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Calendar Time Sampling of High Frequency Financial Asset Prices and the Verdict on Jumps

Marina Theodosiou*

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Abstract

In the current paper, we investigate the bias introduced through the calendar time sampling of the price process of financial assets. We analyze results from a Monte Carlo simulation which point to the conclusion that the multitude of jumps reported in the literature might be, to a large extent, an artifact of the bias introduced through the previous tick sampling scheme, used for the time homogenization the price series. We advocate the use of Akima cubic splines as an alternative to the popular previous tick method. Monte Carlo simulation results confirm the suitability of Akima cubic splines in high frequency applications and the advantages of these over other calendar time sampling schemes, such as the linear interpolation and the previous tick method. Empirical results from the FX market complement the analysis.

Keywords: Sampling schemes, previous tick method, quadratic variation, jumps, stochastic volatility, realized measures, high-frequency data

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JEL Classification: C83, C12, C14, G10.

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*Central Bank of Cyprus. The opinions expressed in this paper are those of the author and do not necessarily reflect the views of the Central Bank of Cyprus or the Eurosystem.

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1 Introduction

The seminal work of Andersen and Bollerslev (1998) has set the route towards the exploitation of high frequency data for the construction of improved ex-post volatility measurements. The realization that squared return innovations over the one-day horizon are contaminated by large idiosyncratic errors which overshadow the true effects of model misspecification, has highlighted the negative implications of the information loss incurred from discarding data at higher frequencies and spurred the proliferation of a vast literature examining intraday return dynamics[1] (Andersen et al., 2004a; Anderesen et al., 2005; Andersen et al., 2000, 2003; Meddahi, 2002; Barndorff-Nielsen and Shephard, 2002).

Using the quadratic variation theory (Karatzas and Shreve, 1991), Andersen and Bollerslev (1998) have shown that; as the observation frequency of returns increases from daily to infinitesimal interval, realized volatility obtained from the cumulative sum of squared observed returns converges in probability to the true latent volatility. This insight points to the estimation of realized volatility using the highest possible frequency available in intraday returns. Nevertheless, when faced with financial market realities, it becomes evident that such a technique is crippled by data limitations and market microstructure effects such as bid-ask spreads (Roll, 1984), nonsynchronous trading effects (Zhou, 1996), discrete price observations (Harris, 1990, 1991), trading mechanisms (Black, 1976; Amihum and Mendelson, 1987) and intraday periodic volatility patterns. Put together, these intraday market conditions render the convergence to the optimal latent volatility infeasible.

Apart from the microstructure effects induced directly in the return process from the market itself and which prevent the use of all available high frequency data, an important source of biases which has been completely overlooked in the literature, is embedded in the method used to construct time homogeneous data, from the time inhomogeneous observed returns. Currently, there exists only a very limited number of methods used for the construction of time homogeneous processes and their potential ramifications for volatility estimation accuracy are yet to be investigated.

In the current paper, we will contribute towards this direction by examining the characteristics of the synthetically created time homogeneous process obtained from the application of calendar time sampling schemes, and specifically the previous tick method. The study is mainly concerned with the analysis of the synthetically constructed return processes in the context of jump detection.

Jump detection has been a recent topic in the literature of volatility modeling which deals with the identification of the discontinuous component (jumps) in the return process. The significant implications of jump exposure on risk management have lead to the propagation of a stream of papers developing techniques for disentangling the continuous part of the process and separately modeling the component attributed to jumps (Barndorff-Nielsen and Shephard, 2004, 2006; Lee and Mason, 2007; Jiang and Oomen, 2008; Aït-Sahalia and

[1] Andersen et al. (2006) for a review.

Jacod, 2009; Podolskij and Ziggel, 2008; Corsi et al., 2008, among many others). The main conclusion from the research is a much stronger than anticipated evidence of frequent jumps in the data.

In the current work, we will demonstrate, through a Monte Carlo simulation, that the large number of jumps detected in the literature by the multi-power variation based tests is due, to a large extend, to the biases induced in the time homogeneous series by the employed sampling method. We propose the use of Akima cubic splines as an alternative method for achieving time homogeneous price processes. We demonstrate the superiority of the proposed method in the context of jump detection by analyzing the size and power properties of two multi-power variation based tests for jumps in relation to the sampling scheme employed in the data.

The paper unfolds as follows. In the proceeding section, a brief review of the existing sampling schemes in the literature is given, with particular focus on calendar time sampling schemes. The effects from the application of the previous tick method on the distributional characteristics of the return process are investigated using empirical data from the FX market. Section three further investigates the weaknesses of the previous tick method in the context of jump detection. Two standard non-parametric tests for jumps based on multi-power variation are applied to data from the FX market and the results are reported. In section four, the idea of cubic spline interpolation is introduced as an alternative to the previous tick method for sampling high frequency data. Section five investigates the size and power properties of the two tests for jumps through a Monte Carlo simulation and for time homogeneous trajectories constructed under the proposed cubic spline interpolation method. The results are compared to those obtained from the application of two other calendar time sampling schemes from the literature; the previous tick method and the linear interpolation method, as well as to the results obtained from the application of tick time sampling. Concluding remarks are given in section six.

2 Constructing Artificial Time Series

The first step, prior to the estimation and analysis of the various volatility estimators, is the construction of the time equidistant price process from the time inhomogeneous observed quotes and transactions in the market. Any consequent results are therefore based on the dynamics and specifications of this artificially constructed price process rather than the observed one. In the literature, there exist three well-established sampling schemes for constructing time homogeneous series; namely the *calendar time sampling* (CTS), *transaction time sampling* (TTS), and *business time sampling* (BTS)[2]. CTS schemes have been the most popular sampling methods in the literature. Their popularity partly stems from the fact that the majority of techniques developed for the estimation of integrated volatility and other volatility estimators, as well as for alleviating microstructure effects, assume

[2]For a discussion, see Dacorogna et al. (2001).

equidistant observations in physical time and consequently require a CTS scheme for their application.

2.1 Calendar Time Sampling Schemes

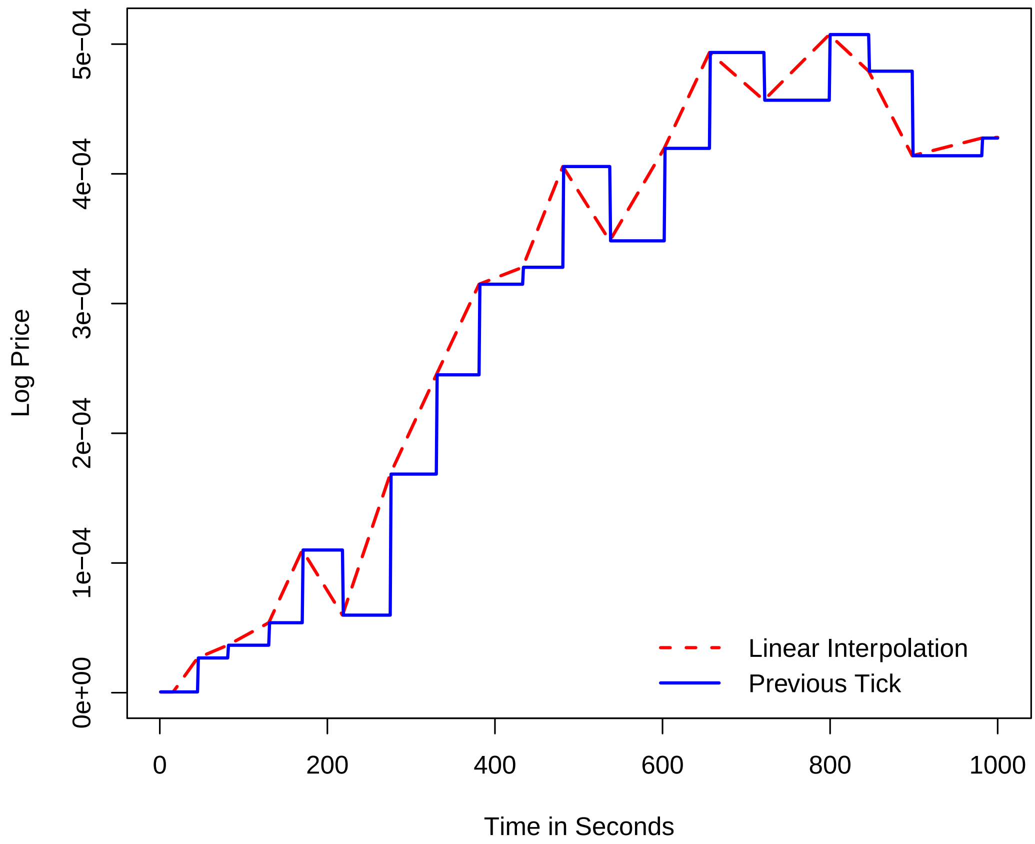

Figure 1: Linear interpolation (dashed line) and previous tick interpolation (solid line).

Under CTS schemes, observations are required to be equidistant in calendar time. Given observed prices at times $t_0 < \cdots < t_N$, one can construct a homogeneous price process at time $\tau \in [t_j, t_{j+1}]$ using:

$$p(\tau) \equiv p_{t_j} \tag{1}$$

$$\tilde{p}(\tau) \equiv p_{t_j} + \frac{\tau - t_j}{t_{j+1} - t_j}(p_{t_j+1} - p_{t_j}) \tag{2}$$

Equation (1) is known as the *previous tick* method. It was introduced by Wasserfallen and Zimmermann (1985) and proclaims that each tick is valid until a new tick arrives. Equation

(2) is known as the *linear interpolation* method and it was introduced later on by Andersen and Bollerslev (1997).

The linear interpolation method produces regularly spaced time series by drawing a straight line between two consecutive observations. Hence, this technique results in smoothed time series and mitigates the effect of sharp price changes. As argued in Barucci and Renó (2002), in some sense, a straight line is the 'minimum variance' path between two observations. Therefore, linear interpolation implicitly assumes a minimum volatility between the observation intervals, introducing in this way an artificial correlation between the subsequent returns of the generated regular time series. In its turn, this leads to a systematic underestimation of volatility, which becomes larger as the sampling frequency decreases (Corsi et al., 2001). Hansen and Lunde (2004, 2006) proved that, in theory, the realized volatility converges in probability to zero as the sampling frequency goes to zero, i.e.

$$\mathrm{RV}^{(m)} \xrightarrow{p} 0 \quad \text{as} \quad m \to 0 \tag{3}$$

Therefore, linear interpolation is not recommended for the construction of artificial price processes and, as a result, the standard technique in the literature has become the previous tick method.

Figure 1 depicts the synthetically constructed time series from the application of the two sampling schemes on a sample data. While the linear interpolation method smooths the data, the previous tick method results in irregular 'step' paths, and thus introduces abrupt changes in the time series. It becomes therefore imperative to investigate how the previous tick sampling scheme and the introduced 'step' effect in the sampled price paths affect the distribution of returns and consequently the estimation accuracy of volatility estimators. This is done in the proceeding section using empirical data from the FX market.

2.2 Investigating the Previous Tick Method

Despite its popularity, the previous tick (hereafter PT) method, like the linear interpolation, also suffers from significant drawbacks when applied to high frequency data. In this section, the effects induced in the artificially created price process from the application of the PT method are investigated using Foreign Exchange data. Specifically, the mid-quotes recorded for the EUR/USD currency pair were employed, covering the sampling period between January 4, 2000 and May 31, 2007, with observations spanning between 21:00 GMT to 21:00 GMT. The data was extracted from the *EBS Market Data* database, which is currently the larger of the two electronic venues that make up the inter-dealer spot FX market, after Reuters. This data has been only recently made available to academic researchers.

Observations recorded between 21:00 GMT on Friday and 21:00 GMT on Sunday are discarded. In addition, days with low trading activity due to public and bank holidays are also excluded from the data. Finally, observations which seemed incompatible with the prevailing market activity are also eliminated using the method described in Barndorff-Nielsen et al. (2008). The resulting return series consisted of 1820 days. Figure 2 depicts

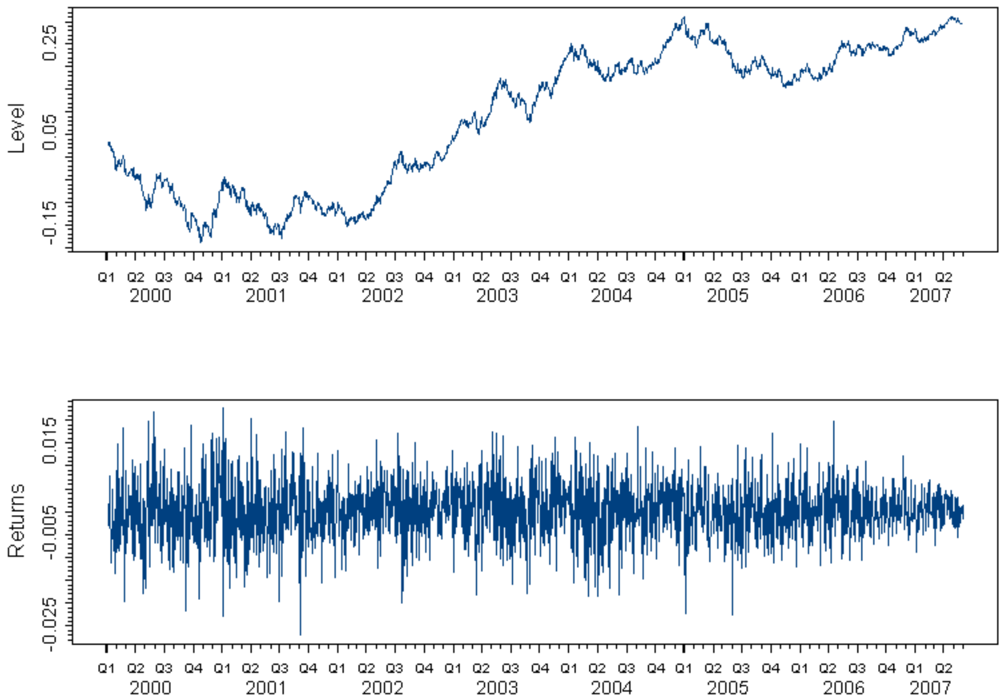

Figure 2: Daily closing prices and daily returns of the currency pair EUR/USD for the period from January 4, 2000 to May 31, 2007.

the daily closing prices and daily returns for the dataset.

In order to investigate the effects from the application of the PT method on the distributional characteristics of the return process, this was applied to the data using various sampling frequencies, ranging from 5 seconds to half an hour. The sample quantiles and sample moments for each resulting return process were computed and summarized in tables 1 and 2 respectively. Also documented is the proportion of consecutive identical mid-quotes in the time series. The first column in each table gives the sampling frequency in seconds used for the construction of the homogeneous series. The proportion of zero returns in the series, for each of the sampling frequencies employed, is given in the penultimate column. The last column documents the proportion of observations in the time homogeneous process relative to the total number of observations in the inhomogeneous time series. The first row in each table gives the same information for the raw time inhomogeneous return process.

Although the sample quantiles for the mid quotes remain constant across the various frequencies, the results for the return process differ greatly, revealing some important properties of the bias induced through the calendar time sampling process. While the distribution of the mid quotes strengthens as the sampling frequency increases, due to the imputation of a large number of identical quotes in the series, the distribution of the return

δ	Min.	1st Qu.	Med.	Mean	3rd Qu.	Max.	% 0s	Prop.Obs
Raw	-0.176	0.000	0	0	0.000	0.223	53.18	100
5	-0.295	0.000	-0.004	0	0.000	0.311	69.83	76.22
30	-0.326	-0.004	0.000	0	0.004	0.460	42.21	12.70
60	-0.397	-0.004	-0.004	0	0.004	0.511	31.37	6.35
120	-0.447	-0.008	-0.004	0	0.008	0.645	22.22	3.18
180	-0.529	-0.008	-0.004	0	0.008	0.543	17.98	2.12
240	-0.529	-0.008	0.000	0	0.011	0.584	15.33	1.59
300	-0.443	-0.012	-0.008	0	0.012	0.539	13.33	1.27
600	-0.542	-0.016	0.000	0	0.017	0.638	9.05	0.64
900	-0.589	-0.023	0.000	0	0.023	0.683	7.41	0.42
1800	-0.724	-0.031	-0.016	0	0.031	0.826	5.20	0.21

Table 1: Sample quantiles for inhomogeneous data (first row) and artificial data constructed at various frequencies using the PT method, for the currency pair EUR/USD and for the year 2006.

series is distorted significantly, becoming heavier in the center with thicker tails. When low frequencies are employed, the value of the minimum recorded return decreases and the maximum recorded return increases while the number of observations found in the center of the distribution decreases as well, thus resulting in a flatter distribution with thinner tails. This is clearly seen in Figure 3 where the histograms of sampled data at five seconds and thirty minutes are plotted adjacently.

The decrease and increase in the values of the minimum and maximum observations is almost threefold for frequencies as low as half an hour. All other sample quantiles also increase dramatically with decreasing frequency. It is worth noting that none of the resulting time series returned sample quantiles close to those of the inhomogeneous return process. The first five quantile moments approach those of the time inhomogeneous series only at the highest sampling frequency of 5 seconds.

The severe transformation of the return distribution directly relates to the method used in the construction of the regular return process. In the application of the PT method, the use of high frequencies leads to the introduction of a large number of identical quotes, significantly high relative to the proportion of observations in the inhomogeneous series. The phenomenon is particularly grave when very high frequencies are used and during the less active trading periods of the day. The induced identical quotes are translated into zero returns which, due to their large number, dominate the distribution and drive its center higher as well as producing dampen lagged covariances[3]. This is shown in Figure 4 where

[3]It has been shown by Zhou (1996) that the negative autocorrelation found in high-frequency data is attributed to noise. Consequently, practitioners have been sampling at low frequencies in order to avoid the negative effects that microstructure noise carries on the estimation of the realized volatility at very high frequencies.

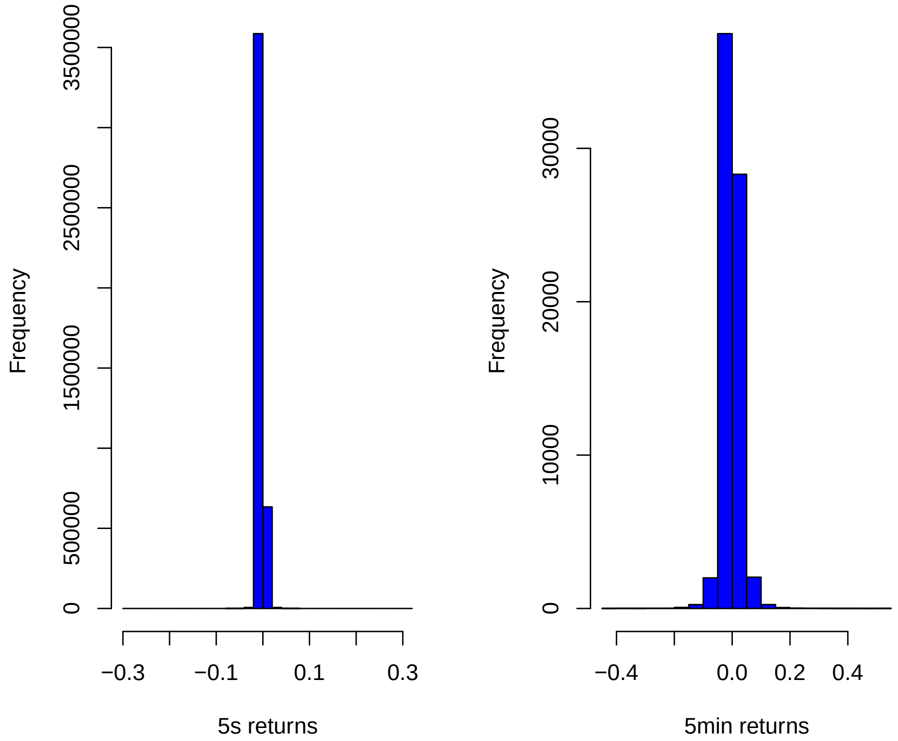

Figure 3: Histograms of homogeneous returns sampled at 5 second intervals (left panel) 30 minutes intervals (right panel).

the autocorrelation function of the inhomogeneous returns for the EUR/USD currency pair is plotted for the year 2006, together with the autocorrelation function of returns sampled at 30 seconds. The first-order autocorrelation increases to around -0.025, for 30 seconds sampled returns, from the -0.2 level obtained for the time inhomogeneous series.

Additionally, the PT method strengthens the impact of periodic daily and weekly effects in the data. This is again the result of introducing a large number of consecutive identical observations, especially during the less active trading hours of the day. The phenomenon is clearly observed in Figure 5 where the autocorrelation function spanning a period of one day is depicted for both the inhomogeneous and thirty seconds sampled returns. While the autocorrelation function of the synthetically constructed regular series appears strongly seasonal, on the left panel of Figure 5, the periodic effect fades as the number of lags increases. In addition, the seasonal patterns corresponding to the hour of the day and the presence of the traders in the three major geographical trading zones, become pronounced only in the time equidistant series.

The presence of a large number of zero returns in the data imposes a strong impact on the tails of the distribution where the true quotes from the data are concentrated. Apart from the marginalization of the former into the tails of the distribution, most importantly,

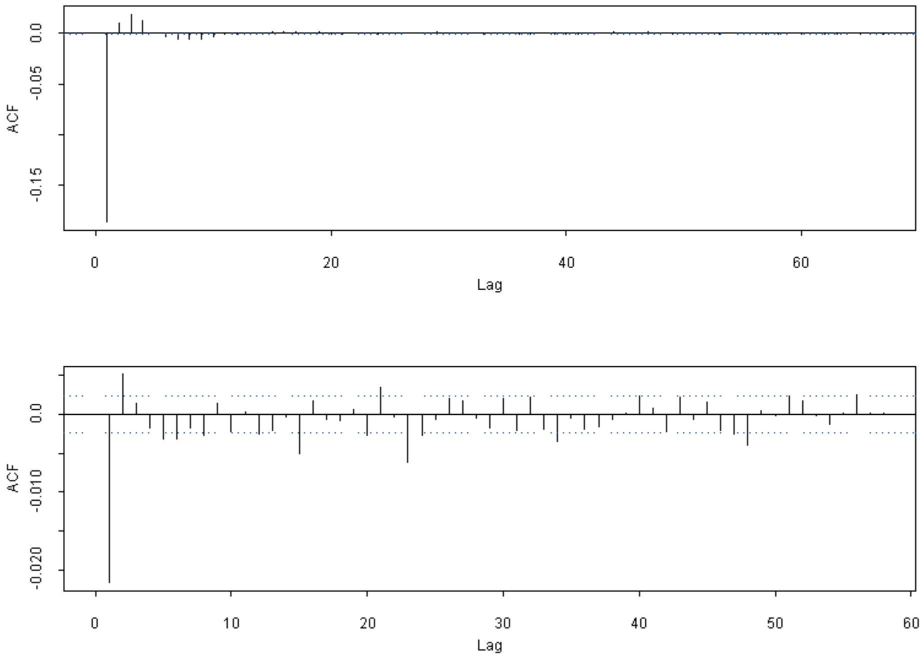

Figure 4: ACF plot of the inhomogeneous returns (upper panel) and homogeneous returns sampled at 30 second intervals (bottom panel).

these are forced to be categorized as outliers. Large series of consecutive zero returns in the data force the subsequent observations (which are the true returns) to look extreme. This is a significant negative effect arising from the application of the PT method, which carries important ramifications for the detection of jumps in the data. It also explains the fatter tails in the synthetically created return processes when high frequencies are used.

Even though, the number of zero returns in the inhomogeneous series is quite high, 53.18%, this number is expected to decrease dramatically in the sampled time series, and in roughly the same proportion as the number of observations in the series. Nevertheless, in the current application the opposite is observed, and the number of zero returns increases to 69.83% for data sampled at five seconds frequency, when the proportion of observations in the sampled data is only 76.09% of the total number of returns in the time inhomogeneous series. The same observation applies for all other frequencies examined.

The results in Table 2 reinforce the conclusions drawn from Table 1 regarding the distribution of the return series. It is clear that as the frequency increases, the kurtosis of the distribution increases dramatically, thus signifying a larger number of observations gathered in the center of the distribution, as well as a larger number of outliers[4]. The kurtosis

[4]This observation was also made in Osler (2005), who found that discreteness in FX rates contributes to excess kurtosis.

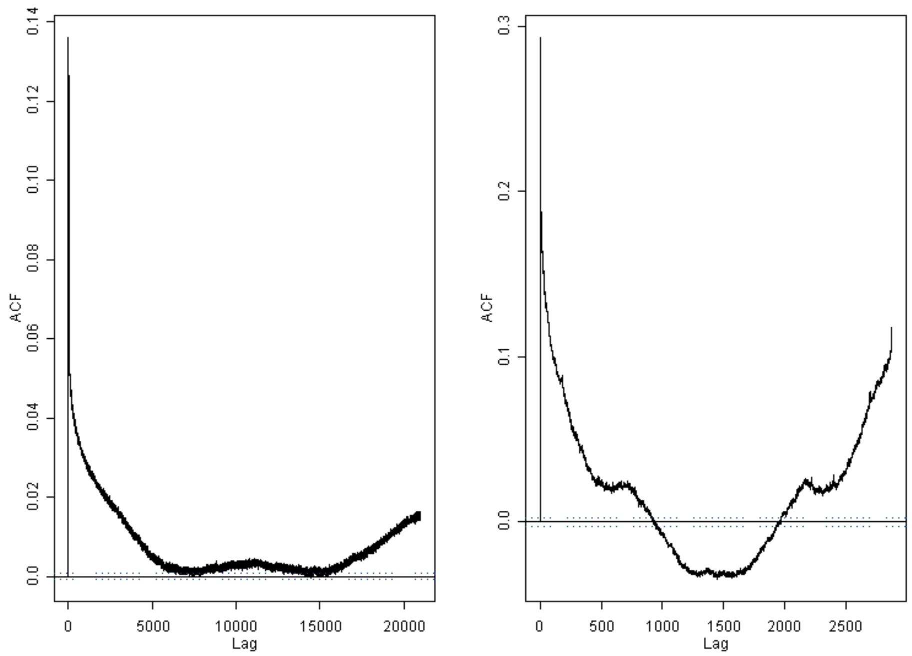

Figure 5: ACF plot of time inhomogeneous absolute returns (left panel) and time homogeneous absolute returns sampled at 30 second intervals (right panel) for lags covering a whole day.

δ	Mean	SD.	Skewness	Kurtosis	% 0s	Prop.Obs
Raw	0	0.004	0.163	19.21	53.18	100
5	0	0.004	0.426	88.80	69.83	76.22
30	0	0.009	0.444	60.99	42.21	12.70
60	0	0.013	0.393	51.39	31.37	6.35
120	0	0.019	0.533	43.46	22.22	3.18
180	0	0.023	0.323	33.41	17.98	2.12
240	0	0.026	0.374	31.14	15.33	1.59
300	0	0.029	0.287	23.50	13.33	1.27
600	0	0.041	0.274	18.06	9.05	0.64
900	0	0.051	0.398	18.42	7.41	0.42
1800	0	0.070	0.354	16.86	5.20	0.21

Table 2: Summary statistics for inhomogeneous data (first row) and artificial data constructed at various frequencies using the PT method.

of the data increases dramatically, almost fivefold, from 19.21 to 88.80, when five-seconds frequency is used and decreases to 16.86 when half an hour sampling frequency is used. Only at fifteen minutes frequency, the kurtosis is almost at the same level as that of the inhomogeneous series (18.42).

The skewness of the distribution also increases significantly at higher frequencies. This is another indication of the creation of artificial outliers in the data and the existence of a large number of zero returns. The first and second moments of the distribution on the other hand, decrease and increase respectively with decreasing frequencies. This effect can again be attributed to the overpowering of the real observations in the data by the zero returns, which drive the mean lower and act in increasing the variability in the data.

It is therefore concluded from the above analysis, that the PT method introduces a large number of zero returns which dominate the artificially constructed time series, predominantly during the less liquid periods in the sample. The impact of zero returns becomes very important for the higher moments of the distribution, but carries a smaller effect on the estimation of realized volatility, for which the quadratic variation theory remains a viable property. It is therefore important to investigate the level of distortions introduced by the large number of zero returns from the application of the PT method on the estimation of other volatility estimators, and consequently, on the performance of the tests in the literature which make use of these estimators to disentangle the discontinuous part of the return process from the continuous one. In the proceeding section the properties of the popular multi-power variation based tests from the literature are investigated in relation to the sampling scheme and sampling frequency employed in the data.

3 Jump Detections and the Previous Tick Method

It is evident from the analysis in section 2, that the PT method artificially introduces a large number of outliers in the data. In order to investigate the potential impact of these outliers on the diffusion process of the artificially constructed return series, in this section the PT method is examined in the context of jump detection. The analysis has been primarily inspired by the large number of jumps detected in the literature. To this day there is no empirical evidence as to whether these jumps can be justified by economic announcements. Some authors have being convinced that jumps are a common feature of financial returns. As stated by Barndorff-Nielsen and Shephard (2006), *"...empirical work points [us] to the conclusion that jumps are common"*. The main contribution of the current paper is therefore to investigate whether jumps are indeed common in financial returns, or an artifact of the synthetically constructed time equidistant return series.

In order to answer the question as to whether the large number of jumps recorded in the literature is biased by the sampling scheme used to construct the time equidistant returns, the jump detection tests of Barndorff-Nielsen and Shephard (2004, 2006) (henceforth BNS) and the tests obtained from the robust volatility estimators of Andersen et al. (2009) are applied to time equidistant FX data constructed using the PT method. The analysis was again carried out using a number of frequencies ranging from thirty seconds to fifteen minutes, in order to also examine the impact of the frequency on the proportion of the jumps detected. In what follows, a brief introduction to the notation of the jump detection tests is given together with the main results.

3.1 Multi-Power Variation Based Tests for Jumps

Let $[Y_\delta]_t$[5] denote the realized quadratic variation (QV) process defined as,

$$[Y_\delta]_t = \sum_{j=1}^{[t/\delta]} y_j^2, \tag{4}$$

where $\delta > 0$ denotes the length of the time span used to construct the homogeneous price process, $t \geq 0$ denotes the day and y_j is the intraday return at sampling point j calculated by,

$$y_j = Y_{j\delta} - Y_{(j-1)\delta}, \quad \text{for} \quad j = 1, 2, \ldots, [t/\delta]. \tag{5}$$

The theory of bipower variation (BPV) is based on the assumption that the quadratic variation process $[Y_\delta]_t$ can be defined as the sum of a purely discontinuous component and a continuous part of the local martingale component of Y, such as,

$$[Y_\delta]_t = [Y_\delta^c]_t + [Y_\delta^d]_t \tag{6}$$

Furthermore, $[Y_\delta]_t$ is assumed to be a member of the Browian semimartingale plus jump class, with zero drift,

$$[Y_\delta]_t = \int_{t-1}^{t} \sigma_s dWs + Z_t \tag{7}$$

where $Z_t = \sum_{j=1}^{N_t} c_j$ denotes the jump process, N is a counting process assumed finite for all t, and c_j are nonzero random variables. σ denotes the cádlág volatility process and W the standard Brownian motion. As shown by BNS, the bipower process can be defined as,

$$\{Y_\delta\}_t^{[1,1]} = \sum_{j=2}^{[t/\delta]} |y_{j-1}||y_j| = \mu_1^2 \int_{t-1}^{t} \sigma_s^2 ds = \mu_1^2 [Y_\delta^c]_t, \tag{8}$$

Consequently, by subtracting the realized bipower variation from the realized quadratic variation process one can obtain an estimate of the discontinuous component of QV,

$$[Y_\delta]_t - \mu_1^{-2}\{Y_\delta\}_t^{[1,1]} = \sum_{j=1}^{N_t} c_j = [Y_\delta^d]_t \tag{9}$$

For the investigation of the existence of jumps in the data, BNS recommend the use of the *adjusted ratio jump statistic* defined by,

$$\hat{J}_{\delta i} = \frac{\delta^{(-1/2)}}{\sqrt{\theta \cdot max(1, \tilde{q}_i/\{\tilde{v}_i\}^2)}} \left(\frac{\tilde{v}_i}{\hat{v}_i} - 1 \right) \tag{10}$$

[5]Throughout the section we will use the same notation as in BNS.

where $\theta = (\pi^2/4) + \pi - 5 \approx 0.6090$, $\tilde{v}_i$ denotes the sequence of T daily realized BPVs,

$$\tilde{v}_i = \frac{M}{M-1}\mu_1^{-2}\sum_{j=2}^{M}|y_{j-1,i}||y_{j,i}|, \qquad i = 1, 2, \ldots, T \tag{11}$$

where $M = 1/\delta$. $\hat{v}_i$ denotes the sequence of T daily realized quadratic variations,

$$\hat{v}_i = \sum_{j=1}^{M} y_{j,i}^2, \qquad i = 1, 2, \ldots, T, \tag{12}$$

and $\tilde{q}_i$ denotes the sequence of daily realized quad-power quarticities,

$$\tilde{q}_i = \frac{M^2}{M-3}\mu_1^{-4}\sum_{j=4}^{M}|y_{j-3,i}||y_{j-2,i}||y_{j-1,i}||y_{j,i}|, \qquad i = 1, 2, \ldots, T \tag{13}$$

Andersen et al. (2004b) also suggested the use of the *realized tri-power quarticity*, in the place of the quad-power quarticity, as a more jump-robust alternative. This is defined by,

$$\tilde{t}_i = \frac{M^2}{M-2}\mu_{4/3}^{-3}\sum_{j=3}^{M}|y_{j-2,i}|^{4/3}|y_{j-1,i}|^{4/3}|y_{j,i}|^{4/3}, \qquad i = 1, 2, \ldots, T \tag{14}$$

Recently, Andersen et al. (2009)(henceforth ADS) have proposed a new set of estimators as an alternative to the bipower variation. As the authors claim, unlike the bipower variation, the new estimators are robust to jumps, microstructure noise, and the occurrence of flat trading in the return process. These are based on the minimum and median of consecutive absolute intraday returns for the estimation of a jump robust volatility measure and are defined by,

$$\hat{v}_i^{(min)} = \frac{\pi}{\pi-2}\Big(\frac{M}{M-1}\Big)\sum_{j=1}^{M}\min(|y_{j-1,i}|, |y_{j,i}|)^2, \quad i = 1, 2, \ldots, T \tag{15}$$

$$\hat{v}_i^{(med)} = \frac{\pi}{6-4\sqrt{3}+\pi}\Big(\frac{M}{M-2}\Big)\sum_{j=2}^{M}\text{med}(|y_{j-2,i}|, |y_{j-1,i}|, |y_{j,i}|)^2, \quad i = 1, 2, \ldots, T \tag{16}$$

The minimum realized volatility estimator ($\hat{v}_i^{(min)}$) is more robust to jumps than the BPV, as it always takes the minimum of two consecutive intraday absolute returns[6]. However, this estimator suffers from the presence of zero returns in the sample. In order to alleviate this problem, the authors suggest the use of the median realized volatility ($\hat{v}_i^{(med)}$).

In the current paper, the *adjusted ratio jump statistics* of BNS and ADS will be implemented using the realized tri-power variation and median realized variance for the two tests

[6]Given that jumps are of finite activity, the probability of having two adjacent jumps will be zero.

respectively, i.e,

$$\tilde{J}_{\delta i} = \frac{\delta^{(-1/2)}}{\sqrt{\theta \cdot max(1, \tilde{t}_i/\{\tilde{v}_i\}^2)}}\left(\frac{\tilde{v}_i}{\hat{v}_i} - 1\right) \tag{17}$$

$$\hat{J}^{(med)}_{\delta i} = \frac{\left(1 - \frac{\hat{v}_i^{(med)}}{\hat{v}_i}\right)}{\sqrt{0.96\frac{1}{M}\max(1, \tilde{q}_i^{(med)}/\hat{v}_i^{(med)})}} \tag{18}$$

where $\tilde{q}_i^{(med)}$ is given by:

$$\tilde{q}_i^{(med)} = M\frac{3\pi}{9\pi + 72 - 52\sqrt{3}}\left(\frac{M}{M-2}\right)\sum_{j=2}^{M}\text{med}(|y_{j-2,i}|, |y_{j-1,i}|, |y_{j,i}|)^4, \quad i = 1, 2, \ldots, T \tag{19}$$

3.2 Jump Tests Results

In this section, the adjusted ratio jump test statistics defined above are applied to the EUR/USD currency pair for the sample period January 2000 to May 2007. The results are shown in tables 3, 4 and 5. For each δ, which denotes the sampling frequency in seconds, the following quantities are calculated and recorded in Table 3. The sum of the first five autocorrelations in the synthetic returns is recorded in the second column of the table. The third, fourth, fifth and sixth columns record the average value of the daily quadratic variation, daily bipower variation, daily minimum realized volatility and median realized volatility estimators respectively. The average daily tri-power quarticity and median realized quarticity are shown in columns seven and eight. Finally columns nine and ten list the proportion of zero returns in the data and the proportion of observations in the synthetic returns relative to the original inhomogeneous data, respectively.

The results in the second column of Table 3 reinforce the conclusions from section two that the PT method weakens the first order autocorrelation of the synthetic return series when compared to the raw data. At the highest frequency of five seconds, the first five autocorrelations sum up to -0.09 compared to -0.200 in the raw data. Furthermore, this sum increases significantly with decreasing frequency.

In terms of the daily average volatility estimators, it is observed that the bipower variation is lower than the minimum and median realized volatility at the highest frequencies of 5 and 30 seconds but is much higher than the other two estimators at lower frequencies of one minute and beyond. The lower value of the bipower variation at the highest frequencies of 5 and 30 seconds might be the result of the large number of zero returns found in the return process at those frequencies, which biases downwards the bipower variation estimator.

When the daily average volatility estimators are compared to the corresponding quantities estimated for the raw data, it is evident that these are much lower. This observation once more highlights the negative effect that zero returns carry on the calculation of the jump robust volatility estimators. The realized quarticity estimators, on the other hand,

EUR/USD 01/04/2000 - 05/31/2007									
δ	ACFs	QV	BPV	MinRV	MedRV	TPQ	MedRQ	% 0s	% Obs
Raw	-0.200	0.616	0.551	0.714	0.639	0.0001	0.0001	43.88	100
5	-0.090	0.470	0.356	0.428	0.435	0.0001	0.0002	65.58	83.00
30	0.032	0.397	0.339	0.346	0.347	0.0008	0.0008	38.26	13.83
60	0.010	0.400	0.351	0.346	0.345	0.0014	0.0012	28.06	6.92
120	-0.022	0.408	0.362	0.347	0.349	0.0021	0.0022	19.61	3.46
180	-0.040	0.414	0.369	0.350	0.352	0.0025	0.0022	15.67	2.31
240	-0.048	0.416	0.372	0.352	0.354	0.0034	0.0030	13.38	1.73
300	-0.044	0.415	0.370	0.348	0.351	0.0027	0.0030	11.72	1.38
600	-0.017	0.410	0.367	0.343	0.346	0.0046	0.0048	7.87	0.69
900	-0.006	0.406	0.357	0.333	0.336	0.0050	0.0058	6.39	0.46
1800	0.002	0.392	0.341	0.314	0.316	0.0073	0.0076	4.39	0.23

Table 3: Results from the application of the adjusted ratio jump statistic to the time series obtained using the PT method.

behave very similarly to those obtained for the time inhomogeneous series at the highest frequency of five seconds, while at the lower frequencies employed, these are much larger.

It becomes clear in column nine of the table, where the percentage of zero returns in the time homogeneous and time inhomogeneous series is recorded, that the number of zero returns dominates the series at the highest frequency of five seconds, while this decreases disproportionately to the number of observations in the data at lower frequencies.

δ	% Diff (BPV)	% Diff (MinRV)	% Diff (MedRV)
5	24.26	8.94	7.45
30	14.61	12.85	12.59
60	12.25	13.50	13.75
120	11.27	14.95	14.46
180	10.87	15.46	14.98
240	10.58	15.38	14.90
300	10.84	16.14	15.42
600	10.49	16.34	15.61
900	12.07	17.98	17.24
1800	13.01	19.90	19.39

Table 4: Percentage difference of jump robust volatility estimators (BPV, MinRV, MedRV) against the quadratic variation.

Table 4 records the percentage difference between the quadratic variation and the jump robust volatility estimators recorded in columns 4, 5 and 6 of Table 3. This quantity is a preliminary indicator of the level of the discontinuous component in the data. The results illustrate that, on average, 13.02% of the days in the sample have jumps when the bipower variation is considered as a jump robust estimator. This number is 15.14% and 14.58% for the minimum and median realized volatility estimators. However, despite the fact that on average, across all sampling frequencies, the minimum and median realized volatility result in a slightly larger proportion of jumps detected, for the highest frequencies of 5 and 30 seconds, the percentage difference between these and the quadratic variation is much smaller than that between the bipower variation, while the reverse is observed for lower frequencies, greater or equal to one minute. This can again be attributed to the presence of a large number of zero returns in the data which biases the bipower variation downwards, while the median and minimum realized volatilities remain by construction more robust to the presence of flat trading in the data.

EUR/USD 01/04/2000 - 05/31/2007						
	% Rejection ($\tilde{J}_{\delta i}$)			% Rejection ($\hat{J}_{\delta i}^{(med)}$)		
δ	5%	1%	0.1%	5%	1%	0.1%
5	99.67	99.51	99.29	91.04	84.12	72.09
30	96.98	93.19	88.08	87.36	75.27	57.09
60	87.80	75.66	59.73	78.19	61.26	41.26
120	70.93	52.80	33.30	67.31	46.48	26.10
180	56.10	37.42	20.66	56.76	36.26	19.51
240	49.34	29.89	15.05	47.75	29.51	16.59
300	43.79	24.84	13.35	44.12	25.71	11.81
600	31.21	17.14	7.69	30.93	18.30	9.12
900	27.42	15.11	6.87	27.97	16.65	7.75
1800	23.52	12.97	5.77	23.96	13.90	6.21

Table 5: The level of rejections of the test statistic applied to the time series obtained using the PT method and for 5%, 1% and 0.1% significant levels.

Table 5 documents the results from the application of the two test statistics in equations (3.17) and (3.18) for three significant levels; 5%, 1% and 0.1%. The results indicate that the percentage of time series for which the test statistics are rejected decreases dramatically as the sampling frequency decreases. This instability might be due to the bias induced through the large number of zero returns in the series. These zero returns drive the quantities of bipower variation and tri-power quarticity downwards, and thus, cause the test statistics to be heavily downward biased, particularly at high frequencies. In addition, the $\tilde{J}_{\delta i}$ test statistic results in a much larger number of jumps detected than the $\hat{J}_{\delta i}^{(med)}$ test statistic, which is more robust to microstructure noise and zero returns in the process, for all significant levels examined and for frequencies higher than three minutes.

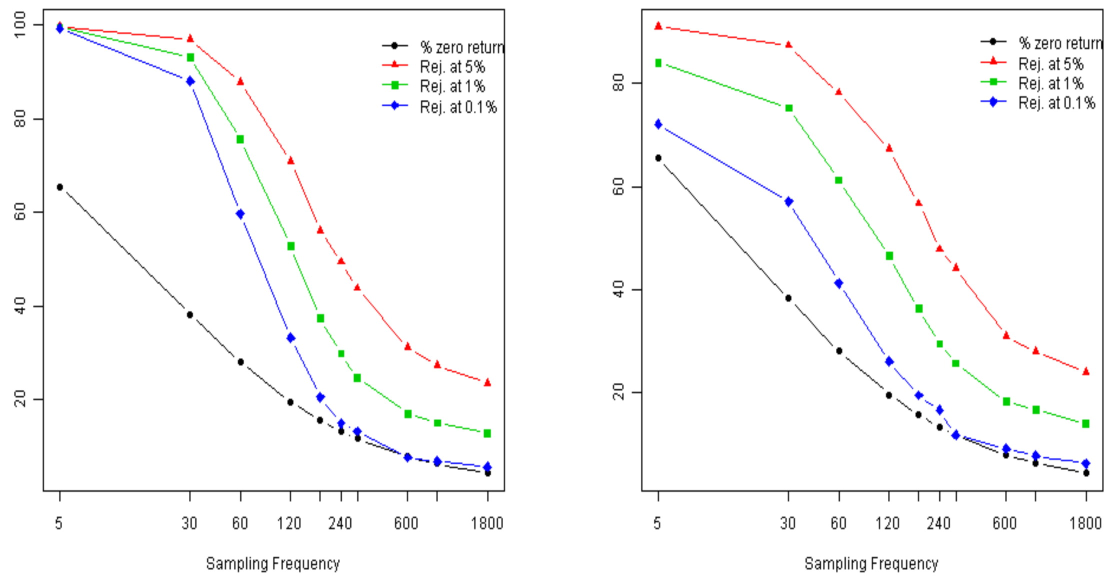

Figure 6: Rejection percentages for the $\tilde{J}_{\delta i}$ test statistic (left panel) and $\hat{J}_{\delta i}^{(med)}$ test statistic (right panel) plot together with the percentage of zero returns in the artificial data set at the various sampling frequencies examined (this is denoted by the solid line with filled circles).

The high correlation between the percentage of zero returns in the data and the number of jumps detected, as evidenced by Figure 6 where the percentage rejections of the two test statistics are plotted at the various sampling frequencies examined, together with the percentage of zero returns in the data, might be an indication that there is indeed a strong causal relation between the induced zero returns and the number of jumps detected by the test statistics. Furthermore, the much smaller percentage of jumps indicated by the difference between the realized volatility and the jump robust volatility estimators in Table 4, than the one detected by the application of the jump tests to the data, hint that the jump statistics might be upward biased.

We believe that the 'step' paths produced by the PT method might be potentially the main factor leading to the detection of a very large number of jumps at high frequencies from the application of the ratio jump statistics. In the proceeding section, we propose the use of Akima cubic splines as an alternative to the PT method for sampling high frequency data. The analysis focuses on the advantages of Akima cubic splines in the context of jump detection.

4 Cubic Spline Interpolation

Interpolating splines and their mathematical representations were first explained in Schoenberg (1946a,b). In the early seventies, Cox (1972); de Boor (1972); Gordon and Riesenfeld (1974) employed splines as a method for designing free form curves and surfaces. They consequently found extensive applications in the automotive and aerospace industry (Ahlberg et al., 1967; Boehm and Farin, 1984; de Boor, 1986) and more recently in computer graphics, image processing and image reconstruction applications (Keys, 1981; Hou and Andrews, 1978; Unser et al., 1991; Parrot et al., 1993; Jensen and Anastassiou, 1993).

Spline interpolation is based on the concept of fitting continuous-time functions through discrete time points. These functions are constructed in the form of polynomials which satisfy certain continuity conditions and boundary constraints, and present some important advantages over other interpolation techniques. Amongst them, is the stability of the resulted series and the simplicity of the calculations involved in their construction. The set of linear equations which need to be solved for the construction of the polynomials are very well-conditioned on the observed time points and, therefore, the polynomial coefficients can be calculated precisely. This allows the calculation scheme to remain stable, even for large number of points in the series. Furthermore, spline-based interpolating methods are very simple and easy to implement in any software package.

Amongst the various spline interpolation techniques, cubic-spline interpolation has been one of the most popular in the literature. This particular type of splines was found to outperform other interpolating schemes in a number of empirical applications. Parker et al. (1983), in an early paper comparing various interpolation techniques for image resampling, concluded that cubic splines resulted in better results than the other interpolating functions examined. Meijering (2000) in another comparison study of interpolation techniques applied to medical images, also found that cubic splines resulted in a considerable reduction of the interpolation error when compared to other interpolation techniques. The same conclusions were drawn in the work of Truong et al. (2000); Jiao and Heath (2004).

Nevertheless, cubic splines suffer from the frequently encountered problem of overshoots and oscillations when data is interpolated at points in the vicinity of steep gradients, produced by rapidly changing data values (Fried and Zietz, 1973; Wolberg and Alfy, 2002). To overcome this problem, a number of alternative methods have been suggested in the literature in the work Fritsch and Carlson (1980); Fritsch and Butland (1993); Akima (1970, 1991); Hyman (1983); Huynh (1993). In the current paper, the cubic splines-based interpolation method developed by Akima (1970) is implemented for the time homogenization of the prices process prior to the application of the jump test statistics.

4.1 Akima Interpolation

Akima (1970) proposed an alternative method to cubic splines to alleviate the interpolating series from overshoots and oscillation, which determines polynomial coefficients locally.

Thus, the coefficients of the cubic polynomial are fixed between every pair of successive data points with specified slopes. These slopes are determined locally as the weighted average of the slopes of the line segments connecting the interpolated point with the points in its proximity. Given that no functional form for the whole curve is assumed, and that the interpolated value is determined by the value of the items on its periphery rather than by the value of all of the data points, Akima's proposed method results in improved interpolation accuracy and oscillations are consequently avoided.

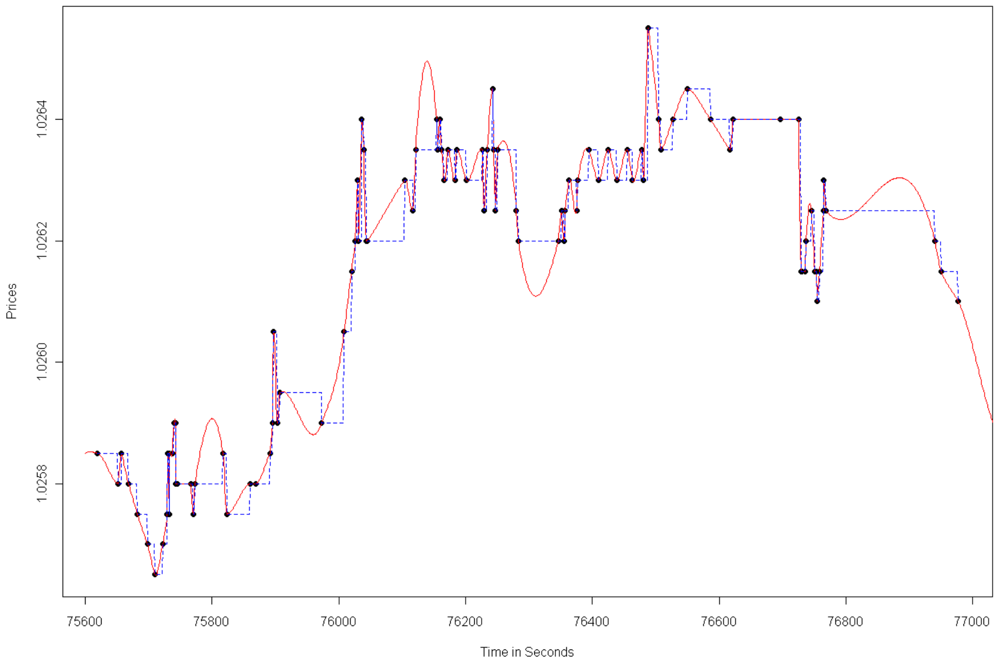

Figure 7: Example of interpolated series using Akima interpolation (solid line) and the PT method (dashed line).

For a set of observed points x_i, $1 \leq i \leq N$, the Akima interpolation function is defined by,

$$s(x) = a_0 + a_1(x - x_i) + a_2(x - x_i)2 + a_3(x - x_i)3, \qquad x_i \leq x \leq x_i + 1 \tag{20}$$

The coefficients of the cubic polynomial (a_0, a_1, a_2, and a_3) for each interval $[x_i, x_i + 1]$ are determined using the function values at the end points of the interval ($s(x_i)$ and $s(x_{i+1})$) and their first derivatives ($s(x_i)'$ and $s(x_{i+1})'$),

$$a_0 = s(x_i), \tag{21}$$

$$a_1 = s(x_i)', \tag{22}$$

$$a_2 = -[2(s(x_i)' - m_i) + (s(x_{i+1})' - m_i)]/(x_{i+1} - x_i), \tag{23}$$

$$a_3 = [(s(x_i)' - m_i) + (s(x'_{i+1} - m_i)]/[(x_{i+1} - x_i)^2], \tag{24}$$

where m_i is the slope of the line segment connecting points x_i and x_{i+1} defined by,

$$m_i = (s(x_{i+1}) - s(x_i)/(x_{i+1} - x_i). \quad (25)$$

Figure 7 depicts an interpolated price process using Akima interpolation (solid line) and the PT method (dashed line). It is illustrated in the graph that the Akima interpolation method results in a more 'natural' looking process than the PT method, while at the same time it avoids the creation of artificial flat pricing, which is seen in the 'step' effect of the PT sample line. Another important advantage of the Akima interpolation method, when this is applied to financial data is that, unlike other cubic spline techniques, it does not impose a continuity condition on the derivatives of the interpolated process. A condition which is inconsistent with the asset pricing theory, which employs models based on Brownian motions.

The Akima interpolation method was applied to time homogenize the price process of the currency pair EUR/USD for the period between January 2000 and May 2007, as it was done in the previous section using the PT method. The results from the application are reported in tables 6 and 7.

EUR/USD 01/04/2000 - 05/31/2007									
δ	ACFs	QV	BPV	MinRV	MedRV	TPQ	MedRQ	% 0s	% Obs
Raw	-0.200	0.616	0.551	0.714	0.639	0.0001	0.0001	43.88	100
5	-0.036	0.450	0.420	0.452	0.436	0.0002	0.0002	24.28	83.00
30	0.004	0.428	0.389	0.383	0.382	0.0009	0.0010	12.25	13.83
60	-0.016	0.427	0.389	0.377	0.375	0.0015	0.0013	8.24	6.92
120	-0.044	0.429	0.389	0.371	0.372	0.0023	0.0024	5.27	3.46
180	-0.059	0.431	0.389	0.368	0.370	0.0026	0.0023	4.07	2.31
240	-0.064	0.432	0.387	0.366	0.369	0.0035	0.0041	3.40	1.73
300	-0.058	0.428	0.384	0.363	0.365	0.0034	0.0040	2.99	1.38
600	-0.024	0.417	0.375	0.352	0.354	0.0048	0.0053	1.96	0.69
900	-0.010	0.410	0.362	0.338	0.342	0.0050	0.0065	1.57	0.46
1800	0.001	0.394	0.342	0.316	0.318	0.0073	0.0077	1.19	0.23

Table 6: Various volatility estimators obtained from data sampled at various frequencies using the Akima interpolation method.

Akima interpolation, like the PT method, results in lower first-order autocorrelations. However, Akima interpolation produces 72.2% less zero returns than the PT method, across all the sampling frequencies examined. The smaller number of zero returns induced by the Akima interpolation method is reflected in the values of the volatility estimators in columns 3, 4 and 5 of Table 6. These are higher for the series constructed under Akima interpolation than for those constructed under the PT method. The difference in the values of the various

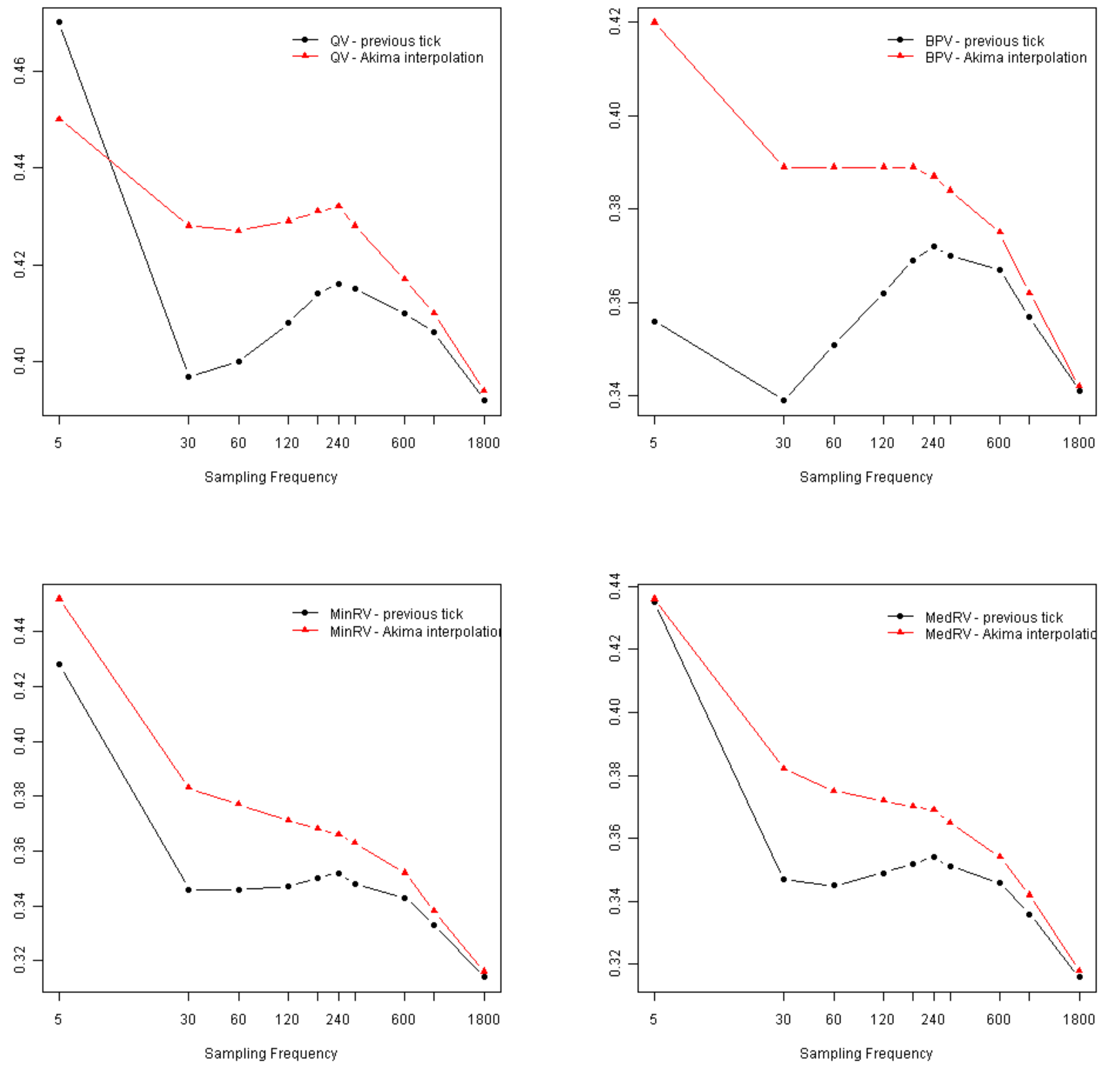

Figure 8: Volatility estimators under the Akima Interpolation (solid lines with triangles) and PT method (solid line with circles). The quadratic variation estimator (QV) is depicted in the upper right plot and the bipower variation estimator (BPV) in the upper left. The minimum variance volatility estimator (MinRV) is depicted in the bottom right plot and the minimum variance (MedRV) estimator in the bottom left plot.

volatility estimators relative to the sampling method employed is visually illustrated in the four plots of Figure 8.

It is observed from Figure 8 that the largest difference between the two interpolating schemes is realized in the estimation of the bipower variation, which is also the volatility estimator the most sensitive to the presence of zero returns in the data. On the other hand, the MedRV and MinRV estimators behave very similarly under the two sampling schemes. This is again due to the robustness of these estimators to zero returns in the data.

The percentage difference between the quadratic variation and the jump robust volatility estimators in columns 4, 5 and 6 of the table is 9.94%, 13.33% and 13.38% respectively, compared to 13.02%, 15.14% and 14.58% for the series constructed under the PT method. Therefore one should expect a smaller number of jumps in the synthetic returns under the Akima interpolation than in the return series constructed under the PT method.

Table 7 confirms the hypothesis formulated from the results in Table 6. The return process constructed using the Akima interpolation resulted in a much smaller number of jumps than the one constructed using the PT method. This is clearly depicted in Figure 9 for the two jump test statistics and for 1% significance level.

In the proceeding section, a Monte Carlo simulation is carried out with the scope of discriminating the impact of the artificially induced zero returns on the level of jumps detected in the synthetically constructed return processes. The two interpolating methods presented in the paper are implemented together with the linear interpolation method. A comparison is carried out in terms of the size and power of the various jump test relative to the sampling scheme employed for the time homogenization of the return series.

EUR/USD 01/04/2000 - 05/31/2007						
	% Rejection ($\tilde{J}_{\delta i}$)			% Rejection ($\hat{J}_{\delta i}^{(med)}$)		
δ	5%	1%	0.1%	5%	1%	0.1%
5	92.53	87.25	80.77	60.33	43.90	28.30
30	87.91	78.08	62.53	80.88	64.56	43.79
60	73.02	55.00	36.59	69.62	50.11	31.48
120	60.71	40.77	22.58	59.95	38.46	20.71
180	48.41	31.04	16.92	51.26	30.77	15.71
240	44.23	26.76	13.46	45.49	27.09	14.56
300	40.00	22.53	11.87	41.37	23.63	10.27
600	29.78	16.10	7.53	30.27	17.36	8.63
900	26.26	14.67	6.32	27.36	16.10	7.25
1800	23.46	12.47	5.71	24.12	13.57	6.48

Table 7: The level of rejections of the test statistic applied to the time series obtained using Akima interpolation and for 5%, 1% and 0.1% significant levels.

5 Monte Carlo Simulation

5.1 Simulation Design

The scope of this section is the investigation of the performance of the BNS and ADS test statistics, in terms of size and power, under three different calendar time sampling schemes, namely the Akima interpolation, linear interpolation and the PT method. An extensive Monte Carlo simulation experiment is carried out to shed some light on the potential impact

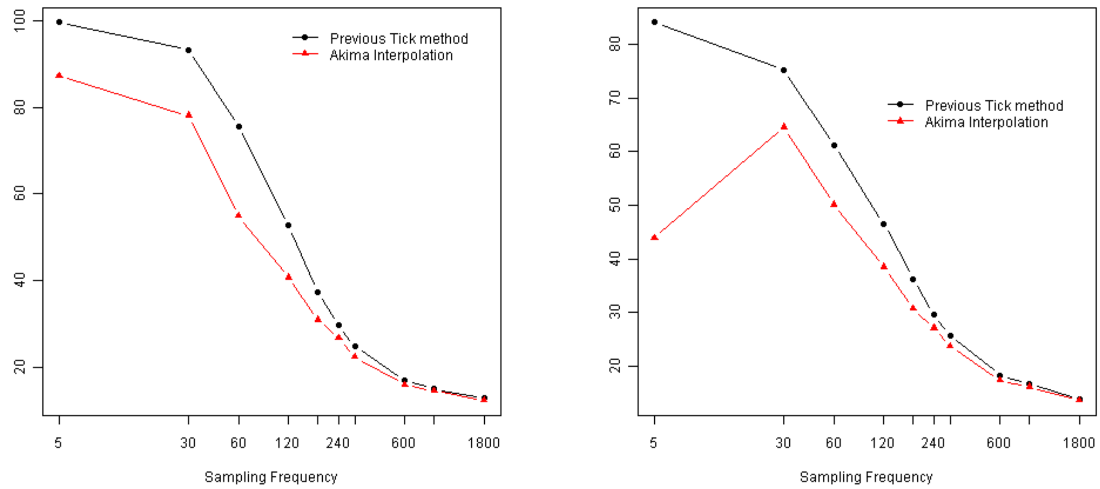

Figure 9: Percentage of jumps detected by the $\tilde{J}_{\delta i}$ test statistic (left panel) and $\hat{J}_{\delta i}^{(med)}$ test statistic (right panel) in the synthetic return process constructed using Akima interpolation (solid line with triangles) and that using PT method (solid line with circles) at various sampling frequencies and at 5% significant level.

of consecutive zero returns, artificially induced in the price process through the use of the PT sampling scheme, on the test statistics of the BNS and ADS jump tests.

As shown in the previous section, the Akima interpolation does not induce any artificial zero returns in the time-homogeneous price process. Consequently, it is interesting to see if the proportion of jumps detected is sensitive to the method used for the time-homogenization of the price process, and therefore the presence of a large proportion of zero returns. For completion of the analysis, the results obtained from the homogenization of the price process after the application of the linear interpolation are also reported.

5.1.1 Mean Intraday Durations

For the purpose of applying the three calendar time sampling schemes to the simulated trajectories of prices, the realistic assumption was made that transactions arrive in the market at random times. The intraday durations of the transactions were simulated under three different scenarios for the mean intraday duration. The first scenario consisted of mean intraday durations of 5 seconds, typically the mean intraday duration observed in the FX market. The second scenario consisted of mean intraday durations of 15 seconds, typically the empirical mean intraday duration observed in the S & P 500 futures market. Finally, the third scenario consisted of mean intraday durations of 30 seconds, to represent the empirical mean intraday duration of less liquid assets in the market.

In order to construct an empirically relevant intraday duration pattern, a cubic spline was fitted in the intraday duration pattern of each day in the sample of the EUR/USD currency pair, and the average duration across each point in time was obtained for the

1820 days in the sample. This was scaled accordingly to obtain the required mean intraday durations of 5, 15 and 30 seconds.

Figure 10 illustrates the general shape of the diurnal pattern found in the FX data, which spans for 24 hours, from 21:00 GMT of the previous day to 21:00 GMT of the next day. Duration is higher after 21:00 GMT and falls after 23:00 GMT, when the Far East markets open. The second decrease in duration observed in the graph is caused by the European markets opening at around 7:00 GMT. Finally, after the opening of the US markets, at around 14:00 GMT, duration reaches its lowest level. Duration therefore decreases significantly when there is an overlap in the trading hours of the most active markets such as Tokyo, New York and London, and it starts increasing again when markets close.

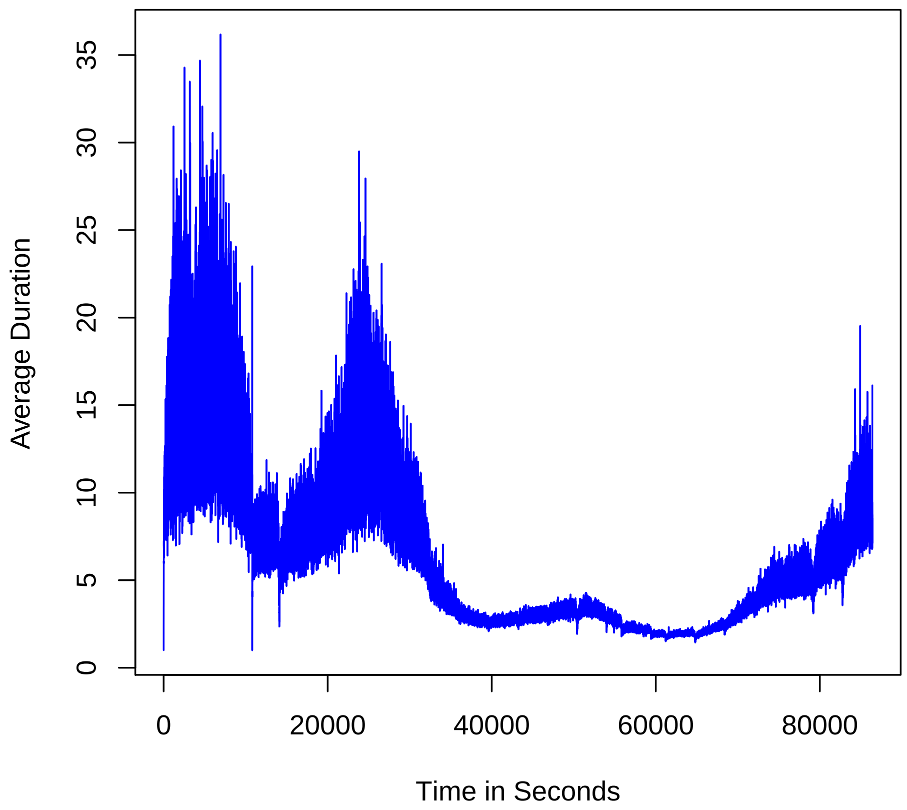

Figure 10: Approximate intraday duration pattern in EUR/USD currency pair, estimated using cubic spline interpolation.

5.1.2 Simulated Sample Paths

The simulated sample paths were constructed using three different data generating processes; the Heston stochastic volatility model, the log-linear one factor stochastic volatility model and the log-linear two factor stochastic volatility model. These are defined as,

- **Heston Stochastic Volatility Model:**

$$\begin{aligned} \mathrm{d}S_t &= S_t\mu\mathrm{d}t + \sqrt{v_t}\mathrm{d}W_t^{(p)} \\ \mathrm{d}v_t &= \alpha_v(\theta - v_t)\mathrm{d}t + \xi\sqrt{v_t}\mathrm{d}W_t^{(v)} \\ Cov[W_t^{(p)}, W_t^{(v)}] &= \rho dt \end{aligned}$$

- **Log-Linear One-Factor Stochastic Volatility Model (LL1F):**

$$\begin{aligned} \mathrm{d}S_t &= \mu\mathrm{d}t + \exp[\beta_0 + \beta_1 v_t]\mathrm{d}W_t^{(p)}, \\ \mathrm{d}v_t &= \alpha_v v_t\mathrm{d}t + \mathrm{d}W_t^{(v)}, \\ Cov[W_t^{(p)}, W_t^{(v)}] &= \rho dt \end{aligned}$$

- **Log-Linear Two-Factor Stochastic Volatility Model (LL2F):**

$$\begin{aligned} \mathrm{d}S_t &= \mu\mathrm{d}t + \mathrm{sexp}[\beta_0 + \beta_1 v_{1t} + \beta_2 v_{2t}]\mathrm{d}W_t^{(p)}, \\ \mathrm{d}v_{1t} &= \alpha_{v1} v_{1t}\mathrm{d}t + \mathrm{d}W_t^{(v1)}, \\ \mathrm{d}v_{2t} &= \alpha_{v2} v_{2t}\mathrm{d}t + [1 + \beta_{v2} v_{2t}]\mathrm{d}W_t^{(v2)}, \\ Cov[W_t^{(p)}, W_t^{(v1)}] &= \rho_1 dt \\ Cov[W_t^{(p)}, W_t^{(v2)}] &= \rho_2 dt \end{aligned}$$

where S_t denotes the spot volatility at time t and μ the (risk neutral) drift. v_t, v_{1t} and v_{2t} are stochastic volatility factors, ξ is the volatility of the volatility, $W^{(p)}, W^{(v)}, W^{(v1)}$, and $W^{(v2)}$ are standard Brownian motions, α_v the mean reversion rate and θ the long run variance. The process v_{1t} is a standard Gaussian process, while v_{2t} exhibits a feedback term in the diffusion function. The spliced exponential function sexp ensures a solution to LL2F exists (see Chernov et al., 2003, for details).

For the simulation of the sample paths from the Heston stochastic volatility model, the initial parameters documented in Zhang et al. (2005) were used where $\alpha_v = 5$, $\mu = 0.05$, $\theta = 0.04$, $\xi = 0.5$ and $\rho = -0.4$. For the LL1F and LL2F stochastic volatility models, the parametrization employed in Huang and Tauchen (2005) was followed. Hence for the LL1F model the parameters used were: $\mu = 0.03$, $\rho = -0.620$, $\beta_0 = 0$ and $\beta_1 = 0.125$. Three different scenarios of mean reversion were considered covering slow, moderate and fast mean reversion ($a_v = -0.00137, -0.100, -1.386$). The corresponding parameters for the LL2F stochastic volatility model were $\mu = 0.03$, $\beta_0 = -1.2$, $\beta_1 = 0.04$, $\beta_2 = 1.5$, $a_{v1} = -0.00137$, $a_{v2} = -1.386$, $\beta_{v2} = 0.25$, $\rho_1 = -0.3$ and $\rho_2 = -0.3$.

The Heston stochastic volatility model is often used in the literature as a benchmark model in studies of volatility modeling. The log-linear one factor stochastic volatility model exhibits more erratic volatility paths than the Heston stochastic volatility model. It has been studied by Chernov et al. (2003), together with the two factor stochastic volatility models (LL2F). Under the LL2F specification the volatility process becomes more erratic than under the LL1F specification, and is also characterized by larger price movements.

Chernov et al. (2003) claim that the log-linear one-factor volatility model is not able to capture well all the moments of the distribution of financial assets, and that the two factors in LL2F model, one being extremely persistent and the other strongly mean reverting with feedback in volatility, are necessary for better capturing the dynamics of asset returns. There has been extensive support in the literature for the above claim, such as in the work of Chacko and Viceira (2005); Engle and Lee (1999); Gallant et al. (1999); Alizadeh et al. (2002); Bollerslev and Zhou (2002).

The Euler discretization scheme was used for simulating the sample paths from the three data generating processes described above. For each of the three stochastic volatility models, a total of 10000 trading days were simulated with variable number of transactions in each day. The number of daily transactions is dependent on the mean intraday duration assumed in the sample. The simulated daily paths span for 24 hours, so as to imitate transactions in the FX market. Each simulated day was sampled using five different sampling frequencies, ranging from 30 seconds to 15 minutes. The proceeding section discusses the results from the Monte Carlo simulation described above.

5.2 Simulation Results

In this section, we investigate the performance of the BNS and ADS jump tests in relation to the interpolating method used for the construction of the time equidistant price processes. The results on tick time are also shown[7].

5.2.1 Size Analysis

Table 8 and Table 9 summarize the results for the size analysis of the BNS and ADS test respectively for five different sampling frequencies; namely 30 seconds, 1 minutes, 2 minutes, 5 minutes and 15 minutes and for 1% significant level (results for other significant levels are found in the appendix).

For 15 and 30 seconds mean intraday durations (MID), the frequency at which the BNS test attains a size close to the nominal was 1 and 2 minutes respectively under the Akima spline and linear interpolations and 5 minutes or lower under the PT sampling scheme. The same conclusions are drawn from the simulated trajectories with 5 seconds MID. Under the Akima spline and linear interpolation methods, the BNS test already converges to the

[7]It should be noted that the two tests considered were constructed under the assumption of equidistant time steps, therefore it is not clear whether the asymptotics of the tests still remain viable under this type of specification.

nominal size at 30 seconds sampling frequencies, while under the PT sampling scheme, the test remains heavily oversized for sampling frequencies higher than 2 minutes.

The results are similar for the simulated paths constructed under the Heston stochastic volatility model and the log linear one factor stochastic volatility model with slow, medium and fast mean reversion. For the trajectories constructed under the log linear two factor volatility model, it is observed that due to the very erratic nature of the sample paths, the test becomes oversized at the lower frequencies under the Akima and linear interpolation sampling schemes, while under the PT method, the BNS test remains heavily oversized at all sampling frequencies considered.

Hence, the BNS converges to the nominal size at higher frequencies for time series constructed under the Akima interpolation method than under the PT method. The test performs similarly when applied to time series constructed under the linear interpolation method to those constructed under the Akima interpolation method; although in the latter case the size properties of the test are slightly better and these are less undersized under the Akima interpolation sampling scheme than under the linear interpolation sampling scheme.

Thus, it can be concluded that the large number of zero returns introduced in the time series from the application of the PT method causes the BNS test to be oversized at high frequencies . The sensitivity of the test to the presence of zero returns in the time series constructed under the PT method is further highlighted from the results obtained across the sample paths simulated assuming different mean intraday duration patterns. It is observed that the longer the mean intraday duration in the series, the more biased is the test to the presence of zero returns, resulting in extreme size results reaching 100% for 1% nominal 30 seconds sampling frequency and 30 seconds mean intraday duration, and decreasing to around 70% for 15 seconds mean intraday duration and to 2% for 5 seconds mean intraday duration. Figure 11 depicts the size results from the application of the BNS tests on trajectories simulated under the LL1F stochastic volatility model with medium mean reversion, for the three sampling schemes and three MID patterns. It is obvious from the plot that the test statistic applied to trajectories sampled under the PT method results in poor size results when compared to the results obtained for the other two sampling schemes.

However, under the Akima and linear interpolation methods the sample paths over smoothed at the highest frequencies, thus resulting in the test being undersized. Nevertheless, the test attains better size properties at higher frequencies under the two aforementioned interpolation methods than under the PT method.

The results obtained for the ADS test are slightly better under all three sampling schemes, compared to the results obtained for the BNS test. However, the improvement is not significant at the highest frequencies, and the ADS test is still undersized under the Akima spline and linear interpolation and extremely oversized under the PT method for 15 and 30 seconds MID. However, the distortions in the size obtained under the PT sampling scheme are a lot smaller for the ADS test than the BNS test, especially for the trajectories

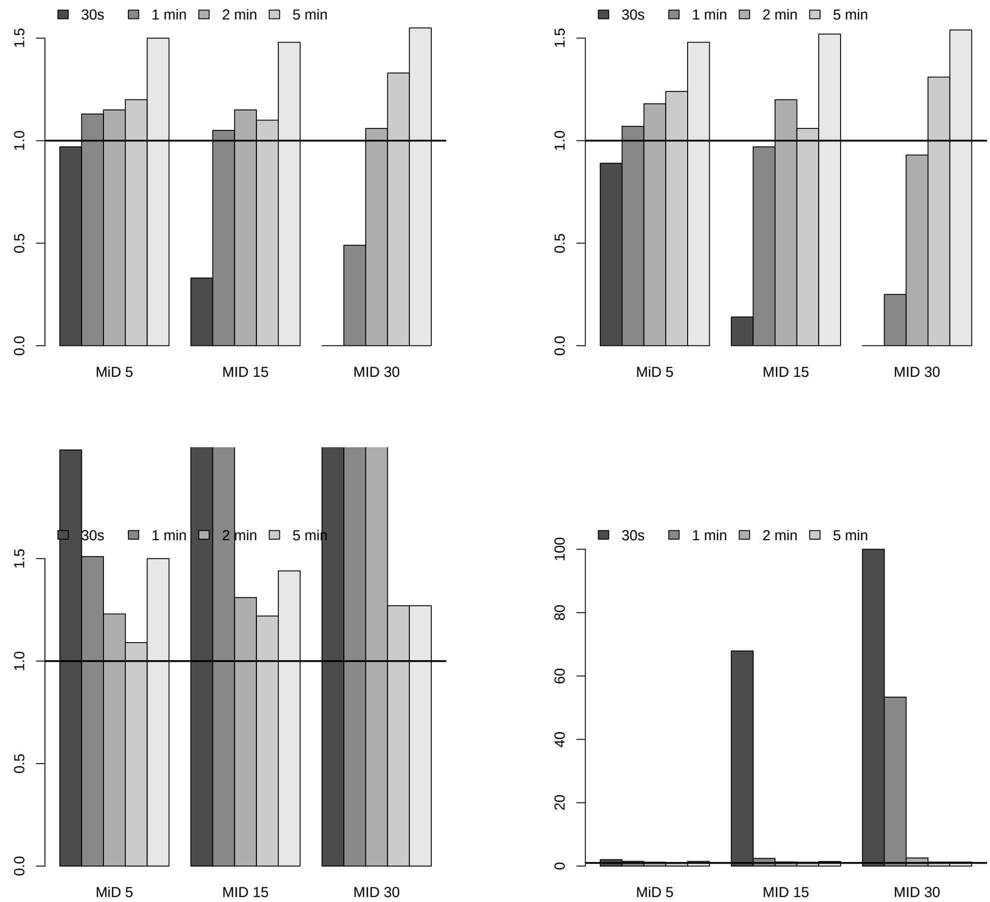

Figure 11: Size results for the BNS test applied to simulated paths from the LL1F stochastic volatility model with medium mean reversion sampled under three sampling schemes; Akima interpolation (upper left panel), linear interpolation (upper right panel) and previous tick method (lower panel) for three different MID patterns. The lower left panel presents the results using the same scale for the y-axis as the one used for the construction of the upper panel plots.

simulated under the LL2F stochastic volatility model.

Hence, the ADS test is capable of correcting, to a large extent, the bias induced through the large number of clustered zero returns from the application of the PT method. Nevertheless, the correction is not adequate and the test attains a good size only at low frequencies, lower than those obtained under the Akima spline and linear interpolation sampling schemes.

It can therefore be concluded that the MID of asset returns and the sampling scheme employed for the time homogenization of the price series play a pivotal role for the performance of the various tests for jumps based on multi-power variation. Overall, it is deduced from the analysis that the Akima spline and linear interpolation give the opportunity to sample at higher frequencies in deep and liquid markets, where as shown by Chaboud et al. (2009), microstructure noise is less of an issue. On the other hand, the PT method requires the practitioner to go to lower frequencies, even at liquid markets like the S & P Futures market, in order to attain good size properties for the two aforementioned jump tests. In addition, the lower the liquidity in the data, the lower the sampling frequency required by the tests to attain good size and power properties. Hence, for 30 seconds MID, the Akima spline and linear interpolations attain good size and power at 2 minutes, while the sampling frequency is 5 minutes or lower for the PT method.

Table 10 documents the results from the application of the two test statistics on the raw simulated trajectories, assuming a tick time sampling scheme for the data. Under the tick time specification, both tests are significantly oversized for all the five models considered, with the ADS test over-rejecting slightly more jumps. The distortions get smaller as one moves from 5 to 15 and from 15 to 30 seconds MID. Nevertheless, as mentioned before, it remains to be seen if the asymptotics of the two tests still apply under this type of sampling specification.

5.2.2 Power Analysis

In order to investigate the power properties of the two tests, we introduce Poisson type jumps into the process and analyze the capabilities of the BNS and ADS test to detect the right jumps, with respect to the sampling scheme employed in the data. The jumps are assumed to be normally distributed with variance σ^2 and constant jump intensity λ. In the current analysis three different types of jumps were considered, ranging from large and infrequent ($\lambda = 0.05$, $\sigma^2 = 2.5$) to small and frequent jumps ($\lambda = 1$, $\sigma^2 = 0.5$).

The results from the power analysis are presented in tables 11-20 for 1% significant level[8]. For every sampling frequency and sampling scheme employed in the analysis, a two by two matrix is recorded, with the power of the test listed on the diagonal of the matrix. The percentage of correctly detected jumps is given on the upper diagonal entry of the matrix, while the percentage of correctly detected days with no jumps is given on the lower diagonal entry of the matrix. The percentage of falsely detected jumps is given on the lower

[8]Results for 5% and 0.1% significant levels are found in the Appendix.

Multipower Variation Ratio Test (BNS)									
	MID: 5 seconds			MID: 15 seconds			MID: 30 seconds		
Freq.	Akima	Linear	PT	Akima	Linear	PT	Akima	Linear	PT
Heston									
30s	1.09	1.10	1.98	0.28	0.13	69.39	0.00	0.00	100.00
1 min	1.28	1.28	1.33	1.04	0.87	2.69	0.36	0.18	53.90
2 min	1.13	1.07	1.11	1.18	1.09	1.32	1.06	0.97	2.64
5 min	1.26	1.25	1.21	1.29	1.29	1.32	1.23	1.27	1.29
15 min	1.26	1.27	1.21	1.26	1.28	1.26	1.28	1.29	1.17
LL1F (slow)									
30s	0.97	0.93	1.84	0.25	0.08	68.20	0.00	0.00	100.00
1 min	0.97	0.94	1.25	0.94	0.83	2.52	0.47	0.26	53.34
2 min	0.94	0.90	0.98	0.99	0.93	1.12	0.87	0.87	2.57
5 min	1.18	1.16	1.20	1.25	1.22	1.11	1.11	1.18	1.25
15 min	1.28	1.34	1.28	1.31	1.33	1.31	1.29	1.30	1.36
LL1F (medium)									
30s	0.97	0.89	2.03	0.33	0.14	67.91	0.00	0.00	100.00
1 min	1.13	1.07	1.51	1.05	0.97	2.44	0.49	0.25	53.33
2 min	1.15	1.18	1.23	1.15	1.20	1.31	1.06	0.93	2.58
5 min	1.20	1.24	1.09	1.10	1.06	1.22	1.33	1.31	1.27
15 min	1.50	1.48	1.50	1.48	1.52	1.44	1.55	1.54	1.27
LL1F (fast)									
30s	1.20	1.08	1.90	0.31	0.12	68.36	0.00	0.00	100.00
1 min	0.83	0.82	1.20	0.77	0.71	2.53	0.39	0.13	52.31
2 min	1.10	1.03	1.07	1.05	1.07	1.09	0.82	0.74	2.31
5 min	1.25	1.21	1.43	1.25	1.21	1.23	1.17	1.23	1.34
15 min	1.21	1.19	1.19	1.22	1.24	1.24	1.20	1.25	1.22
LL2F									
30s	1.34	1.28	2.37	0.64	0.39	46.66	0.07	0.03	96.99
1 min	1.82	1.79	2.00	1.47	1.29	3.03	0.82	0.57	38.70
2 min	1.81	1.88	1.99	1.93	1.91	2.22	1.69	1.58	3.13
5 min	2.54	2.65	2.61	2.57	2.60	2.60	2.51	2.57	2.83
15 min	3.67	3.67	3.70	3.66	3.55	3.73	3.58	3.50	3.53

Table 8: **Size Analysis:** BNS test at 1% significant level.

triangular and the percentage of jumps not detected by the test statistic is given on the upper triangular of the matrix.

MedRV Ratio Test (ADS)									
	MID: 5 seconds			MID: 15 seconds			MID: 30 seconds		
Freq.	Akima	Linear	PT	Akima	Linear	PT	Akima	Linear	PT
Heston									
30s	1.21	1.18	2.16	0.55	0.18	26.60	0.02	0.00	100.00
1 min	1.08	1.08	1.23	0.94	0.89	2.72	0.56	0.25	17.41
2 min	0.97	1.01	1.02	1.03	1.07	1.30	1.11	0.97	2.04
5 min	1.02	1.02	1.07	1.01	1.01	1.07	1.02	0.98	1.02
15 min	1.08	1.09	1.14	1.05	1.04	1.05	1.07	1.08	1.04
LL1F (slow)									
30s	1.22	1.11	2.06	0.44	0.18	24.68	0.00	0.00	100.00
1 min	0.91	0.90	1.18	0.88	0.72	2.31	0.48	0.33	17.37
2 min	0.88	0.89	1.03	0.86	0.85	0.96	0.86	0.77	1.89
5 min	1.04	1.03	1.16	1.09	1.13	1.02	1.13	1.08	1.18
15 min	1.07	1.10	1.09	1.16	1.16	1.18	1.14	1.14	0.99
LL1F (medium)									
30s	0.96	0.79	1.97	0.50	0.23	24.91	0.00	0.00	100.00
1 min	0.97	1.01	1.11	0.94	0.86	2.21	0.74	0.49	16.99
2 min	1.00	0.98	0.95	0.96	0.95	1.13	0.88	0.77	2.04
5 min	1.05	1.08	1.16	1.04	1.10	1.14	0.98	0.99	1.04
15 min	1.09	1.09	1.12	1.14	1.13	1.23	1.07	1.09	1.20
LL1F (fast)									
30s	1.01	0.91	1.97	0.39	0.20	24.93	0.01	0.00	100.00
1 min	1.04	0.98	1.08	0.83	0.79	2.21	0.62	0.34	16.62
2 min	0.97	1.00	0.91	0.88	0.80	1.05	0.91	0.74	1.93
5 min	1.04	1.06	1.03	1.07	1.03	1.00	1.08	1.05	1.08
15 min	1.06	1.05	1.01	0.95	0.98	0.94	0.98	1.04	0.89
LL2F									
30s	1.35	1.27	2.50	0.85	0.54	18.42	0.17	0.05	87.38
1 min	1.75	1.64	1.69	1.65	1.41	2.73	1.04	0.70	14.52
2 min	1.76	1.78	1.88	1.83	1.79	2.07	1.89	1.72	2.81
5 min	2.66	2.68	2.66	2.50	2.58	2.68	2.49	2.57	2.81
15 min	3.38	3.36	3.35	3.36	3.38	3.34	3.28	3.32	3.25

Table 9: **Size Analysis:** ADS test at 1% significant level.

In terms of jumps falsely detected, the results conform to those from the size analysis summarized in tables 8 and 9. Under the Heston stochastic volatility models, the two

	BNS			ADS		
	5%	1%	0.1%	5%	1%	0.1%
Mean Intraday Duration: 5 seconds						
HESTON	14.53	3.95	0.66	16.34	5.17	0.84
LL1F(slow)	14.70	4.26	0.95	16.65	5.41	0.85
LL1F(med.)	14.85	4.52	0.79	16.91	5.19	0.82
LL1F(fast)	14.37	4.27	0.77	17.02	4.89	0.86
LL2F	12.02	3.49	0.54	12.86	3.44	0.60
Mean Intraday Duration: 15 seconds						
HESTON	10.42	2.67	0.36	10.97	2.90	0.47
LL1F(slow)	10.50	2.92	0.44	11.12	3.15	0.54
LL1F(med.)	9.96	2.86	0.47	10.80	2.89	0.39
LL1F(fast)	10.12	2.46	0.34	11.04	3.11	0.47
LL2F	8.90	2.42	0.35	9.30	2.40	0.39
Mean Intraday Duration: 30 seconds						
HESTON	9.70	2.62	0.40	10.10	2.82	0.40
LL1F(slow)	9.39	2.48	0.51	9.63	2.45	0.40
LL1F(med.)	9.07	2.36	0.27	9.53	2.54	0.40
LL1F(fast)	9.23	2.65	0.52	9.96	3.10	0.57
LL2F	9.28	2.62	0.51	9.32	2.60	0.55

Table 10: **Size Analysis:** Tick time.

tests manage to detect a very large proportion of the true jumps in the sample for all three types of jumps investigated, large, medium and small size jumps. The percentage of correctly detected jumps is slightly smaller for the the LL2F stochastic volatility model and decreases further modestly for the LL1F stochastic volatility models with medium and fast mean reversions. However, when simulating from the LL1F stochastic volatility model with slow mean reversion, the performance of both the tests depreciates considerably.

As the MID becomes longer, the percentage of the jump correctly detected by the BNS test decreases significantly for all stochastic volatility models with the exception of the Heston stochastic volatility model. The reduction in the number of correctly detected jumps with longer MID is more severe for the results obtained for the ADS test. A depreciation in the performance of the two tests is also observed relative to the size and intensity of the jumps in the data. The percentage of correctly detected jumps decreases with decreasing jump size and increasing jump intensity, with the exception of the results obtained for the LL1F stochastic volatility models with slow mean reversion. Under this specification, the two tests detect a much larger number of true small sized jumps than of true large and medium sized jumps. For the LL1F stochastic volatility models with medium and fast mean reversions, on the other hand, the number of true small sized jumps detected by the two tests is much smaller than the number of correctly detected medium and large sized jumps. Furthermore, this number decreases considerably with decreasing sampling frequency. However, it is to be expected that as one samples at lower frequencies, small jumps in the data might disappear, as the probability of having two consecutive small jumps increases considerably at those lower frequencies, due to the large intensity of small jumps in the data.

With regards to the sampling scheme used in the data, as also observed in the size analysis, the tests applied to data sampled under the Akima spline and linear interpolation sampling schemes attain a level of falsely detected jumps close to the significant level at higher frequencies than the tests applied to data constructed under the PT sampling scheme. In addition, at those sampling frequencies at which the tests give a size close to the nominal, the tests applied to data constructed under the Akima spline interpolation result in a higher percentage of true jumps detected than the tests applied to data constructed under the linear interpolation and PT sampling schemes. The difference in the performance of the tests under the three sampling schemes increases considerably for medium and small sized jumps. These results indicate that, the Akima and linear interpolations, apart from allowing the practitioner to sample at higher frequencies without introducing artificial biases into the data, they also allow for the tests to perform a more robust detection of small jumps in the data than the PT sampling scheme. This is due to the fact that the PT method, in addition to introducing artificial jumps in the data, it also causes the test statistics to be downward biased, due to the large number of induced zero returns in the data, which cause the underestimation of the various volatility estimators, such as the BPV and thus prevent the test statistics from detecting small and frequent jumps in the series.

Furthermore, the disparity in the performance of the two tests relative to the sampling scheme employed in the data becomes more apparent with longer MID. This observation again highlights the sensitivity of the tests to the liquidity in the data, when this is sampled using calendar time sampling schemes. For 30 seconds MID, the two tests implemented on data constructed under the PT sampling scheme, detect an extremely large proportion of false jumps at the highest frequencies considered, reaching 100%. For data constructed under the Akima and linear interpolation, on the other hand, the two tests detect a smaller percentage of true jumps at the highest frequencies than at lower frequencies. Hence, the percentage of true jumps detected increases as the frequency decreases in the case of the two aforementioned sampling schemes for low liquidity data series. The maximum number of true jumps detected is attained at those frequencies for which the test also attains a size close to the nominal.

It was also observed that the ADS test results in a higher percentage of jumps that remain undetected under the Akima spline and linear interpolation sampling schemes, especially at the highest frequencies considered in the analysis, than under the PT method. This is due to the fact, that unlike the PT, the Akima spline and linear interpolation do not introduce additional zero returns in the data, and therefore the correction provided by the ADS test is redundant when applied to data sampled under these two sampling schemes. Apart from being redundant, it also biases the estimation of the jump-robust volatility measures downwards, thus resulting in the under-detection of the true jumps in the data.

Interestingly, even under the LL2F stochastic volatility model, the tests applied to data obtained under both the Akima spline and linear interpolation sampling schemes detect only a very small proportion of false jumps in the data, unlike the results obtained on data sampled under the PT scheme, where at the 30 seconds MID, and even for the largest jumps, the test detects a much larger proportion of false jumps, especially at the highest frequencies.

Overall, it can be concluded that the Akima interpolation followed by the linear interpolation provide a more robust alternative to the PT method for sampling data at time equidistant points prior to the implementation of multi-power variation based jump tests. Contrary to the PT method, the two aforementioned interpolation techniques do not significantly distort the distribution of returns, and thus allow the test statistics to perform well at high frequencies and correctly detect the majority of jumps in the data.

Tables 21-22 present the results from the power analysis of the two tests on tick time data. In accordance with the results obtained for the size analysis, it is observed that both tests tend to find more jumps under tick time, above the nominal level, than under the calendar time sampling schemes. However, it is noted that the tests are not able to detect a large proportion of the small size jumps, while this proportion is much smaller for large and medium jumps. This was also observed under the calendar time sampling schemes. Again, the ADS test does slightly worse than the BNS test.

It is important to observe however, that the results are very similar for all three MID considered, a fact that highlights the sensitivity of the tests when these are applied to data sampled under the calendar time sampling schemes.

Multipower Variation Ratio Test (BNS)																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	98.99	1.01	98.99	1.01	99.20	0.80	98.59	1.41	98.39	1.61	99.80	0.20	95.37	4.63	92.35	7.65	100.00	0.00
	NJ	1.14	98.86	0.98	99.02	2.40	97.60	0.31	99.69	0.08	99.92	69.49	30.51	0.00	100.00	0.00	100.00	100.00	0.00
1min	J	98.79	1.21	98.79	1.21	98.79	1.21	97.99	2.01	97.79	2.21	98.99	1.01	97.99	2.01	97.59	2.41	99.80	0.20
	NJ	1.08	98.92	1.09	98.91	1.18	98.82	0.96	99.04	0.81	99.19	2.55	97.45	0.36	99.64	0.16	99.84	54.67	45.33
2min	J	98.59	1.41	98.59	1.41	98.59	1.41	97.99	2.01	97.99	2.01	98.39	1.61	98.39	1.61	98.19	1.81	98.59	1.41
	NJ	1.07	98.93	1.02	98.98	0.96	99.04	0.88	99.12	0.89	99.11	1.24	98.76	0.99	99.01	0.78	99.22	2.50	97.05
5min	J	98.39	1.61	98.39	1.61	98.39	1.61	97.99	2.01	97.99	2.01	98.19	1.81	97.99	2.01	97.99	2.01	98.39	1.61
	NJ	1.13	98.87	1.12	98.88	1.05	98.95	1.15	98.85	1.08	98.92	1.10	98.90	1.08	98.92	1.07	98.93	1.14	98.86
15min	J	96.58	3.42	96.58	3.42	96.58	3.42	96.58	3.42	96.58	3.42	96.38	3.62	96.78	3.22	96.78	3.22	96.58	3.42
	NJ	1.19	98.81	1.19	98.81	1.14	98.86	1.22	98.78	1.21	98.79	1.34	98.66	1.23	98.77	1.24	98.76	1.34	98.66
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	96.57	3.43	96.47	3.53	96.78	3.22	96.47	3.53	95.85	4.15	99.27	0.73	92.52	7.48	89.30	10.70	100.00	0.00
	NJ	1.11	98.89	1.03	98.97	2.53	97.47	0.22	99.78	0.08	99.92	68.84	31.16	0.21	99.79	0.20	99.80	100.00	0.00
1min	J	96.47	3.53	96.57	3.43	96.37	3.63	96.26	3.74	96.16	3.84	96.68	3.32	95.53	4.47	95.12	4.88	98.75	1.25
	NJ	1.26	98.74	1.27	98.73	1.29	98.71	0.81	99.19	0.72	99.28	2.27	97.73	0.58	99.42	0.40	99.60	54.94	45.06
2min	J	95.85	4.15	95.64	4.36	95.43	4.57	95.22	4.78	95.22	4.78	95.22	4.78	95.02	4.98	94.91	5.09	95.64	4.36
	NJ	1.08	98.92	1.10	98.90	1.21	98.79	0.93	99.07	0.93	99.07	1.06	98.94	1.22	98.78	1.02	98.98	2.60	97.40
5min	J	94.81	5.19	94.70	5.30	94.81	5.19	94.50	5.50	94.50	5.50	94.29	5.71	93.77	6.23	93.67	6.33	94.39	5.61
	NJ	1.16	98.84	1.20	98.80	1.20	98.80	1.10	98.90	1.03	98.97	1.16	98.84	1.32	98.68	1.34	98.66	1.45	98.55
15min	J	92.63	7.37	92.73	7.27	92.73	7.27	92.63	7.37	92.52	7.48	92.42	7.58	92.42	7.58	92.42	7.58	92.63	7.37
	NJ	1.22	98.78	1.21	98.79	1.21	98.79	1.18	98.82	1.21	98.79	1.18	98.82	1.24	98.76	1.20	98.80	1.24	98.76
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	95.79	4.21	95.79	4.21	96.09	3.91	94.81	5.19	94.13	5.87	99.61	0.39	90.31	9.69	87.38	12.62	100.00	0.00
	NJ	1.16	98.84	1.08	98.92	1.89	98.11	0.41	99.59	0.13	99.87	70.06	29.94	0.89	99.11	0.86	99.14	99.99	0.01
1min	J	95.30	4.70	95.11	4.89	95.30	4.70	94.72	5.28	94.81	5.19	96.09	3.91	93.84	6.16	93.54	6.46	99.02	0.98
	NJ	1.38	98.62	1.33	98.67	1.54	98.46	1.07	98.93	0.89	99.11	2.77	97.23	1.24	98.76	1.14	98.86	54.97	45.03
2min	J	93.84	6.16	93.74	6.26	94.13	5.87	93.74	6.26	93.54	6.46	94.13	5.87	93.35	6.65	93.05	6.95	94.81	5.19
	NJ	1.30	98.70	1.26	98.74	1.09	98.91	0.95	99.05	0.99	99.01	1.24	98.76	1.60	98.40	1.59	98.41	3.16	96.84
5min	J	92.27	7.73	92.27	7.73	92.17	7.83	92.37	7.63	92.37	7.63	92.47	7.53	92.17	7.83	92.17	7.83	92.07	7.93
	NJ	1.10	98.90	1.15	98.85	1.19	98.81	1.12	98.88	1.07	98.93	1.11	98.89	1.43	98.57	1.44	98.56	1.56	98.44
15min	J	86.20	13.80	86.20	13.80	86.20	13.80	86.50	13.50	86.69	13.31	86.01	13.99	86.50	13.50	86.40	13.60	86.30	13.70
	NJ	1.46	98.54	1.45	98.55	1.38	98.62	1.24	98.76	1.25	98.75	1.28	98.72	1.36	98.64	1.28	98.72	1.35	98.65

Table 11: **Power:** Heston stochastic volatility model at 1% significant level (BNS).

		MedRV Ratio Test (ADS))																	
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	94.77	5.23	92.15	7.85	98.79	1.21	85.51	14.49	79.48	20.52	99.40	0.60	71.03	28.97	63.38	36.62	100.00	0.00
	NJ	1.12	98.88	1.07	98.93	2.15	97.85	0.43	99.57	0.15	99.85	26.21	73.79	0.02	99.98	0.00	100.00	100.00	0.00
1min	J	96.38	3.62	94.97	5.03	98.79	1.21	91.95	8.05	88.53	11.47	98.79	1.21	83.90	16.10	80.89	19.11	99.40	0.60
	NJ	1.13	98.87	1.07	98.93	1.19	98.81	1.09	98.91	0.95	99.05	2.49	97.51	0.56	99.44	0.34	99.66	17.76	82.24
2min	J	96.98	3.02	96.38	3.62	98.79	1.21	94.77	5.23	92.76	7.24	98.79	1.21	89.74	10.26	88.13	11.87	98.59	1.41
	NJ	0.99	99.01	0.98	99.02	0.83	99.17	0.97	99.03	0.87	99.13	1.27	98.73	1.14	98.86	1.00	99.00	2.16	97.84
5min	J	97.79	2.21	97.18	2.82	98.39	1.61	96.58	3.42	96.18	3.82	98.39	1.61	96.18	3.82	95.77	4.23	98.39	1.61
	NJ	1.04	98.96	1.04	98.96	1.01	98.99	0.99	99.01	0.98	99.02	0.89	99.11	1.01	98.99	1.05	98.95	1.08	98.92
15min	J	96.98	3.02	96.78	3.22	97.18	2.82	95.98	4.02	95.77	4.23	96.78	3.22	96.78	3.22	96.58	3.42	97.18	2.82
	NJ	1.27	98.73	1.28	98.72	1.17	98.83	1.16	98.84	1.17	98.83	1.21	98.79	1.13	98.87	1.16	98.84	1.13	98.87
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	92.00	8.00	89.72	10.28	96.78	3.22	84.53	15.47	79.34	20.66	98.13	1.87	71.24	28.76	64.07	35.93	100.00	0.00
	NJ	1.25	98.75	1.18	98.82	2.37	97.63	0.34	99.66	0.12	99.88	24.78	75.22	0.21	99.79	0.20	99.80	100.00	0.00
1min	J	93.98	6.02	93.35	6.65	96.57	3.43	91.59	8.41	89.10	10.90	96.37	3.63	84.74	15.26	81.00	19.00	97.30	2.70
	NJ	1.13	98.87	1.10	98.90	1.24	98.76	0.94	99.06	0.83	99.17	2.35	97.65	0.71	99.29	0.43	99.57	18.05	81.95
2min	J	93.87	6.13	93.56	6.44	95.43	4.57	93.25	6.75	91.90	8.10	95.43	4.57	90.03	9.97	87.54	12.46	95.64	4.36
	NJ	0.92	99.08	0.93	99.07	0.98	99.02	0.93	99.07	0.86	99.14	1.20	98.80	1.20	98.80	1.01	98.99	2.28	97.72
5min	J	94.50	5.50	94.29	5.71	94.81	5.19	93.67	6.33	93.04	6.96	94.50	5.50	91.69	8.31	91.38	8.62	94.81	5.19
	NJ	1.17	98.83	1.15	98.85	1.07	98.93	1.04	98.96	1.03	98.97	1.17	98.83	1.07	98.93	1.01	98.99	1.12	98.88
15min	J	92.63	7.37	92.42	7.58	92.73	7.27	92.11	7.89	92.00	8.00	92.83	7.17	91.59	8.41	91.17	8.83	92.63	7.37
	NJ	1.25	98.75	1.23	98.77	1.15	98.85	1.16	98.84	1.17	98.83	1.07	98.93	1.20	98.80	1.14	98.86	1.41	98.59
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	91.10	8.90	89.33	10.67	96.58	3.42	86.50	13.50	82.78	17.22	98.24	1.76	76.42	23.58	67.71	32.29	100.00	0.00
	NJ	1.17	98.83	1.05	98.95	2.18	97.82	0.47	99.53	0.21	99.79	25.72	74.28	0.95	99.05	0.84	99.16	99.97	0.03
1min	J	93.35	6.65	92.95	7.05	95.11	4.89	91.00	9.00	89.33	10.67	95.30	4.70	87.18	12.82	83.27	16.73	96.87	3.13
	NJ	1.36	98.64	1.33	98.67	1.49	98.51	1.16	98.84	0.97	99.03	2.70	97.30	1.54	98.46	1.31	98.69	19.06	80.94
2min	J	93.25	6.75	92.86	7.14	94.13	5.87	92.56	7.44	91.78	8.22	94.62	5.38	90.51	9.49	88.65	11.35	94.62	5.38
	NJ	1.17	98.83	1.16	98.84	1.21	98.79	1.21	98.79	1.17	98.83	1.09	98.91	1.58	98.42	1.56	98.44	3.05	96.95
5min	J	91.10	8.90	91.10	8.90	91.39	8.61	90.70	9.30	90.61	9.39	91.78	8.22	90.31	9.69	90.12	9.88	91.49	8.51
	NJ	1.08	98.92	1.07	98.93	1.10	98.90	0.95	99.05	0.97	99.03	0.98	99.02	1.37	98.63	1.35	98.65	1.39	98.61
15min	J	86.69	13.31	86.59	13.41	86.79	13.21	86.99	13.01	86.99	13.01	87.08	12.92	86.50	13.50	86.59	13.41	86.79	13.21
	NJ	1.05	98.95	1.05	98.95	1.08	98.92	1.02	98.98	1.04	98.96	1.10	98.90	1.21	98.79	1.24	98.76	1.27	98.73

Table 12: **Power:** Heston stochastic volatility model at 1% significant level (ADS).

Multipower Variation Ratio Test (BNS)

		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	75.65	24.35	76.26	23.74	79.48	20.52	73.84	26.16	72.84	27.16	94.97	5.03	64.19	35.81	61.57	38.43	100.00	0.00
	NJ	0.88	99.12	0.81	99.19	2.00	98.00	0.24	99.76	0.09	99.91	67.59	32.41	0.02	99.98	0.00	100.00	100.00	0.00
1min	J	72.03	27.97	72.03	27.97	73.84	26.16	72.23	27.77	71.43	28.57	74.45	25.55	68.01	31.99	66.20	33.80	90.54	9.46
	NJ	0.98	99.02	0.99	99.01	0.88	99.12	0.96	99.04	0.87	99.13	2.59	97.41	0.46	99.54	0.26	99.74	53.60	46.40
2min	J	69.82	30.18	69.82	30.18	69.22	30.78	68.41	31.59	67.81	32.19	70.02	29.98	67.81	32.19	66.60	33.40	71.63	28.37
	NJ	1.00	99.00	0.97	99.03	1.05	98.95	1.13	98.87	1.08	98.92	1.19	98.81	0.77	99.23	0.69	99.31	2.46	97.54
5min	J	63.18	36.82	62.78	37.22	61.77	38.23	63.18	36.82	61.97	38.03	63.38	36.62	62.17	37.83	62.37	37.63	63.58	36.42
	NJ	1.09	98.91	1.08	98.92	1.13	98.87	1.13	98.87	1.15	98.85	1.23	98.77	1.03	98.97	1.01	98.99	1.32	98.68
15min	J	55.33	44.67	55.33	44.67	55.94	44.06	55.53	44.47	55.53	44.47	55.73	44.27	55.33	44.67	55.33	44.67	54.93	45.07
	NJ	1.16	98.84	1.17	98.83	1.17	98.83	1.13	98.87	1.06	98.94	0.95	99.05	1.15	98.85	1.17	98.83	1.01	98.99
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	74.97	25.03	74.66	25.34	76.12	23.88	72.48	27.52	71.44	28.56	93.46	6.54	66.15	33.85	63.14	36.86	100.00	0.00
	NJ	1.39	98.61	1.27	98.73	2.14	97.86	0.24	99.76	0.11	99.89	68.35	31.65	0.17	99.83	0.15	99.85	100.00	0.00
1min	J	72.07	27.93	71.96	28.04	73.00	27.00	71.75	28.25	71.55	28.45	73.73	26.27	69.68	30.32	69.16	30.84	90.55	9.45
	NJ	1.39	98.61	1.28	98.72	1.49	98.51	1.04	98.96	0.97	99.03	2.64	97.36	0.58	99.42	0.40	99.60	53.24	46.76
2min	J	68.74	31.26	69.06	30.94	69.26	30.74	68.54	31.46	68.43	31.57	70.20	29.80	68.22	31.78	67.81	32.19	70.09	29.91
	NJ	1.11	98.89	1.08	98.92	1.16	98.84	1.00	99.00	0.95	99.05	1.31	98.69	1.06	98.94	1.00	99.00	3.04	96.96
5min	J	64.07	35.93	63.97	36.03	64.28	35.72	64.38	35.62	64.28	35.72	65.01	34.99	63.66	36.34	63.66	36.34	64.49	35.51
	NJ	1.05	98.95	1.07	98.93	1.17	98.83	0.95	99.05	0.91	99.09	0.93	99.07	0.97	99.03	0.91	99.09	1.16	98.84
15min	J	58.57	41.43	58.57	41.43	58.88	41.12	58.67	41.33	58.46	41.54	59.29	40.71	58.88	41.12	58.98	41.02	58.77	41.23
	NJ	1.29	98.71	1.26	98.74	1.29	98.71	1.31	98.69	1.26	98.74	1.31	98.69	1.24	98.76	1.18	98.82	1.31	98.69
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	93.05	6.95	92.95	7.05	94.72	5.28	92.27	7.73	91.59	8.41	99.22	0.78	89.04	10.96	86.11	13.89	100.00	0.00
	NJ	1.25	98.75	1.04	98.96	2.03	97.97	0.18	99.82	0.08	99.92	67.10	32.90	0.82	99.18	0.80	99.20	99.99	0.01
1min	J	92.07	7.93	91.98	8.02	92.47	7.53	92.37	7.63	91.98	8.02	93.64	6.36	91.88	8.12	91.49	8.51	98.24	1.76
	NJ	1.24	98.76	1.16	98.84	1.39	98.61	1.09	98.91	0.99	99.01	2.46	97.54	1.26	98.74	1.08	98.92	54.74	45.26
2min	J	90.51	9.49	90.31	9.69	90.51	9.49	90.41	9.59	90.41	9.59	91.00	9.00	90.61	9.39	90.12	9.88	91.78	8.22
	NJ	1.11	98.89	1.10	98.90	1.19	98.81	1.08	98.92	1.14	98.86	1.46	98.54	1.59	98.41	1.50	98.50	3.02	96.98
5min	J	88.06	11.94	87.96	12.04	88.06	11.94	87.96	12.04	87.87	12.13	87.96	12.04	87.77	12.23	87.48	12.52	88.26	11.74
	NJ	1.29	98.71	1.27	98.73	1.24	98.76	1.27	98.73	1.25	98.75	1.25	98.75	1.62	98.38	1.65	98.35	1.54	98.46
15min	J	80.92	19.08	80.72	19.28	80.63	19.37	80.72	19.28	80.82	19.18	80.82	19.18	81.02	18.98	81.02	18.98	80.72	19.28
	NJ	1.69	98.31	1.70	98.30	1.69	98.31	1.73	98.27	1.73	98.27	1.55	98.45	1.79	98.21	1.82	98.18	1.60	98.40

Table 13: **Power:** LL1F stochastic volatility model with slow mean reversion at 1% significant level (BNS).

MedRV Ratio Test (ADS))																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	74.04	25.96	73.64	26.36	78.07	21.93	67.81	32.19	64.59	35.41	87.32	12.68	54.53	45.47	48.49	51.51	100.00	0.00
	NJ	1.13	98.87	1.06	98.94	1.83	98.17	0.45	99.55	0.23	99.77	24.74	75.26	0.02	99.98	0.00	100.00	100.00	0.00
1min	J	72.23	27.77	71.43	28.57	72.43	27.57	68.61	31.39	67	33	75.05	24.95	62.58	37.42	60.76	39.24	81.09	18.91
	NJ	1.05	98.95	1.09	98.91	1.09	98.91	1.00	99.00	0.92	99.08	2.62	97.38	0.63	99.37	0.34	99.66	17.44	82.56
2min	J	69.01	30.99	68.61	31.39	69.82	30.18	68.01	31.99	67.40	32.60	70.22	29.78	63.78	36.22	62.58	37.42	70.62	29.38
	NJ	1.01	98.99	1.04	98.96	1.01	98.99	1.06	98.94	1.08	98.92	1.09	98.91	0.94	99.06	0.80	99.20	2.00	98.00
5min	J	64.19	35.81	63.38	36.62	63.98	36.02	62.17	37.83	62.17	37.83	63.38	36.62	62.98	37.02	62.78	37.22	63.58	36.42
	NJ	1.15	98.85	1.15	98.85	1.19	98.81	1.13	98.87	1.13	98.87	1.05	98.95	1.10	98.90	1.08	98.92	1.37	98.63
15min	J	56.34	43.66	56.34	43.66	56.54	43.46	56.14	43.86	55.94	44.06	56.74	43.26	56.34	43.66	56.74	43.26	56.14	43.86
	NJ	1.14	98.86	1.15	98.85	1.28	98.72	1.16	98.84	1.17	98.83	1.13	98.87	1.10	98.90	1.17	98.83	1.04	98.96
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	71.65	28.35	70.30	29.70	75.18	24.82	64.69	35.31	62.20	37.80	85.57	14.43	52.96	47.04	47.66	52.34	100.00	0.00
	NJ	1.37	98.63	1.26	98.74	2.15	97.85	0.45	99.55	0.23	99.77	25.46	74.54	0.18	99.82	0.15	99.85	100.00	0.00
1min	J	70.51	29.49	70.30	29.70	72.38	27.62	69.37	30.63	67.81	32.19	73.73	26.27	63.55	36.45	60.23	39.77	80.27	19.73
	NJ	1.13	98.87	1.14	98.86	1.39	98.61	1.01	98.99	0.89	99.11	2.27	97.73	0.68	99.32	0.49	99.51	17.04	82.96
2min	J	68.12	31.88	67.91	32.09	69.47	30.53	66.77	33.23	66.87	33.13	69.57	30.43	64.80	35.20	63.03	36.97	70.09	29.91
	NJ	1.13	98.87	1.14	98.86	1.21	98.79	0.85	99.15	0.75	99.25	1.16	98.84	0.91	99.09	0.93	99.07	2.06	97.94
5min	J	64.49	35.51	64.59	35.41	64.80	35.20	64.59	35.41	64.80	35.20	65.52	34.48	64.07	35.93	63.14	36.86	65.52	34.48
	NJ	1.25	98.75	1.23	98.77	1.31	98.69	0.97	99.03	0.97	99.03	1.12	98.88	1.24	98.76	1.23	98.77	1.35	98.65
15min	J	60.23	39.77	60.23	39.77	59.92	40.08	59.81	40.19	59.71	40.29	59.92	40.08	60.44	39.56	59.92	40.08	60.12	39.88
	NJ	1.10	98.90	1.08	98.92	1.17	98.83	1.03	98.97	1.02	98.98	1.01	98.99	1.08	98.92	1.13	98.87	1.13	98.87
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	87.08	12.92	85.23	14.77	94.32	5.68	79.84	20.16	74.07	25.93	96.97	3.03	67.22	32.78	59.49	40.51	100.00	0.00
	NJ	1.07	98.93	1.01	98.99	1.84	98.16	0.47	99.53	0.20	99.80	25.07	74.93	0.90	99.10	0.79	99.21	99.96	0.04
1min	J	89.82	10.18	88.75	11.25	92.66	7.34	85.62	14.38	83.46	16.54	94.03	5.97	80.43	19.57	75.24	24.76	96.09	3.91
	NJ	1.14	98.86	1.14	98.86	1.29	98.71	0.96	99.04	0.80	99.20	2.33	97.67	1.37	98.63	1.11	98.89	18.86	81.14
2min	J	89.53	10.47	88.94	11.06	90.70	9.30	87.57	12.43	86.99	13.01	91.10	8.90	85.52	14.48	82.97	17.03	91.59	8.41
	NJ	1.17	98.83	1.14	98.86	1.16	98.84	0.97	99.03	1.04	98.96	1.44	98.56	1.63	98.37	1.46	98.54	2.87	97.13
5min	J	87.18	12.82	86.89	13.11	87.77	12.23	87.08	12.92	86.40	13.60	88.06	11.94	86.30	13.70	85.62	14.38	87.77	12.23
	NJ	0.99	99.01	1.05	98.95	1.04	98.96	0.96	99.04	0.97	99.03	1.14	98.86	1.34	98.66	1.34	98.66	1.56	98.44
15min	J	80.72	19.28	80.43	19.57	80.43	19.57	80.53	19.47	80.53	19.47	80.92	19.08	80.53	19.47	80.04	19.96	81.02	18.98
	NJ	1.15	98.85	1.17	98.83	1.17	98.83	1.24	98.76	1.20	98.80	1.23	98.77	1.24	98.76	1.24	98.76	1.27	98.73

Table 14: **Power:** LL1F stochastic volatility model with slow mean reversion at 1% significant level (ADS).

Multipower Variation Ratio Test (BNS)																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	92.56	7.44	92.35	7.65	92.96	7.04	90.54	9.46	89.74	10.26	98.99	1.01	84.91	15.09	80.68	19.32	100.00	0.00
	NJ	0.98	99.02	0.97	99.03	2.09	97.91	0.27	99.73	0.12	99.88	67.65	32.35	0.00	100.00	0.00	100.00	100.00	0.00
1min	J	91.35	8.65	90.95	9.05	91.95	8.05	90.14	9.86	89.54	10.46	92.76	7.24	88.33	11.67	87.53	12.47	96.78	3.22
	NJ	1.03	98.97	0.95	99.05	0.93	99.07	0.81	99.19	0.78	99.22	2.37	97.63	0.38	99.62	0.23	99.77	53.52	46.48
2min	J	88.13	11.87	88.33	11.67	88.93	11.07	87.73	12.27	86.92	13.08	88.73	11.27	86.32	13.68	85.92	14.08	89.74	10.26
	NJ	1.10	98.90	1.16	98.84	1.29	98.71	0.95	99.05	0.95	99.05	1.14	98.86	1.09	98.91	0.89	99.11	2.62	97.38
5min	J	85.51	14.49	85.51	14.49	85.51	14.49	85.51	14.49	85.31	14.69	85.51	14.49	85.11	14.89	84.51	15.49	85.11	14.89
	NJ	1.10	98.90	1.08	98.92	1.12	98.88	1.18	98.82	1.15	98.85	1.14	98.86	1.24	98.76	1.25	98.75	1.10	98.90
15min	J	78.67	21.33	78.47	21.53	77.87	22.13	77.67	22.33	77.87	22.13	78.47	21.53	78.47	21.53	78.67	21.33	78.67	21.33
	NJ	1.14	98.86	1.15	98.85	1.16	98.84	1.15	98.85	1.19	98.81	1.02	98.98	1.12	98.88	1.10	98.90	1.18	98.82
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	88.16	11.84	87.95	12.05	90.03	9.97	86.40	13.60	84.94	15.06	97.92	2.08	78.30	21.70	74.45	25.55	100.00	0.00
	NJ	1.22	98.78	1.05	98.95	2.15	97.85	0.29	99.71	0.10	99.90	68.27	31.73	0.20	99.80	0.19	99.81	100.00	0.00
1min	J	86.40	13.60	86.50	13.50	87.33	12.67	86.09	13.91	85.67	14.33	88.27	11.73	83.70	16.30	82.24	17.76	96.78	3.22
	NJ	1.22	98.78	1.25	98.75	1.36	98.64	0.98	99.02	0.81	99.19	2.41	97.59	0.70	99.30	0.46	99.54	53.75	46.25
2min	J	84.42	15.58	84.22	15.78	84.53	15.47	84.11	15.89	84.01	15.99	84.74	15.26	83.28	16.72	82.04	17.96	85.98	14.02
	NJ	1.18	98.82	1.17	98.83	1.20	98.80	1.14	98.86	1.15	98.85	1.38	98.62	1.14	98.86	1.06	98.94	2.98	97.02
5min	J	78.61	21.39	78.50	21.50	79.02	20.98	78.40	21.60	78.19	21.81	78.92	21.08	77.78	22.22	77.47	22.53	79.34	20.66
	NJ	1.27	98.73	1.21	98.79	1.31	98.69	1.12	98.88	1.10	98.90	1.13	98.87	1.25	98.75	1.23	98.77	1.38	98.62
15min	J	68.85	31.15	69.06	30.94	68.74	31.26	68.95	31.05	68.85	31.15	68.85	31.15	68.64	31.36	68.64	31.36	68.74	31.26
	NJ	1.67	98.33	1.74	98.26	1.62	98.38	1.56	98.44	1.57	98.43	1.74	98.26	1.72	98.28	1.77	98.23	1.58	98.42
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	74.46	25.54	73.87	26.13	77.89	22.11	70.74	29.26	68.59	31.41	95.30	4.70	60.86	39.14	55.68	44.32	100.00	0.00
	NJ	1.23	98.77	1.05	98.95	1.99	98.01	0.23	99.77	0.14	99.86	68.02	31.98	0.90	99.10	0.87	99.13	99.99	0.01
1min	J	70.25	29.75	70.55	29.45	71.23	28.77	68.59	31.41	67.32	32.68	73.39	26.61	65.75	34.25	64.09	35.91	91.59	8.41
	NJ	1.01	98.99	1.06	98.94	1.41	98.59	1.11	98.89	1.02	98.98	2.47	97.53	1.39	98.61	1.19	98.81	53.17	46.83
2min	J	63.80	36.20	64.19	35.81	64.87	35.13	64.29	35.71	63.99	36.01	65.36	34.64	63.31	36.69	62.82	37.18	66.93	33.07
	NJ	1.15	98.85	1.19	98.81	1.17	98.83	1.06	98.94	1.02	98.98	1.21	98.79	1.66	98.34	1.57	98.43	3.07	96.93
5min	J	51.57	48.43	51.37	48.63	51.47	48.53	51.57	48.43	51.08	48.92	51.76	48.24	51.57	48.43	52.05	47.95	51.66	48.34
	NJ	1.12	98.88	1.15	98.85	1.15	98.85	1.14	98.86	1.11	98.89	1.11	98.89	1.39	98.61	1.41	98.59	1.54	98.46
15min	J	31.31	68.69	31.41	68.59	32.09	67.91	31.60	68.40	31.51	68.49	31.60	68.40	32.09	67.91	32.39	67.61	32.00	68.00
	NJ	1.59	98.41	1.64	98.36	1.53	98.47	1.63	98.37	1.63	98.37	1.45	98.55	1.79	98.21	1.74	98.26	1.57	98.43

Table 15: **Power:** LL1F stochastic volatility model with medium mean reversion at 1% significant level (BNS).

MedRV Ratio Test (ADS))																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	90.74	9.26	88.93	11.07	93.56	6.44	83.50	16.50	79.07	20.93	96.78	3.22	71.83	28.17	64.99	35.01	100.00	0.00
	NJ	1.07	98.93	0.99	99.01	1.93	98.07	0.47	99.53	0.24	99.76	24.18	75.82	0.02	99.98	0.00	100.00	100.00	0.00
1min	J	90.54	9.46	89.74	10.26	91.75	8.25	86.72	13.28	85.51	14.49	92.35	7.65	81.09	18.91	78.27	21.73	94.77	5.23
	NJ	0.97	99.03	0.94	99.06	0.97	99.03	0.92	99.08	0.73	99.27	2.18	97.82	0.55	99.45	0.26	99.74	17.66	82.34
2min	J	88.33	11.67	87.73	12.27	88.93	11.07	87.12	12.88	86.32	13.68	89.54	10.46	82.70	17.30	80.68	19.32	89.54	10.46
	NJ	1.02	98.98	1.03	98.97	1.13	98.87	1.08	98.92	1.01	98.99	1.05	98.95	1.01	98.99	0.87	99.13	2.10	97.90
5min	J	85.92	14.08	85.92	14.08	86.12	13.88	85.51	14.49	85.11	14.89	86.72	13.28	84.91	15.09	84.51	15.49	86.52	13.48
	NJ	0.95	99.05	0.97	99.03	0.93	99.07	0.99	99.01	0.97	99.03	0.93	99.07	0.99	99.01	0.96	99.04	0.89	99.11
15min	J	80.68	19.32	80.89	19.11	81.09	18.91	80.89	19.11	80.68	19.32	81.29	18.71	80.89	19.11	80.89	19.11	80.89	19.11
	NJ	1.21	98.79	1.20	98.80	1.17	98.83	1.08	98.92	1.10	98.90	1.01	98.99	1.18	98.82	1.12	98.88	1.13	98.87
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	86.60	13.40	85.46	14.54	90.13	9.87	81.10	18.90	77.26	22.74	94.50	5.50	70.92	29.08	62.41	37.59	100.00	0.00
	NJ	1.27	98.73	1.21	98.79	2.19	97.81	0.63	99.37	0.21	99.79	25.25	74.75	0.23	99.77	0.19	99.81	100.00	0.00
1min	J	85.36	14.64	85.05	14.95	86.92	13.08	84.53	15.47	82.66	17.34	87.44	12.56	80.37	19.63	76.32	23.68	90.86	9.14
	NJ	1.17	98.83	1.18	98.82	1.48	98.52	1.12	98.88	0.86	99.14	2.42	97.58	0.72	99.28	0.54	99.46	18.28	81.72
2min	J	84.22	15.78	84.01	15.99	84.94	15.06	83.39	16.61	83.18	16.82	85.36	14.64	82.35	17.65	80.89	19.11	85.67	14.33
	NJ	1.22	98.78	1.25	98.75	1.25	98.75	0.98	99.02	1.01	98.99	1.17	98.83	1.16	98.84	1.07	98.93	2.38	97.62
5min	J	79.96	20.04	79.65	20.35	80.06	19.94	79.65	20.35	79.34	20.66	80.27	19.73	78.40	21.60	77.57	22.43	79.85	20.15
	NJ	1.17	98.83	1.20	98.80	1.17	98.83	0.92	99.08	0.91	99.09	1.03	98.97	1.03	98.97	1.04	98.96	1.27	98.73
15min	J	70.51	29.49	70.51	29.49	70.51	29.49	70.20	29.80	70.09	29.91	70.09	29.91	70.09	29.91	70.20	29.80	69.78	30.22
	NJ	1.27	98.73	1.29	98.71	1.37	98.63	1.17	98.83	1.12	98.88	1.15	98.85	1.15	98.85	1.17	98.83	1.35	98.65
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	73.39	26.61	72.50	27.50	78.38	21.62	69.28	30.72	65.75	34.25	87.18	12.82	60.08	39.92	51.76	48.24	100.00	0.00
	NJ	1.02	98.98	0.96	99.04	2.04	97.96	0.42	99.58	0.23	99.77	24.68	75.32	1.06	98.94	0.92	99.08	99.96	0.04
1min	J	70.25	29.75	70.16	29.84	71.92	28.08	68.40	31.60	67.12	32.88	73.58	26.42	65.46	34.54	63.80	36.20	82.49	17.51
	NJ	1.12	98.88	1.17	98.83	1.18	98.82	1.10	98.90	0.90	99.10	2.38	97.62	1.54	98.46	1.31	98.69	18.48	81.52
2min	J	63.99	36.01	64.19	35.81	64.97	35.03	63.99	36.01	63.41	36.59	64.87	35.13	62.72	37.28	61.64	38.36	67.81	32.19
	NJ	1.19	98.81	1.18	98.82	1.11	98.89	0.99	99.01	0.92	99.08	1.29	98.71	1.77	98.23	1.72	98.28	2.88	97.12
5min	J	53.13	46.87	53.33	46.67	52.94	47.06	52.25	47.75	52.45	47.55	52.74	47.26	53.03	46.97	52.84	47.16	53.33	46.67
	NJ	1.05	98.95	1.08	98.92	1.12	98.88	1.01	98.99	1.00	99.00	1.05	98.95	1.47	98.53	1.51	98.49	1.41	98.59
15min	J	36.79	63.21	36.79	63.21	36.01	63.99	36.30	63.70	36.40	63.60	35.71	64.29	35.91	64.09	36.11	63.89	35.81	64.19
	NJ	1.20	98.80	1.23	98.77	1.18	98.82	1.20	98.80	1.24	98.76	1.18	98.82	1.33	98.67	1.36	98.64	1.43	98.57

Table 16: **Power:** LL1F stochastic volatility model with medium mean reversion at 1% significant level (ADS).

Multipower Variation Ratio Test (BNS)																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	93.16	6.84	92.96	7.04	94.16	5.84	92.35	7.65	90.95	9.05	98.39	1.61	84.31	15.69	80.48	19.52	100.00	0.00
	NJ	1.07	98.93	0.96	99.04	1.96	98.04	0.18	99.82	0.02	99.98	68.49	31.51	0.00	100.00	0.00	100.00	100.00	0.00
1min	J	89.94	10.06	89.74	10.26	91.15	8.85	89.74	10.26	88.93	11.07	91.15	8.85	86.72	13.28	86.12	13.88	98.79	1.21
	NJ	1.02	98.98	1.01	98.99	1.27	98.73	0.92	99.08	0.78	99.22	2.55	97.45	0.48	99.52	0.21	99.79	53.11	46.89
2min	J	87.73	12.27	87.93	12.07	88.93	11.07	88.33	11.67	88.13	11.87	89.13	10.87	85.71	14.29	84.91	15.09	89.34	10.66
	NJ	0.99	99.01	1.01	98.99	1.09	98.91	0.99	99.01	1.03	98.97	1.19	98.81	1.12	98.88	0.95	99.05	2.80	97.20
5min	J	85.11	14.89	85.11	14.89	85.11	14.89	84.71	15.29	84.51	15.49	84.71	15.29	84.71	15.29	84.51	15.49	85.31	14.69
	NJ	1.28	98.72	1.28	98.72	1.33	98.67	1.36	98.64	1.38	98.62	1.32	98.68	1.35	98.65	1.36	98.64	1.44	98.56
15min	J	77.67	22.33	77.67	22.33	77.67	22.33	77.46	22.54	77.26	22.74	77.26	22.74	77.46	22.54	77.87	22.13	78.07	21.93
	NJ	1.29	98.71	1.29	98.71	1.30	98.70	1.28	98.72	1.27	98.73	1.22	98.78	1.26	98.74	1.20	98.80	1.10	98.90
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	88.58	11.42	88.47	11.53	89.82	10.18	86.60	13.40	85.46	14.54	98.34	1.66	78.09	21.91	74.25	25.75	100.00	0.00
	NJ	1.38	98.62	1.27	98.73	2.11	97.89	0.30	99.70	0.06	99.94	68.31	31.69	0.15	99.85	0.15	99.85	100.00	0.00
1min	J	87.12	12.88	87.02	12.98	88.16	11.84	85.77	14.23	85.36	14.64	88.58	11.42	84.22	15.78	82.97	17.03	95.95	4.05
	NJ	1.41	98.59	1.43	98.57	1.50	98.50	1.12	98.88	0.98	99.02	2.39	97.61	0.72	99.28	0.44	99.56	53.37	46.63
2min	J	83.49	16.51	83.18	16.82	83.59	16.41	82.66	17.34	82.45	17.55	84.63	15.37	82.04	17.96	81.83	18.17	84.94	15.06
	NJ	1.36	98.64	1.38	98.62	1.49	98.51	1.11	98.89	1.08	98.92	1.23	98.77	1.26	98.74	1.20	98.80	2.62	97.38
5min	J	79.34	20.66	79.75	20.25	80.17	19.83	78.92	21.08	78.82	21.18	78.82	21.18	78.50	21.50	78.40	21.60	78.92	21.08
	NJ	1.39	98.61	1.37	98.63	1.52	98.48	1.22	98.78	1.24	98.76	1.29	98.71	1.26	98.74	1.35	98.65	1.39	98.61
15min	J	68.33	31.67	68.12	31.88	68.95	31.05	68.43	31.57	68.43	31.57	68.43	31.57	68.43	31.57	68.22	31.78	69.68	30.32
	NJ	1.35	98.65	1.35	98.65	1.33	98.67	1.18	98.82	1.16	98.84	1.27	98.73	1.34	98.66	1.32	98.68	1.35	98.65
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	75.15	24.85	74.27	25.73	77.40	22.60	71.04	28.96	68.59	31.41	96.97	3.03	61.74	38.26	56.26	43.74	100.00	0.00
	NJ	0.95	99.05	0.87	99.13	1.87	98.13	0.23	99.77	0.09	99.91	67.99	32.01	0.84	99.16	0.82	99.18	99.99	0.01
1min	J	71.72	28.28	71.82	28.18	72.21	27.79	69.28	30.72	68.88	31.12	74.76	25.24	67.61	32.39	66.24	33.76	92.37	7.63
	NJ	1.04	98.96	1.06	98.94	1.18	98.82	0.96	99.04	0.86	99.14	2.42	97.58	1.30	98.70	1.20	98.80	54.66	45.34
2min	J	66.05	33.95	65.75	34.25	65.95	34.05	66.44	33.56	65.95	34.05	66.63	33.37	64.09	35.91	63.60	36.40	68.98	31.02
	NJ	1.06	98.94	1.12	98.88	1.08	98.92	0.95	99.05	0.96	99.04	1.10	98.90	1.58	98.42	1.58	98.42	3.10	96.90
5min	J	50.98	49.02	50.98	49.02	51.37	48.63	52.25	47.75	52.25	47.75	51.47	48.53	51.57	48.43	52.25	47.75	52.35	47.65
	NJ	1.24	98.76	1.21	98.79	1.14	98.86	1.08	98.92	1.08	98.92	1.15	98.85	1.50	98.50	1.54	98.46	1.56	98.44
15min	J	32.49	67.51	32.49	67.51	32.29	67.71	33.17	66.83	33.07	66.93	32	68	32.78	67.22	33.17	66.83	33.17	66.83
	NJ	1.37	98.63	1.34	98.66	1.37	98.63	1.33	98.67	1.31	98.69	1.31	98.69	1.47	98.53	1.50	98.50	1.62	98.38

Table 17: **Power:** LL1F stochastic volatility model with fast mean reversion at 1% significant level (BNS).

		MedRV Ratio Test (ADS))																	
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	90.74	9.26	89.34	10.66	94.97	5.03	85.31	14.69	80.68	19.32	96.58	3.42	72.03	27.97	65.39	34.61	100.00	0.00
	NJ	0.90	99.10	0.82	99.18	1.98	98.02	0.43	99.57	0.14	99.86	24.51	75.49	0.01	99.99	0.00	100.00	100.00	0.00
1min	J	89.54	10.46	89.13	10.87	91.15	8.85	86.52	13.48	85.31	14.69	91.15	8.85	80.89	19.11	77.26	22.74	93.76	6.24
	NJ	1.06	98.94	1.05	98.95	1.06	98.94	1.00	99.00	0.87	99.13	2.37	97.63	0.56	99.44	0.29	99.71	17.66	82.34
2min	J	87.32	12.68	87.12	12.88	88.93	11.07	86.92	13.08	86.32	13.68	88.93	11.07	82.90	17.10	81.29	18.71	89.54	10.46
	NJ	1.06	98.94	1.06	98.94	1.01	98.99	1.10	98.90	1.09	98.91	1.04	98.96	1.01	98.99	0.90	99.10	1.99	98.01
5min	J	86.12	13.88	86.12	13.88	86.92	13.08	84.71	15.29	84.71	15.29	86.32	13.68	84.91	15.09	84.31	15.69	85.92	14.08
	NJ	1.04	98.96	1.04	98.96	1.02	98.98	1.09	98.91	1.04	98.96	1.04	98.96	1.15	98.85	1.09	98.91	1.20	98.80
15min	J	79.07	20.93	79.07	20.93	79.07	20.93	78.67	21.33	79.28	20.72	78.87	21.13	79.28	20.72	79.48	20.52	79.28	20.72
	NJ	1.08	98.92	1.10	98.90	1.10	98.90	1.14	98.86	1.10	98.90	1.17	98.83	1.25	98.75	1.20	98.80	1.09	98.91
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	86.29	13.71	85.15	14.85	90.24	9.76	81.31	18.69	76.95	23.05	94.50	5.50	71.34	28.66	62.62	37.38	100.00	0.00
	NJ	1.15	98.85	1.12	98.88	2.11	97.89	0.51	99.49	0.22	99.78	25.04	74.96	0.15	99.85	0.15	99.85	100.00	0.00
1min	J	86.50	13.50	86.09	13.91	87.95	12.05	85.05	14.95	83.49	16.51	88.47	11.53	81.52	18.48	77.26	22.74	92.32	7.68
	NJ	1.28	98.72	1.18	98.82	1.27	98.73	1.00	99.00	0.89	99.11	2.21	97.79	0.80	99.20	0.50	99.50	17.61	82.39
2min	J	83.70	16.30	83.28	16.72	84.53	15.47	83.07	16.93	82.87	17.13	84.84	15.16	81.10	18.90	80.06	19.94	85.67	14.33
	NJ	1.13	98.87	1.13	98.87	1.32	98.68	0.98	99.02	0.91	99.09	1.00	99.00	1.13	98.87	1.04	98.96	2.07	97.93
5min	J	80.48	19.52	80.27	19.73	80.48	19.52	80.27	19.73	79.96	20.04	81.41	18.59	80.06	19.94	79.23	20.77	81.52	18.48
	NJ	1.27	98.73	1.23	98.77	1.35	98.65	1.16	98.84	1.18	98.82	1.22	98.78	1.35	98.65	1.29	98.71	1.38	98.62
15min	J	70.72	29.28	70.61	29.39	70.92	29.08	70.51	29.49	70.40	29.60	70.51	29.49	70.82	29.18	70.40	29.60	70.82	29.18
	NJ	0.94	99.06	0.94	99.06	0.89	99.11	0.97	99.03	0.91	99.09	0.86	99.14	0.95	99.05	1.00	99.00	0.95	99.05
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	73.78	26.22	72.11	27.89	77.50	22.50	69.47	30.53	66.24	33.76	89.14	10.86	59.78	40.22	52.74	47.26	100.00	0.00
	NJ	0.84	99.16	0.81	99.19	1.93	98.07	0.33	99.67	0.19	99.81	24.13	75.87	0.97	99.03	0.90	99.10	99.97	0.03
1min	J	72.11	27.89	71.72	28.28	72.60	27.40	69.96	30.04	68.49	31.51	75.24	24.76	67.42	32.58	64.77	35.23	83.76	16.24
	NJ	1.05	98.95	1.09	98.91	1.35	98.65	1.02	98.98	0.87	99.13	2.25	97.75	1.59	98.41	1.37	98.63	18.52	81.48
2min	J	65.66	34.34	65.26	34.74	65.66	34.34	64.58	35.42	64.58	35.42	67.32	32.68	63.50	36.50	63.21	36.79	69.18	30.82
	NJ	1.00	99.00	0.99	99.01	1.06	98.94	0.88	99.12	0.97	99.03	1.17	98.83	1.74	98.26	1.58	98.42	2.91	97.09
5min	J	53.62	46.38	53.62	46.38	53.52	46.48	53.42	46.58	53.42	46.58	53.82	46.18	54.11	45.89	54.31	45.69	54.50	45.50
	NJ	0.94	99.06	0.99	99.01	1.02	98.98	0.88	99.12	0.88	99.12	0.99	99.01	1.34	98.66	1.37	98.63	1.63	98.37
15min	J	36.01	63.99	36.30	63.70	36.20	63.80	36.20	63.80	36.11	63.89	36.20	63.80	36.99	63.01	37.18	62.82	36.50	63.50
	NJ	1.00	99.00	0.98	99.02	0.97	99.03	1.02	98.98	1.01	98.99	1.02	98.98	1.26	98.74	1.28	98.72	1.27	98.73

Table 18: **Power:** LL1F stochastic volatility model with fast mean reversion at 1% significant level (ADS).

		Multipower Variation Ratio Test (BNS)																	
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	94.37	5.63	94.16	5.84	95.57	4.43	93.56	6.44	93.36	6.64	97.38	2.62	87.73	12.27	84.71	15.29	100.00	0.00
	NJ	1.35	98.65	1.28	98.72	2.35	97.65	0.67	99.33	0.43	99.57	46.13	53.87	0.13	99.87	0.06	99.94	97.12	2.88
1min	J	92.96	7.04	93.16	6.84	93.56	6.44	92.76	7.24	92.76	7.24	93.76	6.24	91.75	8.25	90.95	9.05	96.98	3.02
	NJ	1.62	98.38	1.62	98.38	1.88	98.12	1.66	98.34	1.63	98.37	3.50	96.50	1.00	99.00	0.78	99.22	38.83	61.17
2min	J	92.96	7.04	93.16	6.84	92.76	7.24	92.35	7.65	91.95	8.05	92.35	7.65	90.95	9.05	90.74	9.26	93.16	6.84
	NJ	1.88	98.12	1.84	98.16	1.88	98.12	1.80	98.20	1.74	98.26	2.06	97.94	1.79	98.21	1.54	98.46	3.54	96.46
5min	J	88.53	11.47	88.53	11.47	88.73	11.27	88.73	11.27	88.73	11.27	88.33	11.67	88.93	11.07	88.73	11.27	88.73	11.27
	NJ	2.57	97.43	2.55	97.45	2.78	97.22	2.64	97.36	2.64	97.36	2.63	97.37	2.76	97.24	2.70	97.30	2.85	97.15
15min	J	83.50	16.50	83.50	16.50	83.50	16.50	83.70	16.30	83.70	16.30	83.10	16.90	83.90	16.10	83.90	16.10	83.50	16.50
	NJ	3.85	96.15	3.80	96.20	3.76	96.24	3.85	96.15	3.98	96.02	3.83	96.17	3.63	96.37	3.64	96.36	3.78	96.22
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	87.02	12.98	87.02	12.98	87.02	12.98	85.15	14.85	83.39	16.61	93.87	6.13	77.05	22.95	72.48	27.52	99.79	0.21
	NJ	1.38	98.62	1.42	98.58	2.37	97.63	0.58	99.42	0.35	99.65	46.70	53.30	0.20	99.80	0.17	99.83	96.99	3.01
1min	J	84.01	15.99	84.22	15.78	84.01	15.99	83.49	16.51	83.07	16.93	85.05	14.95	81.41	18.59	80.48	19.52	91.48	8.52
	NJ	1.35	98.65	1.36	98.64	1.65	98.35	1.32	98.68	1.16	98.84	3.07	96.93	0.82	99.18	0.64	99.36	39.31	60.69
2min	J	80.89	19.11	80.89	19.11	81.31	18.69	80.06	19.94	80.17	19.83	80.79	19.21	79.96	20.04	79.75	20.25	82.04	17.96
	NJ	1.89	98.11	1.87	98.13	1.87	98.13	1.86	98.14	1.87	98.13	1.94	98.06	1.80	98.20	1.64	98.36	3.47	96.53
5min	J	76.43	23.57	76.43	23.57	76.43	23.57	75.70	24.30	75.70	24.30	75.80	24.20	74.77	25.23	74.66	25.34	76.22	23.78
	NJ	2.81	97.19	2.73	97.27	2.50	97.50	2.49	97.51	2.59	97.41	2.25	97.75	2.38	97.62	2.46	97.54	2.49	97.51
15min	J	66.87	33.13	66.77	33.23	67.19	32.81	67.39	32.61	67.39	32.61	67.19	32.81	65.94	34.06	65.84	34.16	66.87	33.13
	NJ	3.19	96.81	3.15	96.85	3.30	96.70	3.25	96.75	3.25	96.75	3.08	96.92	3.30	96.70	3.19	96.81	2.99	97.01
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	86.79	13.21	86.50	13.50	88.06	11.94	85.71	14.29	84.83	15.17	94.81	5.19	80.92	19.08	77.59	22.41	99.71	0.29
	NJ	1.27	98.73	1.23	98.77	2.29	97.71	0.51	99.49	0.28	99.72	45.12	54.88	0.74	99.26	0.70	99.30	96.67	3.33
1min	J	85.42	14.58	85.23	14.77	85.52	14.48	85.52	14.48	85.52	14.48	87.38	12.62	83.76	16.24	83.27	16.73	92.27	7.73
	NJ	1.67	98.33	1.70	98.30	1.92	98.08	1.55	98.45	1.44	98.56	2.96	97.04	1.67	98.33	1.41	98.59	39.41	60.59
2min	J	82.78	17.22	82.58	17.42	83.27	16.73	81.70	18.30	81.70	18.30	83.46	16.54	81.90	18.10	81.70	18.30	83.46	16.54
	NJ	1.80	98.20	1.82	98.18	1.92	98.08	1.92	98.08	1.93	98.07	2.19	97.81	2.15	97.85	2.26	97.74	3.92	96.08
5min	J	77.69	22.31	77.59	22.41	78.18	21.82	77.69	22.31	77.59	22.41	78.18	21.82	77.30	22.70	77.40	22.60	77.79	22.21
	NJ	2.66	97.34	2.62	97.38	2.56	97.44	2.51	97.49	2.54	97.46	2.67	97.33	2.77	97.23	2.87	97.13	3.29	96.71
15min	J	67.91	32.09	68.00	32.00	68.20	31.80	67.81	32.19	67.91	32.09	68.20	31.80	67.71	32.29	67.71	32.29	68.10	31.90
	NJ	3.95	96.05	3.87	96.13	3.76	96.24	3.90	96.10	3.90	96.10	3.85	96.15	4.10	95.90	4.11	95.89	3.91	96.09

Table 19: **Power:** LL2F stochastic volatility model at 1% significant level (BNS).

MedRV Ratio Test (ADS))																			
		Mean Intraday Duration: 5 seconds						Mean Intraday Duration: 15 seconds						Mean Intraday Duration: 30 seconds					
		Spl		Lin		PT		Spl		Lin		PT		Spl		Lin		PT	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
$\lambda = 0.05$ $\sigma^2 = 2.5$																			
30s	J	91.55	8.45	89.54	10.46	95.57	4.43	83.50	16.50	79.07	20.93	96.38	3.62	70.82	29.18	63.78	36.22	99.60	0.40
	NJ	1.29	98.71	1.20	98.80	2.29	97.71	0.89	99.11	0.62	99.38	18.67	81.33	0.13	99.87	0.04	99.96	87.07	12.93
1min	J	92.96	7.04	91.75	8.25	94.16	5.84	88.13	11.87	86.52	13.48	94.37	5.63	82.09	17.91	78.87	21.13	95.77	4.23
	NJ	1.62	98.38	1.56	98.44	1.70	98.30	1.47	98.53	1.39	98.61	2.90	97.10	0.94	99.06	0.66	99.34	13.87	86.13
2min	J	90.54	9.46	89.94	10.06	92.15	7.85	89.34	10.66	88.53	11.47	91.75	8.25	84.51	15.49	82.49	17.51	92.35	7.65
	NJ	1.67	98.33	1.66	98.34	1.70	98.30	1.69	98.31	1.66	98.34	1.76	98.24	1.76	98.24	1.69	98.31	2.78	97.22
5min	J	88.73	11.27	88.33	11.67	88.93	11.07	87.53	12.47	87.53	12.47	89.34	10.66	88.13	11.87	87.93	12.07	89.54	10.46
	NJ	2.36	97.64	2.29	97.71	2.37	97.63	2.41	97.59	2.47	97.53	2.62	97.38	2.46	97.54	2.56	97.44	2.38	97.62
15min	J	84.51	15.49	84.31	15.69	84.71	15.29	84.10	15.90	83.90	16.10	84.51	15.49	83.90	16.10	84.10	15.90	83.90	16.10
	NJ	3.59	96.41	3.61	96.39	3.42	96.58	3.54	96.46	3.54	96.46	3.59	96.41	3.40	96.60	3.37	96.63	3.33	96.67
$\lambda = 0.1$ $\sigma^2 = 1.5$																			
30s	J	83.28	16.72	81.93	18.07	86.50	13.50	78.30	21.70	73.42	26.58	90.65	9.35	66.36	33.64	58.57	41.43	98.96	1.04
	NJ	1.57	98.43	1.32	98.68	2.11	97.89	0.81	99.19	0.53	99.47	18.44	81.56	0.34	99.66	0.20	99.80	88.03	11.97
1min	J	81.62	18.38	81.52	18.48	83.80	16.20	80.27	19.73	78.82	21.18	84.42	15.58	75.70	24.30	72.27	27.73	87.33	12.67
	NJ	1.53	98.47	1.50	98.50	1.38	98.62	1.49	98.51	1.31	98.69	2.92	97.08	1.04	98.96	0.82	99.18	14.86	85.14
2min	J	80.06	19.94	79.75	20.25	81.31	18.69	80.37	19.63	79.44	20.56	82.04	17.96	77.36	22.64	75.60	24.40	81.93	18.07
	NJ	2.08	97.92	2.15	97.85	2.01	97.99	1.79	98.21	1.80	98.20	1.98	98.02	1.80	98.20	1.67	98.33	3.03	96.97
5min	J	77.15	22.85	76.95	23.05	77.67	22.33	75.70	24.30	76.22	23.78	77.36	22.64	74.56	25.44	74.04	25.96	76.74	23.26
	NJ	2.41	97.59	2.37	97.63	2.35	97.65	2.20	97.80	2.17	97.83	2.21	97.79	2.24	97.76	2.14	97.86	2.33	97.67
15min	J	67.81	32.19	67.71	32.29	67.60	32.40	67.19	32.81	67.08	32.92	67.91	32.09	67.19	32.81	66.87	33.13	67.39	32.61
	NJ	3.02	96.98	2.98	97.02	2.98	97.02	2.92	97.08	2.89	97.11	2.99	97.01	2.94	97.06	2.95	97.05	3.09	96.91
$\lambda = 1$ $\sigma^2 = 0.5$																			
30s	J	84.05	15.95	82.29	17.71	87.96	12.04	78.47	21.53	73.58	26.42	91.88	8.12	69.18	30.82	60.67	39.33	98.14	1.86
	NJ	1.33	98.67	1.25	98.75	2.28	97.72	0.82	99.18	0.52	99.48	18.06	81.94	0.92	99.08	0.79	99.21	86.83	13.17
1min	J	84.74	15.26	84.34	15.66	86.01	13.99	81.02	18.98	79.45	20.55	86.40	13.60	77.59	22.41	74.36	25.64	89.33	10.67
	NJ	1.69	98.31	1.67	98.33	1.73	98.27	1.63	98.37	1.51	98.49	3.12	96.88	1.96	98.04	1.57	98.43	14.90	85.10
2min	J	81.21	18.79	80.92	19.08	82.39	17.61	80.04	19.96	79.55	20.45	82.09	17.91	78.57	21.43	77.40	22.60	82.29	17.71
	NJ	1.65	98.35	1.59	98.41	1.76	98.24	1.66	98.34	1.72	98.28	1.86	98.14	2.18	97.82	2.13	97.87	3.22	96.78
5min	J	76.81	23.19	76.71	23.29	76.91	23.09	76.22	23.78	76.22	23.78	76.61	23.39	76.71	23.29	75.93	24.07	76.71	23.29
	NJ	2.73	97.27	2.74	97.26	2.43	97.57	2.58	97.42	2.60	97.40	2.44	97.56	2.84	97.16	2.80	97.20	2.64	97.36
15min	J	67.71	32.29	67.91	32.09	68	32	68.10	31.90	68.20	31.80	68.30	31.70	67.71	32.29	67.81	32.19	67.81	32.19
	NJ	3.58	96.42	3.53	96.47	3.54	96.46	3.46	96.54	3.37	96.63	3.59	96.41	3.71	96.29	3.73	96.27	3.73	96.27

Table 20: **Power:** LL2F stochastic volatility model at 1% significant level (ADS).

Multipower Variation Ratio Test (BNS)		Heston		LL1F (slow)		LL1F (medium)		LL1F (fast)		LL2F	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
Mean Intraday Duration: 5 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.98	3.02	83.67	16.33	93.95	6.05	94.76	5.24	94.96	5.04
	NJ	4.15	95.85	3.95	96.05	4.01	95.99	4.15	95.85	3.57	96.43
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	94.79	5.21	76.88	23.13	90.63	9.38	90.52	9.48	90.31	9.69
	NJ	4.13	95.87	3.85	96.15	3.95	96.05	4.21	95.79	3.60	96.40
$\lambda = 1 \quad \sigma^2 = 0.5$	J	58.00	42.00	38.57	61.43	50.21	49.79	50.12	49.88	50.61	49.39
	NJ	4.50	95.50	4.09	95.91	3.68	96.32	4.94	95.06	3.57	96.43
Mean Intraday Duration: 15 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.77	3.23	81.65	18.35	91.73	8.27	92.94	7.06	92.74	7.26
	NJ	2.67	97.33	2.33	97.67	2.54	97.46	2.28	97.72	2.58	97.42
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	94.69	5.31	71.15	28.85	86.15	13.85	86.88	13.13	87.08	12.92
	NJ	2.71	97.29	2.32	97.68	2.52	97.48	2.40	97.60	2.61	97.39
$\lambda = 1 \quad \sigma^2 = 0.5$	J	56.59	43.41	35.03	64.97	45.02	54.98	45.43	54.57	47.40	52.60
	NJ	2.77	97.23	2.30	97.70	2.36	97.64	2.74	97.26	2.99	97.01
Mean Intraday Duration: 30 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.57	3.43	78.63	21.37	90.73	9.27	91.94	8.06	91.73	8.27
	NJ	2.79	97.21	2.63	97.37	2.49	97.51	2.74	97.26	2.62	97.38
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	94.27	5.73	68.33	31.67	84.38	15.63	85.10	14.90	86.25	13.75
	NJ	2.72	97.28	2.58	97.42	2.46	97.54	2.83	97.17	2.68	97.32
$\lambda = 1 \quad \sigma^2 = 0.5$	J	55.94	44.06	34.41	65.59	41.53	58.47	42.05	57.95	45.54	54.46
	NJ	2.11	97.89	2.19	97.81	2.88	97.12	2.50	97.50	2.72	97.28

Table 21: **Power Analysis:** BNS test at 1% significant level and tick time.

		MedRV Ratio Test (ADS))									
		Heston		LL1F (slow)		LL1F (medium)		LL1F (fast)		LL2F	
		J	NJ	J	NJ	J	NJ	J	NJ	J	NJ
Mean Intraday Duration: 5 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.98	3.02	83.87	16.13	94.35	5.65	94.76	5.24	94.56	5.44
	NJ	4.87	95.13	5.17	94.83	5.20	94.8	5.12	94.88	3.95	96.05
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	95.00	5.00	77.29	22.71	90.00	10.00	90.00	10.00	90.52	9.48
	NJ	4.96	95.04	5.11	94.89	5.18	94.82	5.27	94.73	3.89	96.11
$\lambda = 1 \quad \sigma^2 = 0.5$	J	58.80	41.20	39.18	60.82	50.39	49.61	49.90	50.10	50.46	49.54
	NJ	4.80	95.20	5.27	94.73	4.25	95.75	5.46	94.54	3.48	96.52
Mean Intraday Duration: 15 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.77	3.23	81.85	18.15	91.73	8.27	92.14	7.86	92.54	7.46
	NJ	3.00	97.00	2.98	97.02	2.74	97.26	2.80	97.20	2.75	97.25
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	94.48	5.52	71.35	28.65	86.15	13.85	87.71	12.29	86.88	13.13
	NJ	3.06	96.94	2.99	97.01	2.80	97.20	2.79	97.21	2.87	97.13
$\lambda = 1 \quad \sigma^2 = 0.5$	J	56.79	43.21	35.55	64.45	45.18	54.82	45.19	54.81	46.72	53.28
	NJ	3.05	96.95	2.91	97.09	2.66	97.34	3.10	96.90	2.85	97.15
Mean Intraday Duration: 30 seconds											
$\lambda = 0.05 \quad \sigma^2 = 2.5$	J	96.57	3.43	77.82	22.18	90.73	9.27	91.53	8.47	91.53	8.47
	NJ	2.81	97.19	2.88	97.12	2.78	97.22	2.81	97.19	2.96	97.04
$\lambda = 0.1 \quad \sigma^2 = 1.5$	J	94.38	5.63	67.92	32.08	84.69	15.31	84.38	15.63	85.63	14.37
	NJ	2.81	97.19	2.84	97.16	2.73	97.27	2.85	97.15	2.83	97.17
$\lambda = 1 \quad \sigma^2 = 0.5$	J	55.70	44.30	34.30	65.70	41.56	58.44	42.09	57.91	45.26	54.74
	NJ	2.36	97.64	2.72	97.28	2.58	97.42	2.85	97.15	3.13	96.87

Table 22: **Power Analysis:** ADS test at 1% significant level and tick time.

6 Conclusion

In the current paper, the distributional properties of the time equidistant return series constructed under the previous tick method were examined for a number of different sampling frequencies, extending from 30 seconds to 15 minutes. The purpose of the analysis was to investigate the extent to which the sampling scheme alters the dynamics of the return series and the potential impact of these alterations on the estimation of various volatility estimators. The main conclusions from the analysis suggest that the previous tick method results in large samples of identical quotes or transactions which, when translated into zero returns, dominate the distribution of the return series driving its center higher, and thus leading to the marginalization of the true observations in the tails of the distribution. The distortions in the dynamics of the return process from the introduction of these zero returns were confirmed to be quite severe, particularly for the higher moments of the distribution, such as the kurtosis.

As an immediate consequence of the dominating presence of successive zero returns in the data, the estimated bipower variation is severely underestimated. Given that this quantity, together with the integrated quarticity, form the basis for the ratio jump test statistic of Barndorff-Nielsen & Shephard (2004, 2006) and Andersen, Dobrev & Schaumburg (2009), any results about the presence of jumps in the data are expected to be seriously biased.

In order to investigate the extend at which the artificially induced zero returns bias the estimation accuracy of these volatility estimators, and consequently the performance of the two jump tests, an alternative sampling scheme, the Akima spline interpolation method, was proposed which avoids the induction of zero return in the sampled data and this was compared to the PT and linear interpolation sampling schemes. The three sampling schemes were applied to data simulated under three different liquidity scenarios. The results from the application of the jump test statistics on the sampled data were analyzed in terms of size and power and relative to the sampling scheme employed in the data. The properties of the tests under tick time were also investigated. It was concluded from the results that the two multi-power variation based tests are extremely sensitive to the liquidity in the data and that uncovering the most appropriate sampling frequency is crucial for the performance of the tests under all three different sampling schemes examined.

From the extensive Monte Carlo Simulation, it was concluded that the Akima spline and linear interpolation attain better size and power properties at higher frequencies than the previous tick method, with the Akima spline interpolation being slightly better. Finally, it was proven that the majority of the jumps found by the test under the previous tick sampling scheme are false, especially at frequencies higher than 5 minutes.

A promising direction for future research would be the development of jump test statistics which operate under the assumption of non-equidistant time intervals, but this is beyond the scope of this paper.

References

J.H. Ahlberg, E.N. Nilson, and J.L. Walsh. *The Theory of Splines and their Applications*. New York, London: Academic Press. Princeton University, 1967.

Yacine Aït-Sahalia and Jean Jacod. Testing for jumps in a discretely observed process. *Annals of Statistics*, 37(1):184–222, 2009.

H. Akima. A new method of interpolation and smooth curve fitting based on local procedures. *J. ACM*, 17(4):589–602, 1970.

H. Akima. Algorithm 696: Univariate interpolation that has the accuracy of a third-degree polynomial. *ACM Trans. Math. Softw.*, 17(3):367, 1991.

S. Alizadeh, M. Brandt, and F.X. Diebold. Range-based estimation of stochas- tic volatility models. *Journal of Finance*, 57:1047–1092, 2002.

A. Amihum and H. Mendelson. Trading mechanisms and stock returns: An empirical investigation. *Journal of Finance*, 42(3):533–555, 1987.

Torben G. Anderesen, Tim Bollerslev, and Nour Meddahi. Correcting the errors: Volatility forecast evaluation using high-frequency data and realized volatilities. *Econometrica*, 73: 279–296, 2005.

T.G. Andersen and T. Bollerslev. Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance*, 4:115–158, 1997.

T.G. Andersen, T. Bollerslev, and N. Meddahi. Analytic evaluation of volatility forecasts. *International Economic Review, Journal of Applied Econometrics*, 45(1079-1110), 2004a.

T.G. Andersen, T. Bollerslev, P.F. Christoffersen, and F.X. Diebold. *Handbook of Economic Forecasting*, chapter Volatility and Correlation Forecasting, pages 778–878. G. Elliott, C.W.J. Granger, and Allan Timmermann (eds.), Amsterdam: North Holland, 2006.

Torben G. Andersen and Tim Bollerslev. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 39(4):885–905, November 1998.

Torben G. Andersen, Tim Bollerslev, Francis X. Diebold, and Paul Labys. Exchange rate returns standardized by realized volatility are (nearly) gaussian. *Multinational Finance Journal*, 4:159–179, 2000.

Torben G. Andersen, Tim Bollerslev, Francis X. Diebold, and Paul Labys. Modeling and forecasting realized volatility. *Econometrica*, 71(2):579–625, March 2003.

Torben G. Andersen, Tim Bollerslev, and Francis X. Diebold. Some like it smooth, and some like it rough: Untangling continuous and jump components in measuring, modeling and forecasting asset return volatility. Working Paper, University of Pennsylvania, 2004b.

Torben G. Andersen, Dobrislav Dobrev, and Ernst Schaumburg. Jump robust volatility estimation. Northwestern University, 2009.

Ole E. Barndorff-Nielsen and Neil Shephard. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society B*, 64:253–280, 2002.

Ole E. Barndorff-Nielsen and Neil Shephard. Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics*, 2(1):1–48, 2004.

Ole E. Barndorff-Nielsen and Neil Shephard. Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics*, 4(1):1–30, 2006.

Ole E. Barndorff-Nielsen, Peter R. Hansen, Asger Lunde, and Neil Shephard. Designing realized kernels to measure the ex-post variation of equity prices in the presence of noise. *Econometrica*, 76(6):14811536, 2008.

E. Barucci and R. Renó. On measuring volatility and the garch forecasting performance. *Journal of International Financial Markets, Institutions and Money*, 12:183–200, 2002.

F. Black. The pricing of commodity contracts. *Journal of Financial Economics*, 3:167–179, 1976.

W. Boehm and G. Farin. A survey of curve and surface methods in cagd. *CAGD*, 1(1): 1–60, 1984.

T. Bollerslev and H. Zhou. Estimating stochastic volatility diffusions using conditional moments of integrated volatility. *Journal of Econometrics*, 109:33–65, 2002.

P. A. Chaboud, B. Chiquoine, E. Hjalmarsson, and M. Loretan. Frequency of observation and the estimation of integrated volatility in deep and liquid financial markets. *Journal of Empirical Finance*, 17(2):212–240, March 2009.

G. Chacko and L.M. Viceira. Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets. *Review of Financial Studies, Oxford University Press for Society for Financial Studies*, 18(4):1369–1402, 2005.

Mikhail Chernov, A. Ronald Gallant, Eric Ghysels, and George Tauchen. Alternative models for stock price dynamics. *Journal of Econometrics*, 116:225–257, 2003.

F. Corsi, G. Zumbach, U.A. Müller, and M. Dacorogna. Consistent high precision volatility from high-frequency data. *Economic Notes*, 30(2):183–204, 2001.

Fulvio Corsi, Davide Pirino, and Roberto Renò. Volatility forecasting: The jumps do matter. Università di Siena, 2008.

M.G. Cox. The numerical evaluation of b-splines. *Journal of the Institute of Mathematics and its Applications*, 10:134–149, 1972.

M.M. Dacorogna, R. Gençay, U. Müller, R.B. Olsen, and O.V. Pictet. *An Introduction to High-Frequency Finance*. London Academic Press, 2001.

C. de Boor. On calculating with b-splines. *Journal of Approximation Theory*, 6:50–62, 1972.

C. de Boor. *Geometric Modeling*, pages 131–148. G. Farin ed., SIAM, Philadelphia PA, 1986.

R. Engle and G. Lee. *Cointegration, Causality and Forecasting*, chapter A Long-Run and Short-Run Component Model of Stock Return Volatility. Edited by R. Engle and H. White, Oxford University Press, 1999.

J. Fried and S. Zietz. Curve fitting by spline and akima methods: Possibility of interpolation error and its suppression. *Phys Med Biol.*, 18(4):550–558, 1973.

F.N. Fritsch and J. Butland. A method for constructing local monotone piecewise cubic interpolants. *SIAM J. Sci. Statist. Comput.*, 5:300–304, 1993.

F.N Fritsch and R.E. Carlson. Monotone piecewise cubic interpolation. *SIAM J. Numer. An.*, 17:238–246, 1980.

A.R. Gallant, C.T. Hsu, and G. Tauchen. Using daily range data to calibrate volatility diffusions and extract the forward integrated variance. *The Review of Economics and Statistics, MIT Press*, 81(4):617–631, 1999.

W Gordon and R. Riesenfeld. *Computer Aided Geometric Design*, chapter B-Spline Curves and Surfaces, pages 95–126. Barnhill-Riesenfeld (eds.), Academic Press New York, 1974.

Peter H. Hansen and Asger Lunde. Realized variance and market microstructure noise. *Journal of Business and Economic Statistics*, 24(2):127–161, April 2006.

P.R. Hansen and A. Lunde. An unbiased measure of realized variance. Working Paper, Aarhus School of Business, 2004.

L.E. Harris. Estimation of stock variances and serial covariances from discrete observations. *Journal of Financial and Quantitative Analysis*, 25:291–306, 1990.

L.E. Harris. Stock price clustering and discreteness. *Review of Financial Studies*, 3:389–415, 1991.

H.S. Hou and H.C. Andrews. Cubic splines for image interpolation and digital filtering. In *IEEE Trans. Acoust. Speech Signal Process. ASSP-26*, volume 6, pages 508–517, 1978.

Xin Huang and George Tauchen. The relative contribution of jumps to total price variation. *Journal of Financial Econometrics*, 3(4):456–499, 2005.

H.T. Huynh. Accurate monotone cubic interpolation. *SIAM J. Numer. An.*, 30:57–101, 1993.

J.M. Hyman. Accurate monotonicity preserving cubic interpolation. *SIAM Journal of Statistics and Computation*, 4:645–654, 1983.

K. Jensen and D. Anastassiou. Subpixel edge localization and the interpolation of still images. In *IEEE Trans. Image Process*, volume 4, pages 285–295, 1993.

George J. Jiang and Roel C. A. Oomen. Testing for jumps when asset prices are observed with noise a swap variance approach. *Journal of Econometrics*, 144(2):352–370, 2008.

X. Jiao and M.T. Heath. Common-refinement-based data transfer between non-matching meshes in multiphysics simulations. *International Journal for Numerical Methods in Engineering*, 61:2402–2427, 2004.

Ioannis Karatzas and Steven E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, 2 edition, 1991.

R. Keys. Cubic convolution interpolation for digital image processing. In *IEEE Trans. Acoust. Speech Signal Process, ASSP-29*, volume 6, pages 1153–1160, 1981.

Sang-Hyop Lee and Andrew Mason. Who gains from the demographic dividend? forecasting income by age. *International Journal of Forecasting*, 23(4):603–619, 2007. ISSN 0169-2070. doi: http://dx.doi.org/10.1016/j.ijforecast.2007.07.004.

Nour Meddahi. A theoretical comparison between integrated and realized volatility. *Journal of Applied Econometrics*, 17:479–508, 2002.

E. Meijering. Spline interpolation in medical imaging: Comparison with other convolution-based approaches. In *Signal Processing X: Theories and Applications Procedures*, volume 4, pages 1989–1996, 2000.

C.L. Osler. Stop-loss orders and price cascades in currency markets. *Journal of International Money and Finance*, 24:219–241, 2005.

A. Parker, R. Kenyon, and D. Troxel. Comparison of interpolating methods for image resampling. In *IEEE Trans. Medical Imaging MI-2*, volume 1, pages 31–39, 1983.

R.W. Parrot, M.R. Stytz, R. Amburn, and D. Robinson. Toward statistically optimal interpolation for 3-d medical imaging. In *IEEE Eng. Med. Biol.*, volume 12, pages 49–59, 1993.

Mark Podolskij and Daniel Ziggel. New tests for jumps: A threshold-based approach. CREATES Research Papers, 34, School of Economics and Management, University of Aarhus, 2008.

R. Roll. A simple implicit measure of the effective bid-ask spread in an efficient market. *The Journal of Finance*, 39(4):1127–1139, 1984.

I.J. Schoenberg. Contributions to the problem of approximation of equidistant data by analytic functions, part a: On the problem of smoothing or graduation, a first class of analytic approximation formulas. *Quarterly of Applied Mathematics*, 4:45–99, 1946a.

I.J. Schoenberg. Contributions to the problem of approximation of equidistant data by analytic functions, part b: On the problem of smoothing or graduation, a second class of analytic approximation formulas. *Quarterly of Applied Mathematics*, 4:112–141, 1946b.

T.K. Truong, L.J. Wang, I.S. Reed, and W.S. Hsieh. Image data compression using cubic convolution spline interpolation. In *IEEE Trans on Image Processing*, volume 9, pages 1988–1995, 2000.

M. Unser, A. Aldroubi, and M. Eden. Fast b-spline transforms for continuous image representation and interpolation. In *IEEE Trans. Pattern Anal. Mach. Intell.*, volume 13, pages 277–285, 1991.

W. Wasserfallen and H. Zimmermann. The behavior of intraday exchange rates. *Journal of Banking and Finance*, 9:55–72, 1985.

G. Wolberg and I. Alfy. An energy minimization framework for monotonic cubic spline interpolation. *Journal of Computational and Applied Mathematics*, 143(2):145–188, 2002.

Lan Zhang, Per A. Mykland, and Yacine Aït-Sahalia. A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Assocation*, 100(472):1394–1411, December 2005.

B. Zhou. High-frequency data and volatility in foreign-exchange rates. *Journal of Business and Economic Statistics*, 14(1):45–52, 1996.