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## Macroeconomic Policy Interaction under EMU: A Dynamic Game Approach

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### *Abstract*

In this article, we study macroeconomic stabilization in the Economic and Monetary Union (EMU) using a dynamic game approach. With the aid of a stylized macroeconomic model, this article analyzes the transmission and interaction of national fiscal policies and monetary policy of the European Central Bank (ECB) in the EMU. A special focus is on the effects of labor market institutions in the participating countries and of the introduction of fiscal stringency criteria like those imposed in the Stability and Growth Pact.

The European Union (EU) countries started the Economic and Monetary Union (EMU) on January 1, 1999. With the EMU, 11 EU countries replaced their national currencies and national monetary policy autonomy with a common currency, the Euro, and a common monetary policy that is designed and implemented by the European Central Bank (ECB). Consequently, the monetary policy of the ECB will be directed in principle at stabilizing aggregate—i.e., EMU wide—macroeconomic fluctuations and not at country-specific conditions. The primary objective of the ECB is to maintain price stability in the EMU. Subject to the condition that price stability is not endangered, a second target is the

stabilization of general macroeconomic conditions. A consequence of a common monetary policy may be that asymmetric effects result in cases where countries face different business cycle conditions or asymmetric structural configurations of their economies. See, e.g., Hughes Hallett, and Piscitelli (1999) for a detailed analysis of such asymmetric transmissions of the common monetary policy of the ECB.

Fiscal management, on the other hand, remains predominantly a national competence under the EMU. The Maastricht Treaty of 1991 and the Pact for Stability and Growth, which was concluded in Dublin in December 1996 and drafted with the Treaty of Amsterdam on June 16, 1997, advocate fiscal stringency. The fiscal restrictions are motivated by fears that large fiscal deficits will undermine the credibility of the low-inflation commitment of the ECB, since the ECB might eventually be forced to bail out insolvent governments in order to prevent a collapse of the monetary and financial system in the EMU. In addition, high deficits are likely to exert upward pressures on interest rates in the EMU. In other words, high deficit spending by one country could entail costs for the other EMU members—costs that the spending country itself fails to internalize. To prevent excessive fiscal deficits, the Stability and Growth Pact introduces financial sanctions in case countries do not comply with its rules. The provisions of this Pact advocate balanced budgets in the longer run and specify a ceiling for deficit spending of 3% of GDP for each EMU member. A violation of the ceiling will trigger warnings and eventually penalties (unless exceptional circumstances can be evoked).<sup>1</sup>

The EMU countries have given up the monetary policy instruments in the design of their macroeconomic stabilization policies. This loss by itself is likely to increase the need for fiscal policy flexibility when countries face a recession, since the adjustment burden will now be relegated to the fiscal policymaker. The presence of (1) asymmetric macroeconomic shocks, (2) asymmetric transmissions of symmetric macroeconomic shocks, (3) asymmetric transmissions of the monetary policy of the ECB, (4) asymmetric transmissions of fluctuations of the Euro exchange rate, and (5) the absence of a sizeable federal fiscal budget in the EU that would provide automatic stabilization in case a country is hit by adverse conditions could be additional reasons why, in certain circumstances, EMU countries would require a high degree of fiscal flexibility at the national level. Hence, under the EMU, situations may arise where the need for flexibility and the requirement of fiscal stringency are in conflict with each other.

The various spillovers of national fiscal policies and the monetary policy of the ECB may generate additional inefficiencies if the macroeconomic policymakers implement their policies in a noncooperative manner. Demertzis et al. (1999) point out that a perceived lack of accountability and the fact that the ECB is confronted with 11 countries that pursue their individual fiscal policies and labor market strategies could make it less likely that cooperative monetary and fiscal policies will be implemented and sustained in the EMU. Unproductive conflicts and mutually frustrating monetary and fiscal policies could result. Increasing

the accountability of the ECB and improving commitment possibilities of the policymakers could alleviate such policy conflicts under EMU if coordination of monetary and fiscal policies fails to materialize.

The risks of instability in the EMU because of strict adherence to the Stability and Growth Pact and uncoordinated macroeconomic policies have not always been fully realized. In the light of these complications with monetary and fiscal policymaking under EMU, an important issue is therefore the design and implementation of macroeconomic stabilization policies under EMU.

This article analyzes, with the aid of a stylized macroeconomic model, the design and interaction of monetary and fiscal stabilization policies in the EMU using a dynamic game approach. We elaborate further the analysis of macroeconomic policy design in the EMU and the role of labor market institutions therein. In particular, we add a complete modeling of the goods and labor markets that feature rigidities in the short run, and study their adjustment. Using disequilibrium analysis, we make a sharp distinction between a Keynesian and a (Neo-) Classical unemployment regime.

Macroeconomic stabilization in the EMU is modeled as a differential game between the ECB and the fiscal authorities. We consider feedback information patterns in the differential game of macroeconomic stabilization between the fiscal authorities and the ECB.<sup>2</sup> In addition, the effects of coordination of monetary and fiscal policies in the EMU are analyzed. Cooperation allows internalization of the externalities and spillovers associated with macroeconomic policy design in the EMU. These spillovers and externalities exacerbate outcomes in the noncooperative feedback Nash case as the players try to shift the adjustment burden to the other players. A comparison of the outcomes under noncooperative and cooperative macroeconomic policies is made. In the set of cooperative equilibria, we concentrate on the Nash bargaining case, which appears to be the most realistic characterization of the bargaining problem connected with cooperation.

The model also incorporates the stringency requirements on monetary and fiscal policies associated with a conservative and independent ECB and the Stability and Growth Pact, respectively. Since we include a complete modeling of the labor market in the model, we can also address issues of labor market institutions and labor market reforms in the EMU. In particular, nominal and real wage rigidities play a crucial role in the model since they impact upon the ability of the labor market to adjust to the equilibrium rate of (un)employment, in particular under a situation of Classical unemployment.<sup>3</sup> We consider the effects of asymmetric structures where we focus on asymmetries in labor market institutions that may exist in the EMU.

Our analysis extends and complements the existing literature that models macroeconomic policy design in the EMU. This literature has focused on various related topics:

1. Macroeconomic adjustment and policy design in the transition towards EMU with an emphasis on the effects of the monetary and fiscal convergence

criteria of the Maastricht Treaty. Von Hagen and Lutz (1995), Jensen and Jensen (1995), and Barrell and Sefton (1997) are insightful examples that address this convergence issue.

2. Comparing adjustment in the pre-EMU and in the EMU regime, with an emphasis on the likely costs and benefits of a common currency. Currie et al. (1992), Hughes Hallett and Vines (1993), Lane and Gros (1994), and Fair (1998), among others, discuss this issue. In this literature, the question of whether the EMU is likely to constitute an optimal currency area is addressed. It is argued that if macroeconomic shocks in the EMU will have a large asymmetric component, if there are important price and wage rigidities, if labor mobility is low, and if there is little automatic stabilization from a federal fiscal budget, it becomes less likely that the EMU constitutes an optimal currency area.
3. Problems of macroeconomic policy coordination in the EMU. Issues of macroeconomic policy coordination and cooperation attracted a large interest in the macroeconomic literature of the 1980s and 1990s. Petit (1979), Bryant et al. (1988), McKibbin and Sachs (1991), and Nordhaus (1994), e.g., provide detailed discussions on macroeconomic policy transmission and coordination in dynamic macroeconomic models. The coordination issue concerns both the coordination between monetary and fiscal policy in the national economy and the coordination of macroeconomic policies between countries. Many of the issues and results in this literature apply also to the EMU, which features one common monetary policy of the ECB and national fiscal policies of the individual countries. Under EMU, conflicts could arise about the individual fiscal policies that countries are pursuing or about how the national fiscal authorities and the ECB should adjust national fiscal policies and the common monetary policy. Hughes Hallett and Ma (1996) compare outcomes in the EMU under (1) coordination of monetary and fiscal policies with (2) uncoordinated monetary and fiscal policies and (3) a case where monetary policy is assigned to price stability and fiscal policy to output growth. It is found that uncoordinated policies cause particular problems in cases where countries are not symmetric. The fiscal–monetary assignments can be problematic in case of symmetric countries.
4. The importance of labor market institutions and labor market reforms for the functioning of EMU. The EMU countries are all characterized—although to considerably varying degrees—by inflexible labor markets that feature wage rigidities and considerable structural unemployment due to institutional rigidities and skill mismatches. An important question, consequently, is how the EMU will function in the presence of such imperfections in the labor market and if labor market reforms will succeed in increasing labor market flexibility. Empirical work by Demertzis and Hughes Hallett (1998) finds that the combination of asymmetric shocks, asymmetric labor market institutions, and asymmetric transmission mechanisms has led to increasingly divergent structures in the EU that may produce large inefficiencies if

common policies are pursued. Agell (1999) argues that not all labor market rigidities are equally damaging to unemployment and that there are considerable differences in the effects across OECD countries. This result also explains why some labor market reforms will work in some cases but not in other cases. Calmfors (1998) and Bertold and Fehn (1998) address the question of whether the EMU could affect the incentives to undertake structural labor market reforms inside the EMU and in small countries outside the EMU but with strong ties to the EMU. Against the widely held view that the EMU could promote labor market reforms, their game-theoretic analysis argues that EMU may reduce the incentives to undertake labor market reforms in the EMU but may increase reform in small countries outside the EMU. The result depends very much, however, on the specific character of the model, which concentrates on the credibility problem of monetary policy. Compared to national monetary policies, the ECB will be less responsive to an achieved reduction in the structural unemployment level by a labor market reform in a country. Therefore, they argue, incentives to undertake such reforms are reduced in EMU countries.

The outline of the article is as follows. Section 2 introduces a simple dynamic macroeconomic model of the EMU. The interaction and transmission of fiscal policies and monetary policy implemented by the ECB are studied to illustrate the most important aspects of the model. The roles of labor market institutions and of fiscal stringency requirements are considered. Section 3 considers some theoretical issues concerning the (noncooperative) feedback Nash equilibrium and the calculation of the (cooperative) Nash bargaining solution of the dynamic stabilization game that is seen to occur in the EMU. Section 4 studies a numerical example, and Section 5 concludes.

### **1. Macroeconomic stabilization policies in the EMU**

To model macroeconomic adjustment and the possible dynamic stabilization conflicts that may arise in EMU between monetary and fiscal policies, we extend a recently developed approach by Neck and Dockner (1995), who analyze the dynamic interaction of the monetary authorities of two symmetric countries. We extend this two-country model to a setting of a monetary union, implying centralized monetary policy. In addition, we consider the effects of fiscal stabilization policy in such a setting of a monetary union and the effects of fiscal stringency requirements. Finally, we introduce labor market adjustment and institutions and study the effects of asymmetric structures of the labor market institutions in the participating countries.

The macroeconomic model underlying our analysis is a much modified and extended version of the familiar Mundell–Fleming model.<sup>4</sup> A complete modeling of goods and labor market is undertaken, featuring price and wage rigidities and

rationing schemes. In this way, we can analyze dynamic adjustment processes in the EMU and the role of macroeconomic policies therein.

We assume that the EMU consists of two countries, country 1 and country 2, and has been fully implemented. This assumption implies that national currencies have been replaced by a common currency and national central banks by the ECB (and hence that the exchange rate has disappeared as an adjustment instrument). The capital markets are fully integrated, and we abstain from any country-risk premium, implying that any nominal interest differential is arbitrated away instantaneously.<sup>5</sup> Furthermore, we assume that there is no labor mobility between both EMU parts and that goods and labor markets adjust sluggishly. Finally, we ignore the interaction of the EMU area with non-EMU countries.<sup>6</sup> The economic structure of the two-country EMU is given by the following equations:

Country 1	Country 2
$y_1^d(t) = \delta_1 s(t) - \gamma_1 r_1(t) + \rho_1 y_2(t) + \eta_1 f_1(t)$	(1a) $y_2^d(t) = -\delta_2 s(t) - \gamma_2 r_2(t) + \rho_2 y_1(t) + \eta_2 f_2(t)$ (1b)
$y_1^s(t) = \phi_1 n_1(t)$	(2a) $y_2^s(t) = \phi_2 n_2(t)$ (2b)
$y_1(t) = \min\{y_1^d(t), y_1^s(t)\}$	(3a) $y_2(t) = \min\{y_2^d(t), y_2^s(t)\}$ (3b)
$s(t) = p_2(t) - p_1(t)$	(4)
$r_1(t) = i_E(t) - \dot{p}_1(t)$	(5a) $r_2(t) = i_E(t) - \dot{p}_2(t)$ (5b)
$m_E(t) - p_E(t) = \kappa y_E(t) - \lambda i_E(t)$	(6)
$\dot{p}_1(t) = \xi_1 (y_1^d(t) - y_1^s(t)) + v_1 \dot{w}_1(t) + \zeta_1 \dot{p}_2(t)$	(7a) $\dot{p}_2(t) = \xi_2 (y_2^d(t) - y_2^s(t)) + v_2 \dot{w}_2(t) + \zeta_2 \dot{p}_1(t)$ (7b)
$\dot{w}_1(t) = \mu_1 \dot{p}_1(t) - \sigma_1 u_1(t)$	(8a) $\dot{w}_2(t) = \mu_2 \dot{p}_2(t) - \sigma_2 u_2(t)$ (8b)
$n_1^d(t) = -\pi_1 (w_1(t) - p_1(t))$	(9a) $n_2^d(t) = -\pi_2 (w_2(t) - p_2(t))$ (9b)
$n_1^s(t) = \tau_1 (w_1(t) - p_1(t))$	(10a) $n_2^s(t) = \tau_2 (w_2(t) - p_2(t))$ (10b)
$n_1(t) = \min\{n_1^d(t), n_1^s(t)\}$	(11a) $n_2(t) = \min\{n_2^d(t), n_2^s(t)\}$ (11b)
$u_1(t) = n_1^s(t) - n_1^d(t)$	(12a) $u_2(t) = n_2^s(t) - n_2^d(t)$ (12b)

where  $y_i$  denotes real output of country  $i$ ,  $i = \{1, 2\}$ ;  $p_i$ , the output price level;  $r_i$ , the real interest rate;  $f_i$ , the real fiscal deficit that the fiscal authority of country  $i$  chooses;  $w_i$ , the nominal wage;  $n_i$ , employment; and  $u_i$ , the unemployment rate. Moreover,  $s$  measures the competitiveness of country 1 vis-à-vis country 2, since it is defined as the output price differential.  $m_E$  denotes the amount of nominal balances of the common currency that the public holds, and  $i_E$  denotes the common nominal interest rate. All variables, except interest rates, are in logarithms and expressed as deviations from steady-state. The superscript  $d$  denotes demand, the superscript  $s$  supply. The price level and output in the EMU are defined as weighted averages of output prices and outputs of the two countries, i.e.,  $p_E(t) := \omega p_1(t) + (1 - \omega) p_2(t)$  and  $y_E(t) := \omega y_1(t) + (1 - \omega) y_2(t)$ , in which  $\omega$  and  $1 - \omega$  denote the relative sizes of the economies of country 1

and country 2 in the total EMU economy. All the parameters are assumed to be nonnegative. A dot above a variable denotes its time derivative.

Except for the interest rates and the unemployment rates, all the variables are in logarithms and are expressed as deviations from their long-run noninflationary equilibrium. In the long run, the EMU countries converge to their long-run noninflationary equilibrium (growth path), where output is equal to its long-run equilibrium level (implying that  $y(\infty)$  and  $u(\infty) = 0$ ), which is unaffected by monetary and fiscal policies. So our model describes how, in the short and medium run, macroeconomic adjustment takes place towards these equilibrium values. Note also that in our model the fiscal deficit is defined as the cyclical deficit component, i.e., the deviation from the structural deficit. The effects of a reduction in the structural deficit therefore fall outside the scope of our analysis: including such structural changes in the fiscal deficit would introduce nonlinearities in our model and possibly dramatically affect our entire analysis.

Equation (1) gives the aggregate demand for goods as a function of intra-EU competitiveness, the real interest rate, foreign output, and the fiscal deficit. In principle, we allow here for asymmetries in the transmission of fiscal policies in both countries, since an important debate exists in the literature about such asymmetries between countries in the transmission of fiscal policies (cf. the recent discussion about Keynesian and non-Keynesian effects of fiscal consolidation). Equation (2) expresses the aggregate supply of goods as a function of the amount of labor employed in the production process, where it has been assumed in the model that the amount of capital remains at its equilibrium in the short run. According to (3), actual output is rationed by the short side of the goods market. The role of rationing is also present in (7), according to which output prices adjust to some extent to any excess demand or supply in the goods market. In addition, cost-push factors such as wage increases and increases of foreign prices may affect domestic output prices. Equation (4) defines the competitiveness of the EMU countries relative to each other. This intra-EU competitiveness variable is one of the driving variables of the adjustment processes in the EU economy. The definition of real interest rates is given in (5) as the difference between the nominal interest rate and (expected) inflation.<sup>7</sup> The demand for the common currency is given by (6) and depends on output in the currency union and the common interest rate. The money market is cleared by the common interest rate such that money demand equals the supply of Euro base money,  $m_E$ , which is set by the ECB.

The Phillips mechanism is reflected in (8), which relates wage inflation to price changes and the unemployment rate  $u$ . The first component reflects the price compensation in wage formation, whereas the second component reflects the moderating effect of unemployment on wage increases. We do not assume a priori that there is full wage indexation ( $\mu = 1$ ). In principle, we also allow for the case of imperfect indexation ( $\mu < 1$ ). The Keynesian Neoclassical synthesis underlying our model causes nonneutral effects of imperfect indexation. In the long run, the Phillips curves are vertical again, i.e., independent of the value

of  $\mu$  or other structural model parameters. Labor demand,  $n^d$ , according to (9), is assumed to be negatively related to the real wage. Labor supply,  $n^s$ , according to (10), is a positive function of the real wage. Employment in (11) is determined by a similar rationing scheme as in the goods market.<sup>8</sup> The rate of unemployment in (12), finally, equals the difference between labor supply and labor demand.

Both economies are connected by a number of channels through which price and output fluctuations in one part transmit themselves to the other part of the EMU. Output fluctuations in both economies transmit themselves partly to the other EMU part through the import channel. Therefore, the relative openness to each other of both economies, as measured by  $\rho_i$ , creates an important interdependency between both economies. Price differences between the foreign and domestic economy affect relative intra-EU competitiveness  $s(t)$  and, therefore, export demand in both economies. Also, through their effect on the demand for the common currency in the common money market, output and price fluctuations in the domestic economy have repercussions on the foreign economy. Moreover, domestic price fluctuations are caused by both domestic components—domestic excess demand and wage changes—and by foreign prices.

The common monetary policy implemented by the ECB and the fiscal policies implemented by the national fiscal authorities affect real (output and employment) and nominal (prices and wages) adjustment in both economies through these various macroeconomic interdependencies. These interdependencies imply that the fiscal authority of country 1, the fiscal authority of country 2, and the ECB are involved in a dynamic game on macroeconomic stabilization in the EMU.

Combining (1)–(12) yields, after some rewriting and under the absence of a regime switch, the following first-order, four-dimensional linear differential equation, with as state variables the price level,  $p_i(t)$ , and wages,  $w_i(t)$ , in both countries, and as control variables the policy instrument of the ECB,  $m_E(t)$ , and of the fiscal authorities in both countries,  $f_1(t)$  and  $f_2(t)$ :

$$\dot{x}(t) = Ax(t) + B_1v_1(t) + B_2v_2(t) + B_3v_3(t), x(0) = x_0, \quad (13)$$

$$y(t) = Cx(t) + D_1v_1(t) + D_2v_2(t) + D_3v_3(t), \quad (14)$$

where  $x := \begin{pmatrix} p_1 \\ p_2 \\ w_1 \\ w_2 \end{pmatrix}$ ,  $v_1 := f_1$ ,  $v_2 := f_2$ ,  $v_3 := m_E$ , and  $y := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ . Having modeled the economies of both EMU countries and having derived the adjustment dynamics of output and prices over time, we still need to determine the monetary and fiscal policies and to derive their dynamic adjustment over time as a consequence of the interaction between the macroeconomic policymakers. To do so, we need to specify objective functions of the players. We assume that the players have quadratic objective functions. Fiscal authorities are assumed to care about stabilization of inflation, output, unemployment, and fiscal deficits



in their country, i.e.,

$$J_{F_1} := \frac{1}{2} \int_0^{\infty} \{ \dot{p}_1^2(t) + \alpha_1 y_1^2(t) + \beta_1 u_1^2(t) + \chi_1 f_1^2(t) \} e^{-\theta t} dt, \quad (15a)$$

$$J_{F_2} := \frac{1}{2} \int_0^{\infty} \{ \dot{p}_2^2(t) + \alpha_2 y_2^2(t) + \beta_2 u_2^2(t) + \chi_2 f_2^2(t) \} e^{-\theta t} dt. \quad (15b)$$

$\alpha$ ,  $\beta$ , and  $\chi$  are the relative weights that the fiscal authorities attach to output, unemployment, and deficit stabilization, respectively.  $\theta$  measures the rate of time preference. Inflation, output, and unemployment stabilization are standard arguments in the objective functions of macroeconomic policymakers. The assumption that the fiscal authorities value a budget balance can reflect the notion that high deficits, while beneficial to stimulate output, are not without costs: these, to some extent, crowd out private investment and lead to debt accumulation that has to be serviced in the future by lower government spending and higher taxes. Deficits in the loss function also reflect the possibility that excessive deficits in the EMU will be subject to sanctions, as proposed in the “Excessive Deficit Procedure” of the Treaty of Maastricht on the European Union (art. 104c) and its more recent extension into the Stability and Growth Pact. Therefore, countries will prefer low fiscal deficits to high deficits.<sup>9</sup>

The ECB also features inflation and output objectives. In particular, we assume that it cares about inflation,  $\dot{p}_E(t)$ , output,  $y_E(t)$ , unemployment, and the money stock,  $m_E(t)$ , in the EMU:

$$J_E := \frac{1}{2} \int_0^{\infty} \{ \dot{p}_E^2(t) + \alpha_E y_E^2(t) + \beta_E u_E^2(t) + \chi_E m_E^2(t) \} e^{-\theta t} dt. \quad (16)$$

Also, in the case of monetary policy, we assume that policy activism is not without costs and is therefore disliked by the ECB: in case  $\chi_E > 0$ , higher monetary policy activism—while beneficial in stabilizing output and price fluctuations—entails welfare losses in itself. The ECB is not directly concerned with the level of the fiscal deficits. The ECB does care directly about the level of its own instrument, the common money supply, since the ECB dislikes changes from the initial level. This could reflect costs associated with monetary policy activism, as sometimes proposed in the monetary policy literature: other things being equal, policymakers prefer a constant level of their instrument rather than to undertake changes all the time.

We want to consider the dynamic stabilization game in the context of a situation where the European countries are in a recession. This situation implies a negative output gap, i.e.,  $y_i(0) < 0$ , and unemployment, i.e.,  $u_i(0) > 0$ . Hence, we need to analyze how policy instruments, output, and prices adjust over time as a result of the dynamic interaction between macroeconomic policymakers in the EMU. Also, it will be of interest to consider how the degree of fiscal stringency and the asymmetric settings of both economies will affect macroeconomic stabilization policies and adjustment. In this dynamic interaction, one

can consider a number of different strategic and informational concepts. We restrict ourselves here to the Nash strategy, where players act noncooperatively and implement their equilibrium strategy simultaneously using feedback information patterns (i.e., the feedback Nash equilibrium) versus the cooperative Nash bargaining solution. In the latter, the noncooperative Nash equilibrium is featured as the strategic fallback position to which players turn in case cooperation breaks down. The next section gives a more detailed theoretical analysis and derives the feedback Nash and Nash bargaining equilibrium strategies in the linear quadratic setting.

## 2. Some theoretical issues

In the preceding section, we have defined a problem in which three parties (henceforth called players) try to minimize their individual quadratic performance criterion subject to a linear dynamic system. First, consider the case in which the players act noncooperatively, and assume that each player plays a Nash strategy, that is, no player can improve his outcome by altering his decision unilaterally. We focus on feedback Nash strategies in the case of linear system dynamics and quadratic objectives of the players. In this case, the information available to the individual players consists of the initial state and the current state of the game (memoryless perfect state information). This implies that the proposed policy in the feedback case is strongly time consistent—that is, the players have no reason at any future stage of the game to deviate from the adopted policy, even if there have been deviations in the past from the actions dictated by the original policy. An equilibrium concept that has these requirements is the feedback Nash equilibrium defined by, e.g., Starr and Ho (1969) and Başar and Olsder (1999). Since, according to this equilibrium concept, the players can react to the policies of the other players at any stage, its economic relevance is usually believed to be larger than that of the open-loop Nash equilibrium concept, where the players determine their optimal policy at the beginning of the planning period and stick to these policies throughout the planning period.

From (13) we have that each player controls a different set of inputs to a single system, described by a differential equation. The general structure of the system we consider is as follows:

$$\dot{x} = Ax + B_1v_1 + B_2v_2 + B_3v_3, \quad x(0) = x_0, \quad (17)$$

where  $x$  is the  $n$ -dimensional state of the system;  $v_i$  is an  $m$ -dimensional (control) vector that player  $i$  can manipulate;  $x_0$  is the initial state of the system; and  $A$ ,  $B_1$ ,  $B_2$ , and  $B_3$  are constant matrices of appropriate dimensions.

The performance criterion that player  $i = 1, 2, 3$  aims to minimize is

$$J_i(v_i, v_2, v_3) := \frac{1}{2}x(t_f)^T K_{if}x(t_f) + \int_0^{t_f} g_i(x, v_1, v_2, v_3) dt, \quad (18)$$

where

$$g_i(x, v_1, v_2, v_3) := \frac{1}{2} \left( x^T(t) v_1^T(t) v_2^T(t) v_3^T(t) \right)^T F_i \begin{pmatrix} x(t) \\ v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}, \quad \text{with}$$

$$F_i := \begin{pmatrix} Q_i & P_i & L_i & S_i \\ P_i^T & R_{1i} & N_i & T_i \\ L_i^T & N_i^T & R_{2i} & V_i \\ S_i^T & T_i^T & V_i^T & R_{3i} \end{pmatrix}, \quad (19)$$

is semipositive definite,  $K_{if}$  is semipositive definite, and  $R_{ii}$  is positive definite,  $i = 1, 2, 3$ . Throughout the present article, we assume that the following matrix  $G$  is invertible:

$$G := \begin{pmatrix} R_{11} & N_1 & T_1 \\ N_2^T & R_{22} & V_2 \\ T_3^T & V_3^T & R_{33} \end{pmatrix}. \quad (20)$$

The theoretical problem analyzed in this section is to find a feedback Nash equilibrium of the game sketched above. With respect to this game, a number of additional remarks must be made. First, in the literature,  $F_i$  is usually assumed to be diagonal. However, our study requires the analysis of a generalized performance criterion involving cross-terms as well. Note that cross-terms directly show up here. To illustrate: note that output in country 1 is directly influenced by output in country 2 (and vice versa); see (1a). Therefore, if fiscal authorities in country 1 are concerned about output in their country, they are indirectly also concerned about output in country 2; see (1b). The more output in country 1 is influenced by output in country 2, the more the authorities prefer active stabilization policies in country 2. Such effects are represented by the off-diagonal submatrices in matrix  $F_i$ . Note that cross-terms directly show up when one derives (18) from (15) and (16). The diagonal submatrix  $R_{21}$  measures, e.g., how strongly player 1 is concerned about player 2 pursuing a fiscal disequilibrium strategy (see also Fershtman et al., 1987).

Another remark concerns the chosen length of the planning horizon. Often, in the literature, the considered length is chosen infinite. From a computational point of view, it is usually easier to calculate and analyze the equilibrium strategy for the infinite than for a finite planning horizon. However, as shown by Weeren et al. (1999) in a study of the scalar two-player case with an infinite planning horizon (see also Engwerda, 2000), multiple feedback Nash equilibria can occur that all stabilize the closed-loop system. In addition, the dynamic stability of the equilibria was studied in Weeren et al. (1999), and it turned out that there are no arguments to discriminate between these equilibria. Given this

basic problem, we restrict ourselves here to an analysis of the finite planning-horizon problem, where we choose the length of the planning horizon such that the corresponding equilibrium policy becomes virtually independent of  $t_f$ . Consequently, the appropriate feedback strategies used in the analysis become stationary. Obviously, this property cannot be guaranteed a priori, and it would be nice to have some a priori arguments from which one could conclude this property beforehand. The analysis of this question, however, goes beyond the scope of the present article and is therefore ignored here. Furthermore, we assume that the planning horizon is so long that the discounted utility at the end of the planning horizon is negligible. So, the  $K_{if}$  are chosen to be zero ( $i = 1, 2, 3$ ).

In our analysis, the following set of coupled, symmetric, Riccati-type differential equations plays a crucial role:

$$\begin{aligned} \dot{K}_i = & -A^T K_i - K_i A + K_i (B_1 B_2 B_3) G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \\ S_3^T + B_3^T K_3 \end{pmatrix} \\ & + (P_1 + K_1 B_1 L_2 + K_2 B_2 S_3 + K_3 B_3) G^{-1} \begin{pmatrix} B_1^T \\ B_2^T \\ B_3^T \end{pmatrix} K_i \\ & - (I - (P_1 + K_1 B_1 L_2 + K_2 B_2 S_3 + K_3 B_3) G^{-1}) F_i \\ & \times \begin{pmatrix} I \\ -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \\ S_3^T + B_3^T K_3 \end{pmatrix} \end{pmatrix}, K_i(t_f) = K_{if}, i = 1, 2, 3. \end{aligned} \quad (21)$$

Let  $K_i(t)$  satisfy this set of Riccati equations and assume that the players use the strategy

$$\begin{pmatrix} v_1^*(t) \\ v_2^*(t) \\ v_3^*(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1(t) \\ L_2^T + B_2^T K_2(t) \\ S_3^T + B_3^T K_3(t) \end{pmatrix} x(t). \quad (22)$$

Due to the assumption of invertibility of  $G$ , we can prove the following theorem (see the Appendix):

**Theorem 1.** *The linear quadratic differential game (17)–(18) has a feedback-Nash equilibrium for every initial state whenever the set of Riccati differential equations (21) has a solution. Moreover, the equilibrium strategy is then given by (22).*

Note that existence of a solution of the set of Riccati differential equations (21) is only a sufficient condition to conclude that the finite planning horizon has a

solution. Moreover, the invertibility assumption is made in order to simplify the analysis. In case this assumption is not made, one should proceed along the lines of Başar and Olsder (1999, pp. 322–333) to find the equilibrium strategies.

Once we have obtained the feedback Nash equilibrium, this noncooperative solution can be used as a threatpoint in an axiomatic bargaining situation. In the literature, several axiomatic solutions are proposed in which the egalitarian solution, the Nash bargaining solution, and the Kalai–Smorodinsky solution are the most popular ones. As suggested by Douven (1995), we choose the Nash bargaining solution because it allows for an interpretation of the bargaining weights in terms of relative bargaining power, and it also allows for the maximization of gains from cooperation.<sup>10</sup> From axiomatic bargaining theory,<sup>11</sup> we know that the Nash bargaining solution is the unique solution satisfying Pareto optimality, anonymity (which means that the solution is independent of the name of the players), scale invariance, and multilateral stability. Multilateral stability means that the Nash bargaining solution has the property that if one player gets his Nash payoff and the remaining players have to negotiate again on the outcome of the game, then the outcome of this subproblem will be the same for these players as they would have obtained in the original game.

To determine this Nash bargaining solution, we first show how any Pareto-efficient strategy can be calculated. To that end, we introduce a new factorization of matrix  $F_i$ . Let

$$Q_i^c := Q_i, \quad S_i^c := (P_i \quad L_i \quad S_i) \quad \text{and} \quad R_i^c := \begin{pmatrix} R_{1i} & N_i & T_i \\ N_i^T & R_{2i} & V_i \\ T_i^T & V_i^T & R_{3i} \end{pmatrix}.$$

Then

$$F_i := \begin{pmatrix} Q_i^c & S_i^c \\ S_i^{cT} & R_i^c \end{pmatrix}.$$

Since our performance criteria  $J_i$  are convex, by the supporting hyperplane theorem, every Pareto-efficient cost vector  $J^*$  is associated with a vector of positive weights  $\psi$  (with  $\sum_{i=1}^3 \psi_i = 1$ ) such that  $J^* = \min\{\sum_{i=1}^3 \psi_i J_i\}$ . Consequently, we can use the standard linear quadratic regulator theory to find the set of all cooperative equilibrium strategies. Using the results from, e.g., chapter 16 of Lancaster and Rodman (1995) we have the following:

**Theorem 2.** *The equilibrium strategy corresponding with the cooperative bargaining problem in which player  $i$  is attached bargaining weight  $\psi_i$ ,  $i = 1, 2, 3$  is given by*

$$\begin{pmatrix} v_1^*(t) \\ v_2^*(t) \\ v_3^*(t) \end{pmatrix} = -R_\psi^{-1}(B^T M(t) + S_\psi)x(t), \quad (23)$$

where  $B := (B_1 \ B_2 \ B_3)$ ;  $R_\psi := \sum_{i=1}^3 \psi_i R_i^c$ ;  $S_\psi := \sum_{i=1}^3 \psi_i S_i^c$ ; and  $M$  is the solution of the Riccati differential equation,

$$\dot{M}(t) = \tilde{A}^T M(t) + M(t) \tilde{A} - M(t) B R_\psi^{-1} B^T M(t) + \tilde{Q}; \quad M(t_f) = 0. \quad (24)$$

Here  $\tilde{A} := A - B R_\psi^{-1} S_\psi^T$  and  $\tilde{Q} := \sum_{i=1}^3 \psi_i Q_i^c - S_\psi R_\psi^{-1} S_\psi^T$ . The corresponding minimal total performance costs are  $x^T(0)M(0)x(0)$ .

Note that we made the assumption in this theorem that the terminal costs in the performance criteria are zero (i.e.,  $K_{if} = 0$ ). Furthermore, the general formulation of this theorem presupposes that some regularity conditions are met (e.g., the invertibility of  $R_\psi$ ). Since these conditions are met in our case, we choose not to state them explicitly here. In general, the Nash bargaining solution can be identified with the point on the Pareto frontier at which the product of the differences between the Pareto optimal losses and the losses at the Nash threatpoint is maximized. In particular, this implies the following equivalent characterization (for a proof, see, e.g., Douven, 1995, Appendix A.2).

**Theorem 3.** *At the Nash bargaining solution, the following relationship holds between the individual losses of the players, denoted by  $J_i^{NB}$ ; the threatpoint  $d$ , with components  $d_i$ ; and the uniquely determined corresponding bargaining weights, denoted by  $\psi_i^{NB}$ :*

$$\psi_j^{NB} = \frac{\prod_{i \neq j} (J_i^{NB} - d_i)}{\sum_{i=1}^3 \prod_{k \neq i} (J_k^{NB} - d_k)}. \quad (25)$$

This relationship is exploited in the next algorithm to find the Nash bargaining solution.

**Algorithm 1.** *Determination of the Nash bargaining solution*

**Step 1: (initialization)** Calculate the individual performance criteria under the feedback Nash equilibrium strategy. Denote these performances by  $d_i$ .

Choose  $\psi_i = \frac{1}{3}$ .

**Step 2:** Compute the with  $\psi_i$  corresponding equilibrium strategy, according to Theorem 2, and the associated individual performances. Denote these by  $J_i^*$ .

**Step 3:** Check whether, for all  $i$ , the inequalities  $J_i^* \leq d_i$  are satisfied. If not, then there is an  $i_0$  for which  $J_{i_0}^* > d_{i_0}$ . In that case, update  $\psi_{i_0} := \psi_{i_0} + 0.01$ , update  $\psi_i := \psi_i - 0.005$ , for  $i \neq i_0$  and return to Step 2.

**Step 4:** Calculate

$$\psi_j^N = \frac{\prod_{i \neq j} (J_i^* - d_i)}{\sum_{i=1}^3 \prod_{k \neq i} (J_k^* - d_k)}.$$

**Step 5:** If  $|\psi_i^N - \psi_i| < 0.01$ ,  $i = 1, 2, 3$ , then terminate the algorithm and set  $\psi^{NB} = \psi^N$ . Else  $\psi_i := 0.8\psi_i + 0.2\psi_i^N$ ; recalculate the  $J_i^*$ 's and return to Step 2.

Step 3 is used to make sure that we always start in the bargaining set, i.e., the set where for all players  $J_i^*$  is to be preferred over  $d_i$ . If we were to skip this step, we could be stuck with a nonadmissible  $\psi$  in Step 4. Finally, in Step 5, we use the update  $\psi_i := 0.8\psi_i + 0.2\psi_i^N$  instead of the more intuitively appealing update  $\psi_i := \psi_i^N$ . This is to prevent too large steps in the update process, which again might result in a vector  $\psi$  for which the inequalities  $J_i^* \leq d_i$  might no longer be satisfied. Up to now, we do not have a proof that this algorithm always converges. In our simulation study, however, the algorithm worked well, and convergence was obtained in quite a few iterations. An alternative approach to calculate the Nash bargaining solutions is to maximize directly the product of the gains over the noncooperative Nash case. Douven (1995) finds, however, that the algorithm described above is faster and more reliable than this traditional method.

### 3. A simulation study

The variables in the objective functions  $J_i$ ,  $i = \{1, 2, E\}$ , can be rewritten in terms of the state variables and the instruments. Before we can find the equilibrium of the dynamic stabilization game, we first have to rewrite model (13)–(16) as a standard three-player game corresponding with (17)–(19). To that end, introduce  $\tilde{x}(t) := e^{-\frac{1}{2}\theta t}x(t)$ ,  $\tilde{y}(t) := e^{-\frac{1}{2}\theta t}y(t)$  and  $\tilde{v}(t) := e^{-\frac{1}{2}\theta t}v_i(t)$ . We see that the above minimization problems (13)–(17) can be rewritten as

$$\begin{aligned} \min_{\tilde{v}_i} \quad J_i &:= \frac{1}{2} \int_0^\infty \left\{ (\tilde{x}^T(t) \tilde{v}_1^T(t) \tilde{v}_2^T(t) \tilde{v}_3^T(t))^T F_i \begin{pmatrix} \tilde{x}(t) \\ \tilde{v}_1(t) \\ \tilde{v}_2(t) \\ \tilde{v}_3(t) \end{pmatrix} \right\} dt, \quad i = 1, 2, E, \\ \text{s.t.} \quad \dot{\tilde{x}} &= \left( A - \frac{1}{2}\theta I \right) \tilde{x} + B_1 \tilde{v}_1(t) + B_2 \tilde{v}_2(t) + B_3 \tilde{v}_3(t), \quad \tilde{x}(0) = x_0. \end{aligned} \quad (26)$$

We can now readily apply the analytical apparatus developed in Section 3 to the dynamic game between the fiscal players and the ECB on macroeconomic stabilization in the EMU. We concentrate on numerical simulation of a specific example. Obviously, this implies that the exact numerical outcomes are specific to the numerical values of the model parameters that are chosen. However, we are more interested in the general adjustment patterns that are generated. The linearity of the model and a rough sensitivity analysis suggest that the general picture of macroeconomic adjustment provided by our example is indeed representative.

The model enables us to consider adjustment in a considerable number of macroeconomic settings. In particular, we distinguish between (1) a symmetric case in which all model parameters are the same in both countries and (2) an asymmetric case where countries differ in some structural model parameters. In particular, we focus on asymmetries with reference to labor market characteristics, namely, asymmetries with reference to wage indexation and labor market flexibility. These asymmetries and their possible consequences appear to be the most relevant in the context of macroeconomic adjustment under EMU. Furthermore, we distinguish between (1) a noncooperative policy regime and (2) a cooperative policy regime. In the latter regime, the national fiscal policies and the common monetary policy of the ECB are coordinated. Finally, because of the disequilibrium analysis we used in modeling labor and goods market adjustments, a distinction can be made between (1) a (Neo-) Classical unemployment regime and (2) a Keynesian unemployment regime. As we will see, all these different cases will have considerable effects on macroeconomic adjustment and macroeconomic policy design in the EMU.

We use the following values for the structural model parameters in the symmetric case that will serve as our basic reference point:  $\gamma_i = 0.2$ ,  $\delta_i = 0.3$ ,  $\rho_i = 0.3$ ,  $\eta_i = 1$ ,  $\kappa = 1$ ,  $\lambda = 0.1$ ,  $\xi_i = 0.25$ ,  $v_i = 0.7$ ,  $\zeta_i = 0.8$ ,  $\mu_i = 0.9$ ,  $\sigma_i = 0.2$ ,  $\pi_i = 0.7$ ,  $\tau_i = 0$ ,  $\phi_i = 0.75$ ,  $\omega = 0.5$ , and  $\theta = 0.1$ . Concerning the preference weights in the objective functions of the players, the following values have been assumed:  $\alpha_1 = \alpha_2 = 1.5$ ,  $\alpha_E = 0.6$ ,  $\beta_1 = \beta_2 = 3$ ,  $\beta_E = 0$ ,  $\chi_1 = \chi_2 = 0.8$ ,  $\chi_E = 0.5$ . In this case, the ECB's primary focus is price stability, and a lower weight is attached to output stabilization. Instrument stability also carries some weight. The fiscal authorities are primarily concerned with output and (un)employment stabilization but face constraints on the use of their instruments.

Of crucial importance for the dynamic adjustment in the EMU are also the initial values of the state variables of the model, which will determine the initial type of disequilibrium that countries face. We assume that the EMU starts in a situation of wages and prices at disequilibrium levels:  $p_1(0) = p_2(0) = 0.005$  and  $w_i(0) = w_2(0) = 0.01$ . In that case, an initial excess supply in the labor market and an initial excess demand in the goods market result. In the literature on disequilibrium theory, this outcome is generally known as the Classical unemployment regime. Figure 1 displays the adjustment of output, prices, wages, unemployment, competitiveness, fiscal deficits, and the common money supply that results in this symmetric base-case scenario.

Adjustment in the feedback Nash case is indicated by solid lines. Adjustment in the Nash bargaining equilibrium is given by dotted lines and is shown as a basic reference for outcomes in the cooperative case. In this symmetric setting, macroeconomic adjustment in both countries is also symmetric. The initial disequilibrium level of prices and nominal wages depresses output and employment significantly. Adjustment forces, however, are activated by this initial disequilibrium in goods and labor markets. With aggregate demand exceeding aggregate supply of goods and unemployment in the labor markets, prices and



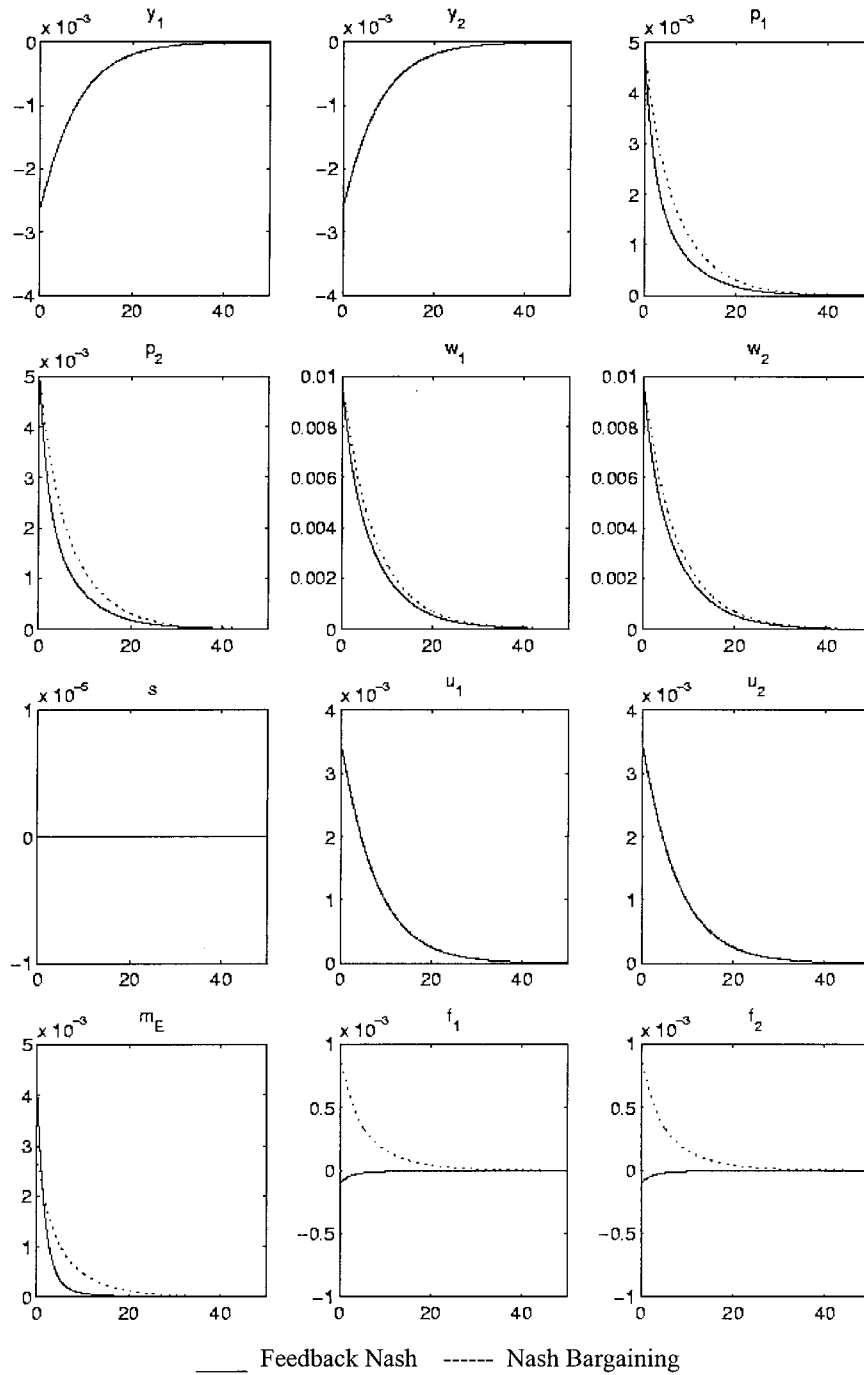


Figure 1. Base scenario; Classical regime.

wages start to adjust downward towards their equilibrium values. Real wages decrease, which fosters output and employment adjustment towards equilibrium. The initial adjustment for the real variables (output and employment) is (slightly) higher in the cooperative case than in the noncooperative case. After  $t = 20$ , adjustment is, however, faster in the noncooperative case.

Under the Classical unemployment regime, monetary and fiscal policies have only effects on output and employment via price and wage adjustment. The common money supply is expanded to stimulate real wage adjustment: the implied higher inflation depresses real wages if indexation is imperfect. In the Nash bargaining case as well, the fiscal authorities implement an expansionary policy and share in the adjustment burden. In the noncooperative feedback Nash case, however, a contractionary fiscal policy is chosen as the fiscal authorities attempt to shift the adjustment burden to the ECB. The low effectiveness and the direct costs—as reflected by the weight  $\chi$  in the loss functions of the policymakers—from active stabilization policies restrain the policymakers from pursuing very active stabilization policies, and we see that the policy impulses remain modest. We will come back to this issue in the final example we study.

In the first (I) line of Table 1, we indicate the welfare losses that result in the noncooperative ( $J^{Nash}$ ) and cooperative ( $J^{NB}$ ) cases, and the bargaining weights ( $\psi^{NB}$ ) in the cooperative decision making on monetary and fiscal policies.

By definition, policy coordination reduces the welfare losses since externalities from individual policies are internalized in the cooperative case and not ignored as in the noncooperative case. Obviously, in this symmetric case, the bargaining weights in the individual countries are equal. The ECB's bargaining weight turns out to be more than double the bargaining weight of an individual country. In the Classical regime with excess demand in the goods market and with limited effects of fiscal and monetary policies on real variables, the gains from policy coordination are also limited, as Table 1 indicates.

Strong effects result from a change in the labor market parameters. Figure 2 gives the adjustment patterns that result in the symmetric case if  $\sigma$  increases from 0.2 to 0.8, implying that wages react much more strongly to unemployment.

A higher degree of wage flexibility increases the adjustment capacity of the economies. Within 10 periods, all adjustment has virtually been achieved, whereas in the base case this occurred only after 25 periods, which also indicates the importance of the flexibility of the labor market. Wages, unemployment, and output adjust considerably faster than in the base case of Figure 1 because of the improved flexibility of the labor market. The fiscal and monetary policy strategies remain very similar to those observed in the base case. According to the second (II) line of Table 1, higher labor market flexibility in the EMU entails considerable welfare gains both under noncooperative and cooperative macroeconomic policy design. From this perspective, it is interesting to note that currently, in various EU countries, attempts to reform labor markets and institutions are undertaken that aim at increasing the flexibility of the labor

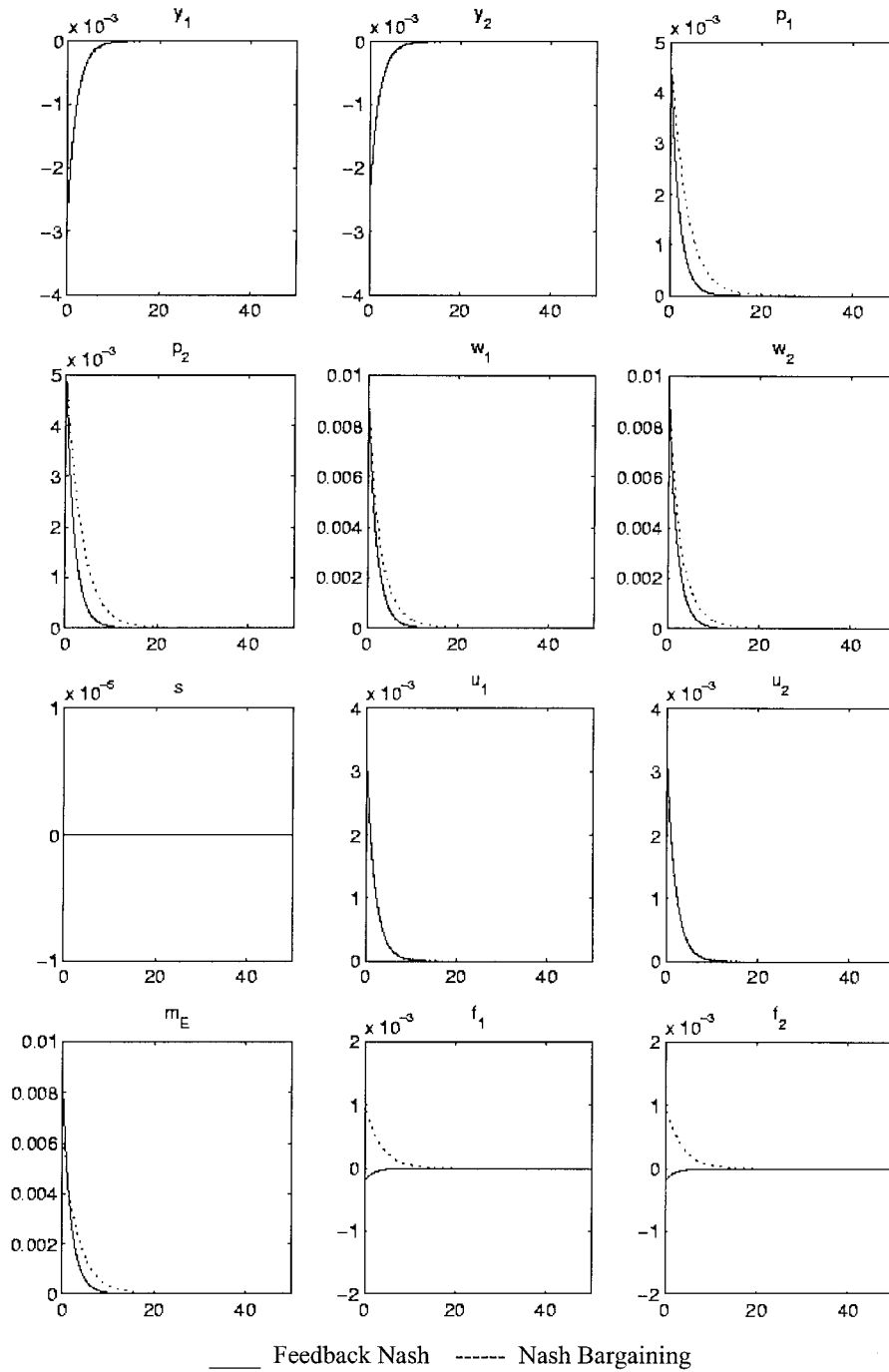


Figure 2. More flexible labor markets ( $\sigma_1 = \sigma_2 = 0.8$ ); Classical regime.

**Table 1.** Noncooperative ( $J^{NASH}$ ) and cooperative costs ( $J^{NB}$ ) (both times  $10^{-5}$ ) and relative bargaining power ( $\psi^{NB}$ ) in the Nash bargaining solutions

Case	Country 1	Country 2	ECB
I $J^{NASH}$	11.21	11.21	1.19
$J^{NB}$	10.97	10.97	1.07
$\psi^{NB}$	0.239	0.239	0.520
II $J^{NASH}$	4.30	4.30	3.62
$J^{NB}$	4.03	4.03	3.36
$\psi^{NB}$	0.326	0.326	0.348
III $J^{NASH}$	11.56	19.88	0.92
$J^{NB}$	11.13	16.34	0.69
$\psi^{NB}$	0.338	0.041	0.622
IV $J^{NASH}$	11.03	4.44	2.22
$J^{NB}$	10.76	4.17	2.03
$\psi^{NB}$	0.292	0.285	0.423
V $J^{NASH}$	85.80	85.80	71.33
$J^{NB}$	71.95	71.95	70.00
$\psi^{NB}$	0.081	0.081	0.838
VI $J^{NASH}$	70.49	70.49	18.33
$J^{NB}$	28.74	28.74	11.57
$\psi^{NB}$	0.1211	0.1211	0.7579

*Note:* Case I, baseline scenario; Case II, higher wage flexibility ( $\sigma_1 = \sigma_2 = 0.2$ ); Case III, asymmetry in the wage indexation parameter ( $\mu_1 = 0.9, \mu_2 = 0.1$ ); Case IV, asymmetry in wage flexibility ( $\sigma_1 = 0.2, \sigma_2 = 0.8$ ); Case V, Keynesian unemployment; Case VI, more fiscal and less monetary flexibility ( $\chi_1 = \chi_2 = 0.8, \chi_E = 3.5$ ).

market. In the cooperative case, higher flexibility of the labor market increases the bargaining weight of the fiscal players at the cost of the ECB.

Having focused so far on the symmetric case, we now turn to asymmetric settings. In particular, we focus on asymmetries of labor market institutions and their consequences for macroeconomic adjustment and policies, since these are generally assumed to be the most significant in the context of EMU. Figure 3 gives adjustment in the presence of an asymmetry in the wage indexation parameter  $\mu$ . We assume that wages in country 1 are indexed to a much larger extent than in country 2,  $\mu_1 = 0.9$  (as before in the symmetric case of Figure 1), and  $\mu_2 = 0.1$ .

This assumption produces considerable asymmetries in the adjustment of both countries. Country 1 displays a quicker adjustment of wages, prices, output, and employment than country 2, particularly under noncooperative

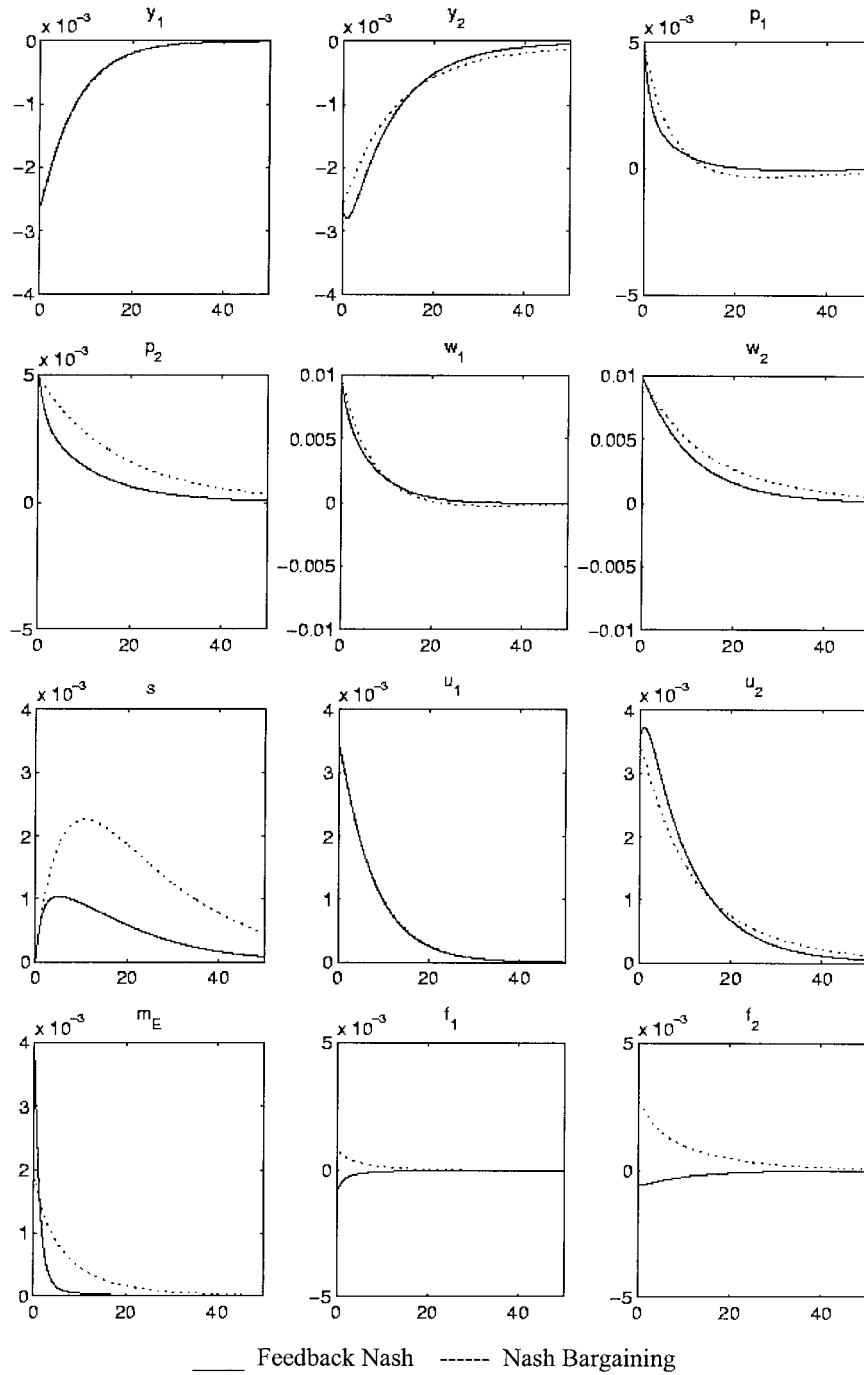


Figure 3. Asymmetric wage indexation ( $\mu_1 = 0.9, \mu_2 = 0.1$ ); Classical regime.

policies. Country 1 is more competitive than country 2 over the complete adjustment period, in particular during the first half of it. For country 2, it also becomes more important whether noncooperative or cooperative macroeconomic policy design prevails, as the larger difference in welfare losses between both policy regimes and a very low bargaining weight in the third (III) line of Table 1 indicate. Relatively large cooperation gains can be established for country 2 and for the ECB.

Even stronger are the effects from the second labor market asymmetry that we analyze in Figure 4. We assume in this second asymmetric case that the countries are symmetric, except that the labor market of country 2 is considerably more flexible than that of country 1, since we assume that  $\sigma_1 = 0.2$  and  $\sigma_2 = 0.8$ .

In this case, the labor market in country 2 reacts much more strongly to unemployment than in country 1. Wages, unemployment, prices, and output adjust much more slowly in country 1, resulting in a considerable difference in welfare losses between the two countries, according to the fourth (IV) line of Table 1. Competitiveness of country 1 remains negative over the entire adjustment cycle. This simulation in particular suggests the importance of labor market flexibility—and possible asymmetries therein—for macroeconomic adjustment in the EMU. The last two examples with asymmetric settings also support the finding of Hughes Hallett and Ma (1996) that asymmetries tend to increase the scope for policy cooperation: for all players, the asymmetric cases display larger gains from cooperation than in the symmetric base case.

Until now, the simulations started with an initial disequilibrium that resulted in output being supply side determined both initially and during the remainder of the adjustment. In this case, policies basically only influence adjustment to the extent that they foster real wage adjustment. Also, we found that the issue of whether macroeconomic policies were coordinated or not did not have a strong impact, except perhaps in the case where the economies are less indexed. A different and potentially stronger role for macroeconomic policies and their coordination results, however, if output is demand determined. In that case, policies can directly influence output and employment.<sup>12</sup> The next two cases consider such a demand-determined regime in the goods market. The initial wages and prices are now below their long-run equilibrium ( $w_1(0) = w_2(0) = -0.005$ ,  $p_1(0) = p_2(0) = -0.01$ ), entailing Keynesian unemployment and excess supply of goods. Figure 5 gives adjustment in the base case but with this set of initial price and wage levels.<sup>13</sup>

In this demand regime, monetary and fiscal policies have a stronger impact on the adjustment, since they directly control aggregate demand for goods and thereby employment. The fiscal authorities who care in particular for output and unemployment stabilization undertake a significant expansionary fiscal impulse. The ECB, on the other hand, which is very concerned about price stability, implements a monetary contraction to limit the price increases, even if this

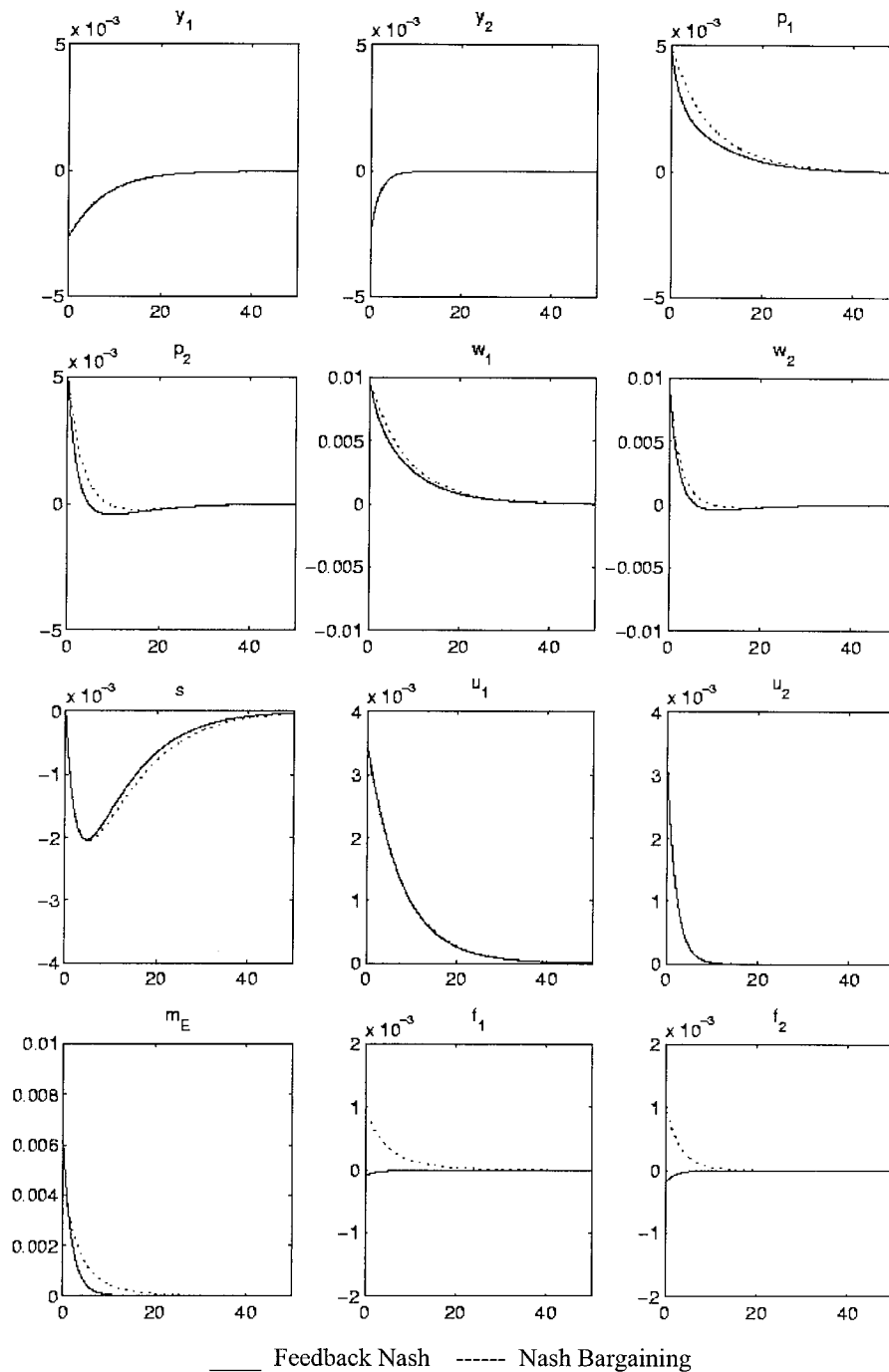


Figure 4.  $\sigma_1 = 0.2, \sigma_2 = 0.8$ , asymmetric wage flexibility; Classical regime.

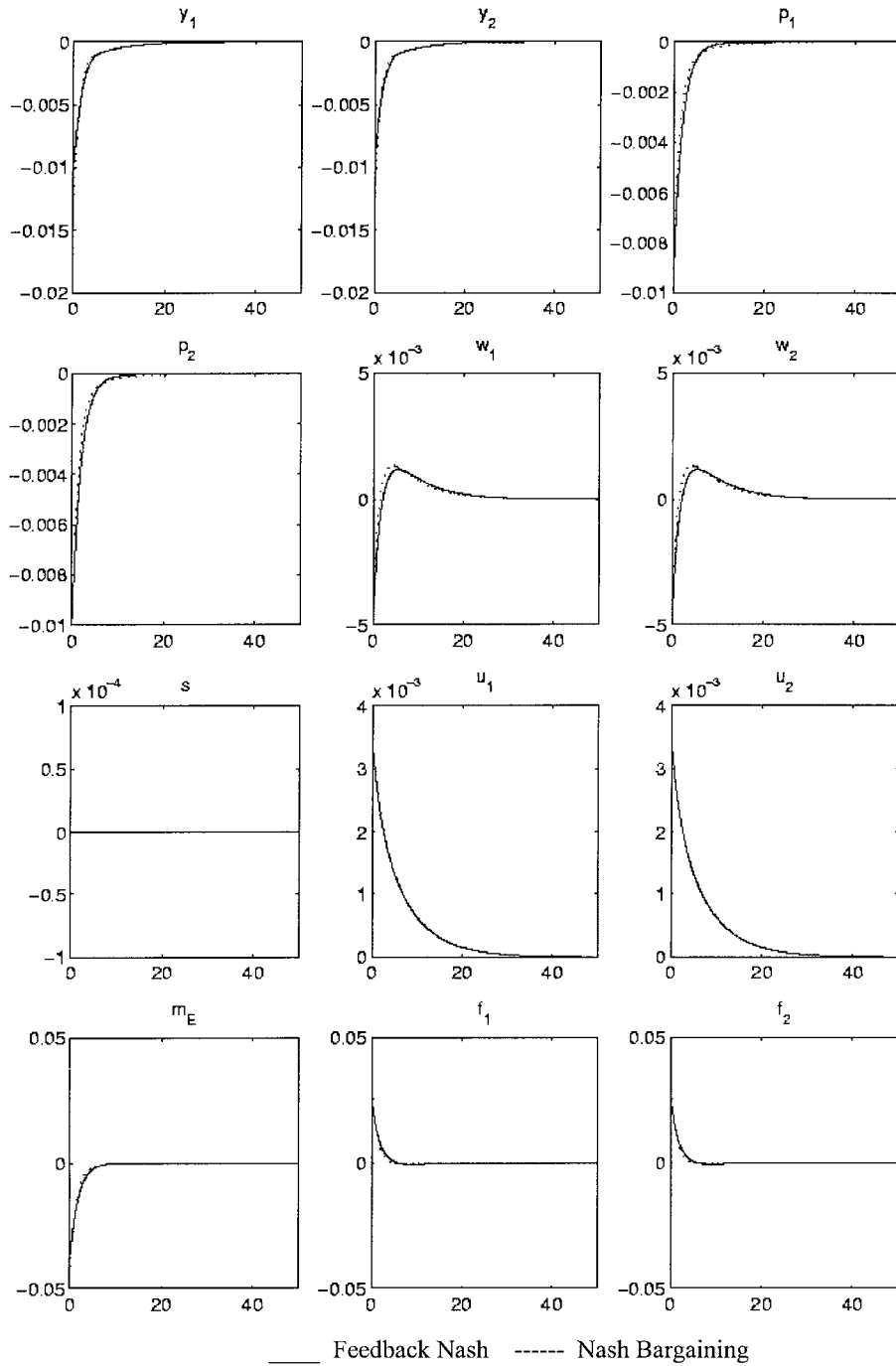


Figure 5. Base scenario; Keynesian regime.



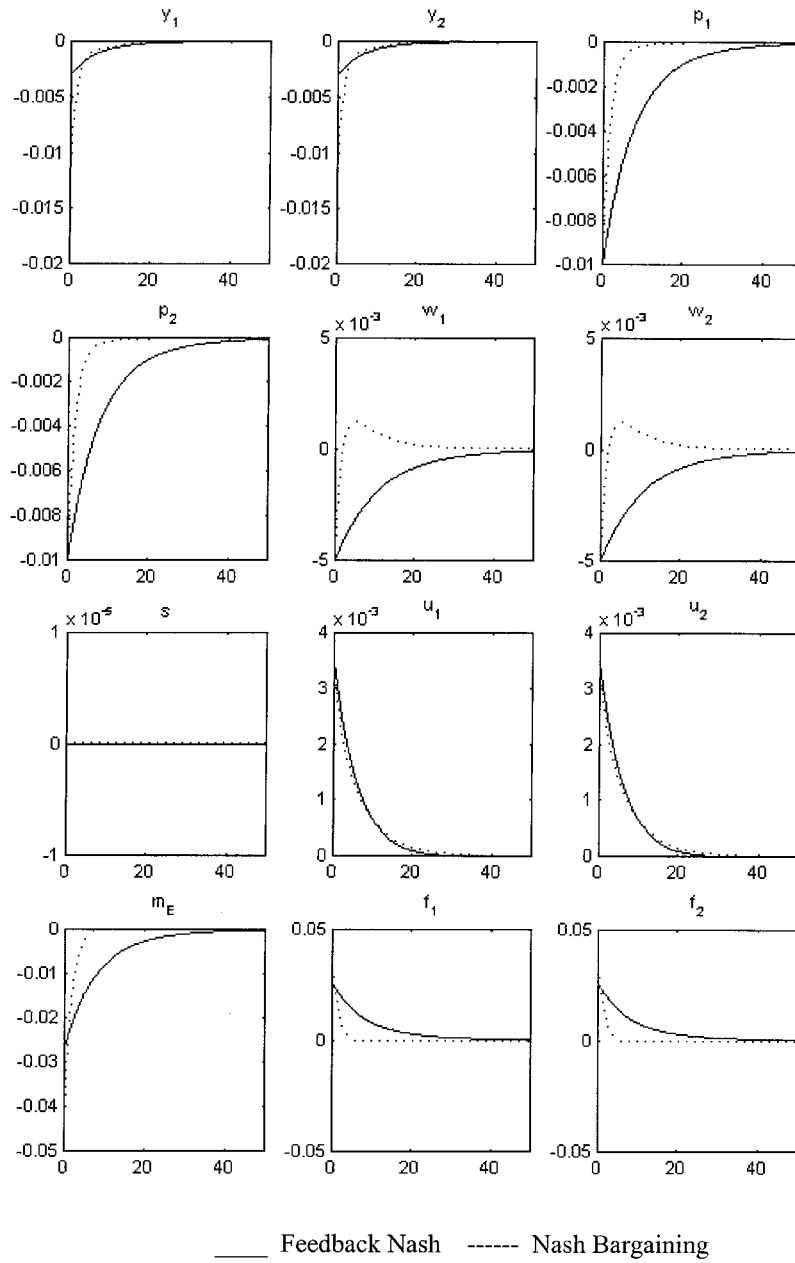


Figure 6. More fiscal and less monetary flexibility; Keynesian regime.

contraction hampers output and unemployment growth. This considerable difference in relative preferences also implies that cooperation has limited effects in this example: the more similar players are in their preferences, of course, the greater becomes the scope for effective policy cooperation.

Compared to the Classical regimes of Figures 1–4, the bargaining weight of the ECB has considerably increased—to the disadvantage of the fiscal authorities, as indicated in the fifth (V) line of Table 1. Note also the larger welfare losses in the Keynesian unemployment regime as compared with the Classical unemployment regime. These costs result in particular because of the much higher degree of policy activism in the Keynesian regime. Note also, however, that the direct comparison of welfare losses across both regimes is not valid in a strict sense, since we are comparing two fundamentally different macroeconomic regimes, e.g., macroeconomic policies are more powerful in the Keynesian regime.

Figure 6 gives the adjustment for the case in which the fiscal and monetary flexibility parameters are changed.  $\chi_1$  and  $\chi_2$  are decreased from 0.8 to 0.25, implying that fiscal policy activism is much less restricted by fiscal stringency requirements and less costly for the authorities.  $\chi_E$  is increased from 0.5 to 3.5, making monetary policy activism more costly for the ECB.

As indicated in Section 2, we interpret the fiscal flexibility parameter as a stylized representation of the Stability and Growth Pact. A lower value of  $\chi_{1,2}$  in this interpretation implies a less strict interpretation of the Stability and Growth Pact. The higher degree of fiscal flexibility enables the fiscal authorities to exercise more fiscal policy activism. Accordingly, fiscal deficits are expanded more in the feedback case and in the Nash bargaining case. The lower monetary flexibility implies here that the ECB implements a less restrictive monetary policy as before. This reduces the adjustment burden for the fiscal policymakers. Consequently, the welfare losses given in line (VI) of Table 1 are considerably lower (compared with the base case of the Keynesian regime in line (V)), both for the fiscal authorities who face a smaller adjustment effort and even for the ECB, which has less instrument costs, since it is more costly to implement a sharp monetary contraction. The new bargaining weights imply that the more flexible fiscal players gain influence in the cooperative decision-making problem.

#### 4. Conclusions

The EMU combines a centralized monetary policy with decentralized fiscal policies. Fiscal stringency requirements are introduced to reduce negative spillovers that could result from excessive deficits. In such a setting, many complications are likely to arise when macroeconomic stabilization policies are designed in the EMU. This article has characterized the problem of macroeconomic stabilization under EMU as a dynamic game between the ECB and national fiscal authorities. Using a dynamic game approach, we have analyzed

how the monetary policy of the ECB and national fiscal policies interact and are transmitted. We have focused on the feedback Nash and the Nash bargaining equilibria of this dynamic game, where the optimal strategies of the players are implemented simultaneously and in a noncooperative and cooperative way, respectively. Moreover, we have analyzed how a looser and a stricter interpretation of the monetary and fiscal stringency requirements of the Maastricht Treaty affects macroeconomic outcomes in the EMU. The analysis has also provided insights into the effects of labor market conditions on macroeconomic outcomes under EMU.

More specifically, we have simulated optimal noncooperative and cooperative solutions divided over six scenarios: four scenarios in a supply-determined Classical unemployment environment and two in a demand-determined Keynesian unemployment environment. Under the Classical unemployment regime, monetary and fiscal policies have only indirect effects on output and employment via price and wage adjustments. Expansionary monetary and fiscal policies increase excess demand and thereby price adjustment. Real wages are affected if indexation is imperfect, which fosters output and employment adjustment towards equilibrium. Common for the Classical unemployment scenarios is that the (noncooperative) feedback Nash strategy selects a contractionary fiscal policy as the national fiscal authorities attempt to shift the adjustment burden to the ECB, while the Nash bargaining strategy selects an expansionary fiscal policy where the fiscal authorities are prepared to share in the adjustment burden. The first two Classical unemployment scenarios considered symmetric cases: a base scenario and a labor market flexibility scenario with a more efficient functioning of the Phillips mechanism. The other two Classical unemployment scenarios have considered two labor market asymmetries: one with an asymmetric wage indexation scheme and one with asymmetric wage flexibility characteristics between the two countries. We have shown that labor market institutions and possible asymmetries in labor market institutions will have a very strong role on adjustment in the EMU if this Classical regime prevails. The issue of policy coordination and flexibility, while nontrivial, is of secondary importance in those cases (except for the national fiscal deficits, the common money supply, and intra-EU competitiveness).

Finally, the last two scenarios have investigated symmetric Keynesian unemployment regimes. In this rationing regime, monetary and fiscal policies have direct effects on output and employment. This outcome implies stronger effects from macroeconomic stabilization policies and stronger welfare effects of lack of coordination and/or flexibility of macroeconomic policies. We have concentrated on a situation with a very inflation-averse ECB and with fiscal authorities who are mainly concerned with output and employment stability. In that case, the Keynesian regime is potentially vulnerable to serious policy conflicts between monetary and fiscal authorities in the EMU: a contractionary monetary policy of the ECB increases the burden for the fiscal authorities to stabilize output and employment. Expansionary fiscal policies, on the other

hand, increase the burden of inflation stabilization for the ECB. We have analyzed how policy cooperation and restrictions on monetary and fiscal policy activism can be used to alleviate such macroeconomic policy conflicts in the EMU.

Two important aspects are not incorporated into the analysis: (1) We do not analyze the case where fiscal authorities coordinate their fiscal policies internally. The question can be raised as to what extent fiscal coordination by itself can already produce the gains from full policy coordination in the EMU. (2) We disregard the interaction with countries outside the EMU area. Adding this aspect will give more insight into the effects of the interaction of the EMU area with the rest of the world—and in particular the effects of fluctuations in the Euro exchange rate on macroeconomic adjustment and policy design in the EMU. Note, however, that the E(M)U economy as a whole, like that of the U.S, is a relatively closed economy, i.e., intra-EU trade and interaction dominate extra-EU trade and interaction. Adding such an external dimension to our analysis may change the incentives to coordinate stabilization policies in two ways: firstly, policy coordination inside the EMU can be helpful to deal with external effects; and secondly, the effectiveness of the fiscal and monetary policies is likely to be less in the presence of an external sector. These aspects are left for future research and may modify a number of conclusions from the current analysis.

## 5. Appendix: Proof of Theorem 1 on the feedback Nash equilibrium

### *Proof of Theorem 1:*

Consider the minimum cost-to-go from any initial state  $\bar{x}$  and any initial time  $t$ , as described by the so-called value function

$$V_i(t, \bar{x}) = \min_{v(s), t \leq s \leq t_f} \left\{ \frac{1}{2} x(t_f)^T K_{if} x(t_f) + \int_t^{t_f} g_i(x, v_1, v_2, v_3) ds \right\}, \quad (27)$$

with boundary condition  $V_i(t_f, \bar{x}) = \frac{1}{2} x^T K_{if} x$ . It is well known (see, e.g., Başar and Olsder, 1999, p. 243) that if there exists a continuously differentiable function  $V(t, x)$  satisfying the so-called Hamilton–Jacobi–Bellman (partial differential) equation

$$-\frac{\partial V_i(t, x)}{\partial t} = \min_v \left\{ \frac{\partial V_i(t, x)}{\partial x} (Ax + B_1 v_1 + B_2 v_2 + B_3 v_3) + g_i(x, v_1, v_2, v_3) \right\}, \quad (28)$$

subject to the boundary condition  $V_i(t_f, \underline{x}) = \frac{1}{2} x^T K_{if} x$ , then the pointwise minimization on the right-hand side of this equation (28) generates the optimal

strategy. Now, assume  $V_i(x, t) = \frac{1}{2}x^T K_i(t)x$ . Then (28) becomes:

$$\frac{1}{2}x^T \dot{K}_i x + \min_{v_i} \left\{ x^T K_i(t)(Ax + B_1 v_1 + B_2 v_2 + B_3 v_3) + \frac{1}{2}(x^T(t) v_1^T(t) v_2^T(t) v_3^T(t))^T F_i \begin{pmatrix} x(t) \\ v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} \right\} = 0. \quad (29)$$

Differentiation of the bracketed term with reference to  $v_i$  yields the next three equations:

$$\begin{aligned} x^T P_1 + v_1^T R_{11} + v_2^T N_1^T + v_3^T T_1^T + x^T K_1 B_1 &= 0, \\ x^T L_2 + v_1^T N_2 + v_2^T R_{22} + v_3^T V_2^T + x^T K_2 B_2 &= 0, \\ x^T S_3 + v_1^T T_3 + v_2^T V_3 + v_3^T R_{33} + x^T K_3 B_3 &= 0. \end{aligned} \quad (30)$$

From this we deduce, under the assumption that matrix  $G$  is invertible, that

$$\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1(t) \\ L_2^T + B_2^T K_2(t) \\ S_3^T + B_3^T K_3(t) \end{pmatrix} x(t). \quad (31)$$

Substitution of these expressions into (13) yields then the advertised set of coupled Riccati differential equations. Note that at this stage we only considered the plausibility of the presented formulae. To provide a formal proof of the theorem, one can proceed in a manner similar to, e.g., Başar and Olsder (1999), pp. 328–329.

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### Notes

1. The Stability and Growth Pact specifies the details of the Excessive Deficit Procedure of the Maastricht Treaty. An annual fall in real GDP of 2% or more is considered an exceptionally severe

downturn, and the deficit-to-GDP ratio can be above the 3% limit as long as a recession of this magnitude persists (although it must remain close to the reference value). If the decline in real GDP is between 0.75% and 2%, an exception can also be invoked if the Council of Ministers concurs.

2. In Engwerda et al. (1998, 1999), open-loop strategies were assumed that are computationally more accessible but admittedly less realistic than feedback strategies as a characterization of macroeconomic policy design.
3. Deeper aspects of the labor market, such as institutional features like social security, minimum wage regulation, structural skill mismatches, and active labor market programs that together determine structural unemployment, are disregarded in our analysis.
4. See, e.g., McKibbin and Sachs (1991) and Whitley (1992) for a detailed discussion on motivation, methodology, and structure of macroeconomic(etr)ic multicountry models that are based on the Mundell–Fleming model.
5. Given our focus on short-run macroeconomic stabilization in the EMU, our analysis ignores issues of intertemporal solvency and the associated possibility of risk premia. Note also that the introduction of government debt dynamics and risk premia would imply intrinsically nonlinear dynamics in the model, in which case we could not apply the technical apparatus of linear quadratic (LQ) differential game theory.
6. As noted by the referee, these simplifying assumptions impose important limitations on our analysis and may therefore lead to biases in its conclusions. For example, the growth effect of the fall in the Euro versus outside countries in its first two years is not present in the model. This fall may also have been fostered by a lack of coordination between monetary and fiscal policies in the Euro area. Ignoring this fall may exaggerate the importance attributed by the model to coordination.
7. Real interest in both EMU countries can therefore differ during the adjustment to long-run equilibria. Inflation expectations are rational—which implies, in the presence of uncertainty in the model, that all agents are endowed with perfect foresight.
8. These rationing schemes in the goods and labor markets are taken from the New Keynesian disequilibrium theory, initiated by Drèze (1975), Bénassy (1975), and Malinvaud (1977). See, e.g., German (1985) and Picard (1993) for a detailed overview of disequilibrium theory and dynamics.
9. A very detailed analysis of the procedures, motivations, and implications of the Stability and Growth Pact is given by Buti et al. (1997) and Eichengreen (1997), who suggest it might successfully deliver at the same time sufficient fiscal discipline and fiscal flexibility.
10. Note that we do not allow for coalition formation or side payments in this case of policy cooperation.
11. See, e.g., Lensberg (1988), Theorem 3.
12. This traditional role of monetary and fiscal policies as demand management policies and in the context of the EMU is the focus of Engwerda et al. (1999).
13. The set of model parameters underlying Figure 5 is therefore the same as the one underlying Figure 1.

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