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Q INVESTMENT MODELS, FACTOR COMPLEMENTARITY AND MONOPOLISTIC COMPETITION

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Abstract

The observed fact that firms invest even if capacities are not fully employed does not fit well into most standard formalizations of optimal firm behavior. In this paper, the q investment approach is adapted to an imperfectly competitive economy where the representative firm is assumed to face demand uncertainty. Nominal rigidities and short-run factor complementarity are imposed as sufficient conditions to allow for the coexistence of investment and excess capacity. Since capacities are underemployed, marginal q is shown to diverge from average q. Finally, excess capacity subsists at steady state which makes it more than a short-run phenomenon.

Key Words

Tobin's q; Investment; Monopolistic Competition; Quantity Rationing Model

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1 INTRODUCTION

The observed fact that firms invest even if capacities are not fully employed does not fit into most standard formalizations of optimal firm behavior. The major part of the literature in investment theory is concerned with perfectly competitive markets where prices are market-clearing, thus ruling out any excess capacity. This is the case both in Jorgenson's (1963) model and q investment models (see Hayashi (1982) and Abel and Blanchard (1983)). More recent papers analyze investment in imperfectly competitive markets with factor substitutability and price flexibility, allowing the firm to work at full capacity.[1] Investment and excess capacity can coexist if any of these assumptions is removed.

After the papers of Akerlof and Yellen (1985), Mankiw (1985) and Blanchard and Kiyotaki (1987), the introduction of nominal rigidities into monopolistically competitive economies has opened new horizons in macroeconomics, allowing for new microfoundations for Keynesian economics. Nevertheless, the existence of involuntary unemployment needs some type of real rigidity in addition to nominal rigidities. This is a well-known problem for "New Keynesian" economists. In the same way, involuntary excess capacity also requires more than nominal rigidities. This point was stressed by Malinvaud (1987) (1989), who analyzes the problem of firms operating with complementary factors and having to decide about capacities under demand uncertainty;[2] if there is short-run factor complementarity, capacity is determined by investment decisions, and these decisions are taken before demand is known. Thus, if realized demand falls below expectations, excess capacity is possible as an ex post undesired outcome.

In Licandro (1990), we stressed the role of price rigidities and factor

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[1] Schiantarelli and Georgoutsos (1990) solve an intertemporal problem in the tradition of q investment models where the firm is assumed to belong to a monopolistically competitive economy. They show that without nominal rigidities price behavior in monopolistic competition introduces a gap between marginal and average q since "...the firm...(has)...to lower the price of additional output produced by a new machine in order to sell it."

[2] More precisely, in Malinvaud (1987) excess capacities are formalized without price rigidity, in which case they are a desired outcome.

complementarity under demand uncertainty for explaining investment decisions. The main result was that in our model marginal q is not equal to average q. This is so *even if* Hayashi's (1982) conditions for equality, which he formulated for a purely competitive model, hold. The difference between marginal q and average q is explained by the expected degree of capacity utilization. When the firm does not know demand with certainty, an increase in capacities has a less than one for one effect on expected production. Moreover, the elasticity of expected production to capacities is a positive function of the degree of capacity utilization. This theoretical result accords with the observed fact that firms invest even if capacities are not fully employed, and supports the intuitive perception that the more capacities approach full utilization the more firms invest.

To generate excess capacities, as stated above, we require that factors be complements at least in the short run. Licandro (1990) has imposed a strong assumption about technology, namely a Leontief specification for the production function. This assumption allows to understand the investment process without having to pay attention to long-run factor substitution effects. In this paper the firm is assumed to produce with a constant-returns-to-scale production function and to face adjustment costs over all production factors, such as in Pindyck and Rotemberg (1983). The introduction of long-run substitutability allows for more than the standard average q effects on investment.

Finally, this paper is related to the "aggregation over micromarkets in disequilibrium" approach.[3] This literature is based on rationing models and considers the economy as the aggregate of a large number of micromarkets, each being in a capacity-constrained or in a demand-constrained equilibrium. In macroeconometric models built on this approach, excess capacity is a natural situation and the degree of capacity utilization is an important endogenous variable. In the final part of this paper, it will be shown that q investment models under factor complementarity and monopolistic competition give formal microfoundations to a particular form

[3] The idea was initially proposed by Muellbauer (1978) and Malinvaud (1980). Lambert (1988) introduces sufficient conditions to approximate aggregate production by a CES function of aggregate capacities and aggregate demand. A macro-model was first estimated by Sneessens and Drèze (1986) and now serves as basis for the "European Unemployment Programme" (see Drèze and Bean (1990)).

of the "aggregation over micromarkets in disequilibrium" model.

2 DEMAND, CAPACITY CONSTRAINTS AND DUC

The representative firm is assumed to operate in a monopolistically competitive market and to face the following demand function:

$$(1) \qquad YD_t = \left(\frac{p_t}{P_t}\right)^{-\varepsilon} \overline{YD}_t u_t;$$

where YD is firm's demand, p is firm's price, P is the aggregate price level, $\overline{YD}$ is expected aggregate demand, and u is a stochastic demand shock, which is assumed i.i.d. with unit mean and given variance. The firm fixes its price one period before the stochastic demand term becomes known. In addition, because all firms are assumed to be identical both prices, p and P, are equal at equilibrium.

The production function, denoted by F(K,L), is assumed linear-homogeneous in capital and labor. Standard assumptions about marginal productivities are imposed. The firm faces adjustment costs on both factors. It decides in period t-1 investment and hiring for period t: for the investment decision, this can be justified by the existence of "delivery lags"; the labor decision is treated in a symmetric way since the process of hiring also takes time. Under these conditions potential output, denoted by YP and representing total capacities, is given by

$$(2) \qquad YP_t = F(K_t, L_t);$$

In period t the firm cannot produce more than capacities and YP is thus an upper bound on production.

Under the previous assumptions, excess capacity and excess demand are possible outcomes in any given period as long as capacities and price are determined one period before the stochastic demand shock becomes known. This implies that production, denoted by Y, is

$$Y_t = \min\{YD_t, YP_t\},$$

so that the level of production depends on which of both constraints is binding. Let us now define the degree of capacity utilization; this is given by the ratio of production to capacities, that is

(3) $$DUC_t = \frac{Y_t}{F(K_t,L_t)}.$$

As stated above, the firm is assumed to face adjustment costs when implementing new equipment and when hiring or firing workers, and to decide at period t-1 about labor and capital for period t. Moreover, the model allows for underutilization of total capacity. However, nothing up to now guarantees that both factors will be underutilized in equal proportions when the firm produces less than total capacities: we need to put more structure on the problem to have short-run factor complementarity. We thus add the assumption that to have different rates of utilization for the two inputs entails infinite costs. This obviously implies that the utilization rate will be DUC for each of them.

Before solving the theoretical problem, let us introduce some preliminary considerations. When there is demand uncertainty, the firm is not sure that all increases in capacities will be used in production. For this reason, the elasticity of expected production to capacities is smaller than one, unless excess capacity were impossible. A better understanding of this point can be obtained by looking at the definition of expected production,[4]

(4) $$E_t(Y_s) = \overline{YD}_s \int_0^{x_s} u_s\, g(u_s)\, du_s + YP_s \int_{x_s}^{\infty} g(u_s)\, du_s,$$

$$\text{where} \quad x_s = \frac{YP_s}{\overline{YD}_s}$$

and $g(u_s)$ is the density function for the stochastic demand term u_s.

Let Φ_p and Φ_d denote the elasticities of expected production to potential output and expected demand, respectively. They can be derived from equation (4) and written as

(5) $$\Phi_{p,s} = \frac{1}{E_t(DUC_s)} \int_{x_s}^{\infty} g(u_s)\, du_s \quad \text{and} \quad \Phi_{d,s} = \frac{\overline{YD}_s}{E_t(Y_s)} \int_0^{x_s} u_s\, g(u_s)\, du_s.$$

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[4] This problem, which has been extensively analyzed, is related to limited dependent variable models. In Quandt (1988) the expected minimum condition is studied in various contexts. Lambert (1988) analyzes the problem in the context of aggregation over micromarkets in disequilibrium.

The elasticities of expected production to both expected demand and capacities sum to one. This follows directly from equations (4) and (5), and implies that *expected production is linear-homogeneous in capacities and expected demand.* Since Φ_p and Φ_d are non-negative and sum to one, they are both smaller than or equal to one and at least one of them is strictly smaller than one. The meaning of these properties is straightforward. Given the demand distribution, an increase in capacities has a less than one for one effect on expected production since the new units have some likelihood of not being utilized. The same is true for an increase in expected demand, given capacities. But if both expected demand and potential output increase in the same proportion, expected production also increases in this proportion.

A particular case will be important in what follows. As shown by Lambert (1988), when the stochastic demand term is lognormally distributed, expected production can be approximated by a CES function of expected demand and potential output. That is

$$(4') \qquad E_t(Y_s) = \left(E_t(YD_s)^{-\rho} + YP_s^{-\rho}\right)^{-\frac{1}{\rho}}.$$

From equation (4') we can easily see that the elasticities Φ_p and Φ_d verify the two following relations:

$$(5') \qquad \Phi_{p,s} = E_t(DUC_s)^{\rho} \quad \text{and} \quad \Phi_{p,s} + \Phi_{d,s} = 1.$$

The coefficient ρ is an inverse function of the variance of the stochastic demand term u and for plausible values of this variance ρ is greater than one. The elasticity Φ_p is an increasing function of expected DUC. From now on, we assume that u is lognormally distributed and that equations (4') and (5') verify.

3 THE ADJUSTMENT COST FUNCTION

The literature on adjustment cost assumes that investment costs depend on investment and capital. In this paper, we suggest a slightly different assumption, which seems better adapted to our problem. Since the firm is assumed not necessarily to produce at full capacity, the degree of capacity utilization appears as an intuitive component of the adjustment cost function; total investment cost are defined as $\Psi(I,K,DUC)$.

Moreover, it is frequently supposed that the investment cost function is linear-homogeneous in investment and capital. For this reason and following the considerations in Licandro (1990), we assume that

$$\Psi(I_s, K_s, DUC_s) = \Omega\left(\frac{I_s}{K_s}\right) K_s DUC_s. \tag{6}$$

If capacities are fully utilized the adjustment cost function becomes linear-homogeneous in I and K, just as is generally supposed. When capacities are partially utilized, condition (6) imposes that the firm faces adjustment costs which are below full-capacity costs. This assumption implies that at full capacity the firm finds it more difficult —in terms of the availability of resources within the firm— to implement new equipment than when some machines and workers are used only partially.

The $\Omega(.)$ function is assumed incresing and convex, and it is normalized as follows:

$$\Omega(\delta) = \frac{\delta}{DUC^*} \quad \text{and} \quad \Omega'(\delta) = \frac{1}{DUC^*}.$$

where DUC* represents expected DUC at steady state. Indeed, the investment rate is equal to the depreciation rate in steady state, which provides a natural point of normalization for $\Omega(.)$. When the firm expects to use capacities at its steady state level and invests just to replace the depreciated capital, the normalization assumption for $\Omega(.)$ implies that there are no adjustment costs, i.e., investment spending is equal to $p^I\delta K$. The condition on $\Omega'(\delta)$ corresponds to the standard condition for the unicity of marginal q at steady state.

In the same way, let us define the labor adjustment cost function as

$$\Lambda(H_s, L_s, DUC_s) = \Theta\left(\frac{H_s}{L_s}\right) L_s DUC_s, \tag{7}$$

where H represents net hiring. The $\Theta(.)$ function is convex and normalized as follows:

$$\Theta(0) = \frac{1}{DUC^*} \quad \text{and} \quad \Theta'(0) = 0.$$

The condition on $\Theta(0)$ implies that at steady state labor costs are equal to the wage bill. The condition $\Theta'(0) = 0$ imposes that labor adjustment cost be minimum when net hiring is zero.

4 OPTIMALITY CONDITIONS

4.1 OPTIMAL PROGRAM AND FIRST-ORDER CONDITIONS

As assumed above only demand is random and all other variables are known by the firm. This assumption allows us to concentrate on the consequences of demand uncertainty. Moreover, since random demand shocks are not autocorrelated the optimal path for controls is nonstochastic. With profits being a linear function of DUC as the only stochastic variable, expected profits can be written as a linear function of expected DUC. Let us define π as "profits at full capacity" and write it as

(8) $$\pi_s = \left[p_s F(K_s,L_s) - w_s \Theta\left(\frac{H_s}{L_s}\right) L_s - p_s^I \Omega\left(\frac{I_s}{K_s}\right) K_s\right],$$

where w_s is the wage rate and p_s^I is the investment good price. Time s expected profits are equal to π_s times expected DUC_s.

At time (t-1) the firm decides about prices, investment and hiring for period t in an intertemporal maximization program. Expectations are taken with reference to the information set at (t-1) and the optimization problem is written as

(9)

$$V_t = \underset{\{p_s,K_s,L_s\}_{s=t}^{\infty}}{\text{Max}} \sum_{s=t}^{\infty}\left[p_s F(K_s,L_s) - w_s \Theta\left(\frac{H_s}{L_s}\right) L_s - p_s^I \Omega\left(\frac{I_s}{K_s}\right) K_s\right] E_{t-1}(DUC_s)(1+r)^{-(s-t)},$$

where

(10) $$E_{t-1}(DUC_s) = \left[\left(\frac{F(K_s,L_s)}{E_{t-1}(YD_s)}\right)^{\rho} + 1\right]^{-\frac{1}{\rho}};$$

(1') $$E_{t-1}(YD_s) = \left(\frac{p_s}{P_s}\right)^{-\varepsilon} \overline{YD}_s;$$

(11) $$K_s = I_s + (1 - \delta) K_{s-1};$$

(12) $$L_s = H_s + L_{s-1},$$

with given initial values K_{t-1} and L_{t-1}.[5]

The Lagrangean for this problem is

$$\mathcal{L}_{t-1} = \sum_{s=t}^{\infty}\left[p_s F(K_s,L_s) - w_s \Theta\left(\frac{H_s}{L_s}\right) L_s - p_s^I \Omega\left(\frac{I_s}{K_s}\right) K_s\right] E_{t-1}(DUC_s)(1+r)^{-(s-t)}$$
$$- \sum_{s=t}^{\infty} \mu_s \left(K_s - I_s - (1-\delta) K_{s-1}\right)(1+r)^{-(s-t)} - \sum_{s=t}^{\infty} \lambda_s \left(L_s - H_s - L_{s-1}\right)(1+r)^{-(s-t)}.$$

and the first-order conditions are

(13) p: $$\frac{\partial \mathcal{L}_{t-1}}{\partial p_t} = \pi_t \frac{\partial E_{t-1}(DUC_t)}{\partial p_t} + E_{t-1}(Y_t) = 0\,;$$

(14) I: $$\frac{\partial \mathcal{L}_{t-1}}{\partial I_t} = - p_t^I \Omega'_t E_{t-1}(DUC_t) + \mu_t = 0\,;$$

(15) H: $$\frac{\partial \mathcal{L}_{t-1}}{\partial H_t} = - w_t \Theta'_t E_{t-1}(DUC_t) + \lambda_t = 0\,;$$

(16) K: $$\frac{\partial \mathcal{L}_{t-1}}{\partial K_t} = 0;$$

(17) L: $$\frac{\partial \mathcal{L}_{t-1}}{\partial L_t} = 0.$$

Implicit in the optimality condition (13) is, as in Sneessens (1987), a price equation where the firm charges a markup on marginal production cost. The price equation takes the form:

(18) $$p_t = \left(1 - (\varepsilon\, \Phi_{d,t})^{-1}\right)^{-1} \cdot \left(\frac{w_t \Theta_t L_t + p_t^I \Omega_t K_t}{F(K_t,L_t)}\right).$$

Since labor and investment costs are a linear function of E(Y), marginal cost is equal to average cost. In determining average cost, the adjustment costs are taken into account by the firm as well as the wage bill and the

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[5] In order to have more symmetry between labor and capital, one could introduce some attrition of labor force via an exogenous separation rate.

purchasing price of investment goods. This equation stresses the influence of demand pressures in the markup rate, through the elasticity of expected production to expected demand, denoted by Φ_d. As in monopolistic competition models, the elasticity of production with respect to prices must be greater than one, i.e., $\varepsilon\Phi_d > 1$, to avoid infinite prices.[6]

Let α be the investment rate. The optimality condition for investment is derived from condition (14) and states

(19) $$q_t = \Omega'(\alpha_t)\, E_{t-1}(DUC_t).$$

In standard q theories the optimal investment rate is a function of marginal q, represented by the ratio of μ to p^I. Since μ is the shadow price of the capital stock, the variable q represents the marginal q for total capital. However, in our case marginal investment costs, given by the right hand side of equation (19), depend also on expected DUC. This can be reconciled with the standard result if we divide both sides by expected DUC and interpret the ratio betwenn q and expected DUC as the marginal q associated to the effectively used capital. Under these considerations, the rate of investment is a function of the marginal q for the *effectively used* capital.

Similarly, from equation (15) we can deduce an optimal path for the hiring ratio, i.e., the ratio of hiring to total labor. The optimal hiring ratio, denoted by β, verifies

(20) $$\omega_t = \Theta'(\beta_t)\, E_{t-1}(DUC_t).$$

ω represents the marginal value of one unit of labor divided by the wage rate. Equation (20) says that the marginal cost of hiring –given by the right hand side– is equal to marginal ω. This is formally equivalent to the condition we derived for investment. As in equation (19), the marginal cost of hiring is weighted by expected DUC because only this proportion of total employees is effectively working.

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6 As long as $\varepsilon\Phi_d < 1$, the firm is induced to set very high (infinite) prices. Since prices go to infinite, the demand faced by the firm goes to zero implying that Φ_d approches unity. This is contradictory with the assumption that ε is greater than one and implies that $\varepsilon\Phi_d > 1$ must verify.

Combining equations (14) and (16) we deduce the Euler equation for capital:

(21) $$p_t^I K_{t-1}\,(1-\delta)\,\Omega_t^{\prime}\,E_{t-1}(DUC_t) = p_{t+1}^I K_t\,(1-\delta)\,\Omega_{t+1}^{\prime}\,E_{t-1}(DUC_{t+1})\,(1+r)^{-1}$$

$$+\left(p_t F_{K,t} K_t - p_t^I \Omega_t K_t\right) E_{t-1}(DUC_t) - \Phi_{d,t}\,\pi_t \frac{F_{K,t} K_t}{F_t} E_{t-1}(DUC_t).$$

Solving recursively forward we obtain

(22) $$p_t^I K_{t-1}\,(1-\delta)\,\Omega_t^{\prime}\,E_{t-1}(DUC_t) =$$

$$\sum_{s=t}^{\infty}\left(p_s F_{K,s} K_s - p_s^I \Omega_s K_s\right) E_{t-1}(DUC_s)\,(1+r)^{-(s-t)}$$

$$-\sum_{s=t}^{\infty}\Phi_{d,s}\,\pi_s \frac{F_{K,s} K_s}{F_s} E_{t-1}(DUC_s)\,(1+r)^{-(s-t)}.$$

Equation (22) states the well-known condition for optimal investment: the marginal cost of investment, on the left hand side, must be equal to the marginal value of capital, on the right hand side. It differs from the standard condition because expected DUC is not necessarily equal to one. In the particular case where the firm expects to fully use its capacities, the elasticity Φ_d becomes zero and equation (22) reduces to the standard condition. But it is an extreme case in which capacities for all future periods are so small –or expected demand is so large– that the probability of an excess capacity is zero. Note that dividing the right hand side of (22) by $p^I_t\, K_{t-1}\,(1\text{-}\delta)$ we obtain a explicit expression for marginal q.

Similarly, combining equations (15) and (17) we deduce the Euler equation for labor:

(23) $$w_t L_{t-1}\,\Theta_t^{\prime}\,E_{t-1}(DUC_t) = w_{t+1} L_t \Theta_{t+1}^{\prime}\,E_{t-1}(DUC_{t+1})\,(1+r)^{-1}$$

$$+\left(p_t F_{L,t} L_t - w_t \Theta_t L_t\right) E_{t-1}(DUC_t) - \Phi_{d,t}\,\pi_t \frac{F_{L,t} L_t}{F_t} E_{t-1}(DUC_t).$$

As for the capital stock we solve forward to deduce

(24)

$$w_t L_{t-1}\, \Theta_t^{\prime}\; E_{t-1}(DUC_t) \;=$$

$$\sum_{s=t}^{\infty}\left(p_s F_{L,s} L_s - w_t \Theta_s L_s\right)\; E_{t-1}(DUC_s)\,(1+r)^{-(s-t)}$$

$$-\sum_{s=t}^{\infty}\Phi_{d,s}\,\pi_s \frac{F_{L,s} L_s}{F_s}\, E_{t-1}(DUC_s)\,(1+r)^{-(s-t)}.$$

Equivalently to equation (22) for the capital stock, equation (24) represents the optimality condition that the marginal cost of hiring must be equal to the marginal value of labor. Dividing the right hand side of (24) for $w_t\, L_{t-1}$ we obtain the explicit form of marginal ω.

4.2 Average Q and Investment Equation

Hayashi (1982) shows that, in competitive markets, linear homogeneity in the installation function in addition to linear homogeneity in the production function are sufficient conditions for the equality between marginal q and average q. Licandro (1990) shows that in the presence of both price rigidity and factor complementarity this proposition does not hold, because capacities do not have a unit elasticity with respect to expected production. When we add hiring adjustment costs to investment adjustment costs, this result still holds but a new point must be stressed.

As a consequence of adjustment costs the hiring process has the same type of effect on the firm's value as the investment process. If we sum the two marginal values from equations (22) and (24), we deduce the total marginal value of all factors. It is

(25)

$$\left(p_t^I K_{t-1}\,(1-\delta)\;\Omega_t^{\prime} + w_t L_{t-1}\Theta_t^{\prime}\right) E_{t-1}(DUC_t) \;=\; \sum_{s=t}^{\infty}\Phi_{p,s}\,\pi_s\, E_{t-1}(DUC_s)\,(1+r)^{-(s-t)}.$$

When the firm expects to use all capacities and in the absence of labor adjustment costs, equation (25) becomes the standard condition for q models, that is

$$p_t^I K_{t-1}(1-\delta)\,\Omega_t^I(\alpha) = \sum_{s=t}^{\infty} \pi_s (1+r)^{-(s-t)} = V_t,$$

where α is the investment rate, i.e., investment divided by the capital stock. Nevertheless, the extreme case of E(DUC) = 1 is associated with a zero probability of demand constraint, which requires very large expected demand or very low capacities. Since capacities are controlled by the firm in the long run and profits are non-negative, in general the firm is interested in expanding capacities to avoid too large an excess demand, bounding expect DUC away from unity in the long run.[7]

Defining average q, denoted by Q, as

$$Q_t = \frac{V_t}{p_t^I K_{t-1}(1-\delta)}, \tag{26}$$

and taking into account equations (19) and (20), we can derive from equation (25) the relation between marginal and average q:

$$q_t = Q_t \widehat{\Phi}_p - \frac{\omega_t}{k_{t-1}(1-\delta)} \frac{w_t}{p_t^I} \tag{27}$$

where k is the capital-labor ratio and

$$\widehat{\Phi}_{p,t} = \sum_{s=t}^{\infty} \Phi_{p,s} \frac{\pi_s E_{t-1}(DUC_s)(1+r)^{-(s-t)}}{V_t}. \tag{28}$$

Note that in each period, the elasticity Φ_p is weighted by time t profits divided by the value of the firm. This weight has the property of a probability measure, since the value of the firm is the addition of all future profits. Thus $\widehat{\Phi}_p$ is a mean value of all future elasticities of expected production to capacities and it will be called the "mean potential output elasticity."

In equation (27) we can see that the neoclassical q model is modified in two different ways. First, because the firm faces two types of adjustment costs the total marginal value of the firm is given by the sum of both the

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[7] Furthermore, as implied by footnote 5, Φ_d must be strictly positive, so that expected DUC is strictly smaller than one.

marginal value of capital and the marginal value of labor. Even if the firm does not face uncertain demand constraints marginal q differs from average q in the second term of the right hand side, which depends on the marginal value of labor. Second, the existence of demand uncertainty and capacity constraints implies a "mean potential output elasticity" $\widehat{\Phi}_p$ below or equal to unity. Without adjustment costs on hiring ($\omega = 0$) the results of Licandro (1990) hold. Finally, marginal q is equal to average q as in Hayashi (1982), when there are no adjustment costs on hiring nor any demand uncertainty ($\widehat{\Phi}_p = 1$).

Combining equation (27) with the optimal investment condition (19) and the optimal hiring condition (20), we obtain the following investment equation:

$$(29) \qquad \Omega'(\alpha_t) = Q_t \frac{\widehat{\Phi}_{p,t}}{E_{t-1}(DUC_t)} - \frac{w_t}{p_t^I} \frac{\Theta'(\beta_t)}{k_{t-1}\,(1-\delta)} \,.$$

In equation (29) investment depends on two terms: in the first term average q is multiplied by the "mean potential output elasticity" $\widehat{\Phi}_p$, –function of the sequence of expected DUCs– and divided by time t expected DUC; the second term is negative and depends on relative factor prices, the capital-labor ratio and the hiring ratio. Note that when there are no labor adjustment costs the second term vanish and when capacities are fully employed the first term is equal to average q.

Equation (29) can be transformed to have an equivalent expression for the hiring ratio. But we cannot deduce α and β from equation (29) and the equivalent condition for labor, since both conditions are the same. The problem is the following. On the one side, we have average q, which follows from the firm's *total* value, obtained by the use of both factors *together*. On the other side, we have *two* marginal values, one for each factor. As it is impossible to separate the contribution of each factor to total value, we cannot deduce these two marginal values from the single relation giving average q. This appears clearly in equation (27). However, equations (21) and (23) can be combined in a different way, and this can allow us to determine the ratio of marginal productivities.

4.3 MARGINAL AND AVERAGE PRODUCTIVITIES

We combine equations (21) and (23) to determine the relation between marginal productivities:

$$(30) \qquad \frac{F_{L,t}}{F_{K,t}} = \frac{UCL_t}{UCK_t}.$$

where the user cost of labor, denoted UCL, is equal to

$$UCL_t = w_t \frac{L_{t-1}}{L_t} \Theta_t' E_{t-1}(DUC_t) - w_{t+1} \Theta_{t+1}' E_{t-1}(DUC_{t+1}) (1+r)^{-1}$$
$$+ w_t \Theta_t E_{t-1}(DUC_t) \; ;$$

and the user cost of capital, denoted UCK, is

$$UCK_t = p_t^I \frac{K_{t-1}}{K_t} (1-\delta)\, \Omega_t' \; E_{t-1}(DUC_t) - p_{t+1}^I (1-\delta)\, \Omega_{t+1}' \; E_{t-1}(DUC_{t+1}) (1+r)^{-1}$$
$$+ p_t^I \Omega_t \; E_{t-1}(DUC_t) \, .$$

As in standard investment theory the user cost of capital can be decomposed into two terms: the marginal gain from investing now rather than next period, and total current investment costs. The same interpretation can be given to the user cost of labor. Consider the particular case where the firm expects to fully use its capacities. In this case, equation (30) becomes

$$(30') \qquad \frac{F_{L,t}}{F_{K,t}} = \frac{w_t \frac{L_{t-1}}{L_t} \Theta_t' - w_{t+1} \Theta_{t+1}' (1+r)^{-1} + w_t \Theta_t}{p_t^I \frac{K_{t-1}}{K_t} (1-\delta)\, \Omega_t' - p_{t+1}^I (1-\delta)\, \Omega_{t+1}' (1+r)^{-1} + p_t^I \Omega_t} \, .$$

In (30'), the ratio of marginal productivities is equal to the ratio of the user costs of factors. When the firm expects to use its capacities only partially, as in equation (30), the user cost of both factors takes a more general form. Nevertheless, this general form depends on the assumptions about the adjustment cost function: when both adjustment cost functions do not depend on DUC, the general solution is equation (30') even if expected DUC is smaller than one.

Since we assume a constant returns to scale production function, the ratio of marginal productivities is a function of the capital-labor ratio k, which is determined by equation (30). In particular, with a Cobb-Douglas technology, equation (30) can be combined with the production function to yield the average productivities:

(31) $$\frac{F(K_t,L_t)}{K_t} = a_k\left(\frac{UCL_t}{UCK_t}\right)^{\gamma-1},$$

(32) $$\frac{F(K_t,L_t)}{L_t} = a_l\left(\frac{UCL_t}{UCK_t}\right)^{\gamma},$$

where a_k and a_l are unimportant constants and γ is the elasticity of capital in the production function. This is a standard result in production theory: with a Cobb-Douglas technology average productivities are log-linear functions of relative factor costs.

5 THE AGGREGATE ECONOMY

5.1 AGGREGATION

Following Sneessens (1987), let us assume that aggregate demand is given; its distribution among firms depends on relative prices and on a random term. Assuming in addition that all firms are ex-ante identical, each of them sets the same price, so that the relative price in fact vanishes. This implies that aggregate demand is indeed represented by $\overline{YD}$ in equation (1), and all that is left to distribute demand unequally between firms is the stochastic term u. This can be interpreted in a simple way: the representative firm faces demand uncertainty because it does not know its future position in the demand distribution. In this sense, demand uncertainty is just firm-specific and there is no aggregate demand uncertainty.

Aggregate capacities can be set equal to the representative firm's potential output. Furthermore, aggregate production and aggregate DUC are equal to the representative firm's expected production and expected DUC. Since the distribution of aggregate demand among firms is assumed lognormal, aggregate production, denoted by $\overline{Y}$, is equal to

(33) $$\overline{Y}_t = \left(\overline{YD}_t^{-\rho} + YP_t^{-\rho}\right)^{-\frac{1}{\rho}}.$$

Aggregate DUC is denoted $\overline{DUC}$ and Φ_p and Φ_d represent the elasticities of aggregate production to aggregate demand and aggregate capacities, respectively. Moreover, all the optimality conditions for the representative firm can be reinterpreted as aggregate conditions.

5.2 STEADY STATE

In steady state, capital and labor remain constant. From the law of motion for capital in equation (11), we deduce that the steady state investment rate is simply equal to the rate of depreciation, i.e., $\alpha^* = \delta$, since net investment is zero. For the same reason, from the law of motion for labor in equation (12), we deduce that net hiring is zero at steady state, i.e., $\beta^* = 0$.

The optimality condition for investment given by equation (19) implies that steady state marginal q is given by

(34)
$$q^* = 1.$$

Equation (34) reflects Tobin's (1969) proposition that capital does not grow when marginal q is equal to one.

The steady state value for marginal ω is given by equation (20) and is equal to zero, since the adjustment cost function $\Theta(\beta)$ is assumed to have a minimum at $\beta = 0$, i.e.,

(35)
$$\omega^* = \Theta'(0) = 0.$$

Combining both marginal values as in equation (29) we deduce the relation between marginal and average q at steady state:

$$q^* = Q^* \Phi_p^* = Q^* (\overline{DUC}^*)^{\rho}.$$

At steady state marginal q differs from average q only in the elasticity of aggregate production to capacities, which is equal to the aggregate degree of capacity utilization to the power ρ.

Another interesting relation is verified at steady state. It concerns the ratio of marginal productivities:

$$\frac{F_L^*}{F_K^*} = \frac{w}{p^I\left(\frac{r+\delta}{1+r}\right)} \quad (36)$$

Equation (36) says that at steady state the ratio of marginal productivities is equal the ratio of factor user costs. As could be expected, the steady state value of the user cost of labor is equal to the wage rate. The user cost of capital takes the standard form

$$UCK^* = p^I\left(\frac{r+\delta}{1+r}\right). \quad (37)$$

Thus in this case the steady state user cost of capital equals financing costs plus depreciation costs. Both marginal productivities depend only on the capital-labor ratio, denoted by k, since there are constant returns to scale. Equation (36) thus determines the steady state capital-labor ratio, k*.

Finally, we can deduce the steady state value Φ_d^* for the elasticity of aggregate production to aggregate demand:

$$\Phi_d^* = \frac{A^*}{\varepsilon(A^*-1)+1}; \quad (38)$$

$$\text{where} \quad A^* = \frac{w\,L^* + p^I\left(\frac{r+\delta}{1+r}\right)K^*}{\left(w\,L^* + p^I\delta\,K^*\right)} > 1.$$

A* is the steady state ratio of the user cost of total factors divided by total costs. Moreover, since A* is greater than one and ε is greater than one and finite, the Φ_d^* elasticity is strictly positive and strictly smaller than one. Dividing the numerator and denominator of A* by L*, we see that A* and thus Φ_d^* depend only on k*, which has been determined above. Finally, $\overline{DUC}^*$ is then positive and smaller than one, implying that the aggregate economy produces with excess capacity at steady state.

5.3 The Aggregate Economy

Under these aggregation assumptions, the aggregate model associated with the behavior of the representative firm is presented in Table 1. It is similar in structure to the "aggregation over micromarkets" model of Sneessens and Drèze (1986). The ratio of average productivities depend on relative user costs. Aggregate production is a CES function of (exogenous) aggregate demand

and of potential output. Aggregate prices are set as a markup over marginal cost, the markup rate itself being a function of demand pressures. The main innovation is in the investment equation. The investment rate depends on profitability (through average q and the degree of capacity utilization) and on factor substitutability (directly through relative factor prices and indirectly through the hiring and capital-labor ratios). Moreover, the firms are assumed not to experience any labor shortage.

6 CONCLUSIONS

Demand uncertainty and factor complementarity have been introduced in the literature to improve the foundations of q investment models and to stress the role of the utilization of capacities in explaining investment behavior. In Licandro (1990) we have analyzed the relation between marginal and average q in the case of a Leontief technology. The main result was that marginal q is smaller than average q, the difference being explained by the expectations about the degree of capacity utilization. As in more standard q models, investment was shown to depend on marginal profitability, represented here by *two* variables: average q and the degree of capacity utilization.

In this paper, adjustment costs on investment and hiring are imposed to model short-run factor complementarity when the production function has constant returns to scale. Labor is modeled in the same way as capital. The introduction of labor adjustment costs implies that labor behaves as a stock. When there is no demand uncertainty, since both factors carry with them adjustment costs, the value of the firm is equal to the marginal value of total factors (capital stock and labor). It implies that, in addition to average q, investment depends on factor substitutability, represented by relative factor prices, the hiring ratio and the capital-labor ratio. This result does not depend on demand uncertainty and can be obtained by simply adding hiring adjustment costs to the neoclassical q model.

At steady state, the main result is that capacities are not fully employed, implying that firms are induced to have capacities greater that expected production. In addition, since adjustment costs on labor are assumed to be minimum when there is no net hiring, at steady state the marginal value of labor is zero. Since marginal q is equal to average q times the elasticity of production to capacities and capacities are underemployed, marginal q is strictly smaller than average q at steady state. In this sense, the introduction of long-run factor sustitutability does not modify the outcome of models where technology is assumed clay-clay.

For empirical problems this way of modeling firm behavior allows for a more formal derivation of the macroeconomic models based on the "aggregation over micromarkets in disequilibrium" principle. The present paper has been a first attempt in this direction. In a deeper investigation, a formal solution of the difference equation system would be required and it would have to be compared with a dynamic simulation of the Sneessens and Drèze (1986) model.

TABLE 1

Factor Productivities [8]

$$\ln\left(\frac{\overline{Y}_t}{K_t}\right) = a_0 + (\gamma-1)\ln\left(\frac{UCL_t}{UCK_t}\right) - \ln\left(\overline{DUC}_t\right)$$

$$\ln\left(\frac{\overline{Y}_t}{L_t}\right) = b_0 + \gamma\ln\left(\frac{UCL_t}{UCK_t}\right) - \ln\left(\overline{DUC}_t\right)$$

Production

$$\overline{Y}_t = \left(\overline{YD}_t^{-\rho} + YP_t^{-\rho}\right)^{-\frac{1}{\rho}}$$

$$\overline{DUC}_t = \frac{\overline{Y}_t}{YP_t}$$

$$YP_t = F(K_t, L_t)$$

$$\Phi_{p,t} = \overline{DUC}_t^{\rho}$$

Prices

$$P_t = \left(1 - \left(\varepsilon\,\Phi_{d,t}\right)^{-1}\right)^{-1}\left(\frac{\left(w_t\Theta(\beta_t)\,L_t + p_t^I\,\Omega(\alpha_t)\,K_t\right)\overline{DUC}_t}{\overline{Y}_t}\right)$$

[8] Since the production function has constant returns to scale, both average productivities have the same equation and only the capital-labor ratio can be determined in this block.

Hiring, Investment and Capital

$$\beta_t = 1 - \frac{L_{t-1}}{L_t}$$

$$\Omega'(\alpha_t) = Q_t \frac{\widehat{\Phi}_{p,t}}{\overline{DUC}_t} - \frac{w_t}{p_t^I} \frac{\Theta'(\beta_t)}{k_{t-1}(1-\delta)}$$

$$K_t = (1 + \alpha_t - \delta) K_{t-1}$$

Firm's Market Value and Φ_p Elasticity

$$Q_t = \frac{V_{t-1}}{p_t^I K_{t-1}(1-\delta)}$$

$$V_{t-1} = \sum_{s=t}^{\infty} \pi_s \overline{DUC}_s (1+r)^{-(s-t)}$$

$$\pi_s = \left[p_s F(K_s, L_s) - w_s \Theta(\beta_s) L_s - p_s^I \Omega(\alpha_s) K_s\right]$$

$$\widehat{\Phi}_{p,t} = \sum_{s=t}^{\infty} \Phi_{p,s} \frac{\pi_s \overline{DUC}_s (1+r)^{-(s-t)}}{V_{t-1}}$$

Factor User Costs

$$UCL_t = w_t \frac{L_{t-1}}{L_t} \Theta'_t E_{t-1}(DUC_t) - w_{t+1} \Theta'_{t+1} E_{t-1}(DUC_{t+1}) (1+r)^{-1} + w_t \Theta_t E_{t-1}(DUC_t) ;$$

$$UCK_t = p_t^I \frac{K_{t-1}}{K_t} (1-\delta)\, \Omega'_t\, E_{t-1}(DUC_t) - p_{t+1}^I (1-\delta)\, \Omega'_{t+1}\, E_{t-1}(DUC_{t+1}) (1+r)^{-1} + p_t^I \Omega_t\, E_{t-1}(DUC_t) .$$

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