Efficient Liability Rules for Multi-Party Accidents with Moral Hazard *

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Abstract

The economic analysis of tort law is extended to multi-party accidents with unobservable actions. Due to the requirement of no punitive damages, the problem resembles a team production problem. It is shown that asymmetry in the agents' impact on the stochastic damage function can be exploited to improve ex ante incentives. This implies departures from the proportional rule, based on the statistical information contained in the circumstances of the accident. If a noisy monitoring technology is introduced, then monitoring can add enough stochastic identifiability among injurers so as to restore efficiency.

Keywords : liability rules, multiple tortfeasors, punitive damages, team production

JEL Classification : D82, G22, K13, K32

1. Introduction

Multi-party accidents are in theory and practice an important problem for the design of efficient liability rules. By multi-party accidents (or, synonymously, by multiple causation) we mean that harm has been generated by the actions of several agents whose contributions are non-separable. The present paper extends the economic analysis of tort law to multiple causation with unobservable actions.[1]

There is a substantial body of literature showing that multiple causation does not constitute a serious obstacle to efficiency as long as the court is fully informed about the circumstances of an accident.[2] The case of a bilateral accident where the injurer's and the victim's activities both have an impact on the expected damage is the best illustration.[3] Granted that the liability rule will sufficiently penalize the parties for any deviation from the optimal levels of care, efficiency will be obtained. It is well known that negligence rules as well as rules of strict liability with contributory negligence will satisfy this condition and implement due care standards at the first best level.

The most detailed contribution on multi-party accidents is due to Lewis Kornhauser and Richard Revesz. In a series of articles, they undertake a comprehensive analysis of the problem, by focusing on detailed comparisons of different sharing rules (and their efficiency impacts). Their papers are primarily motivated by the disposal of hazardous waste and questions raised by the Comprehensive Environmental Response, Compensation and Liability Act (CERCLA).[4] They provide important taxonomies of different rules of apportionment (Kornhauser and Revesz [1989])[5] and extend their analysis to potentially insolvent actors (Kornhauser and Revesz [1990]) and the effects of settlements (Kornhauser and Revesz [1994a],[1994b]).

By contrast, only very few contributions have examined problems of *asymmetric information.* These articles all focus on situations of hidden information (adverse selection) with respect to avoidance costs. Building on the well-known results for the single injurer-case, Kornhauser and Revesz also analyze the situation of uncertainty about the optimal care

[1] We will henceforth call injurers' actions care levels. In formal terms, our analysis applies equally well to care levels as to activity levels, as long as there is a problem of observability.

[2] See e.g. Landes and Posner [1980], Landes and Posner [1987, 199 ff.], Shavell [1987, 164-167], Kornhauser and Revesz [1989].

[3] It is straightforward to show that bilateral accidents are a special case of multicausal damages for which the same logic applies.

[4] 42 U.S.C. §§9601-9675 [1982] & Supp.IV 1986.

[5] By distinguishing, in particular, between rules of (a) full liability versus partial liability, (b) unitary share versus fractional share and (c) fixed share versus proportional share.

levels.[6] EMONS and SOBEL [1991] and FEESS and HEGE [1997] show that efficiency can considerably be increased by applying Bayesian mechanisms.

The purpose of this paper is to analyze multi-party accidents with *unobservable activity or care levels (moral hazard)*. Our paper is, to the best of our knowledge, the first analysis explicitly addressing the moral hazard problem.[7] As in Kornhauser and Revesz, our work is motivated by environmental problems like the disposal of hazardous waste. Imagine a situation in which several agents deposit their wastes on a single landfill. Furthermore, the time lag between the dumping and the occurrence of environmental harm may be long. It may then be impossible or at least uneconomically expensive to figure out which manufacturer has dumped what amount of hazardous waste.[8] This analysis is not only relevant for environmental problems. Other typical examples are mass collisions on highways where actions can neither be observed ex ante nor fully reconstructed ex post. Also, law suits against partnerships (e.g. against consultants or accountants) can be interpreted as multi-party "accidents".

Designing an efficient liability rule for multi-party accidents would be a trivial task even with unobservable actions if punitive damages were not excluded. If there is a single injurer, exclusion of punitive damages means that the compensation must not exceed the harm. Extended to multiple injurers, it must mean that none of the injurers and no subgroup of injurers pay punitive damages. To see that it would be easy indeed to design an efficient liability rule including punitive damages, let us recall the basic idea of strict liability in the single-injurer case: under strict liability, the injurer will always pay for the total damage, which obviously leads her to choose the efficient care levels. This is because by maximizing her individual utility, the injurer faces the same loss function that any decision maker maximizing social welfare would face. So the injurer's optimal marginal utility will coincide with the optimal marginal social welfare. Translating this idea into a multi-injurer framework obviously suggests the following liability rule which FINSINGER and PAULY [1990] call the "double liability rule": each injurer should individually pay for the total damage. Clearly, the resulting care levels will be efficient. The beauty and simplicity of such an arrangement

[6] See KORNHAUSER and REVESZ [1989, 863-870.]

[7] Kornhauser and Revesz point out that their liability rule would be able to cope with moral hazard because damages are apportioned independent of action levels, but they do not investigate whether their liability rule is the best possible in a moral hazard environment. This question is investigated in the current paper.

[8] Mechanisms where only aggregate emissions are observable are examined in MERAN and SCHWALBE [1987], XEPAPADEAS [1991] and KRITIKOS [1993]. Their basic idea is to use penalties whenever aggregate emissions exceed the predefined critical level. Thus, payments are based on emissions, not, as in our model, on harm.

has led many economists to argue (indirectly) in favor of relaxing the no punitive damages condition.[9]

However, we do not follow this approach since in tort systems all over the world, punitive damages are restricted to cases of reckless conduct, e.g. drunken driving.[10] Exceptions seem to be most common in the United States.[11] Several US States are now imposing levies on punitive damages awards. We are not aware of tendencies towards relinquishing no punitive damages outside the United States. And even in the US, there is now a clear tendency to limit the use of punitive damages, heralded by recent product liability law suits.[12] The present paper therefore strictly excludes the use of punitive damages.

The fundamental dilemma can then be described as follows.[13] On the one hand, holding each injurer liable for the total damage must inevitably lead to punitive damages: with n injurers and total harm x, punitive damages of $(n-1)\cdot x$ will be collected. On the other hand, if each injurer i pays only a fraction of the total harm, then care levels will be inefficiently low since each injurer will only take a part of total harm into consideration.

The problem of designing a non-punitive and efficient liability rule becomes transparent if multi-party accidents are interpreted as team production problems or partnership problems. In a team production problem, several "partners" join forces to obtain an output described by a joint output function, but neither they nor a principal or social planner can observe the actions. So rewards and punishments can only be levied in reaction to the observable joint output, not conditional on the (unobservable) individual contributions. In addition, rewards and punishments are restricted by *budget-balancing ex post* which is a condition commanding that they sum up to the total joint output. But now interpret the joint output function as the joint damage function describing the prior accident risks of multiple injurers, and interpret the output as the actual accident loss (in monetary terms) which occurs. Then the analogy should be clear. More precisely, because accidents are outcomes which are uncertain ex ante, a team production model with a stochastic joint output function must be considered.

Holmström [1982] started the literature on team production problems by analyzing partnerships among symmetric and risk-neutral partners (symmetry means here that the impact on the joint production function is symmetric). He shows a fundamental non-existence

[9] See e.g. Polinsky and Che [1991].

[10] See e.g. Shavell [1987, 146].

[11] However, Landes and Posner [1987, 304] report that even in product liability lawsuits, punitive damages have been used in less than five percent of all successful suits. Also, there seems to be no correlation with multiple causation. Other studies find a more frequent use of punitive damages.

[12] See the 1996 Supreme Court decision in *BMW of North America, Inc. v. Gore* 116 S.Ct.1589, 134 L.Ed. 2d 809 (1996).

[13] See e.g. Shavell [1987, 177].

result: an efficient sharing rule respecting budget-balancing ex post does not exist. Now budget-balancing ex post is a strictly weaker condition than no punitive damages (under budget-balancing ex post, subsidies to some partners are admissible while under punitive damages, they must be ruled out). So if a balanced liability rule does not exist, then a rule without punitive damages will not exist, either. Since then, various modifications and extensions of Holmström's model have been suggested which give rise to more encouraging results. For our purposes, the single most important of these contributions is due to LEGROS and MATSUSHIMA [1991]. They consider the stochastic team production problem and introduce asymmetry among agents. They derive conditions for the existence of efficient sharing rules and show that asymmetry among agents alleviates the task considerably. Also, the conditions found for existence are not only sufficient, but also necessary.

The contribution of Legros and Matsushima is important for the present paper because in multi-party accidents, defendants have frequently an asymmetric impact on the joint damage function. To see this, consider again the example of hazardous wastes dumping. It is fairly unlikely that, on a waste site, the size or the composition of all injurers' deposits should be absolutely homogeneous, and that the court should have no clue as to the likely differences among injurers. For example, just the easily observable industrial activities of the defendants give substantial, albeit imprecise, hints as to the heterogeneity of possible deposits. It seems natural to assume that the court gets at least some inaccurate information of the potential waste disposal of the various injurers, i.e. which injurer has access to what sort of hazardous waste. It remains however an important scope of uncertainty about the quantities (the "activity levels") of toxic wastes contained in various injurers' deposits (hence unobservability of activities). This is the situation we have in mind in the present paper. Thus, we investigate under what circumstances efficient liability rules for multi-party accidents can be found, given that punitive damages are excluded and injurers have (most likely) an asymmetric impact on the accident.

Our analysis restricts attention exclusively to strict liability meaning that the total loss is always paid in full by the injurers.[14] An important reason for this is that it is not at all clear how negligence standards can be defined if care levels are unobservable.

We propose a liability rule based on the idea that asymmetries between injurers can be exploited. Loosely speaking, asymmetry means that deviations of the injurers from their first best care levels have a distinct impact on the probability distribution function over outcomes. We show that the power of incentives can be increased in this case and that the liability rule may be efficient if each injurer has to pay a disproportionate share of those

[14]Strict liability is the equivalent to budget-balancing ex post in the team production literature.

outcomes which were more likely her fault than the fault of others, in the sense that if the defendant in question was less careful than she should have been, than the probability for the accident outcome has risen more than it would have if another injurer had been less careful. By applying this idea in an optimal way, we find, roughly speaking, that efficient liability rules exist as long as the asymmetry across injurers is sufficiently large.

This finding should be contrasted with the most prominent interpretation of the strict liability rule in multi-injurer accidents, the proportional rule. The proportional rule or constant splitting rule assumes that each injurer i pays a constant fraction α_i of the losses. Shavell argues that this is "the natural analogue to strict liability in the single-injurer context"(Shavell [1987, 177]). Note that the constant splitting rule would be mostly inefficient in our model. This is a direct consequence of the team production literature. An important insight of our paper is precisely that, as long as agents are asymmetric, other liability rules can do better than the constant splitting rule. Note that there seem to be no juridical obstacles to our proposal to use all available information and to let the division of damages vary with the level of harm. For example, subsequent claims for contribution can be based on negligence (and they frequently are).

Shavell's main motivation to propose the proportional rule is that it can cope with decentralized information: the court does not need information on the joint damage function or individual avoidance costs. While decentralization certainly has many merits, the results of this paper point to the weak side of decentralization: non-decentralized rules may be efficient in cases where decentralized are not. While this is well-known for the case of perfect information, the innovation of the present paper is to give a clear account of the trade-off between decentralization and efficiency in the context of asymmetric information.

In practical terms, our analysis implies that departures from the constant splitting rule are recommendable[15] whenever injurers could have had an asymmetric impact on the accident probability. The recommended rule for such departures is the following: only those injurers who, by their lack of care, could have increased the prior probability for the accident should pay damages. If some injurers had more discretion to increase the prior probability than others, than they should assume a disproportionately larger share (and possibly all) of the losses. In general, one cannot say whether they should pay for all of the losses or only for a larger share of them because this depends on the specific joint damage function or probability distribution in question.[16] Both cases are possible.

[15]Recall that the constant splitting rule cannot be efficient for multi-party accidents. This explains why we say digressions are recommended.

[16]We do not confine the analysis to particular probability distributions because we want to derive results

For the purposes of tort law reform, the recommended liability rule could be phrased as follows. The liability rule is based on the proportional rule as the default, but gives the court the discretion to make an individual injurer pay up to the full amount of the losses rather than a proportional share. Such departure require that (1) such departures be only permitted for those injurers whose lack of care could have increased the prior probability of the actual accident and that (2) injurers pay a higher share than those falling in the first category only if their lack of care could have increased the accident probability more than the lack of care of the other injurers falling in the first category. This gives the court sufficient discretion to apportion damages in an efficient way, depending on the circumstances of the accident (including the joint damage function) and yet permit for a rational agent to correctly anticipate the apportionment for each of the possible accidents she considers when choosing her care level. This illustration should convince the reader that our liability rule would satisfy the rule of law, i.e. the stipulation that the liability rule be universally applicable and well-known to agents prior to the accident. The idea that a fixed liability rule leaves room for contingent apportionment is not really different from the practice under the established liability rules: under the negligence rule e.g., the concrete meaning of "due care" is contingent on the case and must be fixed in each verdict.

As an extension, this paper addresses also intermediate cases between pure moral hazard and perfect information. While pure moral hazard, i.e. the assumption that the injurers' action are completely unobservable,[17] is a useful benchmark, it is certainly not a complete description of the possibilities. In environmental liability problems for example, some information about the actions of injurers is often available. The level of emissions can in principle be monitored, and many emissions (like point source air pollution) are monitored at the source. Surveillance of potentially harmful activities is costly, however. So naturally, monitoring tends to be incomplete, or left to random checks. Thus, in the context of accident law, it seems natural to extend the analysis from moral hazard to include also endogenous information acquisition. The most interesting extension appears to look at some form of "noisy monitoring": the more precise and complete the surveillance, the higher the cost. For example, assume that the environmental agency chooses the precision of the monitoring technology and that monitoring costs are increasing in the precision of the signal. The paper shows that information on a possible deviation should be used swiftly to further increase the power of incentives, by making the apportionment dependent on the signal obtained from

which are valid in the broadest possible way.

[17] This does not exclude that the court can retrieve some information on the likely tortfeasor indirectly, through the statistical posterior that can be constructed from the actual accident. This is what happens in our model.

monitoring. If this is done, then very little additional information may be needed to obtain an efficient liability rule.

Thus, the upshot of our analysis is a strict liability rule which departs from the proportional rule by using all the information available. This result could be interpreted as exhibiting some similarity with a strict liability rule with contributory negligence. One has to be careful with this statement because strictly speaking, it is as much unclear how to apply strict liability with contributory negligence if the care levels are not observable as it is unclear how to apply the negligence rule. Unobservability does not exclude, however, that the court uses all statistical information on the behavior of the agents that can be obtained from the circumstances of the accident. Recall that this is precisely the fundamental idea of our liability rule: the incentives to choose the first best care levels stem from the fact that the agents are fully aware that the apportionment of damages will depend on the outcome, because the latter conveys valuable statistical information on the behavior of the injurers. It is in this sense that our rule contains elements of contributory negligence. These elements become even more important if the court can, albeit noisily, gather information on the action levels of the injurers. Therefore, the recommendation derived from this paper is that the liability rule should increasingly emphasize the contributory negligence component as the precision of the statistical information on the care levels grows.[18] In the logic of our analysis, it is no surprise that this rule is the one which puts as much incentive power as possible on any indication of negligence. The crucial remaining difference to the standard interpretation of strict liability with contributory negligence is that information on negligence is probabilistic information rather than hard facts.

It should be noted that by focusing on strict liability, we ignore the possibility to exempt some outcomes from liability, as the negligence rule does. This possibility could potentially be useful in the context of multiple causation. Looking at this possibility would raise a host of interesting questions, for example whether low- or high-damage outcomes should be exempted, whether there should be full or partial exemption and whether the exempted outcomes form a convex set. Addressing these questions would require a different and much less general model; this is beyond the scope of the present paper.

We emphasize that our analysis relies on the assumption that no other information problems besides moral hazard are present. This means two assumptions in particular: first, the

[18] Because agents know, when choosing care levels, that the apportionment will depend on the information contained in the outcome, our liability rule will remind the reader of revelation or Bayesian mechanisms. Note, however, the difference: revelation mechanisms typically address adverse selection problems, whereas team problems deal with moral hazard. For the importance of revelation mechanisms for accident law, addressing the problem of unobservable avoidance costs, see Feess and Hege [1997].

avoidance cost functions of all injurers are known (no adverse selection). Second, we tacitly assume that the court as well as the various injurers know and understand the stochastic damage function. So problems of information with respect to the links between causes and effects are ruled out. In environmental liability problems, for example, a major problem is often to establish sufficiently hard evidence (even in stochastic terms) between emissions and their subsequent impact on the environment.

Compared to the literature on the stochastic team production problem[19], our analysis has two main innovative features. First, the application to liability rules requires to look at state-contingent restrictions. That is to say, no punitive damages implies that the restriction on aggregate liability payments is different for every single accident outcome. This is because total contributions always add up to the total loss and therefore vary with the level of losses. By contrast, the restrictions in Legros and Matsushima are constant. Second, the analysis of endogenous information acquisition (noisy monitoring) in team production is new. On a minor level, our paper presents a novel explanation for the sufficiency part of the result. This approach is very helpful as a tool to partially characterize the efficient rule: only those injurers should be held liable who could have increased the probability of the actual accident (compared to the first best allocation). A final innovation is our limit inefficiency result: as the number of injurers increases, the existence of an efficient rule becomes more and more precarious.

The paper is organized as follows: section 2 presents the model. Section 3 derives the existence result for non-punitive liability rules. In section 4, a graphical intuition is given and the applicable knowledge for the construction of efficient liability rules is collected. A limit inefficiency result is briefly motivated in section 5. Noisy monitoring is introduced in section 6. We conclude in section 7.

2. The model

There is a finite set of potential injurers, $N = \{1, \ldots, n\}$, each controlling a separate care level $a_i \in A_i$.[20] A_i is a finite set with cardinality T_i: $A_i = \{a_i^1, \ldots, a_i^{T_i}\}$. Throughout, all a_i are unobservable. We assume that $a_i^j > 0, \forall a_i \in A_i$, i.e. the court knows for sure that each

[19]Besides Legros and Matsushima [1991], notably Legros and Matthews [1993] and Fudenberg, Levine and Maskin [1994].

[20]Risk-increasing activities are often differentiated according to whether the adjudication can be conditioned on them (level of care) or not (activity level). We are exclusively concerned with unobservable actions which obviously do not lend themselves to conditioning, and hence refer in our model only to a single activity variable.

injurer has been responsible for some harmful action.[21] While we are looking for liability rules that implement the first best strategy profile in pure strategies, we will have to take into account the possibility that injurers choose deviations in mixed strategies. Let M_i be the set of mixed strategies of player i, with a typical element denoted as $\sigma_i \in M_i$. Define $A = \times_{i \in N} A_i$, and define $M = \times_{i \in N} M_i$.

The joint strategy profile $a \in A$ induces a stochastic outcome, an aggregate damage or harm in monetary terms, whose realizations are drawn from the finite set $X = \{x_1, \ldots, x_h, ...\}$. Actions and harm can be thought of as being multidimensional.[22] Each $a \in A$ induces a probability measure $p(a) = (p_1(a), \ldots, p_h(a), ...)$ over X, where $p_h(a)$ denotes the probability that x_h is realized when a is the strategy profile. Analogously, we define $p(\sigma)$ as the probability distribution function when the mixed strategy profile σ is played. Let $Ex(a) = \sum_h p_h(a)x$ be the expected damage if a is the action profile. $p(a)$ completely describes the joint damage function of the agents. This function is assumed to be non-separable, i.e. the expected damage $Ex(a)$ cannot be rewritten as the sum of individual damage functions.

$l_i(x_h)$ denotes the contribution owed by injurer i if damage x_h is realized. Define a *liability rule* as a function $l(x_h)$: $X \to \mathbb{R}^n$, where

$$l(x_h) = \begin{pmatrix} l_1(x_h) \\ \vdots \\ l_n(x_h) \end{pmatrix}$$

Thus, a liability rule is a vector determining a contingent contribution for each injurer i and for each outcome x_h. So far, a liability rule is nothing but a transfer mechanism. We can safely ignore victims as agents in this model. Following standard practice in Law and Economics, whenever victims can influence $p(a)$, they will be incorporated as agents.

The timing can be represented as follows:

- Stage 1: a social planner proposes a liability rule $l(x_h)$ trying to maximize social welfare.
- Stage 2: action levels are chosen simultaneously by the injurers.
- Stage 3: the stochastic damage is realized and payments are made.

[21] The latter assumption excludes that an injurer could claim not having been involved in the accident at all. Note that it would be difficult to hold an injurer responsible if his participation in the accident cannot be established. The assumption implies, in technical terms, that the standard individual rationality condition familiar from agency theory can be left aside. Our results would become more restrictive if individual rationality were a concern, as we show in a note which is available from the authors. Formal results on individual rationality are also provided by Legros and Matsushima [1991].

[22] Mathematically, this is unimportant by the finiteness of A and X.

All agents are assumed to have VNM-utility functions which are quasi-linear in money. Injurer i's utility is given by:

$$v_i(a) = u_i(a_i) - \sum_{x_h \in X} p_h(a) l_i(x_h)$$

where $u_i(a_i)$ is a (concave) function reflecting the direct utility from the care level, and where $v_i(a)$ denotes i's expected utility if the strategy profile a is played. Analogously, $v_i(\sigma)$ denotes i's expected utility according to the mixed strategy profile σ.

In Law and Economics, the use of social welfare functions with equal weights is standard. Thus, social welfare is given by adding up individual utilities of injurers and victims (damages awarded to plaintiffs reduced by the harm in monetary terms):

$$\begin{aligned} W(a) &= \textstyle\sum_{x_h \in X} \{\sum_{i \in N} [(u_i(a_i) - p_h(a) l_i(x_h)] - p_h(a)[x_h - \sum_{i \in N} l_i(x_h)]\} \\ &= \textstyle\sum_i u_i(a_i) - Ex(a) \end{aligned}$$

The desired allocation is *first best* if $a^* \in \arg\max_a W(a)$. A liability rule is *efficient* if it implements a first best allocation as a subgame perfect equilibrium. A liability rule is *non-punitive* if (i) each injurer pays a non-negative contribution, $l_i(x_h) \geq 0 \;\; \forall i, \forall x_h$, and if (ii) $\sum_i l_i(x_h) \leq x_h, \;\; \forall x_h$. Note that this implies that no individual injurer and no subgroup pay more than x_h. A liability rule is *strict* if moreover the sum of contributions of the injurers add up to the total harm x_h, $\sum_i l_i(x_h) = x_h, \;\; \forall x_h$. Thus, a strict and non-punitive liability rule is simply a division of x_h among the injurers such that no injurer receives a subsidy. Throughout, we restrict attention to strict liability rules in this sense.

3. Efficient and non-punitive rules of strict liability

In order to state a concise condition for the existence of a non-punitive and strict liability rule, it will be helpful to introduce the following definition of deviation n-tuples Ψ which are *sets* of strategy profiles:

Definition 1: *An n-tuple of strategy profiles Ψ is called a* deviation n-tuple*, if it has the form:* $\Psi \equiv ((a^*_{-i} \backslash \sigma_i))_{i \in N}$.

A deviation n-tuple Ψ consists of n strategy profiles; each of these n strategy profiles is assigned to one of the n players, on a one-for-one basis. In the strategy profile assigned to player i, all players except i choose their first best care level. Agent i chooses a deviation to the care level σ_i. In other words, a deviation n-tuple regroups a set of strategy profiles such that exactly one agent deviates at a time, and a deviation is contained for every agent. Note that there is a one-to-one correspondence between the set of mixed strategies M and the set of deviation n-tuples.[23] Abusing notation, we refer to M also as the set of deviation n-tuples. Recall that $p(\sigma)$ is the probability density function induced by $\sigma \in M$.

We present and interpret the first main result of this paper:

PROPOSITION 1: *There exists a strict liability rule which is non-punitive and efficient if and only if:*

$$\forall \Psi \in M : \quad \sum_{i \in N} [u_i(\sigma_i) - u_i(a_i^*)] \leq \sum_h x_h \max_{k \in N} p_h(a_{-k}^* \backslash \sigma_k) - Ex(a^*). \tag{1}$$

Proof: See the Appendix. □

Condition (1) in words: for any deviation n-tuple, the sum of utility gains must be smaller than the difference between the damage outcomes weighted with the maximum probability induced by any of the deviations and the expected damage in equilibrium.

4. Interpretation and characterization

The first subsection provides an intuition for the necessity part of the result. The intuition for the sufficiency part is more intricate and relegated to the next subsection. The last subsection collects the insights on the characterization of efficient liability rules.

4.1. Interpretation: necessity

As mentioned above, we now present an intuition for the necessity part of Condition (1). Recall that there are infinitely many deviation n-tuples Ψ and consider one arbitrarily chosen Ψ. Now define

$$\sum_h \left(p_h(a_{-i}^* \backslash \sigma_i) - p_h(a^*) \right) l_i(x_h)$$

This expression gives the expected increase in the liability payment of agent i if she deviates from the first best allocation to the specific mixed strategy σ_i included in Ψ and will therefore be called the *punishment potential* for agent i. A necessary condition to deter any given

[23] Recall that mixed strategies are only needed for the formal existence result; the efficient allocation which is implemented is in pure strategies.

deviation is that the punishment potential induced by this deviation is at least as large as the utility gain $u_i(\sigma_i) - u_i(a_i^*)$. Moreover, define $\sum_i \sum_h \left(p_h(a^*_{-i}\backslash\sigma_i) - p_h(a^*)\right) l_i(x_h)$ as the *aggregate punishment potential* for all agents. Obviously, to deter each deviation included in Ψ, the aggregate punishment potential must at least be as large as the aggregate utility gain $\sum_{i\in N}[u_i(\sigma_i) - u_i(a_i^*)]$.

To understand the necessity part, it is helpful to look first at deviation n-tuples of a specific sort which we call *symmetric simulations.* A formal definition is as follows:

Definition 2: *A deviation n-tuple* Ψ *is called* a symmetric simulation *if* $\forall i, j \in N$, *and for all strategy profiles* $(a^*_{-i}\backslash\sigma_i)$, $(a^*_{-j}\backslash\sigma_j)$ *in* Ψ, $p_h(a^*_{-i}\backslash\sigma_i) = p_h(a^*_{-j}\backslash\sigma_j)$.

In plain words, a symmetric simulation describes deviations from the first best allocation for agents i and j (where i and j are chosen arbitrarily) such that each damage occurs with exactly the same probability. That is to say, a symmetric simulation is a deviation n-tuple where each of the n strategy profiles induces the same probability distribution function over X. So with respect to symmetric simulations, all agents appear to be equally likely as having deviated from the desired action level. There is no way that the court identifies or excludes a specific agent as the deviator, not even in stochastic terms.

Consider an example with only two injurers and three possible levels of harm, $x_0 = 0$, $x_1 > 0$ and $x_2 > 0$. A symmetric simulation means that the increases in the probabilities for x_1 and x_2 are exactly the same if injurer 1 respectively injurer 2 deviates from the first best allocation to her mixed strategy included in Ψ. So $p_h(a^*_{-1}\backslash\sigma_1) = p_h(a^*_{-2}\backslash\sigma_2)$ for both levels of harm $x_h = x_1$ or $x_h = x_2$. This means that the aggregate punishment potential is independent of the division of the damages. So nothing can be gained by an intelligent division of x_1 and x_2 between the two injurers.

Thus, if Ψ is a symmetric simulation, if in other words the probability distribution function over X is identical for the deviations of all agents, then the term $\sum_h x_h \max_{k\in N} p_h(a^*_{-k}\backslash\sigma_k)$ is identical for all agents. Therefore, for symmetric deviation n-tuples, Condition (1) can simply be written as

$$\sum_{i\in N}[u_i(\sigma_i) - u_i(a_i^*)] \leq Ex(a^*_{-j}\backslash\sigma_j) - Ex(a^*) \text{ for arbitrary } j. \tag{2}$$

It is then clear that Proposition 1 defines in fact a necessary condition for the existence of a non-punitive liability rule: to deter each individual agent, a punishment potential in the amount of her utility gain is needed; aggregating over all agents leads to (2) for that deviation n-tuple.

Next, we consider deviation n-tuples Ψ which are not symmetric simulations. For our example with two injurers, this means that the increase in the probabilities p_1 for damage x_1 and p_2 for x_2 are not identical if injurer 1 or 2 deviates to σ_i. Suppose that p_1 (p_2) is larger if injurer 1 (2) deviates and that both damages are divided equally between the injurers. The aggregate punishment potential can then be increased by raising $l_i(x_1)$ for injurer 1 and $l_i(x_2)$ for injurer 2.

This is exactly the idea underlying Condition (1): Consider a deviation n-tuple where the inequality (1) is just binding. The right hand side of the inequality can be called the *maximum punishment potential*, as every damage is weighted with the maximum of the probability differences that are induced by the deviation n-tuple.[24] Obviously, the actual aggregate punishment potential will have to be (weakly) smaller than the maximum punishment potential as every agent will calculate the increase in her expected damages by using the probability differences that she induces on her own. If inequality (1) is violated, then the maximum punishment potential will be smaller than the aggregate utility gain, and there must be at least one agent whose deviation is profitable. Therefore, Proposition 1 states a necessary condition. If there is a symmetric simulation, then conditions (1) and (2) are identical, i.e. the maximum punishment potential and the aggregate punishment potential are the same.

Note that our intuition for the necessity part does by no means imply sufficiency since we have not shown that a single liability rule can be found that is simultaneously good enough for *all* possible deviation n-tuples (see section 4). The Appendix contains a formal proof demonstrating that a non-punitive liability rule which implements the efficient pure strategy profile a^* exists in fact precisely when Condition (1) is satisfied.

4.2. Interpretation: sufficiency

In this section, we propose a graphical intuition which helps to understand why condition (1) is not only necessary, but also sufficient. The graphical approach should also help to understand heuristically how to approach the *characterization* of efficient rules in the next subsection. In our graphical illustration, there are only two possible outcomes, the "low" damage x_1 and the "high" damage x_2. *Figure 1* depicts this situation. The expected damage is precisely the plane connecting the monetary damage levels of each of the corners of this

[24]Given non-punitiveness, damages have a lower and an upper bound: $0 \leq l_i(x_h) \leq x_h$, $\forall i, x_h$. This implies that the maximum punishment potential is indeed identical to the maximum aggregate punishment potential, which can be achieved only when those agents pay damages which achieve the maximum probability difference. This relation does not hold when non-punitiveness is not required.

graph: $Ex = x_2$ if $p_2 = 1$, etc. In our two-dimensional picture, this plane is the line connecting x_1 and x_2.

It is convenient to imagine that agents choose directly *induced probabilities* rather than actions. To understand what we mean by induced probabilities, recall that every strategy profile a induces a certain probability density function $p(a)$. In our example of just two outcomes, the probability distribution function $p(a)$ can be completely represented with just the probability p_2 of outcome x_2 to occur, since $p_1 = 1 - p_2$. Thus, we can imagine that agents pick directly a probability value p_2 rather than actions. In this sense, consider first the efficient induced probability $p_2(a^*)$ induced by the first best allocation a^*. Through deviations, agent i can induce probabilities $p_2 = p_2(a^*_{-i} \backslash a_i)$ where $a_i \in A_i$. For the remainder of this section, we will stick to this convention of analyzing induced probabilities.

The essence of our explanation consists in comparing utility gains and increases in expected damage levels of individual deviations. Starting from the target allocation $p_2(a^*)$, we can draw the individual utility gains associated with a deviation to p_2. The fact that agents can mix their strategies implies that the set of utility gains which are attainable from deviations is convex. In other words, the frontier of attainable utility gains $u_i(p_2)$ is a concave function over the range of $p_2 \in [0, 1]$. Adding up concave functions will again yield a concave function. Hence the aggregate utility gain under any deviation profile will be concave, too (see $\sum_i u_i$ in *Figure 1*).

[insert Figure 1 about here.]

We consider first the case of symmetric simulations. As explained above, condition (2) will then replace condition (1). That is to say, $\sum_i [u_i(\sigma_i) - u_i(a_i^*)] \leq Ex(a^*_{-i} \backslash \sigma_i) - Ex(a^*)$ is required. In terms of *Figure 1*, this means that the slope of the concave surface $\sum_i u_i(p_2)$ must be smaller than the slope of the line $Ex(p_2)$.

The intuition can now be completed by showing that liability assignments $l_i(p_2)$ exist which deter every possible deviation p_2 and which can be “financed” out of the available punishment potential $Ex(p_2)$. Such a liability assignment will be, in terms of our graphical representation, a line connecting the individual liability shares $l_i(x_1)$ and $l_i(x_2)$. This is depicted as the $l_i(p_2)$-line in *Figure 1.* By virtue of the concavity of the $u_i(p_2)$-curve, agent i's possible deviations are all successfully deterred as long as the $l_i(p_2)$-line can be drawn as non-intersecting to $u_i(p_2)$. This is depicted in *Figure 1* by the tangential line $l_i(p_2)$. Note that the function of utility gains $u_i(p_2)$ has a smaller slope in all directions (in other words, it is decreasing faster and increasing less rapidly) than the $l_i(p_2)$-line of punishments. Next, one can draw the function of utility gains $u_2(p_2)$ for agent 2 on top of the punishment plane

$l_1(p_2)$ of agent 1, as shown in *Figure 2*. The sum of the two functions $l_1(p_2)$ and $u_2(p_2)$ will again be concave. In that fashion, punishment planes $l_i(p_2)$ can be constructed in such a way that the surfaces of utility gains of all successive agents are separated. If the right $l_i(p_2)$-planes are chosen, then the minimal slope of the punishment plane for the last agent is given by the maximum of the aggregate utility gain, $\max_{p_2} \sum_i u_i(p_2)$. The quintessential point is the following: If the condition of Proposition 1 is met, then the slope of the sum of these separating planes will be smaller than the slope of the $Ex(p_2)$-plane of expected damage.[25]

We turn next to the case where a symmetric simulation does not exist. This means that, starting from the optimal allocation $p_2(a^*)$, we are now looking at directions of deviations which are not feasible for all agents. In other words, $p_2(a^*)$ is not an interior point of the set of probability distributions that all agents can induce. For at least some agents, it must be a boundary point of their sets of deviations. Suppose now an increase of the induced probability p_2 is feasible for just a subset $K \subset N$ of the agents. The available punishment potential is given by the slope of the $Ex(p_2)$-plane in the direction of an increase of p_2. If this slope is steeper than the aggregate utility gains of the agents in K (those who can move in that direction), then $l_i(p_2)$-planes can be found which deter anyone from inducing an increase in p_2 and whose slope adds to less than the slope of $Ex(p_2)$. Thus, the feasible deviations can be deterred if:

$$\sum_{i \in K} [u_i(p_2) - u_i(p_2(a^*))] \leq Ex(p_2) - Ex(p_2(a^*)) \tag{3}$$

Note that the summation is done only over those agents i which are contained in the set K (i.e. those who can increase p_2). Corresponding conditions can be formulated for all directions of deviations.

Now look at any deviation n-tuple Ψ which is not a symmetric simulation. The punishment potential available to deter any of the deviations in Ψ depends on whether the deviation in question induces a probability distribution which is a symmetric simulation or not. If it is, then the punishment potential for this deviation is given by condition (2). If it is not, then only those agents compete for the assignment of a positive share out of the losses x_2 which can actually increase p_2. In the extreme case where no deviation is a symmetric simulation, condition (1) states nothing else than that an efficient rule exists as long as the punishment potential for each deviation is sufficient to deter those who can actually produce this deviation.

[25]The reader familiar with elementary topology will recognize a separation theorem behind our argument.

[insert Figure 2 about here.]

Now we can put the pieces together. Condition (1) states that the aggregate utility gain of Ψ must be smaller than the weighted sum of outcomes x_h, each weighted with the maximum induced probability in Ψ. So this statement combines our reasoning on symmetric simulations and on absence of symmetric simulations in a joint statement. Recall that for symmetric simulations, condition (1) collapses into (2). Otherwise, the induced probability $\max_i p_h(a^*_{-i} \backslash \sigma_i)$ determines the punishment potential. When does this give an aggregate punishment potential which is actually higher than the one in condition (2)? This is only the case if there are asymmetries among injurers, in the sense that a probability increase cannot be induced by all injurers.

4.3. Characterization of the efficient rule

What insights can be obtained for the *characterization* of efficient liability rules that we can derive from our interpretation? This question is of course of great importance for the practitioner. While we cannot offer a complete characterization of the efficient rule within the general framework of the model (this would require a parametric specification of the joint damage function), the interpretation provides some valuable insights.

The important insight is that one should first test for the presence of *symmetric simulations*. That is, one should ask whether, starting from the desired allocation a^*, all injurers could have increased the probability of the actual accident x_h or not. The symmetric simulation test is accepted if all of them could have done so. This is the case if, in the space of induced probability distributions, moving into the direction of the actual accident is a symmetric simulation. The insight for the actual form of the liability assignment is in this case: apportion in such a way that individual contributions follow the marginal utilities in the critical symmetric deviation n-tuple, i.e. the deviation n-tuple with minimal slackness according to inequality (2).

The symmetric simulation test is rejected if increasing the induced probability of the actual accident was not feasible for all injurers. In this case, it follows that those injurers who cannot move into the direction of the actual accident should not pay any contributions. *Only those injurers who can increase the probability of the actual accident should pay damages.* To see why, recall that our interpretation hints that the result is best interpreted in terms of induced probabilities: the necessary and sufficient condition is that for each change of induced probabilities, the available punishment potential must be larger than the aggregate utility gain. But note that allocating the punishment potential exclusively to those injurers who could have increased the probability of the actual accident is a sufficient deterrence.

A second remark refers to the actual calculation of the splitting rule. This remark is more a caveat. One might be led to think that determining the corresponding minimum punishment potential $l_i(x_h)$ that must be allotted to i in order to deter all those pure strategy deviations that increase the probability of x_h is sufficient. However, this depends in turn on the splitting rule of other outcomes. Moreover, it must be certain that the $l_i(x_h)$ is not so big that it actually violates i's incentive constraints against deviations which decrease the induced probability of x_h. So the calculation is not trivial and can in general not be separated from those of other outcomes.

5. Limit inefficiency

There is an important implication of our results in section 3 concerning the number of injurers. They imply a *"limit inefficiency" result*, meaning that the larger the economy, the less likely it is that the mechanism yields efficiency.

We will argue informally, using a standard technique which is the technique of replicating individuals. Consider condition (1). To demonstrate the limit inefficiency result in a simple manner, assume that agents $1, ..., n$ are replicated repeatedly. Obviously, while the left-hand side of condition (1) grows proportionally in the number of replications, the right-hand side remains constant. So, for any damage function, there must be a finite number of replications where inefficiency is inevitable.

It follows that, in the presence of moral hazard, tort law can efficiently resolve environmental problems only if the number of injurers is small or if the asymmetry among injurers grows in proportion to their number. For global environmental problems like the depletion of the atmosphere, which undoubtedly have elements of multiple causation and moral hazard to them, liability rules are of little help, at least if unobservability of actions is a real concern.

6. Noisy monitoring

We have argued that allocations are much more likely to be implementable if lucrative deviations are stochastically identifiable. That begs for the following complementary analysis: suppose the court can obtain additional information on the likely action levels that injurers have chosen. For example, suppose an environmental agency requires that potentially harmful emissions be monitored. Taken for granted that monitoring is imprecise, when is it nonetheless sufficient for a stochastic identification of the injurers? How would the feasible allocations improve, and how should the environmental agency monitor?

We describe the monitoring technology as follows. The monitoring technology delivers a noisy signal θ_i on the care levels a_i. The choice variable of the environmental agency is the quality by which i's action is monitored. We suppose that this quality is determined by the expenditure c_i on monitoring of i. We assume that the relationship between monitoring expenditure and signal quality is monotonic. We can express the signal quality directly by c_i. Let $f(\theta_i|a_i, c_i)$ denote the density of θ_i if a_i is chosen and c_i is the quality of monitoring. The monitoring costs are assumed to influence $f(\theta_i|a_i, c_i)$ in the following way: if $c_i^1 < c_i^2$ then $f(\theta_i|a_i, c_i^2)$ is a mean-preserving spread of $f(\theta_i|a_i, c_i^1)$. We assume that the distribution of θ_i is independent from the distribution of x_h, for a given action profile a. Also, θ_i is independently distributed from the signals obtained for other injurers, i.e. θ_i does only depend on i's action a_i and on c_i.

The vector $\theta = (\theta_1, ..., \theta_n)$ denotes the signal on all agents and $c = (c_1, ..., c_n)$ denotes the vector of qualities of observation for each agent. Let $f(\theta|a, c)$ denote the joint probability of θ being the joint observation on all agents if a is the profile of action levels and c is chosen. Note that independence of the signals implies that $f(\theta|a, c) = \Pi_i f(\theta_i|a_i, c_i)$. In particular, if agent i deviates from the efficient action level, then she will affect only the probability of the signal reporting on her own action choice, not those of other injurers: we can write
$f(\theta|a_{-i}^* \backslash a_i, c) = f(\theta_i|a_i, c_i) \cdot f(\theta_{-i}|a_{-i}^*, c_{-i})$.

The information available to the court is now a joint realization of x_h and of the signal θ; both are informative on the likely actions of injurers. The liability rule $l(x_h, \theta)$ should therefore depend on both outcome x_h and signals θ. The timing of the model is now as follows:

- Stage 1: the environmental agency chooses c and the court proposes a liability rule $l(x_h, \theta)$.
- Stage 2: action levels are chosen simultaneously by the injurers.
- Stage 3: the stochastic damage is realized.
- Stage 4: the monitoring signal θ is observed by the environmental agency and delivered to the court.
- Stage 5: payments are made according to $l(x_h, \theta)$.

The social welfare function will now take into account monitoring costs:

$$W(a,c) = \sum_i u_i(a_i) - Ex(a) - c$$

Both the court and the environmental agency maximize social welfare. The optimal choice of c and $l(x_h, \theta)$ will then solve the following problem:

$$\begin{array}{lll} \max_{l(x_h,\theta),c} W(a,c) & = \sum_i u_i(a_i) - Ex(a) - c & \\ \text{s.t.} & l(x_h,\theta) \text{ is strict and non-punitive} & \\ & a_i = \arg\max_{a_i}\{u_i(a_i) - E[l_i(x_h,\theta)]\} & \end{array} \tag{4}$$

We should add that our result below would not change if decision-making of the environmental agency and the court were sequential rather than simultaneous; nor does it matter which of the two institutions decides first. Suppose then that monitoring level c and liability rule $l(x_h, \theta)$ are chosen so as to maximize problem (4). We denote the optimal monitoring level by c^*. Our result demonstrates the condition under which the solution will be an efficient liability rule (i.e. it implements a^*, the efficient allocation defined in Section 2):

Proposition 2: *The liability rule will be efficient if and only if the signal quality c^* is sufficiently informative, i.e. if the condition:*

$$\begin{array}{l} \sum_{i\in N}[u_i(\sigma_i) - u_i(a_i^*)] \leq \\ \sum_h x_h \ \max_{k\in N}\left[f(\theta_k|\sigma_k, c_k^*)p_h(a_{-k}^*\backslash\sigma_k) - f(\theta_k|a_k^*, c_k^*)p_h(a^*)\right] f(\theta_{-k}|a_{-k}^*, c_{-k}^*). \end{array} \tag{5}$$

is satisfied for all deviation n-tuples $\Psi \in M$.

Proof: See the Appendix. □

Thus, an efficient liability rule will only exist if monitoring is sufficiently informative at the optimal level of monitoring expenditure. Condition (5) says that the maximum punishment potential is now given by a weighted sum of accident losses where each outcome is weighted with the maximum probability that the joint realization of θ and x_h has been generated by a deviation contained in the deviation n-tuple under consideration.

An important aspect of this result is that the optimal monitoring policy and the choice of the liability rule are interdependent: the expenditure of the environmental agency should also reflect how the monitoring results can be used in court; i.e. the optimal monitoring policy must reflect that the court is restricted to use non-punitive liability rules.

In a loose sense, one can motivate the result as a variation on the old Beckerian theme of optimal punishment which suggests that the optimal policy for a costly auditing technology would be to minimize the number of audits and to maximize the punishment in case a deviation is detected.

To provide an intuition, it is easiest to directly follow up on the intuition proposed in section 4. Recall that the liability rule $l(x_h, \theta)$ can depend on all components. If there is monitoring, then the court has a description on the outcome state comprising x and θ_i (the signal of the agent i who is monitored). Whatever the informational precision of the signals, we can rewrite the incentive inequalities in terms of this extended state space (x_h, θ). Each agent has only an impact on the distribution of the signal concerning her action, not the signal for the others (recall that the mean of the signals is always the true care level). It follows that there are no symmetric simulations over the extended outcome space (x_h, θ).

The trouble is that the no punitive damages condition puts a cap on the maximal available punishment. In fact, the condition of no punitive damages has quite a bit of bite in this context. If there were no cap on punishments, then just excluding symmetric simulations over the extended state space (x_h, θ) is enough to restore efficiency. To illustrate this point, we investigate, as a pure thought experiment, the optimal liability rule if the no punitive damages condition were not a concern. The court could then levy more than x_h on any subgroup of injurers. Suppose that $\underline{c}_i$ is the minimum level of monitoring such that a signal θ_i is obtained whose distribution changes continuously in the choice of action a_i. Suppose that just a single agent is monitored at minimal precision $\underline{c}_i$. Let $\underline{c} \equiv \min_{i \in N} c(0, ..., 0, \underline{c}_i, 0, ..., 0) > 0$ denote the lowest possible expenditure at which monitoring of a single injurer is possible. Then there always exists an efficient liability rule:

Proposition 3: *There always exists an efficient liability rule satisfying $\sum_{i \in N} l_i(x_h) = x_h$ (but not necessarily satisfying no punitive damages) where the environmental agency will spend not more than the minimum $\underline{c}$ on monitoring.*

Proof: See the Appendix. □

Proposition 3 analyzes a liability rule which collects just the harm x_h. Nevertheless, this is not a realistic rule since $l_i < 0$ is not excluded, i.e. some injurers could effectively receive a subsidy in the event of an accident. Still, it is instructive as it shows that the scope of endogenous information acquisition is restricted by the no punitive damages condition. Minimal monitoring is sufficient if punitive damages can be levied because it delivers enough additional information so that no deviation profile is a symmetric simulation any more.

But no punitive damages is the relevant case, as we argued in the introduction. In this case, the $l_i(x_h, \theta)$-planes cannot assume arbitrarily steep slopes. Therefore, signals must be sufficiently informative. Still, the way the maximum punishment potential reacts to the change in the action level of an injurer can now depend on the expected variation in the extended state space (x_h, θ). If signals are informative enough so that the maximum punishment potential is larger than the aggregate utility gain of a deviation n-tuple, an efficient liability rule can be found.

7. Conclusion

For practitioners of environmental liability, multi-party accidents coupled with unobservable (or insufficiently observable) care levels of the injurers are a frequent problem. This setting was analyzed. We showed that the problem is related to the theory of team production. We investigated the existence of efficient non-punitive and strict liability rules when deviators can be stochastically identified. No punitive damages can be reconciled with efficiency only if agents are very asymmetric. One can summarize: if deviations cannot be identified in stochastic terms, no punitive damages imposes a severe restriction on the possibility that tort law is an efficient remedy for multi-party accidents with moral hazard. Also, the larger the number of injurers, the less powerful an instrument is tort. By virtue of the conditions necessary, there is no hope that future research would find more favorable results.

There are, however, remedies beyond resorting to punitive damages. We showed that even a noisy monitoring technology can restore unconstrained efficiency if it delivers additional injurer-specific information of sufficient precision. In a companion paper (Feess and Hege [1996]), we analyze another remedy: intermediaries can potentially improve the situation because their dealings with injurers are not subject to no punitive damages. In the liability context, the role of the intermediary is naturally assumed by insurance companies. We show that observability of insurance contracts is sufficient to obtain an efficient allocation.

Appendix

Proof of Proposition 1 :

The proof proceeds in four steps: In Step 1, the incentive constraints for the liability rule being an equilibrium are defined. In Step 2, the restrictions of non-punitiveness and strict liability are added and all conditions are put together in matrix form. In Step 3, the Theorem of the Alternatives is used to derive the existence conditions with respect to mixed strategies and deviation n-tuples Ψ. Step 4 demonstrates that the conditions derived are in fact identical with Proposition 1.

Step 1: For any liability rule $l(x_h)$, the efficient allocation a^* is a Nash-equilibrium if and only if

$$u_i(a_i^*) - \sum_h l_i(x_h) p_h(a^*) \geq u_i(a_i) - \sum_h l_i(x_h) p_h(a_{-i}^* \backslash a_i) \quad \forall i,\ \forall a_i \tag{A.1}$$

or, equivalently:

$$\sum_h l_i(x_h) \left(p_h(a_{-i}^* \backslash a_i) - p_h(a^*) \right) \geq u_i(a_i) - u_i(a_i^*) \quad \forall i,\ \forall a_i \tag{A.2}$$

System (A.1) defines the incentive compatibility constraints for all injurers. As a first step towards the matrix representation of the system of constraints, we define $\mathbf{P}_i$ as the $T_i \times h$-matrix where the (j,h) - entry is defined by:

$$P_i(j,h) = p_h(a_{-i}^* \backslash a_i^j) - p_h(a^*). \tag{A.3}$$

So $\mathbf{P}_i$ is a matrix that incorporates the probability differences for each damage and each action a_i, given a_i^* compared to a^*. Moreover, we define $\mathbf{u}_i$ as the $T_i \times 1$ vector of i's utility gains by deviating:

$$\mathbf{u}_i = \begin{pmatrix} u_i(a_i^1) - u_i(a_i^*) \\ u_i(a_i^2) - u_i(a_i^*) \\ \vdots \\ u_i(a_i^{T_i}) - u_i(a_i^*) \end{pmatrix}$$

So the incentive compatibility constraints for all actions of agent i can be written as

$$\mathbf{P}_i \mathbf{l}_i \geq \mathbf{u}_i$$

Recall that the complete system of constraints must also include non-punitiveness and strict liability. To take into account strict liability (total liability payments must add up to total losses), we define the liability payments of an arbitrarily chosen injurer j as the difference between total harm and the liability payments of all other injurers[26]:

$$\sum_h \left(x_h - \sum_{i \neq j} l_i(x_h) \right) \left(p_h(a^*_{-j} \backslash a_j) - p_h(a^*) \right) \geq u_j(a_j) - u_j(a^*_j) \quad \forall a_j \tag{A.4}$$

Recall that $x_h - \sum_{i \neq j} l_i(x_h)$ is the liability payment of injurer j if damage x_h occurs.

Step 2: Now let $\mathbf{P}$ be the following $\sum_j T_i \times (n-1)h$ - matrix:

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \mathbf{P}_2 & & & & & & 0 \\ \vdots & & \ddots & & & & & \vdots \\ 0 & & & \mathbf{P}_{j-1} & & & & 0 \\ -\mathbf{P}_j & -\mathbf{P}_j & \cdots & -\mathbf{P}_j & -\mathbf{P}_j & \cdots & -\mathbf{P}_j & -\mathbf{P}_j \\ 0 & & & & \mathbf{P}_{j+1} & & & 0 \\ \vdots & & & & & \ddots & & \vdots \\ 0 & & & & & & \mathbf{P}_{n-1} & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \mathbf{P}_n \end{pmatrix} \tag{A.5}$$

$\mathbf{P}$ includes the probability differences $P_i(j,h) = p_h(a^*_{-i} \backslash a_i) - p_h(a^*)$ for all injurers and all strategies. Note that the entry for injurer j guarantees that the sum of liability payments add up to total harm. However, non-punitiveness not only requires that $\sum_i l_i(x_h) = x_h$ for all possible outcomes but also that no injurer be subsidized. In order to rewrite the complete system of constraints (including incentive compatibility, non-punitiveness and strict liability) in matrix form we will use the following definition:

$$\mathbf{P}' = \begin{pmatrix} \mathbf{P} \\ \mathbf{E} \end{pmatrix} \tag{A.6}$$

[26] It can be verified (omitted here) that the choice of j is indeed irrelevant for the set of solutions.

where

$$\mathbf{E} = \begin{pmatrix} \mathbf{I} & & & \\ & \mathbf{I} & & \\ & & \ddots & \\ & & & \mathbf{I} \\ -\mathbf{I} & -\mathbf{I} & \cdots & -\mathbf{I} \end{pmatrix}$$

where I denotes the identity matrix. Next, (still needed for the matrix representation) similar definitions are made for the possible damage outcomes and the vectors of utility gains. Therefore let $\mathbf{x}$ be the $h \times 1$-vector of damage outcomes:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_l \end{pmatrix}$$

We define $\mathbf{u}$ as the $\sum_i T_i \times 1$-vector of stacked net utility gains:

$$\mathbf{u} = \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_j - \mathbf{P}_j\mathbf{x} \\ \vdots \\ \mathbf{u}_n \end{pmatrix}$$

Finally, we define $\mathbf{u}'$ by expanding $\mathbf{u}$ in a similar way as $\mathbf{P}'$:

$$\mathbf{u}' = \begin{pmatrix} \mathbf{u} \\ 0 \\ \vdots \\ 0 \\ -\mathbf{x} \end{pmatrix}$$

We are now in a position to write all constraints in matrix form. A non-punitive, strict and efficient liability rule exists if and only if there exist $\mathbf{l}$ such that:

$$\mathbf{P}'\mathbf{l} \geq \mathbf{u}' \tag{A.7}$$

So $\mathbf{P}'\mathbf{l} \geq \mathbf{u}'$ is nothing but a short-cut version of all constraints. Note that the top part $\mathbf{P}\mathbf{l} \geq \mathbf{u}$ ensures incentive compatibility and the summing constraint imposed by strict liability, while the bottom part $\mathbf{E}\mathbf{l} \geq \begin{pmatrix} 0 \\ -\mathbf{x} \end{pmatrix}$ excludes subsidies and thus ensures non-punitiveness.

Step 3: The matrix form (A.4) permits a direct application of the following result on the existence of solutions in systems of linear inequalities (known for example as the Theorem of FAN [1956] or the Theorem of the Alternatives):

LEMMA 1: *Theorem of the Alternatives (Fan):*
Let P be a $m \times n$-matrix, $u \in \mathbb{R}^m$. Then one and only one of the following alternatives holds:
1. $\exists l \in \mathbb{R}^n$ s.t. $P \cdot l \geq u$
2. $\exists \lambda \in \mathbb{R}^M_+$ s.t. $\lambda \cdot P = 0$ and $\lambda u > 0$.

The following definition will be used: Let $A = \{\alpha \mid \alpha \mathbf{P}' = 0\}$. Then $\alpha' \in A$ is called a *critical condition w.r.t. A* if $\alpha' \mathbf{u} \leq 0 \Rightarrow \alpha u \leq 0 \; \forall \alpha \in A$.

Lemma 1 implies that: $\exists \mathbf{l} \in \mathbb{R}^{(n-1)l}$ s.t. $\mathbf{P}'\mathbf{l}' \geq \mathbf{u}'$ if and only if: $\nexists \lambda$ s.t. $\lambda \mathbf{P}' = 0$ and $\lambda \mathbf{u}' > 0$. If $\lambda = 0$ then the condition is satisfied trivially. For any $\lambda \neq 0$, let $\lambda(a_i)$ be the entry for action a_i of argument i. Define $z \equiv \max_{i \in N} \sum_{a_i \in A_i} \lambda(a_i)$. Let $\alpha = \frac{1}{z}\lambda$. Note that $z \geq 0$, $\alpha \geq 0$. Hence for any $\mathbf{P}'$, $\mathbf{u}'$:

$$\lambda \mathbf{P}' = 0 \quad \text{and} \quad \lambda \mathbf{u}' > 0 \quad \Leftrightarrow \quad \alpha \mathbf{P}' = 0 \quad \text{and} \quad \alpha \mathbf{u}' > 0.$$

Step 4: The proof will be finished by showing: (B) $\alpha' \mathbf{P}' = 0$ and (C) $\alpha' \mathbf{u}' > 0$ are equivalent to (1).

Partition α' as:

$$\alpha' = \begin{pmatrix} \alpha \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_{j-1} \\ \mathbf{g}_{j+1} \\ \vdots \\ \mathbf{g}_n \\ \mathbf{g}_j \end{pmatrix}^T, \quad \text{where:} \quad \mathbf{g}_i = \begin{pmatrix} \mathbf{g}_i(x_1) \\ \vdots \\ \mathbf{g}_i(x_h) \\ \vdots \end{pmatrix}, \quad i = 1, .., n \tag{A.8}$$

We add the following notation: Let $\alpha(a_i) = \frac{1}{z}\lambda(a_i)$, let $\alpha_i = \left(\alpha(a_i^1), ..., \alpha(a_i^{T_i})\right)$, let $\rho_i = \sum_{a_i \in A_i} \alpha(a_i)$. Note that $0 \leq \rho_i \leq 1$, $\forall i, j$. Let $\sigma_i = \alpha_i + (1 - \rho_i)\mathbf{a}_i^*$, and let $\sigma = (\sigma_1, ..., \sigma_n)$.

Note that $\sigma \in \mathbb{R}_+^{\sum_i T_i}$, and $\sum_{a_i} \sigma_i(\alpha_i) = 1$, $\forall i$. Hence σ can be interpreted as a vector of mixed strategies. Next, note that $\alpha_i \mathbf{P}_i = \sigma_i \mathbf{P}_i - (1-\rho_i)\mathbf{a}_i^* \mathbf{P}_i = \sigma_i \mathbf{P}_i$, as $\mathbf{a}_i^* \mathbf{P}_i = 0$. Hence, we can without loss of generality normalize α to the mixed strategy σ in the partition of α'. Using this replacement, (B) and (C) can be rewritten as:

$$\forall x_h \in X,\ \forall i \neq j,\ p_h(a_{-i}^* \backslash \sigma_i) - p_h(a_{-j}^* \backslash \sigma_j) = g_j(x_h) - g_i(x_h) \tag{B'}$$

$$\sum_i [u_i(\sigma_i) - u_i(a_i^*)] - \sum_{x_h \in X} g_j(x_h) x_h - \sigma_j \mathbf{P}_j \mathbf{x} \leq 0. \tag{C'}$$

Now the LHS of (C') is maximized when $g_j(x_h)$ is minimized. Hence the α' which incorporates the minimal $g_j(x_h)$ corresponds to the critical condition. Note that, as in Proposition 2, for any n-tuple of α_i's, there are always non-negative numbers $g_i(x_h)$, $i \in N$, s.t. (B') is satisfied. To satisfy (B') under the restriction $g_i(x_h) \geq 0$ $\forall i \in N$, $i \neq j$, $\forall x_h$, $g_j(x_h)$ must satisfy:

$$g_j(x_h) \geq p_h(a_{-i}^* \backslash \sigma_i) - p_h(a_{-j}^* \backslash \sigma_j) \qquad \forall i \neq j, \quad \forall x_h \in X. \tag{A.9}$$

This implies for the $g_j(x_h)$ in the critical condition:

$$g_j(x_h) = \max_{i \in N} p_h(a_{-i}^* \backslash \sigma_i) - p_h(a_{-j}^* \backslash \sigma_j) \qquad \forall x_h \in X. \tag{A.10}$$

Plugging (A.10) into (C') gives condition (1). □

Proof of Proposition 2 : The technique of this proof follows closely the proof of Proposition 1. We will outline only the major steps, the remainder follows by analogy. Let $\mu_{\theta,h}(a,c) = f(\theta|a,c) \cdot p_h(a)$ be the joint probability of the observation (x_h, θ) conditional on the action profile a. Let $\mathbf{H}_i$ be the matrix whose (j,h) entry is, in analogy to (A.3):

$$H_i(j,h) = \mu_{\theta,h}(a_{-i}^* \backslash a_i^j, c) - \mu_{\theta,h}(a^*, c) \tag{A.11}$$

Thus, $\mathbf{H}_i$ contains T_i lines and a different column for each possible observation (θ, h). Let $\mathbf{H}$ and $\mathbf{H}'$ be defined in analogy to $\mathbf{P}$ in (A.5) and $\mathbf{P}'$ in (A.6), respectively. Then any $\hat{\mathbf{l}}$ such that:

$$\mathbf{H}'\hat{\mathbf{l}} \geq \mathbf{u}'$$

will be an non-punitive, strict and efficient liability rule. We finish the proof by showing: $\nexists \hat{\alpha}$ s.t. (D) $\hat{\alpha} H' = 0$ and (E) $\hat{\alpha} u' > 0$. The vector $\hat{\alpha}$ can be partitioned as α' in (A.8), as both vectors are of equal length. Denote the support of θ by Θ. Then, in analogy to (B') and (C'), conditions (D) and (E) can be rewritten as:

$$\forall(x_h,\theta)\in X\times\Theta,\ \forall i\neq j,\ \mu_{\theta,h}(a^*_{-i}\backslash\sigma_i,c)-\mu_{\theta,h}(a^*_{-j}\backslash\sigma_j,c)=g_j(x_h)-g_i(x_h) \qquad (D')$$

$$\sum_i[u_i(\sigma_i)-u_i(a^*_i)]-\sum_{x_h\in X}g_j(x_h)x_h-\sigma_j\mathbf{P}_j\mathbf{x}\leq 0. \qquad (E')$$

In analogy to (A.10), we can thus express $g_j(x_h)$ as:

$$g_j(x_h)=\max_{i\in N}\mu_{\theta,h}(a^*_{-i}\backslash\sigma_i,c)-\mu_{\theta,h}(a^*_{-j}\backslash\sigma_j,c) \qquad \forall(x_h,\theta)\in X\times\Theta \qquad (A.12)$$

Plugging (A.12) into (E') gives:

$$\sum_i[u_i(\sigma_i)-u_i(a^*_i)]-\sum_{x_h\in X}[\max_{i\in N}\mu_{\theta,h}(a^*_{-i}\backslash\sigma_i,c)-\mu_{\theta,h}(a^*_{-j}\backslash\sigma_j,c)]x_h-\sigma_j\mathbf{P}_j\mathbf{x}\leq 0. \qquad (A.13)$$

Now use the fact that $\mu_{\theta,h}(a,c)=\Pi_i f(\theta_i|a_i,c_i)\cdot p_h(a)$ by the stochastic independence of $p_h(a)$ and $f(\theta|a,c)$. Plugging into (A.13) gives condition (5). □

Proof of Proposition 3 : The proof is by construction. Recall that $l(x_h,\theta_i)$ can depend on all components of x and θ_i. (We have only a signal θ_i on the single injurer i who is monitored). To obtain the desired liability rule, construct first a function $g(\theta_i,c_i)$ for each i and c_i such that

$$a^*_i=\arg\max_{a_i}\left\{u_i(a_i)-(n-1)\left[\int g(\theta_i,c_i)f(\theta_i|a_i,c_i)d\theta_i-Ex(a^*_{-i}\backslash a_i)\right]\right\} \qquad (A.14)$$

Consider the following liability rule. For all agents except i, the liability rule will be $l_j(x,\theta_i)=x-g(\theta_i,c_i)$. The liability rule for agent i will be $l_i(x,\theta_i)=(n-1)g(\theta_i,c_i)-(n-2)x$. To demonstrate balancedness, note that summing up shows that $\sum_i l_i(x,\theta_i)=x$. To demonstrate efficiency, recall that for agent i, the efficient action a^*_i is implemented by construction. Next, note that for agents $j\neq i$, the allocation a^* is implemented because $g(\cdot)$ is independent of their action; thus, the variable part of their liability shares is x which obviously leads to the efficient choice.

We will next show that the function $g(\theta_i,c_i)$ will always exist, i.e. for any level of information c_i. To see this, consider any $g'(\theta_i,c_i)$ such that $g'(\theta_i,c_i)\cdot f(\theta_i|a_i,c_i)$ is minimized for $a_i=a^*_i$. Clearly $g'(\cdot)$ exists because $f(\theta_i|a_i,c_i)$ changes continuously in a_i. Suppose $g'(\theta_i,c_i)$ would not satisfy the incentive inequalities underlying condition (A.14). But then take any multiple y of $g'(\theta_i,c_i)$. Because there are no bounds for admissible liability rules, there is no limit on y. Thus, $y<\infty$ must exist satisfying conditions (A.14). It follows that it suffices to have any signal θ_i at hand, regardless of the precision and the number of monitored agents. But then the claim that the minimal choice of monitoring inputs must be optimal is immediate. $\square$

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