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Optimal Advertising with Traded Raw and Final Goods: The Case of Variable Proportions Technology

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An optimal advertising investment rule is derived for a vertically related, competitive market with traded final and raw goods and a processing sector characterized by variable proportions technology and nonconstant returns to scale. An equilibrium displacement framework incorporating conditional factor demands is used to account for the elasticity of substitution between agricultural and nonagricultural inputs to the marketing channel. Simulation for the Canadian beef industry in the post-WTO environment demonstrates how optimal advertising intensity falls as export demand elasticities for beef and live cattle become more elastic. Results show the optimal advertising intensity ranges between 0.05% and 0.22% of farm-level market revenue.

Key words: advertising, derived demand, trade, variable proportions technology

Introduction

Recently, an initiative forwarded by the Canadian Cattleman's Association would coordinate generic advertising and promotion efforts on the part of stakeholders in the Canadian beef industry. As trade plays an important role in determining the economic well-being of Canada's beef cattle industry, advertising and promotion investment must account for the role of live cattle and beef trade in shaping the domestic market place. It should be recognized, though, that transformation of live cattle to beef involves application of a substantial amount of nonagricultural inputs to a traded raw agricultural input (live cattle). The net result is a final good (beef) that is traded and whose sale is promoted at the retail level in Canada through cattle producer investment in market expansion activities, with a historical focus on generic advertising.

Because investment in advertising is pursued with the intent of maximizing cattle producers' surplus, one may suspect that trade of both the final and agricultural goods, and substitution possibilities between agricultural and nonagricultural intermediate inputs, may affect the optimal level of investment. The objective of this study is to derive an optimal investment rule for advertising in a vertically related, competitive market with traded final and agricultural goods and a processing sector characterized by variable proportions technology and nonconstant returns to scale.

The importance of the elasticity of substitution between agricultural and nonagricultural inputs to the processing sector has been demonstrated in previous studies (e.g.,

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Gardner; Mullen, Wohlgenant, and Farris; Wohlgenant 1989, 1993, 1999a; Holloway). Wohlgenant (1993) showed that if the elasticity of substitution between agricultural and nonagricultural inputs is strictly positive, then producers of agricultural commodities should prefer investment of checkoff funds in research related to primary production rather than in market promotion. However, this result must be viewed with caution as it is based on an assumption that the vertical shift in supply resulting from research investment equals the vertical shift in demand resulting from advertising investment.[1] When fixed proportions are assumed (i.e., when the elasticity of substitution equals zero), Wohlgenant found producers should be indifferent between investment in research related to primary production and market promotion. Furthermore, producers of the agricultural input were shown to benefit more from research related to primary production and market promotion than from research-induced changes in the cost of marketing inputs.

Kinnucan (1997) determined that ignoring the role of marketing intermediaries, by omitting the role of the elasticity of substitution, results in an overstatement of farm-level returns to advertising. Note, in drawing these conclusions, both Wohlgenant (1993) and Kinnucan (1997) ignore trade at the farm and retail levels of the market.

However, several studies have explored the role of trade when producers of the agricultural good finance generic advertising activities at the retail level (e.g., Alston, Carman, and Chalfant; Piggott, Piggott, and Wright; Ding and Kinnucan; Kinnucan 1999; and Kinnucan, Xiao, and Yu). Moreover, Kinnucan (1999) derived an optimality rule for cooperative advertising investment in an open, competitive economy that nests earlier optimality rules developed by Dorfman and Steiner and by Nerlove and Waugh. Still, Kinnucan's framework assumes fixed-proportions technology and considers only a single market level. Kinnucan, Xiao, and Yu relaxed the fixed-proportions assumption and incorporated a processing sector, but considered trade only at the farm level and assumed constant returns to scale.

This analysis generalizes the work of Kinnucan and his colleagues by adding retail-level trade and relaxing the constant-returns-to-scale assumption. In fact, the optimal investment rule obtained below nests those previously derived by Dorfman and Steiner; Nerlove and Waugh; Kinnucan (1997); and Kinnucan, Xiao, and Yu. In some respects, this study also extends the work of Wohlgenant (1993) by accounting for retail- and farm-level trade and relaxing the assumption of constant returns to scale, albeit without explicit incorporation of investment in research. Last, this paper extends one of Kinnucan's (1997) results wherein the optimal advertising intensity varies inversely with the farmer's share of the consumer's food dollar, but only when substitution possibilities at the retail level are stronger than substitution possibilities faced by processors.

Economic Framework

Because producers typically finance generic advertising efforts via a checkoff, and such efforts are pursed in a collective manner through a representative commodity agency, it is assumed a commodity agency manager seeks to maximize producers' surplus

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[1] As one reviewer pointed out, equal effectiveness of investment in primary production-related research and market promotion is unlikely to hold. Moreover, when the elasticity of substitution is nonzero, a preference for primary production-related research over market promotion becomes an empirical issue. Interested readers are directed to Chang and Kaiser, and to Wohlgenant (1999b) for further discussion.

through appropriate choice of generic advertising at the retail level. This advertising investment decision is made in isolation from all other investment decisions, such as primary production-related research, investment in financial instruments, and so on. Specifically, the optimality rule calculates the optimal, lump-sum generic advertising investment. As such, the relevant optimization problem is an unconstrained problem which assumes away investment in other alternatives.[2] As will be noted, however, the analysis follows the historical developments in this literature and accounts for the opportunity cost of invested funds—namely, investment in primary production-related research.

To begin, denote producers' surplus as:

$$(1) \qquad \pi = w_X X_S - \int_0^{X_S} f^{-1}(\tau)\,d\tau - \psi A,$$

where π is producers' surplus, w_X is the farm price of the agricultural input, X_S is the quantity supplied of the agricultural input, $f^{-1}(\tau)$ is the inverse farm-supply equation,[3] ψ is the share of the checkoff funds raised from domestic sale of the agricultural input, and A is producers' advertising investment.[4] To determine optimal generic advertising in a competitive environment, equate the marginal value product of advertising to the marginal cost of advertising and solve for optimal advertising, A^*, or alternatively solve for the optimal advertising intensity, AI^*, defined as the ratio of A^* to market revenue.

Because equation (1) includes the producers' cost of advertising, it already measures the net benefits realized by producers. Consequently, the net *marginal* value product of generic advertising can be derived by totally differentiating (1) and solving for the following:

$$\frac{d\pi}{dA} = \frac{w_X X_S}{A} \frac{d\ln(w_X)}{d\ln(A)} - \psi.$$

Following Nerlove and Waugh, set the net marginal value product equal to the marginal return on the advertising investment in its next-best alternative, denoted by ρ, and solve for AI^*:

$$(1') \qquad AI^* = \frac{A^*}{w_X X_S} = \frac{1}{(\psi + \rho)} \frac{d\ln(w_X)}{d\ln(A)}.$$

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[2] A reviewer indicated the optimization problem faced by the agency's manager is a constrained one—specifically, one where producers' surplus is maximized subject to an expenditure constraint requiring the sum of expenditure on alternative investments to be less than or equal to the funds made available through checkoff financing. Such an approach would assume the agency committed a priori to spend a fixed amount of money and then had to allocate this amount over different alternatives. A different approach is taken here, in which the goal is to determine the optimal lump-sum investment for generic advertising, which could then be added to the lump-sum investment for the other alternatives to determine total financing needs.

[3] Research could be included as a shift parameter in the inverse supply function. For the sake of analytical tractability, and to make the connection to previous literature as clear as possible, research has not been included in this function. In this regard, research could be viewed as a fixed value that has been absorbed into the inverse supply function, meaning a change in the level of research could change the optimal level of advertising investment, but only to the extent that the marginal cost curve shifts.

[4] To better focus attention on extending earlier models to account for trade at the farm *and* retail levels, the incidence parameter resulting from checkoff financing of generic advertising has been ignored. This is not to suggest such an incidence parameter is unimportant, but rather to squarely focus attention on the main issue of the study. In fact, ignoring the incidence parameter means the optimal intensities may be underestimated. Chang and Kinnucan provide a complete discussion on the tax-incidence of an advertising checkoff.

Inclusion of ρ allows for a direct accounting of the return to alternative investment opportunities, such as investment in research related to primary production. An increase in the returns to research, as manifest through an increase in ρ, will lower the optimal advertising intensity. Furthermore, the optimal advertising intensity, AI^*, depends on the proportional response of the agricultural input's price, w_X, to a 1% change in advertising. In turn, the value of w_X changes in response to changes at the retail level, but such retail-level changes must first pass through the marketing channel (see Kinnucan 1997). Consequently, it is important not to ignore the structure of production in the marketing channel. Kinnucan, Xiao, and Yu address this issue, but only assume trade at the farm level and constant returns to scale.

The remainder of this section derives an optimal advertising rule using an equilibrium displacement model that accounts for trade in both final and raw agricultural goods, with variable proportions processing technology using two inputs (agricultural and nonagricultural), and nonconstant returns to scale.

A static, competitive, deterministic economic environment in the context of a large, open economy is assumed. Demand for the final good (Y_D) depends on the final good's price (P) and advertising through the demand function:

$$Y_D = D(P, A). \tag{2}$$

Following Ball and Chambers, the processing sector is represented by a well-behaved aggregate production function:

$$Y = g(X, B),$$

where X and B represent agricultural and nonagricultural inputs, respectively. As Gardner points out, the aggregation of nonagricultural inputs into a bundle, B, assumes the relative prices of the nonagricultural inputs in the bundle are constant. The corresponding dual cost function to this production function is defined as:

$$C\left(w_X, w_B, Y\right) = \min_{X,B}\left\{w_X X + w_B B \mid g(X, B) = Y\right\},$$

where C is the cost of producing Y units of output, and w_B is the unit price of the nonagricultural input. Market equilibrium requires equality of marginal cost and price:

$$P = MC\left(w_X, w_B, Y\right). \tag{3}$$

Assuming the law of one price holds, trade of the final good, Y_T, is represented by:

$$Y_T = T(P), \tag{4}$$

where the function $T(\cdot)$ assumes the role of either an export demand or import supply function. Market clearing in the final good's market requires:

$$Y_T \equiv Y - Y_D. \tag{5}$$

The market for the agricultural input reflects trade of the raw agricultural good and the structure of the processing technology via derived demand for the agricultural good.

Following Mullen, Wohlgenant, and Farris, demand for the agricultural input is expressed as a conditional factor demand derived by applying Shephard's lemma to the processing sector's cost function:

$$X_D = C_X(w_X, w_B, Y), \tag{6}$$

where $C_X(w_X, w_B, Y)$ is the partial derivative of the cost function with respect to the price of the agricultural input, and is assumed to be homogeneous of degree zero in prices. Market supply of the agricultural input is a function of its price alone:

$$X_S = f(w_X). \tag{7}$$

Trade of the agricultural input, X_T, is defined as a function of the agricultural good's price:

$$X_T = h(w_X). \tag{8}$$

As with equation (4), equation (8) can be either an export demand or import supply function. Finally, equation (9) provides a market-clearing condition for the agricultural input market:

$$X_T \equiv X_S - X_D. \tag{9}$$

Equations (2)–(9) completely define a vertically related market with traded final and agricultural goods, but also allow for variable proportions technology and nonconstant returns to scale. In this model, a change in the level of advertising, which is exogenous to the system of equations defined by equations (2)–(9), will bring about a new equilibrium.[5] Such equilibrium displacement provides the means by which one can determine the proportional change in farm price to a change in advertising. To begin, logarithmically differentiate equations (2)–(5):

$$\text{dln}(Y_D) = -\eta\,\text{dln}(P) + \beta\,\text{dln}(A), \tag{2'}$$

$$\text{dln}(P) = \gamma_X \text{dln}(w_X) + \gamma_B \text{dln}(w_B) + \gamma_Y \text{dln}(Y), \tag{3'}$$

$$\text{dln}(Y_T) = e\,\text{dln}(P), \tag{4'}$$

$$\text{dln}(Y_T) = \frac{Y}{Y_T}\text{dln}(Y) - \frac{Y_D}{Y_T}\text{dln}(Y_D), \tag{5'}$$

where $\eta(\geq 0)$ and $\beta(\geq 0)$ are the own-price and own-advertising elasticities of demand for the final good, respectively; $\gamma_X(\geq 0)$ is the farm-to-retail price transmission elasticity; γ_B is the price transmission elasticity from the nonagricultural market to the final goods market; γ_Y is the elasticity of marginal cost with respect to output; and e is the own-price

[5] Once equation (1′) is included in the model, advertising becomes an endogenous variable. Also note the amount spent on such activities ultimately depends on the mechanism by which financing is raised. Typically, funds are raised through checkoff levies assessed on the sale of the agricultural input. In this case, ψA will depend upon the price of the agricultural input, the supply of the agricultural input, or some combination of the two. For analytic tractability, however, attention in this study is focused on lump-sum financing. Interested readers are directed to Freebairn and Alston, who provide insight into the impact of financing mechanisms for generic advertising in industries without supply control.

trade elasticity for the final good. In a competitive market, γ_X and γ_B can also be interpreted as the elasticity of marginal cost with respect to the price of the farm and nonagricultural inputs. Substituting (2′), (3′), and (4′) into (5′) and solving for dln(Y) gives:

$$\text{(10)} \qquad d\ln(Y) = \Xi^{-1}\Big[\big(k_R e - \eta\big)\big(\gamma_X d\ln(w_X) + \gamma_B d\ln(w_B)\big) + \beta d\ln(A)\Big],$$

where $\Xi = 1 + k_R - \gamma_Y(k_R e - \eta)$, and $k_R = Y_T/Y_D$. By definition, k_R is bounded to (0, 1] in the case of an exporter and [−1, 0) in the case of an importer. Furthermore, the trade elasticity is negative (positive) for an exporter (importer). Consequently, the term $-k_R e$ is always positive.

Attention is now focused on the agricultural input. Logarithmically differentiating equation (6), using the Allen decomposition of conditional factor demand elasticities and the homogeneity property of factor demands (see Allen, p. 504, for details), results in:

$$\text{(6′)} \qquad d\ln(X_D) = -\sigma s_B d\ln(w_X) + \sigma s_B d\ln(w_B) + \theta d\ln(Y),$$

where $\sigma(\geq 0)$ is the elasticity of substitution between agricultural and nonagricultural inputs, s_i is the ith input's cost share (s_X can be interpreted as the farmer's share of the consumer's food dollar), and θ is the elasticity of derived demand for the agricultural input with respect to output of the final good. Notice that alternative returns-to-scale assumptions can be made by varying the value of θ. Increasing returns to scale occur when $\theta < 1$, constant returns to scale when $\theta = 1$, and decreasing returns to scale when $\theta > 1$.

Next, logarithmically differentiate equations (7), (8), and (9):

$$\text{(7′)} \qquad d\ln(X_S) = \varphi d\ln(w_X),$$

$$\text{(8′)} \qquad d\ln(X_T) = \xi d\ln(w_X),$$

$$\text{(9′)} \qquad d\ln(X_T) = \frac{X_S}{X_T} d\ln(X_S) - \frac{X_D}{X_T} d\ln(X_D),$$

where $\varphi(\geq 0)$ is the own-price supply elasticity for the agricultural input, and ξ is the own-price trade elasticity for the agricultural input.

Following Wohlgenant (1993), assume the supply of nonagricultural input is perfectly elastic. Further assuming this supply curve does not shift means the price of the nonagricultural input is constant, and so $d\ln(w_B) = 0$. Furthermore, $\gamma_X = s_X$ when supply of the nonagricultural input is perfectly elastic (Kinnucan 1997, p. 195). Making these substitutions, setting $s_B = 1 - s_X$, then substituting equation (10) into (6′), and then (6′), (7′), and (8′) into (9′) results in:

$$\Big\{\Xi\big((1 + k_F)\varphi + \sigma(1 - s_X) - k_F\xi\big) - \theta(k_R e - \eta)s_X\Big\}d\ln(w_X) = \theta\beta d\ln(A),$$

where $k_F = X_T/X_D$. By definition, k_F will be bound to (0, 1] for an exporter and [−1, 0) for an importer. Furthermore, the farm-level trade elasticity is negative (positive) for an exporter (importer). Consequently, $-k_F\xi$ is always positive. The proportional response of the farm price with respect to retail advertising can now be rewritten as:

$$\text{(11)} \qquad \frac{d\ln(w_X)}{d\ln(A)} = \frac{\theta\beta}{D},$$

where $D = \Xi((1 + k_F)\varphi + \sigma(1 - s_X) - k_F\xi) - \theta(k_R e - \eta)s_X$.

Substituting equation (11) into equation (1′) produces a formula for the optimal advertising intensity when the final and agricultural goods are traded and the technology exhibits variable proportions and nonconstant returns to scale:

$$AI^* = \frac{A^*}{w_X X_S} = \frac{\theta\beta}{(\psi + \rho)D}. \tag{12}$$

In the following section, attention is next focused on the analysis of the optimal advertising intensity rule.

Analysis of the Optimal Advertising Intensity Rule

Equation (12) is a very general expression of an optimal advertising intensity which nests several special cases that have appeared in the literature. Table 1 shows these special cases. Consider first the case of a single market level in the absence of a marketing intermediary. This market structure can be represented in equation (12) by assuming a fixed proportion technology ($\sigma = 0$), constant returns to scale ($\theta = 1$ and $\gamma_Y = 0$ for convenience), and demand elasticities are measured at the farm level. The Dorfman-Steiner rule for a price- (or quantity-) setting monopolist results when one assumes fixed supplies ($\varphi = 0$) and no trade ($k_F = 0$). Relaxing the fixed-supplies assumption results in the Nerlove-Waugh optimal advertising intensity rule without supply control. Relaxing the fixed-supplies and no-trade assumptions results in Kinnucan's (1999) optimal advertising rule for a traded good.

Return now to the market scenario that includes a marketing intermediary with variable proportions technology. By assuming constant returns to scale ($\theta = 1$, $\gamma_Y = 0$) and no trade ($k_R = k_F = 0$), one obtains the optimal advertising intensity illustrated in Kinnucan (1997) and attributed to Wohlgenant (1993). Take note of the fact that the Hicks-Allen industry input demand elasticity [i.e., $\sigma(1 - s_X) + \eta s_X$] is present in the denominator of this case. If trade does not occur at the farm level ($k_F = 0$), then an optimal advertising intensity not previously reported in the literature results. Finally, assuming no trade at the retail level ($k_R = 0$), as assumed in Kinnucan, Xiao, and Yu, results in the optimal intensity rule shown in the last row of table 1.

Before proceeding, note the role of the elasticity of derived demand for the agricultural good with respect to output (θ). From equation (12), the partial derivative of AI^* with respect to θ is $\beta\Xi((1 + k_F)\varphi + \sigma(1 - s_X) - k_F\xi)/(D^2(\psi + \rho))$, which will be positive for economically interesting values of the arguments of the expression. All other things equal, the optimal investment intensity increases in θ. This means that as returns to scale change from increasing (i.e., $\theta < 1$) to constant (i.e., $\theta = 1$) to decreasing (i.e., $\theta > 1$), the optimal advertising intensity rises. This is consistent with the findings of Nerlove and Waugh, who showed increasing returns to scale reduce the optimal advertising intensity compared to instances where there are constant returns to scale.

Likewise, AI^* increases in γ_Y. Because θ appears in the numerator of (12), its sign is also important. Since θ measures the elasticity between the agricultural good and the final good, it stands to reason that an increase in output of the final good will increase derived demand for the agricultural good, and so θ is assumed to be positive.

Table 1. Special Cases of the Optimal Advertising Intensity (AI^*) Shown in Text Equation (12)

Description	$AI^* =$	Supporting Literature
One Market Level ($\sigma = 0, s_X = 1$):		
▸ Fixed supplies and no trade ($\varphi_X = k_R = k_F = 0$)[a]	$\frac{\beta}{\eta(1+\rho)}$	Dorfman-Steiner (1954) optimal advertising intensity rule
▸ No trade ($k_R = k_F = 0$)[a]	$\frac{\beta}{(\varphi+\eta)(1+\rho)}$	Nerlove-Waugh (1961) optimal advertising intensity rule
▸ Trade	$\frac{\beta}{((1+k_F)\varphi+\eta-k_F\xi)(\psi+\rho)}$	Optimal advertising intensity rule derived by Kinnucan (1999)
Two Market Levels:		
▸ No trade at either market level ($k_R = k_F = 0$)[a]	$\frac{\beta}{(\varphi+\sigma(1-s_X)+\eta s_X)(1+\rho)}$	Uses the same assumptions as Wohlgenant (1993)
▸ No trade at farm market level ($k_F = 0$)[a]	$\frac{\beta}{((1+k_R)(\varphi+\sigma(1-s_X))-(k_R e-\eta)s_X)(1+\rho)}$	Not previously reported in the literature
▸ No trade at retail market level ($k_R = 0$)	$\frac{\beta}{((1+k_F)\varphi+\sigma(1-s_X)-k_F\xi+\eta s_X)(\psi+\rho)}$	Uses the same assumptions as Kinnucan, Xiao, and Yu (2000)

[a] No trade is assumed, so $\psi = 1$.

In what follows, it is convenient to assume constant returns to scale (i.e., $\theta = 1$, and $\gamma_Y = 0$), in which case the optimal advertising intensity can be written as:

$$(13) \qquad AI^* = \frac{\beta}{\left[(1+k_R)\left((1+k_F)\varphi+\sigma(1-s_X)-k_F\xi\right)-(k_R e-\eta)s_X\right](\psi+\rho)}.$$

The term in the brackets of the denominator of equation (13) can be rewritten as:

$$(14) \qquad (1+k_R)\left((1+k_F)\varphi-\sigma-k_F\xi\right)+s_X\left(\eta-\sigma(1+k_R)-k_R e\right).$$

Kinnucan (1997), who assumed no trade at either the farm or retail levels, found optimal advertising intensity is inversely related to s_X, provided $\eta > \sigma$ [to see this in equation (13), set $k_R = 0$]. However, when trade of the final good occurs (i.e., $k_R \neq 0$), the optimal advertising intensity will be inversely related to s_X, provided $\eta - k_R e > \sigma(1+k_R)$. It is the net effect of domestic demand and the trade function, relative to substitution possibilities in the processing sector and the relative magnitude of trade, which determines whether AI^* increases or decreases in s_X. Stated another way, the optimal advertising intensity is inversely related to the farmer's share of the consumer's food dollar if and only if $\eta > \sigma(1+k_R) + k_R e$.

Consider what happens for a fixed nonzero value of the elasticity of substitution. If an import position is held, the retail substitution possibilities needed for $\eta > \sigma(1+k_R) + k_R e$ are weaker than those implied by Kinnucan's no-trade case since $(1+k_R)$, which is less than unity in an import position, diminishes the value of σ, and $k_R e$ is negative.

However, if an export position is held, then $(1 + k_R)$, which is greater than unity in an export position, magnifies the value of σ, while $k_R e$ is still negative—meaning the retail substitution possibilities may have to be weaker or stronger than Kinnucan's no-trade case for $\eta > \sigma(1 + k_R) + k_R e$ to hold.

Two specific cases will help illustrate the relationship between AI^* and the farmer's share of the consumer's food dollar. Consider first a Leontief technology. Since $\sigma = 0$, the last term in (14) is $s_X(\eta - k_R e)$, where $\eta - k_R e > 0$ for all nonzero values of η, k_R, and e. Consequently, with a Leontief technology, AI^* varies inversely with s_X regardless of whether an import or export position is held. Such a result is in accordance with Kinnucan (1997), who observed, "... industries that account for a modest share of the total cost of the finished product will have a stronger incentive to promote ... than industries that account for a relatively large share of total retail value" (p. 197).

Second, consider a Cobb-Douglas technology. Now, $\sigma = 1$, and the last term in (14) can be written as $s_X[\eta - 1 - k_R(1 + e)]$. The relationship between AI^* and s_X now depends on whether an export or import position is held, and on the magnitude of the demand and trade elasticities and the value of k_R. To see this, note retail demand for most food products is thought to be inelastic, so $\eta - 1 < 0$. If an import position is held, $k_R(1 + e) < 0$, which implies $\eta - 1 - k_R(1 + e)$ has an ambiguous sign, and thus the relationship between AI^* and s_X is also ambiguous. If an export position is held, the sign and magnitude of $k_R(1 + e)$ depend on the magnitude of e:

$$k_R(1+e) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \ \forall \ e \in \begin{Bmatrix} (-1, 0] \\ \{-1\} \\ (-\infty, -1) \end{Bmatrix}.$$

If $e \in [-1, 0]$, then $k_R(1 + e) \geq 0$, which implies $\eta - 1 - k_R(1 + e) < 0$, and so AI^* increases in s_X.

Thus, when retail demand is inelastic, the processing technology is Cobb-Douglas, and when faced with nonelastic export demand, the incentive to invest in advertising will be stronger in industries where the farmer's share of the consumer's food dollar is large. If export demand is elastic, then $k_R(1 + e) < 0$, which implies $\eta - 1 - k_R(1 + e)$ has an ambiguous sign, and thus AI^* has an ambiguous relationship with s_X.

Further qualitative results can also be derived. For instance, the optimal advertising intensity falls as farm supply becomes more elastic, as expected, because the incentive to invest in advertising is diminished when farm price changes little with shifts in demand. The optimal intensity also falls as the technology moves from no substitution possibilities ($\sigma = 0$) to perfect substitutes ($\sigma = \infty$). This occurs because, as the elasticity of substitution increases toward infinity, the elasticity of derived demand for the agricultural input becomes perfectly elastic and the incentive to invest in advertising again disappears because shifts in retail demand have a diminished (or no) effect on farm-level prices, ceteris paribus.

Notice also, as the elasticity of trade with respect to the farm price (ξ) becomes perfectly elastic, the optimal intensity approaches zero. Regardless of whether the farm market is in an import or export position, as the trade curve faced by domestic producers becomes more elastic, the incentive to invest in advertising disappears. The same result holds for the elasticity of retail trade with respect to the retail price. Even if derived demand were less than perfectly elastic, a perfectly elastic retail demand curve implies the optimal advertising intensity is zero. Consequently, if one market level is characterized as small and open to trade, it does not pay to advertise. Similar results have been reported previously by Alston, Carman, and Chalfant, and by Kinnucan (1999), but for models assuming only one market level.

The behavior of (13) is next considered by simulating its value under a variety of assumptions. A deterministic approach is first taken to highlight the pattern of the optimal investment intensity as retail- and farm-level trade elasticities vary. The optimal investment rule is then simulated using the probabilistic approach forwarded by Davis and Espinoza, and by Zhao et al. Such an approach allows one to account for the random nature of elasticities used to parameterize the optimal investment rule. Finally, the impact of alternative returns-to-scale assumptions is also explored.

Numerical Simulation

To better understand the properties of the optimal advertising intensity rule, one would like to know how the optimal intensity changes as various parameters change. Numerical simulation is conducted to show the range of values for AI^* based on "best-guess" estimates of elasticities for the Canadian beef cattle complex in a post-World Trade Organization (WTO) environment.

For convenience, the optimal advertising intensity will be reported as a percentage, that is, $AI^* \times 100$. As reliable estimates of e and ξ are not available and Canada holds a net export position in both live cattle and beef, e and ξ will assume the values -10, -5, or -1. These values are thought to be reflective of a wide array of demand responses for exports of Canadian beef and live cattle. The farmer's share of the consumer's food dollar (s_X) takes the value 0.57, which is the same value used by Wohlgenant (1993).

Use of the U.S. cost share in modeling a Canadian market is justified on the grounds that processing sectors and cattle markets in Canada and the United States are linked through trade and have evolved in a similar pattern. Values for k_R and k_F are set at 0.11 and 0.31, respectively, which are averages using 1995–98 data obtained from Agriculture Canada's *Livestock Market Review*. The opportunity cost of funds invested in advertising is set to reflect the returns to beef cattle research. Widmer, Fox, and Brinkman report an internal rate of return to investment in beef cattle research ranging from 59% to 66%. Given this range, ρ is set equal to 0.6, which reflects a 60% opportunity cost of invested funds in beef-cattle research. Historically, only Canadian cattle producers have financed investment in generic advertising, so ψ is set equal to unity.

Following Davis and Espinoza, and Zhao et al., the remaining elasticities needed to calculate the optimal advertising intensity are defined in terms of an underlying probability distribution. Each distribution is repeatedly sampled to generate a large number of values used to calculate the optimal intensity for each set of draws. Care must be taken, however, to ensure the chosen distributions generate theoretically consistent draws (e.g., nonnegative elasticities of substitution).

Goddard and Griffith reported own-price and advertising elasticities for beef in Canada equal to -0.23 and 0.004, respectively. Since $-\eta < 0$ and $\beta > 0$, one can assume distributions for $-\eta$ and β with respective means of -0.23 and 0.004. Because η and β are assumed positive, and typically thought to be inelastic, a beta distribution is assumed for each. Beta distributions rely on two defining parameters that can be

Table 2. Elasticity and Parameter Values

Elasticity/Data (in absolute value)	Symbol	Assumed Value/Mean
Retail Market:		
▸ Own-price demand elasticity [a]	(η)	0.23
▸ Own-advertising demand elasticity [a]	(β)	0.004
▸ Own-price export demand elasticity	(e)	{-10, -5, -1}
▸ Trade's share of retail demand [b]	(k_R)	0.11
Farm Market:		
▸ Elasticity of substitution [c]	(σ)	0.72
▸ Live cattle cost share [d]	(s_X)	0.57
▸ Own-price elasticity of supply [e]	(φ)	0.43
▸ Own-price export demand elasticity	(ξ)	{-10, -5, -1}
▸ Trade's share of farm demand [b]	(k_F)	0.31
Other:		
▸ Producer's share of contribution to checkoff	(ψ)	1.0
▸ Opportunity cost of invested funds	(ρ)	0.6

[a] Source: Goddard and Griffith
[b] Source: Agriculture Canada, *Livestock Market Review* (based on average of annual values over 1995–98)
[c] Source: Wohlgenant (1989)
[d] Source: Wohlgenant (1993)
[e] Source: Cranfield and Goddard

specified to match the desired mean of the distribution. For our purposes, $\eta \sim Be(3, 10)$ and $\beta \sim Be(2, 498)$, with corresponding means of 0.2307 and 0.004.[6]

Cranfield and Goddard noted own-price supply elasticities for live cattle in western Canada equal to 0.431. This value is consistent with supply elasticities reported in the literature (see Marsh for a review of these values). The own-price supply elasticity is assumed to be nonnegative, and to follow a gamma distribution, $\varphi_X \sim \Gamma(0.656, 0.656)$, with mean value 0.430.[7]

Unfortunately, estimates of the elasticity of substitution for the Canadian beef-processing sector are unavailable in the literature. However, Wohlgenant (1989) reports a substitution elasticity estimate for the U.S. beef and veal sector equal to 0.72. Given similarities in Canadian and U.S. beef processing sectors, this value will be used in defining an exponential distribution for the elasticity of substitution: $\sigma \sim \exp(0.72)$, which has a mean value of 0.72.[8] Table 2 summarizes the data and elasticities used to parameterize the optimal investment rule.

Prior to discussing the distribution of the optimal intensity, attention focuses on how changes in e and ξ affect the intensity, but with price, advertising, supply, and substitution elasticities fixed at their respective means, and all other values set as discussed previously. Figure 1 shows the optimal advertising intensity (stated in percentage terms)

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[6] The term $z \sim Be(\text{arg}_1, \text{arg}_2)$ indicates the random variable z is distributed as a beta distribution with defining parameters, arg_1 and arg_2.

[7] The term $z \sim \Gamma(\text{arg}_1, \text{arg}_2)$ indicates the random variable z is distributed as a gamma distribution with defining parameters, arg_1 and arg_2.

[8] The term $z \sim \exp(\text{arg})$ indicates the random variable z is distributed as an exponential distribution with defining parameter, arg.

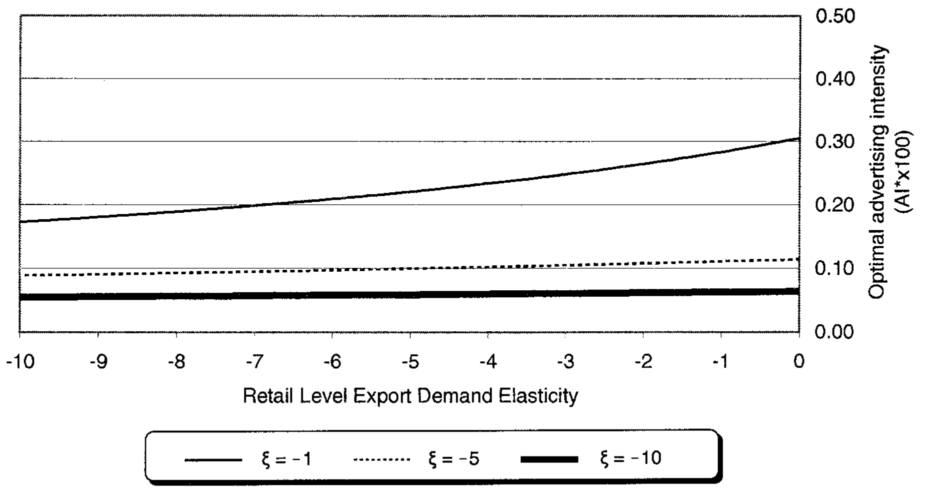

Figure 1. Value of the optimal advertising intensity (stated in percentage terms) at different values of *e* as ξ varies

for different values of ξ when *e* varies continuously between 0 and -10. As expected, the optimal intensity falls as the elasticity of demand for Canadian beef exports becomes more elastic. Specifically, as the elasticity of demand for Canadian beef exports approaches $-\infty$, Canada becomes a small, open market, in which case the retail price will not change with shifts in the retail demand curve, and so it does not pay to advertise. A retail price increase is necessary for farmers to benefit from retail advertising because this increases derived demand for the agricultural input, causing farm price to increase (assuming a large open economy), and with it producer surplus.

Notice also that the magnitude of the optimal advertising intensity falls as the elasticity of demand for live cattle exports, ξ, becomes more elastic (compare the graph in figure 1 of the optimal advertising intensity when $\xi = -1$ to that when $\xi = -10$). Thus, a change in the elasticity of export demand for the agricultural good has a direct bearing on the magnitude of the optimal intensity, but so too does the elasticity of export demand for the final good.

This last point is further highlighted in figure 2, which shows the optimal advertising intensity (in percentage terms) when ξ varies continuously between 0 and -10, and *e* assumes the values -1, -5, or -10. For a given value of *e*, the optimal advertising intensity falls as the demand for Canadian live cattle exports becomes more elastic. Again, this finding is consistent with the fact that in a small, open market, it does not pay to advertise. Just as in figure 1, the magnitude of the optimal advertising intensity falls as the export demand elasticity for beef becomes more elastic. Figures 1 and 2 also demonstrate that even when one level of a market is large (i.e., exports can affect world price), it does not pay to advertise if the other market level is small and open (i.e., exports cannot affect world price).

Figure 3 presents histograms of the optimal investment intensity (again, in percentage terms). Values underlying these histograms were calculated by first making 1,000 draws from the assumed distributions for η, β, φ, and σ. For each of the drawn quadtuple, AI^* is then calculated at the assumed values of k_R, s_X, k_F, ψ, and ρ, with $e = -5$, and ξ assuming the values of -1, -5, or -10. The key result here is that as the export

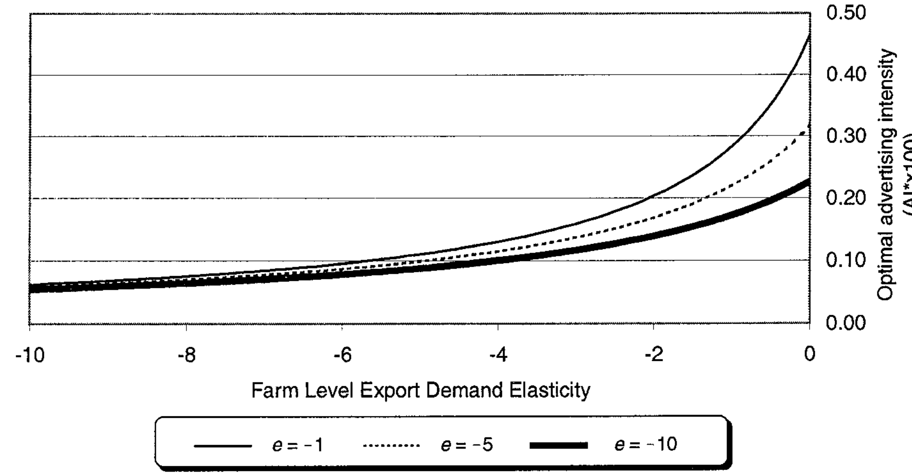

Figure 2. Value of the optimal advertising intensity (stated in percentage terms) at different values of ξ as e varies

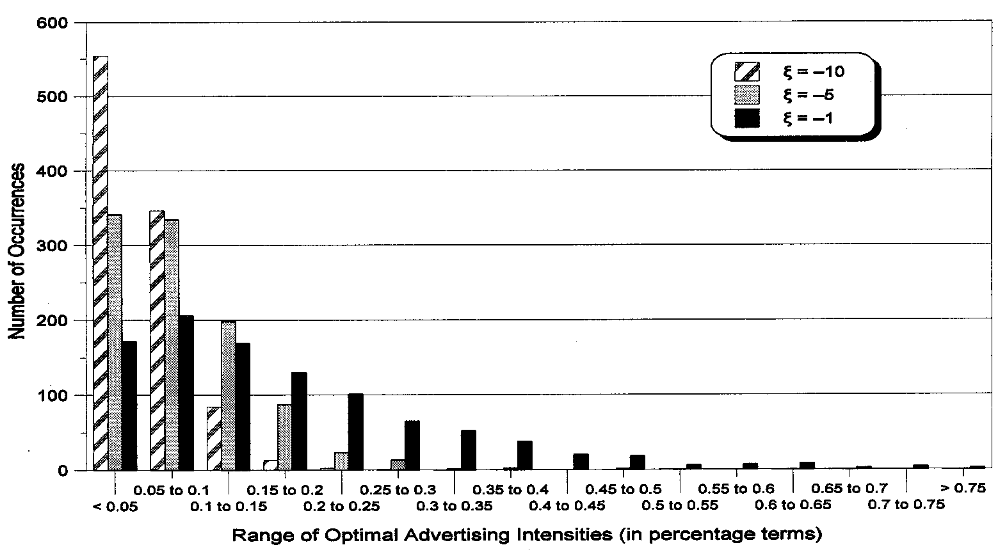

Figure 3. Histograms of the optimal advertising intensities (in percentage terms) when $e = -5$ and ξ varies

demand elasticity for live cattle becomes less elastic, the distribution of optimal intensities becomes more dispersed. While not shown, identical qualitative results occur when $e = -1$ and $e = -10$. In fact, the increased dispersion of the optimal advertising intensities was more noticeable as beef's export demand elasticity became less elastic. As both export demand elasticities become less elastic, the value of the denominator in equation (13) falls, thereby increasing the optimal advertising intensity.

Table 3 reports summary statistics for the values of $AI^* \times 100$ calculated in the stochastic simulation. As expected, the mean value of the optimal advertising intensity increases as the elasticity of export demand for live cattle becomes less elastic and as

Table 3. Summary Statistics from Stochastic Simulation of the Optimal Advertising Investment Rule

Description	e = -10, ξ = -10	e = -10, ξ = -5	e = -10, ξ = -1
Mean	0.049	0.075	0.137
Standard deviation	0.033	0.053	0.105
Skewness	1.310	1.347	1.505
Kurtosis	3.084	3.178	3.629
Lower bound of 95% Chebyshev confidence interval	-0.101	-0.162	-0.334
Upper bound of 95% Chebyshev confidence interval	0.198	0.313	0.608
Maximum *p*-value	0.472	0.499	0.587

Description	e = -5, ξ = -10	e = -5, ξ = -5	e = -5, ξ = -1
Mean	0.052	0.084	0.172
Standard deviation	0.036	0.060	0.138
Skewness	1.313	1.363	1.621
Kurtosis	3.091	3.223	4.045
Lower bound of 95% Chebyshev confidence interval	-0.108	-0.183	-0.447
Upper bound of 95% Chebyshev confidence interval	0.212	0.350	0.790
Maximum *p*-value	0.475	0.509	0.649

Description	e = -1, ξ = -10	e = -1, ξ = -5	e = -1, ξ = -1
Mean	0.055	0.092	0.217
Standard deviation	0.038	0.066	0.187
Skewness	1.316	1.381	1.811
Kurtosis	3.098	3.272	4.916
Lower bound of 95% Chebyshev confidence interval	-0.115	-0.205	-0.620
Upper bound of 95% Chebyshev confidence interval	0.225	0.388	1.054
Maximum *p*-value	0.478	0.520	0.744

Note: Summary statistics have been computed using the simulated values of the optimal advertising intensity stated in percentage form.

the elasticity of export demand for beef becomes less elastic, ceteris paribus. Standard deviations echo figure 3, in that the distribution of AI^* becomes more dispersed as the trade elasticities become less elastic. Furthermore, measures of skewness are positive and increase as trade elasticities become less elastic.

Means and standard deviations in table 3 were also used to calculate 95% Chebyshev confidence intervals. Because the means and standard deviations increase as the trade elasticities become less elastic, so too does the width of each confidence interval. For example, when $e = -10$ and $\xi = -10$, the 95% confidence interval is (-0.10, 0.19), in contrast to (-0.62, 1.05) when $e = -1$ and $\xi = -1$.

The lower bounds for all confidence intervals in table 3 are negative, which means, at the 95% level of significance, the mean optimal advertising intensity (stated in percentage terms) is not statistically different from zero. However, this result is a direct consequence of not accounting for the truncation of the assumed distributions when calculating the confidence intervals.

An alternative to conducting distribution-free statistical analysis is to follow Davis and Espinoza and compute the largest p-value for which a tested null hypothesis would be rejected. This p-value, referred to as the maximum p-value, tests the null hypothesis that $AI^* \times 100 = 0$. Table 3 shows the maximum p-values ranging from 0.472 (when $e = -10$ and $\xi = -10$) to 0.744 (when $e = -1$ and $\xi = -1$). Moreover, for a given value of e (ξ), the maximum p-value falls as ξ (e) becomes more elastic. Hence, the less elastic the trade elasticities become, the more likely it is the optimal advertising investment will be significantly larger than zero—a reflection of the earlier work of Alston, Carman, and Chalfant, and of Kinnucan (1999).

Finally, table 4 presents the optimal advertising intensities (in percentage terms) as θ varies across 0.5 (i.e., increasing returns to scale), 1 (i.e., constant returns to scale), and 2 (i.e., decreasing returns to scale), while all other values are held fixed either at their assumed values or respective means. As the processing technology moves from increasing to constant to decreasing returns to scale, the optimal advertising intensity rises. This finding is expected given the analytical results stated above, and also provides numerical support for the connection between the optimality rule developed here and that of Nerlove and Waugh.

Differences in the magnitude of the optimal intensities are noteworthy, however. For instance, with $\xi = -1$ and $e = -1$, the optimal advertising intensity rises from 0.089% (of farm-level market revenue) to 0.294% as the scale measure increases from 0.5 to 2. Such discrepancies in the optimal intensity suggest that accurate measurement of returns to scale is critical to providing industry with information needed in planning optimal generic advertising campaigns.

Conclusions

This study sought to derive an optimal investment rule for producer-funded advertising in a vertically related, competitive market with traded final and agricultural goods and a processing sector characterized by variable proportions technology and nonconstant returns to scale. The optimal rule was developed from the perspective of a commodity agency manager seeking to maximize producers' surplus through appropriate choice of generic advertising at the retail level.

The optimal advertising intensity depends on the proportional change in farm price to advertising. Because the impact of advertising must first pass through the marketing channel, the proportional change in farm price to advertising was derived using an equilibrium displacement framework which incorporated marketing intermediaries. By using a dual approach to representing marketing intermediaries, explicit account was taken of the elasticity of substitution between agricultural and nonagricultural inputs to the marketing channel. Moreover, the equilibrium displacement model also incorporated scope for trade of the final and agricultural goods.

The resulting optimal advertising intensity rule is very general and nests earlier optimal advertising intensity rules derived by Dorfman and Steiner; Nerlove and Waugh; and Kinnucan (1999), as well as rules based on the same assumptions made by Wohlgenant (1993) and by Kinnucan, Xiao, and Yu. Furthermore, an optimal advertising rule not previously reported in the literature is nested in the more general model.

Assuming constant returns to scale, it was shown that the optimal advertising intensity has an inverse relationship with the supply elasticity of the agricultural input, the

Table 4. Optimal Advertising Intensities (in percentage terms) Assuming Increasing, Constant, and Decreasing Returns to Scale

	$\theta = 0.5$ (Increasing Returns to Scale)		
	$\xi = -10$	$\xi = -5$	$\xi = -1$
$e = -10$	0.026	0.041	0.074
$e = -5$	0.027	0.043	0.081
$e = -1$	0.028	0.045	0.089
	$\theta = 1$ (Constant Returns to Scale)		
	$\xi = -10$	$\xi = -5$	$\xi = -1$
$e = -10$	0.048	0.073	0.121
$e = -5$	0.052	0.080	0.142
$e = -1$	0.054	0.087	0.166
	$\theta = 2$ (Decreasing Returns to Scale)		
	$\xi = -10$	$\xi = -5$	$\xi = -1$
$e = -10$	0.084	0.119	0.177
$e = -5$	0.094	0.140	0.227
$e = -1$	0.104	0.163	0.294

elasticity of substitution between agricultural and nonagricultural inputs, and trade elasticities for the final and agricultural goods. A condition needed for the optimal advertising intensity to vary inversely with the farmer's share of the consumer's food dollar was identified.

This condition generalizes an earlier result found by Kinnucan (1997). However, the relationship between the optimal advertising intensity and the farmer's share of the consumer's food dollar was shown to be ambiguous in general terms, but to vary with the magnitude of the own-price demand elasticity, the elasticity of substitution, the trade elasticity, and relative trade volume.

Simulation was used to explore the properties of the optimal advertising intensity when applied to the Canadian beef cattle industry in the post-WTO environment. Results are in agreement with those of Alston, Carman, and Chalfant: as the elasticity of demand for exports becomes more elastic, the optimal advertising intensity falls. Moreover, the probability of rejecting the null hypothesis that the optimal advertising intensity equals zero falls as the trade elasticities become more elastic. Assuming constant returns to scale, and depending on the value of the export demand elasticities, the mean optimal advertising intensities from the simulation range between 0.05% and 0.22% of farm revenue.

It is important to recognize, however, that the reported optimal advertising intensities do not reflect the tax incidence of the checkoff needed to raise money for investment in advertising. As such, results from Chang and Kinnucan imply the reported intensities are lower bounds on intensities that reflect a tax-shifting effect of checkoff financing. Freebairn and Alston recently summarized optimal advertising investment rules which incorporated such tax-shifting effects.

Values obtained from Agriculture Canada's *Livestock Market Review* place farm-level market revenue for fed cattle at approximately Can$3.39 billion, suggesting an optimal advertising investment between Can\$2.84 million and Can\$7.36 million. The wide range of optimal investment level underscores the importance of trade elasticities and highlights the need for further research to provide reliable estimates of trade elasticities not only for beef and cattle, but for all traded agricultural and food products for which producers invest in generic advertising.

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