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PROMOTING SALES THROUGH COUPONS IN OLIGOPOLY UNDER IMPERFECT PRICE INFORMATION

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Abstract

This paper analyzes sales promotions through coupons in an oligopoly under imperfect price information. Firms can send out coupons that either communicate the price (CCPs) or not (CNCPs). By offering rebates or advertising their prices at distant locations, firms can attract consumers from their rivals' markets. A unique symmetric pure strategy equilibrium is shown to exist where sellers do send out coupons that offer positive rebates. Thus, contrary to the literature, sales promotions are permanent. In the equilibrium with CNCPs, prices, advertising efforts and firms' profits are higher than in the equilibrium with CCPs. However, the equilibrium with CNCPs is not robust if we allow firms to choose the type of coupons as well. In contrast, the equilibrium with CCPs is always robust.

Keywords: Sales Promotions, Coupons, Oligopoly, Imperfect Information, Price Discrimination

JEL Classification: D43, D83

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1 Introduction

Sales promotions are techniques that stimulate consumers to purchase from a specific retail store or to try a particular product. Several methods are used by firms to promote their sales: price promotions, demonstrations, free samples, premiums, point-of-sale displays, etc. Price promotions are typically temporary price discounts offered by firms from time to time. Coupons, shelf-price reductions, and mail-in rebates are the usual marketing devices through which sellers carry out their price promotions. Most of the reports indicate that coupons are extensively used as a sales promotion tool[1]. For instance, Strazewski (1986) reports that 95 % of US sellers have used coupons to promote their sales in 1984. A coupon is a certificate that entitles the holder to redeem its value for money or, occasionally, goods. Coupons are distributed through several ways: free-standing inserts (FSIs), print advertising, direct mailing, on or inside the packages, and in stores.

Recently, the industrial organization literature has offered three alternative explanations of the role of coupons. First, coupons can be used to induce repeated purchases. A seller, by offering a coupon to those consumers that purchase from his store in the first period, can endogenously create a future switching cost. Thus, consumers have less incentives to switch stores in the second period. As a result, coupons decrease competition.[2] Second, coupons can be used as a retailer stimulation mechanism. If a product is sold through a distribution channel, the manufacturer may directly send out coupons to the consumers to motivate retail participation in the promotion.[3] Third, coupons can serve as a price discrimination device. Sellers, by sending out coupons at distant locations, can induce consumers to switch stores. Contrary to the first case, competition is now fostered.[4]

An important role of the coupons has however been ignored by the literature, namely that they may convey price information as well. To address this issue, we study sales promotions through coupons in an oligopoly under im-

[1] Sellers use coupons to stimulate consumers to try a new or an established product, to increase sales volume quickly, to attract repeated purchases or to introduce new package sizes or features.

[2] Banerjee and Summers (1987) develop a two period model with homogeneous products where in the first period firms offer discounts granted to the first period buyers if they repeat their purchases. In their model coupons become a collusive device. Both period equilibrium prices are found to coincide with the monopoly price. Caminal and Matutes (1990) study coupons in a two period differentiated product duopoly. Competition is again found to be relaxed through the use of coupons.

[3] Gerstner and Hess (1991) consider that this fact can explain why rebates are offered even when all consumers use them and price discrimination does not occur.

[4] See Narasimhan (1984), Caminal (1996) and Bester and Petrakis (1996).

perfect price information. We consider a segmented market where consumers incur transportation costs to venture a distant location, and thus only *local* price information is costlessly available to them.[5] Interestingly, in our analysis market segmentation is not only locational but also informational. If sellers do not advertise their prices, consumers have to travel to find out the price charged by the distant store. In this situation, sellers can attract consumers from the rival's location by either advertising the price and/or offering rebates. For this purpose, firms can issue two kinds of coupons: first, coupons that offer at the same time a rebate and communicate the regular price and second, coupons that only offer a rebate. We call them *coupons communicating the price (CCPs)* and *coupons not communicating the price (CNCPs),* respectively. Both types of coupons serve as a price discrimination device and thus fit into the third category described above. However, CCPs convey also price information. Interestingly, promoting sales through either CCPs or CNCPs leads to different informational structures and thus market equilibria. Issuing and sending out coupons is a costly activity. Since printing the price on a coupon does not represent an extra cost, we assume that CCPs and CNCPs are equally costly.

We first study optimal pricing, rebating and advertising intensity in a market where sellers can promote their sales by sending out only CCPs. Sellers, by simultaneously providing price information and practising price discrimination, may gain consumers from other locations and thus increase their market shares. Due to this twofold role of CCPs, our work is linked to two strands of the literature: that on *informative advertising* and that on coupons as a *price discrimination* device. Contrary to the bulk of the informative advertising literature, we assume that all consumers are well informed about the existence, availability and characteristics of the products; only price information is imperfect.[6] This is in line with Bester and Petrakis (1995) where pure price advertising is studied in a price competition model. Our model is however more general since a CCP offering a zero rebate is nothing else than a pure price ad. On the other hand, Caminal (1996) and Bester and Petrakis (1996) show that coupons act as price discrimination devices in equilibrium. Caminal (1996) considers a monopolist facing heterogeneous consumers and uncertain marginal costs. Bester and Petrakis (1996)

[5]From now on, the market segmentation is referred as *locational* but the analysis also applies to e.g. market segmentation due to consumers' brand loyalty.

[6]See e.g. Butters (1977), Grossman and Shapiro (1984), Tirole (1989) and Stegeman (1991). In most of the papers on informative advertising, sellers are able to attract consumers by informing them about the existence of their products. Here, in contrast, sellers can attract consumers only by advertising a lower price or by offering a rebate because consumers are fully informed about the existence and availability of the products.

study a duopoly where consumers have perfect price information. Due to the locational segmentation however, our assumption that consumers have only local price information seems more reasonable.

We show that there exists a symmetric pure strategy equilibrium with rational expectations where sellers send out CCPs to the distant location offering positive rebates. Contrary to the literature, this finding implies that in a (finitely) repeated market interaction between sellers, under imperfect price information sales promotions are *permanent*. For instance, in Bester and Petrakis (1995) a symmetric pure strategy equilibrium in which both firms advertise their prices fails to exist. The reason is that each individual seller can gain by advertising his price only if he offers a lower price than his competitor's. Obviously, this condition cannot be simultaneously fulfilled for all sellers in a symmetric pure strategy equilibrium. As a result, price promotions can only occur in a mixed strategy equilibrium which, in a dynamic interpretation, generates temporal price dispersion (as each store varies its price over time). This is also in line with Shilony (1977), Varian (1980) and Narasimhan (1988). In our model however, sellers by offering rebates through coupons can simultaneously announce lower prices than the undiscounted rivals' prices in a symmetric pure strategy equilibrium. Hence, price promotions are permanent here. Note that, not only the regular price is permanently advertised, but also discounts are permanently offered.

Second, we study the sellers' optimal strategy when they can send out only CNCPs. Again, a symmetric pure strategy equilibrium with rational expectations is shown to exist. Even though firms cannot advertise their prices, they still send out coupons that offer positive rebates in equilibrium. Hence, price promotions are permanent here as well. Not surprisingly, the higher is the difficulty to propagate information in the industry, the higher are equilibrium prices, advertising efforts and sellers' profits. In fact, the equilibrium with CNCPs involves higher prices, advertising efforts and typically firms profits than the equilibrium with CCPs. Analogously, in the equilibrium with CCPs prices, advertising efforts and commonly sellers profits are higher than those under the perfect price information equilibrium of Bester and Petrakis (1996).

We finally study the robustness of the equilibria with CCPs and CNCPs when firms are allowed to select the type of coupons as well. We show that the equilibrium with CNCPs is no longer an equilibrium when firms can send out CCPs. Contrarily, the equilibrium with CCPs is always robust to sellers' deviations to send out CNCPs. The implication is then that CNCPs should not be observed in this kind of markets.

The remaining of the paper is organized as follows. Section 2 describes the model. Section 3 provides a characterization of the equilibrium with

CCPs and sufficient conditions for its existence. In section 4 we study the equilibrium with CNCPs. The robustness of the equilibria is analyzed in section 5. Section 6 contains comparative statics and welfare results. Section 7 concludes. Finally, an appendix contains all the proofs.

2 The model

Following Bester and Petrakis (1996), we consider an industry with two firms, A and B, located at different regions. Firms produce a homogeneous good at zero cost. The neighborhood of each firm is inhabited by a unit mass of consumers who have unitary demands for the good and a common reservation utility $v > 0$. Each consumer can costlessly visit the store at his home location. However, consumer i has to pay a transportation cost $s_i \geq 0$ to visit the distant store, where s_i is uniformly distributed on $[0, \bar{s}]$ across the population in each region. Consumer i's transportation cost is assumed to be unobservable[7]. Further, to avoid local monopolies, we assume that $v > \bar{s}$, i.e. transportation costs are not too high.

Consumers are aware of the existence, characteristics and availability of the goods. Also, they can costlessly learn the local price by visiting the store at their home location. However, they are not informed about the price charged at the distant location. Consumers are able to obtain this price information either by visiting the distant store at some cost, or if the distant store advertises its price. Firms can thus promote their sales by sending out coupons at their rivals' regions. Under our imperfect price information regime, sellers can issue two types of coupons: (i) coupons offering a rebate and also posting the price of the good (*Coupons Communicating the Price (CCPs)),* and (ii) coupons offering only a rebate (*Coupons Not Communicating the Price (CNCPs)).* Both types of coupons serve as a price discrimination device[8] as sellers can charge different prices to consumers from different locations. CCPs also serve as a price information vehicle[9] as firms are able to advertise their prices to the distant locations. Note that a CCP offering a zero rebate can be considered as pure price advertising.

Formally, seller i can send out coupons to a fraction $\lambda_i \in [0, 1]$ of the population at location j[10], offering them a rebate r_i on its regular price p_i;

[7]Under this assumption firms cannot practise *perfect* price discrimination. Also, there is no arbitrage between consumers in the same region.

[8]as in Bester and Petrakis (1996) and Caminal (1996).

[9]as in Bester and Petrakis (1995)

[10]Note that, in our model sellers by distributing coupons to their home locations cannot increase their profits.

$0 \leq r_i \leq p_i$, $i = A, B$. Without loss of generality, let $0 \leq p_i \leq v$, $i = A, B$. Since individual transportation costs are unobservable, λ_i represents the probability with which a consumer in region j receives a coupon. A consumer then who has received a coupon has to pay $p_i - r_i$ for firm i's good, while those who have not received a coupon have to pay p_i. To reach a fraction of consumers λ_i, seller i incurs at a cost $k(\lambda_i)$, where $k(\cdot)$ is assumed to be increasing and convex and satisfy $k(0) = 0$[11]. These assumptions on the cost function are standard in the literature (see e.g. Butters (1977), Grossman and Shapiro (1984), and Bester and Petrakis (1995, 1996)).

3 The equilibrium with CCPs.

In this section sellers are allowed to send out only coupons that post the regular price of the good. We assume that firms compete by simultaneously choosing their prices, rebates and intensity of couponing. Then firm i chooses its marketing strategy, (p_i, r_i, λ_i), to maximize its profits taking as given its rival's marketing strategy, $(p_j, r_j, \lambda_j), i, j = A, B$. To find firm i's profits we need first to determine its demand. Note that the above strategies define four different groups of consumers: first, those located at A *receiving* a CCP from firm B; second, those located at A *not receiving* a CCP from B; third, those located at B *receiving* a CCP from A; and fourth, those located at B *not receiving* a CCP from A.

First, a consumer located at A who receives a coupon learns both, firm B's regular price and the rebate he will obtain if he purchases the good from that firm. Thus, he buys the good from his home location whenever $p_A \leq p_B - r_B + s$ and $v \geq p_A$. Second, consider a consumer located at A not receiving a coupon from firm B. He purchases from his home location as long as $p_A \leq p_B^e + s$ and $v \geq p_A$, where p_B^e is the price he expects to be charged at location B. Third, a consumer located at B who receives a coupon from A learns the price charged and the rebate offered by firm A. He will switch store and purchase from A if and only if $p_A - r_A + s \leq p_B$ and $v \geq p_A - r_A + s$. Finally, a consumer located at B who does not receive a coupon buys from firm A if $p_A^e + s \leq p_B$ and $v \geq p_A^e + s$, where p_A^e is the price he expects to be charged at location A. Since all consumers purchasing from firm A have to pay a price p_A except those receiving a coupon who only pay $(p_A - r_A)$ for the good, the firm A's profits function is given by:

[11] Note that it is implicitly assumed that sending out CCPs or CNCPs are equally costly activities. This is reasonable since printing the price on a CNCP does not represent an extra cost.

$$\Pi_A(p_A,p_B,r_A,r_B,\lambda_A,\lambda_B) = p_A(1-\lambda_B)\max\left\{0,\min\left\{1-\frac{p_A-p_B^e}{\overline{s}},1\right\}\right\}+$$

$$+p_A\lambda_B\max\left\{0,\min\left\{1-\frac{p_A-(p_B-r_B)}{\overline{s}},1\right\}\right\}+$$

$$+p_A(1-\lambda_A)\max\left\{0,\min\left\{\frac{p_B-p_A^e}{\overline{s}},\frac{v-p_A^e}{\overline{s}},1\right\}\right\}+$$

$$+(p_A-r_A)\lambda_A\max\left\{0,\min\left\{\frac{p_B-(p_A-r_A)}{\overline{s}},\frac{v-(p_A-r_A)}{\overline{s}},1\right\}\right\}-k(\lambda_A). \tag{1}$$

By the symmetry of the problem, firm B's profits are given by an analogous expression. We shall restrict attention to symmetric solutions alone. Particularly, the solution concept used here is the symmetric pure strategy Nash equilibrium with rational expectations.

Definition 1 *A symmetric pure strategy Nash equilibrium with rational expectations is a pair of marketing strategies* $(p_A^*, r_A^*, \lambda_A^*)$, $(p_B^*, r_B^*, \lambda_B^*)$ *satisfying:*
(i) $\Pi_i(p_i^*, p_j, r_i^*, r_j, \lambda_i^*, \lambda_j) \geq \Pi_i(p_i, p_j, r_i, r_j, \lambda_i, \lambda_j)$ *for all* $(p_i, r_i, \lambda_i) \neq (p_i^*, r_i^*, \lambda_i^*)$, $i \neq j$, $i, j = A, B$.
(ii) $p_A^* = p_B^*$, $r_A^* = r_B^*$, $\lambda_A^* = \lambda_B^*$.
(iii) $p_i^e = p_i^*$, $i = A, B$.

Condition (i) says that each marketing strategy is a best reply to the other. Condition (ii) imposes symmetry. Finally, condition (iii) requires consumers' price expectations to be fulfilled in equilibrium. From now on, we denote this symmetric equilibrium by the vector (p, r, λ). The following proposition characterizes the interior equilibrium with CCPs and also provides conditions for its existence.

Proposition 1 *If sellers promote their sales by only sending out CCPs, a unique interior symmetric pure strategy equilibrium with rational expectations exists if and only if (a)* $k'(1) > \overline{s}/9$ *and (b)* $k'(2\overline{s}/3v) < v^2/4\overline{s}$. *Such equilibrium is given by the unique solution to the following system of equations (2)-(4):*

$$p = \frac{2\overline{s}}{3\lambda} \tag{2}$$

$$r = 0.5p \tag{3}$$

$$\frac{p^2}{4\bar{s}} = k'(\lambda) \tag{4}$$

The equilibrium price and "advertising" intensity are shown in Figure 1. The downwards slopping line PP depicts equation (2) and the upwards slopping line KK depicts equation (4). The intersection of these two lines provides the equilibrium price and advertising intensity (point E). The role of the conditions (a) and (b) that guarantee an interior solution can be easily seen in Figure 1. If, on the contrary, the marginal cost of sending coupons to all the consumers at the rival' s region were sufficiently low, sellers would optimally set $\lambda^* = 1$; if, on the other hand, the marginal cost were relatively high even for small advertising intensities, sellers could safely charge a regular price equal to the consumers' valuation in order to appropriate all the home consumers' surplus (with or without sending out coupons to the distant location).

<insert figure 1 here>

In equilibrium the rebate is strictly positive and, in particular, equals half the regular price. As a coupon offering zero rebate is equivalent to pure price advertising in our model, proposition 1 predicts that pure price advertising should not be observed in a locationally segmented market. However, one might argue that handling coupons generates extra costs for the seller, and thus pure price advertising is a significantly cheaper sales promotion activity than sending out coupons[12]. Let, for the moment, consider an extended model where firms can send out both pure price ads and coupons. Assume that sales promotion costs are given by $k(\lambda + \delta\mu)$, where μ is the price ads intensity and $\delta \leq 1$. That is, a seller incurs the same costs by choosing a couponing intensity λ and a higher price advertisements intensity of $(1/\delta)\lambda$. Note first that price ads and coupons cannot coexist in a symmetric pure strategy equilibrium. The intuition is simply that by advertising its price, a seller can attract consumers from the distant location only if he offers a lower price than his rival' s. This however cannot happen simultaneously for all the sellers in a symmetric pure strategy equilibrium (see Bester and Petrakis (1995)). Further, it can be shown that if price advertising is sufficiently cheaper than coupons (δ close to zero), then an individual seller has an incentive to deviate from the equilibrium in Proposition 1. He will certainly gain by lowering its price and advertising extensively its lower price at almost

[12]See e.g. Caminal (1996)

no cost. Hence, if δ is sufficiently small, the equilibrium with CCPs of our model cannot be sustained as an equilibrium in the extended model.

Proposition 1 further predicts that sales promotions are *permanent* whenever price information is imperfect. This is due to the existence of a pure strategy equilibrium where firms promote their sales by sending out coupons every period. This is in sharp contrast with the existing literature where sales are temporary[13]. All these models generate temporal price dispersion in the sense that each store varies its price over time. As a pure strategy equilibrium does not exist, sellers can only advertise their prices with some probability in equilibrium.

4 The equilibrium with CNCPs.

Let us assume now that sellers can promote their sales by sending out only CNCPs. Again, we first determine seller A's demand. As previously, four different groups of consumers can be distinguished: first, those located at A *receiving* a CNCP from firm B; second, those located at A *not receiving* a CNCP from B; third, those located at B *receiving* a CNCP from A; and fourth, those located at B *not receiving* a CNCP from A. Contrary to the previous case, a consumer located at A receiving a coupon from firm B only learns the rebate offered from that firm. Thus, he buys the good from his home location whenever $p_A \leq p_B^e - r_B + s$ and $v \geq p_A$, with p_B^e the price he expects to be charged at location B. A consumer located at A not receiving a coupon from firm B purchase from his home location as long as $p_A \leq p_B^e + s$ and $v \geq p_A$. Analogously, a consumer located at B who has received a coupon from A only learns the rebate offered by firm A. He switches store and buys from firm A if and only if $p_A^e - r_A + s \leq p_B$ and $v \geq p_A^e - r_A + s$. Finally, a consumer located at B who has not received a coupon from A buys from firm A if $p_A^e + s \leq p_B$ and $v \geq p_A^e + s$. Firm A's profits are then given by:

$$\Pi_A(p_A, p_B, r_A, r_B, \lambda_A, \lambda_B) = p_A(1-\lambda_B)\max\left\{0, \min\left\{1 - \frac{p_A - p_B^e}{\overline{s}}, 1\right\}\right\} +$$

$$+p_A\lambda_B \max\left\{0, \min\left\{1 - \frac{p_A - (p_B^e - r_B)}{\overline{s}}, 1\right\}\right\} +$$

$$+p_A(1-\lambda_A)\max\left\{0, \min\left\{\frac{p_B - p_A^e}{\overline{s}}, \frac{v - p_A^e}{\overline{s}}, 1\right\}\right\} +$$

[13]See Shilony (1977), Butters (1977), Varian (1980), Narasimhan (1988), Stahl (1994) and Bester and Petrakis (1995)

$$+(p_A - r_A)\lambda_A \max\left\{0, \min\left\{\frac{p_B - (p_A^e - r_A)}{\overline{s}}, \frac{v - (p_A^e - r_A)}{\overline{s}}, 1\right\}\right\} - k(\lambda_A). \tag{5}$$

Firm B's profits are analogous. The following proposition characterizes the equilibrium in case that firms can promote their sales using CNCPs alone:

Proposition 2 *Whenever sellers can promote their sales by sending out only CNCPs, a unique interior symmetric pure strategy equilibrium with rational expectations exists if and only if $k'(1) > \overline{s}/4$, and $k'(\overline{s}/v) < v^2/4\overline{s}$. This equilibrium is given by the solution to the following system of equations (6)-(8):*

$$p = \frac{\overline{s}}{\lambda} \tag{6}$$

$$r = 0.5p \tag{7}$$

$$\frac{p^2}{4\overline{s}} = k'(\lambda) \tag{8}$$

In Figure 1 the equilibrium price and couponing intensity are given by the intersection of curves $P'P'$ and KK (point E'). Equation (6) is depicted in the line $P'P'$, while equation (8) (which is the same as equation (4)) is given by KK. Note that the line $P'P'$ lies entirely above the line PP. The interpretation of the conditions that guarantee an interior solution is similar to that of the previous section.

The equilibrium with CNCPs has similar properties with the equilibrium with CCPs. First, sellers always offer strictly positive discounts to the consumers located at the rival's region by sending out a positive amount of CNCPs. Second, the rebate equals half the regular price. Finally, sales promotions are *not* temporary since coupons offering rebates are distributed every period. All these then reinforce our conclusions of the previous section.

It is interesting now to compare our results of Propositions 1 and 2 with those under perfect price information, as in Bester and Petrakis (1996). The equilibrium under perfect price information is characterized by three equations (their Proposition 1). Their last two equations coincide with the corresponding equations of both our equilibrium with CCPs and with CNCPs. The only difference lies in the first equation: under perfect price information, the equilibrium price has to satisfy $p = \overline{s}/(1 + 0.5\lambda)$, while under CCPs satisfies equation (2) and under CNCPs equation (6). This is shown in Figure 1 where the line KK is common in all cases, and $P''P''$ depicts the first equation of Bester and Petrakis (1996). Point E' in Figure 1 then gives the equilibrium price and advertising intensity under perfect price information. Note further that $P''P''$ lies entirely below PP.

Interestingly, the optimal rebate equals half the regular price ($r = 0.5p$) under both imperfect and perfect price information. In all these cases, r is chosen to maximize the seller's profits from those consumers who receive a coupon. Since CCPs also convey price information, this part of the profit maximization problem in Proposition 1 is the same as in Bester and Petrakis (1996). On the other hand, a consumer receiving a CNCP does not learn the regular price of the good. Then the optimal discount equals half the expected regular price, which however, under the rational expectations hypothesis, coincides with the regular price.

Further, we observe from Figure 1 that equilibrium prices and sales promotion intensity are higher whenever price information is imperfect (E' lies to the southwest of both E and E'). Obviously, imperfect information reduces price competition. Thus, firms can send out coupons to a higher percentage of consumers at the distant location without having to substantially cut their regular prices. It is worth stressing here that the price differential between the equilibrium with CCPs and the equilibrium under perfect information reduces as the marginal costs of couponing decrease (As $k'(\lambda)$ decreases, the line KK shifts to the right in Figure 1). Firms, facing now lower marginal costs, are able to send out CCPs to a higher fraction of consumers, thus increasing the information level in the market. In fact, if the marginal costs of couponing are sufficiently low, the equilibrium with CCPs converges to that under perfect information. On the other hand, the equilibrium with CNCPs involves higher prices and advertising intensity than the equilibrium with CCPs (E lies to the southwest of E'). The intuition is that firms, by using CCPs instead of CNCPs, spread out at the same time price information into the market, and as a result face a stronger price competition. To avoid then a substantial cut of his regular price, a seller optimally reduces his sales promotion intensity.

Finally, we compare sellers profits in the above three equilibria. Our previous discussion reveals that advertising costs are higher in equilibrium under CNCPs than under CCPs, and the latter are higher than under perfect price information. The same is true for both the regular prices and the rebates, while the opposite holds for advertising intensities. As a result, sellers revenues (and hence profits) are not easily comparable across equilibria. As the following proposition however establishes, equilibrium profits are the highest under CNCPs, followed by those under CCPs and the lowest under perfect price information for a common in the literature family of cost functions.

Proposition 3 *If* $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$, *then equilibrium profits are higher* *under CNCPs than under CCPs, and those are higher than under perfect price information.*

The intuition is that as the information available in the market increases, sellers face stronger price competition, and thus typically obtain lower profits. Note however, that as the market becomes more informed, there is also less need to spend on price ads. Finally, the equilibrium profits under CCPs tend to those under perfect information for sufficiently small marginal costs of advertising, while the equilibrium profits under CNCPs are always higher than under CCPs.

5 Robustness of the Equilibria with CCPs and with CNCPs.

So far we have assumed that sellers can promote their sales either by sending out CCPs or CNCPs. Proposition 3 says that when sellers send out CNCPs typically obtain higher profits than when they send out CCPs. A seller is however free to respond to his rival's marketing strategy by issuing any one of these two types of coupons, or even by issuing both types at the same moment. Under this light, it is reasonable to ask ourselves: Do the equilibria with CNCPs and CCPs survive if sellers can choose the type (or the mix of types) of coupons they issue to promote their sales as well?

We first examine the robustness of the equilibrium with CNCPs. Assuming that his rival follows the equilibrium marketing strategy (Proposition 2), does a seller have an incentive to deviate and send out CCPs instead of CNCPs? Obviously, a seller can gain by such a deviation only if he charges a regular price lower than the equilibrium price. (By charging and communicating a higher price, he is unable to attract additional consumers). It turns out that if the seller substitutes all CNCPs with CCPs, and adjusts properly his regular price and rebate, he can always increase his profits. Therefore, as the proposition 4 states, the equilibrium with CNCPs is never robust.

Proposition 4 *The equilibrium with CNCPs is no longer an equilibrium if sellers are allowed to choose the type of coupons they issue to promote their sales.*

The intuition is that sellers find themselves in a Prisoners' Dilemma. Both sellers would earn higher profits if they could only send out CNCPs. However, an individual seller, by lowering his price and informing consumers at the rival's location about this reduction, is able to attract more customers and thus increase his profits. Of course, this can be achieved by sending out CCPs instead of CNCPs.

We next study the robustness of the equilibrium with CCPs. Assuming that his rival follows the equilibrium marketing strategy (Proposition 1), does a seller have an incentive to deviate by sending out some CNCPs as well?. Obviously, such a deviation can only be profitable if the seller raises his regular price. Let $p_A(> p)$ be the regular price charged by the deviating seller A. Let r_A be the rebate offered by a CCP, and r'_A the rebate on a CNCP. When studying the profitability of such a deviation one however has to be careful about the out of equilibrium expectations of those consumers receiving a CNCP, p^e_A. This is because consumers expect in equilibrium to receive either a coupon communicating the price or nothing at all. Some consumers, to their surprise, receive coupons not communicating the regular price and also offering a rebate different than the equilibrium rebate. Rational consumers, however, know that a deviating seller A will set its rebate on the CNCPs in an optimal way, for any regular price chosen. Hence, those consumers observing a rebate that do not expect on a coupon which does not communicate the price, should infer that seller A deviates by sending CNCPs. Moreover, they should form out of equilibrium beliefs which are rational, that is $p^e_A = p_A$. In fact, these expectations are the only "consistent" beliefs based on their information: A consumer in region B receiving a CNCP observes r'_A, knows the equilibrium price p, and also knows that seller A will set optimally its regular price p_A,taking consumers expectations p^e_A as given, such that $r'_A = 0.5(p_A + p^e_A - p)$ (from the f.o.c. of (27) with respect to r'_A). Conducting then a simple thought experiment, this consumer should be able to infer the only price of seller A which is consistent with his beliefs (see also footnote (19)). This, in turn, implies that consumers receiving a CNCP behave in the same way as consumers receiving a CCP. Hence, if seller A did not have an incentive to deviate by raising his price in the equilibrium with CCPs, he does not have an incentive either to increase its price when sending out CNCPs as well. Therefore, this deviation cannot be profitable.[14]

Proposition 5 *The equilibrium with CCPs remains robust when sellers are also able to choose the type of coupons they use to promote their sales.*

[14]This result also holds for other out of equilibrium expectations. For instance, the equilibrium with CCPs is robust if consumers receiving a CNCP are rather "pessimistic", i.e. they believe that seller A charges the equilibrium price under CNCPs. On the other hand, if these consumers are "optimistic", i.e. they believe that seller A did not print the price on the coupon by mistake (even after observing a different than the equilibrium rebate), then the equilibrium with CCPs is robust under mild convexity conditions on the advertising cost function.

In summary, since the equilibrium with CNCPs is not robust when sellers can send out CCPs, while the equilibrium with CCPs is found to be robust to the introduction of CNCPs, our model predicts that CNCPs should not be observed in locationally-informationally segmented markets.

6 Comparative statics.

Figure 1 allow us to derive some comparative statics results for the robust equilibrium with CCPs[15]. We first study how an increase in the degree of product differentiation affects the equilibrium outcome. Product differentiation in our model is related to transportations costs. An increase in the transportation costs, $\overline{s}$, shifts the PP schedule to the right and the KK schedule to the left in Figure 1. As a result, the equilibrium price is unambiguously raised. This is well known in the literature on product differentiation: competition can be relaxed through product differentiation (see Shaked and Sutton (1982)). The equilibrium couponing intensity increases with the degree of product differentiation, too. By substituting equation (2) into equation (4) we obtain that λ has to satisfy $\overline{s} = 9\lambda^2 k'(\lambda)$, and thus λ increases with $\overline{s}$ (since $k''(\lambda) > 0$). The equilibrium redemption rate, however, decreases with the degree of product differentiation. Since only those consumers whose transportation costs are less or equal than the rebate r will redeem their coupons, the redemption rate equals to $r/\overline{s} = p/2\overline{s} = 1/3\lambda$ [16]. Therefore, although more consumers receive a coupon, a smaller percentage of them redeems it.

Finally, equilibrium profits are equal to $\Pi = p - \lambda p^2/4\overline{s} - k(\lambda) = 5\lambda k'(\lambda) - k(\lambda)$ (as $p = 6\lambda k'(\lambda)$ if we substitute (2) into (4) and also using (4)). Since advertising intensity increases with the degree of product differentiation and $\partial\Pi/\partial\lambda = 4k'(\lambda) + 5\lambda k''(\lambda) > 0$, we conclude that sellers profits are larger, the larger is the degree of product differentiation. The intuition is as follows. As $\overline{s}$ increases, it becomes more difficult to attract consumers from the distant location. Then competition becomes softer and firms can charge higher prices. On the other hand, firms have to increase their advertising efforts to motivate consumers to switch stores. Contrarily to the first effect, the latter fosters competition. However, the first effect dominates the second and, as a result, sellers profits increase.

Next, we study how an increase in the marginal costs of advertising affects

[15] Note that similar comparative statics results can be obtained for the non-robust equilibrium with CNCPs.

[16] Note that such redemption rate is well defined if $\lambda^* \in \left[\frac{1}{3}, 1\right]$. This fact is guaranteed if we assume that $2\overline{s} > v$.

the equilibrium outcome. An increase in the marginal costs, $k'(\lambda)$, shifts the KK schedule to the left, while the PP schedule remains unchanged (Figure 1). This unambiguously raises the equilibrium price and decreases the advertising intensity. In fact, if the marginal costs of advertising are prohibitively high, coupons will not be issued at all, and sellers can extract all the local consumers surplus by charging their monopoly price v[17]. Further, the equilibrium redemption rate increases with the marginal costs of couponing. Even though less coupons are send out, a higher percentage of them is redeemed because they offer a higher rebate. Finally, to study the effects of an increase in the marginal costs of advertising on sellers' profits, we consider the family of costs functions $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$. Our model confirms the well-known result in the informative advertising literature: Seller's profits are higher when the marginal costs of couponing are higher[18].While an increase in the marginal costs, and thus in the costs of couponing, has a direct negative effect on profits, there is also a positive strategic effect: sellers reduce their advertising intensity and, as a result, competition is relaxed. The strategic effect dominates the direct effect, and thus sellers profits increase when couponing becomes more expensive. Thus, in line with the literature, sellers prefer advertising to be illegal in our model, too.

The following proposition summarizes our comparative statics results:

Proposition 6 *a) As the degree of product differentiation increases, equilibrium prices, rebates, advertising intensity and sellers' profits increase, while equilibrium redemption rates decrease. b) As the marginal cost of couponing increases, equilibrium prices, rebates and redemption rates increase, while equilibrium advertising intensity decreases. Further, equilibrium profits increase with m if the couponing costs are $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$.*

Finally, it is interesting to provide some welfare results for the equilibrium with CCPs. Total welfare is defined as the (unweighted) sum of consumer surplus and sellers profits. Since all consumers buy in equilibrium and production costs are zero, it is easy to see that total welfare is equal to gross consumer surplus minus sales promotion costs, i.e. $SW = 2(v - k(\lambda))$. Thus, (net) consumer surplus can be attained by subtracting sellers' profits $(10\lambda k'(\lambda) - 2k(\lambda))$ from the total welfare, i.e.:

$$CS = 2v - 10\lambda k'(\lambda) = 2v - 10\overline{s}/9\lambda \tag{9}$$

[17]This is similar to Bester and Petrakis (1995) where if the advertising costs are sufficiently high, firms do not advertise at all and set their local monopoly price in equilibrium.

[18]See Grossman and Shapiro (1984), Peters (1984), Tirole (1989) and Bester and Petrakis (1995).

(since by (2) and (4), $\lambda k'(\lambda) = \bar{s}/9\lambda$). Both consumer surplus and total welfare decrease with the degree of product differentiation, as equilibrium λ increases with $\bar{s}$. Since sales promotion costs are socially wasteful in our model, total welfare is higher when sellers spend less on advertising. Consumers surplus is also decreasing in $\bar{s}$, because the higher is the degree of product differentiation, the softer is the competition between firms and the higher are the prices charged. On the other hand, as marginal costs of couponing increase, consumer surplus decreases. Sellers not only charge higher prices now, but also send a smaller number of coupons to the distant location. Finally, to study how an increase in the marginal cost of couponing affects total welfare, we again consider the family of cost functions $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$. Proposition 7 summarizes the welfare results:

Proposition 7 *a) Equilibrium consumer surplus and total welfare are decreasing with the degree of product differentiation. b) Equilibrium consumer surplus decreases as the marginal cost of couponing increases. Further, total welfare decrease with* m, *if the advertising costs are of the type* $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$.

As both the marginal and the total costs of sales promotion increase, there is a direct negative effect on total welfare, and an indirect positive effect due to a less intense sales promotion activity. The direct effect, however, dominates and total welfare decreases with the parameter m.

7 Summary and conclusions

In a locationally segmented market, it is reasonable to assume that price information is imperfect in the sense that consumers can costlessly observe only the prices charged at their home location. To learn, however, the price at the distant location, they must either venture the distant location and thus incur a transportation cost, or receive price ads from the distant seller. In this market, coupons may enable sellers to compete for buyers at distant locations.

In this paper we have studied coupons as a sales promotions activity. Since the market is informationally segmented, two types of coupons can be issued at equal costs: (a) coupons communicating the price (CCPs) and (b) coupons not communicating the price (CNCPs). CCPs not only offer a rebate, but also convey price information, while CNCPs only offer a rebate. Then a CCP offering a zero rebate is equivalent to pure price advertising. We have characterized the unique symmetric pure strategy equilibria where sellers can send out only CCPs, or only CNCPs, and we have provided conditions for the existence of interior equilibria. In both equilibria with CCPs and CNCPs, sellers always offer positive rebates (equal to half the regular price) and send out a positive amount of coupons. The existence of a symmetric pure strategy equilibrium implies that, contrary to the literature, price promotions are permanent here. Further, sellers advertise their prices by sending out CCPs, which implies that pure price advertising should not be observed in a locationally segmented market (provided that pure price advertising and couponing costs are of similar magnitude)

Since CNCPs only offer a rebate, they do not spread price information. As a result, equilibrium prices, advertising efforts and sellers profits are higher in the equilibrium with CNCPs than with the CCPs. Thus, sellers would be better off if they could only send out CNCPs. However, the equilibrium with CNCPs is not robust. When sellers are allowed to choose the type of coupons to issue as well, they have always an incentive to deviate from their marketing strategy in the "equilibrium" with CNCPs and send out CCPs instead of CNCPs. In fact, sellers found themselves into a Prisoners' Dilemma situation. In contrast, the equilibrium with CCPs has been shown to be always robust. Thus, our model predicts that price discrimination should take place by sending out CCPs instead of CNCPs whenever price information is imperfect in a locationally segmented market.

CNCPs are however observed in many markets. One might argue that the reason is that sellers are colluding in an infinitely repeated market interaction. However, this does not seem to be a reasonable explanation, as if sellers were able to collude in order to avoid CCPs, they must also be able to collude to avoid coupons whatsoever. In the latter case, sellers are local monopolists and thus attain the highest possible profits. Another argument might be that sellers use CNCPs because consumers have perfect price information as in Bester and Petrakis (1996). However, as we argued above, this does not seem too plausible in locationally segmented markets. Some alternative explanation is thus needed to accommodate this observation. One could introduce some asymmetries into our model. For instance, allowing for different consumers willingness to pay for the good sold in each location, or for different transportation costs, one might obtain richer optimal sellers marketing strategies. Another possibility would be to build an extended model with vertical market relations and study whether manufacturers or retailers have incentives to promote their sales and which type of coupons is more likely to be send out from each of the parties.

Appendix

Proof of proposition 1: We first claim that an interior equilibrium has to satisfy equations (2) -(4). Assume, for the moment, that consumers not getting a coupon believe that the price charged at the distant location is not lower than that at their home location, i.e. $p_j^e \geq p_i$; also that $p_j \geq (p_i - r_i)$, $i, j = A, B$. We shall later show that these assumptions are satisfied in equilibrium. Then firm A's decision problem is: (by symmetry, firm B's decision problem is analogous):

$$\underset{p_A, r_A, \lambda_A}{Max} \; [p_A(1-\lambda_B) + p_A\lambda_B(1-(p_A-(p_B-r_B))/\bar{s}) +$$

$$+\lambda_A(p_A - r_A)(p_B - (p_A - r_A))/\bar{s} - k(\lambda_A)] \tag{10}$$

First order conditions are given by the following three equations:

$$1 - \lambda_B(2p_A - (p_B - r_B))/\bar{s} + \lambda_A(p_B - 2(p_A - r_A))/\bar{s} = 0 \tag{11}$$

$$-\lambda_A(p_B - 2(p_A - r_A))/\bar{s} = 0 \tag{12}$$

$$(p_A - r_A)(p_B - (p_A - r_A))/\bar{s} - k'(\lambda_A) = 0 \tag{13}$$

Imposing symmetry, that is, $p_A = p_B = p^*$, $r_A = r_B = r^*$, and $\lambda_A = \lambda_B = \lambda^*$, and using the rational expectations hypothesis, that is, $p_A^e = p_A$, $p_B^e = p_B$, we obtain the system of equations (2)-(4). Note that all the assumptions made above are satisfied in equilibrium. In fact, $p_B^e \geq p_A$, $p_A^e \geq p_B$, $p_A \geq p_B - r_B$ and $p_B \geq p_A - r_A$. Moreover, it can be checked that the second order conditions are satisfied.

Next, we show that the system of equations (2)-(4) has a unique interior solution if and only if $k'(1) > \bar{s}/9$ and $k'(2\bar{s}/3v) < v^2/4\bar{s}$. Equation (2) defines $p_1(\lambda) = 2\bar{s}/3\lambda$ with $p_1'(\lambda) < 0$, $p_1(0) = \infty$ and $p_1(1) = 2\bar{s}/3$ (PP curve in Figure 1). Equation (4) defines $p_2(\lambda) = (4\bar{s}k'(\lambda))^{0.5}$, with $p_2'(\lambda) > 0$, $p_2(0) < \infty$ and $p_2(1) = (4\bar{s}k'(1))^{0.5}$ (KK curve in Figure 1). From Figure 1 it can be checked that $p_1(\lambda)$ and $p_2(\lambda)$ intersect at an interior point such that $p \leq v$ if and only if $k'(1) > \bar{s}/9$ and $k'(2\bar{s}/3v) < v^2/4\bar{s}$.

Finally, we claim that no firm has an incentive to deviate from the proposed equilibrium. Assume that firm B follows the equilibrium strategy (p, r, λ). Note that if firm A deviates by changing its strategy, consumers not receiving a coupon at location B will still remain uninformed about A's price and thus have no reason to change their expectations. Of course, uninformed consumers at location A have no reason either to change their expectations. We first check whether firm A has an incentive to use an alternative strategy consisting of lowering its price together with some rebate and advertising effort.

There are two cases to consider. First, profits from a deviation such that $0.5p \leq p_A \leq p$ are given by:

$$\Pi_A = p_A(1-\lambda) + p_A\lambda\left(1 - (p_A - 0.5p)/\overline{s}\right) +$$

$$+\lambda_A(p_A - r_A)(p - (p_A - r_A))/\overline{s} - k(\lambda_A) \tag{14}$$

Since these profits are the same as those of equation (10), it is obvious that firm A cannot gain by charging p_A. Second, if firm A charges $p_A < 0.5p$ obtains profits:

$$\Pi_A = p_A + \lambda_A(p_A - r_A)(p - (p_A - r_A))/\overline{s} - k(\lambda_A) \tag{15}$$

The first order condition $\partial\Pi_A/\partial r_A = 0$ implies that firm A would optimally set $(p_A - r_A) = 0.5p$. Using this, the first order condition $\partial\Pi_A/\partial\lambda_A = 0$ reduces to $p^2/4\overline{s} = k'(\lambda_A)$. Therefore, firm A would optimally choose $\lambda_A = \lambda$. To complete the argument, note that $\partial\Pi_A/\partial p_A = 1 > 0$,i.e. profits decrease as the price decreases. Thus, firm A would optimally set $p_A = 0.5p$. But this cannot be a profitable deviation as it has been demonstrated above. Thus, firm A cannot gain by lowering its price.

Consider, next, that firm A deviates by raising its price together with some rebate and advertising effort. Profits from such a deviation are given by

$$\Pi_A = p_A(1-\lambda)\left(1 - (p_A - p)/\overline{s}\right) + p_A\lambda\left(1 - (p_A - 0.5p)/\overline{s}\right) +$$

$$+\lambda_A(p_A - r_A)(p - (p_A - r_A))/\overline{s} - k(\lambda_A) \tag{16}$$

From the first order condition $\partial\Pi_A/\partial r_A = 0$, we get that firm A would optimally set $(p_A - r_A) = 0.5p$. Using this, the first order condition $\partial\Pi_A/\partial\lambda_A = 0$ reduces to $p^2/4\overline{s} = k'(\lambda_A)$. Again, firm A would optimally choose $\lambda_A = \lambda$. It can be checked that $\partial\Pi_A/\partial p_A = (p(1+\lambda) - 2p_A)/\overline{s} < 0$, i.e. profits decrease as the price charged increases. As a result, firm A cannot gain by raising its price. The proof is now complete. ■

Proof of proposition 2: The proof is analogous to the proof of proposition 1. Assume, for the moment, that consumers believe that the price charged at the distant location is not lower than the price charged at their home location, i.e. $p_i^e \geq p_j$; also that $p_i \geq p_j^e - r_j$, $i, j = A, B$. We will later show that these assumptions are satisfied in equilibrium. Firm A's maximization problem can then be written as (firm B's decision problem is analogous):

$$\underset{p_A, r_A, \lambda_A}{Max} \; [p_A(1-\lambda_B) + p_A\lambda_B(\overline{s} - (p_A - (p_B^e - r_B))/\overline{s}) +$$

$$+(p_A - r_A)\lambda_A(p_B - (p_A^e - r_A))/\overline{s} - k(\lambda_A)] \qquad (17)$$

First order conditions are given by the following three equations:

$$1 - \lambda_B(2p_A - (p_B^e - r_B))/\overline{s} + \lambda_A(p_B - 2(p_A^e - r_A))/\overline{s} = 0 \qquad (18)$$

$$-\lambda_A(p_B - p_A - p_A^e + 2r_A)/\overline{s} = 0 \qquad (19)$$

$$(p_A - r_A)(p_B - (p_A^e - r_A))/\overline{s} - k'(\lambda_A) = 0 \qquad (20)$$

Imposing symmetry and using rational expectations we obtain the system of equations (6)-(8). Note that all the assumptions made above are satisfied.

Next, we show that the system of equations (6)-(8) has a unique interior solution if and only if $k'(1) > \overline{s}/4$ and $k'(\overline{s}/v) < v^2/4\overline{s}$. Equation (6) defines $p_3(\lambda) = \overline{s}/\lambda$ with $p_3'(\lambda) < 0$, $p_3(0) = \infty$ and $p_3(1) = \overline{s}$ ($P'P'$ curve in Figure 1). Equation (8) is the same as equation (4) ($p_2(\lambda)$ and KK curve in Figure 1). From Figure 1 it can be checked that $p_3(\lambda)$ and $p_2(\lambda)$ intersect at an interior point where $p \leq v$ if and only if $k'(1) > \overline{s}/4$ and $k'(\overline{s}/v) < v^2/4\overline{s}$.

Finally, we show that none of the firms has an incentive to deviate from the proposed equilibrium. Assume that firm B uses the equilibrium strategy (p, r, λ) given by (6)-(8), while seller A deviates by choosing a $p_A \neq p$. It is here crucial to carefully specify the expectations of the consumers located at B after seller A's deviation. Consumers at location B not receiving a coupon do not observe anything new, thus their expectations remain the same. However, those consumers receiving a coupon should change their expectations as long as they observe a rebate different than the equilibrium one, i.e. $r_A \neq 0.5p$. In fact, consumers know that seller A will set his rebate in an optimal way for any regular price p_A, taking as given consumers expectations, p_A^e. From (19), the optimal rebate must satisfy $r_A = 0.5(p_A + p_A^e - p)$. Conducting a simple thought experiment, consumers are able to infer the regular price charged by seller A[19]. This demonstrates that out of equilibrium expectations of those consumers receiving a coupon must be rational, i.e. $p_A^e = p_A$.

Consider first that firm A deviates by lowering its price such that $0.5p \leq p_A < p$. Profits from such a deviation are given by:

[19] Assume that $p = 10$ and $r = 5$ in the equilibrium with CNCPs. Assume, further, that consumers at B receive a coupon with $r_A = 6$. If, for instance, they form expectations $p_A^e = 12$, then consumers, using equation (19), can infer that p_A must be equal to 10. Thus, their expectations cannot be correct. The only beliefs consistent on their information are then $p_A^e = 11$. In fact, (19) implies that $p_A = 11$, thus satisfying $p_A^e = p_A$.

$$\Pi_A = p_A(1-\lambda) + p_A\lambda\,(1-(p_A-0.5p)/\overline{s}) +$$

$$+\lambda_A(p_A-r_A)(p-(p_A^e-r_A))/\overline{s} - k(\lambda_A) \qquad (21)$$

From our previous argument, $p_A^e = p_A$; hence, the optimal rebate must satisfy $0.5p = (p_A - r_A)$. Further, the advertising effort must satisfy $\partial\Pi_A/\partial\lambda_A = p^2/4\overline{s} - k'(\lambda_A) = 0$, which implies $\lambda_A = \lambda$. As a result, $\partial\Pi_A/\partial p_A = 2\lambda(p - p_A)/\overline{s} > 0$, that is, profits decrease as p_A decreases. Hence, seller A does not gain by such a deviation.

Suppose next, that firm A deviates by lowering its price such that $p_A < 0.5p$. Profits are then:

$$\Pi_A = p_A + (p_A - r_A)\lambda_A(p_B - (p_A^e - r_A))/\overline{s} - k(\lambda_A) \qquad (22)$$

As above, p_A^e must be equal to p_A. Therefore, r_A must again satisfy $0.5p = (p_A - r_A)$. Also, $\partial\Pi_A/\partial\lambda_A = 0$ implies that $\lambda_A = \lambda$. As a result, $\partial\Pi_A/\partial p_A = 1.5 > 0$. Therefore, firm A would optimally choose $p_A = 0.5p$, a deviation which has already been shown not to be profitable.

Finally, consider a deviation where firm A raises its price, $p_A > p$. Profits are then given by:

$$\Pi_A = p_A(1-\lambda_B) + p_A\lambda_B\,(\overline{s} - (p_A - (p_B^e - r_B))/\overline{s}) +$$

$$+(p_A - r_A)\lambda_A(p_B - (p_A^e - r_A))/\overline{s} - k(\lambda_A) \qquad (23)$$

As before, $p_A^e = p_A$. Note that profits are the same as in the case where firm A deviates by choosing $0.5 \leq p_A < p$. Thus, the optimal rebate and advertising effort must satisfy $0.5p = (p_A - r_A)$ and $\lambda_A = \lambda$ and the derivative $\partial\Pi_A/\partial p_A$ must equal $2\lambda(p - p_A)/\overline{s} < 0$. This proves that firm A has no interest in raising its price. The proof is now complete. ■

Proof of proposition 3: Let us denote the optimal advertising efforts under CCPs, CNCPs and perfect price information as λ, λ', λ'', respectively. First, we compare profits under CCPs with those under CNCPs. Using (2)-(4), equilibrium profits under CCPs can be written as $\Pi = 5\lambda k'(\lambda) - k(\lambda)$. Also, using (6)-(8), equilibrium profits under CNCPs are $\Pi' = 3\lambda' k'(\lambda') - k(\lambda')$. For the family of cost functions $k(\lambda) = m\lambda^\alpha$, $\alpha \geq 3$, we have $\Pi' > \Pi \Leftrightarrow \lambda'^\alpha(3\alpha - 1) > \lambda^\alpha(5\alpha - 1) \Leftrightarrow$

$$\lambda'/\lambda > ((5\alpha - 1)/(3\alpha - 1))^{1/\alpha} \qquad (24)$$

From (2)-(4), λ must satisfy $\overline{s} = 9\lambda^2 k'(\lambda)$. Similarly, from (6)-(8), λ' must satisfy $\overline{s} = 4\lambda'^2 k'(\lambda')$. Thus,

$$9\lambda^2 k'(\lambda) = 4\lambda'^2 k'(\lambda') \tag{25}$$

This equation relates the optimal efforts under CCPs and CNCPs. For the particular family of cost functions, one has $\lambda'/\lambda = (9/4)^{\alpha/(1+\alpha)}$. Then by (24), $\Pi' > \Pi \Leftrightarrow 2.25 > ((5\alpha - 1)/(3\alpha - 1))^{(1+\alpha)/\alpha}$. This inequality is clearly satisfied for the case $\alpha = 3$. Since the right hand of the inequality is decreasing in α, it can be checked that $\Pi' > \Pi$ for all $\alpha \geq 3$.

Second, we compare profits under CCPs and under perfect price information. The latter are given by $\Pi'' = (4 + \lambda'')k'(\lambda'') - k(\lambda'')$ (see Bester and Petrakis (1996)). By using a similar procedure as before, and given that λ'' satisfies $\overline{s} = 4(1 + 0.5\lambda'')^2 k'(\lambda'')$ under perfect price information, one obtains that $\Pi \geq \Pi''$ if and only if:

$$(5\alpha - 1)(2 + \lambda)^{\frac{2\alpha}{\alpha+1}} \lambda^{\frac{\alpha(\alpha-1)}{\alpha+1}} - 3^{\frac{2\alpha}{\alpha+1}}(\alpha(4 + \lambda)\lambda^{\alpha-1} - \lambda^{\alpha}) \geq 0 \tag{26}$$

for all $\alpha \geq 3$ and $0 < \lambda < 1$. It is clear that in the extreme case $\lambda = 1$, profits under CCPs and under perfect price information coincide. By plotting the left hand side of inequality (26) one can see that it is always positive. ■

Proof of proposition 4: The proof consists of proposing a deviation and showing that it is profitable for a firm. Suppose firm A deviates by adopting the following strategy: charges $p_A = 0.75p$, substitutes all its CNCPs with CCPs and offers a rebate $r_A = 0.25p$. Profits from a deviation where the price charged is lower than p are given by:

$$\begin{aligned} \Pi_A(\cdot) &= p_A(1 - \lambda) + p_A\lambda(\overline{s} - (p_A - 0.5p))/\overline{s} + \\ &+ (p_A - r_A)\lambda_A(p - (p_A - r_A))/\overline{s} - k(\lambda_A) \end{aligned} \tag{27}$$

Substituting the deviating strategy $(p_A, r_A, \lambda_A) = (0.75p, 0.25p, \lambda)$ in (27) and taking into account that the advertising effort takes place through CCPs instead of CNCPs, we obtain that $\Pi_A = 0.75p + 0.25\lambda k'(\lambda) - k(\lambda)$. It is clear that $\Pi_A > \Pi = p - \lambda k'(\lambda) - k(\lambda)$ because $p = 4\lambda k'(\lambda)$ (from equations (6)-(8)). ■

Proof of proposition 5: Let firm B follow the equilibrium strategy (p, r, λ). Assume that firm A deviates by sending out both CNCPs and CCPs. We allow for a quite flexible deviation: CNCPs and CCPs intensities will be denoted by γ_A and λ_A, respectively. Rebates offered by CNCPs and CCPs will be denoted by r'_A and r_A. Consider first a deviation where firm A raises its price. Profits are then given by:

$$\Pi_A = p_A(1 - \lambda)(1 - (p_A - p)/\overline{s}) + p_A\lambda(1 - (p_A - 0.5p)/\overline{s}) +$$

$$+\lambda_A(p_A-r_A)(p-(p_A-r_A))/\overline{s}+\gamma_A(p_A-r'_A)(p-(p^e_A-r'_A))/\overline{s}-k(\lambda_A+\gamma_A) \quad (28)$$

In the optimal deviation, it must be the case that $\partial\Pi_A/\partial r_A = 0$. Therefore, $p_A - r_A = 0.5p$. Analogously, it must also be that $\partial\Pi_A/\partial r'_A = 0$. Hence, $r'_A = 0.5(p_A + p^e_A - p)$. Using these equalities, profits from raising the price reduce to:

$$\Pi_A = p_A(1-\lambda)(1-(p_A-p)/\overline{s}) + p_A\lambda(1-(p_A-0.5p)/\overline{s})+$$

$$+\lambda_A p^2/4\overline{s} + \gamma_A(p+p_A-p^e_A)^2/4\overline{s} - k(\lambda_A+\gamma_A) \quad (29)$$

The first order conditions are:

$$\frac{\partial\Pi_A}{\partial\lambda_A} = \frac{p^2}{4\overline{s}} - k'(\lambda_A+\gamma_A) = 0 \quad (30)$$

$$\frac{\partial\Pi_A}{\partial\gamma_A} = \frac{(p+p_A-p^e_A)^2}{4\overline{s}} - k'(\lambda_A+\gamma_A) = 0 \quad (31)$$

$$\frac{\partial\Pi_A}{\partial p_A} = 1-(1-\lambda)\frac{(2p_A-p)}{\overline{s}} - \lambda\frac{(2p_A-0.5p)}{\overline{s}} + \gamma_A\frac{(p+p_A-p^e_A)}{2\overline{s}} = 0 \quad (32)$$

The same argument as in proposition 2 shows that the only consistent out of equilibrium expectations for the consumers at location B receiving a CNCP are $p^e_A = p_A$. Using this, it can be checked from (30) and (31) that $\lambda_A+\gamma_A = \lambda$. Further, note from (32) that the marginal revenue from a CNCP is positive, while the marginal revenue from a CCP is zero. Hence, the best for firm A is to issue only CNCPs, i.e. $\gamma_A = \lambda$ and $\lambda_A = 0$. Then from equation (32), we get $p_A = 0.5(p+\overline{s})$ and optimal profits given by (29) can be reduced to $\Pi_A = 0.25(p+\overline{s})(p(1-\lambda)+\overline{s})/\overline{s} - \lambda k'(\lambda) - k(\lambda)$. These profits are higher than equilibrium profits whenever $0.25(p+\overline{s})(p(1-\lambda)+\overline{s})/\overline{s} > p$. Rearranging and using (2), the latter expression is equivalent to $3\lambda^2 - 16\lambda + 4 > 0$. Then seller A's profits from the deviation are higher when $\lambda < 0.262965$. However, given $p_A = 0.5(p+\overline{s})$ and (2)-(4), $p_A > p$ if and only if $\lambda > 2/3 > 0.262965$. Therefore, seller A cannot increase its profits by raising its price and sending out CNCPs as well.

Consider next a deviation where firm A lowers its price. Assume that $0.5p \leq p_A < p$. Profits from such a deviation are given by:

$$\Pi_A = p_A(1-\lambda) + p_A\lambda(1-(p_A-0.5p)/\overline{s}) + \lambda_A(p_A-r_A)(p-(p_A-r_A))/\overline{s}$$

$$+\gamma_A(p_A-r'_A)(p-(p^e_A-r'_A))/\overline{s} - k(\lambda_A+\gamma_A) \quad (33)$$

Clearly, $\partial\Pi_A/\partial r_A = 0$ implies $p_A - r_A = 0.5p$. Further, $\partial\Pi_A/\partial r'_A = 0$ implies that $r'_A = 0.5(p_A + p^e_A - p)$. Again, $p^e_A = p_A$. Therefore, it must be the case that $p_A - r'_A = 0.5p$. Thus, the derivative $\partial\Pi_A/\partial p_A$ reduces to $1 - 2\lambda(p_A - 0.5p)/\overline{s} + 0.5\gamma_A p/\overline{s}$, which is larger than 0 for all γ_A and p_A (as $1 - 2\lambda(p_A - 0.5p)/\overline{s} > 0$). Thus, profits decrease in p_A and hence firm A cannot increase its profits by such a deviation. Assume now that $p_A \leq 0.5p$. Then, profits are given by

$$\Pi_A = p_A + \lambda_A(p_A - r_A)(p - (p_A - r_A))/\overline{s}$$

$$+\gamma_A(p_A - r'_A)(p - (p^e_A - r'_A))/\overline{s} - k(\lambda_A + \gamma_A) \tag{34}$$

As before it must be that $p_A - r_A = 0.5p$ and $p_A - r'_A = 0.5p$. Then, the derivative $\partial\Pi_A/\partial p_A = 1 + 0.5\gamma_A p/\overline{s} > 0$ for all γ_A. Thus, profits decrease in p_A, hence firm A cannot gain by lowering its price and sending out CNCPs. The proof is now complete. ■

Proof of proposition 6: It only remains to prove that profits are decreasing in m. For $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$, the equilibrium advertising intensity is $\lambda = (\overline{s}/9\alpha m)^{(1/1+\alpha)}$. Profits then reduce to $\Pi = m^{1/\alpha+1}(5\alpha - 1)(\overline{s}/9\alpha)^{\alpha/\alpha+1}$. It can be checked that this function is increasing in m. ■

Proof of proposition 7: It only remains to prove that social welfare is decreasing in m. For $k(\lambda) = m\lambda^{\alpha}$, $\alpha \geq 3$, social welfare reduces to $SW = 2v - 2m(\overline{s}/9\alpha m)^{(\alpha/1+\alpha)}$ (using the optimal λ). It can be checked that this function is decreasing in m. ■

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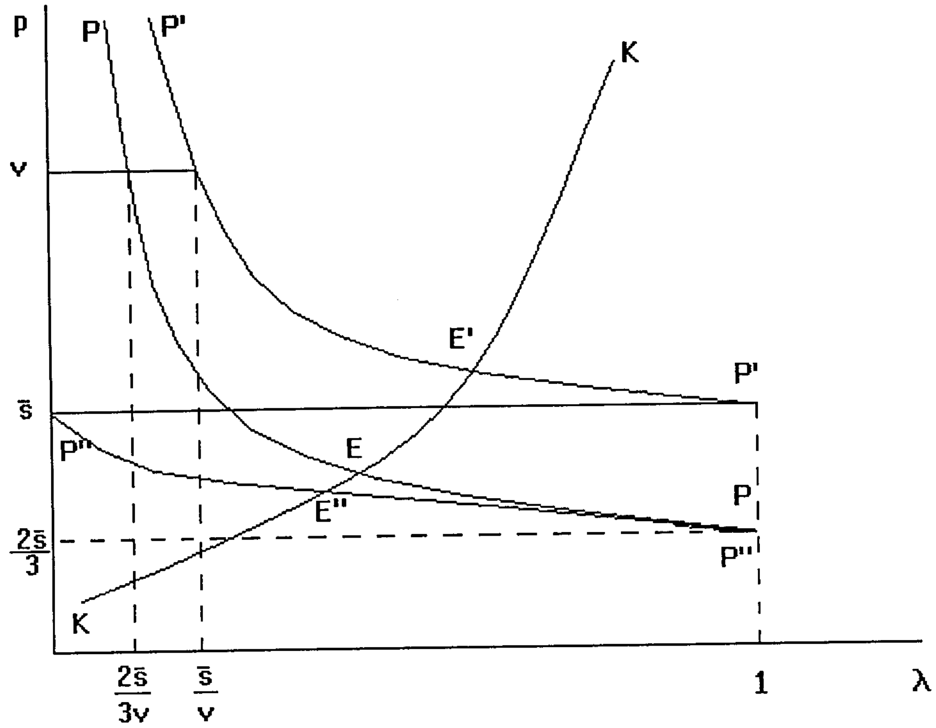

FIGURE 1