Anticipation Effects of Technological Progress on Capital Accumulation: a Vintage Capital Approach*

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Abstract

Due to embodied technological progress new generations of capital goods are more productive. Therefore, in order to study the effects of technological progress, a model must be analyzed in which different generations of capital goods can be distinguished. We determine in what way the firm adjusts current investments to predictions of technological progress. In the presence of market power we show that a negative anticipation effect occurs, i.e. current investments in recent generations of capital goods decline when faster technological progress will take place in the future, because then it becomes more attractive to wait for new generations of capital goods. In case that only investments in new machines are possible, actually a whole wave of anticipation phases arises.
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1 Introduction

This paper studies the effect embodied technological progress has on the firm's capital accumulation process. Embodied technological progress implies that capital goods of later date are more productive. In order to analyze the implications of this feature, a model must be built in which capital stocks of different building years can be distinguished. To do so a vintage capital goods model is developed in which capital goods of younger vintages are more productive.

In a vintage capital goods model productivity of a capital good is completely determined by its age and the year in which it actually operates. In view of technological progress, productivity increases with time, for capital goods of a given age. On the other hand, over time workers get more experienced in working with the same machine over the years, so that the productivity per machine increases with age. For example, due to a learning curve the LCD industry experiences a so-called ramp up time (time needed to start up a production line) with a strongly increasing yield (amount of good products relative to the total amount of products) in the first quarters after the start of production. Hence, a trade-off arises: new machines are relatively productive, because they embody superior technology, but due to learning effects it can still happen that working with old machines leads to higher productivity than working with new machines.

Technological progress is increasingly embodied in new capital goods (see Dekle [14]). To illustrate, Gordon [18] has shown that the relative price of capital has declined fairly steadily and rapidly in the post-war US and other economies. Moreover, Greenwood et al. [19] found that embodied technological progress is the main driver of economic growth. They discovered that in the post-war period in the US about 60% of labor productivity growth was investment-specific. A main example is information technology as noted by Yorokoglu ([32], p.552): "*Information technology capital has a very high pace of technological improvement. Compared with more traditional types of capital, the efficiency of information technology capital has increased much faster over the last few decades. As an example, consider the market for personal computers. IBM introduced its Pentium PCs in the early 1990s at the same price at which it introduced its 286 PCs in the 1980s. Therefore it took less than a decade for the computing technology to improve on the order of 20 times in terms of both speed and memory capacities, without increasing the cost*".

In Chari and Hopenhayn [13] three questions are posed: (i) why are new technologies often adopted so slowly?, (ii) why do people often invest in old technologies even when apparently superior technologies are available?, and (iii) how are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future? To answer the second question, the work by, e.g., Solow et al. [27], Malcomson [24], Benhabib and Rustichini [3], and Boucekkine et al. ([5], [7], [8], and [9]) cannot be used, because there it is only possible to invest in the newest generation of capital goods. The common denominator of the just mentioned contributions is the application of delayed differential equations. Using partial differential equations for the analysis of vintage capital we were able to answer Chari and Hopenhayn's second question, because this allowed us to explicitly introduce the possibility of investing in older capital goods, thus in non-frontier vintages (see Feichtinger et al. [16]). In that paper it was found that investing in older technologies can be preferred because older technologies are cheaper, due to experience effects it is easier to implement them, and due to learning machines can be more efficiently used in the production process over the years. As to the third question of Chari and Hopenhayn, it was found in Feichtinger et al. [16] that the decision to adopt new technologies is not affected by future technological progress. The reason is that in the model of Feichtinger et al. [16] the firm exerts no market power. Hence, increasing its own production does not have an effect on the output prices, so that the NPV's of capital stocks of different age do not influence each other. If the firm does yield market power, in contrast, investing today decreases the output price during the lifetime of the machine in the future. Hence, investing today reduces the NPV of future investments. Purpose of this paper is to analyze the effect of future technological developments on the desirability of current technology investments, while the firm has market power. To do so, we again apply the partial differential equation approach and we assume for simplicity that the firm under consideration is a monopolist.

However, considering market power makes the mathematical analysis much more complicated. In fact, its study requires the application of a new maximum principle for age-structured control systems, that was developed in Feichtinger et al. [17]. The reason is that with market power the output price becomes dependent on production, which in turn depends on the capital goods of all generations present within the firm. Technically, this implies that in the objective an integral term within the revenue function occurs.

The bulk of the literature that applies the capital vintage approach to study the implications of embodied technological progress adopt the dynamic general equilibrium framework. One of the recent papers in this area, Pakko [25], finds that technology shocks are accompanied by initial declines of investment followed by a long transition period of higher growth. Such an initial decline of investment is due to increased future economic depreciation of current investment. That is, an anticipated increase in the rate of future embodied technological progress increases the anticipated rate of economic depreciation of capital and therefore decreases the present discounted value of marginal revenue produced with current investments. In the dynamic general equilibrium set up this economic depreciation is due to the assignment of labor across vintages. That is, the new vintages will draw more labor away in the future from the current vintage and therefore reduce its marginal product.

Our partial set up allows us to analyze the implications of market power leading to marginal revenue being decreasing in production. From an economic point of view this is important, because oligopolistic market structures become more dominantly present nowadays. For instance, Pawlina [26] notes that: "*The extensive process of deregulation taking place in the last decade, combined with a wave of mergers and acquisitions, has resulted in an oligopolistic structure of a large number of sectors. A shift towards such a structure takes place not only in traditional markets (telecommunications, energy, transportation) but also in more competitive industries (fast-moving consumer goods, car manufacturing, pharmaceuticals).*" An oligopolistic market structure also applies to industries like semi conductor (computer chips) and LCD screens, where the supply side only consists of a few major players and where technological progress being embodied in new technologies is of prime importance for investment decision making.

Like Pakko [25], we also found that a technology shock is anticipated on by an investment decline in new machines before the shock occurs. This is because the firm wants to benefit as much as possible from this shock by investing in new machines just after the shock has occurred. The investment decline just before the shock makes that more new capital goods can be purchased after the shock without reducing the output price too much. So, this negative anticipation effect is similar to Pakko [25], but while in Pakko [25] this is caused by a redistribution of labor among the vintages, in our set up the cause is the presence of market power leading to decreasing marginal revenue.

However, in addition to Pakko [25] we found two more effects. *First,* this negative anticipation effect generates a whole wave of alternating positive and negative anticipation effects taking place backwards in time. This wave is also caused by the presence of market power. Take for instance the positive anticipation phase occurring just before the last negative anticipation phase. This one is caused by the fact that during this last negative anticipation phase production is low leading to a high output price. This makes investing more profitable which induces the positive anticipation phase. However, during a positive anticipation phase production is higher and thus price is lower, which in turn leads to another negative anticipation phase occurring just before this positive one. In this way the whole wave of alternating anticipation phases can be explained. In the model we assume that capital goods have a finite life time (note that this is a disadvantage compared to the delayed differential equation approach, where the time to scrap is endogenously determined). The periodicity of the waves is crucially influenced by the length of this finite life time. Also, we find that the anticipation waves are a symmetric image of the echo effects found in papers applying the delayed differential equation approach (Benhabib and Rustichini [3], and Boucekkine et al. [6]).

Second, contrary to the dynamic general equilibrium literature, in our framework the firm can also invest in non-frontier vintages. This makes that the negative anticipation phase occurring just before the technology shock does not necessarily lead to a decline in production, as was the case in Pakko [25]. This is due to the fact that the investment decline in new capital goods is accompanied by increased investments in older capital goods. The advantage is that older capital goods are cheaper and that their lifetime is shorter, so that they can be replaced sooner by the new machines that embody the new technology from after the technology shock. The fact that now the negative anticipation phase does not lead to a decline in production makes that the wave of anticipation phases disappears when investments in non-frontier vintages are also possible.

The contents of the paper is as follows. The model and the necessary optimality conditions are presented in Section 2. Section 3 contains the economic analysis, while Section 4 concludes.

2 The Model

Consider a model of a "new firm", being created at time zero, producing a single good by means of a continuum of vintage capital technologies, with finite life time. The firm has some market power and faces an increasing but concave revenue function. At every period the firm has access to machines in the primary and the secondary market and faces adjustment costs in both markets. Productivity of these machines is influenced by learning and technological progress. As a consequence of the adjustment costs and the presence of learning, the firm adopts the frontier technology slowly.

To analyze the implications of embodied technological progress we have to distinguish between different generations of machines. To do so we explicitly introduce the age of the machine, which is denoted by a. Each machine has a fixed maximal lifetime ω, so that $a \in [0, \omega]$. However, the investment can be negative and the firm has the possibility to sell the machines before the end of the lifetime.

The machines are used to produce goods. The number of goods produced in year t by a machine of age a is given by $f\left(t-a\right)v\left(a\right)$. Since people have to learn how to use new technologies, technological progress must be accompanied by learning (Greenwood and Jovanovic [20]). Here $v\left(a\right)$, with $v' \geq 0$, reflects that due to learning machines can be more efficiently used in the production process over the years[1]. The function $f(t-a)$, with $f' > 0$, captures the fact that due to embodied technological progress new machines are more productive.

If we denote the stock of capital goods of age a at time t by $K(t,a)$, it follows that production at time t is given by

$$Q(t) = \int_0^{\omega} f(t-a)v(a)K(t,a)\,\mathrm{d}a.$$

Due to market power the output price decreases with production, which makes that the firm's revenue, $R(Q(t))$, is concavely increasing in production, so that $R' > 0$, $R'' < 0$. The results obtained in this paper will also hold if the concavity of the revenue function is caused by decreasing returns to capital with respect to production (see, e.g. Benhabib and Rustichini [4]), while the output price is constant[2]. However, for sake of readability, in what follows we stick to the market power interpretation.

Capital stock can be increased by investing. Investments in machines of age a in year t are denoted by $I(t,a)$. In order to invest the firm has to incur acquisition and implementation costs. Costs of acquisition are given by $b(a)I(t,a)$, where $b' < 0$, since older machines are cheaper. The cost of successful implementation of investments is $\frac{c(a)}{2}(I(t,a))^2$. Here it holds that $c' \leq 0$, which exhibits that it is easier to implement older machines because of experience effects (Stenbacka and Tombak [28]). Including such adjustment costs is important (cf. Wirl [30]). As a topical example consider the introduction of computers during the last two decades where possibly the necessary costs of adjusting outweighed so far the entire associated productivity gain (see, e.g., Kiley [23]).

In the model investments in the newest generation of machines, thus with age $a = 0$, are explicitly distinguished and denoted by $I_0(t)$. Analogous to the previous paragraph we get that $b_0 I_0(t)$ are the acquisition costs of these investments, and $\frac{c_0}{2}(I_0(t))^2$ are the costs of successful implementation. Of course, the firm has no experience at all with the installation of the newest generation of machines. Therefore, it holds that $c_0 > c(a)$ for $a > 0$.

Consider at each moment in time, machines of a given age a. It holds that due to embodied technological progress less investments, leading to lower investment expenditures, are needed to produce a given amount of output. Then it can be concluded that in our model the relative price of equipment in terms of output, that is, the cost of investments needed to produce a given amount of output, declines as $f(t-a)$ increases with time. This is in accordance with Greenwood et al. [19], who further state that: "*Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the start and the long run. Concrete examples in support of this interpretation abound: new and more powerful computers, faster and more efficient means of telecommunication, robotization of assembly lines, and so on.*"

[1] Our analysis and results do not employ the monotonicity of v, and therefore they are valid also for other interpretations than learning, where v can be decreasing or non-monotone (see e.g. [16]).

[2] We thank an anonymous referee for pointing this out to us.

As another (initial) control we denote by $K_0(a)$ the investments in capital stock of age a in year 0. The costs are similar to those of $I(t,a)$. If the firm exists already at time $t=0$, then $K_0(a)$ can be viewed as an initial condition, in which case the associated costs are zero.

As distinct from the bulk of the literature using delayed-differential equations (e.g., Benhabib and Rustichini [3], and Boucekkine et al. ([5], [7], [8], and [9]), where it is only possible to invest in the newest generation of capital goods, we apply a partial differential equation approach. The main reason is that in this way we can include the possibility to invest in non-frontier vintages. The evolution law of capital is described by

$$K_t + K_a = I(t,a) - \delta(a) K(t,a), K(t,0) = I_0(t), K(0,a) = K_0(a),$$

where the subscripts denote partial differentiation, and $\delta(a)$ is the depreciation rate of a machine of age a. This specification is based on the fact that time and age move together: machines aged $a+\mathrm{d}t$ in $t+\mathrm{d}t$, are those aged a at time t, minus their depreciation, plus (minus) machines of this vintage bought (sold) in the secondary market at time t. Passing to a limit with $\mathrm{d}t$, one obtains the above equation.

The firm is assumed to maximize the discounted value of the cash flow over an infinite planning period. Denoting the discount rate by r, the resulting age specific dynamic model of the firm is given by[3]

$$\begin{aligned} \max_{I(t,a),I_0(t),K_0(a)} \quad & \left\{ \int_0^\infty e^{-rt} \left[R(Q(t)) - \left(b_0 I_0(t) + \frac{c_0}{2}(I_0(t))^2 \right) \right.\right. && (1)\\ & \left. - \int_0^\omega \left(b(a) I(t,a) + \frac{c(a)}{2}(I(t,a))^2 \right) \mathrm{d}a \right] \mathrm{d}t \\ & \left. - \int_0^\omega \left(b(a) K_0(a) + \frac{c(a)}{2}(K_0(a))^2 \right) \mathrm{d}a \right\} \end{aligned}$$

$$\begin{aligned} K_t + K_a &= I(t,a) - \delta(a)K(t,a), \quad K(t,0) = I_0(t), \quad K(0,a) = K_0(a), && (2)\\ Q(t) &= \int_0^\omega f(t-a)v(a)K(t,a)\,\mathrm{d}a. && (3) \end{aligned}$$

Until now, in the literature only a few papers exist that analyze the vintage differentiation of the capital goods in a complete dynamic optimization framework. The reason is, perhaps, that "full dynamics are notoriously difficult in such models" (Jovanovic [22], pp. 523-524). Contributions offering a full dynamic framework, but then without learning and technological progress, are Barucci and Gozzi [1], and Feichtinger et al. [15]. The same

[3] The precise meaning of optimality for the next problem is given in the Appendix.

holds for Xepapadeas and De Zeeuw [31], in which the composition of capital is studied in connection with environmental policy and the Porter hypothesis[4].

Embodied technological progress is introduced in Barucci and Gozzi [2] and Feichtinger et al. [16], while in both papers the output price is fixed. Due to the latter feature, different predictions of future technological progress have no influence on current investments, since these do not influence the NPV's of later investments. The current paper extends Barucci and Gozzi [2] and Feichtinger et al. [16] by introducing market power. Then current investments increase production which reduces output price and thus the NPV's of future investments. In such a setting it is expected that future embodied technological progress *does* have an influence on the desirability of current investments. The aim of our paper is to determine this influence and thus to provide an answer to Chari and Hopenhayn's third question: how are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future?

Also mathematically, there is a significant difference between the model in Barucci and Gozzi [2] and our model, which is caused by the output-dependent price. The former can be treated by the general maximum principle in Brokate [10] or even by the more specific ones in Chan and Guo [12]. Our model is more complicated because of the nonlinear dependence of the objective function on the integral term in expression (3). We apply the general maximum principle from Feichtinger et al. [17], which, in contrast to that in [10], allows initial (K_0) and boundary (I_0) controls.

2.1 Maximum Principle

To solve the model we apply the maximum principle recently obtained in Feichtinger et al. [17]. The assumptions are stated in a mathematically more precisely manner in the Appendix. First, we introduce

$$
\begin{aligned}
L(t, a, Q, I, I_0, K_0) = e^{-rt} & \left[\frac{1}{\omega} \left(-R(Q) + b_0 I_0 + \frac{c_0}{2} I_0^2 \right) \right. \\
& \left. + b(a) I + \frac{c(a)}{2} I^2 + r \left(b(a) K_0 + \frac{c(a)}{2} K_0^2 \right) \right]
\end{aligned}
$$

in order to represent the objective function in the general form

$$
\min \int_0^{+\infty} \int_0^{\omega} L(t, a, \ldots) \, \mathrm{d}a \, \mathrm{d}t.
$$

[4] In that paper the function $v(a)$ is incorrectly interpreted as technological progress instead of ageing of the capital good (in their model it was imposed that $v' \leq 0$, rather than $v' \geq 0$ as in our model, where, due to learning, productivity may increase with age).

Then the initial, boundary and distributed Hamiltonians, depending on the adjoint variables $\xi(t,a)$ and $\zeta(t)$, which correspond to $K(t,a)$ and $Q(t)$, respectively, take the form:

$$
\begin{aligned}
H_0(a,K_0) &= \int_0^\infty L(t,a,K_0)\,\mathrm{d}t + \xi(0,a)K_0,\\
H_b(t,I_0) &= \int_0^\omega L(t,a,I_0)\,\mathrm{d}a + \xi(t,0)I_0,\\
H(t,a,I) &= L(t,a,I) + \xi(t,a)(I-\delta(a)K(t,a)) + \zeta(t)f(t-a)v(a)K(t,a),
\end{aligned}
$$

where by convention all arguments of L that are skipped, are evaluated along the optimal solution. For example, $L(t,a,K_0) = L(t,a,Q(t),I(t,a),I_0(t),K_0)$.

The maximum principle implies that there exists a solution of the adjoint equations

$$
\begin{aligned}
-(\xi_t+\xi_a) &= -\xi(t,a)\delta(a) + \zeta(t)f(t-a)v(a), \quad \xi(t,\omega)=0,\\
\zeta(t) &= -e^{-rt}R'(Q(t)),
\end{aligned} \tag{4}
$$

such that each component of the optimal control (I, I_0, K_0) minimizes the corresponding Hamiltonian. From equation (4) it is obtained that the co-state variable $\xi(t,a)$, being the shadow price of capital stock, depends on marginal revenue, $e^{-rt}f(t-a)v(a)R'(Q(t))$, and marginal depreciation cost, $\xi(t,a)\delta(a)$.

Introducing the new variable $\lambda(t,a) = -e^{rt}\xi(t,a)$, and substituting $-e^{-rt}R'(Q(t))$ for ζ, we obtain the equation

$$
\lambda_t + \lambda_a = \lambda(t,a)(\delta(a)+r) - R'(Q(t))f(t-a)v(a), \quad \lambda(t,\omega)=0, \tag{5}
$$

and the following first-order optimality conditions:

$$
K_0(a) = \frac{1}{c(a)}(\lambda(0,a)-b(a)), \tag{6}
$$

$$
I_0(t) = \frac{1}{c_0}(\lambda(t,0)-b_0), \tag{7}
$$

$$
I(t,a) = \frac{1}{c(a)}(\lambda(t,a)-b(a)). \tag{8}
$$

3 Economic Analysis

From (5), (8), and (7) it is obtained that the amount of investment is given by

$$
I(t,a) = \frac{1}{c(a)}\left[\int_a^\omega e^{-\int_a^s(\delta(\theta)+r)\,\mathrm{d}\theta} R'(Q(t-a+s))f(t-a)\,v(s)\,\mathrm{d}s - b(a)\right] \tag{9}
$$

for older machines, and

$$I_0(t) = \frac{1}{c_0}\left[\int_0^{\omega} e^{-\int_0^s(\delta(\theta)+r)\,\mathrm{d}\theta} R'(Q(t+s))f(t)\,v(s)\,\mathrm{d}s - b_0\right] \tag{10}$$

for new machines. It follows that the discounted revenue stream over the remaining planning period due to an extra unit of investment at time t, is equal to marginal investment costs. The two equations thus indicate that the firm invests according to a net present value rule such that the net present value of marginal investment equals zero.

The firm disinvests in a situation where the integral in the r.h.s. of (9) is smaller than $b(a)$. In that case the value of the machine is smaller than its market price $b(a)$, so that the firm optimally decides to sell it. Since adjustment costs are symmetric for creation and destruction of capital goods, obsolescence is a gradual process guided by (9). At the moment this vintage reaches its maximal lifetime ω, the remaining capital stock of this vintage is scrapped.

We are primarily interested in how investments in machines of a given age develop over time. As to a first investigation consider investments in new machines, thus with age equal to zero, and obtain from (10) that (for machines with age greater than zero a similar expression is obtained):

$$\begin{aligned} c_0\frac{\mathrm{d}I_0(t)}{\mathrm{d}t} &= \int_0^{\omega} e^{-\int_0^s(\delta(\theta)+r)\,\mathrm{d}\theta} R'(Q(t+s))f'(t)v(s)\,\mathrm{d}s \\ &\quad + \int_0^{\omega} e^{-\int_0^s(\delta(\theta)+r)\,\mathrm{d}\theta} R''(Q(t+s))Q'(t+s)f(t)v(s)\,\mathrm{d}s. \end{aligned}$$

So, there are two effects. The first effect is positive, and shows that embodied technological progress ($f'(t) > 0$) induces delay of the adoption of new technologies, since the productivity of machines increases over time. The second effect reflects that the NPV is dependent on output: during a growth phase the NPV of marginal investment decreases over time because marginal revenue decreases with production. Hence, due to the latter effect investments in machines of a given age reduces over time during a growth phase, while the first effect says that investments should increase because at a later point of time machines will be more productive. Note that during a contraction phase both effects result in postponement of investments.

Our ultimate aim is to provide an answer to the question: how are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future? To do so, we analytically investigate whether an expected increase in technological progress at some moment of time $\hat{t}$ influences the optimal investment policy prior to this moment. Numerical results are provided in the next subsection. In Feichtinger et al. [16] we show that future technological developments have no effect at all on current investments if the firm has no market power. The situation changes, however, in the case of a firm with market power.

We compare two scenarios for technological progress: the first scenario is described by a given technology function $f(t)$ (the benchmark case), while in the second scenario the technology function is $\hat{f}(t)$, where

$$\hat{f}(t) = f(t) \quad \text{for } t \le \hat{t}, \qquad \text{and } \hat{f}(t) > f(t) \quad \text{for } t > \hat{t}. \tag{11}$$

That is, in the second scenario a technological breakthrough takes place at $\hat{t}$. The value

$$\liminf_{t \to \hat{t}+0} \frac{\hat{f}(t) - f(t)}{t - \hat{t}} \tag{12}$$

is a measure for *the level of the technological breakthrough* relative to the benchmark $f(\cdot)$.

We compare the behavior of the firm for these two scenarios. This comparison is aimed to establish how the expectation of a technological breakthrough at a certain time $\hat{t}$ influences the investment behavior of the firm *prior* to $\hat{t}$, versus the case where such a breakthrough is neither expected nor occurring[5]. To make the model more tractable, we only consider investments in new machines, that is, $I(t, a) \equiv 0$. We again allow for positive $I(t, a)$ in the next subsection, where numerical results are presented.

Let $I_0(\cdot)$ and $\hat{I}_0(\cdot)$ be the optimal investments in new machines corresponding to the above two technology functions.

Definition 1. We say that a negative anticipation effect takes place at $\hat{t}$ if

$$\hat{I}_0(t) < I_0(t) \ \text{ for some } \quad t < \hat{t}.$$

We make the following assumptions:

(S1) $b_0 > 0$, $\hat{t} > 3\omega$, and $I_0(\hat{t}) > 0$;

(S2) the learning function $v(\cdot)$ is continuous and $v(s) > 0$ for $s \in (0, \omega)$; the technology function $f(\cdot)$ is positive, Lipschitz continuous and non-decreasing.

Proposition 1 *A sufficiently strong technological breakthrough at time $\hat{t}$ creates a negative anticipation effect at $\hat{t}$.*

[5] An alternative scenario was suggested by an anonymous referee: compare two firms, one of which anticipates the breakthrough at $\hat{t}$, that is, knows $\hat{f}$ in advance, while the other has the same technology function, but is not aware of the breakthrough at $\hat{t}$ beforehand. This implies that before time $\hat{t}$ the latter firm fixes its investment policy as if the breakthrough at $\hat{t}$ will not take place. The investment behavior of these two firms is expected to be different after $\hat{t}$, because the second one has to adjust to the unexpected change of technology. Figure 3 in the next section illustrates this scenario. However, before time $\hat{t}$ — and this is the main concern in the paper — the investment paths of the two firms will be exactly the same as in the two scenarios considered in the main text.

In the Appendix we give a mathematically strict formulation of this and of the next proposition and provide proofs.

Remark. If a technological breakthrough would lead to a jump of $\hat{f}$ at $\hat{t}$, then the negative anticipation effect will take place, no matter how small is the jump. The computational results presented in the next subsection register the negative anticipation effect also in the case where only the steepness of f increases at $\hat{t}$ (without jump). This, however, need not be the case in general, unless the change in the steepness is sufficiently large, as indicated in the formulation of the proposition (see also the mathematically strict formulation in the Appendix).

Economically the occurrence of the negative anticipation phase can be explained by the fact that, given the faster increase of future technological progress, it becomes more attractive to wait for new technologies. As a consequence of this result, the next proposition shows that the anticipation of higher technological progress starting from a point $\hat{t}$ not only leads to underinvestment before the moment $\hat{t}$, but creates a whole wave of alternating negative and positive anticipation phases backwards in time.

Proposition 2 *A sufficiently strong technological breakthrough at time $\hat{t}$ creates a whole wave of alternating positive and negative anticipation phases before the moment $\hat{t}$.*

This backward anticipation wave can be economically explained as follows. Let $[\hat{t}-\beta,\hat{t}]$ be the time interval at which the anticipation effect defined in Definition 1, the existence of which is proved in Proposition 1, takes place. This anticipation effect implies that investments in new machines are reduced during this interval. Therefore, output reduces during the interval $\left[\hat{t}-\beta,\hat{t}\right]$, so that the output price increases there. This raises the NPV of investments in machines from which the lifetime includes this interval, which implies that the incentive to invest in these machines goes up. Then the firm increases investments in those machines that are scrapped somewhere in the neighborhood of $\hat{t}$, so that these machines can be replaced at that time by the more modern machines that arise due to the technological breakthrough taking place at $\hat{t}$. Since here we only consider investments in new machines that have lifetime ω, investments, and thus also production, go up during a time interval containing $\hat{t}-\omega$, and this results in a price dip during this interval. This price dip in turn results in less investments before this "price-dip-interval", and in this way a whole wave of anticipation phases arises.

The above suggests that the scrapping time has an influence on the anticipation wave. Since for Propositions 1 and 2 the analysis is restricted to $I(t,a)=0$, equation (2) collapses to

$$K_t + K_a = -\delta(a)K(t,a),$$

for $a \in [0,\omega]$, and capital jumps to zero at age ω. This is a more general definition of the one-hoss shay assumption than in, e.g., Benhabib and Rustichini [3], in the sense that it allows for some continuous depreciation $\delta(a) \neq 0$. By putting $\delta(a) = \infty$ for $a \geq \omega$, one can avoid the explicit involvement of the maximal life-time ω. That is, in general, the frequency of the anticipation wave depends on the shape of the depreciation function δ, and this applies also to the case $\omega = \infty$, thus where we have no exogenous scrapping age.

The backward anticipation waves are another example of the fact that vintage-specificity may generate oscillatory dynamics. This traces back to Solow et al. [27], in which it was argued that obsolescence of the oldest vintages and their replacement over time may generate persistent fluctuations in investment, and thus in output. These fluctuations may follow the echo principle in the sense that episodes of high (low) investment reproduce themselves in the future at their replacement time. In the literature this is called "replacement echoes". Although Solow et al. [27] did not find these replacement echoes in the model they studied, they were in fact detected in Benhabib and Rustichini [3] and Boucekkine et al. [6]. The difference with our finding is that replacement echoes are everlasting, while our anticipation waves mainly concentrate around the point where the technological breakthrough occurs: the farther away from this point, the smaller the amplitude is.

3.1 Numerical results

Here we provide a numerical analysis[6] departing from the scenario depicted in Table 1.

$R(Q) = p_0 Q - \frac{m}{2} Q^2$	$p_0 = 1$ $m = 0.00008$
ω : economic lifetime of the machine	$\omega = 10$
$f(t-a) = \pi + \frac{t-a+\omega}{n} \log 2$	$\pi = 1$ $n = 6$
$\delta(a) = \frac{2a}{\omega^2} \ln \frac{1}{\kappa}$	$\kappa = 0.2$
r : discount rate	$r = 0.03$
$v(a) = \begin{cases} 2\sqrt{\frac{a}{\nu}} - \frac{a}{\nu} & \text{for } a \in [0, \nu) \\ 1 & \text{for } a \in [\nu, 1] \end{cases}$	$\nu = 5$
b_0 : unit acquisition cost of new machines: 25% from the expected total return	$b_0 = 4.931$
$b(a) = 0.8^{\frac{\omega-a}{\omega}} b_0$: unit cost of a machine of age a	
$\frac{c_0}{2} I_0^2$: adjustment costs of investments in new machines	$c_0 = 1$
$\frac{c(a)}{2} [I(t,a)]^2$: adjustment costs of investments	$c(a) = 1.2 c_0 e^{-a/\omega}$

Table 1. *Parameter values and functional specifications to be used for the numerical analysis.*

[6]To solve the problem (1)-(3) numerically we employ the general approach developed recently by the fourth author and presented in Veliov [29], where more bibliography concerning the numerical analysis can be found.

The learning function $v(a)$ increases steeply at the beginning, reaching the level one (full efficiency) when the age of the technology becomes equal to ν. Then the value one is sustained over the rest of the life time of the technology.

The technological progress function $f(t-a)$ is based on "Moore's law" which says that the memory and arithmetic power of micro-chips develop in an exponential way over time. If the efficiency of a technology doubles every n years (for micro-chips the value of n is about 3 at the moment), and the efficiency parameter of a technology equals 1 at time zero, the efficiency parameter of a particular machine of age a at time t is $2^{(t-a)/n}$. Recently a Philips manager[7] argued that utility (or production) is a logarithmic function of technology in the sense that production increases with one unit in case technology power becomes ten times as large. Or, in other words, production increases with the technology-efficiency parameter in a logarithmic way with base 10. If we further impose that the productivity of the machines build at time $-\omega$ equals π, the following functional form is obtained (cf. Huisman and Kort [21]):

$$f(t-a) = \pi + \frac{t-a+\omega}{n} \log 2.$$

First, consider the situation where the firm can only invest in new machines (thus $I(t,a) = 0$). Two pictures are shown in which the backward anticipation wave occurs, see Figures 1 and 2. For this particular case we choose a specification of f, where $f(t-a) = 3$ as long as vintage $t-a$ is less than 100, while $f(t-a) = 4$ for $t-a > 100$. The economic intuition for the occurrence of this wave is as follows. Just before the technology shock the firm cuts down on investments in new machines, because it becomes more attractive to wait for new generations when future technologies are more efficient. This is the "negative anticipation phase". Since only investments in new machines are allowed, this results in a drop of production, and thus a higher output price. The latter raises the profitability of investments in capital stock for which the lifetime contains the period where we have this higher output price. Therefore, the firm will increase these investments, leading to a positive anticipation phase. However, increased investments imply more production, and thus a lower output price in that interval. This reduces investments in the period just before. Hence, in this way the whole wave of anticipation phases arises. As it can be seen in Figure 1, the anticipation wave is a symmetric image of the echo effects previously discussed in the literature on vintage capital (Benhabib and Rustichini [3] and Boucekkine et al. [6]) in the sense that they are "turned upside down" after the occurrence of the technological breakthrough.

It is clear that the presence of market power, thus implying that the firm's production level influences the price of output, is crucial for the occurrence of this result. This explains why the anticipation wave does not appear in the case of perfect competition (see, e.g., Feichtinger et al. [16]).

[7] Theo Claassen in the Dutch magazine Elsevier (January 24, 1998).

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

Figure 3 shows the difference in investment behavior between an anticipating and a non-anticipating firm, while a technological breakthrough takes place at time $t = 30$. One of the firms anticipates the shock and reduces the investment beforehand in order to invest more after the shock. Also, the backward anticipation wave can be observed. The non-anticipating firm expects that the technological progress will stay as it is, and realizes that a technological breakthrough happens only just at $t = 30$ (see footnote 5). The investment increase after the shock is very modest, since the firm has not reduced its investments before the shock. In this plot we have used a function $R(Q)$ as in the benchmark case, but then with the bigger value $m = 0.001$. After $t = 30$, investments decrease, since, due to the technological breakthrough, keeping the capital stock at the same level leads to more production, and thus to a lower output price.

Next, consider the situation where the firm can invest in different vintages at the same time. From now on we apply the specification of $f(t-a)$ as presented in Table 1. Figure 4 depicts investments in new machines over time for the case that during the first 30 years technology doubles every six years, while after year thirty this happens every three years ($n = 3$ in the specification of $f(t-a)$). Clearly the difference is seen between the cases where the firm has no market power ($m = 0$) and where it has market power ($m = 0.00008$). In the first case the investment rate in the first thirty years is the same as when also after year thirty technology doubles every six years. Hence, no anticipation takes place, because due to the absence of market power the firm's behavior has no effect on the output price, which implies that the NPV's of capital stocks of different generations do not influence each other.

In the market power case we observe a negative anticipation phase: just before year thirty the firm cuts down on investments in new machines, because it becomes more attractive to wait for new generations when future technologies are more efficient. Then just after year thirty the firm benefits from increased technological progress by buying extra new machines. Due to the negative anticipation phase taking place before the technological progress starts to accelerate, the increased production caused by these extra machines does not lead to a too low output price. In addition to Proposition 1, we now have shown that the anticipation effect is also present in case investments in older machines are also possible.

[Figure 4 about here.]

The conclusion of Figure 4 is confirmed in Figure 5. Here we see that just before year thirty the market-power-firm begins to purchase, on average, older machines. These are cheaper and need to be replaced sooner. Then after year thirty the firm switches to buying younger machines. First, some extra ones are bought, which is reflected by the large reduction of the average age just after year thirty, because the firm needs to replace the older capital goods.

The firm without market power still buys the same machines before year thirty, so that before year thirty it does not adjust its investment policy in case technological progress will go faster after year thirty. After year thirty the firm increases size as a reaction to the accelerated technological progress (note that here the firm's increased production does not reduce the output price). This leads to extra investments in new machines, which results in an initial temporary reduction of average age after year thirty.

[Figure 5 about here.]

Figure 6 shows that, despite the reduction of investments in new machines just before year thirty, for the firm with market power production does not decrease at that time. This implies that extra investments in older, and thus cheaper, machines must have taken place there. These machines will be scrapped soon after the time at which technological progress increases, so that they also can be replaced by the very modern machines that will become available after year thirty. This more or less confirms Figure 5 where it can be seen that the average age of machines purchased is large just before year thirty. Since Figure 6 shows that for the case that also older machines are bought production, and thus output price, is not influenced just before year thirty, this anticipation wave (Figure 2) does not occur there.

[Figure 6 about here.]

4 Conclusions

By now we are ready to answer Chari and Hopenhayn's third question: how are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future? The answer differs for the scenarios where only investments in new technologies are possible, or where also investments in older machines are allowed. When approaching a period of rapid embodied technological progress the firm cuts down on investments in new machines in order to wait for the new inventions. In case only investments in new machines are possible, this leads to a fall in production, which in turn creates a whole wave of alternating positive and negative anticipation phases. When also investments in older machines are possible the drop in production does not occur, because the decrease of investments in new machines is compensated by extra investments in older machines. Older machines are cheaper and also their lifetime is shorter so that they can be scrapped at the moment that the more modern machines are invented. So, like in Chari and Hopenhayn [13], also here the firm continues to invest in old technologies when apparently superior technologies are available.

Most contributions adopting a vintage capital goods structure, like the work of Boucekkine et al. ([8], [9], [5], [7]) and Benhabib and Rustichini [3], only allow for investments in new machines. As it became clear from the previous paragraph, including the possibility to invest in older machines in a dynamic model of the firm changes results considerably. The recent maximum principle for age structured control problems in Feichtinger et al. [17] was employed in the analysis and facilitated obtaining new results regarding the firm's capital accumulation, and the effect of embodied technological progress on that.

Further research should explore the robustness of the anticipation effects. However, it can be expected that they will also occur when other production functions or non-stationary demand are considered.

5 Appendix

Everywhere in the paper we assume that all exogenous functions involved in the model are continuous. The revenue function, $R(Q)$, is nonnegative, strongly concave, and has a Lipschitz continuous and bounded derivative[8]. Examples of such functions are $\ln(1+Q)$ and $(1+Q)^{\alpha}-1$ with $\alpha < 1$. Furthermore, it is assumed that r, c_0, $c(a)$, and $f(s)$ are strictly positive, while b_0, $b(a)$, and $v(a)$ are nonnegative. The strict mathematical meaning of the control and state functions (K_0, I_0, I and K, Q, respectively) is standard, but can also be found in [17].

For some controls the value of the objective function could be $+\infty$ (especially in the case of technological progress). Therefore, the optimal value could be $+\infty$, too. Since in this paper we are interested in the immediate effects of change in the data of the problem (rather than in the asymptotic behaviour), the consideration relies on the following property of the optimal solution[9]: $(\hat{I}_0, \hat{I})$ is an optimal investment control for the problem on the infinite horizon $[0, +\infty)$ if and only if for every $\bar{T} > 0$ and for every $\varepsilon > 0$ there exists a $T^* > \bar{T}$ such that for every $T > T^*$ it holds that if $(\hat{I}_0^T, \hat{I}^T)$ is an optimal control for the problem on the finite horizon $[0, T]$, then

$$|\hat{I}_0(t) - \hat{I}_0^T(t)| + |\hat{I}(t,a) - \hat{I}^T(t,a)| \le \varepsilon \quad \text{for every } t \in [0, \bar{T}] \text{ and } a \in [0, \omega].$$

This implies that the solution of the infinite horizon problem can be approximated (uniformly on any compact interval) by the solution of a finite horizon problem, given that the time horizon is sufficiently large. The existence of a solution to the finite horizon problem follows in a standard way from the linear-concave structure of the problem. The maximum principle for the infinite horizon problem in Section 2.1 is derived by applying the general result from [17] to the approximating finite horizon problems.

Everywhere in the paper it is assumed that the optimal capital stock, $K(t,a)$, and the optimal investment in new machines, $I_0(t)$, are nonnegative, and therefore the constraints $K(t,a) \ge 0$ and $I_0(t) \ge 0$ are not explicitly involved in the formulation of problem (1)–(3).

Proposition 1 *(Strict formulation) Suppose that conditions (S1) and (S2) hold for the problem with technology function $f(\cdot)$. Let D be a given positive number. Then there exists a number M such that the negative anticipation effect takes place for every technology function*

[8] The quadratic function $R(Q)$ used in the numerical illustration may take negative values, but obviously it remains positive along the optimal path, thus in the region of interest.

[9] This property can be derived from *weakly overtaking optimality* (see [11]), provided that the technology function f satisfies an appropriate growth condition. In the stationary case (no technological progress), the property holds even with $\varepsilon = 0$ and $T^* = \bar{T} + \omega$.

$\hat{f}(\cdot)$ *which satisfies the relations*

(*i*) $\hat{f}(t) = f(t)$ on $[0, \hat{t}]$, $\quad f(t) \le \hat{f}(t) \le D$ on $(\hat{t}, \hat{t} + 2\omega]$,

(*ii*) *the corresponding optimal control* $\hat{I}_0(t)$ *satisfies* $0 \le \hat{I}_0(t) \le D$ *and* $\hat{K}(t,a) \ge 0$,

(*iii*) $\liminf_{t \to \hat{t}+0} \dfrac{\hat{f}(t) - f(t)}{t - \hat{t}} \ge M.$

Proof. **Step 1.** From the model formulation and the necessary optimality conditions presented in Section 2.1 we obtain the following relations for $t \ge \omega$

$$K(t,a) = e^{-\int_0^a \delta(\theta) d\theta} I_0(t-a),$$

$$I_0(t) = \frac{1}{c_0} \left(\lambda(t,0) - b_0 \right),$$

$$\lambda(t,0) = \int_0^\omega e^{-\int_0^s (\delta(\theta)+r) d\theta} R'(Q(t+s)) v(s) \, \mathrm{d}s f(t),$$

$$Q(t+s) = \int_0^\omega f(t+s-a) v(a) K(t+s,a) \, \mathrm{d}a.$$

Substituting successively, we obtain the integral equation

$$I_0(t) = -\frac{b_0}{c_0} + \int_0^\omega e^{-\int_0^s (\delta(\theta)+r) d\theta} R' \left(\int_0^\omega f(t+s-a) v(a) e^{-\int_0^a \delta(\theta) d\theta} I_0(t+s-a) \, \mathrm{d}a \right) \mathrm{d}s f(t)$$

Changing the variable a in the inner integral we present this equation in the form

$$I_0(t) = -\alpha + f(t) \int_0^\omega \beta(s) R' \left(\int_{t+s-\omega}^{t+s} \gamma(t,s,\tau) f(\tau) I_0(\tau) \, \mathrm{d}\tau \right) \mathrm{d}s, \tag{13}$$

where

$$\alpha = b_0/c_0, \quad \beta(s) = e^{-\int_0^s (\delta(\theta)+r) d\theta}, \quad \gamma(t,s,\tau) = e^{-\int_0^{t+s-\tau} \delta(\theta) d\theta} v(t+s-\tau).$$

Notice that according to (S2) for every $t \ge 0$ the functions $\beta(\cdot)$ and $\gamma(t,\cdot,\cdot)$ are strictly positive almost everywhere in their domains of definition. Moreover, γ is a locally Lipschitz functions of t, uniformly with respect to the rest of the variables belonging to a bounded set.

Step 2. Contrary to the claim of the proposition assume that

$$\hat{I}_0(t) \ge I_0(t) \quad \text{for all} \quad t \in [\hat{t} - 2\omega, \hat{t}]. \tag{14}$$

In particular, according to (S1) it holds that

$$0 < I_0(\hat{t}) \le \hat{I}_0(\hat{t}) = -\alpha + \hat{f}(\hat{t}) \int_0^\omega \beta(s) R' \left(\int_{\hat{t}+s-\omega}^{\hat{t}+s} \gamma(\hat{t},s,\tau) \hat{f}(\tau) \hat{I}_0(\tau) \, \mathrm{d}\tau \right) \mathrm{d}s,$$

so that,

$$\int_0^\omega \beta(s) R'\left(\int_{\hat{t}+s-\omega}^{\hat{t}+s} \gamma(\hat{t},s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\mathrm{d}s \geq \frac{\alpha}{f(\hat{t})} > 0. \tag{15}$$

Step 3. Using (13) we estimate for $t \in (\hat{t}, \hat{+}\varepsilon_0]$ the difference

$$\begin{aligned}
\hat{I}_0(t) - \hat{I}_0(\hat{t}) &= \hat{f}(t)\int_0^\omega \beta(s)R'\left(\int_{t+s-\omega}^{t+s}\gamma(t,s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\mathrm{d}s \\
&\quad -\hat{f}(\hat{t})\int_0^\omega \beta(s)R'\left(\int_{\hat{t}+s-\omega}^{\hat{t}+s}\gamma(\hat{t},s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\mathrm{d}s \\
&= (\hat{f}(t)-\hat{f}(\hat{t}))\int_0^\omega \beta(s)R'\left(\int_{\hat{t}+s-\omega}^{\hat{t}+s}\gamma(\hat{t},s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\mathrm{d}s \\
&\quad +\hat{f}(t)\int_0^\omega \left[\beta(s)R'\left(\int_{t+s-\omega}^{t+s}\gamma(t,s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\right. \\
&\qquad \left. -\,\beta(s)R'\left(\int_{\hat{t}+s-\omega}^{\hat{t}+s}\gamma(\hat{t},s,\tau)\hat{f}(\tau)\hat{I}_0(\tau)\,\mathrm{d}\tau\right)\right]\mathrm{d}s.
\end{aligned}$$

Moreover, condition (iii) of the proposition implies the existence of numbers M and $\varepsilon_0 > 0$ such that

$$\hat{f}(s) \geq f(s) + \frac{M}{2}(s-\hat{t}) \quad \text{for } s \in (\hat{t}, \hat{t}+\varepsilon_0]. \tag{16}$$

Using (15) and the Lipschitz continuity of γ in t, as well as the bounds $\hat{f}(t) \leq D$ and $\hat{I}_0(t) \leq D$, we obtain that

$$\begin{aligned}
\hat{I}_0(t) - \hat{I}_0(\hat{t}) &\geq \frac{\alpha}{f(\hat{t})}(\hat{f}(t)-\hat{f}(\hat{t})) - C_1(t-\hat{t}) \\
&= \frac{\alpha}{f(\hat{t})}(\hat{f}(t)-f(t)) + \frac{\alpha}{f(\hat{t})}(f(t)-f(\hat{t})) - C_1(t-\hat{t}) \\
&\geq \frac{\alpha}{f(\hat{t})}\frac{M}{2}(t-\hat{t}) - C_2(t-\hat{t}) \\
&= (C_3 M - C_2)(t-\hat{t}),
\end{aligned}$$

where $C_1, C_2, \ldots$ are constant depending on the benchmark data and on the number D, but not on the particular function $\hat{f}$.

Step 4. Take an arbitrary $\varepsilon \in (0, \min\{\varepsilon_0, \omega\})$. Then $t^* = \hat{t} - \omega + \varepsilon < \hat{t}$ and according to assumption (14) it holds that

$$0 \leq \hat{I}_0(t^*) - I_0(t^*).$$

Utilizing the formula (13) for f and for $\hat{f}$, and the equality $\hat{f}(t^*) = f(t^*)$ we obtain that

$$0 \le f(t^*) \int_0^\omega \beta(s) \left[R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) \hat{f}(\tau) \hat{I}_0(\tau)\, \mathrm{d}\tau \right) \right. \qquad (17)$$
$$\left. - R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) f(\tau) I_0(\tau)\, \mathrm{d}\tau \right) \right] \mathrm{d}s.$$

Notice that if $s < \omega - \varepsilon$ then the upper bound of the inner integration is $t^* + \varepsilon = \hat{t} - \omega + \varepsilon + s < \hat{t}$. Therefore within the interval of integration it holds that $\hat{f}(\tau) = f(\tau)$ and $\hat{I}_0(\tau) \ge I_0(\tau)$. Since R' is monotonically decreasing, for such s the first value of R' is not greater than the second one in the above expression. Thus

$$0 \le f(t^*) \int_{\omega-\varepsilon}^{\omega} \beta(s) \left[R'(\ldots) - R'(\ldots) \right] \mathrm{d}s.$$

For $s \in (\omega - \varepsilon, \omega)$ we have that $\hat{t} \in (t^* - \omega + s, t^* + s)$ and

$$R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) \hat{f}(\tau) \hat{I}_0(\tau)\, \mathrm{d}\tau \right) = R'\left(\int_{t^*-\omega+s}^{\hat{t}} \ldots + \int_{\hat{t}}^{t^*+s} \ldots \right).$$

Concerning the first integral on the right-hand side we have that $\hat{f}(\tau) = f(\tau)$ and $\hat{I}_0(\tau) \ge I_0(\tau)$. For the second integral we obtain that

$$\hat{f}(\tau) = \hat{f}(\tau) - f(\tau) + f(\tau) \ge \frac{M}{2}(\tau - \hat{t}) + f(\tau)$$

and use the inequality obtained in Step 3:

$$\hat{I}_0(\tau) \ge \hat{I}_0(\hat{t}) + (C_3 M - C_2)(\tau - \hat{t}) \ge I_0(\hat{t}) + (C_3 M - C_2)(\tau - \hat{t}).$$

Thus, for $s \in (\omega - \varepsilon, \omega)$ it holds that

$$R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) \hat{f}(\tau) \hat{I}_0(\tau)\, \mathrm{d}\tau \right) \le R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) f(\tau) I_0(\tau)\, \mathrm{d}\tau + \Gamma(t, s) \right),$$

where

$$\Gamma(t, s) = \int_{\hat{t}}^{t^*+s} \gamma(t^*, s, \tau) \left[\frac{M}{2} \hat{I}_0(\hat{t}) + \frac{M}{2}(\tau - \hat{t})(C_3 M - C_2) + f(\tau)(C_3 M - C_2) \right] (\tau - \hat{t})\, \mathrm{d}\tau.$$

Apparently, the constant M in the formulation of the proposition can be chosen in such a way that $C_3 M - C_2 > 0$, and since for almost every s the values of $\gamma(t^*, s, \tau)$ are strictly positive almost everywhere in τ, we get that $\Gamma(t, s) > 0$ for almost every $s \in (\omega - \varepsilon, \omega)$. Since R' is strictly monotonically decreasing, we obtain that

$$R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) \hat{f}(\tau) \hat{I}_0(\tau)\, \mathrm{d}\tau \right) < R'\left(\int_{t^*-\omega+s}^{t^*+s} \gamma(t^*, s, \tau) f(\tau) I_0(\tau)\, \mathrm{d}\tau \right),$$

Having in mind the strict positivity of β and $f(t^*)$ we come to a contradiction with (17). This contradiction was caused by assumption (14). Therefore this assumption is false, which proves the proposition. Q.E.D.

Proposition 2 *(Strict formulation) For every $t \in [2\omega, \hat{t}]$ for which the difference $I_0(t) - \hat{I}_0(t)$ is nonzero, there exists a point $s \in (t - 2\omega, t)$, such that the difference $I_0(s) - \hat{I}_0(s)$ is also nonzero and has the opposite sign.*

Proof. Let $t \in [2\omega, \hat{t}]$ be a point where, for example, $I_0(t) > \hat{I}_0(t)$. Assume that $I_0(s) \geq \hat{I}_0(s)$ for all $s \in [t - 2\omega, t]$. Then, from the integral equation (13), and since f and $\hat{f}$ coincide at $t - \omega$ and at τ below, we have that

$$0 \leq I_0(t-\omega) - \hat{I}_0(t-\omega) = f(t-\omega) \int_0^{\omega} \left[R' \left(\int_{t+s-2\omega}^{t+s-\omega} \gamma(t-\omega, s, \tau) f(\tau) I_0(\tau)\, \mathrm{d}\tau \right) - \left(\int_{t+s-2\omega}^{t+s-\omega} \gamma(t-\omega, s, \tau) f(\tau) \hat{I}_0(\tau)\, \mathrm{d}\tau \right) \right] \mathrm{d}s.$$

We have $I_0(\tau) \geq \hat{I}_0(\tau)$ in the interval of integration and the inequality is strict in a left neighborhood $\tau = t$, which intersects $(t + s - 2\omega, t + s - \omega)$ if s is close to ω. From the positivity of γ and β and the strict concavity of R' we obtain a contradiction. Thus there must be a point $\tau \in (t - 2\omega, t)$ where $I_0(\tau) < \hat{I}_0(\tau)$. Q.E.D.

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List of Symbols

Latin letters: a, b, c, d, e, f, m, n, p, r, s, t, v, C, D, H, I, K, L, M, R, Q

Greek letters:

α – alpha
β – beta
γ – gamma
δ – delta
ε – epsilon
κ – kappa
λ – lambda
ν – nu
θ – theta
π – pi
τ – tau
ω – omega
ξ – xi
ζ – zeta
Γ – Gamma

Other symbols in mathematical formulas:

d – ordinary differential
∂ – partial differential
$\int$ – integral
∞ – infinity
log – logarithm
ln – natural logarithm

List of Figures

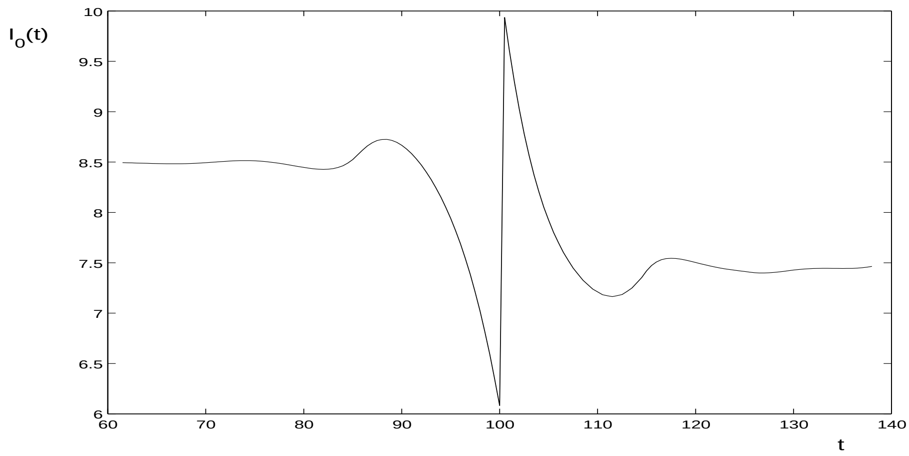

Figure 1: Investment in new machines in case that the technology function $f(t)$ jumps from 3 to 4 at year 100.

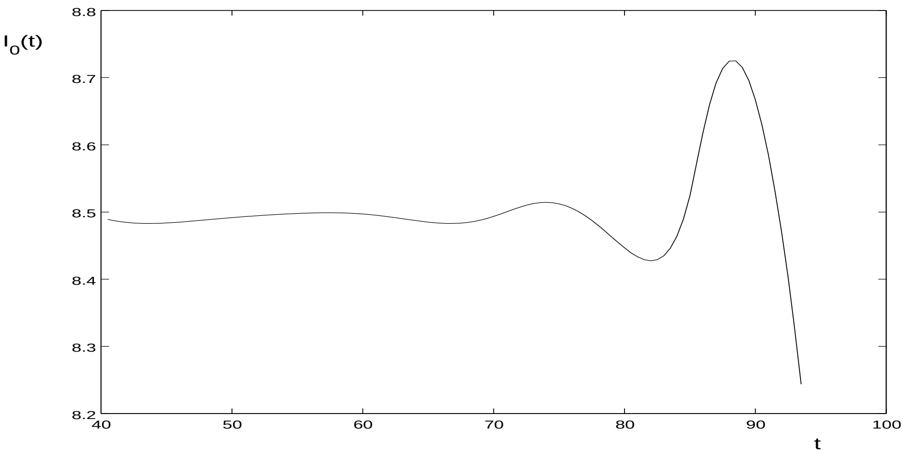

Figure 2: Magnification of a part of the above picture that shows the backward anticipation wave. Without the change of f at time 100, the optimal investment would have a constant value.

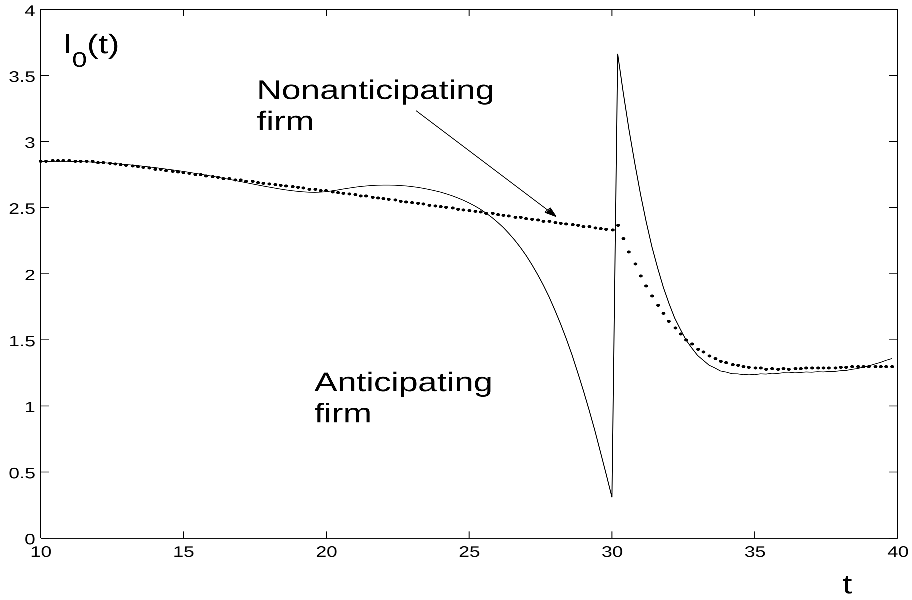

Figure 3: Optimal investment in new machines of a firm that anticipates the technological breakthrough at $t = 30$ (thin solid line), and of a firm that does not anticipate it (bold-dotted line).

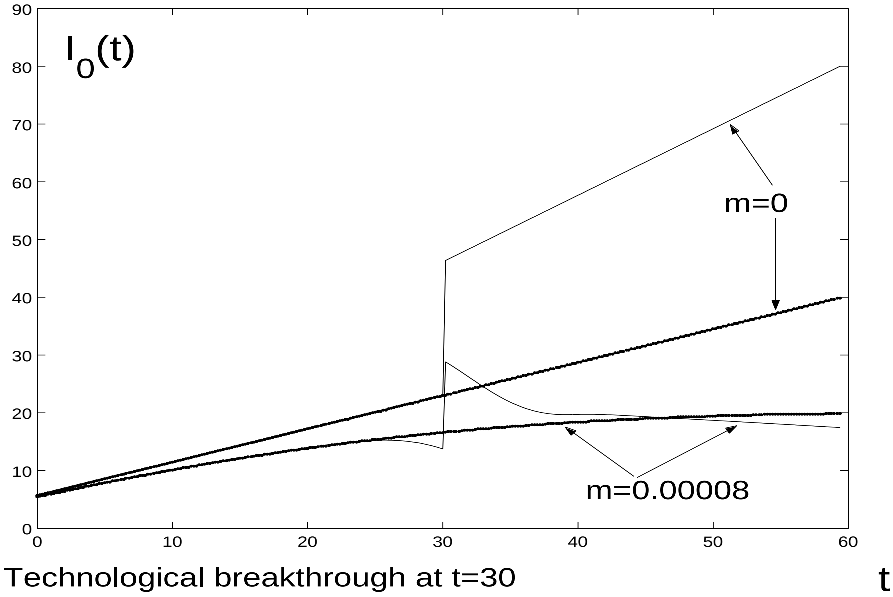

Figure 4: Optimal investment in new machines in the monopolistic ($m > 0$) and in the non-monopolistic ($m = 0$) case. The bold line represents the benchmark case, while the thin line corresponds to the technological breakthrough at $t = 30$.

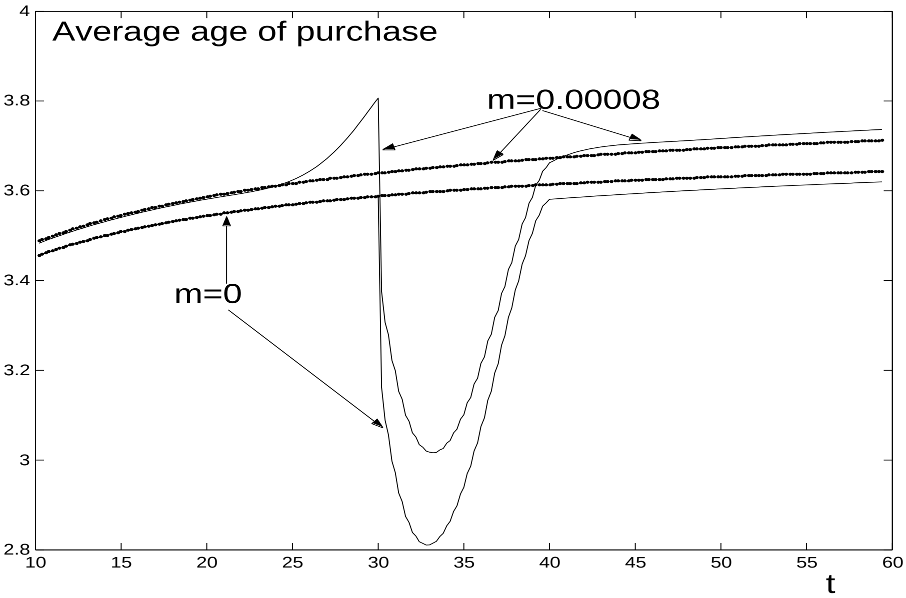

Figure 5: Average age of purchased machines in the monopolistic ($m > 0$) and in the non-monopolistic ($m = 0$) case. The bold line represents the benchmark case, while the thin line corresponds to the technological breakthrough at $t = 30$.

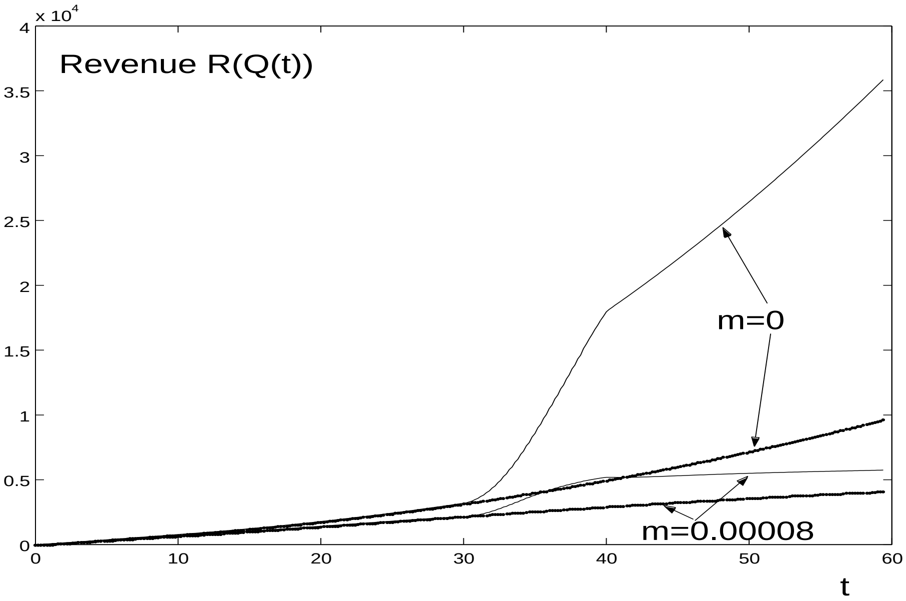

Figure 6: The revenue in the monopolistic ($m > 0$) and in the non-monopolistic ($m = 0$) case. The bold line represents the benchmark case, while the thin line corresponds to the technological breakthrough at $t = 30$.