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SELLING INFORMATION IN EXTENSIVE FORM GAMES *

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Abstract

We consider a situation in wich decision makers in an extensive form game can buy additional information from an information seller before reaching their decisions. Prices for information are selected by the seller. We analyze a variety of scenarios for the price setting process by the seller: the case in which prices are chosen before the game starts (ex-ante pricing), the case in which prices are chosen during the game (ex-post pricing) and the situation in which the seller can pit buyers against each other in determining what information is to be sold. Within the context of ex-ante pricing, we also consider the situation in which the precise information offered to the decision makers is not exogenously given but is selected by the seller.

Keywords: Extensive form games, information.

JEL classification: C72, C73, D82

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1. Introduction

The problem of reaching decisions under uncertainty occurs in many economic situations. In some of these situations, there is an 'outsider' who has access to relevant information which can be sold to the decision makers. For instance, a consultant may have access to data about outputs in a duopoly. An expert may know more about quality than the players in a signaling game[1] whereas a spy may have information about the moves in a strategic interaction between two firms or countries.

A number of papers deal with the role of an information seller in strategic interactions. Sakai (1985) considers a duopoly in which two firms face uncertainty both about their own marginal costs and the marginal costs of their opponent. Both firms can obtain information about these uncertain facts from a market research agency. Sakai investigates how the additional information provided by the agency influences the behavior of the firms. In Admati and Pfleiderer (1986), traders on a speculative market can buy private information about the payoff of a risky asset from a monopolistic information seller. Kamien *et al.* (1990) explores the situation in which an outsider can provide the players of an extensive form game with information about the play of the game. Other relevant papers include Allen (1986), Green (1981), Levine and Ponssard (1977) and Milne and Shefrin (1987).

In nearly all these settings, the information holder is non-strategic: he is present to provide decision makers with additional information but does not have strategic incentives for his behavior.[2] The price of information is usually assumed to be exogenously given. However, it seems relevant in most of the examples above to consider the incentives of the information seller as well. In this paper, we explore several game theoretic models for the situation in which decision makers can buy additional information from a monopolistic information seller. In all of these models the information seller is assumed to be a strategic actor. As a consequence, the prices for information chosen by the seller are no longer given but arise from a strategic interaction between the decision makers and the seller. The difference between the models lies primarily in the way prices are chosen by the seller. Questions that we want to consider are what prices should be chosen by the seller for the information he offers, the income the seller can expect from selling information, and how the outcome is affected by the price setting process.

[1] For a model of agents who can sell information about quality see Lizzeri (1998).

[2] Lizzeri (1998) is an exception. Lizzeri considers a model in which a seller of an object has private information about the quality. A buyer is interested in the purchase of the object. Lizzeri adds to this model a "certification intermediary" who can test the good and learn its quality. The intermediary is a profit maximizing monopolist. In Lizzeri's model, the intermediary moves first, and sets a price for his services and a disclosure rule specifying what information will be revealed to the buyer. The model is thus an example of what we term "ex ante pricing with endogenous signal mechanisms." Lizzeri shows that the intermediary may release less than full information, and that he is able to extract much of the surplus generated by the market.

The situation is modeled as follows. Starting from a finite extensive form game, the *base* game, an *information seller* is added who has access to some information about the play of the game. This might be information about a choice by nature or about the past moves of one or more of the players. In Sections 2 through 5 we assume, for simplicity, that the seller is perfectly informed about all events that take place in the game. Section 6 describes how our model can be extended to the case of a partially informed seller.

The information seller has the ability to sell this information to the players of the base game, who are hence called *buyers*. Formally, the seller offers a finite collection of *signal mechanisms* at each information set. A signal mechanism is a device which assigns to every node of the information set a probability distribution over some finite set of signals. For each signal mechanism the seller chooses a price. The buyer who controls this information set observes the menu of mechanisms and prices and decides which mechanism to buy. To give the buyer the outside option not to buy any information at all, it is assumed that the trivial mechanism (revealing no extra information) is always available for free at every information set. After buying a mechanism, the buyer is provided with the signal produced by this mechanism and then chooses an action. The payoff for the seller is his expected income from selling information. The payoff for a buyer is the difference between the expected payoff in the base game and the expected amount paid for information. The game obtained in this way is called the *extended game*.

To formalize the assumption that both seller and buyers act rationally we assume that the seller and buyers play a sequential equilibrium in the extended game. The highest payoff that can be achieved by the seller in a sequential equilibrium of the extended game is called the *information value* and is used as a measure for the seller's power.

There are several dimensions which have not yet been specified, and along which a sensible model of information selling might vary, depending on the underlying economic situation. First, when and how is the price of a signal mechanism set – at the beginning of the game or when the corresponding information set is reached? Is the price a matter between the seller and the buyer at that information set, or can other buyers be involved? Who else knows the price that is set for a particular mechanism? Finally, what is the information seller able to sell: can he, for example, sell a garbling of his information?

Rather than attempt to analyze all combinations of these assumptions, we content ourselves with an illustrative tour of the various settings. We begin by considering a world in which pricing is a matter solely between the seller and the buyer at a given information set. The scenarios differ in terms of the timing of price setting, and how much latitude the seller has in determining what is to be sold.

With *ex-ante* pricing, the seller chooses prices at the beginning of the game, and these prices are known to everyone before the game begins. Ex-ante pricing thus involves a degree of commitment power on the part of the seller, in the sense that he can commit to prices at the beginning of the game that he may want to change once the game is under way. Such commitment might make sense if, for example, the base game occurs repeatedly with different information buyers, and the same seller (consider for example, a credit reporting agency). In the basic version of *ex-ante* pricing, the set of mechanisms available at each information set is fixed exogenously, and prices can only be conditioned on the a-priori information of the player to whom the information is being sold (the seller may know more, including what he sold to other players, observed at earlier information sets, or even what the outcome of the signals at the current information set are). As we shall see, this is a real restriction. However, it has the advantage that both the seller and buyer know what the price should be at the time of the transaction. Within this setting, we show existence of sequential equilibria, something which is not obvious since prices come from a continuum.

We show by example that the seller would sometimes like to condition his pricing at an information set on information not visible to the buyer, such as the node reached, or what was sold to another player at an earlier information set. This may be desirable even though by doing so, the seller is giving away part of his information for free through the price setting process. We also show that the seller may wish to commit to a mechanism in which the signal is garbled. Effectively, the seller sometimes wants to sell an amount of information which is intermediate between two of the exogenously given signal mechanisms. To capture these possibilities, we extend our model to a very general form of ex-ante pricing in which the seller can endogenously select the set of mechanisms he wants to offer at each information set. In this setting, before the base game begins, the seller can design and offer for sale any set of maps from *his* information to signals, and can let the pricing and availability of these mechanisms depend (in potentially stochastic fashion) on which node has been reached. We show that the seller can restrict himself to offering at every information set exactly one non-trivial mechanism, the signals of which can be interpreted as recommending actions to the buyer. The result thus has a flavor similar to revelation principle arguments (see Myerson (1979) and Dasgupta *et al.* (1979)).[3] Using this result, we are able to show that sequential equilibria with endogenous mechanism selection always exist. Of course, this setting requires a larger degree of commitment on the part of the seller since buyers have to be willing to trust that the seller is charging the price he said he would as a function of the information that is unobservable to the buyer, and similarly, the buyers have to believe in the integrity of the garbling process even though the seller might gain by misrepresenting information.

[3] Because we are working in the extensive form, the result also has a connection to Forges (1986), which studies the effect of introducing communication into games, including the possibility of randomization devices which give correlated but private signals to the various players. Our focus differs from Forges in considering the ability of the seller to add information directly about the play of the game so far at particular decision points, and in explicitly thinking about the incentives of an information seller. It is however the case that one source of the seller's profits in our setting (especially when we turn to unconstrained mechanisms in the penultimate section) can be through providing the sort of signal that Forge discusses.

Under ex-ante pricing, the seller is able to commit to a set of prices (or mechanisms) before the base game starts. In some settings, this does not seem plausible. So, we turn to a model of *ex-post* pricing. In this model, the price of each mechanism is chosen at the moment of sale. This price is known only to the seller and buyer at that information set. So, ex-post pricing involves a very minimal degree of commitment on the part of the seller. We show two examples in which the lowered commitment power relative to ex-ante pricing hurts the information seller. We also argue that the extra flexibility of ex-post pricing compared to the basic version of ex-ante pricing can imply a revenue advantage for ex-post pricing. So, the ranking between the basic form of ex-ante pricing and ex-post pricing is ambiguous. However, because ex-ante pricing with endogenous selection of signal mechanisms incorporates similar flexibility, the information value with endogenous selection of mechanisms is always at least as great as that with ex-post pricing.

Under ex-post pricing the seller might be tempted to reveal part of what he knows at an information set in order to raise the buyer's willingness to pay for the rest. We show an example in which this occurs, and where the seller's inability to commit not to do this lowers his profits as viewed from the beginning of the game.

In the settings discussed so far, information transactions at an information set h are purely between the seller and the player who controls h. However, sometimes the most valuable thing for a seller to do with information at h is to extract money from some player who wants the seller not to reveal information at h. Or, a buyer might wish to publicly precommit to buying (or not buying) information at a particular information set so as to either discourage or encourage other players from playing so that the information set is reached. Under *unconstrained* pricing, we generalize ex-ante pricing by assuming that the seller can set up arbitrarily complicated schemes in which buyers' actions before the play of the base game interact to determine the prices of the mechanisms offered. This may thus seem analytically intractable. However, we exhibit a simple mechanism that can achieve as much for the seller as any unconstrained setup. Using this, we provide a characterization of the information value with unconstrained pricing, and show that the information value in this case is at least as high as in the basic ex-ante case.

In all models described above, we assume that the seller is perfectly informed about all events that take place in the base game, as well as in the extended game. Since this assumption may not be completely realistic in some situations we take a short look at the case of a partially informed seller at the end of this paper. Each of the models can be extended immediately to this situation and similar results would be obtained. A typical phenomenon which arises with an incompletely informed seller is the fact that the buyers' purchase behavior serves as a signaling device to the seller from which he can extract information about the true state of the world. For instance, the seller may learn about a buyer's

type by looking carefully at his purchase behavior in the past.
The paper is organized as follows. In Section 2 we lay out the standard model. The cases of ex-ante and ex-post pricing are analyzed in Sections 3 and 4, respectively. Section 5 considers the case of unconstrained pricing. We conclude by briefly discussing the case of a partially informed seller in Section 6.

2. Model

We begin with the players in a finite extensive form game (called the *base game*) who, whenever they find themselves at one of their information sets, can purchase a *signal mechanism* from a finite set of mechanisms. The signal mechanisms are offered by a monopolistic information seller who sets prices for the mechanisms. We assume that the seller is always perfect informed about the node being reached in the base game. A *signal mechanism* at an information set h is a function assigning to every node $x \in h$ some randomization over a finite set of signals (in some cases, we also allow the randomization to depend on other things the seller knows). Examples include the perfect mechanism, in which each node maps to a distinct signal, the trivial mechanism, in which all nodes map to the same signal, and mechanisms which provide a noisy signal of the node reached. After buying a mechanism, the player observes the signal produced by the mechanism and chooses an action. This gives rise to a new game, called the *extended game,* which is played by the seller and the players of the original extensive form game, who from now on will be called *buyers.*
In order to specify the rules of the extended game, we make the following assumptions:

1. The purchase of a mechanism by one buyer is not observed by other buyers.
2. The signal produced by a mechanism is only observed by the buyer who bought the mechanism.
3. The trivial mechanism is offered at every information set at price zero. In this way, a buyer can always choose not to buy any information at all.

The payoff for the seller in the extended game is the expected income from selling signal mechanisms. The payoff for a buyer is his expected payoff in the base game minus the expected amount paid for signal mechanisms.
We assume that both seller and buyers act rationally. This is formalized by requiring that seller and buyers play a sequential equilibrium of the extended game. In general, there is no unique sequential equilibrium payoff for the seller, which makes it difficult to predict the outcome for the seller. One could think of several ways to overcome this problem, for instance by concentrating on the highest possible payoff for the seller in a sequential equilibrium, or the lowest possible payoff, or the expected income with respect to some probability measure

on the set of sequential equilibria. We think, however, that the first option provides a reasonable measure for the strength of the seller in the extended game. The highest payoff that can be obtained by the seller in a sequential equilibrium of the extended game is called the *information value.*

3. Ex-ante Pricing

Under *ex-ante pricing,* the seller chooses his pricing strategy before the buyers start playing the base game. This pricing strategy is common knowledge among the buyers before the base game starts. Hence, each pricing strategy by the information seller defines a subgame in which at each information set, the buyer who controls that information set chooses what mechanism to purchase and what action to take as a function of the signal received.
We examine two cases which we think of defining the extreme points of a large set of possible models of ex-ante pricing. We call these *basic ex-ante pricing* and *ex-ante pricing with endogenous signal mechanisms.*

3.1. Basic ex-ante pricing

Under *basic ex-ante pricing,* the set of mechanisms available for sale is exogenously specified and assumed to be finite. Further, the price of a mechanism can depend only on the information available to the buyer at the moment the information set in question is reached. The price may, for instance, depend on previous information purchases by the same buyer, but it cannot depend on unobserved information sales to other buyers, or on what else the seller knows at the moment of purchase. Moreover, the seller cannot construct garblings of his basic information. This seems to us a reasonable model of a setting in which the seller has the power to commit to posted prices, but does not have the power to commit to more elaborate schemes (we consider the implications of such power later). An example here would be a credit reporting agency which has the ability to commit to a price for a report about a company but can not change those prices to reflect either who else has bought information about the same company or whether the report contains damaging information about that company.
Formally, the extended game is defined as follows. At every information set h of the base game controlled by buyer i, let i_h be the vector containing (a) the profile of mechanisms bought by i at previous information sets and (b) the profile of signals observed by him at previous information sets. The vector i_h is called the information state and I_h denotes the set of possible information states for buyer i at h. A pure strategy p for the seller is to assign to every information set h and each information state $i_h \in I_h$ a price for each of the mechanisms offered at h.[4] In the remainder of this section, p will simply be called a price vector.

[4] It is sufficient to concentrate on pure strategies for the seller since the prices announced by the seller are observed by all buyers before the base game starts. Therefore, randomizing over

A behavior strategy for buyer i is to assign to each triple (p, h, i_h) where h is controlled by i (a) a randomization over the mechanisms that can be bought at h and (b) a function assigning to each possible signal that can be observed at h a randomization over actions available at h. Note that in the model as described, the seller makes his decisions before the base game starts whereas the buyers reach their decisions at the time the corresponding information set in the base game is reached.

It remains to specify the belief systems for the buyers. Suppose that buyer i finds himself at information set h with information state i_h and has observed price vector p at the beginning of the game. Buyer i does not know at which node in h he is nor does he know which mechanisms have been bought by other buyers at previous information sets and the signals that have been observed by them. The states of the world among which buyer i can not distinguish are therefore triples (x, m, s) where x is a node in h, m is a vector of mechanisms bought by other buyers at previous information sets and s is a corresponding vector of signals observed by these buyers. Let this set of states by denoted by Θ_h. A belief system for buyer i is to specify for each triple (p, h, i_h), where h is controlled by i, some randomization over Θ_h.

Existence of sequential equilibria with basic ex-ante pricing

Under all the situations we consider in this paper, the seller can choose from a continuum of prices (and in some cases from continua on other dimensions as well). Hence, the extended game is not a finite extensive form game and so the existence of sequential equilibria in the extended game is not obvious. We show that in the case of basic ex-ante pricing, a sequential equilibrium can always be found. The key to this construction is that each choice by the seller creates a finite extensive form subgame, where the equilibrium set of the subgame varies upper hemi continuously in the seller's choice.

Theorem 3.1. *Every extended game with basic ex-ante pricing has a sequential equilibrium.*

Proof. At the beginning of the extended game, the seller chooses a price vector p which then becomes common knowledge. By $\Gamma(p)$ we denote the subgame which starts when the buyers observe p. In the subgame $\Gamma(p)$ the seller has no longer an active role since he made all his choices at the beginning. Whenever a buyer is called upon to play at an information set h in $\Gamma(p)$, the set I_h of possible information states is finite. Since there are only finitely many mechanisms, signals and actions at each information set, the subgame $\Gamma(p)$ is a finite extensive form game and therefore has a sequential equilibrium.

prices has no effect in the extended game.

Let $\bar{p}$ be a price high enough to ensure that no buyer is prepared to pay more than $\bar{p}$ for any signal mechanism. (Recall that the trivial mechanism is always available at price zero.) Let P be the set of price vectors in which all prices are at most $\bar{p}+1$. For every $p \in P$, the set $SE(p)$ of sequential equilibria in $\Gamma(p)$ is a non-empty, compact set. Since the seller's payoff depends continuously on the assessment in $\Gamma(p)$, there is thus a sequential equilibrium $(\sigma^p, \beta^p) \in SE(p)$ which induces the highest payoff for the seller among all sequential equilibria in $SE(p)$. Here, σ^p denotes the behavior strategy profile and β^p the belief system of the players in $\Gamma(p)$.

Let $\bar{u}_s = \sup_{p \in P} u_s(\sigma^p, \beta^p)$, where $u_s(\sigma^p, \beta^p)$ is the payoff for the seller induced by (σ^p, β^p). Choose a sequence $(p^k)_{k \in \mathbb{N}}$ in P such that $\bar{u}_s = \lim_{k \to \infty} u_s(\sigma^{p^k}, \beta^{p^k})$. Since P is a compact set, we may assume without loss of generality that p^k converges to some $p^* \in P$. The space of assessments in the subgame $\Gamma(p)$ is the same compact set for every p. Hence, we may assume without loss of generality that $(\sigma^{p^k}, \beta^{p^k})$ converges to some assessment (σ^*, β^*). It is easily checked that $SE(\cdot)$ is upper hemi continuous. Hence (σ^*, β^*) is a sequential equilibrium in $\Gamma(p^*)$.

Consider the assessment of the extended game in which the seller chooses the price vector p^*, the buyers act according to (σ^*, β^*) after observing that p^* has been chosen and play an arbitrary sequential equilibrium in $\Gamma(p)$ after any other price vector $p \neq p^*$. We show that this is a sequential equilibrium of the extended game.

By construction, the seller's payoff is the highest payoff that can be achieved by the seller in any sequential equilibrium of any subgame $\Gamma(p)$ with $p \in P$. Since no buyer is willing to pay more than $\bar{p}$ for information, it follows that it is the highest payoff that can be achieved by the seller in any sequential equilibrium of any possible subgame $\Gamma(p)$. In particular, a sequential equilibrium of $\Gamma(p)$ clearly remains an equilibrium if every element of p that is greater than $\bar{p}+1$ is replaced by $\bar{p}+1$. Therefore, the seller can never improve his payoff by choosing another price vector, which implies that we have a sequential equilibrium of the extended game. □

Since the exhibited equilibrium gives the seller at least as high a payoff as in any other sequential equilibrium, it follows that it supports the information value.

Examples

We next turn to two examples showing that under ex-ante pricing, the seller would benefit from a richer set of alternatives. The first example shows that the seller may want to commit to prices in which the price at an information set depends on information available to the seller but not to the buyer. In particular, we show an example in which the seller may want to condition the price of a mechanism on the node reached at the corresponding information set.

Example 3.1. *Desirability of conditioning price on what the seller knows.* Consider the following game in which the seller offers trivial and perfect information to both player 1 and 2.

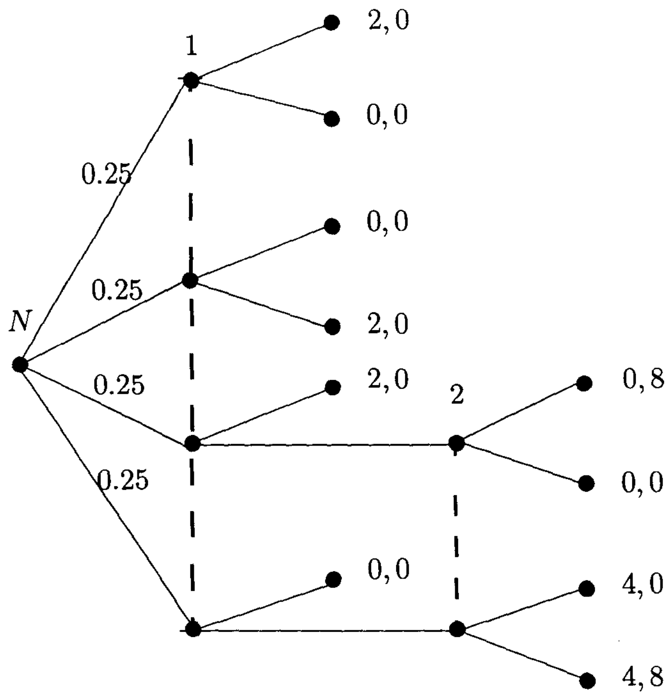

Figure 1

In a world in which the seller can only condition the prices on what the buyer knows, the seller can at most get 2. This payoff is achieved in the sequential equilibrium in which the seller charges 100 to player 1 (therefore forcing player 1 not to buy information), player 1 goes down, the seller charges 4 to player 2 and player 2 buys the perfect mechanism with probability one. It can be verified that it is always optimal for the seller not to sell information to player 1. Since player 2 is at most willing to pay 4 for information, the seller can not expect more than 2 in any sequential equilibrium.
However, if the seller can make the price dependent on the node then he can obtain $2\frac{1}{2}$ in a sequential equilibrium. This can be achieved as follows. The seller charges price 1 to player 1 if one of his two upper nodes is reached and charges price 100 otherwise. Player 1 buys after observing price 1 and does not buy otherwise. The

seller charges price 4 to player 2 who buys the perfect mechanism. Hence, this example illustrates that conditioning the price on what the seller knows but the buyer does not can be beneficial for the seller.

In our next example, it is desirable for the seller to offer a garbling of his information.

Example 3.2. *Desirability of garbling.*

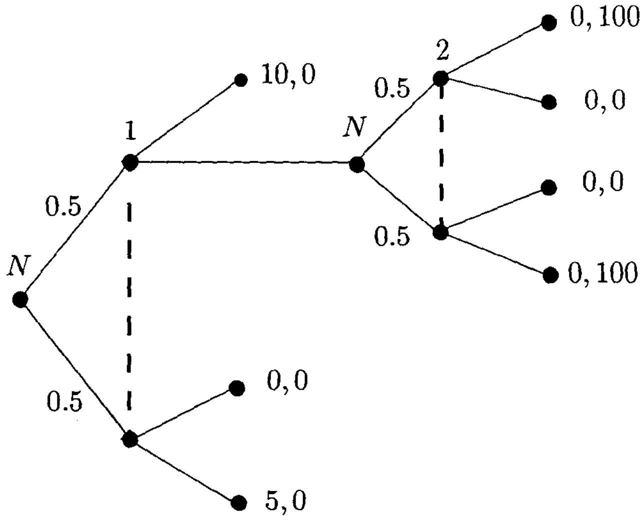

Figure 2

Assume that the seller has available the perfect and trivial mechanisms at each information set. When player 1 buys perfect information, he chooses up at his upper node. When he buys trivial information, he again chooses up. Hence, player 2 is never reached. So the best the seller can do is sell perfect information to player 1 at price 3. However, assume the seller can commit to the following mechanism: If the upper node is reached, then $\frac{1}{2}$ the time I will send you signal "A", and $\frac{1}{2}$ the time I will send you signal "B." If the lower node is reached, I will send you "B". I will sell you this for free. Then, when "B" is received, player 1 updates to probability $\frac{2}{3}$ that he is at his lower node, and so down is a best response. The information seller can then sell player 2 perfect information $\frac{1}{4}$ of the time at price 50, gaining more than 3.

So, if the seller is able to commit to mechanisms more complicated than under basic ex-ante pricing, there is clearly room for him to improve his revenue. There are clearly a plethora of different models which give the seller varying degrees of ability to condition price on hidden information or sell randomizations of his signals. Rather than examine a large number of intermediate models, we turn now to a model of ex-ante pricing in which the seller is given total freedom in this regard.

3.2. Ex-ante pricing with endogenous signal mechanisms

With basic ex-ante pricing we assumed that the set of signal mechanisms offered by the seller at each information set is exogenously given. In this section we analyze the situation where the seller can choose the set of mechanisms he wants to offer. The seller is assumed to be capable of offering *any* possible signal mechanism. That is, he can construct and offer for sale any garbling of his information. For simplicity, we consider the case in which the seller knows exactly which node is reached for each information set of the base game. At each information set the seller selects a randomization over a finite number of *menus*, where a menu consists of a finite set of mechanisms and corresponding prices. We assume that the randomization can be made dependent on everything the seller knows at that point in time, including the node that has been reached. As such, the actual menu offered may serve as a signal to the buyer about which node he is at. We assume that the trivial mechanism is available in every menu and is always offered for free. All the choices by the seller (about the composition of each menu and the probability with which each menu is offered as a function of the seller's information) are made before the actual game starts and are observed by all buyers. The outcome of the randomization over which menu to offer is not: the actual menu chosen by the seller is only observed by the buyer at h and not by the other buyers. This situation can therefore be seen as an extension of the basic ex-ante case where prices can be made dependent on the seller's information. We term this case *ex-ante pricing with endogenous signal mechanisms*, or, more briefly, the *endogenous case.*

We assume that the seller at each point of the game knows the node that has been reached and the outcomes of all information transactions at all information sets previously reached. A pure strategy r for the seller is a function assigning to each information set a randomization over a finite set of menu's, where the randomization is allowed to depend on the node and the sequence of information transactions that have occured until that moment. Here, each menu consists of a finite number of mechanisms and corresponding prices.

At every information set h controlled by buyer i let i_h be the information state for player i containing (a) the menus offered by the seller (as the outcome of some randomization) at i's previous information sets, (b) mechanisms bought by i in the past and (c) signals observed by i in the past. The set of information states

for buyer i at h is denoted by I_h.
Let r be the seller's strategy and M the actual menu chosen at h. A behavior strategy for a buyer assigns to each profile (r, h, i_h, M) where h is controlled by i (a) a randomization over the mechanisms that can be bought at M and (b) a function assigning to each possible signal that can be observed at M a randomization over actions available at h.
At every information set h, let Θ_h be the set of possible states of the world among which the buyer at h can not distinguish. The space Θ_h consists of quadruples $(x, \tilde{M}, m, s)$ where x is a node in h, $\tilde{M}$ is a vector of menus offered to other buyers at previous information sets, m is a corresponding vector of mechanisms bought by these buyers at these information sets and s is a vector of signals observed by them at these information sets. A belief system for buyer i assigns to every profile (r, h, i_h, M), where h is controlled by i, some randomization over Θ_h.
Since the seller can always do whatever he did before under basic ex-ante pricing, and since the seller in both cases chooses before the base game begins, it is clear that ex-ante pricing with endogenous selection of signal mechanisms always yields a higher information value than the base version of ex-ante pricing.
At first sight, a full analysis of this setting seems quite intractable. However, we show that the seller can restrict himself to a so-called *simple* strategy in which at every node of a given information set he offers the same single menu with probability one. This menu consists of only one non-trivial mechanism, which can be interpreted as recommending actions to the buyer.

Definition 3.2. *Say that a seller's strategy is simple if for each h, i_h, there is a single menu M_{h,i_h} which the seller offers with probability 1, where M_{h,i_h} consists of the trivial mechanism (at price 0) plus at most one non-trivial signal mechanism, and where the non-trivial mechanism has signal space equal to the set of actions available at h.*

Note that in a simple strategy, the set of information states i_h at h is just equal to a sequence stating whether or not t/he corresponding buyer has bought non-trivial information at previous information sets and, if so, which actions have been recommended.
Let r be the strategy chosen by the seller before the actual game starts. For every r, the buyers enter a subgame $\Gamma(r)$ in which the seller is no longer a strategic actor since all his actions are already captured by r. Since the subgame $\Gamma(r)$ is finite it contains a sequential equilibrium of the game between the buyers.

Lemma 3.3. *Let (σ, β) be a sequential equilibrium of the buyers in some subgame $\Gamma(r)$ of the endogenous case. Then, there is a simple strategy $\bar{r}$ and a sequential equilibrium $(\bar{\sigma}, \bar{\beta})$ in $\Gamma(\bar{r})$ giving the same payoff for the seller and buyers, and the same distribution over terminal nodes of the base game as (σ, β) in $\Gamma(r)$.*

Proof. Consider the subgame induced by r, and the associated sequential equilibrium of the subgame. We proceed in two steps. First, create an automaton for each h, i_h belonging to i which takes care of viewing the menu offered, deciding on an information purchase, observing the signal, and randomizing over actions according to the specified equilibrium. The automaton does not *choose* actions for the base game. Rather, at each point where i would choose an action from the base game, i's automaton *recommends* an action to player i, where the recommendation is observed only by player i. The automaton is thus a signal mechanism with signal space equal to the action space at h. The signal produced is equal to the randomization over actions that the buyer would have chosen at the same point in time in (σ, β). Note that this randomization over actions already includes the randomization over menu's and the buyer's purchase behavior at h, i_h in (σ, β).
Secondly, we construct a simple strategy for the seller in which at each h, i_h the seller offers exactly one menu containing the trivial mechanism and the automaton. We claim that it is a sequential equilibrium of the induced game for players to always purchase the automaton and follow the recommendations given.
To see that, once one has the automaton purchased, it is optimal to follow the recommended actions, note first that from the point of view of other players, player i's behavior has not changed. And, since i's information in the new game is a garbling of his information in the old, it is clear that if i has a profitable deviation from a recommended action at some information set, then there was also a profitable deviation under the richer original information structure.
In the game created, at the point where h belonging to player i is first reached, i's information i_h consists of whether or not the automaton has been purchased at his previous information sets and a sequence of recommendations at these information sets. Let the price in $\bar{r}$ of the automaton at h, i_h be the expected price that i pays in the original sequential equilibrium at the same point in time. Since we started with a sequential equilibrium of the subgame generated by r, it is optimal for all players to purchase the automaton at each h, i_h at this price. Hence, we have a simple strategy $\bar{r}$ and a sequential equilibrium $(\bar{\sigma}, \bar{\beta})$ in $\Gamma(\bar{r})$ which does not change the expected payoff to the information seller, nor the distribution over the terminal nodes of the base game. □

A key implication of the lemma is that it can be used to show existence of a sequential equilibria.

Theorem 3.4. *The extended game of the endogenous case always has a sequential equilibrium.*

Proof. After the strategy r has been chosen by the seller, the buyers enter the finite subgame $\Gamma(r)$. Let $(\sigma(r), \beta(r))$ be the sequential equilibrium in $\Gamma(r)$ which gives the highest payoff for the seller.[5] We need to show that there is a strategy r^* such that

$$u_s(r^*, \sigma(r^*), \beta(r^*)) = \max_r u_s(r, \sigma(r), \beta(r))$$

where $u_s(r, \sigma(r), \beta(r))$ is the payoff for the seller if he chooses the strategy r and the buyers play $(\sigma(r), \beta(r))$ in $\Gamma(r)$.
By Lemma 3.3 it is enough to show that the *lhs* achieves a maximum over simple strategies r. Further, as we argued before, we may assume that the price $p_{h,i_h} \in [0, \bar{p}+1]$ where $\bar{p}$ is such that no buyer is ever prepared to pay more than $\bar{p}$ for any mechanism. Note that all simple strategies have the same structure: a price for each h, i_h, and a randomization over recommendations for each possible history that has been observed by the seller, where for all simple mechanisms, the history for the seller at a node x consists of whether or not the non-trivial mechanism was purchased at each information set on the path to x, and the recommendation made (everything else is captured by the seller's knowledge that he is at x). There are at each x a finite set of such possible histories. But then it is clear that the set of simple strategies available to the seller is compact (consisting of a finite price vector plus a finite set of maps from the fixed set of histories to randomizations over recommendations). It is also easily seen that the set of sequential equilibria of subgames is *uhc* in the profile chosen by the seller. The result follows. □

Note that in the discussion of Example 3.2, if one interprets message"A" as "choose up" and message "B" as "choose down", then we are in the framework of this result. Note also that this example illustrates the need for more commitment under the endogenous case than under basic ex-ante pricing. It is key that the seller cannot lie about his randomization (which may be tougher to check than it is to see whether the seller misstated an exogenously generated signal). If he could lie, then he would be tempted to say "B" always when the top node is reached, destroying the equilibrium.

4. Ex-post Pricing

Under the two notions of ex-ante pricing we have considered, we have assumed that the seller has the ability to commit at the beginning of the game to a set of prices. In this section, we consider the situation in which the seller does not have this commitment power. Under *ex-post pricing,* the price of a mechanism is chosen at the moment that the corresponding information set is reached, and known only to the player at that information set. This price is allowed to depend on the node that has been reached. The set of mechanisms offered at every information set is fixed. A setting in which it would be natural for price not to depend on the node at the current information set would be if the seller and buyer agree on which mechanism is to be purchased, and only then the outcome of the

[5]Such a sequential equilibrium $(\sigma(r), \beta(r))$ exists since the set of sequential equilibria in $\Gamma(r)$ is compact and the seller's payoff depends continuously on the assessment.

mechanism is realized. So for example, first a home buyer and inspector agree on what will be checked, and only then is information gathered. In other settings, it is clearly more natural to assume that the information seller already knows the state of the world at the time that the information set is reached.

At every information set controlled by buyer i let i_h be the information state containing (a) prices for buyer i mechanisms chosen at earlier information sets, (b) mechanisms bought by him in the past and (c) signals observed by him in the past. A behavior strategy for the seller is to assign to every h, i_h and every node $x \in h$ a randomization $r_{h,i_h}(x)$ over a finite number of price vectors. Here, a price vector contains prices for all mechanisms offered at h. A behavior strategy for buyer i assigns to every h, i_h controlled by i and every price vector p_{h,i_h} chosen by r_{h,i_h} (a) a randomization over mechanisms offered at h and (b) a function assigning to every signal a randomization over actions at h.

At a given information set h, the corresponding buyer is not informed about the prices offered to other buyers at previous information sets, the mechanisms bought there and the signals observed by these buyers. Hence, the space Θ_h of states of the world among which the buyer at h can not distinguish contains quadruples $(x, \tilde{p}, m, s)$ where x is a node in h, $\tilde{p}$ is a list of price vectors offered to other buyers at previous information sets, m is a vector of mechanisms bought at these information sets and s is a vector of signals observed there. A belief system for buyer i assigns to every triple (h, i_h, p_{h,i_h}), where h is controlled by i, a randomization over Θ_h.

This difference between the degree of commitment involved in ex-ante and ex-post pricing can have major consequences for the revenues of the seller. To see this, consider the following examples.

Examples

Example 4.1. *Inability to promise a low price hurts the seller.*

Consider the following extensive form game in which the seller offers the trivial and perfect mechanism at the second information set of player 1.

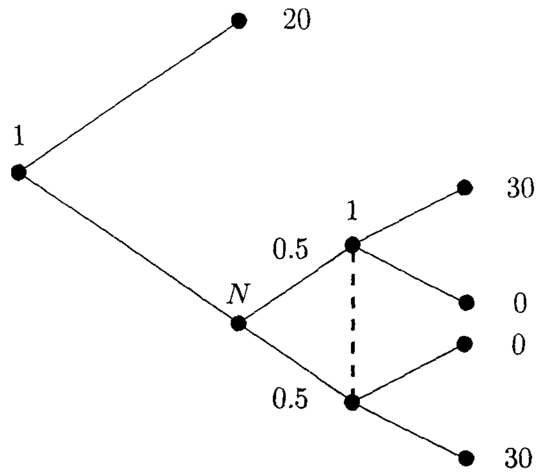

Figure 3

Player 1 is only willing to go down at the beginning if the seller charges at most 10 for the perfect mechanism. However, once the second information set is reached, the perfect mechanism is worth 15 to player 1. With ex-ante pricing, the seller can commit to charge price 10 in a sequential equilibrium since the price is chosen *before* the actual game starts. Therefore, there exists a sequential equilibrium in which player 1 goes down at the beginning and pays price 10 for information. Since the seller can not expect more than 10, the information value with ex-ante pricing is equal to 10.

With ex-post pricing, sequential rationality forces the seller to set the full price 15 for the perfect mechanism since the price is chosen *after* reaching the information set. Knowing this, player 1 will avoid this information set by going up at the beginning. The information value with ex-post pricing is therefore equal to 0.

Example 4.2. *Inability not to sell information in the future hurts the seller at the beginning.*

In the game below, trivial and perfect information is offered at both information sets of player 2.

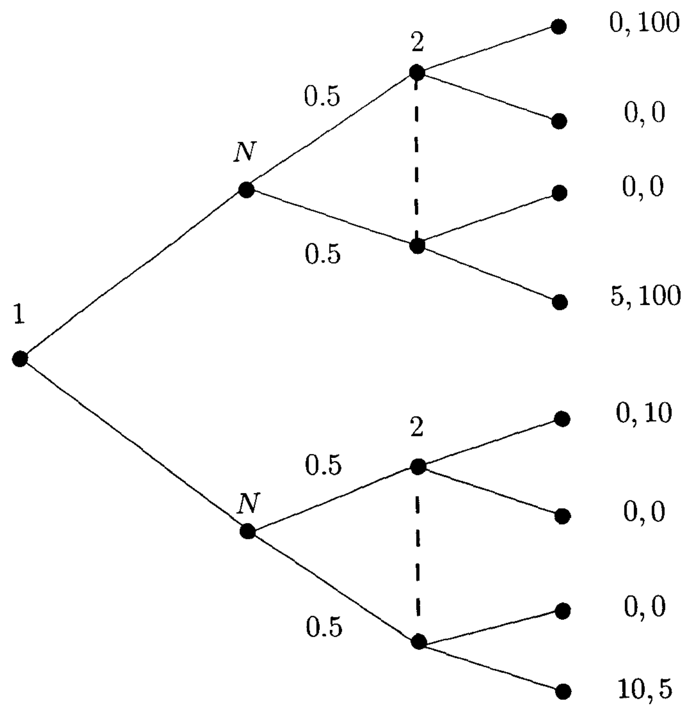

Figure 4

The seller is interested in reaching the upper information set of player 2 since information is most valuable here. This can be done by committing not to sell information to player 2 at his lower information set. Player 2 will then choose up at his lower information set, making it unattractive to player 1 to play down. This enables the seller to reach player 2's upper information set where information can be sold at price 50.

With ex-post pricing, sequential rationality forces the seller to sell information to player 2 at his lower information set at price 2.5. Knowing this, player 1 strictly prefers to go down. Therefore, the seller can not expect more than 2.5 in a sequential equilibrium. So, the seller is hurt by his inability to commit not to sell information to player 2 at his lower information set.

Example 4.3. *Effect of the ability to reveal partial information for free before setting price.*

Up until now, in discussing ex-post pricing, we have assumed that the seller's only communication with the buyer is in setting prices. However, one could imagine that the seller can convey part of his information before setting the price for the rest. For example, a credit reporting agency might send a letter to a potential customer which says "We have the information that a firm you own debt in has lost its AAA credit rating. Here is what it will cost you to get the details." To see the effect of this, consider the following example.

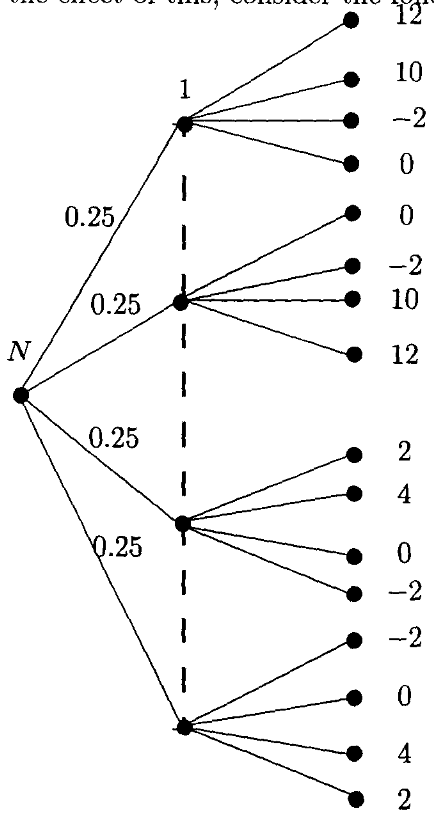

Figure 5

Assume the seller can offer trivial and perfect information. It is easily seen that with no information, the buyer gets an expected payoff of 3, while with perfect information, he can earn 8 in expectation. Hence, in our standard ex-post set up, the seller can obtain 5 in a sequential equilibrium.

Now suppose that the seller is allowed to reveal a part of the information before setting the price. In particular, suppose he can tell the buyer whether or not the play has reached one of the two upper nodes. Now, conditional on being at one of the two upper nodes, the buyer can get a payoff of 6 in expectation by choosing one of his two outer actions. With perfect information, he can earn 12. So, he will pay a further 6 for perfect information. Thus, if the game reaches one of the two upper nodes, it is optimal for the seller to tell player 1 that this is so. However, knowing the strategy of the seller, if player 1 is not told that he is at the two upper nodes, then he will know that he is at one of the two lower nodes. In the latter case, player 1 is only willing to pay 2 for perfect information, since by choosing one of the two inside actions, he gets 2 without further information, while with perfect information he earns 4. Viewed from the beginning of the game, the overall payoff for the seller thus drops to $.5 \cdot (6) + .5 \cdot (2) = 4$. Therefore, the ability to reveal part of the information before setting price hurts the seller.

Comparison of information values

We have seen a number of examples in which revenues were higher with ex-ante than with ex-post pricing. One might be tempted to conclude that ex-ante pricing is always at least as good as ex-post pricing. However, this is false for basic ex-ante pricing. In particular, ex-post pricing allows for two things that basic ex-ante pricing does not: price can depend on the node that has been reached in the information set, and ex-post pricing may involve randomized pricing schemes.[6] Under ex-ante pricing with endogenous signal mechanisms, the seller does have this flexibility. Hence, the result does hold in that case.

Theorem 4.1. *The information value under ex-ante pricing with endogenous signal mechanisms is always greater than or equal to the information value with ex-post pricing.*

Proof. We prove the result by showing that every sequential equilibrium with ex-post pricing induces a payoff for the seller which is less or equal than the information value in the endogenous case.
Now, let (r, σ, β) be a sequential equilibrium of the ex-post case in which $r = (r_{h,i_h}(x))$ is the behavior strategy of the seller, σ is the behavior strategy profile of the buyers and β is the belief system of the buyers. We transform r into a behavior strategy $\bar{r}$ of the ex-ante endogenous case. At every (h, i_h, x) let $P(h, i_h, x)$ be the finite set of price vectors of the ex-post case over which $r_{h,i_h}(x)$ randomizes and let $P(h, i_h) = \cup_{x \in h} P(h, i_h, x)$. Let $\bar{m}_{h,i_h}$ be the mechanism with signal space $P(h, i_h)$ defined by

$$\bar{m}_{h,i_h}(x)(p) = r_{h,i_h}(x)(p)$$

for every node $x \in h$ and every $p \in P(h, i_h)$. Here, $\bar{m}_{h,i_h}(x)(p)$ is the probability that signal p will be sent by $\bar{m}_{h,i_h}$ if node x is reached. Analogously, $r_{h,i_h}(x)(p)$ is the probability that price vector p is chosen by r_{h,i_h} if node x is reached. Let $\bar{p}$ be the expected price that the buyer at h, i_h pays if the seller chooses the randomization r_{h,i_h} over price vectors, beliefs at h are according to β_{h,i_h} and the buyer acts according to σ_{h,i_h}. Let $\bar{r}_{h,i_h}$ be the simple strategy for the seller in the endogenous case putting at every h, i_h and every node $x \in h$ probability

[6] Under basic ex-ante pricing, randomizing over pricing schemes does not produce any effect since the pricing scheme is observed by all buyers before the game starts.

one on the menu with mechanism $\bar{m}_{h,i_h}$ and price $\bar{p}$. Then, from the viewpoint of the buyers, $\bar{r}$ is equivalent to r. Therefore, (σ, β) can be transformed into a sequential equilibrium $(\bar{\sigma}, \bar{\beta})$ of the subgame $\Gamma(\bar{r})$ of the endogenous case. Since $(\bar{\sigma}, \bar{\beta})$ in $\Gamma(\bar{r})$ induces the same payoff to the seller as (r, σ, β) in the ex-post game it follows that the information value of the endogenous case is at least the payoff for the seller in (r, σ, β). □

Existence problem with ex-post pricing

We suspect that the existence of sequential equilibria also holds with ex-post pricing, but have not been able to find a proof. With ex-ante pricing, each strategy the seller chose resulted in a subgame which was a finite extensive form game. We were then able to leverage an existence proof from the properties of the equilibrium correspondence of the subgame as the seller's strategy varied. With ex-post pricing, we can find no analogous construction. It is conceivable that the seller randomizes over many different prices at a given information set, and that buyers use the pricing at their information sets to make inferences about what might have happened at other information sets. Because of this, attempts to prove existence by approximation arguments have foundered on compactness issues.

5. Unconstrained Pricing

Thus far, we have assumed that the seller is constrained to deal with the buyers one by one. In our final model, we allow the information seller to set up complicated interactions between the payments of one buyer and the prices faced by another. For simplicity, we assume here that we are in the basic ex-ante setting (as described in Subsection 3.1). That is, whatever interaction the seller designs to determine prices occurs before the beginning of the base game, and all players know the prices chosen when the base game begins.[7] This ability to condition the prices faced by one player on the actions of the others benefits the seller because for instance, it could be beneficial for player 1 if the seller does not sell information to player 2, and therefore player 1 is prepared to pay a price to the seller for charging player 2 a very high price, and therefore precluding player 2 from buying information. The basic ex-ante setting also implies that the mechanisms offered by the seller are fixed.

As before, the seller cannot force buyers to buy information. So, whatever the form of the interaction, a buyer is assumed always to have the ability to choose to make no payments, and receive no information in the game. As we shall see however, the seller's power is enhanced here by the fact that he can condition

[7] One could also imagine a situation in which the result of the interaction between players is the choice of a mechanism of the form described in the section "Ex-ante pricing with endogenous signal mechanisms." We restrict ourselves to the basic ex-ante pricing setting for simplicity.

the prices charged (and therefore the information sold) to other players on this refusal.

More precisely, before the actual game starts, the seller can set up an extensive form game (with finite or infinite action spaces) played by the buyers. Every terminal node maps to some pair (r, p), where r specifies for every buyer a price which has to be paid in advance to the seller and p is a price vector for the signal mechanisms in the actual game. This extensive form game is called the "pre-game". Since no buyer can be forced to make payments in advance, every buyer should have a strategy in the pre-game which guarantees him an payment vector r in which he does not have to pay anything to the seller. A strategy for the seller is thus to choose a pre-game having these properties.

If a terminal node with payment vector r and price vector p is reached, every buyer pays his part in r and the buyers start playing the actual game in which mechanisms can be bought at prices p.

The form of the pre-game could be very complicated, leading to both analytical intractability and a serious existence question. However, it turns out that a very simple pre-game yields the seller as much revenue as any possible more elaborate pregame meeting our conditions.

Formally, the simple pre-game is defined as follows. The seller specifies for each buyer a price to be paid in advance to the seller. He also specifies which price vector for the mechanisms available to the seller will be chosen as a function of which buyers choose to pay the specified price in the pre-game. The buyers observe this specification and simultaneously decide whether or not to pay the price. All players observe the set of buyers who paid (and hence the price vector chosen) and then the game begins. As before, the trivial mechanism always has to be offered at price zero, regardless of what occurs in the pre-game. The subgame following a certain price vector is the same as the corresponding subgame of the basic ex-ante case following the same price vector. We term this setting *unconstrained pricing.*

Example 5.1. Consider the following game, in which the seller offers the trivial and perfect mechanism to player 2.

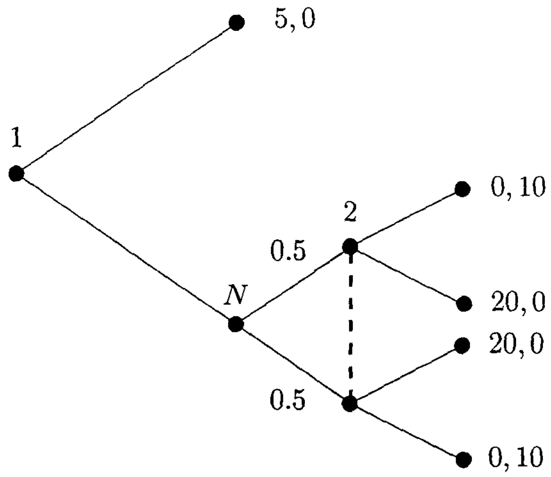

Figure 6

If the seller sells information to player 2 with probability one, player 1 will go up, leaving the players with payoffs 5 and 0 respectively. By not selling information to player 2, player 1 is forced to go down which leads to payoffs 10 and 5 respectively. Hence, both players gain 5 if the seller does not sell to player 2. The seller can exploit this by proposing a contract in which he commits to charge a very high price to player 2 (forcing player 2 not to buy), charges both players 5 for the contract and threatens to offer the perfect mechanism for free (forcing player 2 to buy) if one of the players does not pay. By doing this, the seller earns 10. Since player 1 can guarantee himself a payoff of at least 5 by choosing up, and player 2 can do no worse than 0, it follows that the seller can never get more than 10 by unconstrained pricing. Hence the information value with unconstrained pricing is equal to 10.

With basic ex-ante pricing, the information value is equal to $\frac{5}{2}$. This payoff is attained by offering information at price 5, having player 2 randomize equally between buying and not buying and having player 1 go down. With ex-post pricing, the seller is forced to sell information to player 2 with probability one, and hence player 1 will choose up. Hence, the information value in this case is zero.

Characterization of information value with unconstrained pricing

Before we prove the existence of sequential equilibria and give a characterization of the information value with unconstrained pricing, we need some more definitions. For every price vector p for the mechanisms, let $\Gamma(p)$ be the subgame which follows if the seller chooses the prices according to p. The set of sequential equilibria in $\Gamma(p)$ is denoted by $SE(p)$.[8] For an assessment (σ, β) in $\Gamma(p)$, let $u_i(p, \sigma, \beta)$ be the corresponding expected payoff for buyer i $(i = 1, ..., n)$ and let $u_{n+1}(p, \sigma, \beta)$ be the expected income for the seller induced by p and (σ, β). By

$$\bar{u} = \sup_p \max_{(\sigma,\beta)\in SE(p)} \sum_{i=1}^{n+1} u_i(p, \sigma, \beta)$$

we denote the highest sum of the payoffs (including the payoff of the seller) which could possibly be achieved in any sequential equilibrium of any subgame $\Gamma(p)$. For every buyer i let

$$\underline{u}_i = \inf_p \min_{(\sigma,\beta)\in SE(p)} u_i(p, \sigma, \beta)$$

be the lowest payoff that i can get in any sequential equilibrium of any subgame $\Gamma(p)$.

Theorem 5.1. *With unconstrained pricing a sequential equilibrium always exists. The information value is equal to $\bar{u} - \sum_{i=1}^{n} \underline{u}_i$ and can be supported by a simple pre-game as described above.*

Proof. First, we show that the supremum and the infimum in the definitions of $\bar{u}$ and $\underline{u}_i$ are actually a maximum and a minimum. For every assessment (σ, β) of the subgame $\Gamma(p)$, let

$$u(p, \sigma, \beta) = \sum_{i=1}^{n+1} u_i(p, \sigma, \beta).$$

Let (p^k, σ^k, β^k) be a sequence such that (σ^k, β^k) is a sequential equilibrium in $\Gamma(p^k)$ and

$$\lim_{k\to\infty} u(p^k, \sigma^k, \beta^k) = \bar{u}.$$

We may assume that the prices in p^k are bounded and therefore, without loss of generality, p^k converges to some price vector p^*. Since the space of assessments is the same compact space for every subgame $\Gamma(p)$ we may assume that (σ^k, β^k) converges to an assessment (σ^*, β^*). As can be seen easily, (σ^*, β^*) is a sequential equilibrium in $\Gamma(p^*)$. Since u depends continuously on (p, σ, β) we have that

$$u(p^*, \sigma^*, \beta^*) = \lim_{k\to\infty} u(p^k, \sigma^k, \beta^k) = \bar{u},$$

[8] As before, $\Gamma(p)$ is a finite extensive form game. Therefore, the existence of sequential equilibria in $\Gamma(p)$ is guaranteed. Note also that $SE(p)$ does not depend on what payments where made in the pregame, since such payments affect all nodes in the subgame equally.

which implies that the supremum in the definition of $\bar{u}$ is a maximum obtained at a sequential equilibrium (σ^*, β^*) of the subgame $\Gamma(p^*)$.
In the same way, we can show that the infimum in the definition of $\underline{u}_i$ is a minimum obtained at a sequential equilibrium (σ^i, β^i) of some subgame $\Gamma(p^i)$.

Now, we construct a sequential equilibrium with a simple pre-game which gives the information seller an expected payoff equal to $\bar{u} - \sum_{i=1}^n \underline{u}_i$.
The seller charges every buyer i the price

$$r_i = u_i(p^*, \sigma^*, \beta^*) - \underline{u}_i$$

that has to be paid in advance and every buyer can choose whether or not he wants to make this payment. If everybody pays his price, the seller chooses the price vector p^*. If everybody, except buyer i, pays his price the price vector p^i, as defined above, is chosen. In all other cases, the seller chooses some arbitrary price vector.
Let the strategies for the buyers be as follows. On the equilibrium path, all buyers pay their price in advance, observe price vector p^* and play the sequential equilibrium (σ^*, β^*) in $\Gamma(p^*)$. If everybody pays his price except buyer i, the sequential equilibrium (σ^i, β^i) in $\Gamma(p^i)$ is played. In all other cases, the buyers play an arbitrary sequential equilibrium in the corresponding subgame (as usual, double deviations do not matter).
The payoff for the seller is equal to

$$\begin{aligned}\sum_{i=1}^n r_i + u_{n+1}(p^*, \sigma^*, \beta^*) &= \sum_{i=1}^n [u_i(p^*, \sigma^*, \beta^*) - \underline{u}_i] + u_{n+1}(p^*, \sigma^*, \beta^*) \\ &= \bar{u} - \sum_{i=1}^n \underline{u}_i.\end{aligned}$$

In order to show that this is a sequential equilibrium, it suffices to show that the seller can never obtain a higher payoff by choosing some other (possibly non simple) pre-game. Suppose that the seller deviates to another pre-game. Then, each player always has the option of refusing to make any payments, and then entering a subgame where he earns at least $\underline{u}_i$. And, the expected total payoff to players in any subgame is by definition at most $\bar{u}$. So, in any pregame, and for any play where the expected payment to the seller was higher than $\bar{u} - \sum_{i=1}^n \underline{u}_i$, at least one player must have a strict incentive to deviate.

We have not only proved that the strategies for the seller and buyers constitute a sequential equilibrium of the extended game, but the argument above also shows that the seller can never expect more than $\bar{u} - \sum_{i=1}^n \underline{u}_i$ in *any* sequential equilibrium, which implies that the information value is equal to $\bar{u} - \sum_{i=1}^n \underline{u}_i$. Furthermore, we have shown that the seller can always reach this maximum payoff by setting up a simple pre-game. □

It is clear that the seller benefits from the possibility to condition prices of one player on actions taken by other players. The following result is therefore not surprising.

Theorem 5.2. *The information value with unconstrained pricing is at least as large as the information value with basic ex-ante pricing.*

Proof. In view of the theorem above, it suffices to show that the information value with ex-ante pricing is less or equal than $\bar{u} - \sum_{i=1}^{n} \underline{u}_i$. Consider an arbitrary sequential equilibrium with ex-ante pricing in which the seller chooses the price vector p and the buyers play a sequential equilibrium (σ, β) in the subgame $\Gamma(p)$. The payoff for the seller is therefore equal to

$$u_{n+1}(p, \sigma, \beta) = \sum_{i=1}^{n-1} u_i(p, \sigma, \beta) - \sum_{i=1}^{n} u_i(p, \sigma, \beta) \leq \bar{u} - \sum_{i=1}^{n} \underline{u}_i.$$

which completes the proof. □

The analog to this would have also held if we had worked in terms of ex-ante pricing with endogenous choice of signal mechanisms.

A problem with unconstrained pricing

To avoid openness problems, we needed to give the information seller the ability to specify the sequential equilibrium that followed any given choice of price vector. This has the undesirable feature that it may allow the seller to extract money from the buyers even if he has no relevant information to sell. Consider, for instance, the following game given in normal form.

3, 3	0, 0
0, 0	1, 1

Even though the seller has no relevant information to offer, the information value with unconstrained pricing is equal to 4. This is due to the fact that both buyers are willing to pay 2 to the seller for playing (top, left) on the equilibrium path whereas (bottom, right) will be played off the equilibrium path. In such situations, the role of the seller is one of selecting equilibria rather than selling information, which makes the definition of the information value with unconstrained pricing less attractive in some cases.

6. Partially Informed Seller

Up to this point we assumed that the seller always knows exactly which node of the extensive form game has been reached. As a consequence, he could make

prices contingent on everything that has happened in the game so far. if the rules of the specific model would allow him to.[9] In some situations. however. the seller may only be partially informed about events in the game. Or the seller may not be completely aware of the payoffs received by a player at the end of the game. creating uncertainty about the price this player is willing to pay for information. Our different settings can be extended directly to such situations by assuming that the seller's a-priori information in the base game is given by some partition of the non-terminal nodes. In each of the models considered in this paper. we should add the restriction that the seller's strategy is measurable with respect to this partition.
An additional phenomenon which could occur here is that the buyers' purchase behavior may serve as a signaling device to the seller. revealing some information about the true state of the world in the base game. Think. for instance. of a repeated interaction between an information seller and a buyer. where the seller faces a-priori uncertainty about the buyer's type. The fact whether or not the buyer bought information at a specific price in the past may reveal information about his type. which can be used by the seller to set a better price in the future. In order to illustrate this learning effect. consider the following example.

Example 6.1. At stages $t = 1, 2$ the buyer faces the following decision problem.

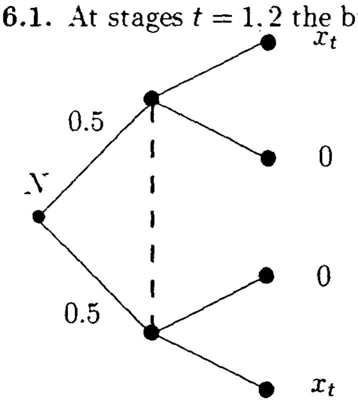

Figure 7

Here. x_t denotes the payoff at stage t. We assume that the buyer can have two possible types: A and B. where $\Pr(A) = \Pr(B) = 0.5$. The payoffs for type A are given by $x_1 = 40$ and $x_2 = 20$. whereas the payoffs for type B are $x_1 = 10$

[9] Recall that the ex-ante model with endogenous signal mechanisms allows the seller to make prices contingent on everything he knows. whereas the basic ex-ante setting prevents the seller to condition prices on events that where not observable to the buyer he is going to sell to.

and $x_2 = 30$ respectively. The seller is not informed about the buyer's type and offers the trivial and perfect signal mechanism at both stages. If we assume that the seller and buyer are in the basic ex-ante setting, the seller's task consists in setting prices p_1 and p_2 (one for each stage) for the perfect mechanism before the game starts. Here, the price p_2 is allowed to depend on the buyer's purchase behavior at stage 1.

Suppose that the seller sets price 20 at the first stage, sets price 10 at the second stage if the buyer bought perfect information at the previous stage and chooses price 15 otherwise. Let the buyer's behavior be as follows. At stage 1, type A buys perfect information whereas type B does not. At stage 2, type A buys at price 10 and type B buys at price 15.

In order to see that the buyer's behavior is optimal, it suffices to consider the buyer's incentives at stage 1. Type A is willing to pay price 20 for perfect information at stage 1, since by not buying he would receive the same expected payoff at stage 1 but would face a higher price for perfect information (15 instead of 10) at stage 2. If type B would buy perfect information at stage 1, he would face a lower price (10 instead of 15) at stage 2, but at stage 1 he would receive net payoff $10 - 20$ instead of 5. So, the loss at stage 1 by buying can not be compensated by the lower price at stage 2.

The price at stage 1 can therefore be used as a screening device to separate both types, and the seller benefits from this extra information at stage 2. The total expected profit for the seller is $(0.5) \cdot 20 + (0.5) \cdot 10 + (0.5) \cdot 15 = 22.5$.

If, on the other hand, the seller would ignore the purchase behavior at stage 1 (meaning that he chooses the same price p_2, independent of what happened at stage 1), he can not expect more than 20. In this case, he would set price 20 at stage 1 and (always) set price 10 at stage 2, having only type A buying at stage 1 and both types buying at the second stage. It is therefore optimal for the seller to learn from the buyer's purchase behavior at stage 1.

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