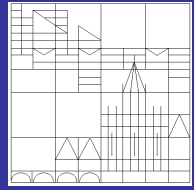University of Konstanz
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Working Effort and Endogenous Job Separations in Search Equilibrium

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Abstract

This paper considers job separations in a search model with labour market matching and moral hazard. Both workers and firms value productive matches and take actions to increase match stability: firms offer a share of match surplus to provide workers with correct incentives and workers take hidden actions (effort) negatively affecting the match separation rate. Heterogeneous productivity draws combined with the moral hazard problem give rise to match-specific endogenous separation rates. Additionally a counteraction of two effects – match stability and match scarcity – explains an observed asymmetric shape of a wage probability density function with a unique interior mode on the support.

JEL classification: J31, J63, J64, M52
Keywords: Matching, separation rate, job stability, effort, wage density

"The next stage appears to be an integration of the market frictions that characterize the DMP (Diamond-Mortensen-Pissarides) model, with efficiency wage models, which can explain wage setting within firms ..."

Olivier J. Blanchard (2008)

1   Introduction

The objective of this study is to analyze the behavior of a model economy with search frictions, moral hazard and endogenous job separation rates. In order to achieve this goal the paper develops a model, where firms face exogenous output shocks, while workers can take hidden actions (effort) to increase stability of the output stream. Unobserved worker actions give rise to the traditional moral hazard problem, so that firms respond by paying efficiency wages. In addition, worker's control over the output stability produces endogenous job separation rates in the model. Endogenous control over the job stability is particularly relevant in a model with search frictions since job search is a time-consuming process and so the separations are costly to workers and firms. The efficiency wage determination mechanism is strongly supported by the empirical evidence. Table 1 presents statistical summary of a large European data set collected by the researchers of a Wage Dynamics Network (WDN). This data set covers more than 17000 of firms across 15 European economies, the results show that about 50% of firms prefer to dismiss workers rather than to reduce base wages in response to an output shock. At the same time one of the two major reasons for avoiding the wage reduction is to maintain high effort and working morale.

This evidence supports a link between worker's effort and wages which is originally suggested in the study by Shapiro and Stiglitz (1984). The starting point of this paper is to introduce this link in a dynamic search and matching framework developed in Mortensen and Pissarides (1994) and Pissarides (2000). In a dynamic setting agents are forward looking and derive value from a match surplus rather than a match income flow. The difference from a static setting is that the match surplus is a function of both the net flow productivity of the match and the match separation rate such that a lower separation rate gives rise to a higher match surplus. This paper proposes a model allowing workers to take hidden actions (effort) that have a negative impact on the match separation rate and therefore extend the expected job duration.

Response to demand shocks Possible strategy	Approval rate %	Reasons for avoiding base wage reductions	Approval rate %
Reduce non-labor costs	38.3	**Lower work morale/less effort**	86
Reduce wages	12.6	Most productive workers leave	86
Base wages	1.9	Regulations/collective bargaining	73
Flexible wage components	10.7	Difficult to attract new workers	72
Reduce amount of labour	49.4	Labor turnover costs increase	70
Permanent employees	15.7	External wages matter	68
Temporary employees	25.0	Reputation suffers	60
Hours worked per employee	8.7	Implicit contract	59

Source: Fabiani S., Galuscak K., Kwapil C., Lamo A., Room T. "Wage Rigidities and Labour Market Adjustment in Europe" (2010). Statistical data: WDN Survey

Table 1: Firms' adjustment strategies to demand shocks

Model predictions can be described in the following way. First, the model incorporates the empirical evidence on efficiency wages and its implications for job stability into the search and matching labour market framework. Here firms leave positive rents to workers in order to motivate them to exert a desired level of effort and profit from an improved match stability and a higher match surplus. Workers bear the cost of effort but face a lower match separation risk. This case can be considered as a corner solution of a bargaining problem where firms have a full bargaining power and job offers are made on the basis of "take-it-or-leave-it". The model is further generalized to characterize an equilibrium with an interior value of the bargaining power. The paper shows that wages in this case can be decomposed into the the bargaining premium and the motivation premium, which would prevail in the absence of bargaining.

Second, the model is extended to the case of heterogeneous jobs. The jobs' heterogeneity is achieved ex-post on the basis of an exogenous productivity distribution. The paper shows that firms in more productive matches offer higher wages to workers, motivate them to exert more effort and indirectly obtain lower separation rates compared to the firms with lower productivity. This mechanism creates a situation where productivity is positively correlated with wages and negatively with separations from a cross-sectional perspective. Strong empirical evidence of a negative relationship between wages and separation rates can be found in Leonard (1987), Anderson and Meyer (1994), Galizzi and Lang (1998) and Christensen et al. (2005). Capelli and Chauvin (1991) explicitly consider the effect of wages on job dismissals, their results suggest that greater wage premiums are associated with lower levels of shirking and dismissals.

Furthermore, this paper presents an analysis of the interaction between the job's scarcity and its stability. In particular, it shows that the inverse relationship between the job's productivity and its separation rate is likely to produce hump-shaped equilibrium wage and productivity distributions even if the initial productivity density is downward-sloping, meaning that the more productive jobs are scarce in the economy. This offers a new explanation of an observed phenomenon of hump-shaped earnings distributions reported in Neal and Rosen (2000), Bontemps, Robin, and Van den Berg (2000), Postel-Vinay and Robin (2002) and Mortensen (2003).

This paper also considers the level of unemployment in search equilibrium with efficiency wages and shows that lower wages do not reduce the equilibrium unemployment rate. This result differs from the classical efficiency wage theory following the study by Shapiro and Stiglitz (1984). Lower wages in search equilibrium with moral hazard have two consequences: (a) firms obtain lower surplus, so the job creation is less intensive, and (b) lower effort is increasing the job separation rate, so the spells of employment are shorter. In the case of heterogeneous jobs the equilibrium unemployment rate depends on the average separation rate and the equilibrium job-finding rate. The effect of a higher reservation productivity on unemployment is traditionally positive, but its explanation is new. Here the positive effect of a lower job-finding rate is partially neutralized by a negative effect of a higher average separation rate resulting from the fact that remaining jobs are better paid and are therefore more stable (survivorship bias).

Finally, this study investigates the question of the optimal unemployment insurance (UI) in an economy with risk averse agents and moral hazard. In the absence of moral hazard Baily (1978) and later Holmlund (1998) show that full unemployment insurance is optimal in an economy with risk averse agents. This result does not however extend to the economy with endogenous search effort among the unemployed, see Hopenhayn and Nicolini (1997) and Fredriksson and Holmlund (2001). In this paper a different aspect of the effect of UI benefits on the decisions of labour market participants is analyzed. It is the unobservable working effort of the employed that is creating a trade-off for the social planner between providing the full unemployment insurance versus the maximum effort incentives. As a result the partial unemployment insurance is optimal: it reduces expenses of the social planner for vacancies and UI benefits due to the fact

that workers exert positive effort and jobs become more stable. In addition, this study shows that the optimal replacement ratio is increasing in the risk aversion of workers and is decreasing in the elasticity of the separation rate with respect to the net flow profit.

The paper is organized as follows. Section 2 contains an overview of the related literature and section 3 presents notation and the model setup. The optimal incentive contracts and the labour market equilibrium are presented in section 4. Section 5 presents an extension of the baseline model to account for jobs heterogeneity. Section 6 contains analysis of the equilibrium efficiency and the optimal unemployment insurance. Section 7 concludes the paper.

2   Overview of the related literature

There are several major directions relating this paper to the existing literature on labour turnover. First, this paper incorporates ideas of a shirking specification of the efficiency wage theory originally developed in Shapiro and Stiglitz (1984) and explored in more details in Akerlof and Yellen (1990) and Lazear (1998). This theory hinges upon the assumption of the inability of employers to costlessly observe worker's effort. The shirking specification of the efficiency wage theory assumes a discrete choice by the worker between shirking and non-shirking strategies under the constraint that a dismissal necessarily follows if a worker is caught shirking. However, dismissals only serve as a discipline device and do not occur in the equilibrium, this is principally different in the present study where negative productivity shocks render the worker unemployed.

Another branch of this literature, namely the turnover specification of the efficiency wage theory developed in Salop (1979), assumes that the labour turnover is costly to the firm; therefore the firm may attempt to reduce separations by offering a higher wage to the worker. In the current research this idea is combined with search frictions and endogenous separation probabilities. MacLeod and Malcomson (1998) merge efficiency wages with a forward looking behavior of agents. Similarly to the current study, they use the idea of job surplus rather than a flow wage to motivate workers to perform and exert effort. The difference occurs in the treatment of separation decisions which in their study are modeled exogenously and are unrelated to worker performance.

A combination of search frictions and agency problems has been originally introduced in Moen and Rosen (2006, 2008). These authors explicitly consider the question of efficiency wages in search equilibrium and develop the setup, where both effort and the match-specific productivity (type) are private information of the worker, so that the model is characterized by a combination of moral hazard and adverse selection problems. Moen and Rosen (2006) show that more high-powered incentive contracts tend to be associated with higher equilibrium unemployment rates. Moen and Rosen (2009) combine incentive contracts and endogenous worker turnover. Their paper deals with a deferred effort compensation and on-the-job search. Allowing workers to search on-the-job creates situations, when workers quit before obtaining their performance related remuneration.

The theory of heterogeneous voluntary separations (quits) arising from search on-the-job was developed in Burdett (1978), Jovanovich (1979), Jovanovich (1984) and Burdett and Mortensen (1998) and is summarized in Mortensen (2003) and Rogerson, Shimer, and Wright (2005). The general idea of these studies is that the probability of an outside offer to exceed the worker's current wage (quit probability) is decreasing in the current wage. These models play a major role in the explanation of the hump-shaped wage and productivity density functions based on wage competition between firms and on-the-job search. The current study is complementary to this group of papers and describes an additional source of job heterogeneity resulting from internal principal-agent problems within a match and relevant for the explanation of unimodal wage and productivity distributions.

Another approach to job destruction in a search and matching general equilibrium framework was introduced in the studies by Mortensen and Pissarides (1994) and Pissarides (2000). According to this approach independent idiosyncratic productivity shocks give rise to an endogenous job destruction rate. Once the productivity falls below the reservation productivity, the firm and the worker simultaneously decide to separate. This links the reservation productivity and the job separation rate, hence making the latter endogenous. Further extensions of this approach such as Rogerson, Shimer, and Wright (2005) allow the job destruction rate to be heterogeneous across firms. It is the explanation of this result and not the result itself that is different in the current study. Mortensen and Pissarides (1994) explain job destruction on the basis of job specific product

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demand fluctuations while this study attempts to extend their approach with an individual worker performance component.

3   Labour market modeling framework

The model is first analyzed in a homogeneous agent framework where the focus is on individual decision making of workers and firms in the presence of asymmetric information. Further in section 5 the model is generalized to account for the ex-post heterogeneity of job matches. This allows to study the properties of a general equilibrium in a labour market characterized by search frictions and firm specific endogenous separation rates.

In section 4 the labour market consists of a continuum of identical workers and firms. Each worker can be found in one of two possible states: employed and exerting nonnegative effort or unemployed and searching for a job. Similarly each firm has a job position which can be either filled with a worker or vacant and searching for a worker. Firms and workers share a common constant discount factor r. In section 5 job matches are heterogeneous with respect to the productivity parameter p drawn from the productivity distribution $F(p)$. Job search is random and undirected and the productivity realization is simultaneously revealed to the worker and firm once a match has been formed. Workers reject job offers below the reservation wage while firms reject productivity realizations below the reservation productivity.

When employed the worker chooses an optimal effort level $e \geq 0$ in response to the contract wage w. Effort is measured on a continuous scale and is not observable to the firm. In addition, workers are risk averse and have instantaneous utility functions of the form: $v(w) - C(e)$, where $v(w)$ is an increasing concave function of flow wage and $C(e)$ is an increasing and convex function of effort. Both functions are normalized to yield a zero instantaneous utility to the worker with zero wage and effort values: $v(0) = 0, C(0) = 0$. In addition, it is assumed that $C'(0) = 0$. Firms are risk-neutral.

Every employment relationship is exposed to a permanent productivity shock reducing the productivity value to zero[1]. The productivity shock arrives with a Poisson arrival rate $s(e)$, which is the separation rate of a match. One of the most important features of the model is that the separation rate is modeled as a decreasing function of worker's effort, meaning that higher effort decreases the probability of a negative productivity shock, i.e. $s'(e) < 0$. Here $e = 0$ implies that the separation rate is equal to it's maximum value $s(0) = \bar{s}$. Once the zero productivity value was drawn the job is destroyed and the worker becomes unemployed. One direct implication of this process is that a present value of output is an increasing function of worker's effort: $\int_0^\infty p \cdot \exp(-s(e))dt$. This expression is a dynamic equivalent of a static concept of a positive relationship between expected output and workers's effort widely used in the moral hazard literature.

The concept of match separation is closely related to that of the job duration. Under the Poisson specification of separation events the expected job duration is inversely related to the separation rate of a match, i.e. $d(e) = 1/s(e)$. This offers an alternative explanation of the effect of hidden actions taken by workers: higher effort decreases the separation rate and has a positive effect on the expected job duration.

Matching between firms and unemployed workers is modeled using the matching function approach. Let u denote the unemployment rate and v - the vacancy rate (expressed as a ratio of vacant jobs to the size of the labour force). Then the number of job matches taking place per unit time and expressed as a fraction of the labour force is given by:

$$m = m(u, v)$$

The matching function is assumed to be increasing in both arguments, concave, and homogeneous of degree 1. The homogeneity assumption is required in order to abstract from the size effects of the labour market and describe the major labour market variables in relative terms. Let θ be the labour market tightness parameter: $\theta \equiv \frac{v}{u}$ - the number of vacancies per unemployed worker. This allows to derive the job arrival rate $\lambda(\theta)$ and the vacancy filling rate $q(\theta)$ as functions

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[1]Throughout the paper it is assumed that the productivity value falls to zero upon a negative productivity shock, however it is sufficient to assume that the new productivity realization is below the worker's reservation wage.

of the labour market tightness parameter θ:

$$\lambda(\theta) = \frac{m(u,v)}{u} = m(1,\theta)$$
$$q(\theta) = \frac{m(u,v)}{v} = m(\tfrac{1}{\theta},1)$$

Wages are determined via the concept of generalized Nash bargaining where both workers and firms account for the expected effort response. There is no commitment, so that wages are continuously renegotiated. In the equilibrium worker rents can be decomposed into the motivation premium and the bargaining premium, where the first one implies leaving job rents to the worker in order to provide him with the correct working incentives. This reflects an essence of the efficiency wage component of the model. Once employed the worker faces a trade-off: exerting more effort at cost $C(e)$ and decreasing the separation risk versus exerting less effort and bearing a high separation risk. Optimal effort level is obtained by equating marginal gains and marginal costs of effort in the course of the worker's surplus maximization strategy.

Employing an efficiency wage determination mechanism in addition to bargaining requires clarification of such an argument as a bonding critique. The idea of the bonding critique is that workers pay a bond or an up-front hiring fee to the firm upon taking a job which may serve as a mechanism to prevent shirking. Therefore bonds or firing fees are often viewed as a substitute for efficiency wages in the part of providing correct incentives to the workers. There are several reasons why bonding is assumed to be prohibited in the model and firms are not allowed to charge an up-front fee.

As noticed in Moen and Rosen (2006) an entrance fee would have to be paid before a worker and a firm learn their match-specific productivity. Once bond value is an interior point in the support of the distribution of job values, a firm may adopt a strategy of leaving the most productive workers and firing the least productive workers in order to collect their bonds. This highlights an emerging moral hazard problem on the side of a firm. Therefore, allowing firms to charge an up-front fee would require extending the model to provide firms with correct incentives which is not a subject of current research. Carmichael (1990) presents a list of potential solutions how to eliminate this moral hazard problem. The most sensible of them is to collect entrance fees into a pension fund and redistribute

to the other workers if shirking occurs. Moreover, Ritter and Taylor (1994) show that bonds can be treated by workers as signals of high chances of bankruptcy. And so the safest firms will have incentives not to charge entrance fees in order to signal a high survival probability.

4 Moral hazard in search equilibrium

4.1 Workers: optimal effort choice

Let U and W denote the present-discounted value of the expected income stream of respectively, an unemployed and an employed worker. When a worker accepts a job at wage w, he chooses an optimal effort level e and keeps the job until a negative productivity shock arrives and the job is destroyed. If a worker rejects the job, he receives unemployment income z and searches again next period. Bellman equations for the unemployed and employed workers are:

$$rU = v(z) + \lambda(\theta)(W - U) \tag{4.1}$$

$$rW = \max_{e \geq 0}\{v(w) - C(e) - s(e)(W - U)\}, \tag{4.2}$$

where $s(e)$ is a job separation rate. An employed worker maximizes the job surplus $(W-U)$ given a wage offer w and a value of unemployment U. The choice variable of worker's maximization problem is effort e chosen in the positive domain $[0, \infty]$ in order to balance the marginal gain of a lower separation rate $s(e)$ and the marginal cost $C(e)$. The first order condition for the worker's optimization problem takes the following form:

$$W - U = \frac{v(w) - rU - C(e)}{r + s(e)} = \left|\frac{C'(e)}{s'(e)}\right| \tag{4.3}$$

Equation (4.3) is an incentive compatibility constraint for a worker and describes the functional relationship between the optimal effort level e and the earned wage w. Optimal effort is also a function of the reservation wage of the worker denoted w_0. Worker's participation constraint implies $W \geq U$, so that the reservation wage w_0 can be obtained from equation $W(w_0) = U$, this means $w_0 = v^{-1}(rU)$. This is true since $e(w_0, w_0) = 0$, meaning that workers choose zero effort in response to the reservation wage w_0. Analysis of the properties of the optimal effort function $e(w, w_0)$ gives rise to the following lemma.

Lemma 1: *(sufficient condition) Consider a risk averse worker with an increasing and convex effort cost function $C(e)$, such that $C(0) = 0$ and $C'(0) = 0$. Then effort $e(w, w_0)$ is an increasing function of the net flow utility $\Delta v \equiv v(w) - v(w_0)$ if $s''(e) \geq 0$. It is also true that $e(w_0, w_0) = 0$.*

Proof: Appendix I.

Let the inequality $s''(e) \geq 0$ in the following be denoted as assumption (A1).

Lemma 1 implies that under assumption (A1) effort $e(w, w_0)$ is an increasing function of wage w for a given reservation wage w_0 and a decreasing function of w_0 for a given wage w. This result is in accordance with the efficiency wage theory which defines efficiency wages as "high wages paid to workers to induce them to put forth more effort" (Lazear (1998, 70)). There is a straightforward economic explanation of this result. A higher value of wage offer w raises the present discounted value of worker's total surplus $W - U$ and therefore increases the marginal benefit of holding this position. A higher marginal benefit of the job allows the worker to increase his effort level in order to equalize the marginal cost and the marginal benefit. Put differently, a higher job surplus implies a higher value loss for the worker in case of the negative productivity shock. In this case the worker is responding by raising effort and reducing the probability of a separation. Condition (A1) also guarantees validity of the first order approach.

Further analysis of wage determination requires a statement about the curvature of the optimal effort function. For this define μ_s – absolute value of the semi-elasticity of the extended discount rate $r + s(e)$ with respect to worker's effort, formally

$$\mu_s = \left| \frac{\partial \ln(r + s(e))}{\partial e} \right| \tag{4.4}$$

Then sufficient conditions for the concavity of the effort function are summarized in lemma 2.

Lemma 2: *(sufficient conditions) Consider a risk-averse worker with an effort function $e(w, w_0)$ given in lemma 1 and an effort cost function $C(e)$ such that $C'''(e) \geq 0$ for $e \geq 0$. Then the effort function $e(w, w_0)$ is concave in the net flow utility $\Delta v \equiv v(w) - v(w_0)$ if the following conditions are satisfied:*

$$\frac{\partial \tilde{\mu}_s}{\partial e} \geq 0 \quad \frac{\partial^2 \tilde{\mu}_s}{\partial e^2} \geq 0 \quad where \quad \tilde{\mu}_s = 1/\mu_s \tag{A2}$$

Proof: Appendix II.

Variable $\tilde{\mu}_s$ is auxiliary, it is an inverse of the semi-elasticity variable μ_s. Assumptions (A2) require the semi-elasticity variable μ_s to be a decreasing function of effort. The semi-elasticity variable μ_s reflects the degree to which worker's actions may influence the separation rate, therefore a lower value of this variable corresponds to the situation of a lower responsiveness of the separation rate to workers' actions and forces workers to exert more effort in order to obtain the desired optimal level of job stability. This implies a positive relationship between effort e and wages w.

Example: linear job duration.

Consider the case, when job duration is a linear function of worker's effort: $d(e) = e + \delta$, where parameter δ denotes the minimum expected job duration corresponding to the case of zero effort ($d(0) = \delta$). This functional assumption gives rise to the inverse relationship between a separation rate and worker's effort taking the following form $s(e) = 1/d(e) = 1/(e + \delta)$. Here the highest separation rate $\bar{s}$ corresponds to the case of the lowest expected job duration so that $\bar{s} = 1/d(0) = 1/\delta$. Note that $s''(e) = 2/(e+\delta)^3 > 0$ so that effort is an increasing function of wage. The inverse of the semi-elasticity variable μ_s is found as:

$$\tilde{\mu}_s = (r(e + \delta) + 1)(e + \delta) \tag{4.5}$$

Investigation of the properties of variable μ_s allows to make a reference about the curvature of the optimal effort function. As follows from expressions

$$\frac{\partial \tilde{\mu}_s}{\partial e} = 2r(e + \delta) + 1 > 0$$
$$\frac{\partial^2 \tilde{\mu}_s}{\partial e^2} = 2r > 0$$

and lemmas 1-2, the optimal effort function is increasing and concave in wage. $\diamondsuit$

For the subsequent analysis it is convenient to use the concept of worker rents R associated with the employment, where $R \equiv W - U$. Applying the envelope theorem to equation (4.3) allows to conclude that the expected worker rent R is an increasing function of the net flow utility Δv and an increasing function of

wage w for a given reservation wage w_0. There are generally three effects of w on the worker's rent. First, there is a direct positive effect on the flow utility $v(w)$. Second, there is a positive effect of w on effort. The implications of this second effect for the worker's rent are twofold: higher effort costs $C(e)$ are combined with a lower job separation rate $s(e)$. However, as effort is optimally chosen by workers these last two effects are mutually neutralized.

4.2   Firms: wage determination

Let J be the present discounted value of expected profit from an occupied job and V the present-discounted value of expected profit from a vacant job. In order to maintain an open position firms incur a vacancy flow cost denoted by c. Consider Bellman equations for an open vacancy and a filled job position:

$$rV = -c + q(\theta)(J - V) \tag{4.6}$$

$$rJ = p - w - s(e)(J - V), \tag{4.7}$$

where $e = e(w, w_0)$ - optimal worker effort function. Equation (4.7) describes a trade-off faced by a firm. For fixed values of p and w_0 a firm bargaining lower wage would enjoy a higher flow profit $p - w$ but should also expect a higher separation rate $s(w, w_0) = s(e(w, w_0))$. In contrast a firm bargaining higher wage would bear a lower flow profit $p - w$ but should also expect a lower separation rate $s(w, w_0)$. Lower separation rate in this case implies improvement in the job stability and a longer expected job duration.

The contract wage w is determined via the concept of generalized Nash bargaining where both bargaining parties account for the optimal effort response of the worker. Outside option of a negotiating worker is to remain unemployed and search for another job, so that the rent of such a worker is given by $R = W - U$. The rent of a firm negotiating with an unemployed worker is given by $J - V$. In addition, the free-entry condition for opening new vacancies implies that competition between firms drives rents from a marginal vacant job to zero: $V = 0$, so that wage w is determined in the following way:

$$\max_w \left[\frac{v(w) - v(w_0) - C(e)}{r + s(e)} \right]^\beta \left[\frac{p - w}{r + s(e)} \right]^{1-\beta} \quad \text{where} \quad e = e(w, w_0) \tag{4.8}$$

Here β denotes the worker's bargaining power and the reservation wage w_0 is treated parametrically. For the interior solution of $e > 0$ the optimal wage equation is given in proposition 1:

Proposition 1: *Suppose firms and workers treat the reservation wage w_0 parametrically, then solution to the optimization problem (4.8) is as follows*:

(a) *For $0 < \beta < 1$ the optimal wage equation is*:

$$J = [1 - \eta_s]\frac{1-\beta}{\beta}\frac{R}{v'_w} \qquad (4.9)$$

where $\eta_s \equiv \partial \ln(r + s(e))/\partial \ln(p - w)$ – elasticity of the extended discount rate $r + s(e)$ with respect to the net flow profit $p - w$.

(b) *For the case $\beta = 0$ the optimal wage equation implies $\eta_s = 1$.*

Proof: The F.O.C. of the objective function (4.8) with respect to w is:

$$J = \left[-\frac{\partial J/\partial w}{\partial R/\partial w}\right]\frac{1-\beta}{\beta}R \qquad \text{where}$$

$$\frac{\partial J}{\partial w} = -\frac{1-\eta_s}{r + s(e)} \qquad \text{and} \qquad \frac{\partial R}{\partial w} = \frac{v'_w}{r + s(e)} \qquad \qquad \Diamond$$

There are a number of implications following from proposition 1. First consider the interior case $0 < \beta < 1$, notice that when workers are risk averse variable v'_w can be interpreted as a "shadow price" of an output unit for the worker. It measures the change in the worker utility value given a unit transfer of output from the firm to the worker. Therefore, equation (4.9) contains worker surplus value expressed in terms of the firm surplus: R/v'_w.

Second, variable η_s is an elasticity of the extended discount rate $r + s(e)$ with respect to the net flow profits $p - w$. Higher net profits $p - w$ imply a lower wage w, this means that workers exert less effort and the separation rate of such a match is higher. In the equilibrium with $0 < \beta < 1$ it should be true that $\eta_s < 1$. This means that optimal wages are set above the level maximizing the firm surplus J which is obtained for $\eta_s = 1$. The situation is depicted in figure 1. Point A in figure 1 corresponds to the case of a monopsonistic labour market with

$\beta = 0$ where firms maximize job surplus J with respect to wage given the optimal effort response by workers. The optimal wage in this case is denoted by $w(p, w_0)$. The bargaining power of a worker in a monopsonistic labour market is zero and therefore the wage takes form of a motivation premium providing incentives for the worker to exert the desired level of effort.
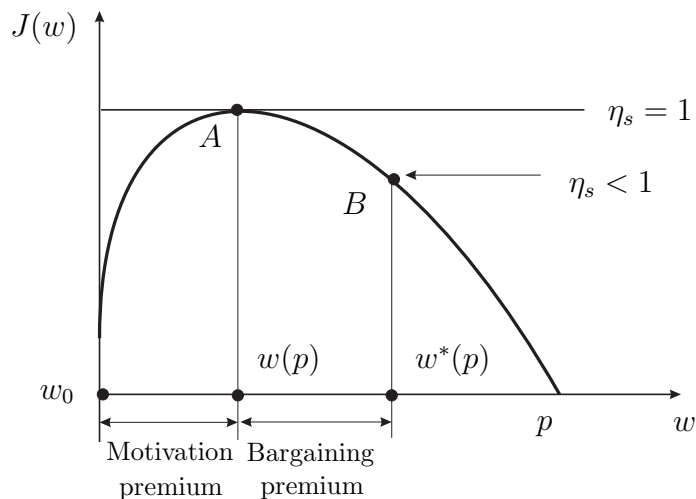

Figure 1: Optimal wage in search equilibrium with moral hazard

Point B in figure 1 corresponds to the more general case $0 < \beta < 1$ where wages are set according to (4.9) and $\eta_s < 1$. The optimal wage function in this case is $w^*(p, w_0)$. In the equilibrium firms pay the bargaining and the motivation premia, and therefore obtain a lower surplus value J compared to the situation with only one motivation premium in a monopsonistic labour market with search frictions. Properties of the search equilibrium with moral hazard and wage bargaining are summarized in proposition 2:

Proposition 2: *Let assumptions (A1) - (A2) be satisfied. Then search equilibrium with moral hazard and wage bargaining ($0 < \beta < 1$) is characterized by a tuple of variables $\{e, w, w_0, \theta\}$ satisfying the worker incentive compatibility constraint (4.3), the optimal wage equation (4.9), the free entry condition $V = 0$, defining variable θ, and the following reservation wage equation*

$$\upsilon(w_0) = \upsilon(z) + \lambda(\theta)R \qquad (4.10)$$

The necessary condition for the equilibrium existence is $p \geq w_0$.

The equilibrium unemployment rate is obtained from the differential equation $\dot{u} = s(e)(1-u) - \lambda(\theta)u = 0$, so that

$$u = \frac{s(e)}{s(e) + \lambda(\theta)}, \quad \text{where} \quad e = e(w, w_0) \tag{4.11}$$

Consider the border case $\beta = 0$ corresponding to the equilibrium with efficiency wages. Classical theory on efficiency wages (see Shapiro and Stiglitz (1984)) predicts that involuntary unemployment may appear in economies with unobservable effort, inducing firms to pay higher wages. High wages paid by firms in order to motivate their employees reduce the demand for labour and can explain the equilibrium unemployment. However, introduced in a model with search frictions, efficiency wages do not increase the number of unemployed. In contrast, paying a lower wage in the economy with search frictions and unobservable effort has two consequences: first, firms' profits fall due to a reduced output stability, so that job creation is less intensive, second, the separation rate of every match in the economy is higher. As a result, the lower job-finding rate $\lambda(\theta)$ and the higher job separation rate $s(e)$ add up to increase the equilibrium unemployment rate.

4.3 Comparative statics

This section considers the implications of an exogenous shift in the productivity parameter p for the optimal wage w. Results obtained in this section are consistent with the empirical findings listed below and will also prove useful for the case of heterogeneous jobs investigated in section 5. Consider the case $\beta = 0$, then equation $\eta_s = 1$ can be alternatively rewritten as

$$p = w + \tilde{\mu}_s/e'_w \tag{4.12}$$

This means that if assumptions (A1)-(A2) are satisfied the right-hand side of this equation is an increasing function of w so that equation $\eta_s = 1$ indirectly implies a positive relationship between the wage and the productivity: $\partial w(p, w_0)/\partial p > 0$. This means that a surplus maximizing firm with a higher productivity p would offer a higher wage w to the worker and enjoy an improved output stability. In this setting the moral hazard problem forces firms to leave rents to their workers in order to induce worker's effort.

The situation is similar for the case $0 < \beta < 1$. It can be shown that the right hand side of equation (4.9) is an increasing function of wage, since both functions $R/v'(w)$ and $\eta_s = \mu_s e'(e)(p-w)$ are increasing in w if assumptions (A1)-(A2) are satisfied. The situation is depicted in figure (2). The left hand side of equation (4.9) is firm's surplus and is a decreasing function of wage in the range $w^*(p, w_0) > w(p, w_0)$. Now consider an exogenous shift in the productivity parameter from p to p' for a given value of the reservation wage w_0. The firm surplus curve shifts outwards in the relevant range $w^*(p, w_0) > w(p, w_0)$, while the curve $R(w)(1 - \eta_s)/v'(w)$ shifts downwards, since variable η_s is increasing in p. Therefore it can be concluded, that wage is an increasing function of the productivity $\partial w^*(p, w_0)/\partial p > 0$:
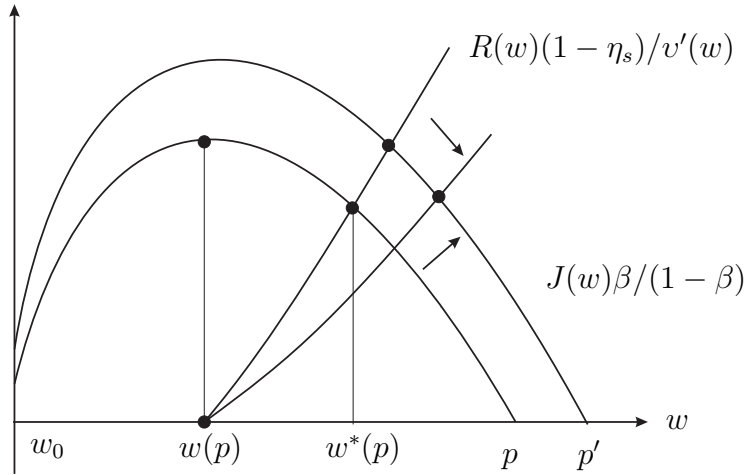

Figure 2: Optimal wage as a function of productivity

This result is consistent with the empirical findings. For example, Hildreth and Oswald (1997, 326) report that "the movements in the degree of firms' financial prosperity are eventually transmitted ... into movements in the pay levels of workers", which means that changes in profitability cause long-run changes in wages. Hildreth and Oswald (1997) estimated the elasticity of wages with respect to the firm's profitability to be approximately 0.02. At the same time Blanchflower, Oswald and Sanfey (1996) estimated the elasticity in the range between 0.02 to 0.05, which means that doubling profitability of a firm will result in up to a 5% increase in wages over several years.

Equation (4.12) also implies that the optimal wage $w(p, w_0)$ is an increasing function of its second argument (for a fixed value of p):

$$0 < \frac{\partial w(p, w_0)}{\partial w_0} < \frac{v'(w_0)}{v'(w)} \tag{4.13}$$

This means that a higher reservation wage w_0 forces firms to pay higher wages. Note further that if more productive firms offer higher wages to workers, meaning that workers' losses in case of a separation are higher, then (ex-ante identical) workers employed in more productive firms would exert more effort and productivity flows of those firms will be more stable on average. Formally

$$\frac{\partial e(w(p), w_0)}{\partial p} = \frac{\partial e(w(p), w_0)}{\partial w} \cdot \frac{\partial w(p)}{\partial p} > 0 \tag{4.14}$$

5 Heterogeneous productivity realizations

5.1 Stationary search equilibrium

Throughout this section every match of a worker and a firm is characterized by a match-specific productivity draw p from an exogenous productivity distribution $F(p)$ with the support in the range $[0, \bar{p}]$. This uncertainty about productivity is meant to reflect diversity of workers and jobs without modeling such heterogeneity explicitly. The productivity realization is simultaneously revealed to the worker and firm once the match has been formed. The matching process is random and undirected. This approach creates an ex-post productivity heterogeneity of jobs and is originally introduced in the study by Pissarides (2000). Also to simplify the representation only the case $\beta = 0$ is considered throughout this section.

In a situation when the productivity is revealed upon a match both unemployed workers and vacant jobs form expectations based on the productivity distribution $F(p)$. The Bellman equations for unemployed workers and vacant jobs adjusted to account for the ex-post productivity heterogeneity can be written as:

$$rU = v(z) + \lambda(\theta) \int \max{(W(p) - U, 0)} dF(p) \tag{5.1}$$

$$rV = -c + q(\theta) \int \max{(J(p) - V, 0)} dF(p) \tag{5.2}$$

Let p_0 denote the reservation productivity, i.e. the minimum productivity level at which the firm will employ the worker. Consider a firm with a productivity draw p_0. Offering the worker wage $w(p_0) > p_0$ will result in a negative profits flow of the firm, hence for the reservation productivity p_0 it must hold that $w(p_0) \leq p_0$ meaning that the firm surplus is nonnegative. At the same time offering the worker wage $w(p_0) < w_0$ will result in the offer rejection, hence for the reservation productivity p_0 it must also hold that $w(p_0) \geq w_0$ meaning that the worker surplus is nonnegative. In general in the equilibrium it must hold that $p_0 = w(p_0) = w_0$ guaranteeing that at the reservation productivity participation constraints are binding for both contracting parties. Here the first part of the equality comes from the formal definition of variable p_0 meaning that the firm surplus is zero at the reservation productivity: $J(p_0) = 0$ or $p_0 = w(p_0)$ from equation (4.7). The second part of the equality comes from the fact that if a firm is offering a wage as high as the productivity draw means that wage $w(p_0)$ is the lowest wage that unemployed workers would accept. This corresponds to the definition of the reservation wage so that $w(p_0) = w_0$. Note also that at the wage offer $w(p_0) = w_0$ the worker would exert zero effort $e(0) = 0$ and the separation rate attains its maximum value of $s(0) = \bar{s}$.

To derive conditions characterizing an equilibrium, consider first surplus equations for a worker and a firm given that both parties follow their optimal surplus maximizing strategies. Using the wage-setting equation (4.12) for the case $\beta = 0$ the firm's surplus can be rewritten as:

$$J(p, w_0) \quad = \quad \frac{p - w(p)}{r + s(e)} = \frac{\tilde{\mu}_s}{e'_w(r + s(e))} \tag{5.3}$$

where $e = e(w(p), w_0)$ and $e'_w = \partial e(w(p), w_0)/\partial w$.

Representation (5.3) allows to make a reference about the major properties of the firm's surplus. First of all, under assumptions (A1)-(A2) firm's surplus is an increasing function of productivity draw p meaning that firms with higher productivity draws attain higher match surplus values. This follows directly from the envelope theorem. On the one hand, a higher productivity draw implies a higher net flow profit $p - w(p)$. On the other hand, high productivity firms pay higher wages and their flow profits are more stable on average. Both effects contribute to the fact that firm surplus $J(p, w_0)$ is an increasing function of p. Additionally

it can be shown that under the same set of assumptions firm's surplus $J(p, w_0)$ is an increasing function of the net flow utility Δv and therefore a decreasing function of the reservation wage w_0 since

$$\frac{\partial \Delta v}{\partial w_0} = v'(w)\frac{\partial w(p, w_0)}{\partial w_0} - v'(w_0) < 0 \quad \Rightarrow \quad \frac{\partial J(p, w_0)}{\partial w_0} < 0 \qquad (5.4)$$

Consider an equilibrium characterized by a set of surplus equations (4.2), (4.7), (5.1), (5.2) and a free-entry condition $V = 0$. In the equilibrium it holds that $p_0 = w_0$ so that the free-entry condition can be expressed as:

$$\frac{c}{q(\theta)} = \int_{p_0} J(p, p_0)dF(p) \qquad (5.5)$$

Equation (5.5) is a job creation (JC) condition and describes a decreasing relationship between the market tightness parameter θ and the reservation productivity p_0, which means that a higher reservation productivity p_0 leads to less job creation. Intuitive explanation of this equation is that the expected vacancy cost on the left-hand side is equated to the expected job profit on the right-hand side (if both sides are divided by $(1 - F(p_0))$). Moreover the left-hand side of equation (5.5) is an increasing function of the market tightness parameter θ. This directly follows from the properties of the matching function described in section 3 and means that a higher value of θ makes it less probable to fill a vacancy and raises the expected vacancy cost. The right-hand side of equation (5.5) is a decreasing function of the reservation productivity p_0. This follows from the fact that a higher reservation productivity p_0 and hence a higher reservation wage w_0, first, reduces the acceptance probability of the worker $1 - F(p_0)$ and, second, forces firms to pay higher wages. Both effects translate into a lower firm surplus.

The surplus of an employed individual can be similarly expressed as:

$$R(p, w_0) = \frac{C'(e)\tilde{\mu}_s}{r + s(e)}$$

where $e = e(w(p), w_0)$. As proved in section 4 surplus $R(p, w_0)$ is a decreasing function of the reservation wage w_0. Then the equilibrium equation for the

reservation productivity can be written as:

$$v(p_0) = v(z) + \lambda(\theta) \int_{p_0} R(p, p_0) dF(p) \tag{5.6}$$

This equation describes an increasing relationship between the market tightness parameter θ and the reservation productivity p_0 (see figure 3). More vacancies increase the job-finding rate $\lambda(\theta)$ and make unemployed workers more choosy: their reservation wage $w_0 = p_0$ rises. Therefore a stationary equilibrium is fully characterized by a tuple of variables (θ, p_0, w, e) where equations (5.5)-(5.6) yield unique equilibrium values of θ and p_0 and equations (4.3) and (4.12) describe the optimal values of contract wage $w(p, p_0)$ and worker's effort $e(p, p_0)$ for every productivity draw $p \in [p_0, \bar{p}]$. The cross-sectional properties of this equilibrium are summarized in the following proposition.
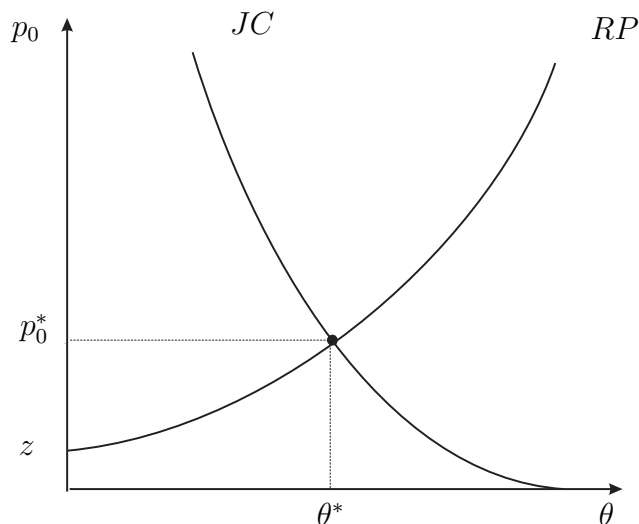

Figure 3: Equilibrium reservation productivity and market tightness

Proposition 3: *In a dynamic general equilibrium model with ex-post job heterogeneity, moral hazard and $\beta = 0$ described by a set of surplus equations (4.2), (4.7), (5.1), (5.2), a free-entry condition $V = 0$, and under a set of assumptions (A1)-(A2) there is a positive cross-sectional correlation of wages, productivity and job durations.*

Proof: A positive cross-sectional correlation of wages and productivity values is implied by equation (4.12). A positive correlation of wages and job durations follows from the worker incentive compatibility constraint (4.3). $\diamond$

The mutual reservation policy of workers and firms implies that the stationary productivity distribution is truncated at point p_0 and productivity draws $p < p_0$ are not observed in the equilibrium. This also has important implications for the stationary earnings distribution and the unemployment rate in the steady-state equilibrium analyzed in the following section.

5.2 Stationary earnings distribution

This section presents analysis of the equilibrium distributions of productivity and earnings in search equilibrium with heterogeneous separation rates. Let $G(p)$ denote the stationary productivity distribution, where $G(p_0) = 0$ and $G(\bar{p}) = 1$, and let $g(p)$ be the corresponding density function such that $g(p) > 0$ for $p_0 \leq p \leq \bar{p}$ and $g(p) = 0$ for $p < p_0$. Then the average job separation rate $s(p_0)$ in the general equilibrium can be written as:

$$s(p_0) = \int_{p_0}^{\bar{p}} s(p, p_0) dG(p), \quad \text{where} \quad s(p, p_0) = s(e(p, p_0)) \tag{5.7}$$

and $e(p, p_0) = e(w(p, p_0), p_0)$ is decreasing in p_0.

Consider a continuum of jobs with a productivity realization p or less and denote it with $E(p)$. In the stationary equilibrium an inflow of workers into this group should be equal to the outflow of workers from this group. The inflow of workers consists of those unemployed individuals drawing the productivity value in the range $[p_0; p]$, hence the inflow of workers is equal to $u\lambda(\theta)[F(p) - F(p_0)]$. The outflow of workers from this group consists of employed individuals who lose their jobs at rates $s(x, p_0) : x \in [p_0; p]$. Therefore the number of jobs with a productivity realization p or less ($E(p)$) obeys the following differential equation:

$$\dot{E}(p) = u\lambda(\theta)[F(p) - F(p_0)] - (1 - u) \int_{p_0}^{p} s(x, p_0) g(x) dx, \qquad p \in [p_0; \bar{p}] \tag{5.8}$$

In a stationary equilibrium $\dot{E}(p) = 0$, so that:

$$u\lambda(\theta)[F(p) - F(p_0)] = (1 - u) \int_{p_0}^{p} s(x, p_0) g(x) dx, \qquad p \in [p_0; \bar{p}] \tag{5.9}$$

Setting $p = \bar{p}$ rewrite equation (5.9) as follows:

$$u\lambda(\theta)[1 - F(p_0)] = (1 - u)s(p_0),\tag{5.10}$$

which is equivalent to the differential equation $\dot{u} = 0$, so that the stationary unemployment rate u is given by:

$$u = \frac{s(p_0)}{s(p_0) + \lambda(\theta)(1 - F(p_0))}\tag{5.11}$$

Inserting equation (5.11) for the stationary unemployment rate into (5.9) yields the following expression:

$$s(p_0)\frac{F(p) - F(p_0)}{1 - F(p_0)} = \int_{p_0}^{p} s(x, p_0)g(x)dx\tag{5.12}$$

In order to obtain the stationary productivity density function $g(p)$ differentiate equation (5.12) with respect to p and use the fact that $g(p) = 0$ for $p < p_0$:

$$g(p) = \frac{s(p_0)f(p)}{s(p, p_0)[1 - F(p_0)]} \quad \text{and} \quad G(p) = \frac{s(p_0)}{[1 - F(p_0)]}\int_{p_0}^{p} f(x)/s(x, p_0)dx$$

for $p \in [p_0; \bar{p}]$.

There are generally two effects driving the transformation of the productivity draw distribution $F(p)$ into the stationary productivity distribution $G(p)$. See figure 4. Both transformations strengthen the fact that the stationary distribution $G(w)$ dominates the initial distribution $F(w)$ ($G(w) \leq F(w)$). First of all, note that for a constant exogenous separation rate $s = s(p, p_0) = s(p_0)$ the density and the distribution functions $g(p)$ and $G(p)$ can be rewritten in the following way:

$$g(p) = \frac{f(p)}{[1 - F(p_0)]} \qquad G(w) = \frac{F(p) - F(p_0)}{1 - F(p_0)}\tag{5.13}$$

The first transformation of $f(p)$ is explained by the reservation policy of workers and implies that the productivity density function $g(p)$ is truncated at $p = p_0$.

The second transformation of $f(p)$ can be explained by differences in job durations $1/s(p, p_0)$ of jobs with different productivity values p. Note that the less productive jobs are less stable and are destroyed at higher intensity rates $s(p, p_0)$
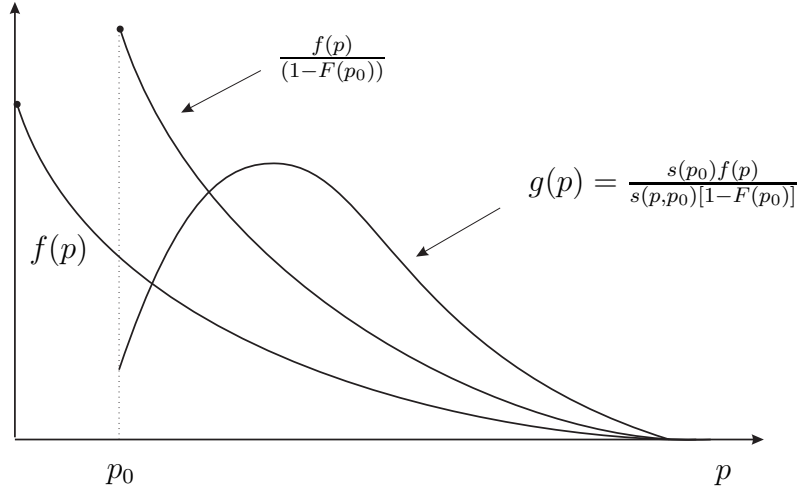
Figure 4: Equilibrium transformation of the productivity distribution

than the more productive jobs. So that jobs with productivity values p such that $s(p, p_0) > s(p_0)$ are destroyed faster than the average and jobs with productivity values p such that $s(p, p_0) < s(p_0)$ are destroyed more slowly than the job with an average separation rate $s(p_0)$.

Now the only parameter to be defined in equations for $g(p)$ and $G(p)$ is the average separation rate in the stationary equilibrium $s(p_0)$. To obtain this parameter value recall that $g(p)$ is a density function of the stationary productivity distribution and therefore should fulfill the following property of the density function:

$$
\begin{aligned}
1 &= \int_{p_0}^{\bar{p}} g(p)dp \\
&= \frac{s(p_0)}{[1 - F(p_0)]} \int_{p_0}^{\bar{p}} f(p)/s(p, p_0)dw
\end{aligned}
$$

This allows to obtain expression for the average separation rate $s(p_0)$:

$$
s(p_0) = \frac{[1 - F(p_0)]}{H(p_0)}, \tag{5.14}
$$

where $H(p_0) = \int_{p_0}^{\bar{p}} f(p)/s(p, p_0)dp$ and is used to simplify the notation.

Note that because $H(p_0)$ is a strictly decreasing function of p_0 the effect of the reservation productivity p_0 on $s(p_0)$ is ambiguous. The positive part of this effect is explained by the fact that a higher reservation productivity $p_0 = w_0$ raises

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the reference income point for the worker and increases thereafter the match separation rate $s(p, p_0)$. This effect translates into a lower stability of jobs and a higher separation rate for every match. The negative part of the effect corresponds to the fact that a higher p_0 reduces the number of successful matches in the economy and therefore has a negative effect on the average separation rate.

The final expression for the stationary unemployment rate can be obtained from equation (6.2) by substituting the expression for the average separation rate:

$$u = \frac{s(p_0)}{s(p_0) + \lambda(\theta)(1 - F(p_0))} = \frac{1}{1 + \lambda(\theta)H(p_0)}$$

This equation is a version of the Beveridge curve describing a negative relationship between unemployment and vacancies for a given value p_0. The structure of this equation shows that a higher reservation productivity p_0 shifts the Beveridge curve outwards due to a lower value of $H(p_0)$. However, an increase in the reservation productivity p_0 is accompanied by a change of the market tightness θ (mutual dynamics of the two variables is presented in figure 3). In general an effect of a higher p_0 on the stationary unemployment rate is ambiguous. Nevertheless, if an original shock to the economy, causing the higher reservation productivity p_0, was such that the market tightness parameter θ decreases (it becomes relatively easier to find a job), then the labour market is characterized by an additional downward movement along the Beveridge curve which unambiguously increases the stationary unemployment rate u in the economy. This sequence of events, for example, takes place in case of a higher unemployment benefit parameter z resulting in a higher income of the unemployed, a higher reservation productivity and a higher stationary unemployment rate in the economy.

Stationary productivity distribution is an important characteristic of the model, however, one may be interested in finding an implied stationary wage (earnings) distribution, first of all for the reason that wage is an observed variable and the model-implied theoretical distribution of wages may then be compared with its empirical counterpart.

Let $k(w)$ denote the probability density of an equilibrium wage distribution such that $k(w) > 0$ for $w \in [w_0, w(\bar{p})]$ and $k(w) = 0$ otherwise. Wages w are defined on the basis of a match-specific productivity draw p. This describes wage as a

function of p: $w(p)$, which is implicitly given in equation (4.12) for the case $\beta = 0$. Using an expression for the probability density of a function of a random variable yields the following equation for the stationary earnings distribution $k(w)$:

$$k(w) \;\; = \;\; \frac{1}{\partial w(p)/\partial p} \cdot g\left(w + \frac{\tilde{\mu}_s}{e'_w}\right) \tag{5.15}$$

$$\text{where} \quad \frac{1}{\partial w(p)/\partial p} \;\; = \;\; 1 + \tilde{\mu}'_s - \tilde{\mu}_s \frac{e''_w}{(e'_w)^2} > 1 \quad \text{for} \quad w \in [w_0, w(\bar{p})]$$

Equation (5.15) shows, that the shape of the wage density function $k(w)$ is defined by the properties of the wage function $w(p)$ and the stationary productivity density function $g(p)$. As shown in section 5.2 the density $g(p)$ of the stationary productivity distribution is likely to have an interior mode on the support $[p_0, \bar{p}]$. In this case if wage is a concave function of productivity, so that $(\partial w/\partial p)^{-1}$ is an increasing function of wage, the wage density function $k(w)$ is likely to have a stronger right shift than the productivity density function $g(p)$.

6 Efficiency and unemployment insurance

6.1 Constrained efficiency

This section considers efficiency properties as well as the optimal unemployment insurance in a decentralized equilibrium with risk averse workers and on-the-job moral hazard. Equilibrium unemployment is an inherent component of all search models and therefore the maximum welfare is never obtained since unemployment is a waste of labour resources. Nevertheless the welfare maximization problem of the social planner can be stated in terms of restricted efficiency, meaning that the social planner is subject to the same matching constraints as market participants.

The first question raised in this section is whether the individual decisions of market participants in a decentralized equilibrium, in particular the equilibrium wage, effort and the market tightness, maximize the social welfare. To simplify the exposition only the case of identical productivity p across jobs is considered throughout this section. The social planner is maximizing the present value of the expected utility of workers net of the effort costs. The welfare function is

then given by:

$$\max_{w,\theta} \int_0^\infty e^{-rt}\Big[uv(z) + (1-u)(v(w) - C(e))\Big]dt, \quad \text{where} \quad e = e(w, w_0)$$

The choice of the social planner is restricted by the resource constraint, meaning that net profits obtained from production $(1-u)(p-w)$ are distributed to cover the costs of job creation $cu\theta = cv$:

$$cu\theta = (1-u)(p-w) \tag{6.1}$$

The unemployment rate differential equation is:

$$\dot{u} = (1-u)s(e) - u\lambda(\theta) \tag{6.2}$$

Note also that if workers were risk neutral the objective function of the social planner would simplify to the expected value of output net of the effort and job creation costs $uz + (1-u)(p - C(e)) - cu\theta$, which is often used in theoretical literature, see Pissarides (2000).

First order conditions of the stated optimization problem extend the result of Hosios (1990), who shows that search externalities resulting from the dependence of the transition probabilities $\lambda(\theta)$ and $q(\theta)$ on the market tightness are not likely to be internalized by the Nash surplus equation, unless a particular value of the bargaining power is assumed. A similar finding is documented in lemma 3 for the case of risk averse workers and on-the-job moral hazard problem:

Lemma 3: *Search equilibrium with risk averse workers, moral hazard and wage bargaining is constrained efficient if* $\beta = \eta_q$, *where*

$$\eta_q = -\frac{\partial q(\theta)}{\partial \theta}\frac{\theta}{q(\theta)} \quad \text{– elasticity of the job filling rate } q(\theta) \tag{6.3}$$

Proof: Appendix III.

6.2 Optimal unemployment insurance

This subsection considers the optimal unemployment insurance in search equilibrium with risk averse workers and moral hazard. As noted in Holmlund (1998):

*"The economics of UI has first and foremost been concerned with positive analysis of the effects of various UI policies. Much less attention has been devoted to the normative issue: what is the **optimal** level of UI benefits in an economy with risk-averse workers?"* (p.130).

Baily (1978) shows that risk aversion of workers implies optimality of the full unemployment insurance $w = z$ in the absence of informational asymmetries. To see this consider the following optimization problem of the social planner, where the unemployment insurance is now a choice variable and the moral hazard problem is omitted from the problem (so that $s(e) = s = const$):

$$\max_{w,z,\theta} \int_0^\infty e^{-rt} \Big[uv(z) + (1-u)v(w) \Big] dt$$

The planner's resource constraint is then modified to include the new type of expenses, namely unemployment benefits uz:

$$cu\theta + uz = (1-u)(y-w) \tag{6.4}$$

Solution to this optimization problem is summarized in proposition 4:

Proposition 4: *The optimal unemployment insurance policy in search equilibrium with risk averse workers implies full unemployment insurance $z = w$, so that the worker net rent R is equal to zero, the optimal wage equation is $J = K + Z$, and the optimal market tightness is given by:*

$$K = \frac{1-\eta_q}{\eta_q} Z, \quad \text{where} \quad K \equiv \frac{c}{q(\theta)} \quad Z \equiv \frac{z}{\lambda(\theta)} \tag{6.5}$$

Proof: Appendix IV.

The optimal wage equation $J = K + Z$ follows directly from the planner's resource constraint and is expressed in terms of the steady state surplus values, where $K + Z$ are expected costs of maintaining one job and providing unemploy-

ment insurance to one worker. These costs are financed by firms profits with a corresponding surplus value J. The costs further are split between the firms and the workers according to the proportion $(1 - \eta_q)/\eta_q$.

Provision of full unemployment insurance is not supported by the empirical evidence, so that the basic search model has been extended in a number of relevant directions. Baily (1978) shows that unemployed workers do not have incentives to search if the full unemployment insurance if provided. This result persists even if private savings of workers are introduced into the model. The explanation for that is the fact that unemployment insurance is a sort of contingent saving, the payment obtains only if the adverse event (job loss) is realized, unlike the precautionary saving which is independent of the event occurrence. Further Shavell and Weiss (1979) in a general framework and Fredriksson and Holmlund (2001) in a search and matching framework show that the optimal unemployment insurance should be decreasing over the unemployment spell in order to motivate unemployed workers to search. In contrast to this, Chetty (2008) shows that 60% of the increase in unemployment durations caused by UI benefits is due to a liquidity effect rather than distortions on marginal incentives to search. This is due to the fact that increases in benefits have much larger effects on durations for liquidity-constrained households.

In this paper a different aspect of the effect of unemployment insurance on the decisions of labour market participants is analyzed. It is the on-the-job effort level workers exert which is dependent on the unemployment insurance. To see this consider the worker's incentive compatibility constraint (4.3). As shown in lemma 1 worker's effort is an increasing function of the net utility flow $v(w) - v(w_0)$, where w_0 is the workers reservation wage obtained as $w_0 = v^{-1}(rU)$. The reservation utility rU is an increasing function of unemployment insurance z, so that worker's effort is negatively related to z. Intuitively a lower job rent R implies a lower punishment for the worker in case of losing the job and therefore reduces worker's incentives to exert effort. The problem of the social planner in this case can be written as:

$$\max_{w,z,\theta} \int_0^\infty e^{-rt} \Big[uv(z) + (1-u)(v(w) - C(e)) \Big] dt, \quad \text{where} \quad e = e(w, w_0)$$

subject to the resource constraint (6.4) and the differential equation for unemployment (6.2). Results are summarized in proposition 5 below.

Proposition 5: *The optimal unemployment insurance policy in search equilibrium with risk averse workers and unobserved effort implies partial unemployment insurance $z < w$, the optimal market tightness θ is obtained from equation $J = K + Z$ and further*

(a.) *the optimal replacement ratio z/w is implicitly given by:*

$$\frac{v'(w)}{v'(z)} = 1 - \eta_s \tag{6.6}$$

(b.) *the optimal surplus split is given by:*

$$K v'(z) = \frac{1 - \eta_q}{\eta_q}(R + Z v'(z)) \tag{6.7}$$

Proof: Appendix V.

Equation (6.6) shows, that full unemployment insurance is suboptimal if asymmetric information concerning worker's on-the-job effort is taken into account. In this setting the social planner is facing a trade-off between providing full unemployment insurance and no effort versus the absence of unemployment insurance with maximum worker's effort. As a result the partial unemployment insurance is optimal: $z < w$. This policy reduces expenses of the social planner for vacancies and unemployment benefits since workers exert positive effort and jobs become more stable. This result is supported in the theoretical literature, for example Brown, Orszag and Snower (2006) in a different framework with taxes find that:

"Lower taxes (uncompensated costs of the employed) and lower transfers (uncompensated benefits of the unemployed) mean greater incentives for job search and work effort. The resulting rise in hiring rates and reduction in firing rates lead to a fall in unemployment. This in turn broadens the tax base and shrinks the number of people requiring support, leading to further reductions in tax rates and unemployment benefit expenditures." (p.19)

In order to obtain an approximated expression for the optimal replacement ratio I use the first order Taylor approximation of function $v'(w)$ around the point z:

$$v'(w) \simeq v'(z) + v''(z)[w - z] \tag{6.8}$$

so that the inverse replacement ratio w/z can be written as:

$$\frac{w}{z} \simeq 1 + \frac{\eta_s}{\rho}, \quad \text{where} \quad \rho = -\frac{v''(z)}{v'(z)}z \tag{6.9}$$

Here ρ is the relative risk aversion coefficient of the unemployed, so that higher risk aversion implies a higher optimal value of the replacement ratio z/w. At the same time note that the elasticity variable η_s shows the sensitivity of the separation rate with respect to the net flow profits $p - w$ and therefore also the sensitivity of the separation rate with respect to the flow wage w. If the dependence of the match separation rate on worker's effort is not recognized, then $\eta_s = 0$ and so the social planner will optimally set $z = w$, which is the case described in proposition 4. Otherwise a higher sensitivity of the separation rate implies a higher marginal gain of providing effort and therefore has a negative effect on the replacement ratio z/w.

7   Conclusions

This paper explores the question of unilateral asymmetric information and endogenous separation rates in a general equilibrium model of labour market characterized by search frictions and matching. The model proposed in the paper combines key features of the efficiency wage theory with the search and matching theory in a spirit of Mortensen and Pissarides (1994). The main (unobserved) variable in the model is worker's effort chosen in response to the contract wage. The model predicts that a higher wage yields a higher job surplus to the worker and consequently results in a higher level of worker's effort.

The key structural assumption of the model is that the distribution of productivity shocks is linked to the worker's effort level in such a way that higher effort raises expected duration of the productivity flow. In this situation a higher value of job surplus imposes a higher penalty for the worker in case of a separation, which necessarily follows after a negative productivity shock. Therefore, in accordance with the predictions of efficiency wage theory, workers employed at higher

wages exert more effort on-the-job. In this setting, different from a shirking specification of efficiency wages, a higher effort level translates into a lower separation probability but does not prevent a separation. Additionally, this model structure guarantees a decreasing relationship between workers' performance and their dismissal probabilities, documented in the empirical literature (see Bishop (1990) and Kwon (2005)).

Wages are determined endogenously in the model using the concept of Nash bargaining generalized to include the worker's incentive compatibility constraint. Efficiency wages are then obtained as a special case for the zero bargaining power parameter. In this setup firms are facing a trade-off between the net flow profit of the job and its separation rate. Similar to the model by Mortensen and Pissarides (1994) the wage dispersion in the economy is a result of an exogenous productivity distribution which implicitly captures the firms and workers heterogeneity. The model predicts that in a more productive match the firm will share the rent with the worker inducing him to exert more effort. Here the rent split resulting from bargaining is amplified by the internal agency problems within the match. This means that the moral hazard problem and contract incompleteness force firms to share the rents even if the bargaining power of workers is equal to zero. Overall, the equilibrium is characterized by a positive correlation of wages and productivity and a negative correlation of wages and job separation rates.

Internal incentive problems between workers and firms combined with a non-degenerate productivity draw distribution create an equilibrium labour market situation with heterogeneous job separation rates. The resulting heterogeneity is such that the more productive jobs are also more stable in expectation. The increasing job stability is interacted with an assumed declining productivity draw distribution, which serves to highlight an increasing scarcity of the more productive jobs. An interaction of job scarcity and its stability is likely to produce the hump-shaped density functions of stationary productivity and wage distributions. This result is consistent with the reported properties of observed earnings and productivity distributions.

One of the final remarks concerns the relationship between the reservation productivity and the stationary unemployment rate. The model predicts that a higher reservation productivity (which itself may result from a higher unemployment

benefit) affects unemployment in a number of ways. First, due to a lower value of the flow utility workers reduce their working effort, which results in a higher probability of negative shocks for all jobs and a higher average separation rate in the economy. Second, a higher reservation productivity in the economy translates into a lower number of successful job matches; this implies a lower job-finding rate and a lower average separation rate (survivorship bias). Nevertheless, due to a mutual neutralization of the last two effects, the model predicts that the total effect of a higher reservation productivity on the stationary unemployment rate is unambiguously positive implying a higher stationary unemployment rate in the equilibrium.

Finally, this paper considers the question of optimal unemployment insurance in an economy with risk averse agents and on-the-job moral hazard. Partial unemployment insurance is optimal in this economy where the social planner is facing a trade-off between incentives provision and unemployment insurance. This paper also shows that the optimal replacement ratio is increasing in the workers risk aversion and decreasing in the elasticity of the separation rate.

8 Appendix

APPENDIX I: Proof of lemma 1.

Rewrite equation (4.3) using a definition $\Delta v \equiv v(w) - v(w_0)$ to obtain:

$$\Delta v = C(e) - \frac{C'(e)}{s'(e)}(r + s(e)) \tag{I-1}$$

Differentiate equation (I-1) with respect to the flow utility surplus Δv to obtain:

$$\frac{1}{\partial e/\partial \Delta v} = -\left[\frac{C''(e)s'(e) - C'(e)s''(e)}{(s'(e))^2}\right](r + s(e)) > 0 \quad \text{if} \quad s''(e) > 0 \tag{I-2}$$

Therefore if $s''(e) > 0$ effort is an increasing function of Δv.

APPENDIX II: Proof of lemma 2.

Using the definition of $\tilde{\mu}_s$ rewrite equation (I-2) in the following way:

$$\frac{1}{\partial e/\partial \Delta v} = C''(e)\tilde{\mu}_s + C'(e)[1 + \tilde{\mu}'_s] \tag{II-1}$$

where $\tilde{\mu}'_s = \partial\tilde{\mu}_s/\partial e$. Then from equation II-1 it follows that the curvature of the optimal effort function $e(\Delta v)$, in particular the sign of the second derivative of effort $e''(\Delta v)$ with respect to the net flow utility Δv, is defined by the sign of the following expression which is the first order derivative of the right-hand side of equation (II-1):

$$-[C''(e) + C'''(e)\tilde{\mu}_s + 2C''(e)\tilde{\mu}'_s + C'(e)\tilde{\mu}''_s] \tag{II-2}$$

Under the assumption $C'''(e) \geq 0$ expression (II-2) is weakly negative if $\tilde{\mu}''_s \geq 0$ and $\tilde{\mu}'_s \geq 0$. This means that conditions (A2) are sufficient for the effort function to be weakly concave in the net flow utility: $e''(\Delta v) \leq 0$.

APPENDIX III: Proof of lemma 3

The current value Hamiltonian for the social planner problem is:

$$
\begin{aligned}
H &= uv(z) + (1-u)(v(w) - C(e)) + \gamma[u\lambda(\theta) - (1-u)s(e)] \\
&+ \alpha((1-u)(p-w) - cu\theta) \quad \text{where} \quad e = e(w, w_0)
\end{aligned}
$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u. The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma \implies \alpha J + R = \gamma \tag{III-1}$$

since

$$R = \frac{v(w) - v(z) - C(e)}{r + s(e) + \lambda(\theta)} \quad \text{and} \quad J = \frac{p - w + c\theta}{r + s(e) + \lambda(\theta)}$$

Maximizing H with respect to w and θ yields:

$$\frac{\partial H}{\partial w} = 0 \implies v'(w) - \alpha[1 + Js'(e)e'(w)] = 0 \tag{III-2}$$

$$\frac{\partial H}{\partial \theta} = 0 \implies \gamma\lambda'(\theta) = \alpha c \tag{III-3}$$

since from the worker incentive compatibility constraint it follows that $C'(e) = -Rs'(e)$.

$$\frac{\partial H}{\partial \theta} = 0 \implies \gamma\lambda'(\theta) = \alpha c \tag{III-4}$$

Expression $Js'(e)e'(w) < 0$ can be alternatively rewritten as $-\eta_s$. Then it follows from equations (III-1)-(III-3) that the optimal social planner solution is characterized by the following surplus splitting equation:

$$R = \gamma - \alpha J = \frac{\alpha J}{1 - \eta_q} - \alpha J \qquad \text{(III-5)}$$

$$Jv'(w) = [1 - \eta_s]\frac{1 - \eta_q}{\eta_q}R \qquad \text{(III-6)}$$

APPENDIX IV: Proof of proposition 4

The current value Hamiltonian for the social planner problem is:

$$
\begin{aligned}
H &= uv(z) + (1 - u)v(w) + \gamma[u\lambda(\theta) - (1 - u)s] \\
&+ \alpha((1 - u)(y - w) - cu\theta - uz)
\end{aligned}
$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u. The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma \quad \Rightarrow \quad \alpha(K + Z) + R = \gamma \qquad \text{(IV-1)}$$

In the steady state the resource constraint of the social planner implies: $J = K + Z$, then equation (IV-1) can be written as $\alpha J + R = \gamma$. Maximizing H with respect to w, z and θ yields:

$$\frac{\partial H}{\partial w} = 0 \quad \Rightarrow \quad v'(w) - \alpha = 0 \qquad \text{(IV-2)}$$

$$\frac{\partial H}{\partial z} = 0 \quad \Rightarrow \quad v'(z) - \alpha = 0 \qquad \text{(IV-3)}$$

$$\frac{\partial H}{\partial \theta} = 0 \quad \Rightarrow \quad \gamma\lambda'(\theta) = \alpha c \qquad \text{(IV-4)}$$

From equations (IV-2)-(IV-3) it follows that $w = z$ so that $R = 0$, while the surplus splitting equation takes the following form:

$$\alpha Z = \gamma - \alpha K = \frac{\alpha K}{1 - \eta_q} - \alpha K \qquad \text{(IV-5)}$$

$$K = \frac{1 - \eta_q}{\eta_q}Z \qquad \text{(IV-6)}$$

APPENDIX V: Proof of proposition 5

The current value Hamiltonian for the social planner problem is:

$$
\begin{aligned}
H &= uv(z) + (1-u)(v(w) - C(e)) + \gamma[u\lambda(\theta) - (1-u)s(e)] \\
&+ \alpha((1-u)(y-w) - cu\theta - uz), \quad \text{where} \quad e = e(w, w_0)
\end{aligned}
$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u. The optimal social planner solution must satisfy:

$$
\frac{\partial H}{\partial u} = -r\gamma \quad \Rightarrow \quad \alpha(K + Z) + R = \gamma \tag{V-1}
$$

In the steady state the resource constraint of the social planner implies: $J = K + Z$, then equation (IV-1) can be written as $\alpha J + R = \gamma$. Maximizing H with respect to w, z and θ yields:

$$
\frac{\partial H}{\partial w} = 0 \quad \Rightarrow \quad \alpha = v'(w) - e'_w(C'(e) + \gamma s'(e)) \tag{V-2}
$$

$$
\frac{\partial H}{\partial z} = 0 \quad \Rightarrow \quad \alpha = v'(z) \tag{V-3}
$$

$$
\frac{\partial H}{\partial \theta} = 0 \quad \Rightarrow \quad \gamma\lambda'(\theta) = \alpha c \tag{V-4}
$$

Workers incentive compatibility constraint can be written as $Rs'(e) = -C'(e)$, then equations (V-2)-(V-3) imply

$$
\frac{v'(w)}{v'(z)} = 1 + Js'(e)e'(w) = 1 - \eta_s \tag{V-5}
$$

and the surplus splitting equation (V-4) becomes:

$$
Kv'(z) = \frac{1 - \eta_q}{\eta_q}(R + Zv'(z)) \tag{V-6}
$$

9 References

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