefore, the importance of the findings in this paper help to share the light on the performance of these two demand specifications when the parameters of the demand functions are assumed to be time-varying. We also include, for comparison purpose, the nonlinear AIDS model (NL-AIDS) and the linear-approximate AIDS model where simple elasticity formulas are used (LA-AIDS/SF).

Tables 2, 3 and 4 provide the true and approximating elasticities of substitution, income elasticities and cross-price elasticities. As mentioned earlier, only the elasticities that have counterparts in the true model are presented. All elasticities in the true model are positive at every single time period. This means that all the goods are substitutes based on the elasticities of substitution. In addition, they are normal goods based on the income elasticities. The result are presented for both specifications of the time-varying parameters (the random walk model and the local trend model).

7.1 Performance of the RM and the LA-AIDS/CF

Both the RM and the LA-AIDS/CF approximated the true time-varying elasticities of substitution with positive values at every time point under the RWM. In addition, the approximating values are close to the true ones within the same utility branch for both demand specifications. On the other hand, while the RM approximated all the three time-varying elasticities of substitution with the correct positive signs at every time period under the

LTM, the LA-AIDS/CF approximated 2 of them with the wrong negative sign (Table 2). The LA-AIDS/CF thus identified goods as complements while they are actually substitutes at every single time period. By comparing the values of the time-varying coefficient elasticities of substitution in Table 2, one realizes that the LA-AIDS/CF produces a poor approximation of the NL-AIDS at every time point.

On the other hand, it appears from Table 3 that the RM correctly classified x_1,x_2 and x_3 as normal goods under both the RWM and the LTM at every time period. In addition, this specification produced a correct classification of the three goods in terms of them being normal necessities and/or luxuries. A notable fact from Table 3 is that the RM produced approximating time-varying income elasticities the values of which are close to the true ones. In contrast, the LA-AIDS/CF performed poorly in recovering the true time-varying income elasticities. Whenever the values of its approximations were positive, they underestimated the true ones. Otherwise, the approximating values of the time-varying income elasticities from this model were negative while the true ones are positive. Finally, the time-varying income elasticities produced by the LA-AIDS/CF are poor approximations of the NL-AIDS.

The RM correctly recovered the signs of the compensated cross-price elasticities (Table 4). The approximating values of the time-varying compensated cross-price elasticities are close to the true ones under both the RWM and the LTM within the same utility branch. The results in Table 4 also show that the LA-AIDS/CF produced approximations of the true time-varying elasticities with negative values, except for $\eta^*_{13,t}$ under the LTM. Even worse, the LA-AIDS/CF produced an approximation of $\eta^*_{23,t}$ with both negative and positive values.

7.2 Performance of the NL-AIDS and the LA-AIDS/SF

The NL-AIDS approximated the true time-varying elasticities of substitution with positive values under the RWM and the LTM. However, the approximating values were not close to the true ones. On the other hand, the model produced approximations of the time-varying income elasticities the values of which tended to be constant over time. Under the LTM, the approximating values of the time-varying income elasticities produced by this specification are very close to one, regardless of the magnitude of the true values. Finally, this model produced compensated cross-price elasticities with

Table 2: Time-varying elasticities of substitution

		Random Walk Model									
	$t =$	1	2	3	4	6	12	24	36	48	60
$\sigma_{12,t}$	True	0.40	0.39	0.39	0.38	0.37	0.27	0.33	0.43	0.34	0.41
	RM	0.30	0.34	0.34	0.32	0.34	0.32	0.37	0.31	0.35	0.33
	NLAI	1.34	1.64	1.64	1.57	1.62	1.62	1.57	1.57	1.65	1.63
	LAISF	-0.37	-0.42	-0.36	-0.23	-0.36	-0.32	-0.48	-0.25	-0.40	-0.36
	LAICF	0.34	0.31	0.34	0.39	0.34	0.36	0.21	0.40	0.35	0.37
$\sigma_{13,t}$	True	0.15	0.60	0.84	1.12	2.00	0.80	0.94	1.39	2.90	2.79
	RM	0.60	0.77	0.75	0.67	0.75	0.68	0.38	0.68	0.75	0.91
	NLAI	0.16	0.20	0.22	0.35	0.20	0.27	0.64	0.25	0.20	0.01
	LAISF	-0.30	-0.26	-0.22	-0.03	-0.25	-0.14	-0.40	-0.18	-0.24	-0.53
	LAICF	0.47	0.48	0.49	0.56	0.48	0.53	0.72	0.52	0.52	0.40
$\sigma_{23,t}$	True	0.91	0.93	0.88	0.64	0.92	0.89	0.25	0.97	1.43	1.17
	RM	0.25	0.29	0.88	0.27	0.28	0.27	0.32	0.25	0.30	0.28
	NLAI	1.93	1.94	1.92	1.83	1.91	1.89	1.95	1.84	1.94	1.95
	LAISF	0.26	0.24	0.28	0.35	0.27	0.30	0.23	0.33	0.26	0.26
	LAICF	0.27	0.24	0.28	0.35	0.28	0.30	0.22	0.34	0.25	0.27

		Local Trend Model									
$\sigma_{12,t}$	True	0.17	0.18	0.20	0.20	0.16	0.14	0.13	0.06	0.09	0.07
	RM	0.13	0.11	0.12	0.14	0.15	0.14	0.11	0.14	0.15	0.18
	NLAI	0.44	0.42	1.10	1.10	1.11	1.07	1.05	1.06	1.07	1.08
	LAISF	-0.91	-1.00	-0.98	-1.04	-1.12	-1.01	-1.09	-1.67	-1.93	-2.51
	LAICF	-0.13	-0.18	-0.17	-0.21	-0.24	-0.17	-0.20	-0.53	-0.68	-1.00
$\sigma_{13,t}$	True	3.05	3.07	2.96	3.01	3.06	2.95	2.65	2.42	2.95	2.45
	RM	0.30	0.28	0.41	0.44	0.41	0.43	1.08	1.31	1.26	1.89
	NLAI	1.06	1.08	1.08	1.09	1.22	1.25	1.59	1.79	1.74	2.30
	LAISF	1.25	1.33	1.46	1.52	1.39	1.48	2.24	2.72	2.59	3.86
	LAICF	1.29	1.38	1.41	1.45	1.36	1.43	2.08	2.49	2.37	3.46
$\sigma_{23,t}$	True	1.71	1.77	1.81	1.86	1.89	1.49	1.37	2.16	2.27	2.85
	RM	0.23	0.19	0.17	0.17	0.22	0.21	0.23	0.27	0.24	0.31
	NLAI	0.53	1.08	1.08	1.08	1.05	1.05	1.02	1.05	1.04	1.06
	LAISF	-0.41	-0.33	-0.35	-0.43	-0.44	-0.57	-0.85	-0.04	-1.43	-1.74
	LAICF	-0.39	-0.38	-0.40	-0.47	-0.48	-0.41	-0.90	-1.09	-1.48	-1.74

Table 3: Time-varying income elasticities

		Random Walk Model									
	$t =$	1	2	3	4	6	12	24	36	48	60
η_{1t}	True	1.039	1.041	1.043	1.062	1.042	1.043	1.136	1.043	1.024	1.028
	RM	1.054	1.028	1.030	1.040	1.031	1.040	1.097	1.042	1.029	1.021
	NLAI	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	LAISF	1.072	1.072	1.072	1.073	1.072	1.072	1.077	1.072	1.072	1.071
	LAICF	0.074	0.075	0.075	0.078	0.076	0.076	0.079	0.075	0.069	0.070
η_{2t}	True	0.449	0.485	0.740	0.435	0.487	0.434	0.489	0.446	0.463	0.456
	RM	0.380	0.441	0.440	0.409	0.487	0.400	0.480	0.384	0.445	0.4187
	NLAI	0.996	0.996	0.996	0.997	0.996	0.996	0.996	0.997	0.996	0.996
	LAISF	0.279	0.255	0.283	0.356	0.286	0.304	0.194	0.347	0.262	0.287
	LAICF	-0.745	-0.777	-0.749	-0.681	-0.748	-0.725	-0.909	-0.675	-0.715	-0.705
η_{3t}	True	0.695	0.645	0.633	0.504	0.635	0.698	0.184	0.716	0.965	0.878
	RM	0.697	0.923	0.890	0.798	0.887	0.806	0.417	0.810	0.891	1.096
	NLAI	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.997	0.997	0.996
	LAISF	0.213	0.241	0.264	0.390	0.248	0.314	0.666	0.289	0.247	0.062
	LAICF	-0.818	-0.799	-0.776	-0.652	-0.795	-0.719	-0.386	-0.739	-0.725	-0.924
		Local Trend Model									
η_{1t}	True	1.000	1.000	0.997	0.995	0.996	0.984	0.996	0.970	0.958	0.967
	RM	0.998	0.994	0.995	0.992	0.992	0.986	0.980	0.974	0.970	0.966
	NLAI	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	LAISF	1.027	1.027	1.028	1.028	1.028	1.028	1.028	0.027	1.027	1.027
	LAICF	0.027	0.027	0.029	0.029	0.029	0.028	0.028	0.028	0.028	0.027
η_{2t}	True	0.544	0.487	0.522	0.566	0.589	0.602	0.568	0.781	0.838	0.968
	RM	0.571	0.580	0.604	0.643	0.669	0.769	0.717	0.882	1.051	1.167
	NLAI	0.991	0.992	0.993	0.984	0.984	0.982	0.983	0.980	0.976	0.973
	LAISF	0.211	0.326	0.203	0.161	0.136	0.039	0.070	-0.134	-0.323	-0.501
	LAICF	-0.794	-0.677	-0.803	-0.846	-0.872	-0.971	-0.940	-0.144	-1.332	-1.508
η_{3t}	True	1.809	2.290	2.491	2.652	2.187	3.175	4.370	10.39	12.15	15.78
	RM	1.932	2.270	2.391	2.597	2.367	2.571	6.523	7.918	7.630	11.50
	NLAI	1.009	1.011	1.011	1.018	1.024	1.031	1.068	1.093	1.085	1.151
	LAISF	0.963	0.953	1.062	1.073	1.063	1.081	1.200	1.272	1.252	1.447
	LAICF	-0.032	-0.043	0.064	0.075	0.065	0.083	0.205	0.272	0.258	0.457

Table 4: Time-varying cross-price elasticities

		Random Walk Model									
	$t =$	1	2	3	4	6	12	24	36	48	60
$\eta^*_{12,t}$	True	0.015	0.015	0.015	0.017	0.014	0.011	0.011	0.018	0.013	0.016
	RM	0.013	0.012	0.012	0.012	0.012	0.012	0.013	0.012	0.012	0.012
	NLAI	0.066	0.065	0.067	0.072	0.067	0.068	0.065	0.071	0.065	0.067
	LAISF	0.016	0.015	0.016	0.021	0.017	0.018	0.010	0.020	0.015	0.017
	LAICF	-0.025	-0.025	-0.025	-0.026	-0.025	-0.026	-0.027	-0.026	-0.026	-0.025
$\eta^*_{13,t}$	True	0.101	0.100	0.100	0.096	0.099	0.118	0.068	0.120	0.158	0.106
	RM	0.039	0.038	0.039	0.039	0.039	0.039	0.041	0.039	0.038	0.038
	NLAI	0.005	0.007	0.009	0.019	0.008	0.012	0.072	0.010	0.007	-0.002
	LAISF	0.027	0.029	0.031	0.042	0.030	0.034	0.101	0.032	0.029	0.0.19
	LAICF	-0.026	-0.026	-0.026	-0.027	-0.026	-0.026	-0.032	-0.026	-0.023	-0.023
$\eta^*_{23,t}$	True	0.043	0.047	0.045	0.039	0.046	0.049	0.029	0.051	0.071	0.047
	RM	2.0e-5	1.1e-5	1.6e-5	1.5e-5	1.9e-5	1.1e-5	2.2e-5	1.4e-5	1.1e-5	2.2e-5
	NLAI	0.201	0.207	0.204	0.198	0.203	0.202	0.289	0.191	0.206	0.191
	LAISF	-0.062	-0.066	-0.061	-0.046	-0.061	-00057	-0.063	-0.051	-0.065	-0.064
	LAICF	-0.062	-0.060	-0.057	-0.046	-0.056	-0.058	-0.023	-0.057	-0.095	-0.082
		Local Trend Model									
$\eta^*_{12,t}$	True	0.008	0.006	0.006	0.006	0.007	0.003	0.002	0.001	0.001	0.001
	RM	0.044	0.048	0.006	0.005	0.004	0.004	0.003	0.003	0.003	0.003
	NLAI	0.013	0.0190	0.016	0.015	0.0360	0.032	0.032	0.026	0.023	0.020
	LAISF	0.024	0.030	-0.004	-0.005	-0.006	-0.007	-0.006	-0.010	-0.014	-0.015
	LAICF	-0.008	-0.008	-0.040	-0.040	-0.040	-0.007	-0.037	-0.036	-0.036	-0.035
$\eta^*_{13,t}$	True	0.056	0.043	0.039	0.036	0.045	0.036	0.013	0.009	0.012	0.005
	RM	0.005	0.004	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.005
	NLAI	0.020	0.015	0.014	0.013	0.018	0.015	0.008	0.006	0.007	0.005
	LAISF	0.025	0.020	0.020	0.161	0.136	0.021	0.018	0.011	0.010	0.008
	LAICF	0.005	0.005	0.006	0.018	0.006	0.006	0.006	0.006	0.005	0.005
$\eta^*_{23,t}$	True	0.031	0.021	0.02	0.020	0.027	0.022	0.008	0.007	0.010	0.005
	RM	0.037	0.028	0.017	0.018	0.018	0.020	0.017	0.020	0.023	0.025
	NLAI	0.058	0.043	0.050	0.048	0.005	0.002	-0.001	-0.001	-0.002	-0.005
	LAISF	-0.020	-0.014	-0.076	-0.084	-0.069	-0.078	-0.075	-0.092	-0.108	-0.122
	LAICF	0.009	0.005	-0.053	-0.059	-0.061	-0.068	-0.068	-0.082	-0.091	-0.105

the correct sign, except for $\eta^*_{13,t}$ under the RWM and $\eta^*_{23,t}$ under the LTM for which both negative and positive values were produced.

The LA-AIDS/SF tended to produce negative values for the time-varying elasticities of substitution, except for $\sigma_{23,t}$ under the RWM, and $\sigma_{13,t}$ under the LTM. Furthermore, this model tended to produce constant values of η_{1t} and fails to capture very high variations in the values of true time-varying income elasticities. Finally, this specification produced time-varying compensated cross-price elasticities with the wrong sign in most of the cases.

7.3 Robustness of the findings

Table 5 shows the time-varying elasticities obtained by using different values of the time-varying parameters in the WS-branch utility function. This new Monte Carlo experiment shows that the previous findings are robust to different values of the time-varying parameters in the true model. For example, the RM model produced approximating time-varying income elasticities the values of which are very close to the true ones. In addition, the model was able to capture the very high values of the time-varying income elasticities. The LA-AIDS/CF produced time-varying income and cross-price elasticities with negative values as in the initial experiment. The NL-AIDS, on the other hand, tended to produce constant values for the time-varying income elasticities.

7.4 Theoretical Regularity

The regularity condition is defined as the non-violation of the negative semi-definiteness of the Slutsky matrix. Rather than being imposed during the estimation procedure, this condition is usually just checked after estimation. In the case of a three-goods demand system, the regularity condition is defined below for both the AIDS and the Rotterdam model. In the AIDS, the Slutsky matrix is negative semi-definite at each time period t if

$$\eta^*_{11t} < 0 \text{ and } \begin{vmatrix} \eta^*_{11t} & \eta^*_{12t} \\ \eta^*_{21t} & \eta^*_{22t} \end{vmatrix} = \eta^*_{11t}\eta^*_{22t} - \eta^*_{21t}\eta^*_{12t} > 0. \tag{7.1}$$

However, for the Rotterdam model one must have

$$\pi_{11t} < 0 \text{ and } \begin{vmatrix} \pi_{11t} & \pi_{12t} \\ \pi_{21t} & \pi_{22t} \end{vmatrix} = \pi_{11t}\pi_{22t} - \pi_{21t}\pi_{12t} > 0. \tag{7.2}$$

Table 5: Time-varying elasticities: Robustness checks

		Random Walk Model					*Local Trend Model*				
	$t =$	1	12	24	36	60	1	12	24	36	60
$\sigma_{12,t}$	True	0.246	0.248	0.205	0.230	0.263	0.116	0.054	0.112	0.044	0.055
	RM	0.534	0.517	0.510	0.485	0.536	0.629	0.648	0.681	0.742	1.041
	NLAI	0.943	0.946	0.948	0.946	0.946	1.109	0.830	0.758	0.755	0.650
	LAISF	0.596	0.615	0.638	0.662	0.604	-0.283	-1.311	-0.643	-0.898	-1.721
	LAICF	1.036	1.035	1.032	1.030	1.035	0.481	0.421	0.254	0.233	-0.098
$\sigma_{13,t}$	True	3.006	3.035	2.770	3.025	2.878	2.992	2.872	2.875	2.900	2.911
	RM	1.300	1.153	1.316	1.204	1.542	0.389	0.603	0.996	1.472	2.016
	NLAI	0.430	0.478	0.418	0.465	0.313	1.210	1.141	1.138	1.220	1.361
	LAISF	0.303	0.366	0.291	0.352	0.155	1.515	1.818	1.977	2.594	3.627
	LAICF	0.905	0.913	0.902	0.910	0.885	1.342	1.552	1.653	2.053	2.720
$\sigma_{23,t}$	True	0.883	0.853	0.619	0.736	0.870	1.113	1.110	1.442	1.472	2.521
	RM	0.476	0.455	0.451	0.422	0.484	0.674	0.699	0.730	0.817	1.149
	NLAI	2.175	2.105	2.103	2.018	2.249	1.133	0.975	0.925	0.971	0.981
	LAISF	1.941	1.887	1.873	1.808	1.987	0.380	0.276	0.080	-0.046	-0.565
	LAICF	1.925	1.870	1.864	1.799	1.974	0.336	0.225	0.022	-0.277	-0.634
η_{1t}	True	1.031	1.033	1.041	1.042	1.032	0.981	0.963	0.967	0.952	0.955
	RM	1.057	1.070	1.059	1.070	1.044	0.989	0.977	0.965	0.960	0.952
	NLAI	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	LAISF	1.060	1.061	1.061	1.061	1.060	1.021	1.020	1.020	1.020	1.020
	LAICF	0.058	0.059	0.059	0.059	0.057	0.021	0.021	0.020	0.020	0.020
η_{2t}	True	0.303	0.290	0.232	0.253	0.312	0.365	0.372	0.485	0.483	0.827
	RM	0.296	0.284	0.280	0.262	0.299	0.478	0.548	0.689	0.736	1.040
	NLAI	0.998	0.998	0.998	0.998	0.998	0.985	0.987	0.982	0.981	0.973
	LAISF	0.571	0.590	0.621	0.646	0.585	0.186	0.138	-0.162	-0.185	-0.694
	LAICF	-0.412	-0.396	-0.370	-0.342	-0.396	-0.834	-0.883	-1.194	-1.213	-1.728
η_{3t}	True	0.965	0.973	0.930	0.946	0.943	2.506	4.489	4.375	8.314	10.64
	RM	0.621	0.544	0.629	0.571	0.747	2.021	3.043	5.058	7.474	10.30
	NLAI	0.998	0.998	0.998	0.998	0.998	1.015	1.046	1.061	1.100	1.163
	LAISF	0.393	0.449	0.381	0.434	0.260	1.210	1.308	1.368	1.596	1.976
	LAICF	-0.581	-0.529	-0.604	-0.547	-0.704	0.220	0.317	0.381	0.613	1.001
$\eta^*_{12,t}$	True	0.009	0.010	0.009	0.010	0.010	0.004	0.002	0.002	0.001	0.001
	RM	0.019	0.019	0.019	0.019	0.018	0.020	0.018	0.015	0.015	0.015
	NLAI	0.036	0.038	0.041	0.044	0.038	0.034	0.023	0.016	0.015	0.009
	LAISF	0.041	0.043	0.046	0.049	0.042	0.015	0.013	0.006	0.006	0.000
	LAICF	0.019	0.019	0.019	0.019	0.018	-0.016	-0.0160	-0.015	-0.015	-0.015
$\eta^*_{13,t}$	True	0.191	0.212	0.172	0.206	0.150	0.074	0.043	0.036	0.023	0.014
	RM	0.084	0.085	0.084	0.085	0.083	0.010	0.010	0.010	0.010	0.010
	NLAI	0.026	0.032	0.025	0.030	0.015	0.030	0.017	0.014	0.009	0.006
	LAISF	0.061	0.067	0.059	0.065	0.049	0.034	0.024	0.021	0.016	0.013
	LAICF	-0.008	-0.008	-0.008	-0.008	-0.008	0.009	0.008	0.008	0.008	0.008
$\eta^*_{23,t}$	True	0.056	0.060	0.038	0.050	0.045	0.027	0.017	0.018	0.012	0.012
	RM	0.028	0.027	0.027	0.025	0.028	0.034	0.030	0.024	0.025	0.034
	NLAI	0.818	0.792	0.730	0.691	0.782	0.034	0.060	0.067	0.061	0.080
	LAISF	0.602	0.582	0.540	0.511	0.577	-0.069	-0.074	-0.101	-0.104	-0.150
	LAICF	0.573	0.548	0.507	0.473	0.553	-0.058	-0.059	-0.072	-0.077	-0.105

Table 6: Regularity index by model and TVC specification

	NLAI		LAISF		LAICF		RM	
Period	RWM	LTM	RWM	LTM	RWM	LTM	RWM	LTM
1	84.3	72.5	47.8	53.3	66.9	51.7	100.0	98.0
2	86.1	71.9	50.0	62.4	64.8	60.3	100.0	98.1
3	87.6	71.9	49.1	40.8	66.6	34.8	100.0	96.1
4	95.3	71.3	56.1	39.5	71.7	32.8	100.0	95.9
6	85.0	95.9	48.7	35.5	66.7	28.6	100.0	95.3
12	91.7	94.8	52.3	33.3	68.0	24.1	100.0	95.5
18	91.8	95.8	60.7	47.8	64.6	30.7	100.0	94.3
24	94.2	95.6	61.4	45.9	61.1	28.2	100.0	93.2
30	92.6	93.3	55.5	37.6	67.6	21.4	100.0	90.9
36	90.9	91.5	48.6	35.8	71.2	18.3	100.0	91.2
42	86.6	90.4	48.1	25.1	68.3	14.1	100.0	91.9
48	88.5	88.2	53.2	27.0	65.3	12.3	100.0	91.8
54	83.0	87.2	46.3	26.6	68.1	11.0	100.0	92.0
60	64.7	85.8	42.8	28.3	67.2	9.8	100.0	92.1

Table 6 reports, at selected time periods, the percentage of replications producing non-violation of the negative semi-definiteness as an index of regularity for the four models.

The Rotterdam model satisfied the regularity condition under the random walk specification in every single replication and at every single time period. The regularity index is thus equal to 100. Under the local trend model specification, the regularity index ranged from 91 to 98 by time period, meaning that a minimum of 91% of the replications per time period satisfied the negative semi-definiteness condition of the Slutsky matrix. On the other hand, the LA-AIDS/CF model achieved a minimum regularity index as low as 9.8 under the local trend model, compared to 60.6 under the random walk model. The maximum proportion of replications per time period that satisfied the regularity condition was also higher under the random walk model (76.0%) than under the local trend model specification (60.3%). In general, the NL-AIDS achieved higher regularity scores compared to the LA-AIDS/CF at each time point.

8 Conclusion

The aim of this paper was to evaluate the ability of the AIDS and the RM to recover true time-varying elasticities derived from the WS-branch utility function. A structural time series model was specified for each demand specification and the time-varying parameters estimated using the Kalman filter. Next, time varying elasticities were computed from the estimated time-varying parameters obtained during the bootstrap procedure. We found that the RM performed better than the LA-AIDS/CF in that it correctly recovered the positive signs of the time-varying elasticities.

The findings in this paper lead to two important implications for the demand analysis with time-varying coefficients. First, with regard to the performance of the LA-AIDS/CF, this model should not be considered as an approximation of the NL-AIDS. It should, in contrast, be considered as a model on its own. This is important since its outcomes may substantially differ from those of the NL-AIDS with regard to the signs and the magnitude of the estimated time-varying parameters and elasticities.

The second implication relates to the choice between an AIDS-type model and the Rotterdam model in empirical demand analysis. An important recommendation is that such a choice be made with respect to the performance of each model to better approximate the properties of an hypothesized true model. However, the results in this paper may be dependent on the structure of the true model and the particular Monte Carlo experiment that was implemented. Therefore, caution should be used in selecting the correct structure to approximate the properties that are contained in a given dataset.

It is noteworthy that the comparison of the performance of models included in this paper mainly focused on how they can approximate the quantitative properties of the true model. However, a broader range of aspects can be considered as well. For example, future research efforts to assess the performance of an AIDS-type model and the Rotterdam model may focus on their forecasting abilities. In the specific case of time-varying parameters, the two models can also be assessed in terms of their performance in producing time series of elasticities that recover the time series properties of the true time-varying elasticities.

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