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QUALITY UNCERTAINTY AND INFORMATIVE ADVERTISING

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Abstract

We consider a single period model where a monopolist introduces a product of uncertain quality. Before pricing and informative advertising decisions take place, the producer observes the true quality of the good while consumers receive an independent signal which is correlated with the true quality of the product. We show that if advertising occurs in equilibrium, there must exist some pooling. We then characterize the constellations of parameters for which advertising occurs in equilibrium: For an advertising full pooling equilibrium to exist, (a) the consumers' valuation for the high quality must be high enough, (b) the informativeness of the market signal must be sufficiently low, (c) the costs of advertising must be high enough and (d) the consumers' priori probability of high quality must be sufficiently high. Existence of an advertising semi-separating equilibrium also requires the three first conditions but, in contrast, the consumers' a priori probability of high quality cannot be too large. When advertising occurs in equilibrium, the adverse selection problem is mitigated. Moreover, the lower are advertising costs, the more intense is the alleviation of that problem.

Keywords: Informative Advertising, Quality Uncertainty, Signaling

* This work was started while I was visiting Free University of Berlin. I am very grateful for the hospitality of its Department of Economics. Specially, I am in debt with Helmut Bester for motivating this research to me and for his posterior advise. All errors are of my exclusive responsibility. Address for correspondence: Departamento de Economía, Universidad Carlos III de Madrid. Calle Madrid, 126; 28903 Getafe, Madrid, Spain. Tel.: ++34 1 624 97 54. FAX.: ++34 1 624 98 75. E-mail: morgonjo@eco.uc3m.es.

1 Introduction

In environments where the quality of the products is unobservable by consumers before they purchase the good (*experience goods*), a substantial amount of advertising is observed.[1] Even though it is not very clear whether the majority of advertising provides direct or indirect information about the quality of the products, the Industrial Organization literature has considered most of these advertising expenditures as directly uninformative. The underlying idea has formerly been proposed by Nelson (1974). In his seminal article, he suggests that advertising of experience goods cannot convey much direct information about their quality. He argues that, since experience qualities are unverifiable before their purchase, sellers' advertisements claiming that they are offering a higher quality product can be misleading and, thus, consumers will disregard them. As a result, he concludes that direct informative advertising of experience goods cannot abound. From his view, however, there are still reasons to observe uninformative advertising as it may indirectly be informative when there exist market mechanisms that positively relate product quality and advertising outlays. Nelson's ideas have more formally been developed by Milgrom and Roberts (1986) and Kihlstrom and Riordan (1984).[2]

In a recent empirical research, however, Caves and Greene (1996) find that quality signaling is not the function of most of the advertising of consumption goods. In contrast, their findings are consistent with advertising that provides verifiable information which buyers learn when products have better features or capabilities. Following this view, we present an advertising model where the quality of the products is unobservable and where sellers can provide verifiable information about their qualities. One possible manner through which sellers can provide such an information is by distributing *free samples* of their products. This quality advertising mechanism has been overlooked by the literature on Industrial Organization. Marketing researchers define free samples as small portions of a newly introduced good that are made available to consumers with the purpose of

[1] *Experience* and *search goods* have formerly been defined by Nelson (1970, 1974). The quality of experience goods can only be ascertained after consumers purchase them while the quality of search goods is learned by consumers upon a simple observation of the products.

[2] Milgrom and Roberts (1986) present a signaling model where the seller chooses both the price and the level of uninformative advertising at the time to introduce an unobservable quality product. In equilibrium, both variables may simultaneously be used as *signals* of quality, that is, consumers can infer the true quality of the product upon the observation of either the price charged and/or the volume of advertising. On the other hand, Kihlstrom and Riordan (1984) offer an alternative explanation for introductory uninformative advertising expenditures. In their model advertising functions as an entry fee into the market for high quality products. Firms not investing in advertising are never considered to produce high qualities.

demonstrating the value of the product. Even though distributing free samples is the most expensive way to introduce a product, it has been shown to be the most effective manner when sellers are very reliable on their products' characteristics.[3] Nevertheless, our model also allows for different advertising mechanisms that provide verifiable information on products' quality such as *point-of-sale* or *point-of-purchase demonstrations.*

We develop an adverse selection model where a monopolist introduces a good whose unobservable quality can be either high or low. Before making any price or advertising decision, the producer observes the true quality of the product and all consumers receive an independent market signal which is positively correlated with the true quality.[4] Then the seller simultaneously sets the price and the informative advertising intensity. Naturally, advertising is costly for the producer. It is assumed that all buyers observe the price charged but, in contrast, the advertising effort is not observable. An individual consumer only observes whether he has received a free sample or not. Hence, only prices may function as signals of quality in our model. Consumers are fully rational: those consumers who receive an advertisement ascertain the true quality of the product and disregard any other signal received while the rest of consumers decide to buy or not taking into consideration the price, the signal observed and also the fact that they have not received a free sample.

We first show that if informative advertising occurs in equilibrium, there must exist some pooling. Indeed, in any separating equilibrium, advertising cannot exist. As it is well known, in a separating equilibrium prices signal quality.[5] In such a case, after observing the price, consumers perfectly learn the true quality of the product and, therefore, advertising expenditures are completely unnecessary. This is in contrast with models where uninformative advertising functions as a quality signal: Milgrom and Roberts (1986) show that uninformative advertising may contribute to the signaling role of prices to achieve separation at minimal cost. In Kihlstrom and Riordan (1984) uninformative advertising is the only manner to signal qualities because firms do not choose their prices. In these models, uninformative advertising expenditures may signal the quality of the products because all consumers observe the amount of advertising expenditures. Contrarily, in our model, some consumers become fully informed about the quality of the product after receiving the informative advertising but unin-

[3]Free samples are widely used to introduce beauty aids, cookies, cleaning products, etc. For instance, Lever Brothers successfully introduced its new Surf detergent by sending out more than 4 million free samples (see Kotler (1994)).

[4]Wolinsky (1983) introduces a similar market signal in his work on prices as signals of qualities. He argues that consumers obtain imperfect but costlessly information about qualities as a by-product of their shopping process.

[5]See Milgrom and Roberts (1986) and Bagwell and Riordan (1991) for models where quality is exogenously given. In Chan and Leland (1982), Wolinsky (1983), Rusell and Ross (1984, 1985) and Riordan (1986) prices signal quality choices.

formed consumers are unable to observe whether a firm advertises or not. Thus, informative advertising cannot function as a signal of quality.[6]

Secondly, we investigate the constellations of parameters for which advertising appears in both full pooling equilibria and partial pooling (or semi-separating) equilibria. It is shown that if the difference between the high and the low quality is not sufficiently large, informative advertising never occurs in any pooling equilibrium. Further, for an advertising full pooling equilibrium to exist, (a) the informativeness of the market signal must be low enough, (b) the consumers' prior probability of high quality must be sufficiently high and (c) the costs of advertising must be high enough. Existence of an advertising partial pooling equilibrium also requires (a) and (c) but, in contrast, the consumers' prior probability of high quality can not be too large. Interestingly, for some parameter constellations, partial and full pooling equilibria coexist. When parameters are such that a semi-separating equilibrium where the low quality seller charges a pooling price with sufficiently high probability exists, then, a higher price accompanied by higher advertising expenditures can also be sustained as a full pooling equilibrium. When the informativeness of the market signal decreases, the set of parameters for which both equilibria coexist vanishes.

It is finally worth noting that in our model some consumers become perfectly informed in a pooling equilibrium. The source of their information is advertising. This is also in contrast to those papers on uninformative advertising as a signal of quality, where consumers are only perfectly informed in a separating equilibrium (or exogeneously). Consumers may here learn the true quality from either the price (in a separating equilibrium) or from advertising (in a pooling equilibrium).[7] Interestingly, the quantity traded when advertising occurs in any type of pooling equilibrium is higher than when advertising is forbidden. As a result, informative advertising is here found to mitigate the adverse selection problem. Further, the lower are the advertising costs, the alleviation of that problem is more intense.[8]

The rest of the paper is organized as follows. Section 2 presents the model. Separating equilibria are analyzed in section 3. In section 4, we investigate full pooling equilibria with and without advertising. Partial pooling equilibria are studied in section 5. Finally, section 6 concludes. Some of the proofs have been relegated to an appendix.

[6] A similar insight can be found in Hertzendorf's (1993) paper, which extends Milgrom and Roberts's by introducing noisy advertising. When consumers cannot perfectly observe a firm's uninformative advertising expenditure, advertising cannot serve as a signal of quality any more. Then, signaling is an exclusive role of prices and, as a result, an advertising separating equilibrium does not exist.

[7] In regard to this, Vettas (1996) develops a similar model where consumers can also learn the true quality of the product from two alternative sources: from the price (if there is separation) and from word-of-mouth communication (if there is separation and/or pooling).

[8] Typically, in adverse selection models the quantity traded tends to be small (see e.g. Akerlof (1970)).

2 The model.

Consider a single period monopoly market where a new product of uncertain quality q is introduced. At the beginning of the trading period, only the producer observes the true quality of the product.[9] For simplicity, there are only two qualities of the good: high quality, q_h, and low quality, q_l. In what follows, the *monopolist when the quality is low* (*high*) will be referred to as the *low* (*high*) *quality seller.* The cost of producing one unit of the high quality good is $c > 0$ while the unitary cost of low quality is normalized to zero. Further, it is assumed that (a) $q_h - c > q_l$ and (b) $q_l - c > 0$. Assumption (a) means that producing the high quality good is socially more efficient. On the other hand, if assumption (b) were not satisfied, then the high quality seller would never mimic his low quality counterpart.

There is a large number of potential consumers whose mass is normalized to unity and each of whom will at most purchase one unit of the product. All buyers have identical reservation values for the products, which are q_h for the high quality and q_l for the low quality good. Prior to purchase and before the seller sets his marketing strategy (price and advertising intensity), none of the consumers is informed about the true quality of the product. Consumers' common prior belief for a high quality product is denoted by β and is common knowledge.

Before any pricing or advertising decision takes place, the producer observes the true quality of the product and all consumers receive an independent signal s about the actual quality. The signal can be either a signal of high (s_h) or low (s_l) quality and is such that:

$$\Pr\{s_h \mid q_h\} = \gamma; \;\; \Pr\{s_h \mid q_l\} = 1 - \gamma. \tag{1}$$

We assume that $\gamma > 0.5$ which implies that the signal s is positively correlated with the true quality. That is, if quality is actually high, the probability of receiving a signal of high quality is higher than the probability of receiving a signal of low quality. If quality is low then it is more likely to receive a signal of low quality than a signal of high quality. On the other hand, we assume that there is enough noise in the market so that the signal is not perfectly correlated with quality, that is $\gamma < 1$. One possible interpretation is that buyers read a number of different consumers reports or newspapers which announce that the product will be introduced into the market and that, on average, a fraction γ of them reports the expected quality of the product correctly. Thus, a fraction γ of the population would receive a correct signal about the actual quality while the rest of the buyers would receive wrong information.[10]

[9] In game theory, this is the typical Nature's movement. By assuming this, we are replacing the "incomplete" information game by a game of "complete" but "imperfect" information (see Harsanyi (1967, 1968)). The problem is thus modelled as one of adverse selection instead of moral hazard.

[10] A similar assumption is found in Wolinsky (1983). He argues that consumers prepurchase

The (high quality) seller can provide perfect information about the true quality through advertising activities.[11] Each consumer is equally likely to receive an advertisement (free sample) of the product through which he is able to ascertain its actual quality. Advertising is costly. For simplicity and computational convenience, we assume that costs of informing a fraction λ of the consumers are quadratic, i.e. $C(\lambda) = 0.5k\lambda^2$, $k > 0$. According to this specification, informing a larger fraction of buyers is more costly and, further, the advertising technology exhibits decreasing returns to scale. These features are standard in the literature on informative advertising.[12] Further, it is implicitly assumed that advertising costs do not depend on the quality advertised. This is reasonable since the low quality seller will never advertise in equilibrium. Furthermore, neither the total amount of money spent on advertising nor the advertising intensity are observable by consumers. Finally, to ensure that the seller's optimal advertising effort is always an interior solution to the corresponding problem, we assume that $k > q_h - c$.

Consumers and firm objectives are assumed to be as follows: consumers, basing their decisions on their quality expectations q_e, maximize their net surplus. On the other hand, the monopolist, taking as given consumers expectations, maximizes profits. Finally, we assume that all features of the model are common knowledge.

If information were complete, in equilibrium the low quality seller would charge $p_l^* = q_l$ and make profits of $\Pi_l^* = q_l$, while the high quality seller would set $p_h^* = q_h$ and obtain profits of $\Pi_h^* = q_h - c$. In what follows, we will refer to these prices and profits as the *optimal prices and profits under complete information.* As $q_h - c > q_l$ (see above) the high quality product is socially more efficient. Thus, under complete information, if the producer were able to choose the quality of the product he would select the high quality.

Under incomplete information, our model defines a signaling game. This game is however non-standard as the marketing strategy of the seller has two components: the price which is observable and the advertising effort which is not observable. Since those consumers not receiving a free sample (uninformed consumers) do not observe whether the seller advertises or not, it is only the price that may signal quality. As usual, we focus on separating, pooling and

observation discloses some information about the quality of the products. He assumes that in the course of a visit to a firm, a consumer gets a signal which depends on the true quality and also on random factors. He assumes however that there is a positive probability that the signal fully reveals the true quality of the product. We rule out this possibility by assuming $\gamma < 1$.

[11]It is obvious that only the high quality seller will have an incentive to spend resources in advertising.

[12]See e.g. Butters (1977) and Grossman and Shapiro (1984). The underlying idea is that an advertisement may fail to reach an uninformed buyer. For instance, if the high quality seller distributes a number m of free samples by inserting them in a number of newspapers or magazines, it is reasonable to think that less than m consumers will become fully informed (as some consumers can at a time buy more than one of these media).

partial pooling (or semi-separating) equilibria. In a separating equilibrium, the high and the low quality sellers choose different prices and, consumers, after observing the price, ascertain the true quality of the product. In contrast, in a full pooling equilibrium, both firms set the same price and consumers cannot ascertain the true quality using only this observation. This feature also appears in a partial pooling equilibrium where the high quality seller always sets the pooling price and the low quality seller randomizes between the pooling price and the optimal price that he would have set under complete information.[13] Since our focus is on advertising we will also distinguish between advertising and non-advertising equilibria. Here, as it is typical in signaling models, a large number of equilibria may arise. There may exist a large number of separating, pooling and partial pooling with or without advertising. The source of this multiplicity is the indeterminacy of the out-of-equilibrium consumers' beliefs. To restrict the class of equilibria we will use the *intuitive criterion.* Intuitively, a proposed equilibrium is intuitive if there does not exist another price, for which the high quality seller is better off while the low quality seller is worse off, when both are considered to sell the high quality product. If this price existed, consumers should correctly infer that only a high quality firm would charge such a price, which makes the high quality seller to profitably deviate by charging this price, and, as a result, the proposed equilibrium to fail.[14]

3 Separating equilibrium.

In a separating equilibrium the high quality and the low quality seller charge different prices. Thus, the price signals the true quality of the product. Let (p_l^*, p_h^*) be a separating equilibrium. Then consumers, after observing price p_l^* (p_h^*) ascertain that the quality is low (high). In other words, consumers, after observing the price charged, disregard any received market signal s and infer the true quality of the product. This characteristic of the separating equilibria allow us to conclude that advertising never occurs in such a class of equilibria. The intuition simply stems from the mere fact that prices signal quality: since all potential consumers would actually purchase the product in equilibrium, advertising the high quality product would only generate additional costs for the seller. Thus:

Proposition 1 *Informative advertising never occurs in a separating equilibrium.*

Note, further, that in a separating equilibrium both sellers' demands are the same as in the full information case, that is:

$$D_i(p) = \begin{cases} 1 & if\ p \leq q_i \\ 0 & \text{otherwise} \end{cases} ;\ i = h, l. \tag{2}$$

[13]As we explain below, a partial pooling equilibrium where the high quality seller randomizes does not exist.

[14]For a formal definition see Cho and Kreps (1987).

The fact that both sellers would serve the entire market in any separating equilibrium leads us to conclude that such type of equilibrium cannot exist in our model. This is due to the fact that all consumers have the same product valuations. Thus, if one consumer buys the high quality product in equilibrium, all of them will also buy it and, therefore, the low quality seller would always have an incentive to mimic his high quality counterpart. The so-called "single-crossing property" is not verified here since sending higher messages (here prices) is not easier for the high quality seller. The next proposition summarizes.[15]

Proposition 2 *A separating equilibrium does not exist in our model.*

4 Pooling equilibrium.

In a pooling equilibrium, both the high and the low quality seller set the same price and consumers are unable to ascertain the true quality using only this observation. Further, consumers expect to receive a free sample from the high quality seller with some positive probability. Let λ_e be the common probability with which consumers expect to be reached by an advertisement of the high quality good. While those consumers receiving a free sample learn the true quality of the product and disregard any signal observed, the rest of them will use all the available information to update their beliefs on quality. Thus, conditional upon observing the price p and a high quality signal s_h, the expected quality of consumers not receiving an advertisement is (by Bayes' rule):[16]

$$q_{eh}(\lambda_e) = \frac{\gamma\beta(1-\lambda_e)q_h + (1-\gamma)(1-\beta)q_l}{\gamma\beta(1-\lambda_e) + (1-\gamma)(1-\beta)} \tag{3}$$

If, on the other hand, consumers observe a low quality signal s_l, they expect the quality to be:

[15]It is important to note that the non-existence of informative advertising in a separating equilibrium is not model specific at all and it is clearly in contrast to those models of uninformative advertising as a signal of quality.

In contrast, the non-existence of separating equilibria is specific to our model. It stems from the fact that all consumers valuations are identical. In fact, the only manner in which consumers valuations differ in our model is through the market signal s. However, in a separating equilibrium consumers disregard such an information. Bagwell and Riordan (1991) obtain separation by considering that consumers' willingness to pay for the high quality are different across them. Assuming this in our model would substantially complicate the rest of the analysis, without adding much to it since our focus is on informative advertising.

[16]Here, the probability of receiving a free sample depends on whether the producer is the high or the low quality seller. Thus, consumers who have not received information will use this fact to update their beliefs on quality by Baye's rule. This is similar to Vettas (1996), where the probability of being informed through word-of-mouth communication also depends on whether the firm is the high or the low type. Thus, those consumers not informed through other consumers update their beliefs taking into account this fact.

$$q_{el}(\lambda_e) = \frac{(1-\gamma)(1-\lambda_e)\beta q_h + \gamma(1-\beta)q_l}{(1-\gamma)(1-\lambda_e)\beta + \gamma(1-\beta)} \tag{4}$$

Therefore, for any price p, the sellers demand $D_i(p, \lambda, \lambda_e)$ depends on the price (p), the advertising intensity (λ) and the consumers expected advertising intensity of the high quality seller (λ_e). Sellers maximize profits $\Pi_i(p, \lambda, \lambda_e)$ taking the expectation λ_e as fixed. Of course, in equilibrium we will require λ_e to be consistent with the actual advertising intensity chosen by the high quality seller (rational expectations hypothesis).

We first derive both sellers demand functions. If the product is actually the low quality one, a fraction γ of the population observes the right signal s_l. These consumers will buy the product as long as $p \leq q_{el}(\lambda_e)$. The rest of consumers, a fraction $1-\gamma$, observes the wrong signal s_h and will then purchase the product whenever $p \leq q_{eh}(\lambda_e)$. Obviously, the low quality seller will not advertise his product at all. Thus, the demand for the low quality product is:

$$D_l(p, 0, \lambda_e) = \begin{cases} 1 & \text{if } p \leq q_{el}(\lambda_e) \\ 1-\gamma & \text{if } q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Analogously, consider that the actual quality is high. Then, a fraction γ of the consumers receives signal s_h while a fraction $1-\gamma$ observes signal s_l. Disregarding, for the moment, the possibility to advertise his product, the high quality seller would serve the entire market for those prices such that $p \leq q_{el}(\lambda_e)$ and, for those in the interval $q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e)$ would obtain a demand of γ. However, the high quality seller can increase his demand by advertising the product as all consumers receiving an advertisement will learn its true quality and thus, will buy the good as long as $p \leq q_h$. Hence, the high quality seller faces the following demand function:

$$D_h(p, \lambda, \lambda_e) = \begin{cases} 1 & \text{if } p \leq q_{el}(\lambda_e) \\ \gamma + \lambda(1-\gamma) & \text{if } q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e) \\ \lambda & \text{if } q_{eh}(\lambda_e) < p \leq q_h \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Given this demand function we can easily compute the high quality seller's optimal advertising effort. If $p \leq q_{el}(\lambda_e)$, the high quality seller serves the entire market because both types of consumers, those receiving the right signal and those observing the wrong one, purchase the product. Therefore, advertising the product would only generate extra costs for the seller. On the other hand, if $q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e)$, none of the consumers observing the wrong signal would purchase unless they receive a free sample. For this interval of prices, the high quality seller chooses $\lambda \in [0,1]$ to maximize his profits $\Pi_h(p, \lambda, \lambda_e) = (\gamma + \lambda(1-\gamma))(p-c) - 0.5k\lambda^2$. From the first order condition it is obtained that $\lambda^* = (1-$

$\gamma)(p-c)/k$ (which also satisfies the second order condition). Finally, if $q_{eh}(\lambda_e) < p \leq q_h$, consumers would not purchase the good unless they ascertain the actual quality. In this case, the monopolist maximizes the function $\Pi_h(p, \lambda, \lambda_e) = \lambda(p-c) - 0.5k\lambda^2$. From the first (and second) order condition it follows that $\lambda^* = (p-c)/k$. Let us emphasize that the assumption $k > q_h - c$ assures that the optimal advertising effort is always an interior solution of the corresponding problem. Summarizing, the high quality seller optimal advertising policy is given by the following lemma:

Lemma 1 *In any pooling equilibrium, given the price p and the expected advertising effort λ_e, the high quality seller optimal advertising strategy is given by:*

$$\lambda^*(p, \lambda_e) = \begin{cases} \frac{(1-\gamma)(p-c)}{k} & \text{if } q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e) \\ \frac{(p-c)}{k} & \text{if } q_{eh}(\lambda_e) < p \leq q_h \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

To illustrate, Figure 1 depicts both sellers' demands. The stepwise function represented by the solid line depicts the low quality seller's demand. The discontinuous schedule represented by the dashed line shows the high quality seller's demand. Observe that there are two flat intervals in the demand for the high quality seller. These correspond to those prices for which the optimal advertising effort is zero. Interestingly, there are also two upward sloping intervals which stem from the fact that the optimal advertising effort is an increasing function of the price within each interval (see lemma 1). The higher is the price, the higher is the surplus the monopolist gets from each unit of good sold and, therefore, the higher are his incentives to advertise.

<insert Figure 1 here>

At this point, it is also worth noting that, since marginal costs of informing a small fraction of consumers are arbitrarily low, the high quality seller will always increase his demand by sending out a small amount of free samples (as long as he does not serve the entire market). From this observation, it turns out that two possible types of equilibria may arise: those where advertising does not occur, which we call *non-advertising pooling equilibria* and those where advertising occurs, which we call *advertising pooling equilibria.* Naturally, in a non-advertising equilibrium both firms will serve the entire market.

In analyzing whether a proposed equilibrium is indeed an equilibrium or not, we have to check for possible profitable deviations. Unfortunately, Bayes' rule does not pin down determinate beliefs off-the-equilibrium path. This means that when a firm deviates from a proposed equilibrium by charging a different price (i.e. sending a disequilibrium "message"), consumers may in general infer any possible expected quality after observing such an out-of-equilibrium price. A proposed equilibrium is then easiestly supported as an equilibrium by assuming that the beliefs formed by consumers after observing a firm deviation are the worst possible, that is, a deviating firm will always be considered to sell the low quality product.[17] To analyze the equilibria, we then need to characterize both sellers' deviation strategies and their profits thereafter. Let p^* be a proposed equilibrium. At worst, when a seller deviates from p^* by charging $\widetilde{p}$, he will be believed to produce low quality with probability 1. Consider first that the deviator is the low quality seller. When he deviates by charging $\widetilde{p}$, he serves the entire market as long as $\widetilde{p} \leq q_l$. Otherwise, he obtains zero demand. Suppose now that the deviator is the high quality seller. He has always the option to advertise his product and, to some extent, diminish the negative effects derived from being considered to produce the low quality with certainty. Of course, he advertises at a level determined by lemma 1. Therefore, he serves the entire market whenever $\widetilde{p} \leq q_l$ and faces demand of $(\widetilde{p}-c)/k$ as long as $q_l < \widetilde{p} \leq q_h$. Otherwise, his demand is zero. We depict these demands in Figure 2. Again, the demand for the low quality good is represented by the solid line while the dashed line represents the high quality seller's demand. There is an upward sloping interval in the high quality seller's demand function. This is due to the fact that for prices such that $q_l < \widetilde{p} \leq q_h$, his optimal advertising effort is an increasing function of the price.

<insert Figure 2 here>

As a result, when consumers believe that quality is certainly low, the best deviating price for the low quality seller is $\widetilde{p} = q_l$. His best deviating profits would then be $\widetilde{\Pi}_l = q_l$. Analogously, the high quality seller has two alternative best deviating strategies, namely, either (a) to charge $\widetilde{p} = q_l$ and not advertise at all which would yield profits of $\widetilde{\Pi}_h = q_l - c$ or (b) to charge $\widehat{p} = q_h$ and advertise at level $\widehat{\lambda} = (q_h - c)/k$ which would yield profits of $\widehat{\Pi}_h = (q_h - c)^2/2k$. We will use these best deviating strategies and profits to characterize the pooling equilibria.

4.1 Non-advertising pooling equilibria.

In any non-advertising equilibria both sellers must serve the entire market. The reason is that informing a small fraction of consumers about his true quality is arbitrarily cheap for the seller; thus, as long as the high quality seller does not serve all the consumers, he will always spend some resources in advertising. The following lemma establishes the set of relevant prices of a non-advertising pooling equilibrium.

Lemma 2 *In any non-advertising pooling equilibrium* $q_l \leq p \leq q_{el}(\lambda_e)$.

Proof. Suppose not, then there are three possibilities. First, if $p < q_l$, the low quality seller would deviate because his profits are strictly increasing for any

[17]Later we will use the well known *intuitive criterion* to restrict the set of possible beliefs.

beliefs in this interval. Second, if $q_{el}(\lambda_e) < p \le q_{eh}(\lambda_e)$, the high quality seller would deviate by advertising the product as $\arg\max_{\lambda} \{(\gamma + (1-\gamma)\lambda)(p-c) - 0.5k\lambda^2\} > 0$. Finally, if $p > q_{eh}(\lambda_e)$, the low quality seller would face zero demand and would then have an incentive to deviate to q_l. ■

For (p^*, λ^*) being a non-advertising equilibrium, in addition, it must be the case that neither the high nor the low quality seller has an incentive to deviate and moreover that $\lambda^* = \lambda_e = 0$. A seller has no incentives to deviate from the proposed equilibrium (p^*, λ^*) as long as he makes higher profits from adopting this strategy than from using his best deviating one. Thus, for the low quality seller one must have that $p^* \ge q_l$ and for the high quality seller it must be true that $(p^* - c) \ge \max\{q_l - c, (q_h - c)^2/2k\}$. The following proposition characterizes the non-advertising equilibria.

Proposition 3 *(p^*, λ^*) is a non-advertising pooling equilibrium if and only if:*
(a) $p^* \ge q_l$
(b) $p^* \le q_{el}(\lambda_e)$
(c) $p^* \ge \frac{(q_h-c)^2}{2k} + c$
(d) $\lambda^* = \lambda_e = 0$.

So far non-advertising pooling equilibria have been characterized. We next turn to their existence. We define:

$$\Psi_1(\beta, \gamma) = \frac{(q_h - c)^2((1-\gamma)\beta + \gamma(1-\beta))}{2((1-\gamma)\beta(q_h - c) + \gamma(1-\beta)(q_l - c))}. \quad (8)$$

Proposition 4 *A non-advertising pooling equilibrium exists if and only if $k \ge \Psi_1(\beta, \gamma)$.*

Proof. $(\Rightarrow)$ Assume $(p^*, 0)$ is a non-advertising pooling equilibrium. Then, by (b) and (c), $p^* - c \ge (q_h - c)^2/2k$ and $q_{el}(0) - p^* \ge 0$. By adding these two inequalities, it is obtained that $q_{el}(0) - c \ge (q_h - c)^2/2k$. Then, by substituting equation (4) into $q_{el}(0)$ and isolating k, we have $k \ge \Psi_1(\beta, \gamma)$.

$(\Leftarrow)$ We show that $(p, \lambda) = (q_{el}(0), 0)$ is a non-advertising equilibrium. First, lemma 1 ensures that optimal advertising when $p = q_{el}(\lambda_e)$ is zero for any λ_e. The low quality seller does not deviate as $q_{el}(0) \ge q_l$. Moreover, the high quality seller does not deviate as long as $q_{el} - c \ge (q_h - c)^2/2k$, which is ensured by the condition that $k \ge \Psi_1(\beta, \gamma)$. ■

The intuition is clear. In a non-advertising pooling equilibrium, the low quality seller mimics his high quality counterpart. The mere fact that consumers are uninformed about the products' quality allows the low quality seller to charge prices above his full information optimal price (q_l) without loosing buyers, and as a result, he makes higher profits. Further, for any price in the relevant range, the low quality seller does not deviate from it as he already serves the entire market.

The fact that all consumers buy at the proposed price also implies, first, that the high quality seller's optimal advertising effort is zero (as there are no gains from informing consumers) and second, that the high quality seller has not incentives to deviate by lowering his price. Finally, for this to be an equilibrium it is necessary that the costs of advertising are sufficiently high. If k were very small, the high quality seller would deviate by charging his full information optimal price disregarding the fact that his product would considered of low quality with probability 1, as he would be able to profitably inform most of the consumers about the true quality at a low cost. If k is high enough, such a deviation is no longer profitable, which is ensured by the condition that $k \geq \Psi_1(\beta, \gamma)$.

<insert figures 3.1 and 3.2 here>

The set of parameters for which a non-advertising pooling equilibrium exists is clearly non-empty. In figures 3.1 and 3.2, we have depicted the function $\Psi_1(\beta, \gamma)$ in the $k - \beta$ and $k - \gamma$ spaces. As we have seen, in a non-advertising pooling equilibrium all consumers buy. This means that the price cannot be higher than the expected quality of those consumers receiving a low quality signal ($q_{el}(0)$). As the consumers' prior probability of high quality (β) decreases, to sustain such an equilibrium, it is necessary that the costs of advertising increase. The reason is that as β decreases, the expected quality $q_{el}(0)$ approaches q_l, and, as a result, the price charged in equilibrium is lower. The high quality seller's incentives to deviate to the strategy $(q_h, \lambda^*(q_h))$ are then higher because his equilibrium profits decrease. The contrary happens when the informativeness of the market signal (γ) diminishes. The price charged in equilibrium increases as γ decreases and, therefore, the equilibrium is easier to sustain.

Finally, notice that any non-advertising equilibrium is intuitive. In fact, assume that p^* is a non-advertising equilibrium. By definition, it satisfies the intuitive criterion if it does not exist another price $\widehat{p}$ such that (a) $\widehat{p} - c > p^* - c$ and (b) $\widehat{p} < p^*$ are verified. Clearly, such a price can never exist.

4.2 Advertising pooling equilibria.

We now turn to study advertising pooling equilibria. In an advertising pooling equilibrium, the price must be high enough so that some consumers do not purchase the product and the high quality seller has an incentive to inform some of them about the true quality. In other words, there must be some consumers that would not buy the high quality product unless they were informed of the true quality. This actually happens for those prices such that $q_{el}(\lambda_e) < p^* \leq q_{eh}(\lambda_e)$ as only those consumers receiving a high quality signal purchase the product (Figure 1). Note also that a price higher than $q_{eh}(\lambda_e)$ cannot be an equilibrium because in that case the low quality seller has zero demand. The following lemma states the set of prices which may plausibly constitute an advertising pooling equilibrium.

Lemma 3 *In any advertising pooling equilibrium* $q_{el}(\lambda_e) < p^* \leq q_{eh}(\lambda_e)$.

For (p^*, λ^*) being an advertising pooling equilibrium, additionally, the advertising effort has to be optimal, that is, λ^* must be equal to $\arg\max_{\lambda} \Pi_h(p^*, \lambda, \lambda_e)$. From lemma 1, then $\lambda^* = (1-\gamma)(p^* - c)/k$. It is also necessary that neither the high nor the low quality seller has an incentive to deviate and, finally, that the consumers' expected advertising intensity of the high quality seller coincides with the actual one. The high quality seller does not deviate from the proposed equilibrium as long as his equilibrium profits, given by $(p^* - c)(\gamma + \lambda^*(1-\gamma)) - 0.5k\lambda^{*2}$, exceed his profits from his best deviating strategy, that is, exceed $\max\{q_l - c, (q_h - c)^2/2k\}$. Analogously, the low quality seller does not deviate if $(1-\gamma)p^* \geq q_l$. The following proposition characterizes the advertising pooling equilibria:

Proposition 5 (p^*, λ^*) *is an advertising pooling equilibrium if and only if:*
(a) $\lambda^* = \lambda_e = \frac{(1-\gamma)(p^*-c)}{k}$
(b) $q_{el}(\lambda_e) < p^* \leq q_{eh}(\lambda_e)$
(c) $(1-\gamma)p^* \geq q_l$
(d) $\gamma(p^* - c) + \frac{(1-\gamma)^2(p^*-c)^2}{2k} \geq \max\left\{q_l - c, \frac{(q_h-c)^2}{2k}\right\}$

To establish existence of an advertising pooling equilibrium we define:
$A = \gamma\beta(1-\gamma)$
$B = A(q_h + c) + k(\gamma\beta + (1-\beta)(1-\gamma))$
$C = Aq_hc + k(\gamma\beta q_h + (1-\beta)(1-\gamma)q_l)$
$X^- = \frac{B-(B^2-4AC)^{\frac{1}{2}}}{2A}$
The following proposition, whose proof is tedious and is thus relegated to the appendix, establishes existence of an advertising pooling equilibrium.

Proposition 6 *An advertising pooling equilibrium exists if and only if:*
(a') $q_h > 2q_l$
(b') $\gamma < \frac{q_h - q_l}{q_h}$
(c') $\beta \geq \frac{k(1-\gamma)q_l}{(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))+k(1-\gamma)q_l}$
(d') $2k\gamma(X^- - c) + (1-\gamma)^2(X^- - c)^2 - (q_h - c)^2 \geq 0$

The intuition behind these conditions is as follows. Consider first the low quality seller. In comparison to his best deviating strategy, in an advertising pooling equilibrium, he charges a higher price ($p^* > q_l$) but sells a lower quantity ($1 - \gamma < 1$). To ensure that the low quality seller does not deviate neither his sales nor his price can be too low. This is assured by conditions (a')-(c'). On the one hand, (a') and (c') ensure that the price charged is not too low. Condition (a') means that the consumers' reservation value for the high quality must be sufficiently larger than their reservation price for the low quality. This is necessary for the buyers' expected qualities to be sufficiently higher than their reservation values for the low quality. This is not enough however. Condition (c') has also to be satisfied. That is, the consumers' prior probability of high quality must also be sufficiently large because otherwise the price charged would be too low. Note that the higher is the prior for the high quality, the higher are consumers' expected qualities and, as a result, consumers' willingness to pay for the products. When either β or q_h is too low, the pooling price is very close to q_l and, as a result, this price cannot be sustained in equilibrium any more. On the other hand, condition (b') assures that equilibrium sales are not too low. It requires that the market is noisy enough (γ small). In fact, in equilibrium, the low quality seller only sells to those consumers who have received the wrong signal (fraction $1-\gamma$), that is, the high quality signal, as those consumers receiving the low quality signal do not buy in a pooling equilibrium. Therefore, the fraction of incorrectly informed consumers through the signal has to be large enough for an equilibrium to exist. Note that condition (a') also assures that the set of γ for which an equilibrium exists is nonempty.

Consider now the high quality seller. The above arguments also allow us to rule out a deviation where the high quality seller lowers his price. If the low quality seller has no incentive to deviate to the price q_l, then the high quality seller has no incentive to deviate to the strategy $(q_l, 0)$ either. The intuition is simply that even if he disregards the possibility of advertising his product, the high quality seller is better off by charging the pooling price. Finally, to ensure that the high quality seller does not deviate by raising his price, condition (d') must be satisfied. This condition requires the cost of advertising to be sufficiently high. If, contrarily, this cost were relatively small, it would always be profitable for the high quality seller to deviate by charging the consumers' reservation price for the high quality and, extensively advertising the product at a low cost. This would also impede the existence of an advertising pooling equilibrium.

To summarize, for an advertising pooling equilibrium to exist, it is necessary that (a) the reservation price for the high quality is large enough in comparison to the low quality one, (b) the informativeness of the signal is sufficiently imperfect, (c) the consumers' prior probability of high quality is large enough and, finally, (d) costs of advertising is sufficiently high.

<insert Figure 4 here>

The set of parameters for which an advertising pooling equilibrium exists is non-empty. In Figure 4 we have depicted the conditions for its existence. The schedule C-C depicts condition (c') while D-D depicts condition (d'). The lower bound $\underline{\beta}$ has been obtained from condition (c'). Of course, the rest of parameters have been chosen to satisfy conditions (a') and (b'). The shaded area then represents the constellation of parameters $k-\beta$ for which informative advertising occurs in a full pooling equilibrium. It is interesting to note that as the consumers' prior probability of high quality (β) decreases, a higher advertising cost is required to sustain the equilibrium. The intuition is again that the price charged decreases as β diminishes because the consumers' willingness to pay decreases. So, from both sellers point of view, a higher parameter k is required to support the equilibrium (both schedules C-C and D-D decrease with k).

Finally, it is important to note that any advertising pooling equilibria satisfy the intuitive criterion. In fact, assume that (p^*, λ^*) is an advertising pooling equilibrium. It satisfies the intuitive criterion if there does not exist another price $\tilde{p}$ for which conditions (a) $\tilde{p} - c > \gamma(p^* - c) + (1-\gamma)^2(p^* - c)^2/2k$ and (b) $\tilde{p} < (1-\gamma)p^*$ are satisfied. In other words, (p^*, λ^*) is an intuitive advertising pooling equilibrium as long as $\gamma(p^* - c) + (1-\gamma)^2(p^* - c)^2/2k + c \geq (1-\gamma)p^*$. Rearranging this, we obtain that it must be the case that $(1-\gamma)^2(p^* - c)^2/2k \geq p^*(1-2\gamma) - c(1-\gamma)$. This is always satisfied because γ >0.5.

5 Partial pooling equilibrium.

In a partial pooling (or semi-separating) equilibrium, the low quality seller randomizes between his optimal perfect information price q_l, and a pooling price p^*, while the high quality seller always sets the pooling price.[18] Let ρ_e denote the consumers' expectation on the low quality seller's probability of charging the pooling price. In a partial pooling equilibrium, the information in the market is as follows: Consumers observing price q_l ascertain that the quality is low. Buyers receiving a free sample from the high quality seller learn that the true quality is high. Finally, those consumers observing the price p^* and a high quality signal s_h and not receiving an advertisement expect quality to be:

$$\tilde{q}_{eh}(\lambda_e, \rho_e) = \frac{\gamma\beta(1-\lambda_e)q_h + (1-\gamma)(1-\beta)\rho_e q_l}{\gamma\beta(1-\lambda_e) + (1-\gamma)(1-\beta)\rho_e} \tag{9}$$

while those buyers receiving a low quality signal s_l form beliefs:

$$\tilde{q}_{el}(\lambda_e, \rho_e) = \frac{(1-\gamma)(1-\lambda_e)\beta q_h + \gamma(1-\beta)\rho_e q_l}{(1-\gamma)(1-\lambda_e)\beta + \gamma(1-\beta)\rho_e} \tag{10}$$

The same type of arguments as in the previous section allow us to find both sellers' demand functions:[19]

$$D_l(p, 0, \lambda_e, \rho_e) = \begin{cases} 1 & \text{if } p \leq \tilde{q}_{el}(\lambda_e, \rho_e) \\ 1-\gamma & \text{if } \tilde{q}_{el}(\lambda_e, \rho_e) < p \leq \tilde{q}_{eh}(\lambda_e, \rho_e) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

[18] It can be easily seen that there is no partial pooling equilibrium where the high quality seller randomizes.

[19] We do not to depict these demands. In fact, they are the same as the demands for the full pooling equilibrium case with a minor difference: $q_{el}(\lambda_e)$ and $q_{eh}(\lambda_e)$ must be $\tilde{q}_{el}(\lambda_e, \rho_e)$ and $\tilde{q}_{eh}(\lambda_e, \rho_e)$ respectively.

$$D_h(p, \lambda, \lambda_e, \rho_e) = \begin{cases} 1 & \text{if } p \leq \widetilde{q}_{el}(\lambda_e, \rho_e) \\ \gamma + \lambda(1-\gamma) & \text{if } \widetilde{q}_{el}(\lambda_e, \rho_e) < p \leq \widetilde{q}_{eh}(\lambda_e, \rho_e) \\ \lambda & \text{if } \widetilde{q}_{eh}(\lambda_e, \rho_e) < p \leq q_h \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Here, we also distinguish between *non-advertising partial pooling equilibria* and *advertising partial pooling equilibria.* By a simple observation of the low quality product demand, we infer that in any advertising partial pooling equilibrium, the pooling price must lie in the interval $\widetilde{q}_{el}(\lambda_e, \rho_e) < p^* \leq \widetilde{q}_{eh}(\lambda_e, \rho_e)$. Otherwise, the low quality seller would never randomize as profits from the pooling price would be either higher or lower than profits from charging q_l. From this fact and since informing a small percentage of consumers is arbitrarily cheap for the seller, it follows that advertising always occurs in any semi-separating equilibrium. These observations are summarized in the following lemma.

Lemma 4 *(a) In any partial pooling equilibrium $\widetilde{q}_{el}(\lambda_e, \rho_e) < p^* \leq \widetilde{q}_{eh}(\lambda_e, \rho_e)$.*
(b) A non-advertising partial pooling equilibrium never exists.

Now, we characterize advertising partial pooling equilibria. Let (p^*, λ^*, ρ^*) be a proposed equilibrium. For the low quality seller to randomize, p^* must be equal to $q_l/(1-\gamma)$. The reason is that the low quality seller must obtain the same profits by charging the pooling price p^* or by setting q_l. His profits from charging q_l are equal to q_l, while by setting the pooling price p^* he makes profits of $(1-\gamma)p^*$. Thus, $p^* = q_l/(1-\gamma)$. Further, this is the unique price which can be supported in a partial pooling equilibrium. At this price, the high quality seller's optimal advertising effort is given by lemma 1 and is equal to $\lambda^* = q_l - (1-\gamma)c/k$. Furthermore, the high quality seller must obtain higher profits from following the equilibrium strategy than from deviating from it, that is, $\gamma(q_l/(1-\gamma) - c) + (q_l - (1-\gamma)c)^2/2k \geq \max\{q_l - c, (q_h - c)^2/2k\}$. Finally, in equilibrium consumers expectations must be correct. The following proposition summarizes the characterization of semi-separating equilibria.

Proposition 7 (p^*, λ^*, ρ^*) *is an advertising partial pooling equilibrium if and only if:*

(i) $p^* = \frac{q_l}{(1-\gamma)}$
(ii) $\lambda^* = \lambda_e = \frac{(q_l-(1-\gamma)c)}{k}$
(iii) $\rho^* = \rho_e$
(iv) $\widetilde{q}_{el}(\lambda_e, \rho_e) < p^* \leq \widetilde{q}_{eh}(\lambda_e, \rho_e)$
(v) $\gamma(p^* - c) + \frac{(1-\gamma)^2(p^*-c)^2}{2k} \geq \max\left\{q_l - c, \frac{(q_h-c)^2}{2k}\right\}$

By rearranging equations (i)-(iv), it is obtained that ρ^* must lie in the interval:

$$(\underline{\rho}, \overline{\rho}) \equiv \left(\frac{\beta(1-\gamma)(q_h(1-\gamma) - q_l)(k - (q_l - (1-\gamma)c))}{k(1-\beta)\gamma^2 q_l},\right.$$

$$\left.\frac{\beta(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))}{k(1-\beta)(1-\gamma)q_l}\right) \tag{13}$$

Of course, since ρ^* is the low quality seller's randomization probability, it has to lie in the interval $(0,1)$. Taking this into account, we can show the following result on existence. The proof is again relegated to the appendix.

Proposition 8 *An advertising partial pooling equilibrium exists if and only if the following conditions (i')-(iv') are satisfied:*

(i') $q_h > 2q_l$

(ii') $\gamma < \frac{q_h - q_l}{q_h}$

(iii') $\beta \leq \frac{k\gamma^2 q_l}{(1-\gamma)(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))+k\gamma^2 q_l}$

(iv') $k \geq \frac{(1-\gamma)((q_h-c)^2-(q_l-(1-\gamma)c)^2)}{2\gamma(q_l-(1-\gamma)c)}$

The intuition behind these conditions is similar to that in the advertising full pooling equilibrium. Consider first the low quality seller. Conditions (i')-(iii') ensure that he does not deviate. Note that conditions (i') and (ii') are the same as conditions (a') and (b') in proposition 7. In contrast, condition (iii') is different and requires that the consumers' prior probability of high quality is not too high. All these three conditions imply that the pooling price charged in a semi-separating equilibrium is neither very low nor very high and that sales are not very low. If the price were very low, the low quality seller instead of randomizing would deterministically charge the optimal price under full information (condition (i')). On the other hand, if the pooling price were too high, then the low quality seller would deterministically charge the pooling price (condition (iii')). Condition (ii') assures that sales are not too low in a semi-separating equilibrium. As in a full pooling equilibrium, demand stems from those consumers who receive the wrong information. So, the noise in the market must be sufficiently high for a partial pooling equilibrium to exist.

Consider now the high quality seller. Condition (i')-(iii') also assure that the high quality seller does not deviate by lowering his price. Condition (iv') is required in order to avoid that the high quality seller profitably deviates by raising its price and extensively distributing free samples. Condition (iv') also requires that the advertising costs must be sufficiently large.

<insert Figure 5 here>

The set of parameters for which a semi-separating equilibrium exists is non-empty. In Figure 5, we have represented conditions (iii') and (iv') in the space $k-\beta$. The rest of the parameters have been chosen to satisfy (i') and (ii'). Condition (iii') is depicted by the schedule C-C while condition (iv') is represented by the line D-D. The shaded area then represents the parameter space for which an advertising partial pooling equilibrium exists. The following remarks are in line: When β is small enough, then it is guaranteed that $0 < \underline{\rho} < 1$. Observe that as β decreases the set $(\underline{\rho}, \overline{\rho})$ shrinks. In the limiting case $\beta = 0$, there is no positive probability for which the low quality seller would be willing to randomize. Contrarily, as β increases, the set $(\underline{\rho}, \overline{\rho})$ is enlarged. In fact, when β approaches 1, both $\underline{\rho}$ and $\overline{\rho}$ tend to infinity. This is the reason for which β cannot exceed the critical value $\overline{\beta}$ (obtained from (iii')) for an equilibrium to exist.

As in the previous equilibria, each advertising partial pooling equilibrium survives the intuitive criterion. In fact, assume that (p^*, λ^*, ρ^*) is an advertising partial pooling equilibrium. By definition, (p^*, λ^*, ρ^*) is intuitive if there does not exist another price $\tilde{p}$ for which (a) $\tilde{p} - c > \gamma(q_l/(1-\gamma) - c) + (q_l - (1-\gamma)c)^2/2k$ and (b) $\tilde{p} < q_l$ are satisfied. In other words, the set $\gamma(q_l(1-\gamma) - c) + (q_l - (1-\gamma)c)^2/2k + c < \tilde{p} < q_l$ must be empty. Condition (v) in proposition 7 guarantees that it is indeed so.

In order to conclude the analysis we consider Figure 6. This Figure has been constructed from figures 4 and 5. The decreasing schedule in region II is the schedule C-C in Figure 4 and the one in region III is the schedule C-C in Figure 5. The critical values $\underline{\beta}$ and $\overline{\beta}$ are also extracted from those figures. For the parameter constellations covered by region I and II, an advertising partial pooling equilibrium exists. Contrarily, an advertising full pooling equilibrium exists for those parameters in regions II and III.

The intuition is as follows: for β close to zero, there exists a partial pooling equilibrium. The low quality seller, in such an equilibrium, charges the pooling price with low probability. As β increases, the probability of charging the pooling price increases. Once β reaches the boundary of region I, also a full pooling equilibrium exists. So, in region II, partial and full pooling equilibria coexist. However, the probability of charging the pooling price goes beyond one when β reaches the boundary of region II. Then, a partial pooling equilibrium cannot exist in region III.

<insert Figure 6 here>

A final observation is that as γ decreases, the region where there may coexist both partial and full pooling equilibria vanishes. The boundaries of both regions become equal in the limiting case of $\gamma = 0.5$.

6 Conclusions

In this paper we have studied the decision of a producer to advertise a high quality product when he introduces the good into a market where consumers are unable to observe the quality of the product without first purchasing it. We have set up an adverse selection model where a monopolist introduces a product of uncertain quality. The product can be of either high or low quality. At the beginning of the trading period, only the producer observes the true quality while the consumers receive an independent market signal which is correlated with the true quality of the good. Then the monopolist sets his price and advertising intensity. Those consumers receiving a free sample learn the true quality of the product while the rest of them update their quality beliefs taking into consideration the price observed, the signal received and the fact that they have not been reached by the advertising campaign.

We have characterized the set of possible equilibria where informative advertising occurs. Informative advertising never occurs in a separating equilibrium because prices convey full information about the quality and then advertising expenditures are then unnecessary. If informative advertising occurs there must exist some pooling. A full pooling equilibrium where informative advertising occurs can only exist when (a) the consumers' valuation for the high quality is sufficiently large, (b) the informativeness of the market signal is low enough, (c) the consumers' prior probability of high quality is sufficiently high and (d) the cost of advertising is high enough. Existence of advertising semi-separating equilibrium also requires (a), (b) and (d) but the consumers' prior probability of high quality must be low enough. We have also showed that when informative advertising appears in equilibrium, the adverse selection problem is mitigated. Moreover, the lower are advertising costs, the more intense is the alleviation of that problem.

7 Appendix

PROOF OF PROPOSITION 6:

We use the following definitions and lemmas. Define:

$A = \gamma\beta(1-\gamma)$

$B = A(q_h + c) + k(\gamma\beta + (1-\gamma)(1-\beta))$

$C = Aq_hc + k(\gamma\beta q_h + (1-\gamma)(1-\beta)q_l)$

$X^+ = \frac{B+(B^2-4AC)^{\frac{1}{2}}}{2A}$

$X^- = \frac{B-(B^2-4AC)^{\frac{1}{2}}}{2A}$

Lemma 5 $B^2 - 4AC > 0$

Proof. Rearranging terms, equation $B^2 - 4AC > 0$ can be rewritten as $a_1k^2 + a_2k + c > 0$, where $a_1 = (\gamma\beta + (1-\gamma)(1-\beta))^2$, $a_2 = -2A(\gamma\beta(q_h - c) - (1-\beta)(1-\gamma)(q_h - 2q_l + c))$ and $a_3 = A^2(q_h - c)^2$. Consider the quadratic equation $a_1k^2 + a_2k + c = 0$. Since $a_1 > 0$, it is a convex quadratic function whose solutions are given by $k = (-a_2 \pm (a_2^2 - 4a_1a_3)^{0.5})/2a_1$. It can easily be shown that $a_2^2 - 4a_1a_3 < 0$; in fact, $a_2^2 - 4a_1a_3 = -(q_h - q_l)\gamma^2(1-\gamma)^3\beta^2(1-\beta)^2(\gamma\beta(q_h - c) + (1-\gamma)(1-\beta)(q_l - c))$, which is clearly negative. Therefore, there is no real solution to the equation $a_1k^2 + a_2k + c = 0$, which implies that $a_1k^2 + a_2k + c > 0$. The lemma then follows. ■

Lemma 6 $X^+ > q_h$.

Proof. We show that $B/2A > q_h$. Then, by using the previous one, the lemma directly follows. $B/2A > q_h$ as long as $A(q_h + c) + k(\gamma\beta + (1-\gamma)(1-\beta)) > 2Aq_h$. This inequality can be rewrite as $k(\gamma\beta + (1-\gamma)(1-\beta)) > A(q_h - c)$. Since $A < \gamma\beta + (1-\gamma)(1-\beta)$ and, by assumption, $k > q_h - c$ it follows that $B/2A > q_h$. ■

Lemma 7 *If* $(q_l - (1-\gamma)q_h) > 0$, *then* $\frac{\beta(q_l-(1-\gamma)q_h)(q_l-(1-\gamma)c)}{\beta(q_l-(1-\gamma)q_h)+(1-\beta)(1-\gamma)q_l)} < q_h - c$

Proof. The left hand side of the inequality is an increasing function of the parameter β. Then, by showing that the inequality holds at worst ($\beta = 1$), the lemma is proved. For $\beta = 1$, the inequality reduces to $(q_l - (1-\gamma)q_h)(q_l - (1-\gamma)c) - (q_h - c)(q_l - (1-\gamma)q_h) < 0$. Rearranging terms, it can be rewritten as $(q_l - (1-\gamma)q_h)(q_l - q_h + \gamma c) < 0$. By using the assumption $q_h - c > q_l$ and the hypothesis $q_l - (1-\gamma)q_h > 0$, it is easily chechked that this inequality is satisfied. ■

The proof of the proposition 6 now follows:

Proof. ($\Rightarrow$) Assume that (p^*, λ^*) is an advertising pooling equilibrium. Then, from proposition 5, it must satisfy equation $p^* \leq q_{eh}(\lambda^*(p^*))$. Solving this inequality for p^*, it is obtained that the equilibrium price must satisfy either $p^* \leq X^-$ or $p^* \geq X^+$. Lemma 5 assures that X^- and X^+ are well defined. In addition, lemma 6 allows us to ignore those prices $p^* \geq X^+$. From (b) and (c) in proposition 5 one has $X^- - p^* \geq 0$ and $p^* \geq q_l/(1-\gamma)$. By adding these two inequalities, it follows that $(1-\gamma)X^- \geq q_l$. This inequality can be rewritten (a bit tedious) as:

$$k(\beta(q_h(1-\gamma)-q_l)-(1-\beta)(1-\gamma)q_l) \geq \beta(q_h(1-\gamma)-q_l)(q_l-(1-\gamma)c) \quad (14)$$

Assume that (a) does not hold, that is, $q_h - 2q_l \leq 0$. Then, since $\gamma > 0.5$, one must have $\gamma \geq (q_h - q_l)/q_h$, that is, $q_h(1-\gamma) - q_l \leq 0$. Otherwise, there would not exist any feasible γ. Then, both sides of the inequality (14) are negative. Rewriting this inequality, it requires that

$$k \leq \frac{\beta(q_l - q_h(1-\gamma))(q_l-(1-\gamma)c)}{\beta(q_l-q_h(1-\gamma))+(1-\beta)(1-\gamma)q_l}. \quad (15)$$

However, lemma 7 shows that

$$\frac{\beta(q_l - q_h(1-\gamma))(q_l-(1-\gamma)c)}{\beta(q_l-q_h(1-\gamma))+(1-\beta)(1-\gamma)q_l} < q_h - c, \quad (16)$$

which, since $k > q_h - c$, constitutes a contradiction. As a result, (a) must be satisfied and, since $\gamma > 0.5$, (b) must also hold. Condition (c) is nothing else than equation (14) properly rearranged.

Finally, by condition (d) in proposition 5, one has that $\gamma(p^* - c) + (1-\gamma)^2(p^*-c)^2/2k \geq (q_h - c)^2/2k$. Since the left hand side of this inequality is strictly increasing in p^* and $p^* \leq X^-$, (d') follows.

($\Leftarrow$) We show that, if (a')-(d') are satisfied, then $(p,\lambda) = (X^-, (1-\gamma)(X^- - c)/k)$ is an advertising pooling equilibrium. First, the optimal advertising intensity follows from substituting X^- into the optimal advertising function given by the lemma 1. Condition (b') ensures that the low quality seller does not deviate. On the other hand, condition (d') assures that the high quality seller does not deviate by using the strategy $(\widehat{p}, \widehat{\lambda}) = (q_h, (q_h - c)/k)$. To complete the proof, we have to show that the high quality seller does not deviate by using the alternative strategy $(\widetilde{p}, \widetilde{\lambda}) = (q_l, 0)$. Profits from using such a strategy are equal to $q_l - c$. From condition (b'), one has that $\gamma(X^- - c) \geq \gamma(q_l/(1-\gamma) - c)$. Since $\gamma > 0.5$, it follows that $q_l - c < \gamma(q_l/(1-\gamma) - c) \leq \gamma(X^- - c)$. Therefore, $\gamma(X^- - c) - (q_l - c) + (1-\gamma)^2(X^- - c)^2/2k \geq 0$; thus, the proposition follows. ■

PROOF OF PROPOSITION 8:

Proof. ($\Rightarrow$) Assume that (p^*, λ^*, ρ^*) is an advertising partial pooling equilibrium. Then, by the hypothesis, it must satisfy $p^* = q_l/(1-\gamma)$, $\lambda^* = \lambda_e = (q_l - (1-\gamma)c)/k$, $\rho^* = \rho_e$ and $\widetilde{q}_{el}(\lambda_e, \rho_e) < p^* \leq \widetilde{q}_{eh}(\lambda_e, \rho_e)$. Combining these equations, it is obtained that ρ^* must lie in the interval

$$(\underline{\rho}, \overline{\rho}) = \left(\frac{\beta(1-\gamma)(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))}{k(1-\beta)\gamma^2 q_l},\right.$$

$$, \frac{\beta(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))}{k(1-\beta)(1-\gamma)q_l}\Bigg). \tag{17}$$

Note here that since ρ^* has to be a positive number, it has to be satisfied that $\gamma < (q_h - q_l)/q_h$. Again, since $\gamma > 0.5$, for existence of advertising partial pooling equilibrium it is necessary that $q_h > 2q_l$. Further, one needs that the intersection between the set $(\underline{\rho}, \overline{\rho})$ and (0,1) is nonempty; indeed (iii') follows from rearranging the inequality $\underline{\rho} < 1$. Finally, by hypothesis, it must be true that $\gamma(q_l/(1-\gamma) - c) + (q_l - (1-\gamma)c)^2/2k \geq (q_h - c)^2/2k$. By isolating k, (iv') follows.

($\Leftarrow$) We show that if (i')-(iv') are satisfied, then

$$(p, \lambda, \rho) = \Bigg(\frac{q_l}{1-\gamma}, \frac{(q_l-(1-\gamma)c)}{k},$$

$$, \frac{\beta(1-\gamma)(q_h(1-\gamma)-q_l)(k-(q_l-(1-\gamma)c))}{k(1-\beta)\gamma^2 q_l}\Bigg)$$

constitutes an advertising partial pooling equilibrium price. First, the advertising intensity is optimal given p and ρ (see lemma 1). Second, (i')-(iii') guarantee that the low quality seller is willing to randomize with a positive but smaller than one probability. Condition (iv') assures that the high quality seller does not deviate by using the strategy $(\tilde{p}, \tilde{\lambda}) = (q_h, (q_h - c)/k)$. To complete the proof, we have to show that the high quality seller does not deviate by using the alternative strategy $(\tilde{p}, \tilde{\lambda}) = (q_l, 0)$. In other words, it must be true that $\gamma(q_l/(1-\gamma) - c) + (q_l - (1-\gamma)c)^2/2k \geq q_l - c$. This inequality is satisfied since $\gamma > 0.5$. (in fact, it is the case that $\gamma(q_l/(1-\gamma) - c) > q_l - c$). ■

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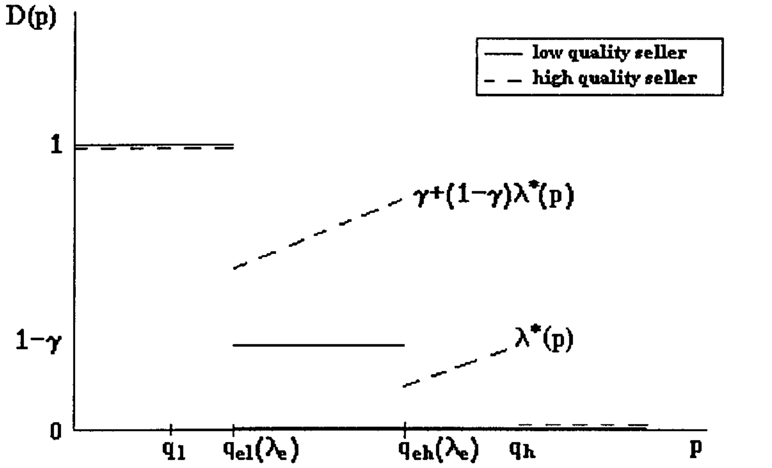

Figure 1: Sellers demands when they are believed to produce the high quality with probability β.

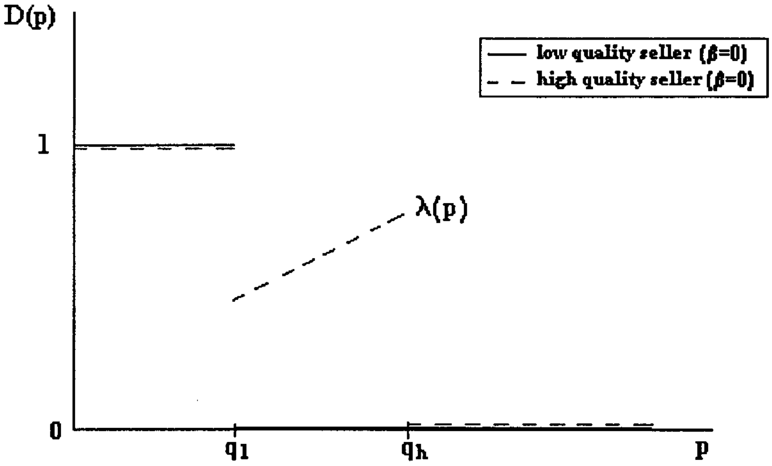

Figure 2: Seller's demands when they are believed to produce the low quality with probability one.

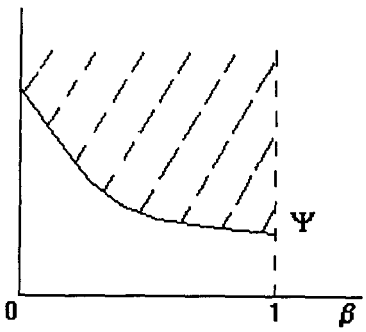

Figure 3.1

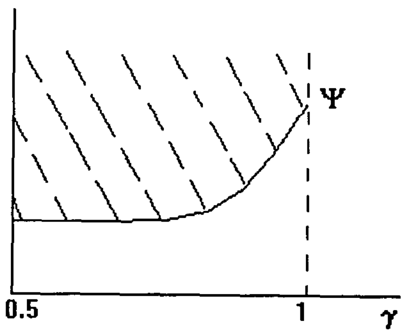

Figure 3.2

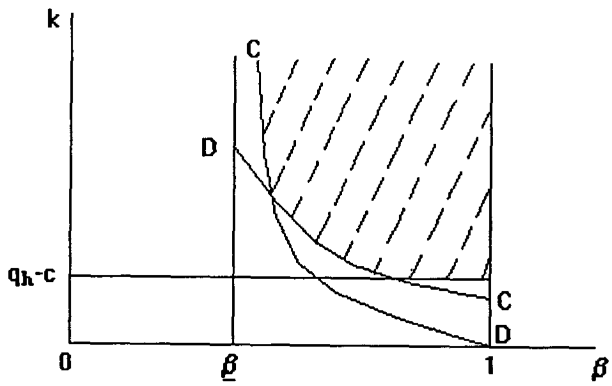

Figure 4

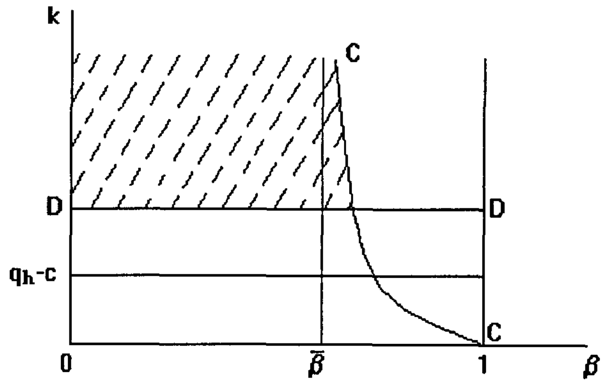

Figure 5

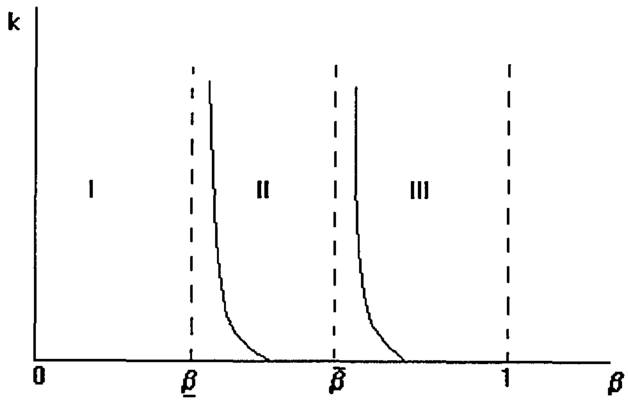

Figure 6

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