Communication, Learning and Optimal Monetary Policy

Mewael F. Tesfaselassie

Communication, Learning and Optimal Monetary Policy

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Universiteit van Tilburg, op gezag van de rector magnificus, prof.dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op vrijdag 16 december 2005 om 10.15 uur door

Mewael Frezghi Tesfaselassie

geboren op 16 december 1971 te Bet Meca, Eritrea.

Promotores: Prof. dr. S.C.W. Eijffinger

Prof. dr. E. Schaling

Acknowledgements

This thesis has grown out of four years of research at CentER, Tilburg University. I would like to thank all those who have made the completion of the thesis possible and my stay at Tilburg University a pleasure.

First, a word of thanks goes to my Ph.D. supervisor, Sylvester Eijffinger. Sylvester has always been around for professional advice and guidance since I joined CentER as a Ph.D. student. He has been open to new ideas and research possibilities. Actually, the fact that the thesis has two different themes is a result of Sylvester's idea of "hedge your position", which has proven right.

I would also like to thank my second supervisor and co-author, Eric Schaling, for his advice, critical judgment, and skillful guidance throughout my research. Without his support I would not have taken the challenge of dealing with the technically difficult chapters of 5 and 6 on learning and optimal control. Eric was kind enough to invite me, in the final stage of the thesis, for a short visit to University of Johannesburg. Our lively professional discussions were aided by the sunny and warm weather of Johannesburg, where work and leisure seemed inseparable. Eric, and his wife Petra, also made sure that I had a wonderful time in Johannesburg. Among other things, the wine tasting evening and dinning at the best seafood restaurants will be memorable.

I am grateful to Marco Hoeberichts, who, as my Master's thesis supervisor, encouraged me to apply for a Ph.D. position at CentER. Marco also takes the credit for initiating our fruitful research collaboration; he has co-authored Chapter 4 on central bank transparency. I thank Marco also for his willingness to join the Ph.D. committee.

Next, I wish to express my thanks to other members of the Ph.D. committee, Jacob de Haan, Bas Werker, and Harry Huizinga for taking their invaluable time to assess and comment on the manuscript.

Although the starting date for my Ph.D. was initially set to January 2000, it had to be postponed until August 2001 due to previous commitments with the University of Asmara. I am grateful to Aart de Zeeuw and CentER for their understanding of the matter and for keeping the Ph.D. offer open for another year.

I have greatly benefited from several discussions I had with Tore Ellingsen during my visit, within the ENTER network, to Stockholm School of Economics between

April and June of 2004. My visit was made possible thanks to the travel grant I had received from CentER and NWO. I thank Tore for enabling me to think more critically about communication issues and for his suggestions on extending chapters 1 and 2.

I would like to acknowledge my colleagues Edwin van der Werf, for writing the Dutch summary (Samenvatting), and Yvonne Adema and Peter van der Windt, for editing the summary, especially in light of the difficult technical words involved in the thesis. All my Ph.D. colleagues and friends who, in some way or another, were always around for a relaxing atmosphere, be it for coffee breaks, lunches, playing soccer or AIO-uitjes, also deserve a word of thanks. I would particularly like to mention Yonas, Biniam, Mussie, Daniel, Petros, G/Michael, Maris, Corrado, Kathi, and Jia for their support and company.

Above all, my deepest appreciation goes to my parents, sisters and brothers for their unconditional support throughout my studies.

Mewael Frezghi Tesfaselassie

Tilburg, October 25, 2005

Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation

Major economies in the world have witnessed a period of low and stable inflation for more than a decade. Many observers of central bank policy acknowledge that this success is mainly due to an important institutional change in monetary policy associated with more political support for central bank independence with clearly defined goals. In other words monetary policy making has become more responsible by committing to long-term price stability, a strategy that is insulated from short-term political control and thus improves the credibility of low-inflation policy. A very visible evidence of this move is the recent adoption of explicit inflation targeting among a number of major central banks around the world, including the Bank of England, Sveriges Riksbank, the Reserve Bank of New Zealand, the Bank of Canada and the Reserve Bank of Australia.

Nevertheless, the achievement of low average inflation does not necessarily mean that the art and science of monetary policy has ceased to be interesting. Even when the average rate of inflation is low, control of the economy is not a trivial matter and at best imperfect for several reasons, including data and model uncertainty, asymmetric information about market sentiments, and uncertainty in the transmission lag from policy instruments to final targets, with exogenous shocks intervening in between. To be sure, the transmission channel of monetary policy to the economy remains a complicated one. Thus, even in an era of stable prices, central banking continues to be what Alan Greenspan, the current chairman of the Federal Reserve Board of the U.S., recently called "a risk management activity."

In this regard, recent policy discussions on central banking commonly speak about new challenges in the conduct of monetary policy. Among other things, the management of private sector expectations stands out to be prominent. As Blinder (1998) points out, monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter like long-term interest rates, stock prices and exchange rates, variables which, by their nature, are forward-looking. Similarly, Svensson (2003b) remarks by saying that "central banks control the output gap and inflation mostly through the private-sector expectations they give rise". Thus "central banks take account of private-sector expectations and treat them very much as independent state variables that they monitor and respond to." In his commentary to Svensson, titled "How Should Monetary Policy Be Conducted in an Era of Price Stability?", Woodford (1999) also discusses the importance of market expectations in the transmission mechanism: "One of the most important issues in the conduct of monetary policy, that should attain particular significance in an era of price stability, is the need to take account of the effects of the central bank's conduct upon private-sector expectations." Moreover, on the role of expectations about future (monetary) policy plans, Woodford says: "... there is every reason to believe that the aspects of economic behavior that are central to the transmission mechanism for monetary policy are critically dependent upon people's expectations, including their expectations regarding future policy."

In practice, the role of forward-looking expectations has also surfaced in the recent policy proposals aimed at getting Japan out of its problems of deflation and liquidity trap. Due to the zero lower bound constraint for the nominal interest rate, these proposals emphasize the need for monetary policy to manipulate private sector inflation expectations by committing to future inflationary policy, including a commitment to keep short-term interest rates very low (nearly zero) for a substantial time in the future.[1, 2]

At the same time, however, there has been a recent surge of interest, perhaps ironically, on bounded (limited) rationality and learning in macroeconomics and its implications for the conduct of monetary policy. The notion of bounded rationality is used to describe behavior when decision makers face a new economic environment in which previous experience is not that helpful. In essence, it is weaker than the assumption of full-information, rational expectations, which was the hallmark of macroeconomics in the 1970s, and more so in the 1980s. This strand of literature maintains that decision makers are far from knowing the true mechanics of economic activity, and contrary to rational expectations, postulates the absence of a commonly understood economic environment. Sargent (1993) says that the goal of macroeconomics based on bounded rationality is to "create theories of transitional dynamics, partly to understand the properties of equilibrium dynamics themselves, and partly to create new dynamics of systems that do not settle down."[3] Specific questions raised in this respect are related to the design

[1] See for e.g. Svensson (2003a).

[2] The emphasis on forward-looking expectations is also observed in other relevant issues of monetary policy design, for example, in discussions of the stability properties of simple policy rules that respond to forward-looking private sector expectations (see for e.g. Evans and Honkapohja, 2002). Some economists also suggest that the smooth behavior of the short-term interest rate observed in practice is linked to the ability of central banks to exploit the forward-looking behavior of markets as a way to affect the long-term interest rates relevant for investment decisions (Goodfriend, 1998).

[3] Clarida et al. (1999) conclude their survey on new developments in monetary policy by

of simple monetary policy rules when the private sector is (adaptively) learning about the economy or about monetary policy rules (e.g. Bullard and Mitra, 2002; Evans and Honkapohja, 2002; Honkapohja and Mitra, 2002). On the other hand, central banks lack complete knowledge of the economic model, including private sector expectations, and have to learn about them based on past policy outcomes.[4]

Against this background, the thesis is organized around two themes that, broadly speaking, emphasize monetary policy making under uncertainty. The first part (chapters 2, 3 and 4) has an institutional nature: it looks at the role of transparency and communication in monetary policy when central banks have private information about the state of the economy and its future developments. The second part of the book (chapters 5 and 6) deals with a technical issue, namely, optimal policy when central banks take account of the degree of uncertainty in the transmission mechanism. To be specific, monetary authorities face uncertainty from lack of perfect knowledge of currently prevailing private sector expectations. It is commonly understood that expectations depend on the performance of the economy, say the rate of inflation, and with asymmetric information the policymaker's problem translates in to one with parameter uncertainty. The question is then how monetary policy reacts optimally to this sort of uncertainty.

The fact that one should recognize that monetary policy is conducted under uncertainty is not new. Dating back to the 1950s and 1960s, monetary economists were aware that any economy at some point or another can suffer from some events (shocks) beyond the control of the central bank. Milton Friedman had emphasized the "long and variable" lags in monetary policy transmission that create difficulties for achieving policy objectives. Moreover, already in the 1950s, Herbert Simon and Henri Theil developed the principle of certainty equivalence that had been central to the study of economic decision making under (additive) uncertainty and later extended by Brainard (1967) to include parameter uncertainty and its impact on optimal policy.

What is new in the recent revival of the role of uncertainty is the explicit consideration of infinite horizon problems with dynamic models, including game-theoretic aspects, which affect the optimal policy choice, and that central banks can take actions now that improves their learning opportunity in the future. Undeniably, these issues make the policy setting more realistic but at the same time the algebra of the optimization problem are usually more demanding and in some cases a resort to numerical approximation is unavoidable.

pointing out the significance of further work on adaptive learning and transition dynamics.

[4]Sargent (1999) and Evans and Honkapohja (2001) among others discuss in detail the role of adaptive learning in macroeconomics. In their book, Evans and Honkapohja (2001) treat the learning approach more systematically and present a number of applications.

1.2 Overview of the Chapters

Throughout the book, asymmetric information and forward-looking expectations play central roles in the determination of macroeconomic outcomes– inflation and aggregate output (or employment). The papers on transparency are based on the microfounded New Keynesian framework, which has become a popular work-horse model for monetary policy analysis.[5] In this framework private sector expectations of future inflation are crucial in determining current period inflation and aggregate output. On the one hand, price setting firms are forward-looking because of imperfect price adjustments. On the other hand, intertemporal consumption (and saving) decisions of households imply that expectations of future output are important for current period aggregate demand. The New Keynesian framework is thus ideal for addressing questions of transparency because forward-looking behavior induces monetary policy to care about the effect of future shocks on current inflation and aggregate output via the expectations channel.

Overview of the Chapters					
Model Features	Chapters on Transparency			Chapters on Learning and Control	
	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6
New Keynesian	√	√	√	-	-
Backward-looking with term structure	√	-	-	√	√
Symmetric uncertainty	-	-	√	-	-
Rational expectations	√	√	√	-	-
Passive learning	-	-	-	√	√
Active learning	-	-	-	-	√
Strict inflation targeting	-	-	-	√	-
Flexible inflation targeting	√	√	√	-	√
Credible policy	√	-	√	√	√

The chapters that deal with learning and control also have models featuring private sector expectations as part of the transmission mechanism. They are extensions of the popular backward-looking macro model of Svensson (1997). The extension uses the term structure of interest rates, an arbitrage relationship between short-term and long-term interest rates, part of which are forward-looking expectations of the long-term interest rate. The model then becomes more elaborate with long-term rates determining aggregate demand and ultimately inflation. Interestingly, the term structure cum backward-looking has forward-looking properties similar to the New Keynesian model. In particular, in both models, private agents are

[5] Clarida et al. (1999) and King (2000) discuss in detail the New Keynesian paradigm.

forward-looking so that future monetary policy decisions are crucial for determining current period policy and macroeconomic outcomes.

1.2.1 Central Bank Forecasts and Communication (ch. 2, 3 and 4)

In recent times an increasing number of central banks have taken steps towards a more transparent monetary policy. The list includes inflation targeting central banks of the UK, Canada, Sweden, Australia, New Zealand, and Switzerland. Among other things, inflation targeting central banks frequently publish what is commonly known as Inflation Reports, one of the official documents used as inputs in policy making and intended to communicate policy objectives and decisions to the public.

There are obvious benefits to the public from truthfully disclosing internal central bank forecasts. Blinder (1998) argues that openness and communication with the public improve the effectiveness of monetary policy as a macroeconomic stabilizer because "central banks generally control only the overnight interest rate, an interest rate that is relevant to virtually no economically interesting transactions." Mishkin (2004) also points out that, not only can transparency help household and business decision makers get a more accurate picture of future developments of the economy, but disclosure of internal forecasts can also help the public understand central bank actions. In other words, immediate release would increase the transparency of central bank policy by showing more of what lies behind its decisions. This could in turn have the advantage of reducing the volatility of financial markets that is associated with speculation about policy motives, as indicated by (Romer and Romer, 2000). One would then expect markets to incorporate the disclosed forecasts in their expectations if they know that these forecasts are more accurate than commercial forecasts. However, despite these benefits, Romer and Romer (2000) cite possible complications in implementing the immediate disclosure of forecasts, saying that immediate disclosure could change the information content of the forecasts since they would attract a lot of attention from the public.

Sparked by the observed trend towards more disclosure of central bank forecasts, a number of recent papers have examined whether there is a theoretical case for transparency. The literature has explored the effects of disclosure in the context of private information about shocks to *current* inflation and output but results are inconclusive. It turns out that what drive the results are (1) the presence (or lack thereof) of credibility problems and (2) the presence (or lack thereof) of forward-looking expectations. Abstracting from credibility issues and using models with static expectations, Cukierman (2001) and Gersbach (2003) find disclosure to be harmful, while by introducing reputation concerns within the same class of models, Geraats (2001) finds disclosure to be beneficial. On the other hand, Jensen

(2000) uses a forward-looking model and gets inconclusive results when central bank credibility plays a role.

The chapters on transparency follow these theoretical discussions. Chapter 2 and chapter 3 deal with the issue of forecast disclosure in the presence of private information about *future* shocks, as opposed to current period shocks, noting that information about future shocks are relevant in a world of forward-looking price setting firms. Following Cukierman (2001) and Gersbach (2003), the analysis in chapter 2 abstracts from credibility issues. In this case, the main result is that with full credibility and common knowledge of central bank targets, advance disclosure of future shocks makes the central bank worse off. As such a credible central bank with private information has the incentive to delay disclosure until after private sector expectations are formed.

Chapter 3 modifies the analysis of chapter 2 in the spirit of Faust and Svensson (2001) and Jensen (2000). First, the model includes unobserved shifts in the central bank's output target. This introduces an inflation bias as the output target can differ from the natural rate. In addition, the timing of events is such that the central bank chooses its policy before private sector inflation expectations are set. In principle, this changes the nature and outcomes of the game since the private sector can infer the output target from observed central bank actions. These modifications turns out to have important consequences since the relevance of disclosing forecasts of future shocks is not clear cut and depends on specific assumptions about the unobserved output target. Specifically, the central bank is better off by withholding its private information about future shocks if the random shift in the output target is directly revealed at the time the future shocks are realized. Otherwise, if the output target has to be indirectly inferred from observed policy decisions of the central bank, then disclosure policy is harmless.

A common result of chapter 2 and chapter 3 is that unlike current period shocks, there is no inherent desire to offset the forecasts of future shocks because these shocks do not have a direct impact on current inflation. This implies that even if current actions of the central bank are observed, say in terms of the current interest rate choice, the public can not infer the central bank's forecasts.

Chapter 4 deviates from the previous two chapters by emphasizing mutual uncertainty between the central bank and the private sector. To motivate mutual uncertainty is not that difficult since in practice central banks spend considerable resources in order to understand prevailing market sentiments and how expectations react to policy and random shocks. This is attested by the collection and dissemination of information by central banks about market sentiments based on different sources, including surveys of commercial forecasters and data from futures markets. However, the transparency literature generally ignores uncertainty on the part of the central bank since the presumption is that the central bank observes private sector expectations and decisions while the private sector is at a disadvantage as a result of not knowing the central bank's preferences and lacking information about the state of shocks. This over simplification is pervasive despite concerns that central bank forecasts of market expectations may have substantial errors (Tarkka and Mayes, 1999; Evans and Honkapohja, 2002).

As such, a more plausible assumption would be that a central bank depends to a large extent on its internal staff forecasts (Honkapohja and Mitra, 2002).[6] At the same time, it is reasonable to assume that the private sector can not perfectly observe the forecasts of the central bank unless the central bank publishes them. If it wishes the central bank can disclose its forecasting procedures and thereby make it easier for the public to infer the judgment errors implied by the forecasting rule. Chapter 4 aims to shed light on the implications of this symmetric uncertainty and communication by the central bank for stabilization policy. The main result in this respect is that communication of assessment errors improves output stabilization at the expense of instability in inflation, thus leading to a variability tradeoff. This tradeoff also has normative implications for policy: a central bank that is sufficiently conservative (in the sense of Rogoff, 1985) improves society's welfare by communicating its assessments. Chapter 4 also gives results for a more general loss function that includes interest rate stabilization and discusses the tradeoff between communication and conservativeness.

1.2.2 Learning, Control and Inflation-Forecast Targeting (ch. 5 and 6)

Chapter 5 and chapter 6 analyze imperfect information and learning about the term structure of interest rates which is embedded in an inflation forecast targeting framework popularized by Svensson (1997).[7] Limited information concerning private sector expectations of the long-term interest rates translates the central bank's problem into one with parameter uncertainty, specifically uncertainty about the degree of persistence in output and inflation. The main question we address is the performance of alternative monetary policy rules when the central bank is faced with the difficult task of simultaneously controlling inflation and estimating (learning) the impact of policy actions.

Introducing imperfect information makes our model similar to some recent studies that deal with the issue of optimal response to an uncertain but possibly learnable economic system. The opportunities for learning about unknown parameters depend on the use of monetary policy instruments to generate data that can speed

[6] Honkapohja and Mitra (2002) argue that due to potentially large errors in observing market expectations, an interest rate rule that responds to surveys of market expectations can lead to large welfare losses.

[7] Svensson remarks that in a more elaborate model, a term structure can be incorporated leading to a richer version of the transmission mechanism. Doing so makes the model more realistic as monetary policy is conventionally viewed as running from short-term interest rates managed by central banks to longer term rates that influence aggregate demand (Goodfriend, 1998). As such Eijffinger et al. (2000b) analyze the role of the term structure in a perfect knowledge setting.

up learning and eventually improve control of the system.[8] Three policy options can be differentiated: the familiar certainty equivalence, myopic policy and dynamically optimal policy. The first two separate the estimation and control part of an intertemporal problem. Both are categorized under passive learning in the sense of disregarding the dynamic link between current decisions and future beliefs about the unknown parameters. The third policy option combines estimation and control and thus represents an active learning policy.

The models of chapter 5 and chapter 6 differ from most of the literature in two respects.[9] First, the structural equations in our model are dynamic even if there was no learning by the central bank. This is due to some inertia in the structural model used in inflation forecast targeting, where future economic conditions depend in part on the current conditions. Second, while the literature typically studies uncertainty about a policy multiplier, the nature of information symmetry in our term structure equation implies it is the persistence parameter in the linear process that is unknown to the central bank.

The literature on learning and control typically constructs the problem around a simple reduced form model where the explanatory variable is the policy instrument (i.e., control variable) whose coefficient has to be estimated at the same time that decisions have to be made about the appropriate level of the instrument variable (say interest rate) that minimize the expected current and future losses from the variability of the dependent variable (say inflation) around a desired target level.

In the presence of policy multiplier uncertainty, a number of papers show that policy under active learning is associated with a more aggressive policy response to new information (higher variability of the policy instrument) compared to the myopic one (e.g Bertocchi and Spagat, 1993; Balvers and Casimano, 1994; Wieland, 1998). The intuition is that, even if there are costs in terms of short-term volatility in the target variable, by actively generating information that improves estimation, policy can recoup the short-run losses by a better control of the economy in the medium to long run. However, this result has been challenged by Ellison and Valla (2001) who call for a less aggressive response by appealing to strategic interactions between the central bank and the private sector, in which case a higher volatility in the policy instrument can lead to volatile inflation expectations, which in turn hinder central bank control of inflation and output.

Chapter 5 introduces the nature of information asymmetry and solves the passive learning problem of a strict inflation targeting central bank. Under strict inflation targeting, monetary policy completely stabilizes predictable fluctuations in inflation, while observable fluctuations are only due to the initial impact of unpredictable shocks and forecast errors. Moreover, the dynamically optimal monetary policy under learning does not deviate from the certainty equivalent and myopic

[8]Such an analysis follows the computationally-oriented dual control literature first popularized in control engineering.

[9]Exceptions are Wieland (1998) and Beck and Wieland (2002). See below.

policies. Thus optimal policy separates estimation and control.

Chapter 6 extends the analysis of chapter 5 to a general case where on the one hand monetary policy faces a tradeoff in stabilizing inflation as well as the rate of interest, the policy instrument, and on the other, the central bank internalizes the effects of current policy choices on its learning possibilities about an unknown degree of persistence in the economy. When variability in the rate of interest enters the loss function, optimal policy deviates from the passive learning rules. This shows that the need for policy to generate higher relative variability in w_{t+1} (measured by coefficient of variation) depends on the state w_t. When the next period's state w_{t+1} deviates a lot from the target due to an unpredictable shock, and thus generates data on its own, optimal policy takes this in to account and thus does not need to actively generate data w_{t+1}, while it does so when the economy is hit by a very small shock. On the other hand, the myopic rule only takes account of the additional source of uncertainty in the persistence parameter, the effect of which is compounded by the magnitude of the state variable w_t. It does not internalize the future benefits in terms of parameter precision because of large deviations of w from the target. Thus it responds linearly and more aggressively than the certainty equivalence rule.

This feature of the myopic rule differs from what one might find when the source of parameter uncertainty lies with the policy multiplier. In that case, policy under the myopic rule tends to be less aggressive than certainty equivalence. Uncertainty about the policy multiplier forces the central bank to be cautious about using its policy instrument freely to stabilize inflation. In our case, the analogous explanation is that, with uncertainty in the persistence parameter, the central bank would like to see less variability in the next period's state variable. Under the myopic rule, this can be achieved only if the policy rate responds aggressively to new information about the state of shocks.

The incentive to deviate from the certainty equivalence policy diminishes for a central bank that gives more attention to inflation stabilization. In that case, the instrument rate can be set optimally to stabilize inflation without much concern about current and future volatilities in the rate of interest. In the limit of strict inflation targeting optimal policy is not affected by uncertainty in the persistence parameter. Thus there is more tendency to probe for a relatively large discount factor (alternatively, for a relatively small discount rate).

Part I

Central Bank Forecasts and Communication

Chapter 2

Central Bank Forecasts and Disclosure Policy

In a simple macro model with forward-looking inflation expectations, this chapter looks into disclosure policy when a central bank has private information on *future* cost-push shocks that potentially disrupt future inflation. It uses a benchmark case where the preferences of the central bank are common knowledge. The basic result is that, as long as the central bank cares about output stabilization, advance disclosure of forecasts of future shocks is harmful to welfare. The intuition behind this negative result is that the public understands that cost-push shocks are not fully stabilized by the central bank because of concerns for output. Thus future inflation is expected to be affected by future shocks. Any advance disclosure of information can destabilize forward-looking inflation expectations and in turn current inflation via the Phillips curve.

2.1 Introduction[1]

In practice, central banks and the private sector spend a lot of resources in their forecasting activities and in assessing the views and forecasts of each other. For some reasons though, central bank forecasts outperform those of the private sector, an indication perhaps of the central bank's superior information about the future state of the economy, including the state of shocks affecting economic activity. In their empirical analysis on differences between commercial and Federal Reserve (Fed for short) forecasts, Romer and Romer (2000) conclude that "the most important finding ... is that the Federal Reserve appears to possess information about the *future* state of the economy that is not known to market participants." (p.455),

[1] An earlier version of chapter 2 and chapter 3 has already been published as a CEPR Discussion Paper (co-authored by Sylvester Eijffinger).

(emphasis ours).[2]

While surveys of private sector (commercial) forecasts, such as the Fed's "Beige Book" and the ECB's Survey of Professional Forecasters, are frequently released, some central banks are reluctant to disclose without delay their own internal forecasts.[3] Recently, some theoretical research has been done on the welfare effects of disclosing in advance central bank information about the state of the economy.[4] The literature has explored this issue in the context of private information about shocks to *current* inflation and output, with mixed results.

This chapter also considers disclosure policy regarding central bank forecasts of shocks. But it deviates from the literature by introducing forecasts of *future* shocks. Information on future shocks is important when expectations are forward-looking. The model used to analyze future shocks is based on the New Keynesian view of the macroeconomy Clarida et al. (see for e.g. 1999); King (see for e.g. 2000); McCallum and Nelson (see for e.g. 2000), where forward-looking inflation expectations influence current period outcomes of inflation. In this case, given the central bank's policy, high variability in inflation expectations (which are conditional on forecasts of future shocks) also implies high variability in current inflation. This makes disclosure policy regarding forecasts of future shocks an interesting issue to study.

Following Cukierman (2001) and Gersbach (2003), the analysis in this chapter abstracts from credibility issues, which are dealt with in chapter 3. The main result is that when the central bank does not suffer from a credibility problem or it's targets are common knowledge, advance disclosure of future shocks makes the central bank worse off. As such the central bank may have the incentive to delay disclosure until after private sector expectations are formed.[5] In turn this may improve stabilization of *current* inflation and output. This negative result accords with that of Cukierman (2001) and Gersbach (2003), which are based on static inflation expectations and decisions are made in a one-shot game.[6]

---

[2] In the case of the Federal Reserve, Romer and Romer (2000) discuss some of the reasons for higher quality forecasts, including inside information about future monetary policy, access to official and unofficial data, and enormous devotion of resources.

[3] In this case, for instance, the Beige Book, which summarizes information gathered by each Federal Reserve Bank through reports from Bank and Branch directors and interviews with key business contacts, market experts and other sources, is published immediately. However, the Fed does not disclose immediately its staff forecasts of the U.S. economy, reported in the "Green Book". The Green Book is made public only with a lag of five years.

[4] In the terminology of Geraats (2001), the release of internal forecasts is part of what she calls *economic* transparency. She discusses several aspects of transparency including political (formal goals, numerical targets), economic (data, models, forecasts), operational (control errors, transmission shocks), procedural (minutes of meeting, voting), and policy (statements, inclination). See also de Haan et al. (2005).

[5] For this result to hold, it must be common knowledge that the central bank has better quality signals about future shocks.

[6] This negative result holds for alternative monetary transmission mechanisms, one of which is a Lucas-type aggregate supply relationship where short run output movements are due to the

The analysis also shows that immediate disclosure of these shocks can have implications different from forecasts of current shocks. In contrast to forecasts of current period shocks, forecasts of future shocks may not be revealed to the public by current policy choices because the central bank refrains from responding to its own forecasts. The central bank may withhold its information about future shocks and imitate the less informed public without the fear of revealing that information by its current actions.

Our discussion proceeds as follows. Section 2.2 describes the model environment for the New Keynesian transmission mechanism, where inflation and output are determined by forward-looking inflation expectations. In section 2.3 we analyze optimal monetary policy under discretion, with and without disclosure of information. Moreover, the effects of secrecy on the behavior of the nominal rate of interest is discussed. In this benchmark case, we show that transparency about future shocks makes the central bank worse off as long as monetary policy aims at other goals besides price stability. In this case adverse supply shocks affect all goal variables, and knowing this, expected movements in future supply shocks make private sector inflation expectations to be more volatile. This effect transmits to *current* prices through expectations of future inflation. It may thus be better from the perspective of the central bank to wait until the information about future supply shocks does not have any value to the private sector. This ensures that public expectations of future shocks are less volatile than when a more accurate information about future shocks is available.[7]

The benchmark case is then modified in some ways. First, instead of discretionary policy, the central bank is assumed to commit credibly to some state contingent rule (Section 2.4). However, this modification does not change the negative result found under the benchmark case. This is then followed by a discussion of policy implementation in terms of targeting rules versus instrument rules. Moreover we raise the practical issue of observability of private sector expectations. In section 2.6, the question of forecast disclosure is analyzed within an alternative transmission mechanism based on Svensson (1997). Concluding remarks are given in section 2.7.[8]

central bank's ability to create surprise inflation. The other variant is the backward-looking macro model (Svensson, 1997) with its main feature of time lags from the policy instrument (the rate of interest) to policy goals (output and inflation). Actually, in the backward-looking model, transparency is bad for welfare only when the central bank cares about interest rate stabilization, on top of inflation and employment. Otherwise transparency does not matter if only inflation and employment are the goals of monetary policy.

[7]In this sense, this paper agrees with the remark by Mishkin (2004) that even if openness is a virtue, for example when central banks are transparent about their long-term inflation goals, some types of transparency may not further social objectives.

[8]In the next chapter, the significance of uncertainty about central bank output target is explored in a multi-period New Keynesian framework with signaling, closely following the work of Jensen (2000).

2.2 Forward-looking Inflation Expectations

As we indicated in the introduction, the New Keynesian view of the macroeconomy gives a prominent role to private sector expectations of future inflation and output in the determination of current inflation and output. A detailed description of the workhorse model can be found, for example, in Clarida et al. (1999) and King (2000).

Important for our analysis is the forward-looking Phillips equation determines inflation given by:

$$\pi_t = \beta E_t^p \pi_{t+1} + \lambda x_t + u_t \tag{2.1}$$

where π is the inflation rate, x is the output gap, and u is a zero-mean stochastic shock to inflation. The shocks are assumed to come from a white noise process, a specification that is common in the transparency literature.[9] The parameters β and λ satisfy $0 < \beta < 1$ and $\lambda > 0$. $E_t^p \pi_{t+1}$ stands for private sector expectations of next period's inflation conditional on available information at time t. Thus inflation depends on forward-looking private sector expectations, the output gap and inflation shock. When prices are sticky, meaning that not all firms can reset their prices in every period, expectations about future prices (and therefore inflation) play an important role in determining the current level of inflation. It is the link between current inflation and expectations of future inflation that differentiates the New Keynesian Phillips curve from the Lucas-type Phillips curve where non-neutrality of monetary policy comes from unexpected (surprise) inflation.

Likewise the dynamics of output demand is governed by a simplified version of the so called intertemporal IS equation:

$$x_t = -\phi(i_t - E_t^p \pi_{t+1}) + v_t \tag{2.2}$$

where i is the nominal interest rate and v is an i.i.d shock to aggregate demand. The parameter ϕ satisfies $\phi > 0$.

The central bank chooses a sequence of current and future short-term nominal interest rates to minimize the expected value of current and future losses arising from variability in inflation and output.[10]

$$E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{2.3}$$

[9] The recent literature on discretion and commitment in a New Keynesian framework sometimes assumes an i.i.d specification (for instance Woodford, 1999).

[10] The intertemporal objective function (2.3) is commonly used in monetary policy analyses. Woodford (1999) has shown that (2.3) represents a second-order approximation to the negative of the utility of a representative agent.

The period t loss function is typically given by

$$L_t = \pi_t^2 + \alpha x_t^2 \tag{2.4}$$

with α denoting the weight the central bank places on output stabilization goal relative to inflation stabilization. For simplicity the target rate of inflation is normalized to zero. Moreover since the central bank targets the equilibrium level of output, we also normalize the output gap target to zero.

For $\beta \to 1$, one can scale the loss function (2.3) by $(1 - \beta)$, which can then be approximated by the unconditional expected value of period t loss (2.4) (see for e.g. Rudebusch and Svensson, 2002):

$$(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t L_t \approx E(L_t) = \sigma_\pi^2 + \alpha \sigma_x^2 \tag{2.5}$$

where σ_π^2 and σ_x^2 are, respectively, the unconditional variances of inflation and the output gap. We will use (2.5) to evaluate the welfare losses arising from the regimes of transparency and non-transparency.

In the model of this section the central bank is assumed to have a more accurate forecast of the cost-push shock u_{t+1} so that the it can can track the shock's development better than the private sector. For simplicity, the central bank has perfect information about the shocks while the private sector receives a noisy signal, s_t, of the shock. Endowing the central bank with full knowledge of the shock is only meant for convenience, and is innocuous to our qualitative result. All we need is for the central bank to do better than the private sector in tracking the movement of future shocks.

Except for information asymmetry regarding u_{t+1}, there is common knowledge of the central bank's loss function, including the targets for inflation and output and the preference parameter α. For the moment we abstract from inflation bias considerations, as the central bank targets equilibrium output, which is not unrealistic given the widely accepted assertions about the prestige of major central banks.[11]

2.3 Disclosure Policy under Discretion

Since the transparency regime is first announced, private sector expectations will be conditional on the announced regime. If u_{t+1} is communicated, this will be

[11]For some forceful arguments against the literature on inflationary bias, see McCallum (1995) and Blinder (1998). In the case of the Fed, Bernanke (2003) and Romer and Romer (2000) discuss the reputation that the Fed has gained over the past two decades. Credibility issues are discussed in chapter 3.

incorporated in the formation of expectations. Otherwise, expectations are formed with knowledge of only the signal s_t:

$$s_t = u_{t+1} + \epsilon_t \tag{2.6}$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is noise that contaminates the signal, and is independent of $u_{t+1} \sim N(0, \sigma_u^2)$. Optimal signal extraction gives $E_t^p u_{t+1} = ks_t$, where $k \equiv \sigma_u^2/(\sigma_u^2 + \sigma_\epsilon^2)$. It is important, for future reference, to realize the fact that the variance of ks_t, given by $k^2\sigma_s^2$, is less than σ_u^2:

$$k^2\sigma_s^2 = k\sigma_u^2 < \sigma_u^2 \tag{2.7}$$

Thus, the variance of private sector expectations of the shock based on the signal is less than what it would be if there was full information about the shock. This is the essence in which the central bank might have an incentive not to reveal its information about the true value of the shock.

We proceed by deriving the equilibrium outcomes under each regime and then compare the resulting losses from each regime. Thus, we can think of the central bank first deciding on revealing its private information, and then the game is played where private sector expectations are formed and the central bank chooses policy taking private sector expectations as given (see Cukierman (2001) for a similar setup with Lucas-type transmission mechanism):

The timing in period t is:

- u_t realizes and is commonly known
- The central bank decides to reveal u_{t+1} or not.
- $E_t^p \pi_{t+1}$ is formed conditional on the disclosure policy of the central bank.
- The central bank chooses the pair $\{x_t, \pi_t\}$.

Under discretionary policy, the central bank minimizes (2.3) period-by-period given private sector expectations, thus the term $E_t^p \pi_{t+1}$ in the Phillips equation (2.1) is taken as a fixed parameter.[12] Since the central bank takes private sector expectations as given, the following optimality condition holds in both transparent and non transparent regimes[13]

$$x_t = -\frac{\lambda}{\alpha}\pi_t \tag{2.8}$$

[12]Thus the timing of events is such that the central bank chooses its interest rate policy for the current period after observing private sector inflation expectations, and current and next period shocks; see for e.g. Cukierman (2001).

[13]For convenience the problem is solved in two steps. Once the optimal paths for inflation and output are known in the first step, the optimal instrument path for the nominal rate can be found from the IS equation (2.2).

According to (2.8), in each period, the central bank contracts (expands) current output in response to a higher (lower) rate of current inflation. In essence, the central bank is reacting to any variable that directly or indirectly affects current inflation. For example if for some reasons inflation expectations increase, given the level of output, current inflation goes up. The optimality rule ensures that this situation does not materialize because the central bank is willing and able to reduce current output to ease the burden of the shock on the current rate of inflation. The above optimality condition is related to what Lars Svensson calls a "targeting rule", a rule expressed in terms of the goal variables (inflation and output), and derived from a well-defined objective function. It differs from an "instrument rule" that describes a reaction function for the nominal rate of interest (the instrument of monetary policy).[14] The next step is to determine private sector inflation expectations. Since the private sector correctly anticipates the targeting rule of the central bank, plug (2.8) in (2.1)

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}E_t^p\pi_{t+1} + \frac{\alpha}{\alpha+\lambda^2}u_t \tag{2.9}$$

This equation shows clearly that the evolution of actual inflation depends on currently held private sector expectations about future inflation and on the current realization of the exogenous shock u_t. In this setting, private sector expectations of π_{t+1} are ultimately determined by their forecasts of u_{t+1}. Thus the role of forecasts of the shocks is clear, and any information that improves the private sector's forecast accuracy with respect to these shocks is valuable. The mechanism by which any private information about forecasts of future shocks affect current inflation outcomes can be shown easily for the simple case where the shocks are white noise with mean zero and finite standard deviation.

Disclosing $u_{t+1} \rightarrow E_t^p\pi_{t+1} = f(u_{t+1}) \rightarrow \pi_t = g(u_{t+1})$

Withholding $u_{t+1} \rightarrow E_t^p\pi_{t+1} = f(s_t) \rightarrow \pi_t = g(s_t)$

With this idea in mind, we can now solve the model for $E_t^p\pi_{t+1}$ and derive the rational expectations equilibrium. We first derive equilibrium inflation and output under a non-transparent regime where knowlede of u_{t+1} is withheld by the central bank.

2.3.1 Equilibrium under a Non-transparent Regime

Under a non-transparent regime, the relevant state variables are u_t and s_t. Using the commonly used method of undetermined coefficients,[15] we start from equation

[14] See for e.g. Svensson (2003b).

[15] McCallum (1983) emphasizes on solving the model using only the fundamentals of the economy (in this case u_t and s_t), avoiding bubble solutions. McCallum calls this the Minimal State

(2.9) and guess that

$$\pi_t = \theta_1 u_t + \theta_2 s_t = \theta_1 u_t + \theta_2(u_{t+1} + \epsilon_t) \tag{2.10}$$

where the coefficient are yet to be determined. Without full disclosure of u_{t+1}, the private sector resorts to its signal in forming expectations of u_{t+1}. Thus inflation expectations are given by:

$$E_t^p \pi_{t+1} = \theta_1 E_t^p u_{t+1} = \theta_1 k s_t \tag{2.11}$$

Next, replace (2.11) in (2.9) to get the following equilibrium level of inflation:

$$\pi_t = \frac{\alpha\beta}{\alpha + \lambda^2}\theta_1 k s_t + \frac{\alpha}{\alpha + \lambda^2} u_t \tag{2.12}$$

Consistency between equation (2.12) and the guessed form (2.10) implies that:

$$\theta_1^* = \frac{\alpha}{\alpha + \lambda^2} \qquad \theta_2^* = \beta k \theta_1^{*2}$$

Then the solution for the output gap follows easily from the f.o.c:

$$x_t = -\frac{\lambda}{\alpha}(\theta_1^* u_t + \theta_2^* s_t) \tag{2.13}$$

We can now compare the resulting inflation expectations of the two parties:

$$\begin{aligned} E_t^c \pi_{t+1} &= \theta_1^* u_{t+1} \\ E_t^p \pi_{t+1} &= \theta_1^* k s_t = \theta_1^* k(u_{t+1} + \epsilon_t) \end{aligned} \tag{2.14}$$

It is easy to see, taking account of equation (2.7), that private sector inflation expectations have less variability compared to that of the central bank.

2.3.2 Equilibrium under a Transparent Regime

Next consider the case of transparency about u_{t+1}, which means that both parties have identical information set. We note that under full disclosure, the relevant state variables are u_t and u_{t+1}. The conjecture takes the form:

$$\pi_t = \theta_1 u_t + \theta_2 u_{t+1} \tag{2.15}$$

Variables (MSV) method.

where again the coefficients are determined later. With full disclosure of u_{t+1}, rational expectations imply

$$E_t^p \pi_{t+1} = \theta_1 E_t^p u_{t+1} = \theta_1 u_{t+1} \tag{2.16}$$

It is easy to show that the equilibrium levels of inflation and output have the same form as their counterparts under a non-transparent regime, except that now u_{t+1} replaces ks_t and $\theta_2^* = \beta\theta_1^{*2}$. Thus we have:

$$\pi_t = \theta_1^* u_t + \theta_2^* u_{t+1} \tag{2.17}$$

$$x_t = -\frac{\lambda}{\alpha}(\theta_1^* u_t + \theta_2^* u_{t+1}) \tag{2.18}$$

We can easily see that current inflation and output levels are affected not only by current period shocks, but also by future shocks that are released to the public. Thus releasing information regarding u_{t+1} makes current inflation and output more volatile. Formally, the welfare effects of disclosure policy is evaluated by substituting the equilibrium levels of inflation and output into the loss function and taking the expected value of the function. [16]

It is straightforward to show that the above negative result also holds when the central bank's loss function includes additional goals, such as concerns for instability in interest rates (see Cukierman (2001), Goodhart (1998) and Woodford (1999a), among others, for discussions of interest rate stabilization).

Summarizing, the solutions for inflation and output depend on the degree of transparency about u_{t+1}. The main culprit for the increased volatility under transparency is the variation in private sector inflation expectations. The central bank would like the private sector to expect that prices will be stable while the central bank knows about upcoming non zero cost-push shocks to prices. The central bank knows the current error in private sector forecasts but is not willing to disclose any information before period $t+1$ arrives or, equivalently, before private sector expectations are set and policy actions taken.

2.3.3 Equilibrium Interest Rate

As it was indicated in the introduction, the transparency literature focuses on disclosure of current shocks. An implication of this is that current policy choices may partly reveal to the public the central bank's private information. In the New Keynesian framework with private information on future shocks, current period action does not give a signal of the central bank's private information for two

[16] In equilibrium, private sector and central bank inflation expectations are $E_t^p \pi_{t+1} = E_t^c \pi_{t+1} = \theta_1^* u_{t+1}$, which differs from the case of secrecy as far as private sector expectations are concerned.

reasons. First, as the Phillips and IS equations show, optimal policy reacts to private sector expectations of inflation and output, which under secrecy do not depend on the central bank's information about u_{t+1}. Second, unlike u_t, which directly affects current inflation irrespective of private sector expectations, the central bank does not need to react to u_{t+1}. As can be seen from the case of secrecy (see (2.12) and (2.13)), the information advantage of the central bank with respect to u_{t+1} is not revealed even ex post. Intuitively, under no disclosure policy, the private sector does not know the realization of u_{t+1}, although it knows that the central bank has that information. The best it can do is therefore to set expectations based on its signals.

To see the implications for the nominal interest rate of not releasing the forecasts of u_{t+1}, use the equilibrium solution for output and private sector expectations in the IS equation and solve for the interest rate rule that implements optimal policy. Ignoring the demand shock v_t for simplicity[17]

$$i_t = E_t^p \pi_{t+1} - \frac{1}{\phi} x_t$$

Thus, in equilibrium, the rate of interest ultimately depends on current shocks and private sector signals of next period shocks. Even if the central bank announces its interest rate target for period t, there is no way that the central bank can reveal its private information by its current actions. This is true even if the private sector knows as much as the central bank about the latter's loss function, including the targets for inflation and output and the relative weight on output stabilization. In this respect, Svensson (2003b) argues that the best way to make the central bank's forecasts observable to the public is by revealing the central bank's model, information, assumptions and judgments. In previous studies on transparency of current shocks, knowledge of the loss function enables the private sector to infer ex post the central bank's private information. In our case, revelation of its loss function may not help the public at all to infer the central bank's private information about future shocks.

2.4 Disclosure Policy under Limited Commitment

The classic theory of time-inconsistency in monetary policy rationalizes the high inflation period of the 1970s by the discretionary behavior of central banks. The term "inflation bias" was coined to underscore the implication of the theory that absent rules based monetary policy, equilibrium inflation turns out to be above the

[17] Observe that the demand shock does not give rise to a tradeoff in stabilizing inflation and output since the implied adjustment in interest rates to changes in demand shock moves output and inflation in the same direction. That is why in equilibrium, output and inflation are independent of the demand shock.

socially optimal level. The reason lies in the temptation of monetary authorities (due to unrealistic output or employment target) to renege on their plans once private sector expectations are set. With forward-looking expectations emphasized by the New Keynesian view of the macroeconomy, we may have not only an inflation bias, but also a ”stabilization bias” as a result of discretionary policy. Even without the inflation bias problem, monetary authorities would like the private sector to believe that policy will be strongly anti-inflationary in the sense of stabilizing inflation but once private sector inflation expectations are manipulated this way, the authorities will have an incentive (if they are free to do so) not to stabilize inflation strongly, contrary to their plans. Knowing this fact, the private sector will set inflation expectations such that the discretionary equilibrium is the only result.

If the central bank can not credibly commit to keep inflation variability low in the future, thereby loses power to anchor inflation expectations, then policy ends up being discretionary, optimizing period by period, *given* expectations. The crucial observation we made in the case of discretionary policy is that the central bank would like to see that fluctuations in private sector inflation expectations are minimized. In this situation the central bank will do anything that makes inflation expectations less variable. If it has private information about future developments of the economy, it will refrain from disclosing those information to the public, as we have shown in the case of cost-push shocks.

This section shows that the undesirable property of transparency about future shocks is not unique to discretionary policy. Even if the central bank were to follow a policy based on some rules, it would still favor secrecy. The reason lies in the fact that transparency always impairs the central bank's ability to stabilize current inflation and output because private sector expectations add volatility to current inflation irrespective of the policy regime.

2.4.1 Commitment for a Transparent Central Bank

A simple way to appreciate the gains from some form of commitment would be to consider a transparent regime about the shock u_{t+1}. The question is then, can the central bank improve stabilization policy if it has the ability to commit to a given policy rule? The answer is, yes. To make it specific, suppose the central bank can commit credibly to a simple policy rule that takes the same form as (2.18). Although this is a sort of limited commitment, as we have constrained the central bank to follow a rule that has a particular form, it serves to show the benefits from commitment. The idea is to see if a transparent central bank can improve welfare by committing to a simple rule within the same class of rules derived under discretion. Thus consider a commitment to the following rule

$$x_t = -Au_t - Bu_{t+1} \tag{2.19}$$

where the weights A and B are to be chosen optimally by the central bank. Then from (2.19) (and see Appendix) private sector expectations for output and inflation follow

$$E_t^p x_{t+1} = -Au_{t+1} \qquad E_t^p \pi_{t+1} = (1 - \lambda A)u_{t+1}$$

These expressions show clearly that the central bank's choice of a particular value for A will directly affect private sector inflation and output expectations, and via the Phillips and IS equations, current inflation and output. Using the expression for $E_t^p \pi_{t+1}$ in the Phillips equation (2.1) the reduced form expression for inflation, under commitment to the simple rule, will be

$$\pi_t = (1 - \lambda A)u_t + (\beta(1 - \lambda A) - \lambda B)u_{t+1} \tag{2.20}$$

Given the choices for the values of A and B, the dynamics of output and inflation is governed by (2.19) and (2.20), respectively. We can now express the expected loss as a function of the parameters A and B

$$EL_t = ((1 - \lambda A)^2 + \alpha A^2 + (\beta(1 - \lambda A) - \lambda B)^2 + \alpha B^2)\sigma_u^2 \tag{2.21}$$

The central bank minimizes (2.21) with respect to A and B with the optimal values given by

$$A^* = \frac{\lambda[\lambda^2 + \alpha(1 + \beta^2)]}{\alpha\lambda^2\beta^2 + (\alpha + \lambda^2)^2} = \Big(1 + \frac{\alpha^2\beta^2}{(\alpha + \lambda^2)^2 + \alpha\beta^2\lambda^2}\Big)\frac{\lambda}{\alpha + \lambda^2}$$

$$B^* = \frac{\alpha\beta\lambda}{\alpha\lambda^2\beta^2 + (\alpha + \lambda^2)^2}$$

The first observation is that both of these coefficients differ from their counterparts under discretion with transparency (see equation (2.18)), showing the central bank could improve up on the discretionary equilibrium by following a simple state-contingent rule that takes the same form as the discretionary solution but with different weights placed on the current versus forecasted shocks. Moreover, as long as $\alpha \neq 0$, that is the central bank cares about output stabilization as well as inflation stabilization, A^* is larger than its corresponding coefficient while B^* is smaller than its corresponding coefficient. This means that under commitment to the simple target rule (2.19) policy responds more aggressively to current shock realizations u_t but less aggressively to upcoming shock innovations u_{t+1}. The intuition for this result is that with partial commitment, a more aggressive policy in terms of contracting aggregate demand in reaction to current shocks leads the private sector to expect aggressive policy in the next period, thus lowering their inflation expectations. This in turn dampens the effect of future shocks on current inflation. Thus the central bank can afford to be less aggressive with respect to future shocks because the private sector does part of the job by adjusting its expectations. Knowing the value of A^*, the reduced-form of private sector inflation expectations is

$$E_t^p \pi_{t+1} = \frac{H\alpha}{\alpha + \lambda^2} u_{t+1} \qquad H \equiv 1 - \frac{\alpha\beta^2\lambda^2}{\alpha\beta^2\lambda^2 + (\alpha + \lambda^2)^2}$$

Since H satisfies $0 < H < 1$, private sector inflation expectations respond less strongly to future shocks than is the case under discretion. This outcome arises from the central bank's commitment to react more strongly to current shocks. If this commitment is credible, the private sector expects a strong reaction to next period shocks when the time arrives. This in turn lowers inflation expectations and current inflation.

For equilibrium inflation we have

$$\pi_t = \frac{\alpha}{(\alpha + \lambda^2) + (\alpha + \lambda^2)^{-1}\alpha\beta^2\lambda^2} u_t + \frac{\alpha^2\beta}{(\alpha + \lambda^2)^2 + \alpha\beta^2\lambda^2} u_{t+1} \tag{2.22}$$

Note that, compared to discretion, a policy of limited commitment results in less variability in the dynamics of inflation (compare (2.17) and (2.22)). This behavior contrasts with output, which is more volatile with respect to the current shock but responds less strongly to next period's shocks. Although this might make one conclude that the net effect of limited commitment on central bank loss function is not clear, it should be obvious that limited commitment improves welfare. Why else would the central bank choose different coefficients under limited commitment although the simple rule (2.19) falls under the class of rules derived from the discretionary solution? For the sake of completeness, however, we compare the expected losses in both regimes. Let T stand for transparency and $i = d(discretion), c(commitment)$

$$EL_i^T = Q_i^T \sigma_u^2$$

$$where \qquad Q_c^T \equiv \frac{\alpha^2(1 + \beta^2) + \alpha\lambda^2}{(\alpha + \lambda^2)^2 + \alpha\beta^2\lambda^2} \qquad Q_d^T \equiv \frac{\alpha(\alpha^2\beta^2 + (\alpha + \lambda^2)^2)}{(\alpha + \lambda^2)^3}$$

Next, evaluate the ratio Q_d^T/Q_c^T

$$\frac{Q_d^T}{Q_c^T} = 1 + \frac{\alpha^3\beta^4\lambda^2}{(\alpha(1 + \beta^2) + \lambda^2)(\alpha + \lambda^2)^3} > 1$$

2.4.2 The Gains from Secrecy under Limited Commitment

What we have shown so far is that given its decision to release internal forecasts, especially about u_{t+1}, to the public, the central bank is able to improve macroeconomic outcomes by credibly committing to a simple rule that reacts to those shocks. But, can the central bank do even better by not releasing information about u_{t+1} and committing to a simpler rule? We can easily show that this is possible. For instance, the central bank will gain by not releasing u_{t+1} and simply announcing the following policy rule:

$$x_t = -Au_t \tag{2.23}$$

To see this, take private sector expectations of next period output:

$$E_t^p x_{t+1} = -AE_t^p u_{t+1} = -Aks_t$$

Moreover, private sector inflation expectations are given by (see Appendix):

$$E_t^p \pi_{t+1} = (1 - \lambda A)E_t^p u_{t+1} = (1 - \lambda A)ks_t$$

Given private sector expectations of inflation and output and the simple rule (2.23) followed by the central bank, inflation will take the form

$$\pi_t = (1 - \lambda A)(\beta ks_t + u_t)$$

Expressing the expected loss as a function of A

$$EL_t = [(1 + \beta^2 k)(1 - \lambda A)^2 + \alpha A^2]\sigma_u^2 \tag{2.24}$$

and minimizing (2.24) with respect to A, it is easy to show that the optimal value of A is $\lambda/[(1 + \beta^2 k)^{-1}\alpha + \lambda^2]$. The central bank prefers this outcome to the case with commitment and information disclosure of u_{t+1}. Thus if the central bank is ever to commit to a simple rule, it will choose not to include u_{t+1} and not be transparent about its realization, showing that the gains from not releasing private information about u_{t+1} is not particular to discretionary settings.

It is possible to generalize the commitment case by considering the unconstrained commitment solution; that is the optimal policy rule under commitment is not constrained to take the functional form of the rule under limited commitment. In that case, it can be shown that the targeting rule is

$$x_t = x_{t-1} - \frac{\lambda}{\alpha}\pi_t$$

which looks similar to the discretionary case, except that now there is an additional lagged term, x_{t-1}, indicating history dependence (the notion of 'timeless perspective' is discussed by Woodford (1999a)). The desirability of secrecy about u_{t+1} holds true also under unconstrained commitment.[18]

2.5 Implementation Issues

Hitherto, our analysis has been based on the presumption that the central bank can target private sector expectations if these forecasts are observable at the time or just before the central bank decides about the current target for the rate of interest. Thus, even if it forms its own (internal) forecasts which are better at tracking the path of future shocks to inflation, the central bank does not actually use them for decision making. This result can be put into perspective by noting that there is a debate about the usefulness of internal central bank forecasts versus private sector forecasts (see for e.g. Hall and Mankiw, 1994; Evans and Honkapohja, 2002; Svensson, 1997).

It is important to understand the central bank's incentives and what it is aiming at. There are two sources of fluctuations in inflation–the exogenous cost–push shocks and private sector inflation expectations, which is influenced in part by central bank policy. Given any policy path followed, the central bank would like to see very small fluctuations in these variables. Of course, the cost-push shocks are exogenous; nothing can be done to prevent their realization, although the central bank may try to neutralize some of the effects of these shocks. With respect to private sector expectations, the problem is more subtle.

2.5.1 Targeting Rules and Real Equilibrium Determinacy

The analysis in the previous sections was in terms of inflation and output and did not involve the question of how optimal policy is implemented using the rate of interest. From practical point of view, the question of policy implementation is very crucial as it raises issues of determinacy of rational expectations equilibria for inflation and output. In particular our interest lies in a situation where private sector inflation expectations might potentially change for no fundamental reason (what is usually referred to as sunspot driven changes in expectations) and how monetary policy responds to this situation. Following Taylor (1993), there has been a lot of discussion on the properties of simple interest rate rules that respond to inflation and output. As summarized by Woodford (1999), to ensure determinacy, the nominal rate must respond more than one-to-one to changes in inflation, a property dubbed as the " Taylor principle " by Woodford. If the Taylor principle

[18]It is also interesting to see that when $\alpha = \lambda$ the above rule is identical to a rule that is derived from a discretionary policy that targets the nominal income growth. For a thorough discussion of nominal income targeting, see Hall and Mankiw (1994).

is fulfilled, then any expectations driven increases (decreases) in inflation is followed by a contractive (expansive) monetary policy, ensuring that such sunspot driven expectations are not realized.

The problem of real indeterminacy arises from the possibility that the central bank fails to respond strongly to variations in private sector expectations that are not related to fundamentals of the economy (see Jensen (2002) for a detailed analysis). Suppose we are in a steady state of zero inflation and output-gap and consider an increase in private sector inflation expectations for no fundamental reason. This is usually referred to as sun-spot driven expectations because by assumption nothing has changed in the fundamental variables (for example new information about future cost-push shocks) that justifies changes in expectations. What happens next to the dynamics of inflation and output depends a lot on how the central bank responds, which in turn is related to its reaction function for the rate of interest. In our case, the central bank is optimizing period by period to ensure that the targeting rule (2.8) is satisfied in every period. Since the private sector knows the targeting rule, it expects the central bank to adjust the rate of interest in every period such that the targeting rule holds. With this idea in mind, the effects of sun-spot driven private sector inflation expectations, other things equal, are felt first on current inflation and output, as we can read from the Phillips and IS equations. But, according to the targeting rule, the central bank responds optimally by contracting current output, where the magnitude of the contraction depends partly on the central bank's preference parameter α. To implement its contractionary policy, the central bank pushes the nominal rate of interest up immediately and with sufficient force. It turns out then that the hypothesized changes in expectations are not fulfilled.

The relevance of this example has been emphasized in recent discussions of the properties of simple monetary policy rules. It has been shown that interest rate rules that respond only to fundamental variables, in our case cost-push and demand shocks, suffer from real equilibrium indeterminacy as they fail to respond to sun-spot driven expectations. In other words, fundamentals based rules assume that expectations are formed based on available information about fundamental shocks, and thus respond to expectations only indirectly (via these shocks). This occurs because the private sector is assumed to follow the rational expectations equilibrium that is derived based on the fundamental shocks only (the MSV method). Thus these rules do not account for out-of-equilibrium changes in expectations. In our case, the targeting rule is key in avoiding indeterminacy from sun-spot driven expectations.

2.5.2 Observability of Private Sector Expectations

Another potential problem relates to the observability of private sector expectations. This issue has been part of the subject of recent discussions in the design of monetary policy rules. For example, in their investigation of determinacy and stability of macroeconomic systems under private sector learning, Evans and Honkapohja (2002) strongly suggest for incorporating private sector forecasts of inflation and output into Taylor-type interest rate rules. However, the authors wonder about whether there are large errors in measuring these expectations. They say that "while private forecasts by different institutions are regularly published, it is not self-evident that these published numbers accurately represent the expectations of the private sector that are relevant for the key private economic decisions."(p.3) In our case, it is common knowledge that the central bank has a better forecast than the private sector about future shocks. The central bank also observes the current rate of inflation and output, thus it can set the interest rate such that the targeting rule is satisfied in every period. Even though the central bank may not observe private sector expectations, this is not needed as long as it can observe the changes in the current levels of inflation and output. In deciding not to release its forecasts, all the central bank needs to know is that it has a better forecast of the shocks than the private sector. The question is then, which assumption is stronger, observability of current private sector expectations or observability of current inflation and output? The more severe problem for the central bank is unobservability of current inflation and output, because in that case these variables would be more variable owing to unobservable shocks to the central bank at the time it makes decisions on optimal interest rates. The problem is of course exacerbated if private sector expectations are also unobserved or there are large observation errors.

2.6 Forward-Looking Interest Rate Expectations

In this section, we analyze the question of releasing forecasts in a modified version of what Cukierman (2001) called the Neo-Keynesian transmission mechanism, referring to the backward-looking model of Svensson (1997). We add a term structure equation to that framework, thereby introducing forward-looking expectations about the level of the long-term real interest rates, which affect aggregate demand and with some lag the rate of inflation.[19] Notwithstanding notational differences, the Neo-Keynesian model that Cukierman (2001) analyzes is given by

$$\pi_{t+1} = \lambda x_t + u_{t+1} \tag{2.25}$$

$$x_{t+1} = -\beta R_t + v_t \tag{2.26}$$

where we have modified the demand equation by using the long-term real rate R_t instead of the short-term real rate r_t. Moreover, in the spirit of Svensson (1997), there is a one-period lag from changes in interest rates to changes in aggregate demand, and a two-period lag between policy actions and actual inflation. The long

[19] Using a slightly different setup, Eijffinger et al. (2004) explore the role of learning in optimal monetary policy.

and short-term interest rates are related by the expectations hypothesis of term structure (see Eijffinger et al. (2004) for details of the term structure equation)

$$R_t = (1-k)r_t + kE_t^p R_{t+1} + \epsilon_t \tag{2.27}$$

$E_t^p R_{t+1}$ is private sector's expectation of the long real rate in period $t+1$ and ϵ_t represents a term-premium.

The loss function depends on inflation and interest rate variability. Including output in the loss function does not change the qualitative results, so we leave it out in order to simplify the algebra. Moreover, what we intend to show is that under full credibility, the non-optimality of disclosing future shocks goes through insofar as the central bank does not focus exclusively on price stability.

$$L_t = \pi_{t+1}^2 + \alpha r_t^2 \tag{2.28}$$

with α represents the degree of central bank concern about interest rate stabilization relative to inflation stabilization.

Suppose as in Cukierman (2001) the central bank has perfect advance information about the three relevant shocks, u_{t+2}, v_{t+1} and ϵ_t, assumed to be white noise. Moreover, in line with our issue of interest, assume also that u_{t+3} and v_{t+2} are known only to the central bank.[20] The first order condition for minimizing the loss function subject to the model constraints is given by

$$\begin{aligned} &E_t\{[-\beta\lambda(1-k)r_t - \beta\lambda k E_t^p R_{t+1} - \beta\lambda\epsilon_t + \\ &\quad +\lambda v_{t+1} + u_{t+2}](-\beta\lambda(1-k)) + \alpha r_t\} = 0 \end{aligned} \tag{2.29}$$

The reaction function for r_t is then

$$r_t = AE_t^p R_{t+1} + B\epsilon_t + Cv_{t+1} + Du_{t+2} \tag{2.30}$$

where

$$A = -\frac{\beta^2\lambda^2 k(1-k)}{\alpha + \beta^2\lambda^2(1-k)^2} \qquad B = -\frac{\beta^2\lambda^2(1-k)}{\alpha + \beta^2\lambda^2(1-k)^2}$$

$$C = \frac{\beta\lambda^2(1-k)}{\alpha + \beta^2\lambda^2(1-k)^2} \qquad D = \frac{\beta\lambda(1-k)}{\alpha + \beta^2\lambda^2(1-k)^2}$$

[20] For simplicity, we ignore knowledge of ϵ_{t+1} by both parties. The analysis can easily be extended to include this effect but it does not add new insights.

With this reaction function and given private sector expectations, we can see that actual inflation will be

$$\pi_{t+1} = A_1 E_t^p R_{t+1} + B_1 \epsilon_t + C_1 v_{t+1} + D_1 u_{t+2} \tag{2.31}$$

where

$$A_1 = -\frac{\beta^2 \lambda^2 k(1-k)}{\alpha + \beta^2 \lambda^2 (1-k)^2} \qquad B_1 = -\frac{\beta^2 \lambda^2 (1-k)}{\alpha + \beta^2 \lambda^2 (1-k)^2}$$

$$C_1 = \frac{\beta \lambda^2 (1-k)}{\alpha + \beta^2 \lambda^2 (1-k)^2} \qquad D_1 = \frac{\beta \lambda (1-k)}{\alpha + \beta^2 \lambda^2 (1-k)^2}$$

Importantly the short rate responds to private sector expectations of the long-real rate. One can observe that instability in private sector expectations induces short rate volatility and the central bank does not fully offset the effects of expectations on inflation two-periods ahead because of its concerns about stabilization of the short-term rate. Ideally, of course, the central bank would like to see stable private sector expectations.

The next step is to solve for private sector expectations of the long rate for the next period. This brings us again to the issue of the release of forecasts by the central bank. In this respect, the similarity to the New Keynesian framework is clear. To see this, use the reaction function (2.30) in the term structure to get an expression for R_t as a function of private sector expectations and the three shocks.

$$R_t = A_2 E_t^p R_{t+1} + B_2 \epsilon_t + C_2 v_{t+1} + D_2 u_{t+2} \tag{2.32}$$

In deriving the rational expectations equilibrium, the relevant state variables depend on whether the central bank releases its information or not. As before, we use the method of undetermined coefficients; first, given that it is common knowledge that the central bank has private information, we guess that

$$R_t = \theta_0 \epsilon_t + \theta_1 v_{t+1} + \theta_2 v_{t+2} + \theta_3 u_{t+2} + \theta_4 u_{t+3} \tag{2.33}$$

where, to anticipate our results, the final values for the coefficients θ_2 and θ_4 depend on disclosure policy of the central bank. In particular, under secrecy these coefficients will be identically zero. The other coefficients will have non-zero values under secrecy or transparency, as they disrupt the economy irrespective of private sector expectations. the state variables are ϵ_t, v_{t+1} and u_{t+2}. Then the release of information about v_{t+1} and u_{t+2} only and private sector rational expectations imply

$$E_t^p R_{t+1} = \theta_1 E_t^p v_{t+2} + \theta_3 E_t^p u_{t+3} \tag{2.34}$$

Based on disclosure policy of the central bank, we have either $E_t^p R_{t+1} = 0$, reflecting the fact that the private sector take the expected value of the i.i.d shocks when no information is provided by the central bank, or $E_t^p R_{t+1} = \theta_1 v_{t+2} + \theta_3 u_{t+3}$ when information is released. Note that, as in the case of the New Keynesian model, the release of v_{t+1} and u_{t+2}, to which the central bank is offsetting in period t, is irrelevant as these do not help the private sector in forecasting the future. This is exactly the sort of problem we encountered under the New Keynesian model. Replacing the resulting private sector expectations under secrecy, the actual law of motion of R_t become simply

$$R_t = B_2 \epsilon_t + C_2 v_{t+1} + D_2 u_{t+2} \tag{2.35}$$

which is independent of the shocks v_{t+2} and u_{t+3}. Moreover, as can be seen from (2.30) and (2.31), the short-term real rate and inflation are not affected by these shocks. Thus release of future shocks can only destabilize inflation and the rate of interest.

2.7 Summary and Conclusion

Some central banks do not disclose their internal forecasts to the public in a timely manner, and even if they did, it is not clear if they would report their true forecasts, or if they adjust them so as to simply follow the markets, as Romer and Romer (2000) indicated in their study of the Federal Reserve of the U.S. where forecasts are published only with a long time lag so that the value of the published information becomes negligible.

Recent theory on transparency has not settled the question about welfare gains from advance disclosure of central bank forecasts. Existing research has analyzed this question assuming private information about *current* shocks, as these shocks have direct impact on *current* economic variables, such as inflation and output, that a central bank is interested in stabilizing. Based on this notion of private information, a few empirical studies on transparency lend support to the argument that disclosure of central bank forecasts can enhance the reputation and flexibility of monetary policy.

This chapter explores the significance of private information on *future* shocks as forecasts of future shocks are crucial when inflation expectations are forward-looking. The main result is that advance disclosure of forecasts of future shocks does not improve welfare, and in some cases not desirable as it impairs stabilization of current inflation and/or output. This result holds when there is no credibility problem or the central bank's output target is common knowledge. Another implication of the model is that, in contrast to forecasts of *current* period shocks emphasized by the literature, forecasts of future shocks may not be revealed to the public by current policy choices because the central bank refrains from responding to its own forecasts. Thus with respect to the signaling role of current policy actions, private information about future shocks has a different policy implication than that of current shocks. While current shocks may be revealed by current central bank actions, this may not be true for forecasts of future shocks. The intuition is that forecasts of future shocks do not influence the setting of current policy if the public is not aware of them; the central bank responds to current shocks only. If it can observe (or infer about) market expectations, the central bank finds it optimal to announce only what the markets already know.

Even though disclosing information seems counter-intuitive, as it improves the accuracy of private sector inflation forecast, the negative result on welfare is a consequence of the central bank having objectives other than price stability. With multiple macroeconomic goals, releasing internal forecasts before the public has currently formed expectations of future shocks, and thus future inflation, can actually impair overall stabilization efforts.

The result about the destabilizing effect of early disclosure of forecasts goes through for some alternative specifications, as long as there is full information regarding central bank preferences. In the case of the New Keynesian model, the results go through for a loss function that includes interest rate stabilization objective, on top of inflation and output; or if the central bank targets nominal income growth, instead of inflation and output, as proposed by some economists. Moreover, whether policy is conducted under discretion or some form of commitment is inconsequential to the main result. Our conjecture is that the results also apply if we drop rational expectations and assume in line with the adaptive learning literature that the private sector and/or the central bank adaptively learn about the structure of the economy, adjusting their forecasts with the arrival of new data. All that is needed for our results is that the central bank has superior forecasts about future supply shocks.

Appendix Expected Inflation under a Policy of Limited Commitment

$$\pi_t = \lambda x_t + u_t + \beta E_t^p \pi_{t+1}$$

$$= E_t^p \sum_{k=0}^{\infty} \beta^k [\lambda x_{t+k} + u_{t+k}]$$

$$= E_t^p \sum_{k=0}^{\infty} \beta^k [\lambda(-A_c u_{t+k} - B_c u_{t+k+1}) + u_{t+k}]$$

$$= E_t^p \sum_{k=0}^{\infty} \beta^k [(1 - \lambda A_c) u_{t+k} - \lambda B_c u_{t+k+1}]$$

$$= (1 - \lambda A_c) u_t - \lambda B_c E_t^p u_{t+1} + \beta[(1 - \lambda A_c) E_t^p u_{t+1} - \lambda B_c E_t^p u_{t+2}] + \cdots$$

Since the shocks are white noise, private sector inflation expectations when the central bank fully discloses the value of u_{t+1} is given by

$$E_t^p \pi_{t+1} = (1 - \lambda A_c) u_{t+1}$$

which is the expression following equation (2.19) in the main text.

Chapter 3

Credibility, Signaling and Disclosure Policy

3.1 Introduction

Following Faust and Svensson (2001) and Jensen (2000), this chapter modifies the policy problem of chapter 2 in two ways. First, the model includes unobserved shifts in the central bank's output target. This introduces an inflation bias as the output target can differ from the natural rate, which for simplicity is normalized to zero. In addition, the timing of events is such that the central bank chooses its policy before private sector inflation expectations are set. The public observes central bank policy decisions that respond in part to the central bank's private information. In principle, this implies that the private sector can infer in part the output target from observed central bank actions.

With these modifications, it turns out that the relevance of disclosing forecasts of future shocks is not clear cut and depends on specific assumptions about the unobserved output target. Specifically, the central bank is better off by withholding its private information about future shocks if the random shift in output target is directly revealed at the time the future shocks are realized. Otherwise, if the output target is not revealed in subsequent periods, but is inferred from observed policy decisions of the central bank, then disclosure policy is irrelevant for current period outcomes. In both cases, however, advance disclosure is not optimal.

As will be shown below, the reason lies in the strong dependence of one-period-ahead private sector inflation forecasts on central bank actions, which induces the central bank to focus exclusively on price stability in subsequent periods. Knowing this incentive, the private sector does not incorporate its forecasts of future shocks in forming expectations of next period's inflation.

However, as in the full information benchmark of the previous chapter, in equilibrium, forecasts of future shocks may not be revealed to the public by current policy choices. If disclosure is not optimal, the central bank can abstain from responding to its own forecasts of the cost-push shocks.

Geraats (2001) shows how disclosure of central bank forecasts can be beneficial when the public also faces uncertainty about the central bank's inflation target. The intuition is that disclosure of forecasts affect the incentives of the central bank to create surprises, as loss of credibility becomes a concern to the central bank in a multi-period framework. This incentive effect depends on the public's ability to make inferences about the unobserved inflation target based on its observed *current* central bank actions. The more information it receives about central bank forecasts of current shocks, the better the public is able to infer the unobserved inflation target and thus the central bank's intentions. It is then in the interest of the central bank to invest in reputation in the early periods in order to have more flexibility in the future in responding to shocks.[1]

Yet, desirability of disclosing current period shocks is disputed when forward-looking considerations take prominence, as in the New Keynesian view of the transmission mechanism (Jensen, 2000). Supposing unobserved output target as the source of inflation bias, Jensen (2000) shows how releasing forecasts of *current* shocks distorts current stabilization policy even though it solves the credibility problem of the sort mentioned in the previous paragraph. If the public observes central bank policy decisions *before* forming inflation expectations, a high degree of transparency about current shocks makes inflation expectations become extremely sensitive to the central bank's current stabilization actions. Confronted with very sensitive inflation expectations, policy tilts heavily toward inflation stabilization at the cost of making output very volatile. Thus disclosing internal forecasts could be undesirable for a central banker who enjoys good initial reputation.

There is one important common feature of the above cited papers. If, from the outset, the central bank's targets for inflation and output are common knowledge, then its forecasts of *current* period shocks can be perfectly inferred from its monetary policy decisions. The reason is that, even if the public does not observe current period shocks directly, the central bank reacts to those shocks, as they disrupt the level of *current* inflation and output that the central bank wants to stabilize.[2]

[1] In a cross-section study using 87 countries Chortareas et al. (2002) find that publication of forecasts reduces average inflation. Geraats and Eijffinger (2004) use time-series data on several aspects of transparency for nine major central banks, based on an index of transparency constructed by Eijffinger and Geraats, and conclude that higher transparency is associated with lower short-term as well as long-term interest rates, thus lending support to the positive reputation effects of releasing forecasts, as argued by Geraats (2001).

[2] Among other things, the paper by Geraats (2001) differs from Jensen (2000) in the effect of direct revelation of the unobserved target. In Geraats (2001), this leads to worse outcomes because the credibility problem would remain unresolved. This result seems at odds with the recent calls for transparency about inflation goals (e.g., Rogoff, 2003). In Jensen (2000), direct revelation of central bank's output target may dominate indirect revelation through the release of internal forecasts if the central bank has good reputation and stabilization policy has more importance.

3.2 A Three-Period Model

Suppose in period t the central bank has private information about the supply shock u_{t+1} while u_t is common knowledge. Somewhat similar to Jensen (2000), the policy game is played for two periods where the Phillips equation for period t is given by

$$\pi_t = E_t^p \pi_{t+1} + \lambda x_t + u_t \qquad t = 1, 2, 3 \qquad u_1 = u_3 = 0$$

In Jensen (2000), u_1 is assumed to be private information of the central bank while (implicitly) u_2 is unknown as of period 1; its value is set to zero as period 2 is interpreted to be the long-run.[3] Since we are interested in analyzing future shocks, suppose instead that in period 1, u_1 is common knowledge ($u_1 = 0$ for simplicity) while u_2 is the central bank's private information. In Period 3, the economy reaches a steady state, with $u_3 = 0$, as period 3 represents the long-run.

Without loss of generality, ignore discounting and take the central bank loss function defined over three periods

$$U = L_1 + L_2 + L_3$$

$$L_t = \pi_t^2 + \alpha(x_t - x_t^*)^2 \qquad t = 1, 2, 3$$

where $x_1^* = 0$ and $x_2^* = x_3^*$, and $x_t^* \sim N(0, \sigma^2)$.

Sequence of events and actions:

- Period 1: central bank knows u_2 and chooses $x_1 \rightarrow E_1\pi_2$ formed $\rightarrow \pi_1$ determined
- Period 2: x_2^* and u_2 realize, and private sector knows $u_2 \rightarrow x_2$ chosen $\rightarrow E_2\pi_3$ formed $\rightarrow \pi_2$ determined
- Period 3: (full information steady state) x_3 chosen $\rightarrow \pi_3$ determined

A permanent shock to the output target occurs in period 2, and is private information of the central bank. Shifts in the output target of the central bank may represent political pressures on the central bank or changes in the composition of the decision making committee of the central bank (see for e.g. Faust and Svensson, 2001). Thus in periods 2, the private sector faces uncertainty about the preference shock and in period 1 about the central bank's forecast of u_2. As in Jensen, monetary policy is discretionary, so the model is solved backwards starting from period

[3] Thus in Jensen only the game in period 1 is relevant for final outcomes.

2. Since the policy horizon is finite and markets are forward-looking, a terminal condition for inflation expectations must be assumed for period 3 (see Jensen (2000) in this regard). As noted above, the economy stays in a full information steady state from period 3 onwards, implying $\pi_3 = \pi_4$, $E_3^p \pi_4 = \pi_4$ and $x_3 = 0$. Consistent with this idea, assume $E_3^p \pi_4 = \frac{\alpha}{\lambda} x_3^*$.[4] Then the central bank minimizes

$$E_3^c[(\frac{\alpha}{\lambda}x_3^* + \lambda x_3)^2 + \alpha(x_3 - x_3^*)^2]$$

with respect to x_3. It is easy to get the solution for x_3, and in turn for π_3

$$x_3 = 0 \qquad \pi_3 = \frac{\alpha}{\lambda}x_3^* \tag{3.1}$$

These are the steady state values for output and inflation, featuring an inflation bias as long as $x_3^* > 0$.

Next consider period 2. As we show below, it turns out that the welfare effects of disclosing u_2 in period 1 depends on how the game is played in period 2. We consider alternative scenarios based on the private sector's knowledge of x_2^* in period 2. In the first case, the private sector directly observes the central bank's output target, and thus the central bank's choice of x_2 does not play a signaling role about the output target. In the second case, the private sector has to infer the output target from the central bank's choice of x_2, giving rise to signaling and incentive effects. Thus the determination of $E_2^p \pi_3$, which follows from the solution for π_3 in (3.1) is crucial.

3.3 Period 2 Output Target Directly Revealed

When x_2^* is common knowledge in period 2, $E_2^p \pi_3 = \alpha x_2^*/\lambda = E_3^p \pi_4$. In other words, private sector inflation expectations are identical in periods 2 and 3. The solution for x_2 is similar to that in period 3, except for the fact that u_2 is not necessarily zero. Analogous to period 3, the solutions for x_2 and π_2 are

$$x_2 = -\frac{\lambda}{\alpha + \lambda^2}u_2 \qquad \pi_2 = \frac{\alpha}{\lambda}x_2^* + \frac{\alpha}{\alpha + \lambda^2}u_2 \tag{3.2}$$

An implication of (3.2) is that, in period 1 inflation expectations depend on the private sector's forecast of u_2. From period 1's perspective both the central bank

[4] As Jensen rightly points out, the exact expression for the terminal condition is not that important for the choice of disclosure policy made in period 1. The particular expression we have chosen for the terminal period simplifies the algebra. This particular inflation expectations can also be derived from an infinite horizon model with full information about the loss function of the central bank.

and the private sector expect x_2^* to take its mean value of zero. It follows that the value of $E_1^p\pi_2$ depends on disclosure policy of the central bank. Under full disclosure of u_2, $E_1^p\pi_2 = \frac{\alpha}{\alpha+\lambda^2}u_2$, while $E_1^p\pi_2 = \frac{\alpha}{\alpha+\lambda^2}ks_1$ if u_2 is not disclosed and the private sector has to depend on its period 1 signal. Note that the problem faced by the central bank in period 1 is identical to the benchmark model of chapter 2, with common knowledge of the central bank's preferences. Anticipating that private sector expectations depend on disclosure policy, the central bank chooses a value for x_1 that minimizes the period 1 loss function.

When withholding u_2, the loss function is:[5]

$$E_1^c[(\frac{\alpha}{\alpha+\lambda^2}ks_1 + \lambda x_1)^2 + \alpha x_1^2]$$

The first order condition with respect to x_1 gives:

$$x_1 = -\frac{\alpha\lambda}{(\alpha+\lambda^2)^2}ks_1 \tag{3.3}$$

implying

$$\pi_1 = (\frac{\alpha}{\alpha+\lambda^2})^2 ks_1 \tag{3.4}$$

On the other hand, with full transparency, x_1 and π_1 are affected by u_2 via $E_1^p\pi_2$. The loss function is:

$$E_1^c[(\frac{\alpha}{\alpha+\lambda^2}u_2 + \lambda x_1)^2 + \alpha x_1^2]$$

and the solutions are:

$$x_1 = -\frac{\alpha\lambda}{(\alpha+\lambda^2)^2}u_2 \tag{3.5}$$

and

$$\pi_1 = (\frac{\alpha}{\alpha+\lambda^2})^2 u_2 \tag{3.6}$$

A comparison of (3.3),(3.4) with (3.5),(3.6) shows that communicating u_2 to the private sector in period 1 makes inflation and output more volatile. This is in line with the result in the full information benchmark (section 2.3 of chapter 2). Thus preference shocks, if observable by the public, do not change the basic message of the full information case without preference shocks. We think that a more interesting case is when the central bank's preference is not directly revealed to the private sector. This gives rise to incentive effects as far as the central bank is concerned, forcing it to react to private sector expectations, which in turn depend on the actions taken by the central bank.

[5]One may wonder if the private sector could get a signal about u_2 from the central bank's choice of x_1. This is not possible as the central bank would never react to u_2 in period 1.

3.4 Period 2 Output Target Indirectly Revealed

Next consider the case where x_2^* is not directly revealed to the public. Then, this time $E_2^p \pi_3 = \alpha(E_2^p x_2^*)/\lambda$ as the private sector has to forecast the value of x^*. Since the relevant state variables are x_2^* and u_2, conjecture the following form for x_2

$$x_2 = h_2 u_2 + h_x x_2^* \tag{3.7}$$

where the coefficients are yet undetermined. Since in period 2 the private sector observes x_2 and u_2, it can make perfect inferences about x_2^* from a signal, s_2, constructed as follows:

$$s_2 \equiv x_2 - h_0 - h_2 u_2 = h_x x_2^* \tag{3.8}$$

It is straightforward to see from (3.8) that private sector expectations of x_2^* given the signal s_2 is given by $E_2^p x_2^* = S_x s_2$ where $S_x \equiv 1/h_x$. Then $E_2^p \pi_3 = \alpha S_x s_2/\lambda$, and the minimization problem for period 2 is:

$$E_2^c[(\frac{\alpha}{\lambda} S_x s_2 + \lambda x_2 + u_2)^2 + \alpha(x_2 - x_2^*)^2]$$

which gives the following first order condition:

$$0 = E_2^c[(\frac{\alpha}{\lambda} S_x s_2) + \lambda x_2 + u_2)(\frac{\alpha}{\lambda} S_x + \lambda) + \alpha(x_2 - x_2^*)]$$

Using the fact that $E_2^c s_2 = h_x x_2^*$, x_2 can be expressed as a function of u_2 and x_2^*. Then the undetermined coefficients must satisfy:

$$h_2 = -\frac{\lambda^2 + \alpha S_x}{\lambda(\alpha + \lambda^2 + \alpha S_x)}$$

$$h_x = \frac{\alpha \lambda^2}{\alpha \lambda^2 + (\lambda^2 + \alpha S_x)^2}$$

The solutions can be found recursively, starting with the equation for h_x, which implies $h_x \rightarrow 0$.[6] It follows, by definition, that $S_x \rightarrow \infty$. This shows that private sector expectations react very strongly to the signal s_2 (which is related one-to-one with central bank action x_2), forcing the central bank not to respond to its preference shock x_2^*. Finally, the first equation gives the solution for h_2:

$$h_2 = -\frac{1}{\lambda}$$

[6] Note here that there is no role for disclosure of forecasts because u_2 is common knowledge.

Combining the above results, equilibrium output and inflation in period 2 are:

$$x_2 = -\frac{1}{\lambda}u_2 \qquad \pi_2 = \frac{\alpha}{\lambda}x_2^* \tag{3.9}$$

The intuition for this result is as follows. The strong dependence of private sector inflation expectations on output signals forces the central bank to care about its reputation. Thus in contrast to the previous case, where the private sector has full information about the output target, the fact that expectations are now very sensitive to the policy action, x_2, induces the central bank not to accommodate the preference shock to its output target. Moreover, the central bank lets the output gap fully absorb the cost-push shock.

One immediate implication of (3.9) is that disclosing central bank forecasts of u_2 prior to the formation of inflation expectations in period 1 is harmless to period 1 outcomes of inflation and output. Disclosing u_2 in period 1 does not affect inflation expectations because of the common knowledge that the central bank would completely offset the effect of u_2 on π_2.

The solutions for x_1 and π_1 in period 1 can easily be derived. First (3.9) implies that $E_1^p\pi_2 = \alpha(E_1^p x_2^*)/\lambda = 0$. Next, the central bank solves for the optimal level of x_1 that minimizes:

$$(0 + \lambda x_1 + u_1)^2 + \alpha x_1^2$$

The equilibrium output and inflation in period 1 are then given by:

$$x_1 = -\frac{\lambda}{\alpha + \lambda^2}u_1 \qquad \pi_1 = \frac{\alpha}{\alpha + \lambda^2}u_1 \tag{3.10}$$

where the central bank optimally stabilizes period 1 shocks. Inflation expectations are firmly anchored at zero due to the anticipated behavior of the central bank, which stabilizes period 2 inflation completely. Thus in this case early disclosure of u_2 to the private sector in period 1 is harmless to the central bank's welfare loss.

3.5 Concluding Remarks

This chapter modifies the analysis in the previous chapter by incorporating unobserved preferences. In addition to asymmetric information about future cost-push shocks, we assume, following Faust and Svensson (2001), that the central bank's output target is private information. Shifts to the output target give rise to the standard inflation bias and associated credibility problem. The idea is that better knowledge about cost-push shocks can help the private sector make better inferences about the unobserved output target. It turns out that the effects of transparency about future cost push shocks depends on whether the central bank's output target is common knowledge or indirectly revealed from future policy actions.

The main result of this chapter is that advance disclosure of forecasts of future shocks does not improve welfare, and in some cases is not desirable as it impairs stabilization of current inflation and/or output. This result holds when there is no credibility problem or the central bank's output target is common knowledge. When there is uncertainty about the central bank's current output target, and this uncertainty is not resolved in the subsequent period, advance disclosure does not matter for current outcomes. The reason lies in the strong dependence of one-period-ahead private sector inflation forecasts on central bank actions, which induces the central bank to focus exclusively on price stability in subsequent periods.

With respect to the signaling role of current policy actions about internal forecasts of cost-push shocks, the results of the previous chapter continue to hold. Forecasts of future shocks do not influence the setting of current policy if the public is not aware of them; the central bank responds to current shocks only.

Obviously, there are some limitations of our analysis, limitations that are also shared by the literature on disclosure policy of central bank forecasts. First, the central bank is assumed to observe private sector expectations without error. Introducing observation errors would put the central bank at a disadvantage, and with very large errors, the central bank may even be forced to be transparent about its private information. Second, there is no strategic manipulation of expectations by the private sector, although it knows that the central bank is responding to private sector expectations. If the private sector knows that the central bank reacts to private sector expectations, strategic behavior becomes more important. Third, on the part of the central bank there could be an incentive to manipulate its private information and truthful revelation may not be feasible. As Romer and Romer (2000) have noted, even if central banks disclose their internal forecasts to the public in a timely manner, it is not clear if they would report their true forecasts, or if they would adjust them so as to simply follow the markets. Fourth, the analysis would be broader if current *and* future shocks remain private information of the central bank.

Chapter 4

Central Bank Communication and Output Stabilization

Some central banks have a reputation for being secretive. A justification for that behavior that we find in the literature is that being transparent about its operations and beliefs hinders the central bank in achieving the best outcome. In other words, a central bank needs flexibility and therefore cannot be fully transparent. Using a forward-looking New Keynesian model, we find exactly the opposite. A central bank that is conservative improves output stabilization by being transparent about the procedures it uses to assess the economy and, especially, about the forecast errors it makes. Under certain conditions transparency by a conservative central bank also improves interest rate stabilization. We also find that higher transparency makes it optimal for the central bank to be more conservative as the benefits from higher transparency in terms of output stabilization are greater the more conservative is the central bank.

4.1 Introduction[1]

Monetary policy makers broadly agree that communication is a very important part of their business. Communication gives central bankers a tool to shape private sector expectations, which are crucial for effective monetary policy. Blinder (1998) argues that openness and communication with the public improve the effectiveness of monetary policy as a macroeconomic stabilizer because: "Central banks generally control only the overnight interest rate, an interest rate that is relevant to virtually no economically interesting transactions. Monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter like long-term interest rates, stock market values, and exchange rates."

[1]An earlier version of this chapter (co-authored by Sylvester Eijffinger and Marco Hoeberichts) has already been published as a CEPR Discussion Paper.

Most theoretical studies of central bank transparency assume some kind of informational asymmetry between the central bank and the private sector. In particular, the central bank has an informational advantage about its own goals (e.g., Cukierman and Meltzer, 1986; Geraats, 1999; Jensen, 2000; Eijffinger et al., 2000a; Faust and Svensson, 2001) or the state of the economy (e.g., Cukierman, 2001; Geraats, 1999; Jensen, 2000; Gersbach, 2003). Within this class of models, the issue of whether it is desirable to communicate private information of the central bank (goals, intentions or forecasts) is far from being settled. A reading of the literature shows that the desirability of communication depends on the specific nature of the model and its information structure.

In all these studies, it is assumed that monetary authorities can observe and respond directly to private sector expectations. While it is unquestionable that central banks can hide their true intentions, and there is some evidence that they have superior information about the economy (see for e.g. Romer and Romer, 2000), we think it is unrealistic to presume that policy makers have precise knowledge of market expectations. A more realistic setting, in the spirit of Tarkka and Mayes (1999) and Evans and Honkapohja (2002), among others, has to assume that the central bank's assessment of private sector expectations is imperfect. Tarkka and Mayes (1999) have incorporated this assumption in a model that features Lucas-type transmission mechanism while Evans and Honkapohja (2002) analyze imperfect observability of private sector expectations in the context of simple monetary policy rules under adaptive learning. Evans and Honkapohja (2002) point out that although survey data on private forecasts of future inflation and output are available to central banks, there are apparent concerns about the accuracy of this data.

This paper studies a case where the information is asymmetric in two ways. First, the private sector has superior knowledge about its own expectations of future inflation and output. The central bank sets its policy based on an imperfect assessment of these private sector expectations. Likewise the private sector can not perfectly observe these assessments made by the central bank unless the central bank publishes them. If it wishes the central bank can provide information about the way its assessment is produced and thereby make it easier for the public to infer the judgment errors made by the central bank (see Tarkka and Mayes, 1999). Our aim is to shed light on the implications of communication by the central bank regarding its assessments on private sector expectations and finally on macroeconomic outcomes. It should be borne in mind that the aim of central bank communication is to reduce uncertainty on the part of the private sector, since these errors are also reflected in the setting of interest rate policy. The presumption is that even if the variance of the assessment error is exogenously fixed, communication of these errors to the public can change public expectations such that under certain conditions overall stabilization is improved.[2]

[2]Expectation formation is the result of a complicated process and involves constantly changing judgments, that is "information, knowledge, and views outside the scope of a particular model"

We look at the effect of communication on the macroeconomic variables that we are mainly concerned with in this model: the rate of inflation and the output gap. Assuming the performance of the economy is measured by a weighted sum of the variability in the rate of inflation and the output gap, the main result is that by communicating with the public its assessments errors, the central bank improves the variability of output at the expense of the variability of inflation, leading to a tradeoff. This tradeoff also has normative implications for policy. As will be shown later, a central bank that is sufficiently conservative (in the sense of Rogoff, 1985) improves society's welfare by communicating its assessment of market expectations of inflation and output.

4.2 The Model

Since the role of communication is more important in a forward-looking framework, we base our analysis on the forward-looking New Keynesian model. A detailed treatment of this model can be found in several recent papers that address monetary policy, including Clarida et al. (1999) and King (2000). The two basic structural relationships (one for inflation and the other for the output-gap) come from a log-linear approximation to a micro-founded dynamic general equilibrium model where aggregate behavior is a result of explicit optimization by households and firms. There is monopolistic competition in the product market and firms face nominal price rigidity, thus giving monetary policy the ability to influence economic activities in the short run.

Inflation is determined by a forward-looking Phillips equation:

$$\pi_t = \beta E_t^p \pi_{t+1} + \lambda x_t + u_t \tag{4.1}$$

where π is the inflation rate, x is the output gap, and $u \sim N(0, \sigma_u^2)$ reflects a cost-push shock to inflation, which is identically and independently distributed (in short i.i.d.). The parameter β captures the discount factor while λ is related to the average frequency of price changes and the elasticity of product demand. These parameters satisfy $0 < \beta < 1$ and $\lambda > 0$. The superscript p in $E_t^p \pi_{t+1}$ stands for private sector expectations. Thus current period inflation depends on private sector expectations of future inflation, current period output gap and the cost-push shock. The equation for inflation is derived from firms' pricing decisions, assuming that not all firms can change their prices in any given period.

The output gap is governed by a forward-looking IS equation, which is a log-linearized Euler equation associated with households' intertemporal consumption

(Svensson, 2005). It is thus natural to assume that assessments of the forecasts of other parties is fraught with errors. See also Mankiw et al. (2002) for an empirical evidence on the existence of substantial disagreements regarding expectations.

and saving decisions. It is given by:

$$x_t = E_t^p x_{t+1} - \phi r_t + v_t \tag{4.2}$$

where r is the real interest rate and $v \sim N(0, \sigma_v^2)$ is an i.i.d. demand shock that captures preference shocks or expected changes in government expenditure. The parameter ϕ is an intertemporal elasticity of substitution of consumption and satisfies $\phi > 0$. Thus the current period output gap depends on private sector expectations of next period's output gap, the real interest rate and the demand shock.

Finally, the real interest rate is determined by the Fisher equation, which links the nominal interest rate with the real interest rate.

$$r_t = i_t - E_t^p \pi_{t+1} \tag{4.3}$$

where i is the nominal interest rate. Combining (4.2) and (4.3) one can rewrite the output gap as a function of private sector expectations and the central bank's policy instrument.

$$x_t = E_t^p x_{t+1} - \phi i_t + \phi E_t^p \pi_{t+1} + v_t \tag{4.4}$$

In each period the central bank optimizes after making an assessment of private sector expectations. It sets i_t that minimizes the expected value of it's period t loss function given by:

$$L_t^c = \pi_t^2 + \alpha x_t^2 \tag{4.5}$$

where α is the relative weight on output stabilization and superscript c stands for central bank.

4.3 Equilibrium under Full Information

Before analyzing optimal policy under asymmetric information, it is useful to consider a simpler, baseline scenario where the central bank has full information about the economy and observes private sector expectations without error. In this case, even if it does not directly observe current inflation and output, the central bank can infer about them from private sector expectations, the prevailing interest rate, and the two shocks.

Following much of the literature, the model is solved assuming that monetary policy is conducted under discretion.[3] The optimality condition in terms of the

[3]Clarida et al. (1999) argue that it is realistic to assume a discretionary monetary policy.

target variables (thus the name *targeting rule*) can be derived easily by minimizing (4.5) subject to the Phillips equation (4.1).

$$x_t = -\frac{\lambda}{\alpha}\pi_t \tag{4.6}$$

According to (4.6), in each period, the central bank contracts (expands) current output in response to a higher (lower) rate of current inflation, with the degree of response depending on the α and λ. Combining the targeting rule with (4.1), (4.2) and (4.3), the nominal interest rate is set according to the following *instrument rule*:

$$i_t = \frac{1}{\phi}\Big\{(\phi + \frac{\beta\lambda}{\alpha + \lambda^2})E_t^p\pi_{t+1} + E_t^p x_{t+1} + \frac{\lambda}{\alpha + \lambda^2}u_t + v_t\Big\} \tag{4.7}$$

The rate of interest responds optimally to private sector expectations and the two shocks. It is straight forward to derive the rational expectations solution by solving for private sector expectations. After substituting (4.6) in (4.1), the rate of inflation is given by,

$$\pi_t = \frac{\alpha\beta}{\alpha + \lambda^2}E_t^p\pi_{t+1} + \frac{\alpha}{\alpha + \lambda^2}u_t \tag{4.8}$$

Thus the dynamics of actual inflation depend on currently held private sector expectations about next period's inflation and on the current realization of the exogenous shock u_t.

To solve for $E_t^p\pi_{t+1}$ and derive the rational expectations equilibrium, note first that the only relevant state variable is u_t. Using the commonly used method of undetermined coefficients,[4] first take a guess for the solution taking the following form:

$$\pi_t = \theta u_t \tag{4.9}$$

from which private sector rational expectations follow

$$E_t^p\pi_{t+1} = \theta E_t^p u_{t+1} = 0 \tag{4.10}$$

Using this result back in (4.8)

$$\pi_t = \frac{\alpha}{\alpha + \lambda^2}u_t \tag{4.11}$$

[4]McCallum (1983) emphasizes solving the model using only the fundamentals of the economy (in this case u_t), thereby avoiding bubble solutions. McCallum calls this the Minimal State Variables (MSV) method.

where consistency of rational expectations imply $\theta = \alpha/(\alpha + \lambda^2)$.

To complete the rational expectations solution under full information, use (4.11) in the optimality condition to get the solution for output

$$x_t = -\frac{\lambda}{\alpha + \lambda^2} u_t \tag{4.12}$$

Thus under full information, equilibrium inflation and output are functions of the cost-push shock. Moreover, as long as $\alpha > 0$ the impact of this shock is partially absorbed by inflation and partially by output.

4.4 Assessment Errors and Disclosure

To this point the central bank was endowed with perfect knowledge of the state of the economy and in particular regarding private sector expectations. A more realistic setting, in the spirit of Tarkka and Mayes (1999) and Evans and Honkapohja (2002), among others, is that the central bank's assessment of private sector expectations about the future output gap and the future rate of inflation is imperfect. Evans and Honkapohja (2002) discuss the issue of observability of current private expectations in the context of the adaptive learning literature. They point out that although survey data on private forecasts of future inflation and output are available to central banks, there are apparent concerns about the accuracy of this data. Although most experts would agree that it is very hard for the central bank to accurately measure the public's expected output gap, opinions differ about the extent to which the central bank is uncertain about the public inflationary expectations (see, however, Mankiw et al., 2002).

One may choose a general setup, where the central bank makes an assessment error in both private sector inflationary expectations and output gap expectations (where variances of these errors may be different). However, as shown in Appendix A, our qualitative results are not changed by focusing only on output gap expectations. To capture asymmetric information, suppose private sector output forecasts and the central bank's assessment of those forecasts are related by

$$E_t^c x_{t+1} = E_t^p x_{t+1} - w_t^x \tag{4.13}$$

where superscript c denotes central bank forecasts. Since for now the central bank is assumed to observe inflation expectations $w_t^\pi = 0$ for all t (Appendix A extends the model to include w_t^π). Importantly, the assessment errors follow a first-order autoregressive process

$$w_t^x = \rho w_{t-1}^x + \eta_t^x \tag{4.14}$$

where the innovations are independently and normally distributed with $\eta_t^x \sim N(0, \sigma_\eta^2)$ and ρ is a measure for the degree of persistence of the assessment errors, satisfying $0 < \rho < 1$. Assessment errors can be persistent if the central bank only sluggishly adjusts its procedures.

Similar to the full information setting, monetary policy is discretionary but now the central bank optimizes period by period based on its internal assessment of private sector expectations. Moreover, since the central bank does not observe current inflation and output, the optimality condition is written with the actual values of inflation and output in (4.6) replaced by the forecasts of these variables.

$$E_t^c x_t = -\frac{\lambda}{\alpha} E_t^c \pi_t \tag{4.15}$$

where the central bank's expectation of the Phillips equation is based on its assessment of private sector inflationary expectations[5]

$$E_t^c \pi_t = \beta E_t^c \pi_{t+1} + \lambda E_t^c x_t + u_t \tag{4.16}$$

Using (4.16) in the optimality condition (4.15)

$$E_t^c x_t = -\frac{\beta\lambda}{\alpha + \lambda^2} E_t^c \pi_{t+1} - \frac{\lambda}{\alpha + \lambda^2} u_t \tag{4.17}$$

Likewise, taking the central bank's expectation of the IS relation, (4.2)

$$E_t^c x_t = E_t^c x_{t+1} - \phi i_t + \phi E_t^c \pi_{t+1} + v_t \tag{4.18}$$

Next combine (4.18) and (4.16) to get the following expression for the nominal interest rate

$$i_t = \frac{1}{\phi}\Big\{\Big(\phi + \frac{\beta\lambda}{\alpha + \lambda^2}\Big) E_t^c \pi_{t+1} + E_t^c x_{t+1} + \frac{\lambda}{\alpha + \lambda^2} u_t + v_t\Big\} \tag{4.19}$$

It can easily be seen that (4.19) has a similar form as its full information counterpart, (4.7). The only difference lies in the expectational terms. Due to imperfect information about price sector output expectations, the central bank uses its own (internal) forecasts when setting the rate of interest. At the same time, unless they are disclosed by the central bank, the private sector does not observe the central bank's forecasts nor the current assessment errors.

[5] We get the optimality condition (4.15) by minimizing the expected value of (4.4) subject to the central bank expectation of the Phillips curve, which is (4.16) below.

$E_t^c \pi_{t+1}$ is only a noisy forecast of $E_t^p \pi_{t+1}$, which means that the central bank commits forecast errors and associated control errors. With (4.13) in mind, (4.19) can then be written alternatively as

$$i_t = \frac{1}{\phi}\Big\{\Big(\phi + \frac{\beta\lambda}{\alpha+\lambda^2}\Big)E_t^p \pi_{t+1} + (E_t^p x_{t+1} - w_t^x) + \frac{\lambda}{\alpha+\lambda^2}u_t + v_t\Big\} \tag{4.20}$$

so that the policy rate i_t is affected by the current assessment error, w_t^x. The public understands the structure of information asymmetry in the economy and so by using (4.20) in (4.2), and the resulting equation in (4.1), it can infer that the actual dynamics of output and inflation is a function of private sector expectations and the assessment error.

$$x_t = -\frac{\beta\lambda}{\alpha+\lambda^2}E_t^p \pi_{t+1} + w_t^x - \frac{\lambda}{\alpha+\lambda^2}u_t \tag{4.21}$$

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}E_t^p \pi_{t+1} + \lambda w_t^x + \frac{\alpha}{\alpha+\lambda^2}u_t \tag{4.22}$$

An interesting feature of (4.21) and (4.22) is that both expressions are free of the term $E_t^p x_{t+1}$. The reduced form solutions are found by first solving for $E_t^p \pi_{t+1}$ using (4.22) and conditional on available information. Equation (4.22) differs from the full information counterpart, (4.8), by the additional term λw_t^x, which captures assessment errors reflecting our asymmetric information setting. Note also that, even if there were no shocks to inflation ($u_t = 0$) inflation and output would still be stochastic owing to the central bank's assessment errors.

4.5 Communication and Expectations

In forecasting future inflation, the public uses its knowledge of past assessment errors (in light of their persistent nature) and any information about current errors that the central bank has disclosed.[6] We assume that the private sector gets a signal of the current period error according to:

$$s_t^x = w_t^x + \varepsilon_t^x \tag{4.23}$$

where from (4.14) the assessment error is normally distributed with a given finite variance, $w_t^x \sim N(0, \sigma_{wx}^2)$, and $\sigma_{wx}^2 = \frac{1}{1-\rho^2}\sigma_{\eta x}^2$.

The degree of communication is measured by the quality of the central bank's disclosed information about its assessment of the public's state of mind. With the

[6] A more general situation arises when past and present assessment errors are hidden variables so that the private sector uses a more general form of Kalman filtering to get optimal forecasts.

variance of the assessment errors, σ^2_{wx}, exogenously fixed, a fuzzy account of the assessment errors by the central bank leads to large noise variance, $\sigma^2_{\varepsilon x}$, since it becomes difficult for the public to infer the unobservable assessment error from disclosed information. Here the public depends completely on available central bank models, procedures, and judgements that produce internal inflation forecasts to make inferences about the assessment errors. In a more complicated scenario one may allow the public to deduce or infer the central bank's private assessment errors from observing the central bank's interest rate decisions.[7]

As in standard signal extraction problems, (4.23) represents an observation equation, where the input signal w^x is contaminated by an independent noise term, ε^x, whose variance, $\sigma^2_{\varepsilon x}$, is affected by the central bank's disclosure policy. Since the central bank controls the signal-to-noise ratio through its communication policy, one can think of the central bank as choosing τ where $\sigma^2_{\varepsilon x} = \tau\sigma^2_{\eta x}$ and $0 \leq \tau < \infty$ (see e.g. Faust and Svensson, 2001). If it opts for noiseless communication with the public, the central bank sets $\tau = 0$, while choosing $\tau \to \infty$ captures the other extreme, where the central bank chooses not to communicate at all. See also Appendix B where τ is related to the capacity of a communication channel, a concept borrowed from information theory.

The public's optimal predictor of w^x_t can be solved using the Kalman filter, where (4.14) is the transition equation and (4.23) is the observation equation. As w^x_{t-1} is in the public's information set, the steady state solution to the optimal predictor for w^x_t is (see e.g Sargent (1987b) and Faust and Svensson (2001))

$$w^x_{t|t} \equiv E^p(w^x_t | s^x_t, w^x_{t-1}) = (1-K)\rho w^x_{t-1} + K s^x_t \tag{4.24}$$

where $K \equiv \frac{P}{P+\sigma^2_\varepsilon}$ is the Kalman gain ($0 \leq K \leq 1$). Here, P is the conditional variance of the optimal predictor and is given by[8]

$$\begin{aligned} P &= \sqrt{\left(\frac{(1-\rho^2)\sigma^2_\varepsilon - \sigma^2_\eta}{2}\right)^2 + \sigma^2_\eta\sigma^2_\varepsilon} - \frac{(1-\rho^2)\sigma^2_\varepsilon - \sigma^2_\eta}{2} \\ &= \left(\sqrt{\frac{((1-\rho^2)\tau - 1)^2}{4} + \tau} - \frac{(1-\rho^2)\tau - 1}{2}\right)\sigma^2_\eta \end{aligned} \tag{4.25}$$

where we have used $\sigma^2_\varepsilon = \tau\sigma^2_\eta$. Then K can be rewritten as

$$K = \frac{2}{1 + \tau(1-\rho^2) + \sqrt{4\tau + (1-\tau(1-\rho^2))^2}} \tag{4.26}$$

[7]When the public is uncertain about central bank goals, (in addition to uncertainty about the assessment errors), communication can play a different role. In this case Faust and Svensson (2001) define transparency as "how easily the public can deduce central bank unobserved goals and intentions from observables."

[8]P is the limit of the conditional variance of the predictor, $P_t = E^p(w^x_t - w^x_{t|t})^2$ which is updated recursively from $P_t = P_{t-1} - \frac{P^2_{t-1}}{P_{t-1}+\sigma^2_\varepsilon} + \sigma^2_\eta$.

Observe that there is a monotonic relationship between K and τ.[9] Thus the optimal choice of the degree of communication can be analyzed in terms of K, from which the optimal choice of τ follows from (4.26).

Equation (4.24) says that in forming expectations about the current assessment error of the central bank, the private sector takes a weighted average of its signal s_t^x and a forecast ρw_{t-1}^x based on the AR(1) series. The weighting factor K in turn depends on the quality of central bank communication. Obviously when the public receives no signal, ($K = 0$), the best available forecast of w_t^x is given by ρw_{t-1}^x.

4.6 Solving the Model

As before, we solve the model by applying the method of undetermined coefficients. First, we conjecture that π_t depends on the cost-push shock, u_t, last period assessment error, w_{t-1}^x, and its innovation, η_t^x, and the noise that is introduced by the central bank's communication policy, ε_t^x:

$$\pi_t = B_{\pi 2} w_{t-1}^x + B_{\pi 4} \varepsilon_t^x + B_{\pi 6} \eta_t^x + B_{\pi 7} u_t \tag{4.27}$$

Then from this follows private sector inflation expectations assuming knowledge of the signal s_t^x, the AR(1)-structure of w_t^x and its previous realization w_{t-1}^x.

$$E_t^p \pi_{t+1} = B_{\pi 2} w_{t|t}^x = B_{\pi 2}((1-K)\rho w_{t-1}^x + K s_t) \tag{4.28}$$

Essential here is that the public, using the signal of current period error and its knowledge of the autoregressive process, is able get an optimal forecast of the unobserved assessment error. Next, after substituting (4.28) in (4.22) and simplifying

$$\begin{aligned}\pi_t \;=\; & \Big(\frac{\alpha\beta B_{\pi 2}}{\alpha+\lambda^2} + \lambda\Big)\rho w_{t-1}^x + \frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2}\varepsilon_t^x \\ & + \Big(\frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2} + \lambda\Big)\eta_t^x + \frac{\alpha}{\alpha+\lambda^2} u_t\end{aligned} \tag{4.29}$$

Consistency of RE requires equalizing the coefficients of (4.29) with those of (4.27).

$$\begin{aligned}B_{\pi 2} &= \frac{\rho\lambda(\alpha+\lambda^2)}{(1-\beta\rho)\alpha+\lambda^2} \\ B_{\pi 4} &= \frac{\alpha\beta\rho\lambda K}{(1-\beta\rho)\alpha+\lambda^2}\end{aligned}$$

[9] Note that when there is no persistence in w, (4.24) collapses to $E(w|s) = Ks = \frac{1}{1+\tau}s$, thus forecasts depend only the current period signal. In this case, $E(w|s) = w$ if $\tau = 0$ while $E(w|s) = 0$ if $\tau \to \infty$.

$$\begin{aligned} B_{\pi 6} &= \lambda + \frac{\alpha\beta\rho\lambda K}{(1-\beta\rho)\alpha+\lambda^2} \\ B_{\pi 7} &= \frac{\alpha}{\alpha+\lambda^2} \end{aligned}$$

Observe that with respect to communication the relevant coefficients $B_{\pi 4}$ and $B_{\pi 6}$ are both nonnegative and directly proportional to K. Thus inflation has more variability with higher degree of transparency.[10]

By using (4.28) in (4.21) (with the solution for B_π's in mind) the equilibrium output process will be

$$x_t = B_{x2}w^x_{t-1} + B_{x4}\varepsilon^x_t + B_{x6}\eta^x_t + B_{x7}u_t \tag{4.30}$$

where

$$\begin{aligned} B_{x2} &= \frac{\rho(\alpha+\lambda^2)}{\alpha+\lambda^2(1-\beta\rho)^{-1}} \\ B_{x4} &= -\frac{\beta\rho\lambda^2 K}{(1-\beta\rho)\alpha+\lambda^2} \\ B_{x6} &= 1-\frac{\beta\rho\lambda^2 K}{(1-\beta\rho)\alpha+\lambda^2} \\ B_{x7} &= -\frac{\lambda}{\alpha+\lambda^2} \end{aligned}$$

Equation (4.30) makes it clear that output (like inflation) responds positively to current innovations to the assessment error η^x_t but (unlike inflation) it responds negatively to the current observation noise ε^x_t.

Next using the reduced forms of (4.29) and (4.30), we can show the effect of changing K (the degree of transparency) on the variability of inflation and output.

$$\sigma^2_\pi = \Big(\frac{B^2_{\pi 2}}{1-\rho^2} + \frac{(1-K)B^2_{\pi 4}}{K(1-(1-K)\rho^2)} + B^2_{\pi 6}\Big)\sigma^2_{\eta x} + B^2_{\pi 7}\sigma^2_u \tag{4.31}$$

$$\sigma^2_x = \Big(\frac{B^2_{x2}}{1-\rho^2} + \frac{(1-K)B^2_{x4}}{K(1-(1-K)\rho^2)} + B^2_{x6}\Big)\sigma^2_{\eta x} + B^2_{x7}\sigma^2_u \tag{4.32}$$

where we have made use of the relations

$$\begin{aligned} \sigma^2_{wx} &= \frac{\sigma^2_{\eta x}}{1-\rho^2} \\ \sigma^2_{\varepsilon x} &= \tau\sigma^2_{\eta x} \\ &= \frac{1-K}{K(1-(1-K)\rho^2)}\sigma^2_{\eta x} \end{aligned}$$

[10]Note that $B_{\pi 7}$ and B_{x7} (see below) are not functions of K.

from (4.23) and (4.26), respectively.

It is not difficult to show that increasing the value of K unambiguously raises the variability of inflation while it reduces the variability of the output gap.[11] Thus increasing K (higher degree of transparency) improves the performance of output at the expense of inflation, and vice versa.

4.7 When Does Society Benefit from Central Bank Communication?

In order to analyze the effect of communication by the central bank about the assessment errors, we allow for the possibility that the society differs from the central banker in the way it weighs the relative benefits from inflation and output stabilization. Moreover, society may have additional social goals, in this case, interest rate stabilization. To formalize this

$$L_t^s = \pi_t^2 + \alpha_s x_t^2 + q_s i_t^2 \tag{4.33}$$

where α_s and q_s represent society's concern about output and interest rate stabilization (relative to inflation), respectively. Taking the unconditional expectation of (4.33), where the unconditional means of π, x and i are equal to the target (zero), we have[12]

$$E[L_t^s] = \sigma_\pi^2 + \alpha_s \sigma_x^2 + q_s \sigma_i^2 \tag{4.34}$$

In the following analysis we first study the case where $q_s = 0$ and allow the possibility that $\alpha_s \neq \alpha$. As we show below, it turns out that given society's preference α_s, and an appointment of a sufficiently conservative central banker, ($\alpha < \alpha_s$), communication makes the society better off.

What we have in mind is a situation where the central bank decides on interest rate policy based on its own weight on output stabilization while the society assigns a higher weight on output stabilization (the central bank is then weight-conservative, as in Rogoff (1985)). Formally, given α_s we can assign a value for α such that communication is worthwhile for society's welfare. The intuition behind this result is as follows. Take, for instance, a positive assessment error in period t. That means the central bank underestimates the expected level of next period's output gap. The policy it has planned is therefore too loose and the interest rate it plans

[11] From (4.31) and (4.32) the partial derivatives of the coefficients of $\sigma_{\eta x}^2$ w.r.t K are positive and negative, respectively.

[12] Note that, as was shown in chapter 2, equation (2.5), for a discount factor very close to one, we can approximate the intertemporal loss function by the expected value of the per-period loss function.

to set too low (see (4.20)). If the public is aware of the fact that the procedure used by the central bank leads to an underestimation of the expected output gap (i.e. this error is communicated) the public will expect this error to persist in the future (this follows from the AR(1) form in (4.28)). It will therefore have higher inflationary expectations. This in turn is picked up by the central bank (with an assessment error, though) and it makes policy tighter than without communication. The opposite reasoning holds for a negative assessment error.

The coefficient of $\sigma_{\eta x}^2$ in (4.31) monotonically increases with K while that in (4.31) monotonically decreases with K. The optimal value of K from society's point view can be calculated in principle, but it turns out to be a very complicated expression. We thus consider the extreme cases of no communication $K = 0$ and full communication $K = 1$. For this case, take the difference

$$E[L_t^s]_{K=1} - E[L_t^s]_{K=0} = \frac{\beta\rho\lambda^2}{((1-\beta\rho)\alpha + \lambda^2)^2} Q\sigma_{\eta x}^2 \tag{4.35}$$

where

$$Q = 2(\alpha - \alpha_s)(\alpha + \lambda^2) - \beta\rho(\alpha^2 - \alpha_s(2\alpha + \lambda^2))$$

Observe that since the coefficient of σ_u^2 does not depend on K, this term vanishes from (4.35). Given the structural parameters of the economy, the sign of the right hand side of (4.35) depends on the sign of Q, that, in turn, depends on the value of α relative α_s. For example, if the central banker shares the same preferences as the society, $\alpha_s = \alpha$, we have $Q = \alpha\beta\rho(\alpha + \lambda^2) > 0$, so that communication is welfare decreasing. It is then clear that when society has a central banker who shares the society's loss function (i.e. the central banker is neither conservative nor liberal compared to society), publication of central bank assessments actually makes the society worse-off. This result also shows that if left to his own decision (in other words, if he is independent in deciding upon publication of forecasts) the central banker would prefer not to reveal his forecasts. Thus in our setup there arises a situation where communication may not be desired by a central banker but may benefit the society. Proposition 1 below summarizes the result.

Proposition 1 *Suppose the public has no preference for interest rate stabilization, ($q_s = 0$). Then communication about the central bank's assessment error of output expectations improves society's welfare if the policymaker is sufficiently conservative, i.e., if*

$$\frac{\alpha_s}{\alpha} > \frac{2\lambda^2 + \alpha(2-\beta\rho)}{\lambda^2(2-\beta\rho) + \alpha(2-2\beta\rho)} > 1$$

Proof: From (4.35)

$$Q < 0 \qquad iff \qquad \frac{\alpha_s}{\alpha} > \frac{2\lambda^2 + \alpha(2-\beta\rho)}{\lambda^2(2-\beta\rho) + \alpha(2-2\beta\rho)}$$

Since the term on the right hand side of the inequality sign is greater than one, the proposition says that for the society to benefit from transparency, it must have appointed a central banker who is sufficiently conservative. The positive effect of communication on stabilization of the output gap is stronger when the central banker is more conservative (small α). On the other hand stabilization of the output gap contributes more to social welfare if society puts more weight on output gap stabilization (large α_s). Thus what matters is the ratio of α_s to α; society benefits more from communication the higher the degree of conservativeness of the central banker. In summary a society that has appointed a very conservative central banker can make itself better off by instructing the central banker to be transparent to the public by publishing the bank's official forecasts.

As an extension of the above analysis, we ask under what conditions communication turns out to be welfare improving when society's welfare depends not only on the variability of inflation and output but also on the nominal interest rate. [13] For this purpose let us fix the level of α_s such that

$$\alpha_s = \frac{2\lambda^2 + \alpha(2-\beta\rho)}{\lambda^2(2-\beta\rho) + \alpha(2-2\beta\rho)}\alpha \tag{4.36}$$

This means that under the case without an additional interest rate goal for the society (i.e. $q_s = 0$), society would be indifferent to central bank communication. Proposition 2 below gives the condition under which communication improves social welfare when we allow the society to care about interest rate stabilization.

Proposition 2 *Suppose society has a preference for interest rate stabilization, ($q_s > 0$), and α_s is given by (4.36), then communication about the central bank's assessment error of output gap expectations improves society's welfare if*

$$\phi < \frac{(2-\rho)(1-\beta\rho)\alpha + (2-\rho(1+\beta-\beta\rho))\lambda^2}{\rho\lambda(\alpha+\lambda^2)}$$

Proof: Assume $q_s > 0$ and (4.36) holds. Then a higher value of K increases society's welfare if and only if σ_i^2 decreases. It is easy to show that in equilibrium, σ_i^2 is inversely proportional to K if and only if the inequality condition is satisfied.

Note that the right hand side of this inequality condition is positive and finite. Given our assumption that $\phi > 0$, what the condition requires is that ϕ should

[13] For discussions of interest rate stabilization as related to instability in financial markets and financial crises, see, for example, Cukierman (2001), p. 61 and the references there in.

not be too large. This makes sense since the effect of more communication on the variability of the nominal interest rate depends on the degree to which private sector expectations of the next period's inflation and output respond to the current assessment errors (see the central bank's reaction function (4.20)). It turns out that as ϕ gets smaller, private sector expectations of output and inflation (see (4.28)) respond less strongly to the (current) assessment error on output expectations.

Another important result of the model concerns the relationship between the degree of transparency and conservativeness of the central bank. The fact that transparency affects the variability of inflation and output in opposite ways has consequences for the optimal degree of conservativeness. For the case without interest rate stabilization goal for the society, the following proposition asserts that a higher degree of transparency increases the optimal degree of conservativeness.

Proposition 3 *Suppose* $(q_s > 0)$ *and* $(K = 0)$*. Then communication about the assessment error will increase optimal conservatism of the central bank if the persistence of the assessment error on the expected output gap is not too large, i.e., if*

$$\rho < \frac{\phi^2(\alpha_s + \lambda^2) + q_s}{q_s(1 + \phi\lambda)}$$

Proof:

$$\frac{\partial \alpha^*}{\partial K}|_{K=0,\alpha=\alpha^*} = -\frac{\beta\rho[\phi^2(\alpha_s + \lambda^2) + q_s][q_s(1 - \rho(1 + \phi\lambda)) + \phi^2(\alpha_s + \lambda^2)]\sigma_{wx}^2}{\phi^4\sigma_u^2}$$

with

$$\alpha^*|_{K=0} = \alpha_s + \frac{q_s}{\phi^2}$$

Then

$$\frac{\partial \alpha^*}{\partial K}|_{K=0,\alpha=\alpha^*} < 0 \qquad iff \qquad \rho < \frac{\phi^2(\alpha_s + \lambda^2) + q_s}{q_s(1 + \phi\lambda)}$$

Note that under the benchmark case where $q_s = 0$, Proposition 3 requires no relevant restrictions on the persistence parameter since in that case we would have that

$$\frac{\partial \alpha^*}{\partial K}|_{K=0,,\alpha=\alpha^*} < 0 \qquad iff \qquad \rho < \infty$$

The intuition behind Proposition 3 is that increased communication (from the zero level) improves stabilization of the output gap. Therefore, better communication (larger K) makes it optimal for the central bank to become more conservative (smaller α).

4.8 Concluding Remarks

It is sometimes argued that central banks need to be secretive in order to maintain flexibility, which enables them to stabilize the economy. In a standard New Keynesian model we arrive at an opposite result. By communicating and being transparent about its procedures that lead to assessment errors of private sector expectations, the central bank is better able to stabilize output gaps than when its assessment errors come as a surprise to the public. The inflation rate, however, will become more volatile. The reason is that the public's reaction to the errors will cause the bank to adjust its interest rate in the direction that helps to stabilize the impact of the error on the output gap. In this case, aggregate demand policy can not be used to stabilize the effect of inflation expectations on current inflation.

A crucial element in our analysis is that, with communication by the central bank, the public is able to forecast the error that the central bank will make in assessing private sector expectations. In our welfare analysis we showed that a sufficiently conservative central bank improves society's welfare by communicating its assessment of private sector expectations. This holds in the benchmark case where society cares only about inflation and output stabilization and in a case where we allow the society to have interest rate stabilization goal on top of inflation and output.

Furthermore, we analyze the relationship between communication and central bank conservativeness. It turns out that when the assessment errors on output gap expectations are not too persistent, a central bank deciding to be more transparent can afford to be more conservative since the benefits from higher transparency in terms of output stabilization are greater the more conservative is the central bank.

Appendix A Assessment Errors on Output and Inflation Expectations

In this section we show that including assessment errors regarding private sector expectations of future inflation yields the same qualitative results as in the main text. We start by modifying (4.20) to allow for assessment errors in inflation expectations.

$$i_t = \frac{1}{\phi}\Big\{(\phi + \frac{\beta\lambda}{\alpha+\lambda^2})(E_t^p\pi_{t+1} - w_t^\pi) + (E_t^p x_{t+1} - w_t^x) + \frac{\lambda}{\alpha+\lambda^2}u_t + v_t\Big\} \quad \text{(A.1)}$$

We can now plug (A.1) in (4.2) and the resulting expression in (4.1) to get

$$x_t = -\frac{\beta\lambda}{\alpha+\lambda^2}E_t^p\pi_{t+1} + (\phi + \frac{\beta\lambda}{\alpha+\lambda^2})w_t^\pi + w_t^x - \frac{\lambda}{\alpha+\lambda^2}u_t \quad \text{(A.2)}$$

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}E_t^p\pi_{t+1} + (\phi\lambda + \frac{\beta\lambda^2}{\alpha+\lambda^2})w_t^\pi + \lambda w_t^x + \frac{\alpha}{\alpha+\lambda^2}u_t \quad \text{(A.3)}$$

Next, solve (A.3) by applying the method of undetermined coefficients. We conjecture that inflation depends on the assessment errors, the observation noises and the inflation shock

$$\pi_t = B_{\pi 1}w_{t-1}^\pi + B_{\pi 2}w_{t-1}^x + B_{\pi 3}\varepsilon_t^\pi + B_{\pi 4}\varepsilon_t^x + B_{\pi 5}\eta_t^\pi + B_{\pi 6}\eta_t^x + B_{\pi 7}u_t \quad \text{(A.4)}$$

Then from this follows private sector expectations analogous to the main text.

$$\begin{aligned} E_t^p\pi_{t+1} &= B_{\pi 1}E(w_t^\pi | s_t^\pi, w_{t-1}^\pi) + B_{\pi 2}E(w_t^x | s_t^x, w_{t-1}^x) \\ &= B_{\pi 1}((1-K)\rho w_{t-1}^\pi + K s_t^\pi) + B_{\pi 2}((1-K)\rho w_{t-1}^x + K s_t^x) \end{aligned} \quad \text{(A.5)}$$

Next substituting (A.5) in (A.3) and simplifying we get

$$\begin{aligned} \pi_t = {} & \frac{\lambda^2(\beta+\phi\lambda) + \alpha(\beta B_{\pi 1} + \phi\lambda)}{\alpha+\lambda^2}\rho w_{t-1}^\pi + (\frac{\alpha\beta B_{\pi 2}}{\alpha+\lambda^2} + \lambda)\rho w_{t-1}^x + \\ & + \frac{\alpha\beta K B_{\pi 1}}{\alpha+\lambda^2}\varepsilon_t^\pi + \frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2}\varepsilon_t^x + \frac{\lambda^2(\beta+\phi\lambda) + \alpha(\beta B_{\pi 1}K + \phi\lambda)}{\alpha+\lambda^2}\eta_t^\pi + \\ & + (\frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2} + \lambda)\eta_t^x + \frac{\alpha}{\alpha+\lambda^2}u_t \end{aligned} \quad \text{(A.6)}$$

Consistency of RE requires the matching of the coefficients in (A.6) with those of (A.4):

$$
\begin{aligned}
B_{\pi 1} &= \frac{\rho\lambda(\alpha\phi+\lambda(\beta+\phi\lambda)}{(1-\beta\rho)\alpha+\lambda^2} \\
B_{\pi 2} &= \frac{\rho\lambda(\alpha+\lambda^2)}{(1-\beta\rho)\alpha+\lambda^2} \\
B_{\pi 3} &= \frac{\alpha\beta K B_{\pi 1}}{\alpha+\lambda^2} \\
B_{\pi 4} &= \frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2} \\
B_{\pi 5} &= \frac{\lambda^2(\beta+\phi\lambda)+\alpha(\beta K B_{\pi 1}+\phi\lambda)}{\alpha+\lambda^2} \\
B_{\pi 6} &= \frac{\alpha\beta K B_{\pi 2}}{\alpha+\lambda^2}+\lambda \\
B_{\pi 7} &= \frac{\alpha}{\alpha+\lambda^2}
\end{aligned}
$$

Now with these set of coefficients for the equilibrium inflation process, we can say something about the effect of changing K (the degree of transparency) on the variability of inflation. [14]

Moreover

$$
\begin{aligned}
\sigma_\pi^2 = &\Big(\frac{B_{\pi 1}^2}{1-\rho^2}+\frac{(1-K)B_{\pi 3}^2}{(1-\rho^2)K}+B_{\pi 5}^2\Big)\sigma_{\eta\pi}^2 + \\
&+\Big(\frac{B_{\pi 2}^2}{1-\rho^2}+\frac{(1-K)B_{\pi 4}^2}{(1-\rho^2)K}+B_{\pi 6}^2\Big)\sigma_{\eta x}^2+B_{\pi 7}^2\sigma_u^2
\end{aligned}
\tag{A.7}
$$

where we have made use of the relation $\sigma_{wj}^2=\sigma_{\eta j}^2/(1-\rho^2)$ and $\sigma_{\varepsilon j}^2=\frac{1-K}{K}\sigma_{wj}^2$ for $j=\pi,x$. It is not difficult to show that increasing the value of K unambiguously raises the variability of inflation. How about the effect of K on output variability?

By using (A.5) in (4.2) the equilibrium output process will be

$$
x_t = B_{x1}w_{t-1}^\pi + B_{x2}w_{t-1}^x + B_{x3}\varepsilon_t^\pi - B_{x4}\varepsilon_t^x + B_{x5}\eta_t^\pi - B_{x6}\eta_t^x + B_{x7}u_t \tag{A.8}
$$

where

$$
\begin{aligned}
B_{x1} &= (\phi+\frac{\beta\lambda}{\alpha+\lambda^2})\rho \\
B_{x2} &= \frac{\rho(\alpha+\lambda^2)}{\alpha+\lambda^2(1-\beta\rho)^{-1}}
\end{aligned}
$$

[14] Note that $B_{\pi 7}$ and B_{x7} (see below) are not functions of K.

$$
\begin{aligned}
B_{x3} &= -\frac{\beta\lambda^2 K B_{\pi 1}}{\alpha+\lambda^2} \\
B_{x4} &= -\frac{\beta\lambda K B_{\pi 2}}{\alpha+\lambda^2} \\
B_{x5} &= \phi + \frac{\beta\lambda(1-KB_{\pi 1})}{\alpha+\lambda^2} \\
B_{x6} &= 1 - \frac{\beta\lambda K B_{\pi 2}}{\alpha+\lambda^2} \\
B_{x7} &= -\frac{\lambda}{\alpha+\lambda^2}
\end{aligned}
$$

Equation (A.8) makes it clear that output (like inflation) responds positively to current assessment errors (w_t^π and w_t^x) but unlike inflation it responds negatively to current observation noises (ε_t^π and ε_t^x).

$$
\begin{aligned}
\sigma_x^2 = &\Big(\frac{B_{x1}^2}{1-\rho^2} + \frac{(1-K)B_{x3}^2}{(1-\rho^2)K} + B_{x5}^2\Big)\sigma_{\eta\pi}^2 + \\
&+ \Big(\frac{B_{x2}^2}{1-\rho^2} + \frac{(1-K)B_{x4}^2}{(1-\rho^2)K} + B_{x6}^2\Big)\sigma_{\eta x}^2 + B_{x7}^2\sigma_u^2
\end{aligned} \tag{A.9}
$$

From (A.9), we observe that, unlike inflation, the variability of output is negatively related to K. Thus it appears that increasing K improves the performance of output at the expense of inflation.

All the relevant B's either monotonically decrease or monotonically increase with K. We consider only the extreme cases of no communication $K=0$ and full communication $K=1$. For this take the difference

$$
\begin{aligned}
EL_{K=1}^s - EL_{K=0}^s &= \frac{\beta\rho\lambda^2(\alpha\phi+\lambda(\beta+\phi\lambda))^2}{[(\alpha+\lambda^2)((1-\beta\rho)\alpha+\lambda^2)]^2} Q\sigma_{\eta\pi}^2 + \\
&+ \frac{\beta\rho\lambda^2}{((1-\beta\rho)\alpha+\lambda^2)^2} Q\sigma_{\eta x}^2
\end{aligned} \tag{A.10}
$$

where

$$
Q = 2(\alpha-\alpha_s)(\alpha+\lambda^2) - \beta\rho(\alpha^2 - \alpha_s(2\alpha+\lambda^2))
$$

Observe that the coefficient of σ_u^2 does not depend on K; thus this term vanishes from (A.10). Given the structure of the economy, the sign of (A.10) depends on the sign of Q that in turn depends on the relative sizes of α and α_s. Therefore it turns out that the general case that includes inflation rate assessment errors has the same condition as the simpler case we considered in the main text.

Appendix B Information Transmission Through a Limited Capacity Channel

In the main text, we took a short cut and assumed that through its communication policy the central bank controls the signal-to-noise ratio by choosing τ in $\sigma_\varepsilon^2 = \tau\sigma_\eta^2$, where the value of σ_η^2 is exogenously fixed but commonly known.

In this appendix we show how the short cut used in the main text can be motivated. To accomplish this goal, we makes use of the notion of "channel capacity" borrowed from the literature on information theory that was first developed by Shannon (1948). The capacity of a noisy channel is typically related to information processing constraints. This constraint places a lower bound on the maximum possible reduction of noise in the signal extraction problem that private agents solve. In particular, the variance of the noise is a function of the capacity of the channel that agents use to encode the input signal into an output signal. The variance of the noise can be expressed in terms of the exogenously given variance of the input signal. This simplifies the algebra as the only parameter in the signal extraction that one needs to keep track of is the capacity of the channel.

Information theory has recently been applied to economics by Sims (1998, 2003). Sims looks at macroeconomic implications of decisions by agents that are constrained by information processing. Adam (2003) follows Sim's approach in a study of optimal monetary policy when firms have private information about shocks hitting the economy. In the framework that is analyzed here, the central bank communicates about its assessment of expectations to the public through an information channel that may be subject to limited capacity. The idea is that the central bank provides the public with the models and procedures used in the central bank's assessments.[15]

The private sector constructs its observation equation that produces s_t, which is observed as a result of w_t being contaminated with a noise ε_t whose variance the central bank can influence indirectly by its choice of the capacity of the channel of communication, C. The private sector is able to extract information about w_t such that the noise generated, ε_t, is designed to have a minimum variance. This makes the signal extraction problem a simple function of the primitive parameter C.

To see this point more clearly, a brief derivation follows. In dealing with the information channel with limited capacity, information theory defines a measure of uncertainty of a random variable, called entropy.[16] The entropy of a random input w, denoted by $H(w)$, is defined as[17]

$$H(w) = -\int_{-\infty}^{\infty} p(w)ln[p(w)]dw$$

[15] Announcements and published reports are also possible sources of information regarding the central bank's assessment of expectations.

[16] This measure has several attractive properties compared to other measures of uncertainty (see Cover and Thomas (1991) for a textbook treatment).

[17] Time-subscripts are dropped for clarity.

where $p(w)$ is a probability density function of w. Shannon has shown that given the variance, σ_w^2, $H(w)$ is maximized when $p(w)$ is a normal density function. In the case that $p(w)$ is normal, we have

$$H(w) = \frac{1}{2}(ln(2\pi e) + ln\sigma_w^2)$$

Assume w is normal, and introduce a second random variable, s. If w and s are not independent, conditioning on s reduces the entropy of w. The information about w obtained by observing s, is called the mutual information, denoted by $I(w, s)$. It is given by

$$I(w, s) = H(w) - H(w|s)$$

In our case the public uses observations on s to inform itself about the input signal w. The capacity of the communication channel C places an upper bound on the maximum attainable mutual information,

$$I(w, s) \leq C$$

It turns out that $I(w, s)$ attains a maximum, so that $I(w, s) = C$, if the private sector is able to construct its signal s in such a way that the noise ε is normal and independent of the input w. This results in a minimum noise variance for a given level of entropy. We assume that the public can encode the input signal such that the minimum noise variance is achieved. This implies that the measurement equation is $s = w + \varepsilon$, and since w and ε are normal, s is also normal. Moreover, based on the measurement equation, $I(w, s)$ can easily be calculated.[18]

First, using a basic theorem from information theory (see, for instance, Cover and Thomas, 1991) we have the following equivalence relation

$$I(w, s) = H(w) - H(w|s) = H(s) - H(s|w)$$

In words, the amount of uncertainty reduction for the two jointly distributed variables is the same whether we use observations on s to infer about w or vice versa. The theorem allows one to use the second (computationally more attractive[19]) expression

$$\begin{aligned} H(s) - H(s|w) &= \frac{1}{2}(ln(2\pi e) + ln(\sigma_w^2 + \sigma_\varepsilon^2)) - \frac{1}{2}(ln(2\pi e) + ln\sigma_\varepsilon^2) \\ &= \frac{1}{2}ln\Big(1 + \frac{\sigma_w^2}{\sigma_\varepsilon^2}\Big) \end{aligned}$$

[18]Note that the measurement equation has the same form as in standard signal extraction, where typically assumes that the variance of ε is exogenously fixed. By contrast, here the measurement equation is not assumed a priori, but is a result of optimal coding under a limited capacity channel.

[19]From equation (4.23) it is easier to compute the conditional probability distribution for $s|w$ than for $w|s$.

Optimal information processing by the private sector implies that channel capacity is used to the maximum, so that the capacity constraint is binding, ($I(w,s) = C$). Thus we have

$$I(w,s) = \frac{1}{2} ln\Big(1 + \frac{\sigma_w^2}{\sigma_\varepsilon^2}\Big) = C$$

where σ_w^2 and C are primitive parameters. It follows that

$$\sigma_\varepsilon^2 = \frac{\sigma_w^2}{e^{2C} - 1} \tag{B.1}$$

or, using (4.23),

$$\sigma_\varepsilon^2 = \tau \sigma_\eta^2 \tag{B.2}$$

where $\tau = \frac{1}{(1-\rho^2)(e^{2C}-1)}$.

Obviously, from equation (B.1) and (B.2), the (minimum) variance of the noise due to partial communication is a negative function of the capacity of the communication channel, C.

Part II

Learning, Control and Inflation-Forecast Targeting

Chapter 5

Strict Inflation Targeting and Passive Learning

5.1 Introduction[1]

One of the fundamental problems in the design of monetary policy is the existence of uncertainty about the true transmission mechanism, running from policy instruments to final target variables. Besides uncertainty associated with exogenous shocks, (often called additive uncertainty), important parameters in the economic structure are usually unknown to monetary authorities and may change over time in an unpredictable manner.

This chapter and the next fit in the recently revived research on the role of parameter uncertainty in optimal monetary policy, first analyzed by Brainard (1967). In particular, it follows some recent studies that deal with the issue of how aggressive monetary policy should be in uncertain economic environment where there is a possibility to learn over time about unknown parameters in the economic system by using monetary policy instruments to generate data that improves future parameter estimation and in turn control of the system.

Our work builds on the backward-looking model of inflation and output determination popularized by Svensson (1997) for monetary policy analysis, where inflation is affected by lagged aggregate demand, which in turn depends on the lagged short-term real interest rate controlled by the central bank. Svensson (1997) remarks that in a more elaborate model, a term structure can be incorporated. Doing so makes the model more realistic as monetary policy is conventionally viewed as running from short-term interest rates managed by central banks to longer term rates that influence aggregate demand (Goodfriend, 1998). This chapter works in that direction by introducing the term structure of interest rates in an otherwise

[1]An earlier version of chapter 5 (co-authored by Sylvester Eijffinger and Eric Schaling) has already been published as a CEPR Discussion Paper and Bank of Finland Discussion Paper.

backward-looking model. By including the term structure, we have a richer version of the transmission mechanism. Moreover, we introduce imperfect information and learning about the term structure of interest rates that could have potential consequences for the conduct of optimal policy.

5.2 Information Asymmetry, the Term Structure and Monetary Policy

The term structure relates short-term and long-term interest rates via the expectations hypothesis, where the private sector forms expectations of next period long-term rates while the central bank has perfect control of the short-term rate. The interaction of the two parties determines current long-term rates that determine the level of aggregate spending. For instance, Goodfriend (1998) points out that this perspective had been exposed by John Hicks's expectations theory of the term structure, where "a central bank's leverage over the longer term rates comes from the fact that the market determines these as the average expected level of short rates over the relevant horizon (abstracting from a term premium and default risk)."[2]

Under symmetric information between the private sector and the central bank, Svensson's characterization of inflation forecast targeting can be extended so as to include the effect of the term structure, and with it, forward-looking private sector expectations. Then, the extended model has the feature that, at the time of making its interest rate decisions, the central bank needs information on private sector expectations of the one-period-ahead long-term interest rate (see Eijffinger et al., 2000b). More importantly, however, one can introduce information asymmetries in the term structure relationship as a result of the central bank's inability to observe private sector expectations perfectly. Moreover, the case of limited information concerning private sector expectations of the long rate gives rise to multiplicative parameter uncertainty so that optimal control by the central bank affects the speed of learning about the unknown parameter and vice versa.

The incentive to learn about the term structure relationship emerges due to asymmetric information, where the central bank does not have full information about private agents's expectations of the long-term interest rates and thus needs to learn about them over time using the latest available data and a forecasting function. The nature of the asymmetric information is such that it induces the central bank to learn the private sector's forecasts, which are based on a commonly known forecasting rule. In particular, although it understands that forecasts of the long-term

[2]Goodfriend also discusses an alternative theoretical perspective of the term structure, one that is based on the Fisher decomposition of the nominal bond rate into expected inflation and an expected real return. In this case, he says "working in the other direction, the long bond rate contains a premium for expected inflation and, thus, is an indicator of the credibility of a central bank's commitment to low inflation."

interest rates are a function of the current state of the economy, the central bank does not know the parameter in the forecasting rule.

We think it is quite reasonable that the central bank has imperfect knowledge of private sector expectations. Actually recent papers on adaptive learning and the performance of simple monetary policy rules take imperfect observability of expectations as a realistic scenario. For example, Honkapohja and Mitra (2002) study determinacy and stability properties of New Keynesian monetary policy models when inflation forecasts are heterogenous and agents are learning adaptively.[3] They note that central banks typically observe private sector inflation expectations with error and that these observation errors can be so large that central banks must rely on their own (internal) forecasts of inflation.

In the adaptive learning literature, central bank learning about unknown parameters of interest is implemented using recursive least-squares, whereby beliefs about the parameters are updated in each period with the arrival of new information about the economy. But this does not go further than incorporating the effects of these updating procedures on the actual law of motion of the economy. Even though one may argue that the hypothesis of adaptive learning relaxes the stronger assumption of rational expectations, and is thus more realistic, it abstracts from the challenge faced by the central bank in real time with the simultaneous problem of controlling the economy and estimating (learning about) important structural parameters. By contrast, what makes active learning more interesting is that it recognizes the fact that the arrival of new information is partly a result of policy actions of the central bank and that the behavior of the central bank is affected by this fact.

5.3 Related Literature

Within the inflation forecast targeting framework, Svensson (1999) allows for multiplier uncertainty in the dynamic linear model of Svensson (1997) and finds that, for a strict inflation targeting central bank and abstracting from learning issues, the optimal interest rate rule is less aggressive to new information than that implied by a policy based on certainty equivalence, a policy that ignores such uncertainty, and sets the level of the instrument based on the latest available estimate of the policy multiplier. That is, multiplier uncertainty calls for a more gradual adjustment of the conditional inflation forecast towards the target. His result is in line with that found in a generic static linear model, first pointed out by Brainard (1967).[4] The reason for a more cautious policy is due to the fact that the conditional variability

[3]The object in the adaptive learning literature is whether simple monetary policy rules that take into account learning considerations lead to what is called expectational stability (E-stability for short) of the rational expectations equilibria.

[4]At the time, Brainard's result was an important contribution to the then existing literature that ignored multiplicative uncertainty.

of the target variable, here inflation, depends positively on the policy instrument, and by being cautious, the policy maker reduces the impact of the variance of the unknown policy multiplier on the variance of the target variable.[5]

However, Schaling (2004) calls for a more aggressive policy response in a framework that modifies Svensson (1997) with a non-linear Phillips curve, which gives rise to uncertainty in the policy multiplier even if the model has only additive demand shocks.[6] Under strict inflation targeting, he shows that policy is more aggressive than implied by certainty equivalence. Thus under multiplier uncertainty, Brainard's result does not always go through when there are inherent non-linearities in the economy.

A drawback of these studies is that optimal policy is conducted on a period by period basis, as there is no role for active learning. This is the case if one assumes that the unknown parameters are randomly drawn independent of past realizations so that parameter uncertainty is renewed every period. As a result, this scenario leaves no scope for the policy maker to learn about the unknown random parameter from past data.

Several recent studies have revived the issues of active learning and optimal monetary policy under multiplier uncertainty.[7] The literature typically constructs the problem of learning and control around a simple regression model where the explanatory variable is also the control variable, a policy instrument such as the interest rate, whose coefficient has to be estimated and at the same time decisions have to be made about the appropriate level of the instrument that minimizes the expected loss from the variability of the dependent variable, say inflation, around a desired target level. Here, the policy maker has the opportunity to learn about the unknown parameters by actively generating information. In particular, the central bank can affect its own learning possibilities through its current choice of the policy instrument. But in doing so it sacrifices short-term goals to carry out experiments.[8] In this case, one has to differentiate between three policy rules– certainty equivalence, myopic, and optimal (Prescott, 1972). The first two rules ignore the dynamic link between learning and control. While the certainty equivalence rule ignores parameter uncertainty, the myopic rule allows for uncertainty surrounding the unknown parameters. On the other hand, the optimal policy incorporates active learning.[9]

[5]In what follows, the terms cautionary, less aggressive, less responsive and less activist are used interchangeably.

[6]The non-linear Phillips curve is assumed to be convex in output. This means that starting from say a zero steady state inflation, a positive output gap is more inflationary than an equal but negative gap is deflationary.

[7]Early economic applications of active learning and control include a monopolist firm that experiments with price and sacrifices current expected profits in order to learn about its demand curve In this case, the process to be controlled is deterministic but unknown to the decision maker. This raises naturally the question of whether learning converges in the limit to the truth.

[8]Bertocchi and Spagat (1993) call this "sacrificing current reward in exchange for information."

[9]Active learning raises two issues: one is related to the computationally oriented literature,

A careful reading of this strand of literature shows that most studies assume the presence of uncertainty in the policy multiplier. The policy multiplier can be modeled as the coefficient on the money supply (Bertocchi and Spagat, 1993; Balvers and Casimano, 1994), the rate of interest (Wieland, 1998; Ellison, 2003), or inflation (Yetman, 2002; Ellison and Valla, 2001). A common feature of these studies is that the linear economic process subject to central bank control is static, as in Brainard (1967). Thus all dynamics in the economy are only due to central bank learning.[10]

Under uncertainty in the policy multiplier, experimentation may require the central bank to be more responsive to new information on the state of the economy (thus higher variability of the policy instrument). This turns out be the case in Bertocchi and Spagat (1993), Balvers and Casimano (1994) and Wieland (1998), who argue that monetary policy should be more responsive to new information and actively seek to generate information even if there are costs in terms of short-term volatility in the target variable. Although the problem is not formulated in terms of monetary policy instrument and targets, Beck and Wieland (2002) also reach a similar conclusion for a decision maker that cares about parameter uncertainty.[11] This conclusion undermines the basic Brainard (1967) result that calls for more cautious policy (implying a lower variability of the policy instrument). What drives these recent results (calling for a more active policy) is the possibility that the central bank can learn about the unknown parameter and thus improve performance in the future, a feature which is absent in the static model of Brainard (1967).

However, Ellison and Valla (2001) argue in favor of a more cautionary policy under active learning by appealing to strategic considerations. They show that strategic interactions between the central bank and the private sector introduce additional costs associated with activist policy. More specifically, while activist monetary policy may generate valuable information, it can lead to volatile inflation expectations that in turn hinder the central bank from holding inflation and output stable around their targets. Ellison and Valla (2001) thus restore the main message of Brainard (1967), although for reasons related to optimal learning.

Our model, although linear, differs from most of the above mentioned studies in two respects. First, the structural equations in our model are dynamic even if one assumes that there is no learning by the central bank. This is due to the presence of endogenous persistence (inertia) in the economy in the sense that future economic conditions depend in part on the current state of the economy.[12] Second, while

which started originally in disciplines such as in control engineering and later adapted to economic applications (e.g., Prescott, 1972). The focus is the comparison of the three alternative decision rules. The second issue is about the asymptotic properties of the beliefs and actions of the decision maker under optimal Bayesian learning (e.g., Kiefer and Nyarko, 1989).

[10]Exceptions are (Wieland, 1998; Beck and Wieland, 2002).

[11]Of course, the target and instrument variables in Beck and Wieland (2002) can readily be adapted to a monetary policy setting.

[12]See e.g., Clarida et al. (1999) for the use of the term endogenous persistence in inflation.

the literature typically studies uncertainty about a policy multiplier, the nature of information symmetry in our term structure equation implies that the persistence parameter in the linear process is unknown to the central bank. In a generic model, Beck and Wieland (2002) analyze uncertainty in the policy multiplier when there is endogenous persistence in the state variable.

For a linear process, Kiefer and Nyarko (1989) have shown that if the variable that is multiplicative to the unknown parameter does not converge to a constant, beliefs converge with probability one to the truth. In our case, this condition is fulfilled since the exogenous random shocks prevent the path of the state variable, which is multiplicative to the unknown persistence parameter in the dynamic process, from settling down to a constant steady state. Neither can policy (which is subject to control lags) fully stabilize the state variable since the random shocks are unpredictable at the time of policy setting. Thus beliefs converge with probability one to the correct values in the limit and current actions of the central bank do not hinder the long run properties of its beliefs (see also section 5.9). With this in mind, we concentrate on characterizing the degree of policy activism under alternative decision rules.[13]

The analysis under learning is conducted under two alternative scenarios. The first assumes that the central bank decides on policy based on a signal about private sector expectations, while in the second scenario, the central bank's regression model is based on a semi-reduced form equation derived from the structural equations. In both cases, the central bank is involved in filtering incoming data on the economy, which helps it improve its knowledge of the unknown parameter. In the limiting case where the random demand and supply shocks have zero variances, the semi-reduced equation becomes deterministic and there is no need for learning. When the signaling equation is active, parameter updating is possible if the state variable is initially away from its target (zero). Sooner or later learning stops since the deterministic state variable moves towards a constant (zero) steady state value (see also section 5.9).

5.4 Plan of chapter 5 and 6

The analysis in chapter 5 is conducted for a monetary policy regime that cares only about inflation variability, that is, strict inflation targeting. One can then compare the results from this exercise with the benchmark model of Svensson (1997). Our

The persistence in inflation is endogenous in the sense that current inflation depends partly on last period's inflation. This contrasts with persistence in inflation that arises as a result of autoregressive behavior in the *exogenous* shocks driving inflation.

[13]By contrast, in a static model with uncertainty about the policy multiplier, which has been considered in most studies, if the policy instrument converges too quickly then beliefs may fail to converge to the correct values as it becomes difficult to estimate the parameter due to very little or no variability in the instrument. Thus present actions of the central bank are important for convergence properties of limit beliefs.

structural equations are similar to those of Eijffinger et al. (2000b), which also extend Svensson (1997) using the term structure equation but without learning considerations.

The remainder of chapter 5 is structured as follows. We first present the inflation forecast targeting framework and discuss the transmission mechanism that includes the term structure of interest rates. Next, the optimal policy rule is derived under perfect knowledge as a benchmark. Then we extend the framework by introducing imperfect knowledge and passive learning. The analysis is conducted under strict inflation targeting, where policy completely stabilizes predictable fluctuations in inflation, while actual (observed) fluctuations are only due to the initial impact of unpredictable shocks and forecast errors as the result of uncertainty in the unknown parameter. We conclude the chapter with a preliminary discussion of optimal policy under active learning. We also touch upon the dynamics of passive learning and convergence of beliefs and using simulations we illustrate that beliefs ultimately converge to the truth.

Chapter 6 builds on the same set of equations and notion of information asymmetry as in chapter 5 but allows for

- a flexible inflation targeting regime and
- active learning by the central bank with uncertainty about the term structure.

The main question is the performance of alternative decision rules for the policy rate, the short-term interest rate. As mentioned previously, we differentiate between the three policy rules: certainty equivalence, myopic policy, dynamically optimal policy, where the first two separate the estimation and control part of the problem and are categorized under passive learning in the sense of disregarding the dynamic link between current decisions and future beliefs about the unknown parameters. Under the optimal policy, the central bank recognizes the tradeoff between estimation and control, and that current policy actions and economic outcomes influence the speed of learning by providing information that may improve future performance.

5.5 The Policy Problem

In this section we describe the inflation forecast targeting framework, with the transmission mechanism that incorporates the term structure of interest rates. First the current period rate of inflation is determined by its lagged level and the lagged output gap.

$$\pi_{t+1} = \pi_t + \alpha_1 z_t - \eta_{t+1} \tag{5.1}$$

where z is the (log of) the output gap with its coefficient $\alpha_1 > 0$. The output gap is defined as the difference between actual output and potential output, which is conveniently normalized to zero, and η is a normally distributed white noise supply shock.

The output gap is autoregressive and is affected by the long-term real rate:

$$z_{t+1} = \beta_1 z_t - \beta_2 R_t + d_{t+1} \tag{5.2}$$

where R is long-term real rate, d is a white noise demand shock that is also assumed to be normally distributed. Moreover, the two coefficients satisfy the restriction $\beta_1 > 0$ and $\beta_2 > 0$. This relationship is similar to the one used by Rudebusch and Svensson (2002). The differences are that here the output gap depends on the long-term real interest rate rather than the short-term real interest rate, and that they consider an additional lagged z term.

Finally, the short-term real interest rate (r_t), which can perfectly be controlled by the central bank,[14] and the long real rate are related by the expectations theory of the term structure

$$r_t = R_t - D(\hat{E}_t R_{t+1} - R_t) \tag{5.3}$$

where $\hat{E}_t R_{t+1}$ denotes private sector (private sector) expectations (where the hat sign '^' is appended to denote possibly nonrational private sector expectations) of next period's long real rate.[15] Here r_t represents the real yield to maturity on a one-period bond which is traded on the interbank money market. This yield must be equal to the (one-period) real holding period return on a long-term bond. The parameter D is defined such that $D + 1$ is equal to what is known as Macaulay's duration (see Eijffinger et al. (2000b) for details). For our purposes it turns out to be convenient to rewrite (5.3) so that the current long real rate is expressed as a linear combination of r_t and $\hat{E}_t R_{t+1}$:

$$R_t = (1 - k) r_t + k \hat{E}_t R_{t+1} + \zeta_t \tag{5.4}$$

where $k \equiv D/(D + 1)$. We have added a normally distributed white noise term ζ_t, where $\zeta_t \sim (0, \sigma_\zeta^2)$, to capture unobserved term premium. One may postulate that the significance of ζ_t increases with k.[16] Thus we may implicity define a function $\zeta_t = k\xi_t$ where ξ_t is another white noise shock. Then $\zeta_t \to 0$ if $k \to 0$ since in that case the term premium vanishes *by construction*, ($\zeta_t = 0$ for all t). In that case, the

[14] The central bank has perfect control over the real short rate, r_t, because $r_t = i_t - E_t \pi_{t+1}$ where $E_t \pi_{t+1}$ is predetermined at time t.

[15] This will be relevant when discussing central bank learning under imperfect knowledge.

[16] Of course, there is no a priori reason for choosing a linear relationship. One might as well assume that ζ_t is proportional to k^2 or use any monotonic relationship that implies $\lim_{k \to 0} \zeta_t = 0$.

duration of the long-term bond is equal to one and there is no distinction between short and long-term interest rates.

The presence of unobserved shocks in the term structure is not that essential when the central bank directly observes $\hat{E}_t R_{t+1}$. Under perfect knowledge, we set $\zeta_t = 0$ for all periods without loss of generality. On the other hand, the presence of unobserved shocks becomes relevant when we examine asymmetric information between the central bank and the private sector, namely, when the central bank can not directly observe $\hat{E}_t R_{t+1}$ (see section 5.8).

For future reference, we iterate (5.4) forward (ignoring the term premium):

$$z_t = \hat{E}_t \sum_{j=0}^{\infty} k^j (1-k)(i_{t+j} - \pi_{t+1+j}) \tag{5.5}$$

It remains to specify the preference of the central bank. The central bank chooses a sequence of current and future short-term nominal interest rates to meet the objective[17]

$$\min_{\{r_\tau\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \frac{1}{2} (\pi_\tau - \pi^*)^2 \tag{5.6}$$

subject to $(5.1), (5.2)$ and (5.4). Here, π^* is the central bank's inflation target and δ is the discount rate with $(0 < \delta < 1)$. The expectations operator E_t refers to the central bank's expectations conditional on the information set in period t. It is obvious that the derivation of the optimal short rate depends on the assumed information structure, including model and data uncertainty faced by the central bank.

5.6 Similarity with the New Keynesian Model

Before we proceed to solve the model for an inflation targeting central bank, it is worthwhile to point out the similarity between this chapter's model, implied by equations (5.1) to (5.4), and the forward-looking New Keynesian (FLNK) model that we employed in chapters 2, 3 and 4. In those chapters we saw the importance of forward-looking inflation and output expectations in the transmission mechanism.

Of course, if the term structure equation is switched off, (i.e., $k = 0$), then there is no distinction between short and long-term interest rates, and there is no role for forward-looking private sector expectations. This takes us back to the backward-looking model of Svensson (1997), which differs markedly also from the micro-founded FLNK models.

[17]Since $E_t \pi_{t+1}$ is predetermined, we write the optimization problem in terms of choosing $\{r_\tau\}_{\tau=t}^{\infty}$.

As pointed out by Clarida et al. (1999), in the FLNK model, the IS equation is obtained by log-linearizing the consumption Euler equation that arises from households' optimal saving decision. Translated into this chapter's notation, the New Keynesian IS equation is given by:

$$z_t = \hat{E}_t z_{t+1} - \beta_2(i_t - \hat{E}_t \pi_{t+1}) + d_t \tag{5.7}$$

This equation differs from equation (5.2) mainly because current output depends on expected future output as well as the (ex ante) short real interest rate. The intuition behind equation (5.7) is that, since households (consumers) prefer to smooth consumption, expectation of higher consumption in the next period (associated with higher expected output) leads them to want to consume more today, which raises current output demand. This implies that higher expected future output raises current output. In turn the negative effect of the real rate on current output reflects intertemporal substitution of consumption. In this respect, the interest elasticity, β_2, corresponds to the intertemporal elasticity of substitution. So, absent a term structure equation, the IS curve of the FLNK model differs in two ways from the purely backward-looking model of Svensson (1997). First, current output depends on expected future output rather than on past output, and second, the parameter β_2 is micro founded, namely it is no longer a free ad hoc parameter but is equal to the intertemporal elasticity of substitution.

Iterating (5.7) forward gives:

$$z_t = \hat{E}_t \sum_{j=0}^{\infty} [-\beta_2(i_{t+j} - \pi_{t+1+j}) + d_{t+j}] \tag{5.8}$$

According to Clarida et al. (1999), equation (5.8) illustrates the degree to which beliefs about the future affect current aggregate activity within the FLNK model. The output gap depends not only on the current real rate and the demand shock, but also on the expected future paths of these two variables.

At this stage it is interesting to compare equation (5.8) with the IS equation in our model that includes the forward-looking term structure equation. To keep things simple, set $\beta_1 = 0$ in our IS equation (5.2) and ignore the term premium. Then, using equation (5.4) in equation (5.2), and taking note of equation (5.5), we have:

$$z_{t+1} = -\beta_2(1-k)\hat{E}_t \sum_{j=0}^{\infty} k^j (i_{t+j} - \pi_{t+j+1}) + d_{t+1} \tag{5.9}$$

Comparing (5.9) with (5.8) from Clarida et al. (1999), we see that in (5.8) the current level of activity depends on private sector expectations about future short real interest rates and the demand shock. In our model - with the term structure switched on, ($k > 0$), the future level of output depends on beliefs about future short real rates.[18] The 'dependent variable' is future rather than current output because of the control lag of monetary policy. Current period policy affects the next year's level of output, not the present level, as in Clarida et al. (1999). So, apart from the fact that beliefs about the future do not include the demand shock, the only difference between our IS curve and the IS curve in the FLNK model is the one-year control lag of monetary policy.

So far the comparison has been made for simple versions of the two models. A similar comparison can also be made when both models incorporate a more complicated interaction of forward and backward-looking elements. For example, a good empirical fit for inflation based on the FLNK model usually includes lagged inflation, resulting in what is known as a hybrid New Keynesian Phillips curve. The hybrid variant reflects some inertia in the rate of inflation and nests the purely FLNK model considered here (see Clarida et al., 1999).

5.7 Optimal Policy under Perfect Knowledge

Under a strict inflation targeting regime, the central bank minimizes the intertemporal loss function (5.6) taking into account the dynamic equations (5.1), (5.2) and (5.4). To get some straightforward results, this section assumes that the central bank can observe and respond directly to private sector expectations and moreover that the private sector and the central bank have rational expectations.[19] In this section, we also set $\zeta_t = 0$ for all t without loss of generality.

The timing of events is such that the central bank chooses its interest rate policy after private sector expectations are set. In the terminology of game theory, the private sector is a Stackelberg leader and the central bank is a Stackelberg follower. As the Stackelberg leader, the private sector sets $\hat{E}_t R_{t+1}$ in the knowledge that the central bank, the Stackelberg follower, will treat private sector expectations as given. The private sector understands the central bank's optimization problem and works out the central bank's interest rate rule, $r(\pi_t, z_t, \hat{E}_t R_{t+1})$; it then uses this information when setting $\hat{E}_t R_{t+1}$. The central bank then observes the realizations of z_t, π_t, and $\hat{E}_t R_{t+1}$ and chooses r_t (through the choice of i_t, given that $\hat{E}_t \pi_{t+1} = E_t \pi_{t+1}$ is predetermined). In a Stackelberg equilibrium, the best response function, $r(\pi_t, z_t, \hat{E}_t R_{t+1})$, minimizes the monetary authority's intertemporal loss given $\hat{E}_t R_{t+1}$. At the same time, private sector expectations are rational given the interest rate rule.

Sequence of events in period t

z_t, π_t realize $\rightarrow$	private sector sets $\rightarrow$	central bank chooses
	$\hat{E}_t R_{t+1}$	$r_t = r(z_t, \pi_t, \hat{E}_t R_{t+1})$

[18] It is easy to see that if the term premium were included, the future level of output would also depend on forecasts of future term premiums.

[19] So, the central bank knows how much policy is 'in the pipeline' according to financial markets.

By definition, under strict inflation targeting, there is no other goal besides inflation stabilization. Moreover, under full information on model parameters and private sector expectations, the additive shocks are the only sources of uncertainty facing the central bank; thus one can invoke certainty equivalence. With no tradeoffs in the loss function and with the presence of only additive shocks, the dynamic optimization problem is equivalent to period-by-period optimization, whereby strict inflation targeting implies setting r_t such that expected inflation two-period ahead is at the desired target. The first-order condition in terms of expected inflation is thus (see Appendix A for details)[20]

$$E_t\pi_{t+2} = \pi^* \tag{5.10}$$

Thus, optimal policy for a strict inflation targeting regime defined by (5.6) implies inflation forecast targeting in the sense of Svensson (1997). That is, the best the central bank can do is to set the short rate such that it expects inflation two-period ahead to be on target. The first-order condition, together with the structural constraints, lead to a closed form solution for the short-term real interest rate:

$$r_t = \frac{1}{\alpha_1\beta_2(1-k)}(\pi_t - \pi^*) + \frac{1+\beta_1}{\beta_2(1-k)}z_t - \frac{k}{1-k}\hat{E}_t R_{t+1} \tag{5.11}$$

This rule is similar to simple (non-optimal) Taylor-type rules, which are widely used for policy analysis, with the exception that now policy also responds directly to $\hat{E}_t R_{t+1}$, the private sector's forecast of the long real rate. This modification is a result of the term structure equation that has been embedded in an otherwise backward-looking model (see also Eijffinger et al., 2000b, 2004). It is easy to check that if $k = 0$ and the term structure equation vanishes, the policy rule collapses to a version of the Taylor rule analyzed by Svensson (1997) (hereafter the Svensson-Taylor rule).[21]

An interesting characteristic of this solution is that the central bank's optimal choice of r_t is inversely related to private sector expectations about its future short rates. For example, if the private sector expects rates to go up (down) in the future, as a consequence current real short rates are lowered (raised) today. The property that if the private sector expects future short rates to go down the central bank raises current short real rate (or talks about raising it) reminds us of the old joke about the Bundesbank (BuBa): "The BuBa is just like cream, the more you stir it, the thicker it gets."[22] The reason for this inverse relationship is that

[20]See also Svensson (1997) and Eijffinger et al. (2004). Even though the first-order condition in terms of $E_t\pi_{t+2}$ is equivalent to that in Svensson (1997), our optimal level of the short rate responds to three state variables– π_t, z_t, and $\hat{E}_t R_{t+1}$, which are part of the central bank's information set. In Svensson (1997) the state variables are π_t and z_t.

[21]Taylor rules are often written in terms of the i_t. Given the definition of r_t one can easily derive the optimal level of i_t from (5.11).

[22]In addition, the Bundesbank always considered the long-term interest rate as a reflection of its credibility.

the central bank's inflation forecast– given the current period inflation rate and the output gap– depends on the present level of the real long-term interest rate. So, an optimal inflation forecast implies an optimal level of the current long real rate. Since the optimal long real rate (consistent with strict inflation targeting) is a weighted average of the current ex ante optimal short rate r_t^* and $\hat{E}_t R_{t+1}$, i.e. $R_t^* = (1-k)r_t^* + k\hat{E}_t R_{t+1}$, where according to equation (5.11) a higher value of $\hat{E}_t R_{t+1}$ necessitates a lower value of r_t^*, and vice versa.[23]

Next the four equation system is solved for a rational expectations equilibrium. Inserting the reaction function (5.11) in the term structure (5.4) implies that R_t has the following reduced form solution:

$$R_t = \frac{1}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{1+\beta_1}{\beta_2} z_t \tag{5.12}$$

Importantly, this solution shows that the effects of private sector expectations of the long rate are completely offset when the central bank can observe and respond to those expectations. We can of course derive the exact form of those expectations. Inserting (5.12) in the output equation gives the reduced form solution:

$$z_{t+1} = -\frac{1}{\alpha_1}(\pi_t - \pi^*) - z_t + d_{t+1} \tag{5.13}$$

Next, forwarding (5.12) one period and using equations (5.13) and (5.1) we get:

$$R_{t+1} = -\frac{\beta_1}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{\beta_1}{\beta_2} z_t + u_{t+1} \tag{5.14}$$

where $u_{t+1} \equiv \frac{1+\beta_1}{\beta_2} d_{t+1} - \frac{1}{\beta_2}\eta_{t+1}$ is a composite white noise shock, independent of policy and the state of the economy. Taking expectations as of time t gives:

$$\hat{E}_t R_{t+1} = -\frac{\beta_1}{\alpha_1\beta_2}(\pi_t - \pi^*) - \frac{\beta_1}{\beta_2} z_t \tag{5.15}$$

This result represents the rational expectations solution under symmetric information, where private sector expectations are consistent with the solution for the long-term real interest rate implied by strict inflation targeting. Alternatively, we can express r_t in reduced form by plugging (5.15) into the reaction function (5.11):

$$r_t = \frac{1+k\beta_1}{\alpha_1\beta_2(1-k)}(\pi_t - \pi^*) + \frac{1+\beta_1(k+1)}{\beta_2(1-k)} z_t \tag{5.16}$$

[23] In other words, if the private sector expects a looser monetary policy in the future, this leads to a tighter policy stance today to compensate.

As can be seen from (5.16) the ex ante real rate, r_t, is now expressed in terms of current inflation and output.[24] In this way we can compare the behavior of r_t in (5.16) with the Svensson-Taylor rule that results by setting $k = 0$. We can easily check that when $0 < k < 1$ and the term structure is relevant, r_t becomes more sensitive to changes in the states π_t and z_t. Moreover the higher the value of k, the more volatile r_t. The intuition for this result is as follows. If k increases the term structure says that private sector expectations become more important in determining R_t, a variable that directly affects aggregate demand. To offset the decrease in the policy leverage on R_t, the central bank needs to be more aggressive in responding to changes in π_t and z_t.

5.8 Imperfect Knowledge and Passive Learning

The previous section looked at a benchmark case of perfect knowledge where the central bank perfectly observes and responds to private sector expectations of the long real rate. Although the model extends Svensson (1997), endowing the central bank with perfect knowledge of private sector expectations is hardly realistic. Central banks typically observe private sector expectations with error and that these observation errors can be large (Evans and Honkapohja, 2002). In practice, the challenge for any central bank is that financial markets are sophisticated in analyzing and predicting future monetary policy actions of the central bank. This increased degree of sophistication makes it harder for the central bank to observe private sector expectations without error. It is thus natural to relax the assumption of perfect knowledge by introducing uncertainty with regards to private sector expectations.[25]

Suppose that the private sector's forecasting function for the long real rate takes the same form as the rational expectations solution under full information, equation (5.15). That is expectations respond to information on the current period state variables- inflation and output. Specifically,

$$\hat{E}_t R_{t+1} = \gamma w_t \tag{5.17}$$

where we have defined a new variable $w_t \equiv -(\pi_t + z_t)$ and for the purpose of tractability we have set $\alpha_1 = 1$ and $\pi^* = 0$. The nature of imperfect information is such that, the central bank knows the private sector's forecasting rule, but not the actual value of γ. Let c_t denote the best forecast of γ and p_t the variance of c_t, (the degree of confidence placed upon c_t), of which more will be said later.[26]

[24] Eijffinger et al. (2000b) also derive a similar reaction function (in ex post terms) for the case of flexible inflation targeting.

[25] We continue to assume that the central bank knows the structural model of the economy, including the parameters.

[26] Under imperfect information, the exact value of γ is not crucial for the results that follow. From the perspective of the central bank, the unknown parameter γ can take any real number.

By positing a simple forecasting function for the private sector, we abstract from the interaction of optimal monetary policy and rational expectations on the part of the private sector. This way, our formulation of the learning rule of the private sector follows the recently revived literature on adaptive expectations, which assumes that the private sector behaves adaptively (see for e.g. Evans and Honkapohja, 2001).

In this case, unlike rational expectations, adaptive learning reduces the burden of computing the equilibrium outcomes of a game between the central bank and the private sector. In a fully specified model, one has to solve for private sector expectations that takes into account the fact that the central bank is learning about the parameter γ. Since the state variables include the central bank's parameter estimates and associated uncertainty, private sector expectations also need to incorporate this fact when forming expectations, which in turn should feed into the central bank's optimal control problem. These considerations would make the problem more realistic but at the cost of high computational complexity. Due to this reason Orphanides and Wieland (2000) and Yetman (2002) also assume adaptive private sector expectations.

When the central bank conducts policy based on passive learning, by construction it disregards the potential tradeoff between estimation and control. In other words, the central bank simply ignores the effect of current policy actions on the degree of precision of future estimates of the unknown parameter, thereby treating control and estimation separately. Formally, the passive learning policy first calculates c_t and p_t and then takes these to be fixed parameters at the stage of optimization, which means that when choosing policy, the dynamic process of these estimates (so called updating equations) are ignored. Likewise, before choosing r_{t+1} in period $t+1$ the central bank updates its belief about γ to c_{t+1}, ignoring the fact that it will have to update this estimate in the future.[27] In this way the central bank fails to internalize the effect of current actions on future beliefs.

Certainty equivalence is a special case of a passive learning policy since it ignores p_t by assumption. Thus, the policy maker does not incorporate p_t even if the effect of p_t on the loss function could in principle be reduced by an appropriate choice of r_t.[28] In this case, the sequence of events can be described as follows:

Sequence of events in period t

z_t, π_t realize $\rightarrow$	private sector sets $\rightarrow$	central bank chooses
	$\hat{E}_t R_{t+1} = \gamma w_t$	$r_t = r(z_t, \pi_t, c_t)$

so that the certainty equivalence policy responds to $c_t w_t$, the best forecast of γw_t. In other words, c_t simply replaces the true parameter γ in the interest rate rule.[29]

[27] In the sense of Sargent (1999), passive learning implies that in any period t the central bank pretends that its current estimate c_t will apply forever, as if it is the true parameter. But the central bank's updating of its estimate in period t+1 falsifies this pretense.

[28] Note that having full confidence in c_t is equivalent to assuming $p_t = 0$.

[29] In general, with flexible inflation targeting, $r_t = r(z_t, \pi_t, c_t, p_t)$. See chapter 6.

$$\begin{aligned} r_t &= \frac{1}{\beta_2(1-k)}\pi_t + \frac{1+\beta_1}{\beta_2(1-k)}z_t - \frac{k}{1-k}c_t w_t \\ &= \frac{1+\beta_2 k c_t}{\beta_2(1-k)}\pi_t + \frac{1+\beta_1+\beta_2 k c_t}{\beta_2(1-k)}z_t \end{aligned} \tag{5.18}$$

So far nothing has been said regarding the estimation procedure for c_t and p_t. The next two sections discuss alternative ways of dealing with asymmetry information and associated estimation of the unknown parameter. In both procedures, the central bank eventually learns γ for the reason given in section 5.3, namely that γ is the coefficient of the state variable w_t, which never settles down as it depends on exogenous stochastic shocks. Thus, the choice of a procedure is inconsequential to the ability of the central bank to learn the persistence parameter eventually.

5.8.1 Learning Based on a Signal

Following Eijffinger et al. (2004) assume that the central bank receives a noisy signal y_t about private sector expectations.[30] The signaling equation takes the following form:

$$y_t = \hat{E}_t R_{t+1} + \epsilon_t = \gamma w_t + \epsilon_t \tag{5.19}$$

where ϵ is a measurement (observation) error, assumed to be independently and normally distributed with mean zero and bounded variance, σ_ϵ^2. The only information available to the central bank when it sets policy at time t is current and past values of y_t and w_t. In this setting the central bank cannot observe separately the two components (γ and ϵ_t), even ex post. A limiting case of (5.19) is the perfect knowledge scenario where $\sigma_\epsilon^2 = 0$.

Sequence of events in period t		
z_t, π_t realize $\rightarrow$	private sector sets $\rightarrow$	central bank observes y_t
	$\hat{E}_t R_{t+1} = \gamma w_t$	and chooses r_t

Here c_t denotes the estimate of γ based on the model (5.19). Then the central bank's best guess of private sector expectations would be $c_t w_t$. If the central bank proceeds by first estimating the model (5.19) and then choosing its interest rate given c_t, we have (5.18).

[30] A real world counterpart of signal processing can be that central banks collect information on private sector interest rate expectations (say from surveys), which is then taken as a signal of the true private sector expectations.

The method by which the revised estimate of γ is obtained may be described as a filtering process, which maps the sequence of prediction errors into a sequence of revisions (see Appendix B).

$$\begin{aligned} c_{t+1} &= c_t + \kappa_{t+1}(y_{t+1} - c_t w_{t+1}) \\ p_{t+1} &= p_t - \kappa_{t+1} w_{t+1} p_t \end{aligned} \tag{5.20}$$

where $\kappa_{t+1} \equiv w_{t+1} p_t F_{t+1}^{-1}$ is commonly referred to as the Kalman gain, the weight assigned to new information coming from the latest forecast error, $y_{t+1} - c_t w_{t+1}$. The Kalman gain in turn depends on the conditional variance of y_{t+1}, given by $F_{t+1} \equiv w_{t+1}^2 p_t + \sigma_\epsilon^2$. As can be seen from (5.20), w_{t+1} is a component of the Kalman gain and thus affects the conditional variance of the parameter estimate, p_{t+1}. Due to the presence of autoregressive behavior in w_{t+1} and p_{t+1}, changes in the current state of the economy, w_t, have persistent effects.

The updating equations in (5.20) represent the learning channel through which a policy decision made in period t, r_t, affects the state variable of period t+1, w_{t+1}, and consequently, beliefs about the unknown parameter (i.e. $c_{t+1} = c(w_{t+1})$ and $p_{t+1} = p(w_{t+1})$).

From (5.20), we see that $\partial\kappa_{t+1}/\partial\sigma_\epsilon^2 < 0$. If the amount of noise that is contaminating the signal diminishes, more weight will be given to new information in the prediction error, $(y_{t+1} - E_t y_{t+1} = y_{t+1} - c_t w_{t+1})$, relative to the previous estimate c_t. Evans and Honkapohja (2002) point out that a policy rule based on a signal may be optimal if the measurement error σ_ϵ^2 is very small. In this case, the policy rule that responds to the signal y_t is given by

$$r_t = \frac{1}{\beta_2(1-k)}\pi_t + \frac{1+\beta_1}{\beta_2(1-k)}z_t - \frac{k}{1-k}y_t \tag{5.21}$$

This rule has the same form as the rule under perfect knowledge, (5.11), but where $\hat{E}_t R_{t+1}$ is replaced with y_t. Note also that, if the variance of the noise is large, the performance of the rule based on (5.22) turns out to be poor; in that case, optimal signal extraction leads to the passive learning rule (5.18).

5.8.2 Learning Based on a Semi-reduced Form Equation

We now describe the setting of monetary policy under passive learning in a way that is comparable to chapter 6, which examines the numerical solution for optimal policy under active learning.[31] First, since the central bank is assumed to have full knowledge of the functional form (5.17), it knows that the term structure equation under imperfect knowledge is,

$$R_t = (1-k)r_t + k\gamma w_t + \zeta_t \tag{5.22}$$

[31] Here we abstract from signaling considerations.

Observe that the unobserved term structure shock, ζ_t, injects additional uncertainty and prevents the central bank from inferring, in any period, the value of γ from (5.22).[32] Next, substituting equation (5.22) in the IS and Phillips curves gives,

$$w_{t+1} = \tilde{\gamma} w_t - \beta_1 z_t + \tilde{\beta} r_t + \nu_{t+1} \tag{5.23}$$

where $\tilde{\beta} \equiv \beta_2(1-k)$ and $\nu_{t+1} \equiv \beta_2\zeta_t - d_{t+1} + \eta_{t+1}$ is a composite white noise shock. The persistence parameter $\tilde{\gamma} \equiv 1 + \beta_2 k\gamma > 0$ is unknown to the central bank because of the unknown parameter γ coming from equation (5.17).[33]

Let $\tilde{c}_t$ denote an estimate of $\tilde{\gamma}$. Now, since $\tilde{\gamma} = 1+\beta_2 k\gamma$, after $\tilde{c}_t$ has been estimated, c_t can be inferred indirectly from $c_t = -(1-\tilde{c}_t)/k\beta_2$.[34] Thus we see that here the procedure estimates γ in two steps. Of course, as noted before, under certainty equivalence, the central bank has, by definition, full confidence on its parameter estimate $\tilde{c}_t$ and indirectly c_t. Thus the functional form of the interest rate rule is still given by (5.18), bearing in mind that now we have a new procedure for estimating c_t.

One may, therefore, argue that with the new procedure, there are no substantive changes regarding the interest rate rule, as it is almost identical to the one given by equation (5.18). The difference lies in the procedure (regression equation and corresponding data) used for estimating c_t. Here, the central bank uses equation (5.23) (where w_t is the dependent variable as of period t) and data up to and including w_t, r_{t-1} and z_{t-1}, while in the signaling case, there is an assumed signaling equation with the signal, y_t as the dependent variable and w_t as the explanatory variable. Note also that besides w_t and z_t, the policy instrument, r_t, appears as an explanatory variable in equation (5.23). By contrast, the signaling equation (5.19) depends only on w_t. Section 5.10 discusses the convergence properties of the parameter estimate under passive learning and for both estimation procedures.

In the case where the central bank uses the regression equation (5.23) to estimate the unknown $\tilde{\gamma}$ arising from unobserved private sector expectations, the latest data

[32]This is true even if the central bank knows the parameter k, and has data on the yield curve (r_t and R_t) and the current state of the economy, w_t.

[33]In fact equation (5.23) is the constraint that the central bank faces in setting r_t that minimizes the variability of π_{t+1}. Thus ultimately, the central bank is interested in getting an estimate of the composite persistence parameter $\tilde{\gamma}$. To see this, rewrite the right side of the Phillips curve in terms of w_{t+1}, $\pi_{t+2} = -w_{t+1} - \eta_{t+2}$. Since in period t the central bank can not influence the additive supply shock, η_{t+2}, minimizing the variability of π_{t+2} is equivalent to minimizing the variability of w_{t+1}; and so one can focus on the variable w_{t+1}. With the additive shock as the only source of uncertainty in the model, the first order condition under perfect knowledge, bearing in mind that now $\pi^* = 0$, continues to hold under certainty equivalence. That condition is $E_t\pi_{t+2} = 0$, which can now be rewritten as $E_t w_{t+1} = 0$. Using (5.23), we can easily back out the interest rate rule consistent with the first order condition.

[34]Although at the outset (5.23) may look like a non stationary process due to the possibility $\tilde{\gamma} > 1$, we have to keep in mind that r_t is not an exogenous variable. Rather, its process is derived from optimal control where it responds *inversely* to w_t and directly to z_t so as to stabilize w_{t+1}. Thus one can assess the (non) stationarity of (5.23) only after taking this feedback effect into account in the reduced form equation for w_{t+1}.

as of period $t+1$ would be (w_{t+1}, w_t, z_t, r_t). Analogous to the previous section, the updating equations for c_{t+1} and its variance p_{t+1} are[35,36]

$$\begin{aligned} \tilde{c}_{t+1} &= \tilde{c}_t + \tilde{\kappa}_{t+1}(w_{t+1} - \tilde{c}_t w_t + \beta_1 z_t - \tilde{\beta} r_t) \\ \tilde{p}_{t+1} &= \tilde{p}_t - \tilde{\kappa}_{t+1} w_t \tilde{p}_t \end{aligned} \tag{5.24}$$

where $\tilde{\kappa}_{t+1} \equiv w_t \tilde{p}_t F_{t+1}^{-1}$ is the Kalman gain. But now, F_{t+1}^{-1} is the conditional variance of w_{t+1}, that is $F_{t+1} \equiv w_t^2 \tilde{p}_t + \sigma_\nu^2$. Note that (5.24) represents a different learning channel. Here, r_t affects w_{t+1}, and consequently, $\tilde{c}_{t+1} = c(w_{t+1})$ but, unlike the signaling case, $\tilde{p}_{t+1}$ is predetermined and known since $\tilde{p}_{t+1} = p(w_t)$ and not $p(w_{t+1})$.

In this setting, policy choices in terms of r_t influence the data generation process, in particular, the sequence of $\{w_t\}$. In turn, the parameter estimate and its variance, which characterize the beliefs of the central bank, depend on the data generated. From the principle of least squares estimation the precision of the estimate $\tilde{c}_t$ depends positively on the sample variance of w_t. That is, one gets a more precise estimate (in other words, a smaller value of $\tilde{p}_t$) when the sample variance of w_t increases, and vice versa. We recognize that the current choice of monetary policy r_t affects w_{t+1}, and via the dynamic equation, w_{t+j}, for $j = 2, 3, 4, ...$ This relationship between r_t, w_{t+1} and $\tilde{p}_{t+2}$ raises a potential tension between the urge to minimize current period expected loss from variability in w_{t+1}, (the control part), and the incentive to get a more precise estimate of the degree of persistence in the economy that would help improve future outcomes (the learning part).

5.9 Passive Learning and Convergence

Obviously one may wonder as to whether the passive learning policy enables the central bank to learn the true parameter in the limit. This question is relevant because as we noted above the variability of the data w_t matters for efficient estimation of unknown coefficient γ. Kiefer and Nyarko (1989) have shown that a policy based on parameter learning may have consequences in terms of the limit behavior (convergence) of beliefs and policy actions. In particular, for a linear regression model without persistence in the state variable, where the policy multiplier is unknown and has to be estimated, they show that beliefs may not converge to the truth if policy actions converge very quickly. Similarly, Kiefer and Nyarko (1989) show that beliefs would converge with probability one to the truth if actions do not converge.

In our case, the linear process for inflation is dynamic, as current period inflation depends partly on the previous period's level of inflation. Moreover, current policy

[35] See also Appendix B.

[36] If the unknown parameter is time-varying, the updating equations can easily be adapted to allow for this variability via the Kalman filter (see for e.g. Harvey, 1992).

does not have complete control over future inflation since the latter is continually hit with unobserved shocks. Current policy can only offset the effects of the predictable components. In other words, the data sequence $\{w_t\}$ never settles down, as it is constantly hit by new unpredictable shocks (see equation (5.23)). Thus beliefs about the unknown parameter γ in (5.19) or (5.23) will converge with probability one to the truth. The upshot is that as long as the policy maker can not completely stabilize w_{t+1}, the sequence $\{w_t\}$ remains stochastic even in the limit, thus generating enough sample variability for estimating the coefficient of persistence.[37]

Using numerical values for the known parameters in the model, Figures 5.1 illustrates the convergence of the estimate of the persistence parameter under the certainty equivalence policy, here denoted by r_t^p.

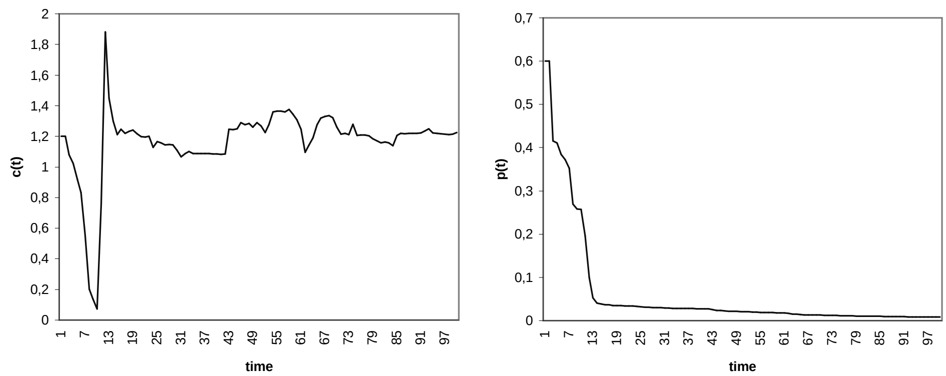

Figure 5.1: Convergence of $\tilde{c}_t$ under the semi-reduced form equation and $r_t = r_t^p$

Our simulations generate 10,000 observations of the parameter estimate and its variance. There is convergence to the true parameter in the passive learning case, although the speed of convergence can be slow. As Kiefer and Nyarko (1989) have shown in a Bayesian updating sense, beliefs converge to the truth with probability 1 if policy actions (which are multiplicative to the unknown parameter) do not converge. Using similar reasoning, the parameter estimate in our case converges to the truth since the sequence $\{w_t\}$ does not converge in the limit, as the process for w_t is constantly hit by unpredictable exogenous shocks. For this reason, those observations of $\tilde{c}_t$ and $\tilde{p}_t$ far in the distant future are not shown.

Again, in the limiting case where the random demand, supply and term structure shocks have zero variances, the semi-reduced equation becomes deterministic. If (5.23) is deterministic, the central bank can easily infer the value of $\tilde{\gamma}$ at time $t+1$

[37] See also Appendix B.

after observing w_{t+1}.[38] Of course the issue of optimal policy becomes uninteresting if there are no shocks that inject dynamics into the system due to belief updating about the unknown parameter. One can also make a similar argument in the case where the signaling equation, (5.19), is used to estimate the unknown parameter, γ. If the demand and supply shocks have zero variances, the right hand side variable, w_t, in (5.19) will also be deterministic. Then, parameter updating becomes impossible if w_t settles down to a constant value and there are no random shocks that constantly move it away from its resting point.

5.10 The Way Forward: Active Learning

Under strict inflation targeting and active learning policy, the cost of experimentation is higher short run variability in the target variable, (inflation). This is true whether the uncertain parameter is multiplicative to the policy instrument or to the state variable (in our case, the persistence parameter).

When there is uncertainty about the policy multiplier, experimentation or probing is associated with more variability in the policy instrument since without variability in the policy instrument precision of the parameter estimate is very low. Along the same lines, when there is uncertainty about the persistence parameter, a more precise estimate requires more sample variability in the state variable, which is multiplicative to the parameter. But what this means for the variability of the policy instrument (whose coefficient is known with certainty) is not clear a priori. This is because we have more variability in w_{t+1} when r_t responds more aggressively or less aggressively (compared to the rule under passive learning) to new information.

The main thing to note in terms of the learning equation, (5.19) or (5.23), is that more (short-term) variability in the sequence $\{w_t\}$ (that in turn improves precision of the parameter estimate) is achieved if the central bank fails to fully stabilize predictable movements in w_{t+1} by responding too little or too much compared to the certainty equivalence rule. Under strict inflation targeting, the passive learning rule completely stabilizes any predictable variations in w_{t+1}.

The value of experimentation in terms of speeding up learning about the true parameter can be shown by two simple exercises. In these exercises, r_t deviates from its level under passive learning policy for the first 50 periods. Initial beliefs are set as follows: ($\tilde{c}_0 = 1.2$, $\tilde{p}_0 = 0.6$), and the parameters are $\gamma = 0.4$, $\beta_1 = 1$, $\beta_2 = 1$, $k = 0.5$, $\sigma_u = \sigma_\epsilon = 0.3$.

First, for 50 periods, the central bank deviates from the passive learning rule by

[38] To see this, take (5.23) one period backward and subtract the resulting expression from (5.23), which gives $w_{t+1} - w_t = \tilde{\gamma}(w_t - w_{t-1}) + \tilde{\beta}(r_t - r_{t-1})$ (ν is now treated as a constant in equation (5.23)). It is then easy to infer the value of $\tilde{\gamma}$ when w_{t+1} is available in period $t+1$. If the demand and supply shocks are constants but the term structure shock is assumed to be random, then ν_t will also be random. What is relevant for the learning dynamics is that ν_{t+1} be a random variable which can not be observed by the central bank.

setting r_t equal to twice the level given by the certainty equivalence rule r_t^p. In other words, the central bank deliberately over reacts to new information about the state of the economy. Second, the central bank follows a rule that reacts less aggressively than is implied by (5.18), say, setting r_t equal to zero irrespective of the state of the economy.

5.10.1 Policy Deviations under the Semi-reduced Form

Figure 5.2 displays, respectively, a series of 100 realizations of the parameter estimate, $\tilde{c}_t$, and its sample variance, $\tilde{p}_t$, under the more aggressive rule. Under this rule, the real interest rate deviates a lot from its equilibrium or neutral level (which here is normalized at zero), compared to the passive learning rule, (Figure 5.1). A simple comparison of Figure 5.1 and Figure 5.2 shows that convergence under the more aggressive rule is considerably faster. This can be seen more clearly from the right side of Figure 5.2, where the variance $\tilde{p}_t$ converges to zero fairly quickly, compared to its path shown in Figure 5.1 for case of the passive learning.

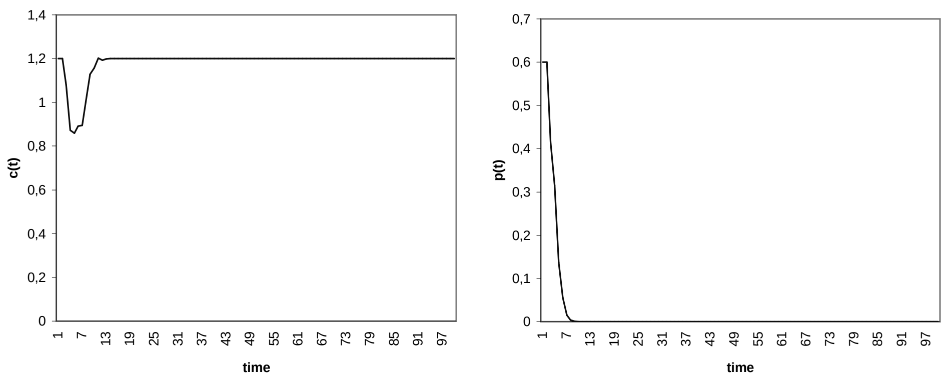

Figure 5.2: Convergence of $\tilde{c}_t$ under the semi-reduced form equation and $r_t = 2r_t^p$

Similar patterns are observed for the case where r_t is much less aggressive, (Figure 5.3). Under this rule, the variability of w_{t+1} goes up compared to that based on passive learning. By setting $r_t = 0$ for the first 50 periods, the central bank fails to offset the predictable effects on w_{t+1} of past and present shocks (reflected in the current period state, w_t), so that the short-term performance of the economy deteriorates. On the other hand the increase in the variance of the sequence $\{w_t\}$ enables the central bank to speed up the convergence of $\tilde{c}_t$ to the true value, ultimately improving its control of inflation in the distant future.

Although not shown here, we find that the larger the deviation of a contemplated rule from r_t^p, the faster the speed of convergence of the parameter estimate.

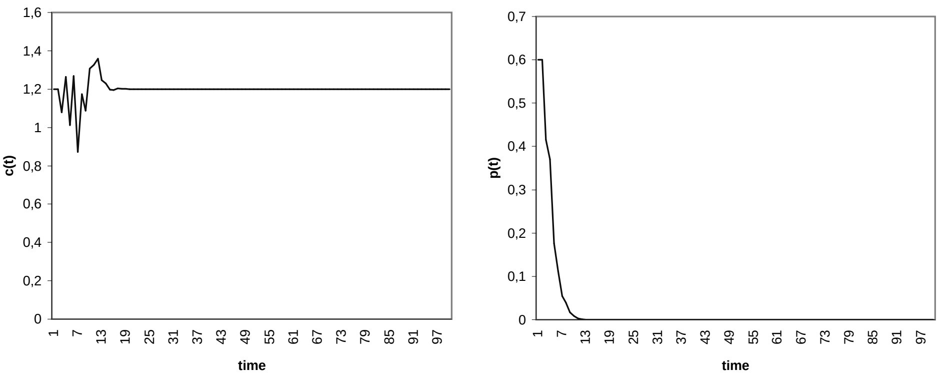

Figure 5.3: Convergence of $\tilde{c}_t$ under the semi-reduced form equation and $r_t = 0$

5.10.2 Policy Deviations under Signaling

Here we repeat the exercise in the previous section for the case where estimation of the unknown parameter is based on the signaling equation. The properties of the parameter estimate and its variance under the signaling equation are similar to the previous section, where estimation was conducted using the semi-reduced form derived from the structural equations. Figures 5.4 and 5.5 illustrate the results that correspond to, respectively, a more aggressive and less aggressive interest rate policy relative to the certainty equivalence rule, (5.18).[39]

Of course there are costs of experimentation in terms of short-term volatility in the target variable, here inflation, as the central bank purposely allows additional inflation variability. The tradeoff that arises from experimenting with this rules shows the need to derive an optimal reaction function for r_t that takes into account the benefits and costs of probing. An optimal policy weighs long-term gains from experimentation against short-term variability.

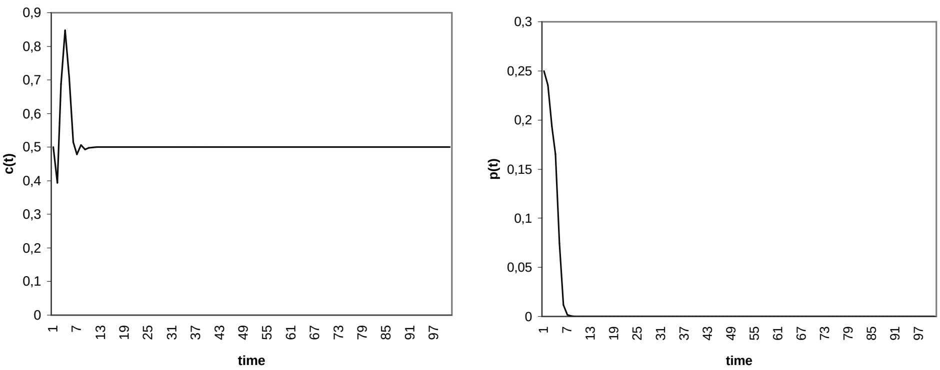

Figure 5.4: Convergence of c_t under the signaling equation and $r_t = 2r_t^p$

[39]To be consistent on notations, in this case initial beliefs are given by ($c_0 = 0.5$, $p_0 = 0.25$).

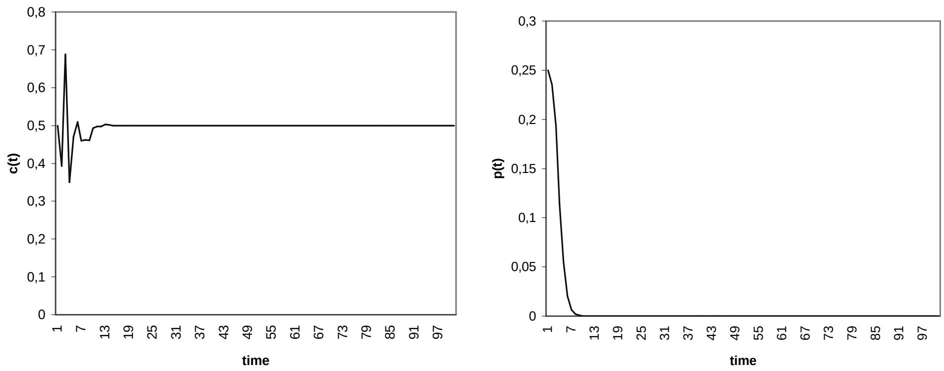

Figure 5.5: Convergence of c_t under the signaling equation and $r_t = 0$

Chapter 6 gives the full numerical solution to the central bank's control and estimation problem.

Appendix A Optimal Policy under Perfect Knowledge

The central bank chooses $\{r_\tau\}_{\tau=t}^{\infty}$ so as to minimize (5.6) subject to (5.1), (5.2) and (5.4). We can reformulate the optimization problem as choosing the indirect control variable $\{u_t\}_{t=0}^{\infty}$ to maximize

$$-E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \frac{1}{2} (\pi_\tau - \pi^*)^2 \tag{A.1}$$

subject to

$$x_{t+1} = x_t + u_t + \xi_{t+1} \tag{A.2}$$

where $x_t = E_t \pi_{t+1}$ is the new state variable, and for now $u_t = \alpha_1 E_t z_{t+1}$ is the new control variable and $\xi_{t+1} = -\eta_{t+1} + \alpha_1 d_{t+1}$. The Lagrangian for this problem is

$$L = E_0 \sum_{t=0}^{\infty} \Big[-\delta^t (x_t - \pi^*)^2 - \delta^{t+1} \mu_{t+1} (x_{t+1} - x_t - u_t - \xi_{t+1}) \Big] \tag{A.3}$$

The first order conditions (with respect to the control and state variables) are

$$\frac{\partial L}{\partial u_t} = \delta E_t \mu_{t+1} = 0 \tag{A.4}$$

$$\frac{\partial L}{\partial x_t} = -(x_t - \pi^*) - \mu_t + \delta E_t \mu_{t+1} = 0 \tag{A.5}$$

First, from equation (A.4), it follows that $E_t \mu_{t+1} = 0$.[40] Then, (A.5) gives $\mu_t = -(x_t - \pi^*) \Rightarrow E_t \mu_{t+1} = -(E_t x_{t+1} - \pi^*) = 0$. The first order condition is then $E_t x_{t+1} = \pi^*$. Since by definition $E_t x_{t+1} = E_t \pi_{t+2}$, the condition can be written as $E_t \pi_{t+2} = \pi^*$, as given by (5.10) in the main text.

[40]Note that this implies that the constraint (A.2) is not binding, which is not surprising given that the loss function has only one argument and the constraint includes only additive shocks.

Appendix B Recursive Updating of Beliefs

We derive the updating equations for sample estimates, c_t and p_t, based on the stochastic model $y_t = \gamma w_t + \epsilon_t$, where γ is an unknown constant and ϵ_t is mean zero i.i.d normal noise variable. The derivation follows Pollock (2002), and is based on the calculus of conditional expectations and associated method of moments.

Let x and y be random variables whose joint distribution is characterized by well-defined first and second-order moments $E(x), E(y), E(x^2)$, and $E(yx)$. Let us define $V(x) = E(x^2) - [E(x)]^2$; $C(y,x) = E(yx) - E(y)E(x)$. Suppose the conditional expectation of y after knowing the realization of x is a linear function of x, given by $E(y|x) = \alpha + \beta x$, where α and γ are fixed parameters. Taking unconditional expectations and invoking the law of iterated expectations, one can show that $\alpha = E(y) - \gamma E(x)$ and $\gamma = \frac{C(y,x)}{V(x)}$.[41] Then it follows that (see Pollock (2002, p.4),

$$E(y|x) = E(y) + C(y,x)[V(x)]^{-1}[x - E(x)]. \tag{B.1}$$

$$V(y|x) = V(y) - C(y,x)[V(x)]^{-1}C(y,x) \tag{B.2}$$

With these preliminaries, we now turn to our stochastic model. Suppose (c_{t-1}, p_{t-1}) have already been estimated. Before observing the value of y_t, these estimates may represent prior beliefs in the sense that we may attribute a prior distribution to $\gamma \sim N(c_{t-1}, p_{t-1})$, where the mean is $c_{t-1} = E(\gamma|\Omega_{t-1})$ and the variance $p_{t-1} = V(\gamma|\Omega_{t-1}) = E[(\gamma - c_{t-1})^2]$. This way of assigning a distribution to γ is in the spirit of a Bayesian prior. The aim is then to derive the estimates $c_t = E(\gamma|\Omega_t)$ and $p_t = V(\gamma|\Omega_t) = E[(\gamma - c_t)^2]$ based on y_t and such that we make the best use of the priors (c_{t-1}, p_{t-1}). First, along the same lines of (B.1)

$$E(\gamma|\Omega_t) = E(\gamma|\Omega_{t-1}) + C(\gamma, y_t|\Omega_{t-1})[V(y_t|\Omega_{t-1})]^{-1}[y_t - E(y_t|\Omega_{t-1})] \tag{B.3}$$

where $\Omega_t = \{\Omega_{t-1}, y_t\} = \{\Omega_0, y_1, ..., y_t\}$, with $\Omega_0 = \{c_0, p_0, \sigma_\epsilon^2\}$.[42] Moreover,

- $E(y_t|\Omega_{t-1}) = c_{t-1}w_t$ is the prediction of y_t before learning its value;
- $y_t - E(y_t|\Omega_{t-1}) = (\gamma - c_{t-1})w_t + \epsilon_t$ is the prediction error;
- $V(y_t|\Omega_{t-1}) = E[(y_t - E(y_t|\Omega_{t-1}))^2] = w_t^2 p_{t-1} + \sigma_\epsilon^2$ is the variance associated with the prediction error; and

[41] For α, this involves multiplying $E(y|x)$ by the marginal density function of x, $f(x)$, and then integrating with respect to x, which by the law of iterated expectations gives $E(y)$. For γ, first multiply $E(y|x)$ by $xf(x)$ and integrate with respect to x to obtain the joint moment $E(xy)$.

[42] With normal initial priors, one can derive a sequence of conditional distributions $N(y_t|\Omega_{t-1})$ for $t = 1, 2, ...$

- $C(\gamma, y_t|\Omega_{t-1}) = E[(\gamma - c_{t-1})(y_t - c_{t-1}w_t)]$ is the covariance of γ and y_t conditional on prior information. By substituting for y_t, the covariance becomes $E[(\gamma - c_{t-1})^2 w_t + (\gamma - c_{t-1})\epsilon_t] = p_{t-1}w_t$.

Then using these results in the recursive equation (B.3) gives:

$$c_t = c_{t-1} + p_{t-1}w_t(w_t^2 p_{t-1} + \sigma_\epsilon^2)^{-1}(y_t - c_{t-1}w_t) \tag{B.4}$$

If we define $F_t^{-1} \equiv w_t^2 p_{t-1} + \sigma_\epsilon^2$, the Kalman gain is $\kappa_t = F_t^{-1} p_{t-1} w_t$. Analogous to (B.2), the updating equation for p_t is:

$$V(\gamma|\Omega_t) = V(\gamma|\Omega_{t-1}) - [C(\gamma, y_t|\Omega_{t-1})]^2 [V(y_t|\Omega_{t-1})]^{-1} \tag{B.5}$$

Again, after similar substitution of the components of (B.5) we have:

$$p_t = p_{t-1} - p_{t-1}^2 w_t^2 F_t^{-1} \tag{B.6}$$

Finally, to arrive at (5.20) of the main text, simply forward (B.4) and (B.6) one period. It is also straightforward to use similar steps as above and derive the updating equations for the semi-reduced equation (5.23).

Appendix C Convergence of the Parameter Estimate under Learning

In this appendix we examine the question of convergence of the parameter estimate $\tilde{c}_t$ to $\tilde{\gamma}$ under least squares learning.

Under imperfect knowledge, the central bank is assumed to understand the fact that private sector expectations are state dependent. Although expectations take the form $\hat{E}_t R_{t+1} = \gamma w_t$, the weight placed on is unknown to the central bank. We saw how this information asymmetry gives rise to a constraint that relates w_{t+1}, $(\equiv -\pi_{t+1} - z_{t+1})$, to w_t, z_t and r_t, that is, $w_{t+1} = \tilde{\gamma} w_t + \beta_1 z_t + \tilde{\beta} r_t + \nu_{t+1}$. The only unknown parameter in this constraint is the composite parameter, $\tilde{\gamma} \equiv 1 + k\beta_2\gamma$. Thus defining the left hand side of the regression model as $y_{t+1} \equiv w_{t+1} - \beta_1 z_t - \tilde{\beta} r_t$, we note that $\tilde{\gamma}$ can be estimated by running a regression of y_{t+1} on w_t:

$$y_{t+1} = \tilde{\gamma} w_t + \nu_{t+1} \tag{C.1}$$

This equation can then be used to obtain an estimate in period t, $\tilde{c}_t$, by using past and present data that includes (y_t, w_{t-1}). Then, from the central bank's point of view, the perceived value of y_{t+1} is $\tilde{c}_t w_t$.

To establish convergence under least squares learning, first we formulate the dynamics of learning by the central bank as a stochastic recursive algorithm. Let $V_t \equiv t^{-1}\sum_{j=0}^{t-1} w_j^2$. Then the central bank's updating of its estimate can be written in recursive least squares

$$\begin{aligned}\tilde{c}_t &= \tilde{c}_{t-1} + t^{-1}V_t^{-1}w_{t-1}(y_t - \tilde{c}_{t-1}w_{t-1}) \\ &= \tilde{c}_{t-1} + t^{-1}V_t^{-1}w_{t-1}(w_{t-1}[T(\tilde{c}_{t-1}) - \tilde{c}_{t-1}] + \nu_t) \end{aligned} \tag{C.2}$$

$$V_t = V_{t-1} + t^{-1}(w_{t-1}^2 - V_{t-1}) \tag{C.3}$$

where $y_t = \tilde{\gamma}w_{t-1} + \nu_t = T(\tilde{c}_{t-1})w_{t-1} + \nu_t$ and T implicitly defines a mapping from the perceived law of motion (PLM) (based on $\tilde{c}_t$) to the actual law of motion (ALM), $\tilde{\gamma}$. Of course, the function $\tilde{\gamma} = T(\tilde{c}_{t-1})$ is a constant, because no matter what one perceives $\tilde{\gamma}$ to be, it is not influenced by the perception.

To use the method of stochastic recursive algorithm, one must first put the system in a standard form, which allows only lagged values of V on the right hand side of the system (see Evans and Honkapohja (2001) for details). Thus define another variable S such that $S_{t-1} = V_t$ and the system now is rewritten as follows

$$\tilde{c}_t = \tilde{c}_{t-1} + t^{-1}S_{t-1}^{-1}w_{t-1}[w_{t-1}(\tilde{\gamma} - \tilde{c}_{t-1}) + \nu_t] \tag{C.4}$$

$$S_t = S_{t-1} + t^{-1}\Big(\frac{t}{t+1}\Big)(w_t^2 - V_{t-1}) \tag{C.5}$$

where now w_t appears in the updating equation for S_t because of the redating of V_t. The system can now be written in standard form with the following definition of variables: $\theta_t = (\tilde{c}_t, S_t)$, $X_t = (w_t, w_{t-1}, \nu_t)$, and $g_t = t^{-1}$.

$$\theta_t = \theta_{t-1} + g_t Q(\theta_{t-1}, X_t, t) \tag{C.6}$$

where the two components of Q are:

$$Q_c(\theta_{t-1}, X_t(\theta), t) = S_{t-1}^{-1}w_{t-1}(w_{t-1}[T(\tilde{c}_{t-1}) - \tilde{c}_{t-1}] + \nu_t) \tag{C.7}$$

$$Q_s(\theta, X_t(\theta), t) = (\frac{t}{t+1})(w_t^2 - S_{t-1}) \tag{C.8}$$

The function Q expresses the way in which the vector of estimates, θ_{t-1}, is revised in line with the latest available data. Here, X_t is the state vector that includes the effects of w_t , w_{t-1} and ν_t, and g_t is a deterministic sequence of 'gains'– i.e. a non-increasing sequence of positive numbers– satisfying $\lim_{t\to\infty} tg_t = 1$.

We are interested in the conditions under which $\lim_{t\to\infty}\theta_t = \tilde{\theta}$, where $\tilde{\theta}$ solves either $EQ(X_t, \tilde{\theta}) = 0$ in the case that X_t is drawn from a distribution that is stationary or $\lim_{t\to\infty} EQ(X_t, \tilde{\theta}) = 0$ in the case that X_t is asymptotically stationary. In our case, the path of the explanatory variable in the regression equation, w_t, is not exogenous since the central bank engages in controlling the path of w_t, and thus the data generating process. We recognize that following the decisions of the central bank regarding its instrument, r_t, we can easily derive the actual law of motion for w_t

$$w_{t+1} = (\tilde{c}_t - \tilde{\gamma})w_t - \nu_{t+1} \tag{C.9}$$

where $(\tilde{c}_t - \tilde{\gamma})w_t$ is the (state dependent) additional control error arising from uncertainty about $\tilde{\gamma}$. Because of this feature, in real time X_t is a function of θ_{t-1}.

The technique of standard stochastic recursive algorithm (SRA), also known as stochastic approximation, has established that the limiting behavior of the sequence $\{\theta\}$ in real time, determined by the stochastic difference equation (C.6), can be approximated by an associated ordinary differential equation (ODE),

$$\frac{d\theta}{d\tau} = \lim_{t\to\infty} EQ(\theta, \bar{X}_t(\theta)) = h[\theta(\tau)] \tag{C.10}$$

where $\bar{X}_t(\theta)$ denotes the dynamics of X_t when we fix the real-time estimate θ_{t-1} at θ. The right hand side term is evaluated with respect to the asymptotic stationary distribution of $\{X_t\}$ and τ denotes "notional" or "artificial" time (see Evans and Honkapohja (2001), pp. 31). The stochastic approximation result states that if the ODE is locally stable around $\tilde{\gamma}$, then $\tilde{\gamma}$ is a possible limit point of the SRA (see p.36 of Evans and Honkapohja, 2001).

The easiest way to proceed is to look at the two components of Q separately. For the first component, which gives the revisions to $\tilde{c}_{t-1}$, by fixing the values of c and S and evaluating the expectation over X_t we get

$$\begin{aligned} h_c(\tilde{c}, S) &= \lim_{t\to\infty} EQ_c(\theta_{t-1}, \bar{X}_t(\theta), t) \\ &= \lim_{t\to\infty} S^{-1} E w_{t-1}(w_{t-1}[T(\tilde{c}) - \tilde{c}] + \nu_t) \\ &= (T(\tilde{c}) - \tilde{c}) S^{-1} E w_{t-1}^2 \end{aligned} \tag{C.11}$$

Analogously, evaluating the second component of Q gives

$$\begin{aligned} h_s(\tilde{c}, S) &= \lim_{t\to\infty} EQ_s(\theta, \bar{X}_t(\theta), t) \\ &= \lim_{t\to\infty} (\frac{t}{t+1}) E(w_t^2 - S) \\ &= E w_t^2 - S \end{aligned} \tag{C.12}$$

where Ew_t^2, the unconditional second moment of w_t, is equal to Ew_{t-1}^2 if the series $\{w_t\}$ is stationary, which holds if $\tilde{c}_t - \tilde{\gamma}$ is less than one in absolute value. That is $\tilde{c}_t$ is sufficiently close to $\tilde{\gamma}$ (the fixed point of interest). The reason is that since we are interested in local convergence, it is necessary that in deriving the ODE, the process (C.9) be asymptotically stationary.

We can write the differential equation for each component of θ

$$\frac{d\tilde{c}}{d\tau} = \frac{\sigma^2}{S}(T(\tilde{c}) - \tilde{c}) \tag{C.13}$$

$$\frac{dS}{d\tau} = \sigma^2 - S \tag{C.14}$$

This system is recursive, so that we can solve the second equation for S, and then use the result in the first equation to find the limit of $\tilde{c}$. The second equation implies $S \to \sigma^2$ from any starting point (it is globally stable). In other words, $\frac{\sigma^2}{S} \to 1$, provided S is different from zero along the path. Hence, the stability of the differential equation system is determined entirely by the stability of the non-homogenous equation for $\tilde{c}$, which has the fixed point

$$\frac{d\tilde{c}}{d\tau} = 0 \Rightarrow \tilde{c} = \tilde{\gamma} \tag{C.15}$$

The slope of the differential equation is -1, so the system is stable near the fixed point $\tilde{\gamma}$.

Chapter 6

Flexible Inflation Targeting and Active Learning

6.1 Introduction

As discussed under the introduction to chapter 5, the term structure of interest rates constitutes an important transmission channel in the monetary transmission, (that is from monetary policy to the economy), by linking the short-term interest rate to the long-term interest rate. The motivation for learning was shown to arise from the assumption that monetary authorities face uncertainty about private sector expectations of the long-term interest rate that is part of the term structure relationship. We then discussed how a passive learning policy is conducted in an inflation forecast targeting framework, leaving the full analysis of active learning and flexible inflation targeting for the present chapter.

The numerical analysis in this chapter builds on the model of chapter 5 and considers the possibility of active learning by a monetary authority about the term structure of interest rates. Regarding the private sector, we maintain the assumption in the previous chapter, namely, that private sector expectations are formed adaptively. This simplifies the analysis so that one can focus on the role of central bank learning in the inflation forecast targeting framework of Svensson (1997). An appealing feature of our model is that, even though private sector expectations are initially non-rational, they become rational in the limit as the central bank learns the persistence parameter and the equilibrium solution is identical to that of the perfect knowledge case.

6.2 Beliefs and Learning

At the time policy actions are taken (in terms of selecting the level of r_t) the central bank does not perfectly observe the realization of $\hat{E}_t R_{t+1}$. Evans and Honkapohja

(2002) point out that in general if one introduces small measurement (observation) errors, a policy based on a signal may continue to be optimal in sense that the deviation from the perfect information case in terms of the resulting welfare loss caused by observation errors is negligible (see equation (5.22) of chapter 5, where the interest rate rule responds directly to the signal on private sector expectations).

With large measurement errors, however, the policy rule that reacts to the available signal about $\hat{E}_t R_{t+1}$ can be very costly. In that case the central bank may choose to get a better forecast of $\hat{E}_t R_{t+1}$ by running its own regression. In chapter 5, we saw that the central bank understands that private sector expectations take the following form:

$$\hat{E}_t R_{t+1} = \gamma w_t \tag{6.1}$$

In practice the main problem for a central bank is that private sector agents– financial analysts, investment bankers etc.– have become more and more sophisticated in analyzing and predicting monetary policy actions of the central bank. This increased degree of sophistication of the private sector makes it harder for the central bank to observe expectations with a good degree of precision. In fact observation errors on private sector expectations could be high.[1]

Since the central bank understands the forecasting rule of the private sector, substituting (6.1) into the term structure equation and in turn the IS and Phillips equations gives the actual law of motion for $w_{t+1} \equiv -(\pi_{t+1} + z_{t+1})$ (see chapter 5). In the previous chapter, we saw that the transmission mechanism is such that at time t, controlling the variability of π_{t+2} is equivalent to controlling the variations in w_{t+1}, as the two differ only by an additive shock to inflation two period ahead. We remember that the actual law of motion of w_{t+1} is given by:

$$w_{t+1} = \tilde{\gamma} w_t - \beta_1 z_t + \tilde{\beta} r_t + \nu_{t+1} \tag{6.2}$$

where $\tilde{\beta} \equiv \beta_2(1-k) > 0$ and ν is a composite shock, that is a linear combination of the structural shocks to inflation, output and the term structure. The persistence parameter $\tilde{\gamma} \equiv 1+\beta_2 k\gamma > 0$ is unknown to the central bank because of the unknown parameter γ (see chapter 5 for details).[2] Actually, (6.2) is the constraint that the central bank faces in setting r_t optimally and, ultimately, what matters for policy is an estimate of $\tilde{\gamma}$, denoted by $\tilde{c}_t$, based on (6.2). Note that as in the learning and control literature, the constraint (6.2) also serves as a regression model to estimate the unknown persistence parameter.

[1] Since potentially γ could be time varying, it may be appropriate to have γ_t with assumptions about its time series properties. Following Beck and Wieland (2002) it is possible to consider a random walk process.

[2] Note that under perfect knowledge, given these positive parameters, the central bank lowers r_t when w_t go up (i.e. when $\pi_t + z_t$ goes down).

When active learning by the central bank is involved, the optimization problem usually gets complicated as the number of state variables increases (due to what is known as the "curse of dimensionality"). In our case, besides the state variable w_t, the current output gap, z_t, also appears as a state in equation (6.2). Together with the beliefs about the mean and variance of the persistence parameter, there would be four state variables in the model. The dual control literature usually uses a constraint similar to (6.2) but with $\beta_1 = 0$ so that next period's state variable, w_{t+1}, depends on the policy instrument, r_t, and possibly lagged values of the state variable; see Beck and Wieland (2002). In our case, setting $\beta_1 = 0$ reduces the number of state variables to three. This makes the dynamic equation comparable to that considered by Beck and Wieland (2002):[3]

$$w_{t+1} = \tilde{\gamma} w_t + \tilde{\beta} r_t + \nu_{t+1} \tag{6.3}$$

Note that in the model of chapter 5, if $\beta_1 = 0$, the perfect knowledge, rational expectations solution gives $\hat{E}_t R_{t+1} = (\beta_1/\beta_2) w_t = 0$. This is inconsequential to the way learning is modeled under imperfect knowledge. The reason is that from the point of view of the central bank, the incentive for learning arises from the fact that γ is *some* unknown parameter, and not from its specific value. In any case, the motivation for central bank learning comes from the knowledge of the functional form of (6.1), where in the case $\beta_1 = 0$, the central bank would be assumed to lack information about the true γ being actually zero.

As before, when period $t + 1$ arrives, the central bank updates its estimate by including the latest available data (w_{t+1}, w_t, r_t) in the regression equation (6.3). Using the widely used method of recursive least squares we have the following updating equations for $\tilde{c}_t$ and its variance, denoted by $\tilde{p}_t$ (see for e.g. Beck and Wieland, 2002; Pollock, 2002).[4]

$$\begin{aligned} \tilde{c}_{t+1} &= \tilde{c}_t + \kappa_{t+1}(w_{t+1} - \tilde{c}_t w_t - \tilde{\beta} r_t) \\ \tilde{p}_{t+1} &= \tilde{p}_t - \kappa_{t+1} w_t \tilde{p}_t \end{aligned} \tag{6.4}$$

where $\kappa_{t+1} \equiv w_t \tilde{p}_t F_{t+1}^{-1}$ is commonly referred to as the Kalman gain, which is the weight assigned to new information coming from the forecast error $w_{t+1} - \tilde{c}_t w_t - \tilde{\beta} r_t$. The Kalman gain in turn depends on the conditional variance of w_{t+1}, (based on information in period t), given by $F_{t+1} \equiv w_t^2 \tilde{p}_t + \sigma_\nu^2$. As can be seen from (6.4), the current state of the economy, w_t is part of the Kalman gain, and affects the path of the conditional variance of the parameter estimate, $\tilde{p}_{t+1}$. Due to the presence of autoregressive behavior in w_{t+1}, changes in the current state of the economy, w_t,

[3] In Beck and Wieland (2002), the only unknown parameter is by assumption $\tilde{\beta}$, which is multiplicative to the control variable r_t.

[4] If the unknown parameter is time-varying, the updating equations can be modified to allow for this variability via the Kalman filter; see for instance Sargent (1999) and Beck and Wieland (2002).

have direct effects on the variability of w_{t+1}, and consequently on the variability of $w_{t+2}, w_{t+3}, ...$[5]

The updating equations in (6.4) capture the idea that central bank learning possibilities about the unknown parameter is influenced by policy decisions made in period t, r_t. The channel work as follows: r_t affects the state variable in period t+1, w_{t+1}, and consequently, beliefs about the unknown parameter (i.e. $\tilde{c}_{t+1} = c(w_{t+1})$ and $\tilde{p}_{t+2} = p(w_{t+1})$). The link between current and future policy choices is established because expected future interest rate decisions by the central bank depend on the expected future state of the economy. From the principle of least squares estimation the precision of the estimate $\tilde{c}_{t+2}$ depends positively on the variance of w_{t+1}. One gets a more precise estimate (in other words, a smaller value of $\tilde{p}_{t+2}$) when the variance of w_{t+1} increases, and vice versa. Since we recognize that the current choice of monetary policy r_t affects $E_t w_{t+1}$, and given predetermined F_{t+1}, the coefficient of variation, defined by the ratio $\sqrt{F_{t+1}}/E_t w_{t+1}$ is also a function of r_t. This relationship between r_t, w_{t+1} and $\tilde{p}_{t+2}$ raises a potential tension between the urge to minimize current period loss from variability in r_t (the control part) and the need to get a more precise estimate of the degree of persistence in the economy that would help improve future outcomes (the learning part).

Under flexible inflation targeting, the central bank conducts monetary policy by adjusting the nominal interest rate, and with predetermined inflation expectations, perfectly controls the short-term real interest rate.[6] The central bank chooses $\{r_\tau\}_{\tau=t}^{\infty}$ to minimize the discounted sum of expected current and future losses, (see chapter 5).

$$\min_{\{r_\tau\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L_\tau \tag{6.5}$$

where $L_t = \frac{1}{2}(w_t^2 + \lambda r_t^2)$, $\lambda > 0$ is the relative weight assigned to the loss from the variability in r_t, the control variable.[7] As before, the discount rate is denoted by δ, $(0 < \delta < 1)$. The expectations operator E_t refers to the central bank's expectations conditional on information set in period t. Note that when $\lambda \to \infty$, the central bank engages in full stabilization of r_t, i.e., $r_t = 0$ for all t.[8]

In a strict inflation targeting regime with passive learning, the central bank minimizes the inter-temporal loss function (6.5) setting $\lambda = 0$ and taking into account

[5] The assumption that the shock is normally distributed with known variance is standard in the learning literature. If the prior belief also have a normal distribution, then the posterior belief is a normal distribution. This property of the posterior belief is convenient when dealing with numerical computations (see Appendix B).

[6] Of course the central bank directly controls the nominal rate, but because of the control lags, the one-period ahead expected inflation is predetermined, so that the central bank has perfect control of the real short rate.

[7] This follows the learning and control literature. See Beck and Wieland (2002) for a detailed discussion.

[8] In chapter 5, section 5.10, we did a simulation to see the consequences for parameter learning of a constant interest rate rule of the form $r_t = 0$, implemented for some initial periods.

the linear dynamic equation (6.3). The optimization problem would give solutions for r_t analogous to the previous chapter, as the problem is solved period-by-period and the first-order condition sets the conditional expectation of inflation two-period ahead equal to the desired target. As we will see below, this is no longer true with flexible inflation targeting ($\lambda > 0$) as future losses and the discount rate play a role in the choice of the current policy rate.

6.3 Passive Learning: Certainty Equivalence vs. Myopic

Before considering the role of active learning in optimal policy, this section solves for optimal policy under passive learning. There are two subcases under passive learning– one that ignores parameter uncertainty (certainty equivalence rule) and the other incorporates parameter uncertainty (myopic rule). Both rules are passive in nature because they ignore the link between policy choices today and future learning that is apparent from the updating equations. From the vantage point of the current period the central bank's belief is not expected to be updated in the future, implying that when choosing current policy, it anticipates the initial belief $(\tilde{c}_t, \tilde{p}_t)$ to remain fixed for all future periods. Consequently, in both cases, the non-linear updating equations drop out of the optimization problem.[9]

The certainty equivalence rule is a special case of the myopic policy rule since under the former $\tilde{p}_t = 0$. The implication of this can be seen by decomposing $E_t w_{t+1}^2$ as follows[10]

$$E_t w_{t+1}^2 = (E_t w_{t+1})^2 + F_{t+1} \tag{6.6}$$

where $E_t w_{t+1} = \tilde{c}_t w_t + \tilde{\beta} r_t$ and under the myopic policy $F_{t+1} = w_t^2 \tilde{p}_t + \sigma_\nu^2$. On the other hand, under certainty equivalence, $\tilde{p}_t = 0 \Rightarrow w_t^2 \tilde{p}_t = 0$, and so F_{t+1} is completely exogenous, depending only on the variance of the additive shock, σ_ν^2.

$$E_t w_{t+1}^2 = (E_t w_{t+1})^2 + \sigma_\nu^2 \tag{6.7}$$

Thus the difference in the way the conditional variance, F_{t+1}, is treated is also reflected in the solution of the dynamic control problem under each case (see below).

[9] Of course, when next period arrives, the bank updates its belief but then expect it to remain fixed from that period on.

[10] Remember, by assumption the additive shock ν_{t+1} is i.i.d. and thus uncorrelated with period t estimation error, $\tilde{\gamma} - \tilde{c}_t$.

6.3.1 The Certainty Equivalence Policy (CER)

Under certainty equivalence the central bank ignores parameter uncertainty. In other words, the central bank is fully confident about its estimate $\tilde{c}_t$ so that its current belief is given by $(\tilde{c}_t, \tilde{p}_t) = (\tilde{c}_t, 0)$. We can also think of this situation as the limiting case of the updating equations, where $\tilde{p}_t = 0$ implies that $\tilde{c}_{t+1} = \tilde{c}_t$. Thus, the state $\tilde{c}_t$ does not change and is independent from the policy instrument. The minimization problem is

$$\min_{\{r_\tau\}_{\tau=t}^{\infty}} E_t\Big[\sum_{\tau=t}^{\infty} \delta^{\tau-t} L_\tau |(w_t, \tilde{c}_t)\Big] \tag{6.8}$$

subject to the linear constraint (6.3). Importantly, with the non linear updating equations ignored, the problem is linear-quadratic, and the derivation of the optimal level of r_t is similar to that under perfect knowledge. To get the certainty equivalence rule, one usually proceeds in two steps. First, solve for the optimal rule assuming perfect knowledge of $\tilde{\gamma}$ and second, simply replace $\tilde{\gamma}$ by its recent estimate $\tilde{c}_t$. Alternatively, first replace $\tilde{\gamma}$ by its recent estimate $\tilde{c}_t$ in the linear constraint (6.3) and then solve the optimization problem taking the estimate as a fixed parameter. In any case, we can rewrite the above minimization problem using recursive dynamic programming and then use the 'guess and verify' method on the value function as in Svensson (1997).

With this in mind, one can write the Bellman equation associated with the minimization of (6.8).[11]

$$V(w_t) = \min_{r_t} \left[L(w_t, r_t) + \delta E_t V(w_{t+1})\right] \tag{6.9}$$

subject to (6.3) with $\tilde{\gamma}$ replaced by its certainty equivalence estimate $\tilde{c}_t$, which at the stage of optimization is understood by the central bank to be a fixed parameter (and not a state variable). Because of the resulting linear-quadratic structure of the minimization problem, the value function will be quadratic in the state w_t.

$$V(w_t) = \mu_0 + \frac{1}{2}\mu w_t^2 \tag{6.10}$$

where the two coefficients remain to be determined. If (6.10) is correct, it follows that:

$$E_t V(w_{t+1}) = \mu_0 + \frac{1}{2}\mu E_t w_{t+1}^2 \tag{6.11}$$

[11]Note that the value function in the Bellman equation does not have time subscript. This is because in infinite horizon problems, we are interested only in the unique time invariant value function, V, and associated unique, stationary policy rule, that result from repeated iterations on the Bellman equation starting from any bounded continuous V_0 (e.g. $V_0 = 0$). Convergence of the value function is guaranteed due to the *contraction mapping theorem* (see Sargent, 1987a). For linear-quadratic control problems, convergence is achieved in a single iteration if V_0 is quadratic.

where $E_t w_{t+1}^2$ follows from equation (6.7). Using (6.11) in (6.9) and taking the first order condition, we get

$$\lambda r_t + \mu\tilde{\beta}\delta(\tilde{c}_t w_t + \tilde{\beta} r_t) = 0 \tag{6.12}$$

which can easily be solved for r_t to give:

$$r_t = -\frac{\mu\delta\tilde{c}_t\tilde{\beta}}{\lambda + \mu\delta\tilde{\beta}^2} w_t \tag{6.13}$$

It is important to observe that, even though we have placed a time subscript on $\tilde{c}_t$, as far as passive learning is concerned, $\tilde{c}_t$ should be thought of as a fixed parameter (not a state variable) and it is not expected to be affected by the current policy choice. This is the sense in which forecasting and control are separated by construction.[12]

In order to identify μ, first differentiate (6.10) with respect to w_t:

$$V_w(w_t) = \mu w_t \tag{6.14}$$

Next, invoking the envelope theorem on the Bellman equation (6.9), and taking note of (6.13):

$$V_w(w_t) = w_t + \delta\mu\tilde{c}_t(\tilde{c}_t w_t + \tilde{\beta} r_t) = f(\mu) w_t \tag{6.15}$$

where $f(\mu) \equiv 1 + \lambda\delta\tilde{c}_t^2\mu/(\lambda + \delta\tilde{\beta}^2\mu)$. For the conjectured value function (6.10) to be correct, it is required that the coefficients of (6.14) and (6.15) have to be equalized:[13]

$$\mu = f(\mu) \tag{6.16}$$

Rearrange (6.16) to get the following quadratic equation for μ:

$$\delta\tilde{\beta}^2\mu^2 + [\lambda - \delta(\tilde{\beta}^2 + \tilde{c}_t^2\lambda)]\mu - \lambda = 0 \tag{6.17}$$

[12]Thus, due to the sequential nature of decision making, the updating of the parameter estimate is kept in the background.

[13]Alternatively, one can work directly with the value function. Substitute (6.13) in (6.9) and match the resulting coefficients with those in the conjectured value function (6.10).

It is easy to check that $\lim_{\mu\to 0} f(\mu) = 1$ and $\lim_{\mu\to\infty} f(\mu) = 1 + \lambda\tilde{c}_t^2/\tilde{\beta}^2$. Thus a unique solution for μ, such that $\mu \geq 1$, is :

$$\begin{aligned} \mu^{ce} &= \frac{1}{2}\Big(1 - \frac{\lambda(1-\delta\tilde{c}_t^2)}{\delta\tilde{\beta}^2} + \sqrt{\Big(1 - \frac{\lambda(1-\delta\tilde{c}_t^2)}{\delta\tilde{\beta}^2}\Big)^2 + \frac{4\lambda}{\delta\tilde{\beta}^2}}\Big) \qquad (6.18) \\ &= \frac{1}{2}\Big(1 - \frac{\lambda(1-\delta\tilde{c}_t^2)}{\delta\tilde{\beta}^2} + \sqrt{\Big(1 + \frac{\lambda(1-\delta\tilde{c}_t^2)}{\delta\tilde{\beta}^2}\Big)^2 + \frac{4\lambda}{\tilde{\beta}^2}}\Big) \end{aligned}$$

where superscript 'ce' stands for certainty equivalence. We see that the estimate of the persistence parameter, $\tilde{c}_t$, which from the perspective of period t is not expected to be updated in the future (as per passive learning), affects the value of the coefficient μ^{ce}. For future reference, note also that $\mu^{ce} \to 1$ as $\lambda \to 0$.

The solution (6.18) is similar to equation (B.6) of Svensson (1997), except that in Svensson (1997) the persistence parameter is known for certainty (and simply set equal to 1) and flexible inflation targeting is defined in terms of inflation and output stabilization. To see the effect of λ on policy responsiveness to w_t, rewrite (6.13), (bearing (6.18) in mind) so that $r_t = -\tilde{c}_t w_t/(\phi + \tilde{\beta})$, where $\phi = \lambda/(\mu^{ce}\delta\tilde{\beta})$; $\partial\phi/\partial\lambda > 0$.[14] Thus given $\tilde{c}_t$, r_t is less responsive to w_t as λ increases. Moreover, as $\lambda \to 0$, $\phi \to 0$ and $r_t = -(\tilde{c}_t/\tilde{\beta})w_t$ (strict inflation targeting). On the other hand, as $\lambda \to \infty$, $\phi \to \infty$ implying $r_t = 0$ (full stabilization of r_t).

6.3.2 The Myopic Policy (MR)

A myopic policy rule differs from certainty equivalence only because the myopic policy takes account of the current degree of uncertainty in the current estimate of the persistence parameter (thus $\tilde{p}_t > 0$). The central bank continues to ignore the fact that current policy can affect future beliefs and so treats $\tilde{c}_t$ and $\tilde{p}_t$ as *fixed* parameters, implying that the only state variable from the central bank's point of view is w_t.[15] The conjecture for the value function is then the same as (6.10) and the first order condition with respect to r_t will take the same form as (6.12). The difference is that now the coefficient μ will be a function of $\tilde{p}_t$ as well as $\tilde{c}_t$.

In identifying μ, we remember that in (6.11), $E_t w_{t+1}^2$ is a function of $\tilde{p}_t$ via (6.6).

[14]Upon simplifying

$$\phi = 1/2\left(\delta\tilde{\beta}^2 - (1-\delta\tilde{c}_t^2) + \sqrt{\Big(\frac{\delta\tilde{\beta}^2}{\lambda} + 1 - \delta\tilde{c}_t^2\Big)^2 + \frac{4\delta^2}{\lambda\tilde{\beta}^2}}\right)$$

from which it is easily seen that $\partial\phi/\partial\lambda > 0$.

[15]One can also think of $\tilde{c}_t$ and $\tilde{p}_t$ as state variables. However, these states do not change and are independent from the policy instrument. This means that, when optimizing, the terms $\kappa_{t+1}(w_{t+1} - \tilde{c}_t w_t - \tilde{\beta} r_t)$ and $\kappa_{t+1} w_t \tilde{p}_t$ on the right hand side of the updating equations drop out.

The counterpart of (6.15) is now

$$\begin{aligned} V_w(w_t) &= w_t + \delta\mu\tilde{c}_t(\tilde{c}_t w_t + \tilde{\beta} r_t) + \delta\mu p_t w_t \\ &= \Big(1 + \frac{\lambda\delta c_t^2 \mu}{\lambda + \delta\tilde{\beta}^2\mu} + \delta\tilde{p}_t\mu\Big) w_t \end{aligned} \tag{6.19}$$

Thus following the steps analogous to the previous section, we match the coefficients of (6.14) and (6.19),

$$\mu = 1 + \frac{\lambda\delta c_t^2 \mu}{\lambda + \delta\tilde{\beta}^2\mu} + \delta\tilde{p}_t\mu \tag{6.20}$$

or

$$(1 - \delta\tilde{p}_t)\mu = 1 + \frac{\lambda\delta c_t^2 \mu}{\lambda + \delta\tilde{\beta}^2\mu} \tag{6.21}$$

where the certainty equivalence case arises if $\tilde{p}_t = 0$, that is if one disregards parameter uncertainty. In finding the solution for μ that satisfies (6.21), we note that the expression on the right side of the equation is identical to the corresponding term under certainty equivalence, but on the left hand side of the equation, the coefficient on μ is $0 \leq 1 - \delta\tilde{p}_t \leq 1$ if $\tilde{p}_t$ is not too large. This holds if, say, the initial parameter uncertainty is such that $\tilde{p}_0 \leq 1$, which is actually not that restrictive if $\tilde{c}_0$ is also not too large. In this case, the prior belief $(\tilde{c}_0, \tilde{p}_0)$ such that $\tilde{p}_0 = \tilde{c}_0/2$ implies high initial uncertainty (see Beck and Wieland (2002)). This means that if we restrict the central bank's belief such that $\tilde{c}_0 \leq 2$, we can reasonably assume as well that $\tilde{p}_0 \leq 1$.[16] With this in mind, we can easily observe that when parameter uncertainty is taken into account, the fixed point for μ, denoted by μ^m, will be larger or equal to its fixed point under certainty equivalence, μ^{ce}, (see Figure 6.1).

The fixed point under the myopic rule is based on the dashed line from the origin and is larger than the fixed point under the certainty equivalent rule (based on the solid line from the origin). Thus the myopic policy is more aggressive that the certainty equivalent rule.

As before, to get the solution for μ, rewrite (6.21):

$$A\mu^2 + B\mu - \lambda = 0 \tag{6.22}$$

where $A \equiv (1 - \tilde{p}_t\delta)\delta\tilde{\beta}^2$, $B \equiv (1 - \tilde{p}_t\delta)\lambda - \delta(\tilde{\beta}^2 + \tilde{c}_t^2\lambda)$. We can check that if $\tilde{p}_t \to 0$, then $A \to \delta\tilde{\beta}^2$ and $B \to \lambda - \delta(\tilde{\beta}^2 + \tilde{c}_t^2\lambda)$, which are the coefficients under the certainty equivalence rule.

[16] For e.g., in Beck and Wieland (2002) the prior belief about the unknown parameter is characterized by a mean of 0.5 and variance 0.25, so that the central bank faces considerable uncertainty.

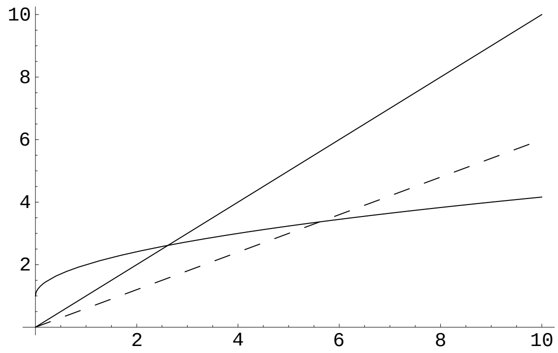

Figure 6.1: Comparing $\mu^{ce} = \mu^{ce}(\tilde{c})$ and $\mu^m = \mu^m(\tilde{c}, \tilde{p})$

The solution for equation (6.22) depends, among other things, on $\tilde{p}_t$.

$$\mu^m = \frac{1}{2}\Big(-\frac{B}{A} + \sqrt{\Big(\frac{B}{A}\Big)^2 + \frac{4\lambda}{A}} \Big) \tag{6.23}$$

$\mu^m \geq 1$ as long as the initial value of $\tilde{p}_0$ is not too large. We know that $\tilde{p}_t$ goes down in magnitude over time as more data about w and r arrive. This is true even for policy under passive learning. See chapter 5 for a discussion of convergence under learning.

Unlike the case of certainty equivalence, $\lim_{\lambda\to 0} \mu^m = 1/(1 - \delta\tilde{p}_t)$. The myopic case collapses to the certainty equivalence only if $\tilde{p}_t \to 0$, which also implies that $\mu^m = \mu^{ce} \to 1$.

Remember that the central bank knows the policy multiplier, $\tilde{\beta}$, with certainty. On the other hand, when choosing its interest rate under the myopic policy, the central bank behaves as if its initial belief, including $\tilde{p}_0 > 0$, will not be updated. In other words, the policy maker currently thinks that he will live with an uncertain estimate now and in the future. This perception somehow exaggerates actual future uncertainty because it neglects the fact that as time goes by, the precision of $\tilde{c}$ will increase ($\tilde{p}$ will decline) with the arrival of new economic data.

What is then the intuition behind a more aggressive rule under the myopic policy? The control problem is dynamic, so the bank expects to incur losses from variability in w_{t+1} (via its effect on $F_{t+2} = \tilde{p}_{t+1}w_{t+1}^2 + \sigma_\nu^2$). Since as of period t the central bank does not internalize the effect of policy on future beliefs, we have $\tilde{p}_{t+1} = \tilde{p}_t$ and $E_tF_{t+2} = \tilde{p}_tE_tw_{t+1}^2 + \sigma_\nu^2 = \tilde{p}_t(E_tw_{t+1})^2 + \tilde{p}_tF_{t+1} + \sigma_\nu^2$, which shows the benefits from a policy that sets E_tw_{t+1} closer to zero. It follows that, given $\lambda > 0$, from the perspective of the myopic policy r_t responds more strongly to deviations of w_t so that E_tw_{t+1} is closer to the target w^* (zero) than implied under the certainty equivalence rule.[17]

[17]Note that p_t scales up the component of the loss function associated with the variability

CER vs MR		
	$\lambda = 0$	$\lambda > 0$
policy rule r_t	CER=MR	CER $\neq$ MR

It is important to note that policy at time t can affect only $E_t w_{t+1}$, which corresponds with the expected loss under certainty equivalence. On the other hand, the conditional variance, F_{t+1}, is independent of r_t since p_t and w_t are predetermined as of time t. In the case where $\lambda = 0$, the myopic rule collapses to the certainty equivalence rule. This result is different from the classic study by Brainard (1967) and other related papers, where the policy multiplier is assumed to be unknown, so that policy can affect the conditional variance component, and the certainty equivalence principle breaks down even when the control variable (policy instrument) does not enter the loss function. What we have shown in this section is that, as long as $\lambda > 0$ the myopic rule differs from the certainty equivalence for the case where the persistence parameter is unknown while the policy multiplier is known with certainty.

6.4 Optimal Policy under Learning

We now formalize the active learning problem, in which the central bank can not separate estimation and control, as future beliefs about the persistence parameter depend on the whole history of the state variables and the rate of interest choices of the central bank, including the current one. Under optimal policy, the policy maker can take actions now such that w_{t+1} is more informative and contributes to a more precise future estimate of the persistence parameter.

As before, the central bank chooses a sequence of current and future short-term nominal interest rates (note that $E_t \pi_{t+1}$ is predetermined) to minimize the intertemporal loss function

$$\min_{\{r_\tau\}_{\tau=t}^{\infty}} E_t \Big[\sum_{\tau=t}^{\infty} \delta^{\tau-t} L_\tau | (w_t, \tilde{p}_t, \tilde{c}_t) \Big] \tag{6.24}$$

But now, the constraint is not just the linear Phillips curve (6.3) but also (and importantly) the non linear updating equations (6.4). The effect of current policy on future beliefs becomes visible from the Bellman equation associated with the dynamic programming problem (6.24) (see Beck and Wieland, 2002):

$$V(w_t, \tilde{c}_t, \tilde{p}_t) = \min_{r_t} \left[L(w_t, r_t) + \delta E_t V(w_{t+1}, \tilde{c}_{t+1}, \tilde{p}_{t+1}) \right] \tag{6.25}$$

of w_{t+1}. The presence of p_t does not matter under strict inflation targeting since the policy instrument is set such that $E_t w_{t+1} = 0$. By contrast, with flexible inflation targeting, $\lambda > 0$ implying that $E_t w_{t+1} \neq 0$. The presence of p_t then requires a more aggressive policy that drives $E_t w_{t+1}$ closer to zero.

subject to (6.3) and (6.4). The second term on the right side of (6.25) is the expected discounted loss from period $t+1$ onwards, with time $t+1$ state vector $(w_{t+1}, \tilde{c}_{t+1}, \tilde{p}_{t+1})$ depending on current period policy actions and state vector via the updating equations. This term thus captures the value of information and is given by

$$E_t V(.) = \int V\Big(w_{t+1}, c(w_{t+1}, \tilde{c}_t, \tilde{p}_t, w_t, r_t), p(\tilde{p}_t, w_t)\Big) f(w_{t+1}|.)dw \qquad (6.26)$$

where $f(w|.)$ is the conditional density function of w_{t+1}. Even if it has to control a linear process (6.3) and its loss function is quadratic in the two arguments, the policy maker faces a non-linear constraint because of the updating equations. Thus the dynamic programming problem falls outside the linear-quadratic formulation that is usually assumed in many economic applications. Fortunately, using a standard contraction mapping argument, Kiefer and Nyarko (1989) have shown the existence of a stationary policy and a unique value function that solve the dynamic programming problem. It is thus possible to use numerical dynamic programming methods to approximate the value function and associated policy rule. Judd (1998) describes extensively the numerical methods for solving Bellman equations, while Wieland (1998), and Beck and Wieland (2002), among others, apply these methods to optimal policy under parameter uncertainty.

As pointed out by Wieland (1998), a drawback of this procedure is the so-called curse of dimensionality, which sets in as the number of state variables becomes large. The reason is that the number of computations increases geometrically with the number of state variables in the optimization problem, which in turn undermines the precision of the numerical approximation. This will not pose a problem for us since we only have three state variables, $\tilde{c}_t$, $\tilde{p}_t$ and w_t.

6.5 Some Numerical Results

Having described the main elements of the policy problem under active learning, we now present some numerical results. We first present our results for benchmark values for the weight on interest rate stabilization, λ, the policy multiplier, $\tilde{\beta}$, the discount factor, δ, and the variance of the composite exogenous shock, σ_ν^2. Then we compare these results with those derived for alternative parameter values.

Parameter configuration

Parameter	Benchmark value	New value
$\tilde{\beta}$	0.5	1
λ	0.5	0.1
δ	0.95	0.75
σ_ν^2	1	0.5

Possible initial beliefs for $\tilde{c}$ are as high as 1.4 and as low as 0.2, while the beliefs about the variance $\tilde{p}$ range from 0.1 to 0.7. The relative degree of confidence in an estimate measured by the coefficient of variation, $\sqrt{\tilde{p}}/\tilde{c}$, takes its lowest value at $(\tilde{c}_0, \tilde{p}_0) = (1.4, 0.1)$ implying $\sqrt{\tilde{p}}/\tilde{c} < 0.23$. In this case, the uncertainty associated with $\tilde{c}$ is quite small. As will be shown below, given the parameter configuration, the role of parameter uncertainty in inducing an active learning policy decreases with the coefficient of variation. Moreover, to get an idea of how initial beliefs about the degree of uncertainty surrounding the parameter estimate matters for policy, 16 alternative pairs of $(\tilde{c}_0, \tilde{p}_0)$ are considered from the sets $\tilde{c}_0 \in \{0.2, 0.6, 1, 1.4\}$ and $\tilde{p}_0 \in \{0.1, 0.3, 0.5, 0.7\}$. All the figures shown below have $(\pi_t + z_t)$ on the horizontal axis. This follows from the definition $\pi_t + z_t = -w_t$. Using the original variables π_t and z_t helps to interpret the results in terms of inflation and output gap.

6.5.1 Results under Baseline Parameters

In this section, we compare the three decision rules- certainty-equivalent, myopic and dynamically optimal rules- given the benchmark parameter values and the central bank's initial beliefs about the unobserved persistence parameter. Figure 6.2 shows the response of interest rate r_t to deviations of the state w_t from its target level of zero, for a specific belief characterized by the mean $\tilde{c}_0 = 1$ and variance $\tilde{p}_0 = 0.5$.

Since monetary policy is conducted under a flexible inflation targeting regime, $(\lambda > 0)$, there is a tradeoff between stabilizing inflation and the short real rate. In this case, all three decisions rules do not completely offset the predictable impact of w_t on w_{t+1}. With the model featuring autoregressive behavior, any random shock to w_t will then have a long lasting effect under the three policy rules. However the degree of gradualism associated with a given level of λ differs from one decision rule to another.

As we saw in the preceding section, the myopic rule is more aggressive than the certainty equivalent rule for all levels of the state variable. The reason is that, under the myopic policy rule, the central bank recognizes that, provided that there is uncertainty surrounding the belief $\tilde{c}_0$, in other words, $\tilde{p}_0 > 0$, the contribution of this source of uncertainty to the variability of w_{t+1} (and subsequently w_{t+2}, w_{t+3}, ...) increases with w_t. In a dynamic setting, stabilizing w_{t+1} also helps minimize the negative impact of this source of uncertainty. Thus the rate of interest has to move more in response to w_t compared to a policy that disregards parameter uncertainty.

Now, as can be seen from Figure 6.2, for low to moderate deviations of the state from the target, the dynamically optimal policy is even more aggressive than the myopic one. But if the deviations are large, the optimal policy responds less aggressively, even compared to the certainty equivalence policy. From the updating equations, the larger the deviation of w_t from zero (due to say an exogenous shock), the smaller $\tilde{p}_{t+1}$ (implying that $\tilde{c}_{t+1}$ is a more precise estimate) leading to a smaller control error when setting r_{t+1}. Thus, in contrast to the myopic policy, the actively learning central bank anticipates future improvements in policy performance as $|w_t|$ increases. This shows that when realized exogenous shocks, ν_t, that ultimately drive data generation for w_t, are large there is less role for deliberate probing by the central bank, more so the larger the deviation of w_t from the target.[18]

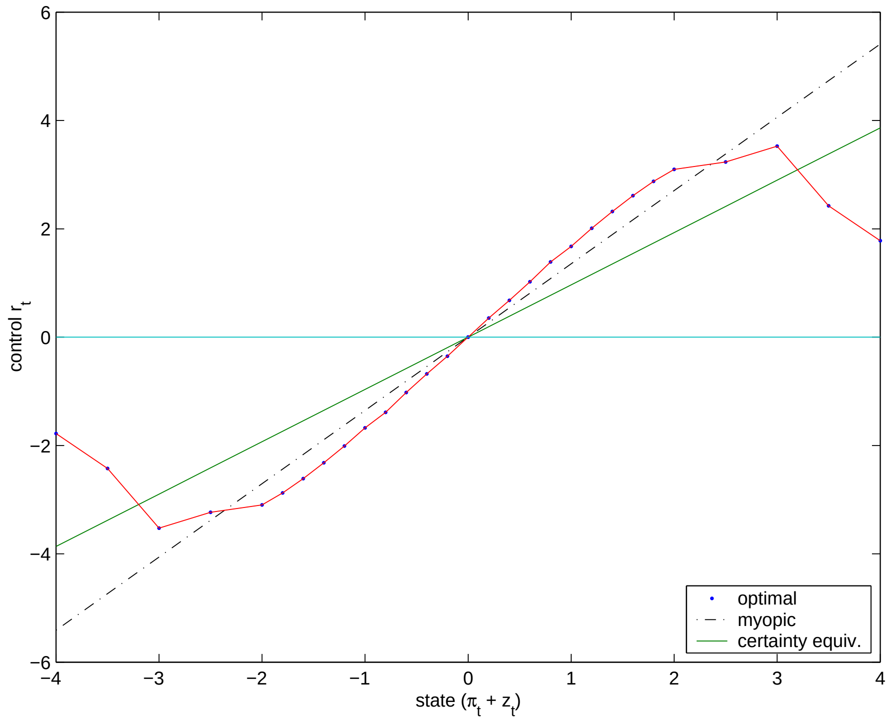

Figure 6.2: The three decision rules for initial beliefs $\tilde{c}_0 = 1$ and $\tilde{p}_0 = 0.5$

The qualitative results shown in Figure 6.2 carry over to other possible initial beliefs about the persistence parameter. In Figure 6.3, 16 alternative plots are shown, each plot corresponding to a specific configuration of the initial belief, $\tilde{c}_0$ and $\tilde{p}_0$.[19] Perhaps not surprisingly, the three rules diverge with the magnitude of $\tilde{p}_0$ and $\tilde{c}_0$. For example, when parameter uncertainty is small (say $\tilde{p}_0 = 0.1$) and there is a small degree of persistence in the economy ($\tilde{c}_0 = 0.2$), the three decision rules tend to be identical. At the other extreme, when both $\tilde{c}_0$ and $\tilde{p}_0$ are large, there are clear divergences between the decision rules.

Next we examine how the incentives for the central bank to deviate from the certainty equivalence and myopic rules may depend on other parameters of interest. As can be expected, the three decision rules are affected by changes in $\tilde{\beta}$, λ and δ.

[18] At the same time, of course, F_{t+1}, the conditional variance of w_{t+1}, increases with w_t^2 but this component of $E_t w_{t+1}^2$ is independent of r_t.

[19] Because the rate of interest responds symmetrically to positive and negative deviations of the state variable from the target (i.e. zero), only positive deviations are shown in the plots.

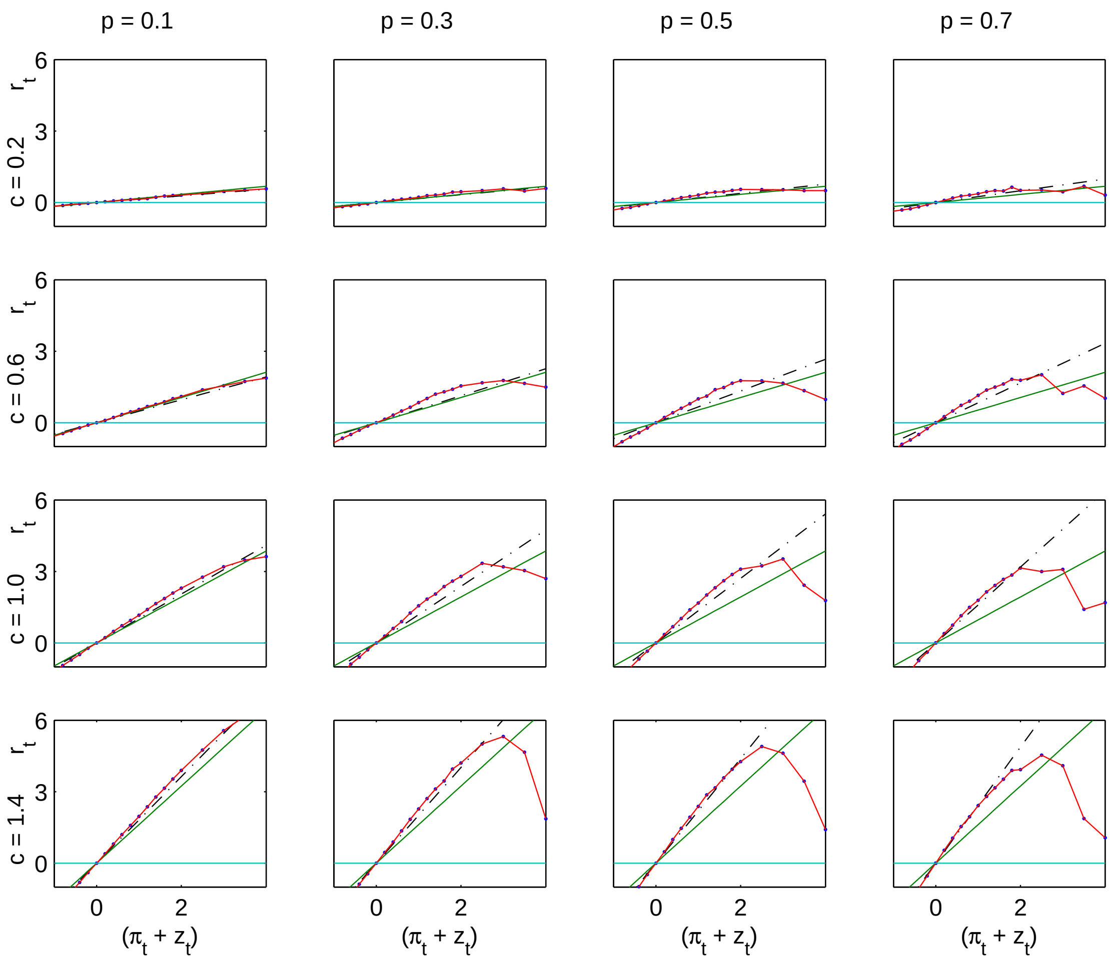

Figure 6.3: The three decision rules for alternative initial beliefs

Thus when analyzing the effect of changes in these parameters, it is not proper to compare directly the optimal policies arising from each parameter setting. Rather, one has to take the difference between the optimal and the myopic policies. On the other hand, a change in σ_ν^2 affects only the dynamically optimal policy. In this case, we compare directly the dynamically optimal rules associated with each value of σ_ν^2.

6.5.2 Policy under Less Volatile Shocks

First, we look at the effect of a decrease in the variance of the exogenous random shock, σ_ν^2. In the benchmark case, the variance was set to 1, while now the variance is reduced by half, standing at 0.5, which implies that, for a given policy path, the economy is inherently more stable as it is subject to less volatile shocks. Note here that unlike the optimal policy, the certainty equivalence and myopic rules are not affected by σ_ν^2. So only the optimal decision rules for the two alternative values of

σ_ν^2 are shown in Figure 6.4 below.

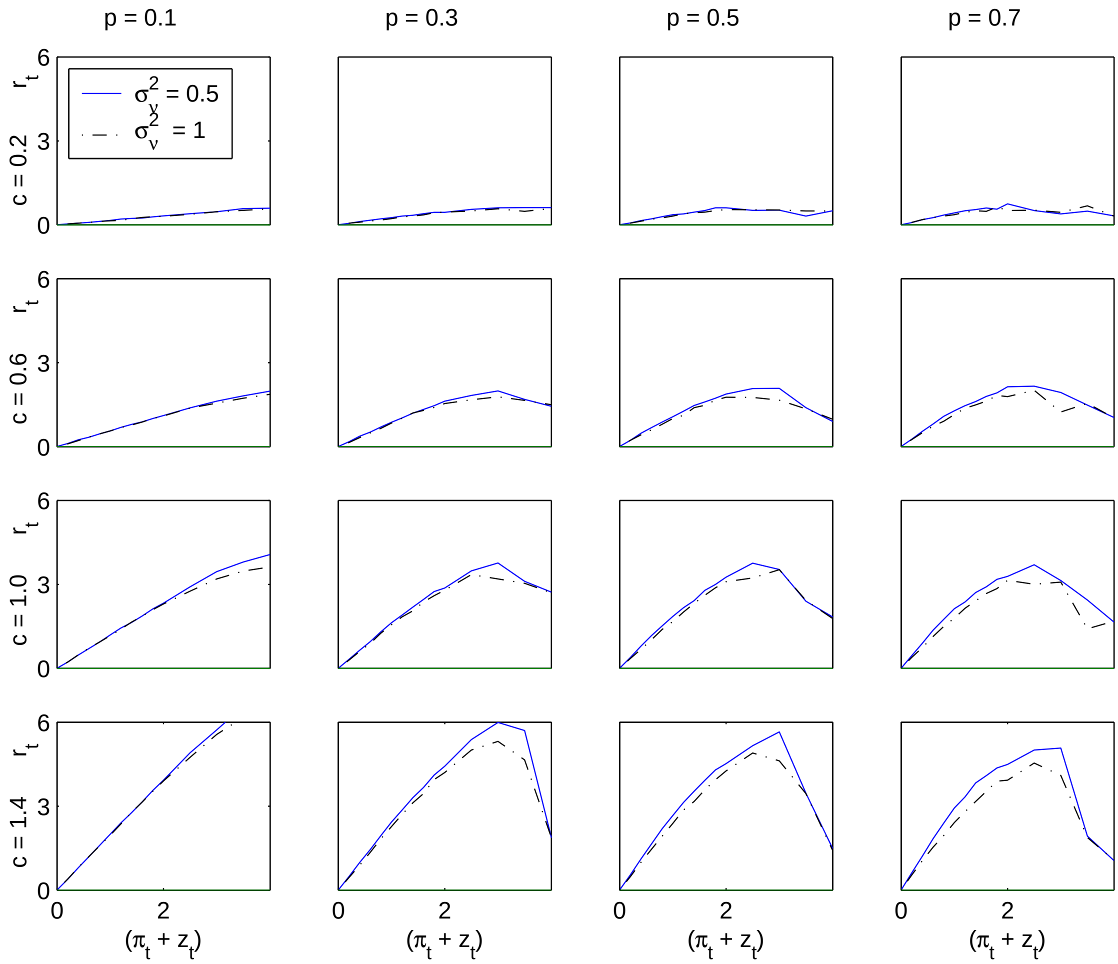

Figure 6.4: Performance of optimal policy ($\sigma_\nu^2 = 1$ vs. $\sigma_\nu^2 = 0.5$)

In each panel of the figure the benchmark case ($\sigma_\nu^2 = 1$) is shown by the dotted line while the solid line arises from the smaller variance of the shock ($\sigma_\nu^2 = 0.5$). It is apparent from most of the panels, optimal policy is more aggressive when the variance σ_ν^2 decreases, that is when the shocks driving the w_t process are less volatile. This is especially the case with large values of $\tilde{c}$ and $\tilde{p}$ shown in the lower right corner of the figure. The intuition is that, with low variability in the shocks, optimal policy needs to actively manage data generation by increasing the (conditional) coefficient of variation in w_{t+1}, defined here as $CV = \sqrt{F_{t+1}}/E_t w_{t+1}$. Since F_{t+1} is predetermined, CV can be increased only by a lower value of $E_t w_{t+1}$, which in turn requires a more aggressive response of r_t to w_t.[20]

[20]Quantitatively, this difference is not large when $\tilde{c}$ or $\tilde{p}$ is small.

6.5.3 Policy Cares More about Inflation Stabilization

Next, it is interesting to see how a different value of λ affects the incentives for the design of optimal policy. We examine optimal policy when the weight placed on interest rate stabilization, λ, decreases. Under the benchmark value $\lambda = 0.5$, and policy gives considerable weight to losses arising from variability in the real interest rate, r_t.

Generally, as λ gets smaller, the central bank responds more aggressively to its information, $(\pi_t + z_t)$. As noted previously, this is true for the optimal policy as well as the myopic and certainty equivalence rules, and is intuitively correct since if λ is small, monetary policy gives high priority to stabilizing inflation at the expense of a more volatile short real rate. When all the decision rules are affected by a change in λ, it does not make sense to compare directly the dynamically optimal policies for alternative values of λ. Rather we take the difference between the optimal and the myopic, or between the optimal and the certainty equivalence rules, for a given level of λ and see how this measure (i.e. the relative performance of the optimal policy) is affected when λ takes another value. In this light, Figure 6.5 compares the relative performance of the optimal policy under the two alternative value of λ and for a wide range of initial beliefs.

In each plot, the solid line depicts the difference between the optimal and myopic policies for $\lambda = 0.1$ while the corresponding measure for the baseline case, $\lambda = 0.5$, is displayed using the dotted line. As can be seen from the figure, there are no very clear results as the initial beliefs and the magnitude of the deviations in the state variable seem to matter. In most of the plots, if $(\pi_t + z_t)$ is close to zero, the difference between optimal and myopic is less pronounced for $\lambda = 0.1$ since in this case the myopic policy already generates enough data by aggressively minimizing $E_t w_{t+1}$ (and thereby increasing CV given that F_{t+1} is predetermined). The role of experimentation increases with λ if deviations of $(\pi_t + z_t)$ are relatively small.

When policy cares much less about inflation stabilization, experimentation is maintained only for small deviations of the state variable.

We get a different picture from the lower right corner, when $\tilde{c}_0$ and $\tilde{p}_0$ are large. In this case, the effect of w_t on $E_t w_{t+1}$ and F_{t+1} is large. There is less role for experimentation under $\lambda = 0.5$ because here the myopic rule is affected by the initial beliefs more than the optimal policy. Moreover, the value of the state at which we have a shift from a more aggressive to a less aggressive optimal policy is smaller under $\lambda = 0.5$ than under $\lambda = 0.1$, where optimal policy remains more aggressive over larger deviations of the state from the target.

As might be expected, we see from the upper left panels that the effect of changes in λ on the extent of experimentation disappears for small values of $\tilde{c}_0$ and $\tilde{p}_0$, the difference between the optimal policy and the myopic almost disappears. As a final remark, we note here that similar results apply if we use the certainty equivalence as the reference policy against which the performance of optimal policy is measured.

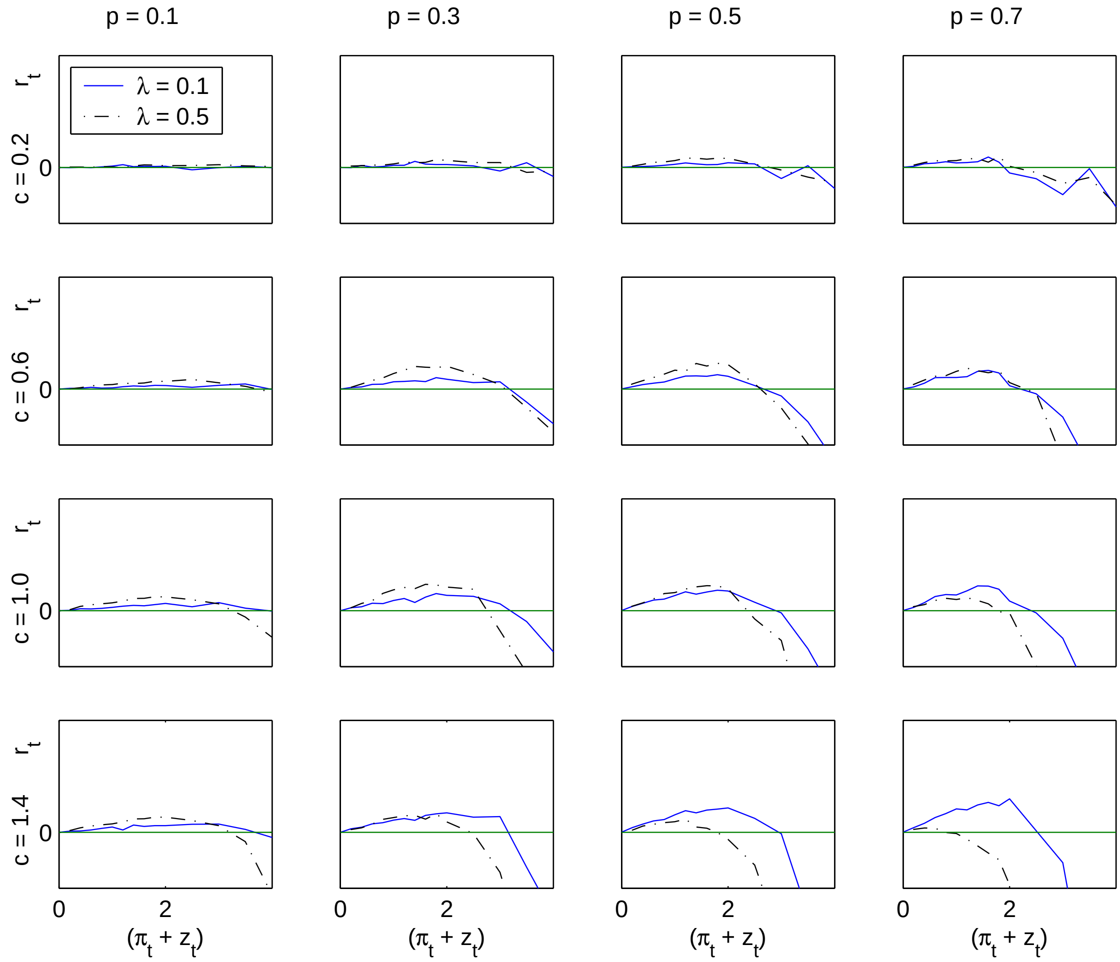

Figure 6.5: Relative performance of optimal policy ($\lambda = 0.1$ vs. $\lambda = 0.5$).

6.5.4 The Effect of a Larger Policy Multiplier

Figure 6.6 shows how the degree of probing is affected by the size of the policy multiplier, $\tilde{\beta} \in \{0.5, 1\}$. The solid line is associated with a larger multiplier, $\tilde{\beta} = 1$, while the dotted line is associated with a smaller multiplier, $\tilde{\beta} = 0.5$. From (6.2), we know that $\tilde{\beta} \equiv \beta_2(1-k)$; thus a smaller value of $\tilde{\beta}$ is associated with a weak leverage of monetary policy on the current long real rate, (from the term structure equation, the weight on the short real rate, $(1-k)$, decreases). In other words, as k gets closer to one, the model becomes more forward-looking since the long term interest rates will be determined mainly by movements in private sector expectations. Due to this fact, the importance of learning increases as well. Since the long real rate in turn determines aggregate demand with a one period lag and ultimately inflation with two period lags, a change in monetary policy has only a small effect if $\tilde{\beta}$ is small. This induces the central bank to be more aggressive in response to shocks in order to have a considerable effect on the economy and stabilize inflation.

So again using the difference between the optimal and the myopic policies as a measure of experimentation, the figure shows that, given small to moderate deviations of $\pi_t + z_t$), experimentation decreases as the value of $\tilde{\beta}$ gets larger.

The plots in Figure 6.6 are very similar to those in Figure 6.4. Thus a larger policy multiplier has effects on the degree of experimentation similar to a smaller weight on interest rate stabilization. For most of the alternative initial beliefs, under the larger value $\tilde{\beta} = 1$ experimentation is maintained over large deviations of the state from the target. The effect on the relative response of the optimal policy of a change in $\tilde{\beta}$ diminishes if $\tilde{c}_0$ or $\tilde{p}_0$ is sufficiently small.

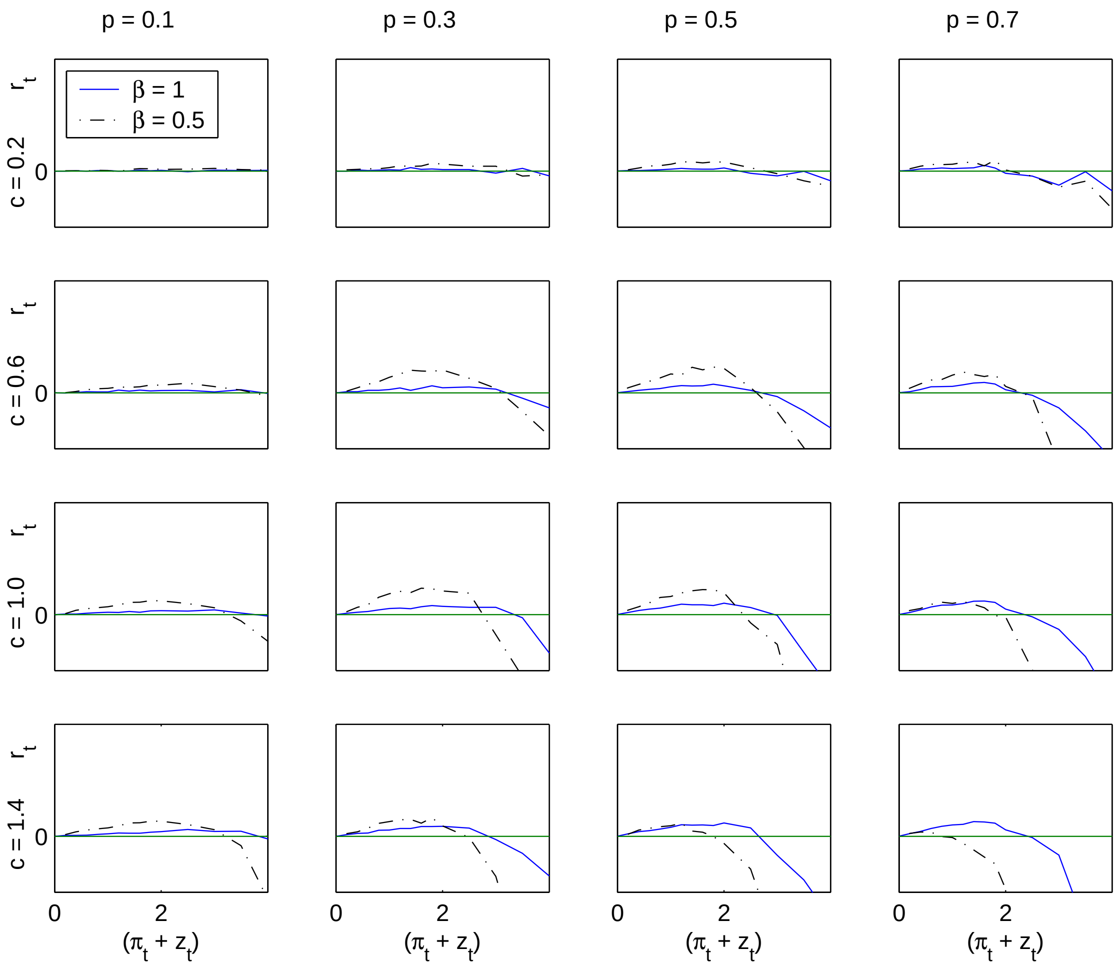

Figure 6.6: Relative performance of optimal policy ($\tilde{\beta} = 1$ vs. $\tilde{\beta} = 0.5$)

6.5.5 Policy Cares Less about the Future

The benefits from experimenting with the policy rate are in terms of reduced variability of the economy in the future from more precise estimates and improved control associated with less uncertainty in the unknown parameter. Under flexible inflation targeting, optimal policy exploits the tradeoff between these future benefits and current costs from large movements in r_t. In this regard, one would expect that the incentive to experiment increases as the central bank gives more weight to expected future losses from variability in $w_{t+\tau}$ and $r_{t+\tau}$, for $\tau = 1, 2, 3....$ Thus there is more tendency to probe for a relatively large discount factor (alternatively, for a relatively small discount rate). Figure 6.7 confirms this intuition about the effect of changes in the discount factor, δ, especially for large initial beliefs about the persistence parameter.

In Figure 6.7, each plot shows the difference between the optimal and myopic policies for two sets of values of the discount factor, $\delta \in \{0.95, 0.75\}$. The baseline value of 0.95 is shown by the solid line and is compared to the smaller value 0.75 given by the dotted line. With a smaller discount factor, the central bank cares less about the future expected losses and thus policy tilts towards stabilizing current expected loss from variability in r_t.

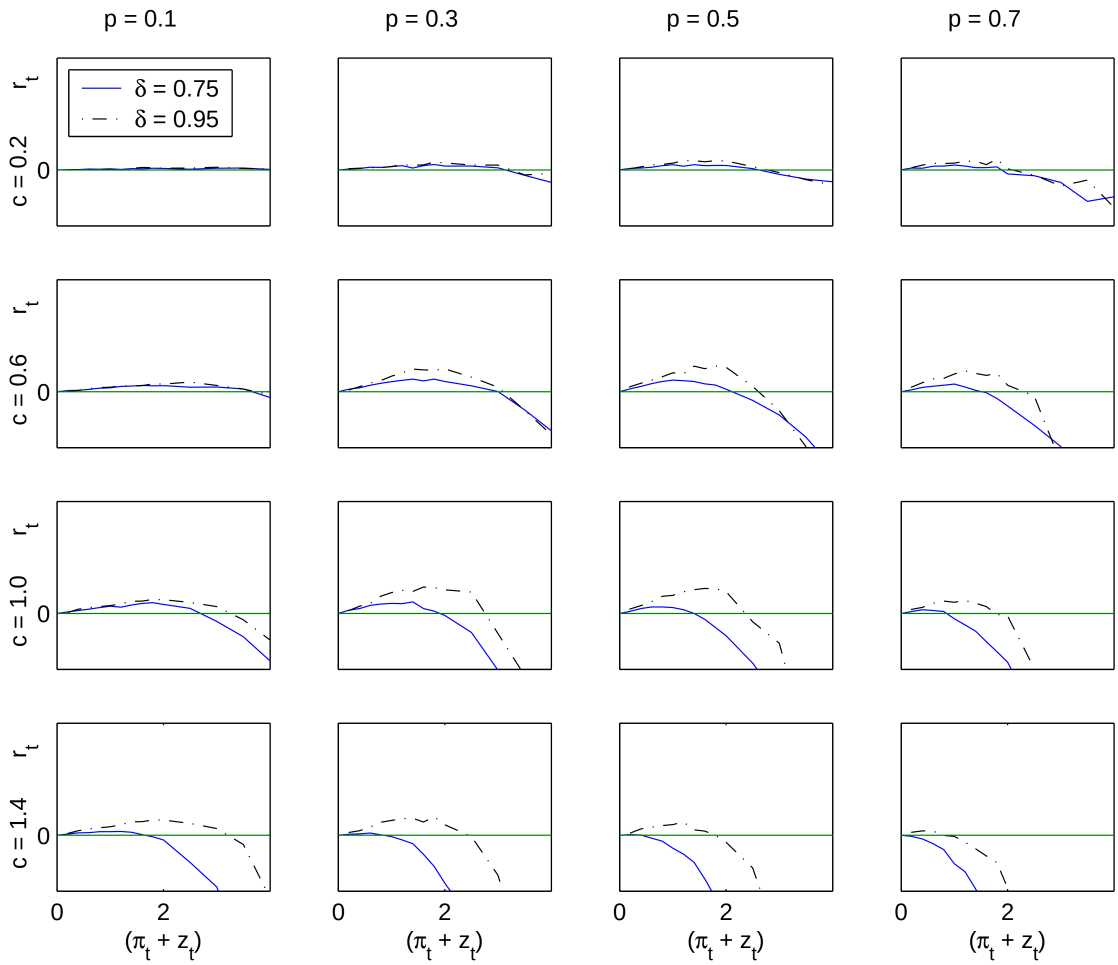

Figure 6.7: Relative performance of optimal policy ($\delta = 0.75$ vs. $\delta = 0.95$)

6.6 Concluding Remarks

In this chapter we set out to extend the analysis of chapter 5 to a general case where on the one hand monetary policy faces a tradeoff in stabilizing inflation as well as the rate of interest, (the policy instrument), and on the other hand, the central bank internalizes the effects of current policy choices on its learning possibilities about an unknown degree of persistence in the economy.

We find that under flexible inflation targeting and uncertainty in the degree of persistence in the economy, allowing for active learning possibilities has effects on the optimal interest rate rule followed by the central bank. For a wide range of possible initial beliefs about the unknown parameter, the dynamically optimal rule is in general more activist, in the sense of responding aggressively to the state of the economy, than the myopic rule for small to moderate deviations in the state variable. On the other hand, for large deviations, the optimal policy is less activist than the myopic and the certainty equivalence policies. This shows that the role of optimal policy in generating variability (increasing the coefficient of variation of the target variable) depends on the current state of the economy. When next period's state of the economy is expected to deviate a lot from the target due to an unpredictable shock, and thus generates data on itself, optimal policy does not need to increase the coefficient of variation of the next period's state w_{t+1}, while it does so when the economy is hit by a very small shock and the state is close to the target.

On the other hand, the myopic rule does not incorporate the future benefits from large deviations of current state from the target and thus responds linearly and more aggressively than the certainty equivalence rule. The intuition for the aggressiveness of the myopic rule lies in the fact that it takes account of the additional source of uncertainty, which is compounded by the magnitude of the state variable, as the unknown parameter is multiplicative to the current state. By acting more aggressively to stabilize the predictable impact of current period shocks on future aggregate demand and inflation, the myopic reduces the impact of uncertainty associated with estimating the persistence parameter.

This feature of the myopic rule differs from what one might find when the source of parameter uncertainty lies with the policy multiplier. In that case, policy under myopic rule tends to be less aggressive than certainty equivalent. Uncertainty about the policy multiplier forces the central bank to be cautious about using its policy instrument freely to stabilize inflation. In our case, the analogous explanation is that, with uncertainty in the persistence parameter, the central bank would like to see less variability in the next period's state variable. Under the myopic rule, this can be achieved only if the policy rate responds aggressively to new information about the state of shocks.

With very small weight on interest rate stabilization, the dynamically optimal policy is still more aggressive than the myopic for small to moderate deviations of

$(\pi_t + z_t)$, though to a lesser extent than when interest rate stabilization is more important. The reason is that the myopic policy is already aggressive enough when inflation stabilization receive high priority. A larger policy multiplier, the coefficient of the real rate, and a higher weight on inflation stabilization have similar effects on the degree of experimentation. In the limit of strict inflation targeting optimal policy is not affected by uncertainty in the persistence parameter.

As a final remark, we note that it is possible to have different assumptions about how private sector expectations are formed. One may for example start out by assuming rational expectations on the part of the private sector. In our case, even though expectations are non-rational in the short-run, they turn out to be rational in the limit since the central bank learns the unknown parameter with probability one. The solution of the model is the identical to the full information rational expectations equilibrium. Moreover, it is possible to allow for misspecification in the central bank's econometric model used to estimate the persistence parameter. The role of misspecification has been analyzed for example in Evans and Honkapohja (2001).

Appendix Numerical Dynamic Programming

The numerical approximation of the dynamically optimal policy under learning is based on the Bellman equation (6.25).

The Bellman equation is non-standard because of the non-linear updating equations, which means no closed-form solution is available. Before solving for the optimal value of r_t, the central bank must first evaluate the expected continuation value, which is a function of next period beliefs, and through the non-linear updating equations, depends on current period states, beliefs and central bank's policy instrument.

We compute the value function and the policy (reaction) function by repeated iterations over (6.25) based on the dynamic programming algorithm of Beck and Wieland (2002). The procedure is implemented by starting with an initial value for V_0 for all grid points over the state vector, $(w, \tilde{c}, \tilde{p})$, and solving the right side of the Bellman equation for the optimal level of r at each grid point.[21] As an initial guess for V_0 one can use the value function arising from the myopic policy and calculate V_1 by applying the operator T to V_0 and update said table. Then the resulting value function is used as the next guess in the iteration. This procedure continues until convergence in value function is achieved. In particular, the iteration over the Bellman equation is repeated until the difference between two successive iterates is sufficiently small. It turns out that, the value function iteration can be slow to converge, thus in order to speed up convergence, policy iterations are conducted after each value function iteration.

Another issue in numerical approximation is interpolation. The grid points at which the value function is calculated are discrete in number. In this case, the values of the function in between grid points are approximated by linear interpolation. The advantage of linear interpolation is that it preserves the shape of the function, positivity and monotonicity. Thus, even though the algorithm only saves a discrete approximation of V_0, when computing values of V_1, linear interpolation guarantees that Blackwell's sufficiency conditions are satisfied and the algorithm remains a contraction mapping.

Of course, one expects to get multiple local maxima because of the non-convexities introduced from the non-linear updating equations. A way to handle this problem is to first undertake a rough grid search, then, based on the minimum found, a golden section search. This procedure ensures that the global minimum is computed more precisely.

The crucial steps in the numerical algorithm are related to evaluating the expected continuation value, given the current state of the economy, w_t, and the central bank beliefs about the unknown degree of persistence in the economy, $\tilde{c}_t$ and associated uncertainty, $\tilde{p}_t$.

[21] The minimization step is not trivial as it requires repeated evaluation of an integral associated with $E_t V(.)$.

This step involves replacing next period beliefs, $\tilde{c}_{t+1}$ and $\tilde{p}_{t+1}$, by the non-linear updating equations. The presence of non-linearity makes it difficult to evaluate the integral (6.26).[22]

Since the random shocks and the prior beliefs about the unknown parameter are assumed to be normally distributed, one can evaluate the integral numerically using the Gauss-Hermite quadrature (GHQ). The GHQ evaluates the integral using a discrete number of points, n, called nodes, with the weights on the nodes optimally chosen to achieve good precision (see Judd, 1998).

Generally, given a normally distributed random variable, $Y \sim N(\bar{y}, \sigma^2)$, the expectation of $f(Y)$ is given by:

$$E\{f(Y)\} = (2\pi\sigma^2)^{-1/2} \int_{-\infty}^{\infty} f(y) e^{-(y-\bar{y})^2/2\sigma^2} dy \tag{A.1}$$

where y is a particular realization of Y. By using the linear change of variables $x = (y - \bar{y})/\sqrt{2}\sigma \Leftrightarrow y = \sqrt{2}\sigma x + \bar{y}$, implying $dy = \sqrt{2}\sigma dx$, we can rewrite (A.1) as follows:

$$\begin{aligned} E\{f(Y)\} &= (2\pi\sigma^2)^{-1/2} \int_{-\infty}^{\infty} f(\sqrt{2}\sigma x + \bar{y}) e^{-x^2} \sqrt{2}\sigma dx \\ &= \pi^{-1/2} \int_{-\infty}^{\infty} f(\sqrt{2\sigma^2} x + \bar{y}) e^{-x^2} dx \\ &\approx \pi^{-1/2} \sum_{j=1}^{n} \omega_j f(\sqrt{2\sigma^2} x_j + \bar{y}) \end{aligned} \tag{A.2}$$

where x_j and ω_j are, respectively, nodes and weights in the GHQ. The nodes and weights for alternative values of n are tabulated (see Judd (1998)).

This result can be adapted to our model, where conditional distribution of w_{t+1} is normal with mean $\tilde{c}_t w_t + \tilde{\beta} r_t \equiv \bar{w}_{t+1}$ and variance $w_t^2 p_t + \sigma_\nu^2 \equiv F_{t+1}$. The expectation of $V(w; .)$ is taken over all possible realizations of w_{t+1}.

$$\begin{aligned} E_t\{V(w_{t+1}; .)\} &= \pi^{-1/2} \int_{-\infty}^{\infty} V(\sqrt{2F_{t+1}} x + \bar{w}_{t+1}; .) e^{-x^2} dx \\ &\approx \pi^{-1/2} \sum_{j=1}^{n} \omega_j V(\sqrt{2F_{t+1}} x_j + \bar{w}_{t+1}; .) \end{aligned} \tag{A.3}$$

See Judd (1998) for a detailed treatment of numerical dynamic programming techniques. Wieland (1998) and Beck and Wieland (2002) also give a brief description of the method.

[22] Note that the expectation of $V(w_{t+1}; .)$ has to be taken over *all* possible realizations of w_{t+1}.

Chapter 7

Summary and Conclusions

Recently, two aspects of the monetary transmission mechanism (both related to expectation formation) have attracted attention and form the basis for the current thesis. The first aspect concerns the greater role of forward-looking expectations in the design of monetary policy (see Blinder, 1998; Woodford, 1999; Svensson, 2003b). For instance, Blinder (1998) points out that monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter like long-term interest rates, stock prices and exchange rates, variables which, by their nature, are forward-looking. Second, some authors have stressed the importance of imperfect knowledge on the part of decision makers due to lack of information about the true model, data uncertainty, or even computational limits of decision makers, which may give rise to possible non-rational behavior, the so called bounded rationality (see Sargent, 1999; Evans and Honkapohja, 2001). In that case, decision makers have to form expectations while adapting to new environments and with the arrival of new information about the economy. A crucial question is how actions taken today may hinder or speed up agents' learning possibilities. Moreover, should monetary policy strategy take into account the learning constraints of the private sector and the central bank itself. This thesis raises some issues related to these two elements of policy design. The first three chapters analyze the desirability of disclosing central bank forecasts while the last two chapters are devoted to optimal learning and control under inflation targeting. A common feature of the transparency and learning parts of the thesis is that forward-looking private sector expectations are important for macroeconomic outcomes. Asymmetric information between the central bank and the private sector is also a key maintained hypothesis throughout the thesis.

7.1 Central Bank Forecasts and Communication

In recent times an increasing number of central banks have taken steps to be more transparent. The list includes the central banks of the UK, Canada, Sweden, Australia, New Zealand, and Switzerland. In general, one may argue that transparency improves the effectiveness of monetary policy (Blinder, 1998), help household and business decision makers get a more accurate picture of future developments of the economy and understand central bank actions (Mishkin, 2004), or reduces the volatility of financial markets that is associated with speculation about policy motives (Romer and Romer, 2000). However, despite these benefits, Romer and Romer (2000) cite possible complications in implementing the immediate disclosure of forecasts, saying that immediate disclosure could change the information content of the forecasts since they would attract a lot of attention from the public. But then why? Is it because disclosing information may worsen overall macroeconomic stabilization?

In this thesis the role of transparency is analyzed within the context of central bank forecasts and when inflation expectations are forward-looking. There is some evidence that central banks have better information about the state of the economy and its future development than is available to firms and households (see for e.g. Romer and Romer, 2000). Based on this maintained hypothesis, chapters 2 and 3 examine whether there is any rationale for disclosing private information about upcoming shocks. Specifically, the issue concerns disclosure policy regarding forecast of *future* shocks, as opposed to current period shocks that the literature has stressed. The motivation for studying future shocks is that information about these shocks is relevant in a world of forward-looking expectations (as in the microfounded New Keynesian framework).

Chapter 2 abstracts from credibility issues and finds that with full credibility, advance disclosure of future shocks is not desirable for a central bank with multiple goals as expectations become more volatile with disclosure hindering current stabilization effort. Chapter 3 modifies the analysis of chapter 2 and allows for unobserved shifts in the central bank's output target and associated credibility problems (Faust and Svensson, 2001; Jensen, 2000). In addition, the central bank chooses its policy before private sector inflation expectations are set so that the private sector can infer the output target from observed central bank actions. It turns out that when credibility is an issue, the relevance of disclosing forecasts of future shocks is not clear cut and depends on specific assumptions about the unobserved output target. To be specific, the central bank is better off by withholding its private information about future shocks if the random shift in output target is directly revealed (no incentive effects) at the time the future shocks are realized. On the other hand, if the output target has to be indirectly inferred from observed policy decisions of the central bank, then disclosure policy is harmless. The reason lies in the strong dependence of one-period-ahead private sector inflation forecasts on central bank actions, which induces the central bank to focus exclusively on price stability in subsequent periods. Anticipating this incentive effects, private sector expectations of next period's inflation remain stable and thus contribute to current period stabilization efforts of the central bank.

A common result of chapter 2 and 3 is that unlike current period shocks, there is no inherent desire to offset the forecasts of future shocks because these shocks do not have a direct impact on current inflation. This implies that even if current actions of the central bank are observed, say in terms of the current interest rate choice, the public can not infer the central bank's forecasts.

Central banks may also lack information on private sector expectations and need to depend on their own assessments of what these expectations are. Since the assessments are private information of the central bank, it is interesting to analyze the consequences of disclosing them to the public. This issue is the subject of chapter 4, which stresses mutual uncertainty between the central bank and the private sector. It aims to shed light on the implications of this symmetric uncertainty and communication by the central bank for stabilization policy. Existing literature generally assumes perfect knowledge of expectations despite concerns about the presence of large assessment errors (see Tarkka and Mayes, 1999; Evans and Honkapohja, 2002).

Chapter 4 shows that communication of assessment errors improves output stabilization at the expense of instability in inflation, thus leading to a variability tradeoff. The intuition for this result is as follows: with full knowledge of assessment errors, private sector expectations react in a way that induces the bank to adjust its interest rate so that output (and aggregate demand) is stabilized. In this case, aggregate demand fails to stabilize the effect of inflation expectations on current inflation. The tradeoff also has normative implications for policy: a central bank that is sufficiently conservative (as in Rogoff, 1985) improves society's welfare by communicating its assessments. This result also holds for a more general loss function that includes interest rate stabilization if the interest rate multiplier is not too large. Finally, we also show that a more transparent central bank can afford to be more conservative since the benefits from higher transparency in terms of output stabilization are greater the more conservative is the central bank.

7.2 Learning, Control and Inflation-Forecast Targeting

A number of major central banks have adopted what is called inflation-forecast targeting. The main features of this regime are extensively discussed in Svensson (1997) and, with some extensions, in Svensson (1999), although the role of the term structure and learning under uncertainty are left out. Chapter 5 and 6 analyze optimal policy under learning in an inflation-forecast targeting framework that incorporates the term structure of interest rates. The model with the term structure is more elaborate and realistic as monetary policy is conventionally viewed as running from short-term interest rates managed by central banks to longer-term rates that influence aggregate demand (Goodfriend, 1998). More importantly, it is shown that limited information concerning private sector expectations of the long-term interest rates translates the central bank's problem into one with parameter uncertainty, specifically uncertainty about the degree of persistence in output and inflation. Then main interest is thus in the performance of alternative decision rules based on passive learning (certainty equivalence and myopic rules) or active learning (dynamically optimal rule). Active learning is especially interesting as the central bank takes account of the simultaneous problem of controlling inflation and estimating (learning about) the degree of persistence in the economy.

The reduced form equation implied by our structural equations differs in two ways from the linear process considered in most studies. First, the reduced form equation is dynamic due to some inertia in the structural model where future economic conditions depend in part on the current period conditions. Second, while the literature typically studies uncertainty about a policy multiplier, the nature of imperfect information in our term structure equation implies that it is the persistence parameter in the linear process that is unknown to the central bank.

Chapter 5 solves the passive learning problem under strict inflation targeting, where monetary policy completely stabilizes predictable fluctuations in inflation. Here, the dynamically optimal policy under learning does not deviate from the certainty equivalent and myopic policies. Thus optimal policy separates estimation and control. Chapter 6 extends the analysis of chapter 5 to a general case of flexible inflation targeting with active learning where the central bank internalizes the effects of current policy choices on its learning possibilities about the unknown persistence parameter. When the rate of interest, which is the policy instrument, enters the loss function, optimal policy deviates from the passive learning rules. Moreover, the need for policy to generate higher relative variability in inflation depends on the state of the economy. When inflation is expected to deviate a lot from the target due to an unpredictable shock, the economy generates data on its own so that policy does not need to actively generate data, while it does so when the economy is hit by a very small shock. On the other hand, the myopic rule only takes account of the additional source of uncertainty in the persistence parameter, the effect of which is compounded by the magnitude of the state variable. Since it does not internalize the future benefits in terms of parameter precision, it responds linearly and more aggressively than the certainty equivalence rule.

This feature of the myopic rule differs from what one might find when the source of parameter uncertainty lies with the policy multiplier, where the myopic rule tends to be less aggressive than certainty equivalence. The reason is quite intuitive. Uncertainty about the policy multiplier forces the central bank to be cautious about using its policy instrument freely to stabilize inflation. In our case, the analogous explanation is that, with uncertainty in the persistence parameter, the central bank would like to see less variability in the next period's state variable. Under the myopic rule, this can be achieved only if the policy rate responds aggressively to new information.

The incentive to deviate from the certainty equivalence policy diminishes for a central bank that gives more attention to inflation stabilization. In that case, the instrument rate can be set optimally to stabilize inflation without much concern about current and future volatilities in the rate of interest. In the limit of strict inflation targeting optimal policy is not affected by uncertainty in the persistence parameter. Thus there is more tendency to probe for a relatively large discount factor (that is, small discount rate).

We note that it is possible to allow for specification errors in the central bank's econometric model used to estimate the persistence parameter. Moreover, one can make alternative assumptions about how private sector expectations are formed, for example assuming rational expectations on the part of the private sector. But in that case the model becomes much more intractable as private sector expectations have to incorporate the active learning problem of the central bank, which does not have a closed-form solution. An appealing feature of our model is that, even though expectations are initially non-rational, they become rational in the limit as the central bank learns the persistence parameter and the equilibrium solution is identical to that of the perfect knowledge case.

Bibliography

K. Adam. Optimal Monetary Policy in the Presence of Imperfect Common Knowledge. *Center for Financial Studies Working Paper*, 12, 2003.

R.J. Balvers and T.F. Casimano. Inflation Variability and Gradualist Monetary Policy. *The Review of Economic Studies*, 61(4):721–738, 1994.

G. Beck and V. Wieland. Learning and Control in a Changing Economic Environment. *Journal of Economic Dynamics and Control*, 26(9), 2002.

B. Bernanke. Remarks by Governor Ben S. Bernanke Before the Money Marketeers of New York University. *Federal Reserve Board*, February 3, 2003.

G. Bertocchi and M. Spagat. Learning, Experimentation and Monetary Policy. *Journal of Monetary Policy*, 32(1):169–178, 1993.

A. Blinder. *Central Banking in Theory and Practice.* MIT Press, Cambridge,MA, 1998.

W. Brainard. Uncertainty and the Effectiveness of Policy. *American Economic Review*, 57, 1967.

J. Bullard and K. Mitra. Learning about monetary policy rules. *Journal of Monetary Economics*, 49(6):1105–1129, 2002.

G. Chortareas, D. Stasavage, and G. Sterne. Does it Pay to be Transparent? International Evidence from Central Bank Forecasts. *Federal Reserve Bank of St. Louis Review*, 84(4):99–117, 2002.

R. Clarida, J. Gali, and M. Gertler. The Science of Monetary Policy: A New-Keynesian Perspective. *Journal of Economic Literature*, 37:1661–1707, 1999.

T. Cover and J. Thomas. *Elements of Information Theory.* Wiley, New York, 1991.

A. Cukierman. Accountability, Credibility, Transparency and Stabilization Policy in the Eurosystem. In C. Wyplosz, editor, *The Impact of EMU on Europe and the Developing Countries*, pages 40–75. Oxford University Press, Oxford, 2001.

A. Cukierman and A. Meltzer. A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information. *Econometrica*, 54(5):1099–1128, 1986.

J. de Haan, S. Eijffinger, and S. Waller. *The European Central Bank: Centralization, Transparency, and Credibility.* The MIT Press, Cambridge, MA, 2005.

S. Eijffinger and P. Geraats. How Transparent are Central Banks? *Forthcoming in the European Journal of Political Economy.*

S. Eijffinger, M. Hoeberichts, and E. Schaling. Why Money Talks and Wealth Wispers: Monetary Uncertainty and Mistique. *Journal of Money, Credit and Banking*, 32, 2000a.

S. Eijffinger, E. Schaling, and M. Tesfaselassie. Heterogeneous Information about the Term Structure, Least-Squares Learning, and Optimal Rules for Inflation Targeting. *CEPR Discussion Paper 4279, Center for Economic Policy Research*, 2004.

S. Eijffinger, E. Schaling, and W. Verhagen. The term structure of interest rates and inflation forecast targeting. *CEPR Discussion Paper*, 2000b.

M. Ellison. The Learning Cost of Interest Rate Reversals. *CEPR*, Discussion Papers 4135, 2003.

M. Ellison and N. Valla. Learning, Uncertainty and Central Bank Activism in an Economy with Strategic Interactions. *Journal of Monetary Economics*, 48(1): 153–171, 2001.

G. Evans and S. Honkapohja. Adaptive Learning and Monetary Policy Design. *Discussion Paper 19/2002,Bank of Finland*, 2002.

G.W. Evans and S. Honkapohja. *Learning and Expectations in Macroeconomics.* Princeton University Press, 2001.

J. Faust and L. Svensson. Transparency and Credibility: Monetary Policy with Unobservable Goals. *International Economic Review*, 42:369–397, 2001.

P. Geraats. Transparency and Reputation: Should the ECB publish its inflation forecasts? *Paper presented at the conference on Monetary Policy-Making under Uncertainty*, December, 1999.

P. Geraats. Why Adopt Transparency? The Publication of Central Bank Forecasts? *ECB Working Paper 41, European Central Bank*, 2001.

P. Geraats and S. Eijffinger. Does Central Bank Transparency Reduce Interest Rates? In *CEPR-ESSIM Meeting Hosted by the Banco de Espana, Tarragona, Spain, 26-29 May, 2004*, 2004.

H. Gersbach. On the Negative Social Value of Central Banks' Transparency. *Economics of Governance*, 4(2):91–102, 2003.

M. Goodfriend. Using the Term Structure of Interest Rates for Monetary Policy. *Federal Reserve Bank of Richmond Economic Quarterly*, 84(3):13–30, 1998.

C. Goodhart. Central Bankers and Uncertainty. In *Keynes Lecture presented to the British Academy*, 1998.

R. Hall and G. Mankiw. Nominal Income Targeting. *NBER Working Paper 4439, National Bureau of Economic Research*, 1994.

A. Harvey. *Time Series Models*. MIT Press, Cambridge, 1992.

S. Honkapohja and K. Mitra. Performance of Monetary Policy with Internal Central Bank Forecasting. *European Central Bank*, Working paper series 127(00):00, 2002.

H. Jensen. The Optimal Degree of Transparency in Monetary Policymaking: The case of imperfect information about the cost-push shock. Mimeo, University of Copenhagen, 2000.

H. Jensen. Monetary Policy Frameworks and Real Equilibrium Indeterminacy. Mimeo, University of Copenhagen, 2002.

Kenneth Judd. *Numerical Methods in Economics*. MIT Press, 1998.

N. Kiefer and Y. Nyarko. Optimal Control of an Unknown Linear Process with Learning. *International Economic Review*, 1989.

R. King. The New IS-LM Model: Language, Logic, and Limits. *Federal Reserve Bank of Richmond Economic Quarterly*, 86(3):45–103, 2000.

N.G. Mankiw, R. Reis, and J. Wolfers. Disagreement about Inflation Expectations. *NBER Working Paper*, No. 9796, 2002.

B. McCallum. On Non-Uniqueness in Rational Expectations Models. *Journal of Monetary Economics*, 11:139–168, 1983.

B. McCallum. Two Fallacies Concerning Central Bank Independence. *American Economic Review*, 85:207–11, 1995.

B. McCallum and E. Nelson. Timeless Perspective Vs. Discretionary Monetary Policy in Forward-Looking Models. *NBER Working Paper 7915, National Bureau of Economic Research*, 2000.

F. Mishkin. Can Central Bank Transparency Go Too Far. *Paper presented at the Reserve Bank of Australia Conference on The Future of Inflation Targeting*, August 9-10, 2004.

Athanasios Orphanides and V. Wieland. Inflation Zone Targeting. *European Economic Review*, 44(7):1351– 1387, 2000.

Stephen Pollock. Recursive Estimation in Econometrics. *Queen Mary University of London, Department of Economics*, Working Paper 462(00):00, 2002.

C. Prescott. The Multi-Period Control Problem Under Uncertainty. *Econometrica*, 40(6):1043–1058, 1972.

K. Rogoff. The Optimal Degree of Commitment to an Intermediate Monetary Target. *The Quarterly Journal of Economics*, 100:1169–1189, 1985.

K. Rogoff. A Case for Inflation Transparency: A Commentary. *Financial Times*, April 23, 2003.

Christina D. Romer and David H. Romer. Federal Reserve Information and the Behavior of Interest Rates. *American Economic Review*, 90(3):429–457, 2000.

G. Rudebusch and L. Svensson. Eurosystem monetary targeting: Lessons from U.S. data. *European Economic Review*, 46, 2002.

T. Sargent. *Dynamic Macroeconmic Theory.* Harvard University Press, 1987a.

T. Sargent. *Macroeconomic Theory, 2nd ed.* Academic Press, New York, 1987b.

T. Sargent. *Bounded Rationality in Macroeconmics.* Oxford University Press Inc., New York, 1993.

T. Sargent. *The Conquest of American Inflation.* Princeton University Press, New Jersy, 1999.

E. Schaling. The Nonlinear Phillips Curve and Inflation Forecast Targeting: Symmetric Versus Asymmetric Monetary Policy Rules. *Journal of Money, Credit and Banking*, 36(3):361–386, 2004.

C. Shannon. A Mathematical Theory of Communication. *The Bell System Technical Journal*, 27:379–423, 1948.

C. Sims. Stickiness. *Carnegie Rochester Conference Series on Public Policy*, 49: 317–356, 1998.

C. Sims. Implications of Rational Inattention. *Journal of Monetary Economics*, 50 (3):665–690, 2003.

L. Svensson. Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets. *European Economic Review*, 41:1111–1146, 1997.

L. Svensson. Inflation Targeting: Some Extensions. *Scandinavian Journal of Economics*, 101(3):337–361, 1999.

L. Svensson. Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others. *Journal of Economic Perspectives*, 17(4):145–166, 2003a.

L. Svensson. What is Wrong with Taylor Rules? Using Judgment in Monetary POlicy through Targeting Rules. *Journal of Economics Literature*, 41:426–477, 2003b.

L. Svensson. Monetary Policy with Judgment: Forecast Targeting. *International Journal of Central Banking*, 1:1–54, 2005.

J. Tarkka and D. Mayes. The Value of Publishing Official Central Bank Forecasts. *Bank of Finland Discussion Paper*, 22, 1999.

J. B. Taylor. Discretion versus Rules in Practice. *Carnegie-Rochester Series on Public Policy*, 39:195–214, 1993.

V. Wieland. Monetary Policy and Uncertainty about the Natural Unemployment Rate. *Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System*, (22), 1998.

M. Woodford. Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability? In *New Challenges for Monetary Policy*. Federal Reserve Bank of Kansas City, 1999.

M. Woodford. Optimal Monetary Policy Inertia. *NBER Working Paper 7261, National Bureau of Economic Research*, 1999a.

J. Yetman. Probing Potential Output: Monetary Policy, Credibility, and Optimal Learning under Uncertainty. *Mimeo, University of Hong Kong*, 2002.

Summary in Dutch

Dit proefschrift gaat in op twee onderwerpen gerelateerd aan het "ontwerpen" van monetair beleid: communicatie als instrument voor stabilisatiepolitiek enerzijds en optimaal monetair beleid met een lerende centrale bank anderzijds.

In het eerste deel van het proefschrift wordt de rol van communicatie als instrument voor stabilisatiepolitiek bestudeerd (hoofdstukken 2, 3 en 4). Hoewel een aantal belangrijke centrale banken stappen heeft gezet om hun voorspellingen over de toestand van de economie openbaar te maken, geven bestaande theoretische studies gemengde resultaten over de wenselijkheid van openbaarmaking. Hoofdstukken 2 en 3 dragen bij aan de bestaande literatuur (die betrekking heeft op het openbaar maken van schokken in de huidige periode) door in te gaan op de onthulling van toekomstige inflatieschokken. Dit is met name interessant wanneer de verwachtingen van de private sector rationeel zijn (zoals in het microgefundeerde Nieuw-Keynesiaanse raamwerk het geval is).

In hoofdstuk 2 wordt aangetoond dat, in het geval van volledige geloofwaardigheid van de centrale bank, het vooraf openbaar maken van toekomstige schokken niet wenselijk is voor een centrale bank die meerdere doelstellingen heeft dan prijsstabiliteit. Verwachtingen gaan in dit geval namelijk meer fluctueren bij de onthulling van voorspellingen en werken zodoende de huidige stabilisatie-inspanningen tegen. In hoofdstuk 3 is een verandering in de economische conjectuurdoelstelling van de centrale bank mogelijk. Deze doelstelling is onbekend bij het publiek en kan dus de geloofwaardigheid van de centrale bank aantasten. Het blijkt dat in dit geval de wenselijkheid van openbaarmaking niet eenduidig is en afhangt van specifieke aannames met betrekking tot de onbekende conjectuurdoelstelling. De resultaten van hoofdstuk 2 gelden ook wanneer de willekeurige verandering in de conjectuurdoelstelling direct onthuld wordt door de centrale bank (die dus geen prikkel heeft zijn gedrag te veranderen) op het moment dat de schokken plaatsvinden. Anderzijds is openbaarmakingbeleid onschadelijk als de conjectuurdoelstelling indirect afgeleid moet worden van waarneembare beleidsbeslissingen. In dit geval zorgt de sterke afhankelijkheid van de inflatievoorspellingen van de private sector van beleidsacties ervoor dat de centrale bank zich in komende periodes uitsluitend op prijsstabiliteit richt. Doordat de private sector op dit prikkel of "incentive"-effect anticipeert, blijven de inflatieverwachtingen van de private sector stabiel, hetgeen bijdraagt aan de stabilisatie-inspanningen van de centrale bank.

Een gezamenlijk resultaat van hoofdstukken 2 en 3 is dat, in tegenstelling tot het geval van schokken in de huidige periode, er geen inherente wens is om de voorspellingen van toekomstige schokken te bestrijden, aangezien deze schokken geen directe gevolgen hebben voor de huidige inflatie. Dit betekent dat zelfs als het publiek het huidige rentebeleid waarneemt, zij de voorspellingen van de centrale bank niet kan afleiden.

Centrale banken schatten regelmatig de verwachtingen van de markten in. De inschattingen bevatten soms grote fouten, deze worden echter in de meeste bestaande studies over het beleid van openheid genegeerd. Hoofdstuk 4 gaat in op het communiceren van inschattingsfouten in een raamwerk met rationele verwachtingen. Het laat zien dat communicatie ervoor zorgt dat conjectuurstabilisatie verbetert, maar dat inflatiestabiliteit verslechtert omdat inflatieverwachtingen zodanig reageren dat ze de centrale bank bewegen tot een renteaanpassing om zodoende de geaggregeerde vraag te stabiliseren. De bank slaagt er echter niet in om het directe effect van inflatieverwachtingen op de huidige inflatie te verkleinen. Het normatieve gevolg is dat een centrale bank, die voldoende conservatief is, de maatschappelijke welvaart verbetert door zijn inschattingen te communiceren. Tevens wordt aangetoond dat een meer transparante centrale bank het zich kan veroorloven om conservatiever te zijn aangezien de voordelen van meer transparantie in termen van conjectuurstabilisatie groter zijn naarmate de centrale bank conservatiever is.

Het tweede deel van dit proefschrift gaat in op optimaal beleid waarbij een centrale bank leert binnen een raamwerk van "inflation-forecast targeting" dat tevens de termijn structuur van de rente bevat (hoofdstukken 5 en 6). Dit onderwerp is vooral van belang omdat een aantal belangrijke centrale banken een beleid van "inflation-forecast targeting" in gebruik hebben genomen. Een model dat tevens de termijn structuur bevat is uitvoeriger en realistischer dan modellen die dit niet incorporeren, omdat men in het algemeen van mening is dat het effect van monetair beleid loopt via de korte rente, die door de centrale bank gecontroleerd wordt, naar de lange rente, die de geaggregeerde vraag en de inflatie benvloedt. Wat belangrijker is, is dat wordt aangetoond dat beperkte informatie over de verwachtingen van de private sector over de lange rente ervoor zorgt dat het probleem van de centrale bank een probleem wordt met onzekerheid over parameters, namelijk onzekerheid over de mate van persistentie in conjectuur en inflatie. In termen van berekeningen zorgt de manier waarop onvolledige informatie in de vergelijking voor de termijnstructuur wordt gemodelleerd ervoor dat de persistentie-parameter in het lineaire proces onbekend is bij de centrale bank, terwijl de literatuur in het algemeen onzekerheid over een beleidsmultiplier bestudeerd.

Onze interesse ligt vooral in de prestaties van alternatieve renteregels met een passief lerende centrale bank (zekerheidsequivalentie en kortzichtige regels) respectievelijk, en een actief lerende centrale bank (een dynamisch optimale regel). Met name een actief lerende centrale bank is interessant omdat de centrale bank dan rekening houdt met de gelijktijdig optredende problemen van enerzijds het beheersen van de inflatie en anderzijds het schatten (en leren) van de mate van persistentie in de economie.

Hoofdstuk 5 richt zich op het probleem van passief leren met een strikte inflatiedoelstelling ("strict inflation targeting"), waarbij monetair beleid voorspelbare schommelingen in inflatie volledig stabiliseert. Hier wijkt het dynamisch optimale beleid met een lerende centrale bank niet af van zekerheidsequivalent en kortzichtig beleid. Optimaal beleid scheidt zodoende het schatten enerzijds en het beheersen van inflatie anderzijds. Hoofdstuk 6 breidt de analyse van hoofdstuk 5 uit naar het algemene geval van een flexibele inflatiedoelstelling ("flexible inflation targeting") met een actief lerende centrale bank waarbij de bank het effect van huidige beleidskeuzes op zijn mogelijkheden om meer te weten te komen over de onbekende persistentie-parameter internaliseert. Wanneer het beleidsinstrument, de rentevoet, onderdeel is van de nutsfunctie van de centrale bank, wijkt optimaal beleid af van de regels bij passief leren. Bovendien, de noodzaak voor een beleid om meer relatieve variatie in inflatie te genereren hangt af van de toestand van de economie. Wanneer verwacht wordt dat door een onvoorspelbare schok de inflatie sterk afwijkt van de doelstelling, genereert de economie zelf data zodat het beleid niet actief data hoeft te genereren terwijl dit wel het geval is als de economie getroffen wordt door een hele kleine schok. Anderzijds houdt de kortzichtige regel alleen rekening met de extra bron van onzekerheid in de persistentie-parameter, waarvan het effect versterkt wordt door de omvang van de toestandsvariabele. Aangezien de centrale bank de toekomstige voordelen in termen van parameter precisie niet internaliseert, reageert ze lineair en agressiever dan in het geval van de regel met zekerheidsequivalentie.

Dit kenmerk van de kortzichtige regel verschilt van wat men zou kunnen vinden wanneer de bron van parameter onzekerheid bij de beleidsmultiplier ligt, waarbij de kortzichtige regel in het algemeen minder agressief is dan zekerheidsequivalentie. De oorzaak is vrij intutief: onzekerheid over de beleidsmultiplier dwingt de centrale bank om voorzichtiger om te springen met zijn beleidsinstrument om inflatie te stabiliseren. In ons geval is de analoge uitleg dat, met onzekerheid over de persistentie-parameter, de centrale bank liever minder variatie in de toestandsvariabele in de volgende periode ziet. Bij de kortzichtige regel kan dit alleen bereikt worden als het beleid agressief op nieuwe informatie reageert.

De prikkel om van het beleid onder de zekerheidsequivalente regel af te wijken neemt af voor een centrale bank die meer aandacht heeft voor inflatie stabilisatie. In dat geval kan het instrument optimaal ingezet worden om inflatie te stabiliseren zonder dat de bank zich veel zorgen hoeft te maken over de huidige en toekomstige volatiliteit in de rentevoet. In het extreme geval van een strikte inflatiedoelstelling ("strict inflation targeting") wordt optimaal beleid niet benvloedt door onzekerheid over de persistentie-parameter. Daarnaast is er een grotere neiging om te experimenteren wanneer de tijdsvoorkeurvoet relatief hoog is (relatief lage discontovoet).

Wel dient opgemerkt te worden dat het mogelijk is om specificatiefouten in het econometrische model, dat de centrale bank gebruikt om de persistentieparameter te schatten, toe te staan. Daarnaast kunnen alternatieve aannames gemaakt worden over hoe de private sector tot haar verwachtingen komt, bijvoorbeeld door rationele verwachtingen aan de kant van de private sector te veronderstellen. Echter, in dat geval is het model veel moeilijker te analyseren omdat de verwachtingen van de private sector het probleem van actief leren van de centrale bank moeten incorporeren, waardoor het model geen gemakkelijk hanteerbare oplossing heeft. Een aantrekkelijk kenmerk van ons model is dat verwachtingen, ondanks dat ze initieel niet rationeel zijn, in de limiet rationeel worden omdat de centrale bank te weten komt wat de persistentieparameter is en de speltheoretische Stackelberg evenwicht identiek is aan de oplossing in het geval van volledige informatie.