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Theory and Methodology

Optimal periodic development of a pollution generating tourism industry ☆

Alfred Greiner [a], Gustav Feichtinger [b], Josef L. Haunschmied [b,*], Peter M. Kort [c], Richard F. Hartl [d]

[a] *Department of Economics, University of Bielefeld, Vienna, Austria*
[b] *Institute of Operations Research and Systems Theory, Vienna University of Technology, Argentinierstrasse 8, 1040 Vienna, Austria*
[c] *Department of Econometrics and Center, Tilburg University, Vienna, Austria*
[d] *Institute of Management Science, University of Vienna, Vienna, Austria*

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Abstract

This paper studies how environmental pollution affects optimal development of the tourism industry over time. The planner has the possibility to stimulate tourism by carrying out service expenditures, like organizing events, advertising, attracting seasonal workers, etc. The positive effect of these expenditures on tourism is negatively influenced by the presence of pollution, since the latter element distracts tourists from visiting a particular region. We show that for a particular scenario service expenditures, tourism as well as pollution exhibit a cyclical development over time. This policy implies that when pollution is high, tourism activities are reduced in order to give the environment a chance to recover. Environmentalists advocate this behavior, but in this paper, we show that this policy is also optimal from a profit maximizing point of view. © 2001 Elsevier Science B.V. All rights reserved.

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True ecotourists find themselves in the age-old tourist bind: they want to see something 'unspoiled', but help to spoil it just by being there. The Economist, January 10th, 1998.

1. Introduction

World population has more than doubled in the 50 years since World War II. Post-war technology, including jet aircraft, industrial and office automation and their concomitant social changes, has altered the pre-war work ethic and shortened the work week.

Paid vacation and travel away from home form now a human right. The fragile environment of our planet is rapidly endangered under the support of greatly increased number of leisured people.

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* Corresponding author. Tel.: +43-1-58801-11926; fax: +43-1-58801-11999.

E-mail address: josef.l.haunschmied@tuwien.ac.at (J.L. Haunschmied).

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Many tourists want an experience with nature. About 50 million visitors a year populating Spanish beaches are significant contributors to air, water and noise pollution. Up to 50 ski-lifts on a single Alpine mountain, impacted by traffic jams 100 km long, with an entire valley floor paved for parking clearly is an overused environment [1].

In many tourist sites, the rewarding phase of development is characterized by a long and intense growth of services that, sooner or later, seriously impact the environment, thus creating rather critical situations.

Snow-making is one example of a service expenditure that aims to raise the number of tourists. In [2], several polluting effects of snow-making are mentioned, like late snow cover, which leads to a shortened growing and grazing season, increased risks of gullying, mudslides and soil erosion, etc. Another example is the investment in the hotel business in Bulgaria and Hungary in order to make these resorts competitive in the West European market [3,4]. In the latter paper, it is argued that if the provincial hotels in Hungary are to enhance their customer base, additional opportunities must be exploited. Such opportunities exist in terms of the arts, countryside and sports pursuits, historic monuments and country parks. Additionally, for the increasing number of flexible, often motorized, self-organizing international visitors as well as the anticipated growing numbers of domestic tourists, there will be the need for reliable, good quality hotel accommodation at moderate cost in both Budapest and throughout the country. In Nepal, it is expected that the number of tourists visiting the Royal Bardia National Park will continue to rise with investment in transport infrastructure [5].

The present paper studies the negative impact of tourists on environmental quality. It assumes that more service expenditures lead to an increased number of tourists which generates more income for the region. At the same time pollution increases with the number of tourists, while environmental pollution in a region deters tourists from visiting that region. This intertemporal trade-off is studied in an optimal control framework. The aim of this research is not to solve a concrete problem by solving an empirically justified model with calibrated parameters. Instead, we intend to gain qualitative insights into the general problem by using a rather simple model.

The rest of the paper is organized as follows. Section 2 introduces the model, which is analysed in Section 3. Section 4 contains a numerical example, and Section 5 discusses the results. Finally, Section 6 concludes the paper.

2. The model and its necessary optimal conditions

The aim of this paper is to study intertemporal optimal service expenditures $I(t)$, measured in Euro, of a tourism planner who invests money in services in order to attract tourists. In general, we omit the time argument t if no ambiguity arises. Examples of tourist services are the organization of events, like concerts, exhibitions or the offer of special programmes, like guided tours in the mountains. Other flow expenditures which attract tourists are the cost for seasonal workers, like waiters, cooks in restaurants and personnel for ski-lifts. A better trained staff costs more but, on the other hand, leads to a higher quality of the services offered for tourists, making the region more attractive. Furthermore, we can think of advertising expenditures and costs of maintaining the present infrastructure.

These types of expenditures will be of particular importance if the region under consideration has been opened up. This means that our model is of particular relevance for regions in which additional infrastructure like hotels or ski-lifts will not raise the number of visiting tourists. Therefore, the stock of infrastructure is taken to be constant. This assumption enables us to focus better on the interaction between tourism and the environment.

More concretely, we assume that the change in the number of tourists $\dot{T}$ is a linear function of the effectiveness of services in the sense of attracting tourists. However, pollution P negatively affects the attractive power of services I, since a polluted environment distracts tourists from coming to this region. We consider a general stock of pollution without investigating pollution in depth. In our model formulation pollution is an indicator of environmental quality measured in physical units.

As to pollution we adopt an S-shaped specification implying that more pollution leads to a relatively strong decline in the growth rate of tourists as long as pollution is low. If the environment is already heavily polluted an additional unit of pollution merely leads to a small decline in the growth rate of the number of tourists, given a certain amount of service expenditures. This assumption can be justified by supposing that tourists visiting a polluted region are primarily interested in the services offered by the tourist region, and to a lower degree in a clean environment. Therefore, additional pollution will not deter many of them from visiting that region. Further, we assume that a given amount of investment is less effective, in case the stock of pollution is higher. The number of tourists, T, then evolves according to

$$\dot{T} = gI\mathrm{e}^{-(vP)^2} - bT, \quad T(0) = T_0 > 0, \tag{1}$$

with $g > 0$ a constant parameter indicating how much one additional Euro raises the change in the number of tourists visiting the region under consideration, other things being equal. The parameter $v > 0$ is a constant parameter determining how much an additional unit of pollution reduces the change in the number of tourists. The higher v, the higher will be the negative effect of an increase in pollution on the growth rate of tourists. The parameter $b \geqslant 0$ gives the decline in the number of tourists due to crowding effects. This means that a region becomes less attractive when lot of tourists visit that region, leading to a decrease in the number of tourists.

Environmental quality is negatively affected by the presence of tourism, implying that the stock of pollution, measured in physical units, rises with the number of tourists visiting the region. Furthermore, there is a natural regeneration process implying that nature is able to absorb a certain amount of polluting activities without being harmed. This is called the absorptive capacity of nature and is supposed to be of the form $\alpha(P) = mP\mathrm{e}^{-P/\bar{P}}$, m, $\bar{P} > 0$. The parameter m is a constant affecting the absolute value of the function $\alpha(P)$. The larger m the larger $\alpha(P)$. Further, the specification of $\alpha(P)$ implies that for values of P lower than $\bar{P}$ the absorptive capacity is low and rises with P or, formulated in a different way, if the environment is relatively clean, i.e., pollution is low, a given number of tourists cause high damages. The absorption capacity reaches a peak for $P = \bar{P}$ and declines again, meaning that nature cannot regenerate if the stock of pollution is high.

Summarizing our discussion from above, the evolution of pollution is given by

$$\dot{P} = \tau T - mP\mathrm{e}^{-P/\bar{P}}, \quad P(0) = P_0 > 0, \tag{2}$$

with $\tau > 0$ a constant parameter giving the rise in pollution due to an additional tourist.

The objective of the tourist planner is to maximize the discounted stream of cash flow generated by the tourism industry. As to the planner we suppose that it consists of representatives of different interest groups, like investors and the local government who decide together about the amount invested in the region under consideration. The cash inflow consists of tourism revenue, pT, while the cash outflow equals the service expenditures, which are assumed to be a quadratic function of the services. This takes into account that the higher the services, the more the overhead generated. Hence, the intertemporal objective becomes

$$\max_I \int_0^\infty \mathrm{e}^{-rt}\left(pT - \left(c_1 I + 0.5c_2 I^2\right)\right) \mathrm{d}t, \tag{3}$$

subject to (1) and (2). We assume that the income per tourist, p, is determined exogenously, which can be justified by the presence of strong competition among different tourist regions. r is the discount rate and should be in the range of the return on investment of comparable investment projects or of the return on capital in the tourism sector.

To find the optimal solution we form the current-value Hamiltonian (for an introduction to the optimality conditions of Pontryagin's maximum principle see [6] or [7]) which is

$$\begin{aligned} H(\cdot) = {} & pT - (c_1 I + 0.5c_2 I^2) + \lambda_1 (gI\mathrm{e}^{-v^2P^2} - bT) \\ & + \lambda_2(\tau T - mP\mathrm{e}^{-P/\bar{P}}), \end{aligned} \tag{4}$$

with λ_i, $i = 1,2$, shadow prices of T and P, respectively. λ_1 has the dimension of Euro per number of tourists and λ_2 has the dimension of Euro per physical units.

The necessary optimality conditions are given by

$$\frac{\partial H(\cdot)}{\partial I} = 0 \iff I = (\lambda_1 g e^{-v^2P^2} - c_1)/c_2, \tag{5}$$

$$\dot{\lambda}_1 = r\lambda_1 - \frac{\partial H(\cdot)}{\partial T}, \tag{6}$$

$$\dot{\lambda}_2 = r\lambda_2 - \frac{\partial H(\cdot)}{\partial P}. \tag{7}$$

If the maximized Hamiltonian was concave in P, the necessary optimality conditions would also be sufficient given that the transversality condition $\lim_{t\to\infty} e^{-rt}(\lambda_1 T + \lambda_2 P) = 0$ is fulfilled. Since, in our model, the maximized Hamiltonian is not globally concave in P, the necessary conditions are not sufficient for optimality. Therefore, the trajectories satisfying the necessary conditions are candidate optimal solutions.

The expenditures for services I are determined by the shadow price of tourists λ_1 and by pollution P. The more an additional tourist increases the cash flow the higher the service expenditures will be. Further, pollution negatively affects the number of tourists and, thus, lowers the cash flow. Therefore, the shadow price of pollution is expected to be negative. Moreover, pollution has a direct negative effect on the service expenditures which can be seen from (5).

3. Analysis of the canonical system

The dynamic behaviour of our model is given by (1), (2), (6) and (7). It should be noted that investment at any moment in time is a function of the shadow price λ_1 and of pollution P and is given by (5). Thus the dynamics is completely described by the following autonomous differential equation system:

$$\dot{T} = g\Big(\big(\lambda_1 g e^{-v^2P^2} - c_1\big)/c_2\Big)e^{-(vP)^2} - bT, \tag{8}$$

$$\dot{P} = \tau T - mPe^{-P/\bar{P}}, \tag{9}$$

$$\dot{\lambda}_1 = (r+b)\lambda_1 - p - \lambda_2\tau, \tag{10}$$

$$\dot{\lambda}_2 = r\lambda_2 + 2\lambda_1\Big(\big(\lambda_1 g e^{-v^2P^2} - c_1\big)\Big/c_2\Big)gv^2Pe^{-v^2P^2} + \lambda_2 m e^{-P/\bar{P}} - \lambda_2 mPe^{-P/\bar{P}}/\bar{P}. \tag{11}$$

Assuming that a stationary point $(T^{\star}, P^{\star}, \lambda_1^{\star}, \lambda_2^{\star})$ exists, the local dynamics around that stationary state is determined by the eigenvalues of the Jacobian matrix at this stationary point. The Jacobian matrix, i.e., the matrix of the first derivatives of (8)–(11) with respect to $(T, P, \lambda_1, \lambda_2)$ at the stationary point, is given by

$$J = \begin{pmatrix} -b & a_{12} & g^2e^{-2v^2(P^{\star})^2}/c_2 & 0 \\ \tau & a_{22} & 0 & 0 \\ 0 & 0 & b+r & -\tau \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix},$$

with

$$a_{12} = \frac{-4g^2\lambda_1^{\star}v^2P^{\star}}{c_2e^{2v^2(P^{\star})^2}} + \frac{2gc_1v^2P^{\star}}{c_2e^{v^2(P^{\star})^2}},$$

$$a_{22} = -\frac{m}{e^{P^{\star}/\bar{P}}} + \frac{mP^{\star}}{\bar{P}e^{P^{\star}/\bar{P}}},$$

$$a_{42} = \frac{-2\lambda_2^{\star}m}{\bar{P}e^{P^{\star}/\bar{P}}} + \frac{2g\lambda_1^{\star}\Big(-c_1 + g\lambda_1^{\star}e^{-v^2(P^{\star})^2}\Big)v^2}{c_2e^{v^2(P^{\star})^2}} + \frac{\lambda_2^{\star}mP^{\star}}{\bar{P}^2e^{P^{\star}/\bar{P}}} + \frac{4c_1g\lambda_1^{\star}v^4(P^{\star})^2}{c_2e^{v^2(P^{\star})^2}}\left(1 - \frac{2g\lambda_1^{\star}}{c_1e^{v^2(P^{\star})^2}}\right),$$

$$a_{43} = \frac{4g^2\lambda_1^{\star}v^2P^{\star}}{c_2e^{2v^2(P^{\star})^2}} - 2g\frac{c_1}{c_2e^{v^2(P^{\star})^2}}v^2P^{\star},$$

$$a_{44} = r + \frac{m}{e^{P^{\star}/\bar{P}}} - \frac{mP^{\star}}{\bar{P}e^{P^{\star}/\bar{P}}}.$$

The eigenvalues of that matrix are given by

$$\mu_{1,2,3,4} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{K}{2} \pm \sqrt{\left(\frac{K}{2}\right)^2 - \det J}},$$

with K defined as

$$K = \begin{vmatrix} -b & g^2e^{-2v^2(P^{\star})^2}/c_2 \\ 0 & b+r \end{vmatrix} + \begin{vmatrix} a_{22} & 0 \\ a_{42} & a_{44} \end{vmatrix} + 2\begin{vmatrix} a_{12} & 0 \\ 0 & -\tau \end{vmatrix}$$

$$= -b(b+r) + a_{22}a_{44} - 2\tau a_{12} \text{ (see [8])}.$$

Looking at the formula for the eigenvalues, one immediately realizes that the eigenvalues are symmetric around $r/2$. Since $r > 0$ holds, this implies that the system is never completely stable (in the sense that all eigenvalues have negative real parts), it can be at best saddle-point stable. From an economic point of view, saddle-point stability means that all variables are constant in the long run. That is there is a constant level of investment which is chosen such that the number of tourists visiting the region remains constant too. Further, the stock of pollution is also constant because the pollution caused by tourists just equals the absorptive capacity of nature. In this case, we may speak of a sustainable development since the environmental quality remains constant in the long run. The transitional behaviour of the variables in case of saddle-point stability is characterized by unimodal time paths if the eigenvalues are real. If the eigenvalues are complex conjugate, however, the variables dispose cyclical oscillations until the stationary point is reached. This means that there are periods with high investment followed by periods with low investment. But, in the long run, investment is constant and just high enough so that the number of tourists is kept at a constant level.

Besides convergence to the stationary state in the long run, the system may show persistent endogenous cycles. This behaviour can be observed if the dynamic system (8)–(11) undergoes a Hopf bifurcation (for a complete statement of the Hopf bifurcation theorem see, e.g., [9]).

Let us find out whether cyclical development of tourism and pollution may be observed in our model. From the formula of the eigenvalues (see, e.g., [8]) we know that $K > 0$ and $\det J > 0$ are necessary conditions for two purely imaginary eigenvalues and, thus, for the emergence of a Hopf bifurcation which leads to stable limit cycles. Looking at the constant K we see that there are two effects which may generate a positive K. First, if $P > \bar{P}a_{22}$ can become positive and, thus, the product $a_{22}a_{44}$ has a positive sign for r sufficiently large. This condition states that pollution must be in the range where an additional unit of this variable leads to a decline in the absorptive capacity of nature. The second effect which may lead to a positive K is $a_{12} < 0$. This condition states that an increase in the level of pollution has a negative impact on the evolution of tourists in the steady state. The determinant is calculated as

$$\det J = -(ba_{22} + \tau a_{12})((b + r)a_{44} + \tau a_{43}) + \tau^2 a_{42} g^2 e^{-2v^2(P^{\star})^2}/c_2.$$

For $a_{22} > 0$, $a_{44} > 0$ and $a_{12} < 0$, $a_{42} > 0$ and $ba_{22} + \tau a_{12} < 0$ are sufficient for a positive sign of the determinant.

The economic interpretation of cyclical strategies in this model is as follows. A high level of services attracts tourists and thus leads to an increase in the number of tourists. However, more tourists also cause more pollution which exerts a negative influence on the number of tourists and the latter will decline. This effect is intensified by the fact that more pollution also reduces the effectiveness of service expenditures, so that they are also reduced. When the number of tourists decreases, pollution declines, which attracts more tourists both directly and indirectly by raising service expenditures. As a consequence, the number of tourists will rise again. Below, we will give a more detailed discussion of the limit cycle using a numerical example.

4. A numerical example

In order to illustrate the possibility of persistent cycles we resort to a numerical example and choose the following parameter values: $v = \sqrt{0.195}$/physical unit, $g = 0.85$ tourist/Euro, $\bar{P} = m = 1$, $\tau = 0.816$ physical units/tourist, $b = 0.075$, $p = 0.1$, $c_1 = 0$, $c_2 = 0.03225$. The discount rate r is the bifurcation parameter.

First, we study the question of how many stationary states exist. To do so, we proceed as follows. We solve Eq. (9), $\dot{P} = 0$, with respect to T giving $T = T(P, \cdot)$ at the stationary state, with the $\cdot$ standing for the parameters which take the values mentioned above. Then we insert $T = T(P, \cdot)$ in Eq. (8), $\dot{T} = 0$, which then depends on λ_1 and P. Solving this equation with respect to λ_1 gives $\lambda_1 = \lambda_1(P, \cdot)$ at the stationary state. Next, we solve equation $\dot{\lambda}_1 = 0$ (10) with respect to λ_2 yielding

$\lambda_2 = \lambda_2(\lambda_1(P,\cdot),\cdot)$ at the stationary state which can be written as $\lambda_2 = \lambda_2(P,\cdot)$. Inserting $\lambda_1(P,\cdot)$ and $\lambda_2(P,\cdot)$ in the left-hand side of Eq. (11) gives the following function, which we define as $f(P,r)$ (computations were done with Mathematica, see [11])

$$
\begin{aligned}
f(P,r) \equiv\ & 0.000147061\mathrm{e}^{-2P+0.39P^2}P^3 \\
& + \frac{1.22549\left(-0.1+0.00410264\mathrm{e}^{-P+0.39P^2}P(0.075+r)\right)}{\mathrm{e}^P} \\
& - \frac{1.22549P\left(-0.1+0.00410264\mathrm{e}^{-P+0.39P^2}P(0.075+r)\right)}{\mathrm{e}^P} \\
& + 1.22549r\left(-0.1+0.00410264\mathrm{e}^{-P+0.39P^2}P(0.075+r)\right).
\end{aligned}
$$

Setting $f(P,r) = 0$ and solving this equation with respect to P gives a stationary state $P^{\ast} = P(r)$ for our dynamic system (8)–(11). The solution of $f(P,r) = 0$ crucially depends on the value of the discount rate r. Fig. 1 shows the combinations of P and r which satisfy $f(P,r) = 0$. In particular, we can identify three different regions:

1. for $r \in (0, r_1)$ there exists one stationary state, (Fig. 1(a));
2. for $r \in (r_1, r_2)$ there exist three stationary states, (Fig. 1(b));
3. for $r \in (r_2, \infty)$ there exists one stationary state, (Fig. 1(c) and (d)),

where $r_1(\approx 0.071)$ and $r_2(\approx 0.1361)$ are the values for r giving exactly two stationary states. In Fig. 1(b) one realizes that there exist three Ps such that $f(P,r) = 0$ holds for an $r \in (r_1, r_2)$ fixed.

In the ensuing analysis, we perform a numeric study of the eigenvalues for sufficiently dense set of discrete values of r and infer general statements from these numerical results. In particular, we are interested in the question of whether endogenous persistent cycles may occur for certain values of the discount rate.

In region 1, we set $r = 0.01$, $r = 0.03$ and $r = 0.07$. With these values for the discount rate the stationary state is saddle-point stable. For example, for $r = 0.03$ the stationary point is given by $T^{\ast} = 0.449177$, $P^{\ast} = 1.08824$, $\lambda_1^{\ast} = 0.00238647$, $\lambda_2^{\ast} = -0.122242$, implying $I^{\ast} = 0.049929$. The

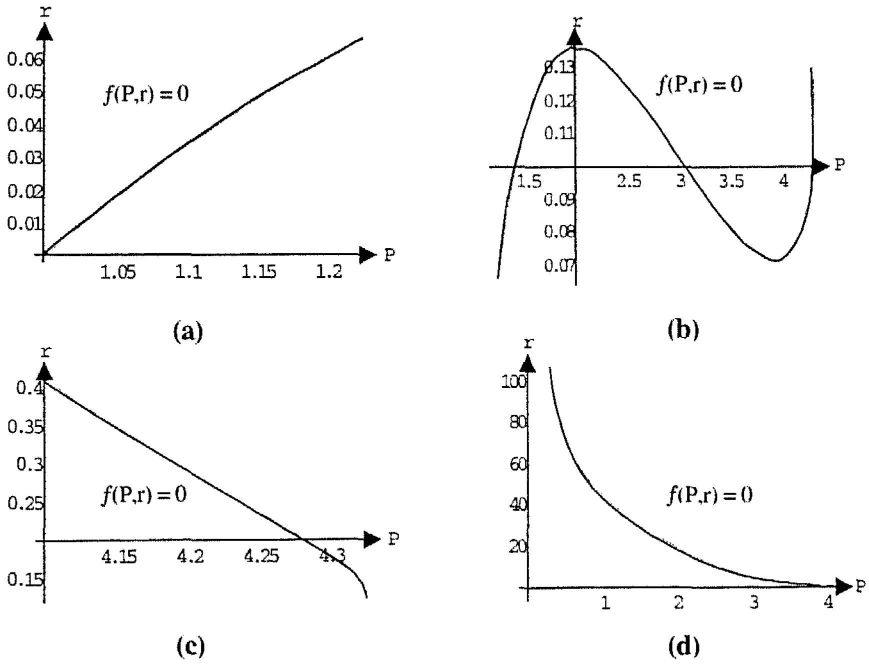

Fig. 1. Combinations of r and P giving a stationary point for (8)–(11).

eigenvalues are given by $\mu_{1,2} = 0.551312 \pm 0.553901\sqrt{-1}$ and $\mu_{3,4} = -0.521312 \pm 0.553901\sqrt{-1}$.

To study the dynamics for region 2, we first set $r = 0.13$. For this value of the discount rate the stationary points are given by $T^{1\star} = 0.0701502$, $P^{1\star} = 4.32483$, $\lambda_1^{1\star} = 0.345777$, $\lambda_2^{1\star} = -0.0356811$, with the eigenvalues $\mu_{1,2} = 0.133889 \pm 0.0959579\sqrt{-1}$ and $\mu_{3,4} = -0.00388928 \pm 0.0959579\sqrt{-1}$. The second stationary point is given by $T^{2\star} = 0.274707$, $P^{2\star} = 2.34966$, $\lambda_1^{2\star} = 0.00792001$, $\lambda_2^{2\star} = -0.120559$, with the eigenvalues $\mu_{1,2} = 0.065 \pm 0.322338\sqrt{-1}$, $\mu_3 = 0.321793$, and $\mu_4 = -0.191793$. The third stationary point, finally, is $T^{3\star} = 0.376025$, $P^{3\star} = 1.72896$, $\lambda_1^{3\star} = 0.00403908$, $\lambda_2^{3\star} = -0.121534$, with the eigenvalues $\mu_{1,2} = 0.335492 \pm 0.303857\sqrt{-1}$ and $\mu_{3,4} = -0.205492 \pm 0.303857\sqrt{-1}$. This implies that the first and third stationary points are saddle-point stable while a one-dimensional invariant stable manifold characterizes the second stationary point. Hence, practically it is not possible to find initial states with an optimal path converging to the second stationary point.

In general, it cannot be determined as to converging to which of the two (saddle-point) stable stationary points yields the higher value of the integral (3). To be exact the existence of two saddle-point stable stationary points does not automatically mean that there are two bounded candidate optimal solutions. All this may depend on the initial conditions as to T and P. On the other hand it is also feasible that the initial values of T and P are such that converging to the two stable stationary states yield the same value for (3), implying that the planner is indifferent between the two stationary states. Up to now such indifference points (first occurrence in the literature cf. [10]) have not often been computed. This field is an important topic for future research.

Varying the discount rate, we observe for $r = r_{\mathrm{crit}_1} = 0.1130763$ a Hopf bifurcation at the first stationary point which gives rise to stable limit cycles (for these computations we used the software LOCBIF [12]). For values of the discount rate smaller than the critical value r_{crit_1} stable limit cycles can be observed. Fig. 2 shows the limit cycle in the P–T–I phase diagram with $r = 0.11295$, i.e., with a discount rate of about 11.3%. For $r = 0.11295$, the first stationary point is given by $T^{1\star} = 0.070462$, $P^{1\star} = 4.31906$, $\lambda_1^{1\star} = 0.340623$, $\lambda_2^{1\star} = -0.044093$. For $r = 0.11295$, the second and third stationary points are unstable and saddle-point stable, respectively, that is the qualitative

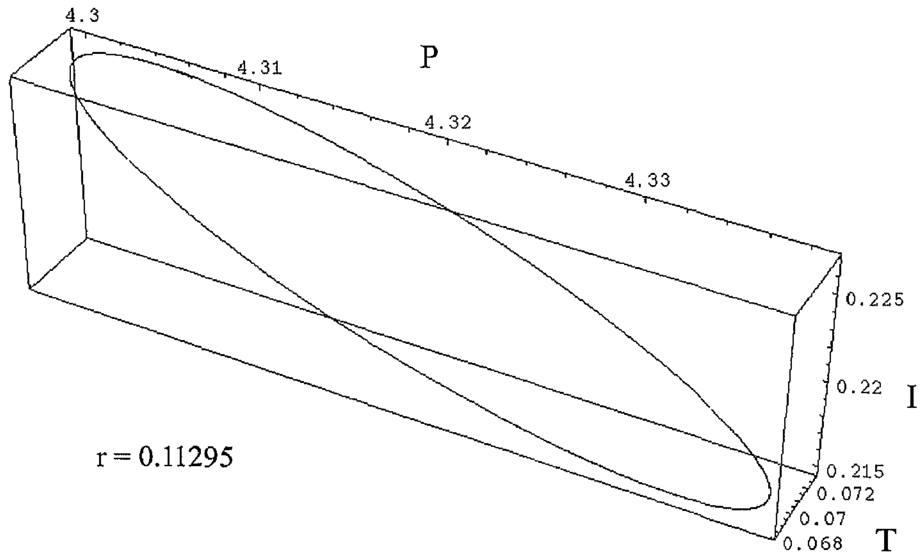

Fig. 2. Phase portrait of the I–T–P-space.

dynamics does not change for these stationary points.

If we further decrease r the first stationary point becomes unstable, that is the real parts of the eigenvalues of the Jacobian evaluated at this rest point become positive. The second and third stationary points remain unstable and saddle-point stable, respectively, when the discount rate is decreased (we computed the eigenvalues with r between $r = 0.075$ and $r = 0.11$ using a step size of 0.005).

In region 3, there exists one stationary point which is saddle-point stable for about $r < 1.812$. Setting $r = r_{\text{crit}_2} = 1.811854$ another Hopf bifurcation can be observed which leads to stable limit cycles. (Interpreting one time period as one year implies that the annual discount rate is about 180%. That is the planner is extremely myopic in this case.).

Discount rates of this or higher size mean practically an infinite discount rate. However, for the sake of completeness we add the mathematical results on large-sized discount values. Increasing the discount rate further, the system becomes unstable and for $r = r_{\text{crit}_3} = 36.47188$ a third Hopf bifurcation occurs which generates stable limit cycles. The cycles are observed for values of the discount rate smaller than the critical value r_{crit_3}. For discount rates larger than r_{crit_3} the Jacobian has complex conjugate eigenvalues with two negative real parts, i.e., it is a saddle-point. For r between 69 and 70 the eigenvalues become real, with two having negative signs and two having positive signs.

5. Discussion of the limit cycle

We now give an economic description of the way in which persistent oscillations come about. We focus on this solution since there the economic interpretations are not trivial. Fig. 3 shows the cyclical time paths of tourist services, the number of tourists and the level of pollution with r set to $r = 0.11295$.

The cycle can be divided into several phases.

Phase 1: We start with a clean environment and a medium number of tourists at $t = t_1$. Since P is small, a given level of service expenditures leads to a large number of tourists. In this short phase all three variables, i.e., P, I, and T increase.

Phase 2: After the services I 'peak' at time t_2, it is reduced in order to reduce costs and to prevent the environmental pollution from rising beyond a (justifiable) level. Due to the still relatively high level of tourist services, the number of tourists, T, still increases, and so does pollution, P. This phase

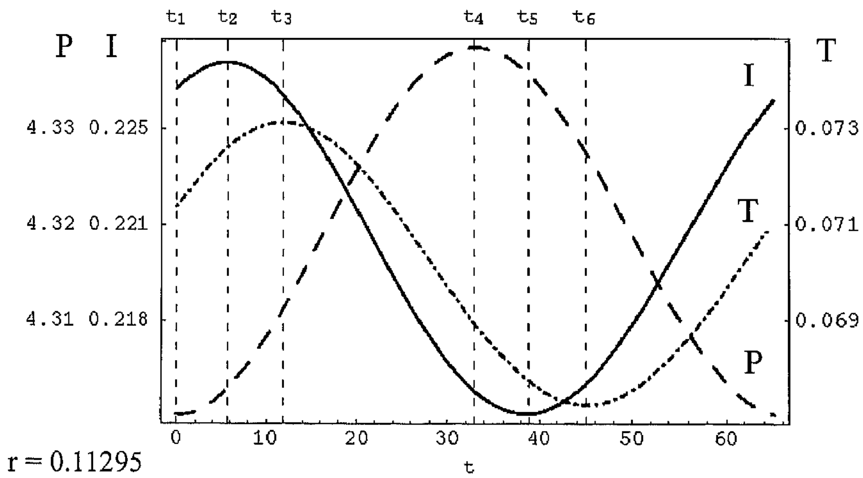

Fig. 3. Time paths of the control variable I, and the state variables T and P.

ends with the 'tourist boom' in t_3. These two phases might be denoted as 'booming regime'.

Phase 3: Now, both I and T fall, while P still increases. Saturation of tourism is reached, but pollution increases until it reaches its peak at time t_4. This holds because the number of tourists is still relatively high so that pollution does not yet decline. Further, a rising level of pollution negatively affects the absorptive capacity of nature which tends to raise pollution.

Phase 4: From now on the development is the mirror image of the first three phases. Therefore, we only give a short description. Shortly after t_4, i.e., in t_5, tourist service expenditures are minimized to counter-act the high pollution. All these variables, I, T, and P decrease during this period.

Phase 5: Due to high pollution and low service expenditures increasingly more tourists are deterred from visiting the region we consider. Thus, at the end of this period the number of tourists reaches its minimum. Definitely, this and the former phases might be denoted as 'declining regime'.

Phase 6: In this period, both I and T increase, while P still decreases. This means that we have a recovery governed by the same 'spirit of launching'. In t_1 the cycle starts anew.

The essence of this 'endless story' is that a profit-maximizing planner will protect the environment by an adequate, i.e., in our case an oscillatory service expenditure policy. Or, to put in other words: even if environmental quality is no explicit target of the planner, it plays implicitly an important role by attracting tourists (or by deterring them from a polluted area).

6. Conclusion

In this paper, we considered the optimal service expenditure policy a tourism planner follows. Assuming that tourists are attracted by tourist services and a clean environment which, however, suffers from a large number of tourists we could demonstrate that an oscillatory service expenditure policy may be optimal. This means that, in the long run, the planner does not invest a constant amount in tourist services. As a consequence, the number of tourists as well as the state of the environment show cyclical oscillations, too. This implies that regions, which are less attractive for some time, may recover and attract tourists in the future. It should be mentioned that the cash flow generated by tourists and, thus, the revenue generated by the tourism industry in this region, is also subject to oscillations given the cycles in the number of tourists.

As to the formulation of our model, other assumptions would make sense, too. First, it could be imagined that a stock of tourist infrastructure attracts tourists. In the model, this would mean that a third state variable must be added. Further, the level of pollution could enter the objective functional directly, i.e., the planner takes into account that a clean environment yields immediate benefits for the population in the region under consideration. We are aware that our version is just one possibility to model the interaction between investment in tourist services, the number of tourists and the state of the environment. But we think that the model, although highly stylized, does make sense and yields insights into the problem studied in this paper. Other, possibly more complex, variants are left for future work.

A last point we want to mention concerns the empirical relevance of our model. Looking at some tourist regions in the Alps it seems to be true that tourist regions are characterized by flourishing phases followed by phases of stagnation. However, to our knowledge there do not exist thorough empirical studies which try to explain this development empirically. This is an open but nevertheless important topic which is also left to future work.

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