Convergence and the Welfare Gains of Capital Mobility in a Dynamic Dixit-Stiglitz World

by

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Abstract
The effects of capital mobility on welfare and the speed of adjustment is studied in a two-country growth model. Research and development (R&D) allows monopolistic firms to improve their productivity level. National and international knowledge spillovers affects the returns to R&D. The two countries considered differ only with respect to the initial productivity level. The country with lowest productivity level gradually catches up with the leading country. There is complete convergence in the long run if there is no capital mobility. Under perfect capital mobility, countries end up with equal long-run productivity levels, but permanent differences in consumption. The speed of convergence is larger with perfect capital mobility than with balanced trade. The difference increases with substitution between product varieties and the rate of intertemporal substitution. Capital mobility harms (benefits) the leader (lagging) country if domestic spillovers are more important than international spillovers.

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1. Introduction

The Dixit-Stiglitz (1977) framework has become a powerful tool to analyse monopolistic competition and market structure in general equilibrium models, in particular in (international) macroeconomics and trade theory. While the Dixit-Stiglitz model was originally phrased in a static context, its importance is as least as large in a dynamic context. In R&D based-models of economic growth, aggregate economic growth is explained from the incentives private firms have to invest in research and development (R&D) [seminal contributions are Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)]. R&D generates blueprints for new product varieties, new production processes, or improved product quality. Firms are only willing to invest in R&D if they can reap some profits. The Dixit-Stiglitz framework has proved to be a most appealing and elegant way of modelling the conditions under which firms can realise profits, which both provide the incentives to innovate as well as the means to pay for the cost of innovation. The key assumption in the Dixit-Stiglitz framework is that each firm produces a good that is differentiated from other goods in the market. The elasticity of substitution in utility between product varieties from different producers captures the degree of product differentiation. This parameter determines the firm's price elasticity and hence the degree of market power it can exert. Because of this link between market conditions and investment incentives, international differences in market conditions determine international differences in growth. A central question in growth theory is what allows countries with relatively low income to grow at a relatively high rate of growth so that they can catch up with the richer countries. Under what conditions do income levels in rich and poor countries converge? How fast is the speed of convergence?

In this paper it is studied how rates of convergence are affected by market conditions as captured by firm's market power. In particular, it is examined whether it becomes easier or harder for poor countries to catch up with rich countries if the degree of product differentiation among firms, and hence market power, is high. The key ingredient of the Dixit-Stiglitz framework is thus linked to the issue of convergence. The role product differentiation plays turns out to depend on whether international capital mobility is assumed or not. I explore the implications not only for the rate of convergence, but also for welfare. It is shown that although a higher degree of substitution (and hence lower market power) causes the rate of convergence to increase under capital mobility, it also implies a larger welfare gap between rich and poor countries.

The model in this paper is a two-country model of endogenous growth based on in-house R&D. There is a given number of firms. By spending on R&D, each of them invests in firm-specific knowledge, which determines the their productivity level. The cost of R&D depends on the firm-specific knowledge stock, as well as on national and international knowledge average knowledge stock. The latter two determinants capture intertemporal knowledge spillovers and result in the familiar research externality, which makes firms invest too little in R&D. The two countries considered differ only with respect to the initial productivity level.

The main results are as follows. There is complete convergence in the long run if there is no capital mobility. Under perfect capital mobility, countries converge to equal long-run productivity levels, but permanent differences in consumption remain. The speed of convergence is larger with perfect capital mobility than with balanced trade. The difference increases with substitution between product varieties. Capital mobility harms (benefits) the leader (lagging) country if domestic spillovers are more important than international spillovers.

The topic of convergence has received a lot of attention in the growth literature. Two strands stand out in this literature. The first strand focusses on growth driven by (human) capital accumulation with one final good produced only and perfect competition. A distinction should be made between closed and open economies. Closed economy models predict convergence between rich and poor countries as long as there is diminishing returns with respect to reproducible capital. Poor countries have low levels of capital and realize high rate of return to investment so that they grow relatively fast (Barro and Sala-i-Martin 1995). If there is constant returns to capital, growth differentials are persistent and there is no convergence (the AK-model, see Rebelo 1991). In the open economy setting, capital is assumed to be mobile. In the simplest version, convergence in productivity levels is immediate since capital flows to the poor country that has accumulated less capital and realizes a high (ex-ante) rate of return. However, this is at odds with empirical research that finds a limited rate of convergence, of about 2 per cent only (Temple, 1999). The introduction of adjustment costs or borrowing constraints makes the rate of convergence limited again (see Turnovsky and Sen, 1995; Barro, Mankiw and Sala-i-Martin 1995). While *financial* capital is internationally mobile, physical (and human) capital have to be accumulated in the country where they are used. Investment in the domestic capital stock takes time and is costly in terms of foregone consumption, even though borrowing from abroad is possible. In the present paper, productive capital stocks (firm specific knowledge) is home-grown, too, which explains why convergence takes time.

The second strand of literature focusses on growth driven by R&D with monopolistic competition and differentiated goods in the spirit of Dixit-Stiglitz. It is found that spillovers of knowledge between countries are important for convergence. If there are no such spillovers and if no inputs in production are traded, countries that start at different productivity levels diverge (Grossman and Helpman, 1991 chapter 8; Fung and Ishikawa, 1992; Feenstra, 1996). With international spillovers, most analyses find that international growth rates converge in the long run (Aghion and Howitt, 1998). In the literature both goods trade and international capital mobility are considered. However, so far it has not been explored how convergence is affected by export demand conditions (as captured by the export price elasticity, which is again related to the elasticity of substitution in the Dixit-Stiglitz framework) and how it is affected by the international capital mobility. The present paper aims at filling this gap.

This chapter is also related to the literature on international interdependency in international macroeconomics, both with respect to the type of models and the focus on how welfare levels of countries are interrelated (see the survey by Lane 2001). While this literature focusses on monetary shocks, this chapter focusses on productivity. It is investigated how initial cross-country differences in productivity affect international capital flows and welfare differences. The model can be interpreted as an analysis how a permanent productivity shock in one country spill over to other countries, how international capital mobility affects the propagation of the shock, and how endogenous R&D and international knowledge spillovers affect the persistence of shocks over time.

The paper is organised as follows. The model is presented in Section 2. In Section 3, the model is reduced to two system of differential equations, one system for the model with capital mobility, the other system for the model without capital mobility. It is shown that the two countries converge in the long run. Section 4 studies how fast the countries converge. Section 5 focusses on welfare. Section 6 concludes. The appendix contains proofs of propositions.

2. A two-country endogenous growth model

2.1. Structure of the model

There are two countries that are characterized by identical preferences, technological opportunities, and primary factor endowments. However, one country, indexed by superscript A, starts at a more *A*dvanced productivity level than country B (also referred to as the *B*ackward country). The central question is whether the two country converge in terms of productivity levels, starting from this initial asymmetry, how fast they converge, and how welfare in the two countries evolves over time.

Each country has one primary factor of production in fixed supply (labour), which is allocated over two activities, production and research. Produced goods are differentiated and each variety is produced by a single monopolistic firm. These firms control and accumulate firm-specific knowledge (as in Smulders and Van de Klundert, 1995). Within each country, there is a continuum of symmetric firms on the unit interval. This allows us to save on notation by formulating the model for a single representative firm. All goods are traded in international markets at zero transport costs.[1]

The structural relationships are given in Table A. Countries are denoted by superscript i = A, B (and if necessary also by superscript j for the other country). Each line in the table represents two equations, one for each country.

Labour productivity in production is denoted by h as appears from eqs. (A.1), relating output X to input L. Firms have an opportunity to increase labour productivity h by performing R&D according to eqs. (A.2). Knowledge can be increased by allocating labour (R) to R&D. Productivity in R&D depends on a fixed coefficient ξ and three sources of knowledge (h^i, $\bar{h}^i$ and $\bar{h}^j$). First, firms build upon specific knowledge accumulated in the past. Second, all firms benefit from knowledge spillovers emanating from other firms in their country. Third, there are knowledge spillovers from abroad. Knowledge spillovers relate to the average level of knowledge in the different economies ($\bar{h}$). Productivity levels may differ across countries, but are identical across firms within a country. For this reason average knowledge levels are equal to the knowledge levels of firms in each country ($\bar{h}^i = h^i$).

Intertemporal preferences in the consumption index C are given in eqs. (A.3). Infinitely-lived households apply a constant utility discount rate ϑ. The relative rate of risk aversion is denoted by ρ so that the elasticity of intertemporal substitution equals $1/\rho$. The consumption index (C) combines consumption of domestically produced varieties (D) and imported varieties (M) by way of a CES sub-utility function, with an elasticity of substitution denoted by $\varepsilon > 1$, eqs. (A.4).

Goods markets clear, see eqs. (A.5). The supply of labour is normalized at one and equals total demand for labour, see eqs. (A.6).

[1] In Van de Klundert and Smulders (2001), the implications of non-traded goods for growth and convergence are considered).

Table A Structural relationships

Technology	$X^i = h^i L^i$	(A.1)
	$\dot{h}^i = \xi (h^i)^{1-\alpha_h-\alpha_f} (\bar{h}^i)^{\alpha_h} (\bar{h}^j)^{\alpha_f} R^i$	(A.2)
Preferences	$U_0^i = \frac{1}{1-\rho} \int_0^{\infty} (C_t^i)^{1-\rho} \, e^{-\vartheta t} \, dt$	(A.3)
	$C^i = \left[(D^i)^{(\varepsilon-1)/\varepsilon} + (M^i)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \quad \varepsilon > 1$	(A.4)
Market clearing	$X^i = D^i + M^j$	(A.5)
	$L^i + R^i = 1$	(A.6)

Endogenous variables:

X	output
h	labour productivity
L	labour in production
R	labour in research
D	consumption domestically produced goods
M	imports
C	aggregate consumption index

Parameters:

α_f	foreign spillover parameter
α_h	domestic spillover parameter
ξ	research productivity parameter
ϑ	utility discount rate
$1/\rho$	elast. intertemporal substitution
ε	elast. intratemporal substitution

All equations apply to i, j = A, B and $j \neq i$.

2.2. Consumer and firm behaviour

The behavioural equations of our model are summarized in Table B. Consumers maximize intertemporal utility over an infinite horizon. The decision problem consists of two stages subject to the usual budget constraints. In the first stage, each consumer decides on the path of aggregate consumption over time. This gives rise to the familiar Ramsey rule, shown in eqs. (B.1). The growth rate of consumption equals the difference between the real consumption rate of interest and the pure rate of time preference, multiplied by the elasticity of intertemporal substitution. In the second stage consumers split total per period consumption spending over domestically produced varieties, eqs. (B.2) and foreign varieties, eqs. (B.3). The price elasticity of demand is equal to ε in all cases considered. Eqs. (B.4) defines the price index of aggregate consumption. Since there are neither transport costs nor international differences in preferences, this index is the same in both countries. By choosing the composite consumption good as the numeraire, we can set the aggregate consumption price equal to one.

Producers maximize the value of firm over an infinite horizon. Each firm faces a downward sloped total demand function for its products as appears from eqs. (B.2) and (B.3). Profit maximization implies that firms set a mark-up over (marginal) cost equal to $\varepsilon/(\varepsilon-1)$, as in eqs. (B.5). Labour demand for R&D follows from setting marginal revenue ($\xi K p_h$) equal to marginal cost (w), eqs. (B.6). The shadow price of firm-specific knowledge p_h is introduced as a Lagrangian multiplier in the maximization procedure.[2] Firms face a trade-off with respect to investing in specific knowledge as appears from the arbitrage conditions (B.7). These conditions say that investing an amount of money equal to p_h in the capital market (the RHS of B.7) should yield the same revenue as investing that same amount of money in knowledge creation. The latter raises labour productivity in the production sector and hence revenue in this sector (first term on the LHS of B.7), it raises also the knowledge base in R&D (second term) and it yields a capital gain (last term).

Finally, eqs. (B.8) imply that domestic net savings are invested in net foreign assets (A). Domestic savings are the sum of the trade balance (in parentheses) and interest receipts on foreign assets. Under perfect capital mobility the rate of interest is uniform across countries ($r^A = r^B$). At the other extreme there is the case of balanced trade or zero mobility implying $A = 0$. Both regimes with respect to the balance of payments will be analysed.

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[2] The maximization problem of firm k in country i can be represented by the following Hamiltonian $H^k = p^k(X^k;\cdot)X^k - w^i(X^k/h^k + R^k) + p_h^k\left[\xi(h^k)^{1-\alpha_h-\alpha_f}(\bar{h}^i)^{\alpha_h}(\bar{h}^j)^{\alpha_f}R^k\right]$, where $p^k(\cdot)$ is the firm's demand function, see (B.2), $X/h = L$ is labour employed in production, see (A.1), the term in square brackets is firm-specific knowledge accumulation $\dot{h}$, see (A.2), and p_h is the co-state variable. The firm's instruments are X^k and R^k and it controls state variable h^k.

Table B Behavioural relationships

Consumer behaviour	$\dot{C}^i / C^i = (1/\rho)\left(r^i - \dot{p}_c^i / p_c^i - \vartheta\right)$	(B.1)
	$D^i = C^i \left(p^i / p_c\right)^{-\varepsilon}$	(B.2)
	$M^i = C^i \left(p^j / p_c\right)^{-\varepsilon}$	(B.3)
where	$p_c = \left[(p^i)^{1-\varepsilon} + (p^j)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} = 1$	(B.4)
Producers behaviour	$p^i = \dfrac{\varepsilon}{\varepsilon - 1} \dfrac{w^i}{h^i}$	(B.5)
	$p_h^i = \dfrac{w^i}{\xi K^i}$	(B.6)
	$p^i \left(\dfrac{\varepsilon - 1}{\varepsilon}\right) L^i + \xi(1 - \alpha_h - \alpha_f)\left(\dfrac{K^i}{h^i}\right) p_h^i R^i + \dot{p}_h^i = r^i p_h^i$	(B.7)
where	$K^i \equiv (h^i)^{1-\alpha_h-\alpha_f} (\bar{h}^i)^{\alpha_h} (\bar{h}^j)^{\alpha_f}$	
Balance of payments	$X^i p^i - C^i p_c + r^i A^i = \dot{A}^i$	(B.8)
Asset market equilibrium	$A^A + A^B = 0$	(B.9)

Symbols

A	net foreign assets	p_c	price index consumption
r	nominal interest rate	p	output price
w	wage rate	p_h	firm's shadow price of knowledge

All equations apply to i, j = A, B and $j \neq i$.

2.3. Semi-reduced model

Table C reduces the model to five key equations. In deriving the equations, it is taken into account that all firms within a country have the same productivity level, $h^i = \overline{h^i}$. The growth rates of h and C are denotes by g and g_C respectively; $a = A/h$ denotes net foreign assets relative to productivity.

Equation (C.1), which is derived from (B.1) and (B.4), restates the Ramsey rule. It represents the relationship between consumption growth and the required rate of return on households' savings. Equation (C.2) combines (B.6) and (B.7). The equation represents the rate of return that firms can maximally pay to households. Equation (C.3) represents labour market equilibrium. It states that the amount of labour not allocated to production, results into productivity growth. The productivity in research depends on the knowledge gap h^i/h^j. A low stock of knowledge relative to the other country induces large spillovers and allows the country to grow faster at a given amount of labour allocated to research. Balance of payments equilibrium is represented by equation (C.4) which combines (A.1) and (B.8). Goods market equilibrium is given by (C.5) which combines (A.1), (A.5) and (B.2)-(B.3). Since preferences are homothetic and prices are the same in both countries, the ratio of consumption of goods produced by A relative to those produced by B is the same in both countries and equals the ratio of A's output relative to B's output.

If the two countries are completely symmetric -- that is, if they have equal productivity levels ($h^A = h^B$), the same allocation of labour, and no foreign assets nor debt ($a = 0$) -- a balanced growth path arises. On the balanced growth path, consumption and output grow at a common growth rate which is endogenous and given by:[3]

$$g_C^i \; (\equiv \dot{C}^i/C^i) \; = \; g^i \; (\equiv \dot{h}^i/h^i) \; = \; \frac{\xi - \vartheta}{\rho + \alpha_h + \alpha_f}, \qquad i = \text{A, B} \tag{1}$$

As usual, growth falls with discount rates, risk aversion, and spillover parameters, but increases with the productivity of R&D. For future use, it is also useful to note that in the steady state we have:

$$\xi L \; = \; \vartheta \; + \; (\rho - 1 + \alpha_h + \alpha_f) g \tag{2}$$

$$r \; = \; \vartheta \; + \; \rho g \tag{3}$$

[3]We restrict the analysis to interior solutions with $g > 0$, which requires sufficiently high productivity in research, $\xi > \vartheta$. If $\xi \leq \vartheta$, no growth occurs in the steady state. Outside the steady state, some convergence in productivity levels will take place if the two country start with a large initial productivity gap, but the more ϑ exceeds ξ, the larger the long-run productivity gap remains.

To analyse the dynamics of the model, we log-linearize around the symmetric steady state in which both countries are identical. Linearized variables are denoted by tildes, $\tilde{x} \equiv$ d ln$x$ = dx/x. We solve for relative variables, that is, ratios of country A's to country B's variables, denoted by superscipt R, $\tilde{x}^R \equiv \tilde{x}^A - \tilde{x}^B = d(x^A/x^B)$. Hence tilded variables superscripted R describe how the variable in country A deviates from that in country B. The only exception is variable $\tilde{a}$ which relates to absolute difference from the steady state, $\tilde{a} \equiv da$. We make this exception because in the symmetric equilibrium $a = A/h = 0$. Table D directly follows from straightforward linearization of the relations in Table C.

In a similar way, the model can be solved for country summations ($\tilde{x}^S \equiv \tilde{x}^A + \tilde{x}^B$ for any variable x), which allows us to analyse the dynamics of the "integrated world economy". Before doing so, the model has to be rewritten model in terms of stationary variables. In particular, we first rewrite the model in Table C such that C/h shows up as a single variable, instead of C and h as separate variables. The model in stationary variables for the integrated economy then has a very simple reduced form:

$$\rho \dot{\tilde{L}}^S = (\xi L)(\rho + \alpha_h + \alpha_f)\tilde{L}^S \tag{4}$$

This single differential equation in $\tilde{L}^S = \tilde{L}^A + \tilde{L}^B$ applies both under perfect capital mobility and under balanced trade. The differential equation is unstable, so that $\tilde{L}^S$ jumps immediately to the steady state. The allocation of labour in the integrated world economy remains unchanged over the entire transition ($\tilde{L}_t^S = 0 \ \forall t$). It implies that all stationary variables in the integrated economy are time-invariant. Hence, $\tilde{g}^S = 0$, so that the knowledge stock of the integrated world economy does not change over time and the summation of knowledge levels must be equal to the summation of the initial deviations from the symmetrical steady state level: $\tilde{h}_t^S = \tilde{h}_0^S \ \forall t$.[4] Also the stationary variable C/h is time-invariant in the integrated world economy, so $\tilde{C}_t^S - \tilde{h}_t^S = 0$. Hence, the results for the integrated world economy can be summarized as:

$$\tilde{L}_t^A + \tilde{L}_t^B = 0 \tag{5}$$

$$\tilde{C}_t^A + \tilde{C}_t^B = \tilde{h}_0^A + \tilde{h}_0^B \tag{6}$$

[4] The solutions for country variables ($\tilde{x}^i$) follow from combining $\tilde{x}^R$ and $\tilde{x}^S$. For instance, $\tilde{L}_t^A = \tilde{L}_t^R/2 = -\tilde{L}_t^B$, and similar for all other stationary variables. For the predetermined knowledge levels, the following holds: $\tilde{h}_t^A = (\tilde{h}_0^S + \tilde{h}_t^R)/2$; $\tilde{h}_t^B = (\tilde{h}_0^S - \tilde{h}_t^R)/2$.

The integrated world economy behaves as a closed economy on a balanced growth path, irrespective of the level of the world knowledge stock and its distribution of over the two countries. How the two countries behave separately depends on whether there is international capital mobility or not and is analysed in the next section.

Table C Key relationships

Ramsey rule	$\rho\, g_C^{\,i} \;=\; r^{\,i} \;-\; \vartheta$	(C.1)
Investment decision	$\xi\left(h^{\,i}/h^{\,j}\right)^{-\alpha_f} L^{i} \;+\; (1-\alpha_h)\hat{h}^{\,i} \;-\; \alpha_f \hat{h}^{\,j} \;+\; \hat{p}^{\,i} \;=\; r^{\,i}$	(C.2)
Labour market equilibrium	$g^{\,i} \;=\; \xi\left(h^{\,i}/h^{\,j}\right)^{-\alpha_f}(1-L^{\,i})$	(C.3)
Balance of payments	$p^{\,i}L^{\,i} \;-\; C^{\,i}/h^{\,i} \;=\; \dot{a}^{\,i} \;-\; (r-g^{\,i})\,a^{\,i}$	(C.4)
Goods market equilibrium	$\dfrac{p^{\,i}}{p^{j}} \;=\; \left(\dfrac{h^{i}L^{i}}{h^{j}L^{j}}\right)^{-1/\varepsilon}$	(C.5)

Notation: $g_C \equiv \hat{C}$ consumption growth; $g \equiv \hat{h}$ productivity growth; $a \equiv A/h$ net foreign assets, scaled by knowledge stock. In all equations j = A, B; i = A, B and $j \neq i$.

Table D Linearized model: country differences

Ramsey rule	$\rho\, \dot{\tilde{C}}^{R} \;=\; r\tilde{r}^{\,R}$	(D.1)
Investment decision	$(\xi L)\left[\tilde{L}^{\,R} \;-\; (2\alpha_f)\,\tilde{h}^{\,R}\right] \;+\; (1-\alpha_h+\alpha_f)\,\dot{\tilde{h}}^{R} \;+\; \dot{\tilde{p}}^{R} \;=\; r\tilde{r}^{\,R}$	(D.2)
Labour market equilibrium	$\dot{\tilde{h}}^{R} \;=\; -(2\alpha_f)\tilde{h}^{\,R} \;-\; (\xi L)\tilde{L}^{R}$	(D.3)
Balance of payments	$L\left(\tilde{p}^{\,R}+\tilde{L}^{\,R}+\tilde{h}^{\,R}-\tilde{C}^{\,R}\right) \;+\; 2(r-g)\tilde{a}^{\,A} \;=\; 2\dot{\tilde{a}}^{A}$	(D.4)
Goods market equilibrium	$\tilde{p}^{R} \;=\; -\dfrac{1}{\varepsilon}\,(\tilde{h}^{R} \;+\; \tilde{L}^{R})$	(D.5)

3. Balanced trade versus capital mobility

From table D, a two-dimensional phase diagram can be derived to find the rate of convergence in productivity. This will be done separately for the regime of perfect capital mobility and that of balanced trade.

3.1 Balanced Trade

In the absence of international capital markets, total production equals total consumption in each country and net foreign assets positions are zero, $\tilde{a} = 0$. From (D.4) and (D.5), national production and consumption are directly linked according to:

$$\tilde{C}^R = \frac{\varepsilon - 1}{\varepsilon} (\tilde{L}^R + \tilde{h}^R) \tag{7}$$

The reduced-form model in relative variables can be compressed to a system of two differential equation in $\tilde{h}^R$ and $\tilde{L}^R$. The result can be presented in matrix notation as:

$$\begin{bmatrix} \dot{\tilde{h}}^R \\ \dot{\tilde{L}}^R \end{bmatrix} = \begin{bmatrix} -(2\alpha_f)g & -(\xi L) \\ -(2\alpha_f)\Omega_{BT} & (\xi L)\Phi_{BT} \end{bmatrix} \cdot \begin{bmatrix} \tilde{h}^R \\ \tilde{L}^R \end{bmatrix} \tag{8}$$

where

$$\Omega_{BT} = \frac{\varepsilon(\vartheta + 2\alpha_f g) + (\rho - 1)g}{\rho(\varepsilon - 1) + 1}, \tag{9}$$

$$\Phi_{BT} = 1 + \frac{\varepsilon(\alpha_h - \alpha_f)}{\rho(\varepsilon - 1) + 1}. \tag{10}$$

The determinant of the matrix in (8) is negative.[5] Therefore, the system of differential equations is saddle-point stable. The corresponding phase diagram is drawn in Figure 1. As appears from eqs. (8) the $\dot{\tilde{h}}^R = 0$ locus slopes downward. The figure depicts a upward sloping $\dot{\tilde{L}}^R = 0$ locus, which applies under realistic parameter assumptions (e.g. $\rho > 1$ and $\alpha_h > \alpha_f$). The stable arm of the saddle path is indicated by the broken line. Its slope is unambiguously positive. Unequal productivity levels give rise to transitional dynamics. For any $\tilde{h}^R \neq 0$, the system moves along to the stable arm and converges to a symmetric

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[5] The determinant equals
$$-(2\alpha_f)(\xi L)(\Phi_{BT} g + \Omega_{BT}) = -(2\alpha_f)(\xi L)\varepsilon[\vartheta + (\rho + \alpha_h + \alpha_f)g]/[\rho(\varepsilon - 1) + 1] < 0.$$

equilibrium in the long run. Suppose $\tilde{h}_0^R > 0$ so that country B lags behind A. During the process of convergence, the leading economy employs more labour in production than the lagging country. The lagging country allocates relatively more labour to R&D, which boosts growth.[6] In the long run there is complete catching up, $\tilde{h}_\infty^R = \tilde{L}_\infty^R = 0$, and each country's productivity expands at the same rate, given by (1).

Insert Figure 1

By standard procedures, we can find analytical solutions for the linearized model. Let λ_{BT} be the absolute value of the negative root of the matrix in (8). This parameter is the adjustment speed that governs the dynamics of all variables in the case of balanced trade. We find:

$$\tilde{L}_\infty^R = \tilde{h}_\infty^R = 0 \ , \quad \tilde{h}_0^R \text{ given,} \tag{11}$$

$$\tilde{L}_t^R = \left(\frac{\lambda_{BT} - (2\alpha_f)g}{\xi L} \right) \tilde{h}_t^R \tag{12}$$

Consumption follows from (7) and (12):

$$\tilde{C}_t^R = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\lambda_{BT} + \xi L - (2\alpha_f)g}{\xi L} \right) \tilde{h}_t^R \tag{13}$$

3.2. Capital mobility

With perfect capital mobility, rates of return are equalized, $\tilde{r}^R = 0$. Equation (D.1) reveals that relative consumption levels are constant over time. Formally, relative consumption follows from integration of (D.1), which gives:

$$\tilde{C}^R = \hat{v}, \tag{14}$$

where $\tilde{v}$ is the constant of integration, which can be solved from the initial conditions of the

[6] It may seem unrealistic that the poor country undertakes more R&D than the rich country. Note, however, that R&D in the model should more broadly interpreted than merely patent development. It encompasses all activities that firms undertake to improve productivity and quality, including for example imitation and reverse engineering.

dynamic model. From table D, with (D.1) replaced by (14), the model can be reduced to the following system of three differential equations:

$$\dot{\tilde{a}}^A = (r-g)\tilde{a}^A + (L/2)\left(\frac{\varepsilon-1}{\varepsilon}\right)(\tilde{h}^R + \tilde{L}^R) - (L/2)\tilde{v} \tag{15}$$

$$\begin{bmatrix} \dot{\tilde{h}}^R \\ \dot{\tilde{L}}^R \end{bmatrix} = \begin{bmatrix} -(2\alpha_f)g & -(\xi L) \\ -(2\alpha_f)\Omega_{CM} & (\xi L)\Phi_{CM} \end{bmatrix} \cdot \begin{bmatrix} \tilde{h}^R \\ \tilde{L}^R \end{bmatrix} \tag{16}$$

where

$$\Omega_{CM} = \varepsilon(\vartheta + 2\alpha_f g) + (\varepsilon\rho - 1)g, \tag{17}$$

$$\Phi_{CM} = 1 + \varepsilon(\alpha_h - \alpha_f). \tag{18}$$

Since both assets and goods are perfectly mobile internationally, allocation of production over the two countries can be separated from the allocation of wealth and consumption. The system in (16) represents the allocation of production, which can be solved for independently from (15), which represents the allocation of consumption. Note that the structure of (16) is similar to the case of balanced trade, see (8). This gives rise to a similar phase diagram and a similar time pattern for relative productivity and labour allocation.[7] Also (12) and (13) apply after replacing λ_{BT} by λ_{CM}, where λ_{CM} be the absolute value of the negative root of the matrix in (16), which is the adjustment speed that governs the dynamics of all variables in the case of capital mobility.

To solve for relative consumption, we use (15) and the boundary values. Substituting the long run results for *L* and *h* into (15) we can solve for the long-run asset position:

$$\tilde{a}^A_\infty = \frac{L/2}{r-g}\tilde{v} \tag{19}$$

Since assets are predetermined, we have:

$$\tilde{a}^A_0 = 0 \tag{20}$$

Since λ_{CM} represents the adjustment speed of the economy, we may write the change of any variable as proportional to the gap between its current value and its long run value. For assets we thus have:

---

[7] Saddlepoint stability again applies, since the determinant equals $-(2\alpha_f)(\xi L)(\Phi_{CM}g + \Omega_{CM}) = -(2\alpha_f)(\xi L)\ \varepsilon[\vartheta + (\rho + \alpha_h + \alpha_f)g] < 0$.

$$\dot{\tilde{a}}_t^A = \lambda_{CM}(\tilde{a}_\infty^A - \tilde{a}_t^A) \tag{21}$$

Substituting the last three results into (15) for $t = 0$, we find:

$$\tilde{v} = \left(\frac{r-g}{r-g+\lambda_{CM}}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)(\tilde{L}_0^R + \tilde{h}_0^R) \tag{22}$$

Relative consumption at all $t > 0$ follows from (14), (22) and (11):

$$\tilde{C}_t^R = \left(\frac{r-g}{r-g+\lambda_{CM}}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\lambda_{CM}+\xi L-(2\alpha_f)g}{\xi L}\right)\tilde{h}_0^R \tag{23}$$

3.3. Consumption and productivity over time

Figure 2 depicts the evolution of consumption and productivity when country A starts at a productivity level ahead that of country B ($\tilde{h}_0^R > 0$). Without capital mobility, the advanced country consumes more than the lagging one, but consumption levels converge over time. Also with capital mobility, the advanced country consumes more than the lagging one, but now this is a permanent situation. Country A maintains higher levels of consumption than country B despite the fact that productivity levels converge. The reason is that country A accumulates foreign assets. Country B uses growing export revenues to service its foreign debt. Note that capital mobility allows both countries to smooth consumption over time.

Insert Figure 2

4. How does monopolistic competition affect convergence?

To examine how quickly productivity levels in the two countries converge, we study the properties of the stable roots of the planar system in (8) and (16), which capture the dynamics under balanced trade and capital mobility respectively. These roots can be written as

$$\lambda_k = [(T_k^2 - 4D_k)^{1/2} - T_k]/2 > 0, \tag{24}$$

where T_k and $D_k < 0$ be the trace and determinant, respectively, of the system in (8) for k = BT (and of that in (16) for k = CM). From this expression we can derive the following (proofs are in the appendix):

Proposition #1: If $\rho \geq 1$, the speed of adjustment (rate of convergence) is faster with capital mobility than with balanced trade.

Remark: In all numerical experiments I tried, I found the same result for $0 < \rho < 1$.

Consumers prefer to smooth consumption. Capital mobility allows a country that is lagging behind to close the productivity gap at higher speed without restraining consumption a lot, by running current account deficits. The lagging country closes the gap with the leading country, since it realizes (ex ante) a higher rate of return and invests more than the leading country. If capital is not mobile internationally, investments have to be financed fully by domestic savings, which is costly for domestic consumers who want to smooth consumption. In this case, the supply of savings is less elastic, which slows down the process of catching up relative to the case in which foreign supply of capital finances catching up.

The adjustment speed with capital mobility equals that without capital mobility only if $\rho = 0$ (since then the matrices in (8) and (16) are identical).[8] In this extreme case, utility is linear in consumption and consumption smoothing no longer plays a role. Supply of savings is perfectly elastic, independent of whether capital mobility applies or not.

The following proposition states how convergence rates change with the elasticity of substitution ε, which can be considered as the monopolistic competition parameter:

Proposition #2: With capital mobility, the rate of convergence increases in ε. Without capital mobility, the rate of convergence decreases (increases) in ε if $\rho > 1$ ($\rho < 1$).

The intuition behind this result is as follows. A larger price elasticity (ε) implies a smaller terms of trade loss from an increase in national output. Hence, the larger the price elasticity, the larger the future gains in terms of export revenues can be reaped by the lagging country when it gradually closes its productivity gap with its trading partner. In other words, a higher price elasticity implies a higher (ex ante) rate of return for a given productivity gap. With capital mobility, higher rates of return attract more foreign capital and thus speed up

[8] We also find $\lambda_{CM} = \lambda_{BT}$ if $\varepsilon = 1$. This is however a degenerate case since the monopoly price is no longer defined. A unitary price elasticity makes the monopolist's revenues independent of productivity levels which removes any incentive to innovation.

convergence. Without capital mobility, a higher rate of return has ambiguous effects on investment since income and substitution effects work in opposite directions. In this case, domestic investment equals domestic savings so that domestic income and substitution effects determine investment. If $\rho > 1$, the intertemporal rate of substitution is low and a rise in the rate of return makes consumers to save less, which slows down accumulation and convergence. In the opposite case of high intertemporal substitution, consumers start saving more and speed up convergence.

Table E Calibration and sensitivity analysis

	ε (5)	ρ (5/3)	ϑ (0.025)	α_f (0.15)	α_h (0.4)	g (0.018)
λ_{CM} (3.4%)	0.4	0.5	0.4	1.1	−0.4	0.6
λ_{BT} (1.0%)	−0.1	−0.2	0.4	1.0	−0.1	0.6
$\tilde{U}^R_{0_{CM}} / \tilde{h}^R_0$ (0.67)	0.2	0.1	0.1	−0.1	−0.0	−0.1
$\tilde{U}^A_{0_{CM-BT}} / \tilde{h}^R_0$ (−0.01)	0.7	−0.3	−0.6	−0.3	1.1	0.6

Values in parentheses: benchmark parameters and results.
Numbers in table: elasticities evaluated at benchmark parameter set.

Table E presents a calibration exercise and sensitivity analysis. The top row presents the benchmark set of parameters. There are six degrees of freedom which I use to calibrate the model to generally accepted outcomes. I fix the growth rate g on its postwar US average. In every numerical experiment, I adjust productivity parameter ξ according to (1) to ensure the selected growth rate. Time preference ϑ and intertemporal substitution $1/\rho$ are not far from accepted views. The price elasticity ε is chosen such that a reasonable profit margin $1/(\varepsilon-1)$ of 25 percent results, which is consistent with a broad range of studies (Obstfeld and

Rogoff, 2000).[9] The benchmark parameters give adjustment speeds under the two alternative regimes that are close to the famous two percent, found in almost all empirical studies (see Sala-i-Martin 1996).[10] I report the results for adjustment speeds in parentheses in the first column, top rows. The other columns of the Table display elasticities for the adjustment speeds evaluated at the benchmark parameter set, with respect to each of the six parameters, holding fixed the other five parameters.

It turns out that the rates of convergence under the alternative regimes differ substantially. Also, we see that the rate of convergence under capital mobility is most sensitive to the growth rate and the foreign spillover parameter. The adjustment speed under balanced trade is fairly robust to changes in parameters.

5. Welfare

Rather than cross-country differences in consumption, as indicated by C^R and discussed above, cross-country differences in welfare are relevant to assess the impact of capital mobility and convergence. In this section it is discussed how much welfare between a lagging and leading country differs and how capital mobility affects this difference.

5.1. Welfare calculus

Intertemporal welfare of the representative consumer in a country depends on the entire path of consumption, see (A.3). Because of this and because in the absence of unexpected shocks consumption is continuous, a change in welfare can be decomposed in a level effect -- the change in initial consumption -- and a growth effect -- the change in the growth rate of consumption over time. Linearizing the intertemporal welfare function (A.3), we find the

---

[9]Note that ε both represents substitution among domestically produced varieties and substitution between home and foreign goods. Empirically, these elasticities differ considerably: export elasticities are considerably below 5. The current model can be easily adapted to separate aggregate export demand elasticities from firm's elasticities of demand, by adding an additional level of nesting in A.4). If we would denote the latter by η and the former still by ε, only (B.5) and (B.7) would change (ε has to be replaced by η in these equations) and all other equations remain the same. Hence, ε should be interpreted as the elasticity of export demand. If we choose ε = 2 and leave other parameters in Table E unchanged, we find $\lambda_{CM} = 2.1\%$, $\lambda_{BT} = 1.1\%$, $\tilde{U}^R_{0_{CM}} / \tilde{h}^R_0 = 0.43$, $\tilde{U}^A_{0_{CM-BT}} / \tilde{h}^R_0 = -0.003$; the elasticities with respect to ε become significantly larger, but other elasticities remain similar to those in Table E.

[10]Recently, however, doubts have started to arise about the robustness of this number. In his survey on the econometrics of convergence, Temple (1999, p. 134) reports that more sophisticated studies find estimates that range between zero and 30 per cent a year.

precise expression:[11]

$$\frac{\mathrm{d}U_0}{(1-\rho)\,U_0} \;\equiv\; \tilde{U}_0 \;=\; \tilde{C}_0 \;+\; \frac{1}{r-g}\left[\left(\frac{r-g}{r-g+\lambda}\right) g\tilde{g}_{C0} \;+\; \left(\frac{\lambda}{r-g+\lambda}\right) g\tilde{g}_{C\infty}\right] \tag{25}$$

where λ is the relevant rate of convergence. The first term is the level effect, the bracketed term is the growth effect. The latter is a weigthed average of the short-run change in growth ($\tilde{g}_{C0}$) and its long-run change ($\tilde{g}_{C\infty}$). The larger the adjustment speed λ, the larger is the weight on the long-run change, since long-run values are approached faster. Note that the change in welfare in (51) is scaled in such a way that the expression can be interpreted as the *equivalent change in permanent consumption*, that is the permanent increase in consumption on a balanced growth path that generates an equivalent change in welfare.

Taking country differences and substituting $g\tilde{g}^R_{Ct} \;=\; \dot{\tilde{C}}^R_t \;=\; \lambda\,(\tilde{C}^R_\infty \;-\; \tilde{C}^R_t)$, we may write for the change in country A’s welfare relative to country B’s welfare:

$$\tilde{U}^R_0 \;=\; \left(\frac{r-g}{r-g+\lambda}\right)\tilde{C}^R_0 \;+\; \left(\frac{\lambda}{r-g+\lambda}\right)\tilde{C}^R_\infty \tag{26}$$

Substituting the solutions for C^R, from (13) and (23), and steady state relations, (2) and (3), we find for the two regimes:

$$\tilde{U}^R_{0_k} \;=\; \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{1+(\alpha_h-\alpha_f)\left(\frac{g}{r-g+\lambda_k}\right)}{1+(\alpha_h+\alpha_f)\left(\frac{g}{r-g}\right)}\right)\tilde{h}^R_0 \tag{27}$$

where $k = BT,CM$ denotes the capital market regime.

5.2. Cross-country welfare differences

The expression in (27) gives the relative consumption differential on the balanced growth path that is equivalent to the differences in welfare stemming from the fact that country B starts at a productivity level that is $\tilde{h}^R_0$ below that of country A. With capital mobility, the expression equals the actual consumption difference, since consumption differentials are permanent and consumption grows at the balanced growth rate. Without capital mobility, the expression is a fraction of the actual short-run consumption differential, since relative consumption levels

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[11] For a step-by-step derivation, see Smulders (1994), page 294.

converge in the long run. Nevertheless, the equivalent consumption differential can be written both with and without capital mobility as in (27), which facilitates the comparison between the two regimes.

Equation (27) reveals some interesting properties of welfare differentials between converging countries:

> Proposition #3: Welfare of the lagging country is below that of the leading country. The welfare differential increases with ε if $(\alpha_h - \alpha_f)$ is sufficiently small.
>
> Proof: The first term in parentheses in (27) increases in ε. By proposition 2, λ in the second term in parentheses changes with ε, but this effect never dominates if $(\alpha_h - \alpha_f)$ is sufficiently small.
>
> Remark: In numerical experiments I only could identify parameter values for which the welfare differential increases with the price elasticity.

Not surprisingly, we find that the country that lags in terms of productivity has lower intertemporal welfare than the leading country. A strong position in international markets, as measured by a low value of the price elasticity ε, mitigates the welfare differences. In other words, market power insulates a country against adverse productivity shocks. The reason is that strongly favourable terms of trade effects counterbalance the negative income effects. This effect would also apply in a static economy (with g = 0), in which the welfare differential would be simply $(\varepsilon - 1)/\varepsilon$ times the productivity difference. The fact that the second term in parentheses is smaller than unity reveals that the process of catching up associated with spillovers and growth reduces the welfare gap below the level that would apply in the static economy.

In Table E some numerical results are presented. It turns out that for the benchmark parameter set a 1 per cent productivity gap causes intertemporal welfare of the lagging country to be 0.67 per cent below that of the leading country, measured in terms of permanent consumption. This number is rather insensitive to any of the parameters.

5.3. The gains from capital mobility

We now turn to the gains from capital mobility. A country benefits from capital mobility if its welfare is higher with capital mobility than without. We have to calculate the differences in welfare for each country under the alternative regimes. We can infer these values from the relative variables. Recall that total consumption in the world economy does not depend on the international capital market regime, see (5)-(6). Therefore, the gains from capital mobility stem only from the change in allocation of consumption over the two countries when moving from balanced trade to capital mobility. In particular, the gain from capital mobility for country A is half of the difference between $\tilde{U}_0^R$ evaluated for capital mobility and $\tilde{U}_0^R$ evaluated for balanced trade. Country B's gain is the same number with opposite sign (see section 3.2). From (27) we find:

$$\tilde{U}^A_{0_{CM-BT}} = -\tilde{U}^B_{0_{CM-BT}} = \frac{1}{2}(\tilde{U}^R_{0_{CM}} - \tilde{U}^R_{0_{BT}}) = \tag{28}$$

$$-\left(\frac{\varepsilon-1}{2\varepsilon}\right)\left(\frac{g}{1+(\alpha_h+\alpha_f)\left(\frac{g}{r-g}\right)}\right)\left(\frac{\lambda_{CM}-\lambda_{BT}}{(r-g+\lambda_{BT})(r-g+\lambda_{CM})}\right)(\alpha_h-\alpha_f)\,\tilde{h}_0^R$$

The expression in (28) gives the welfare premium of capital mobility (in terms of permanent consumption) for country A. It reveals the winners and losers from international capital mobility as stated in the following proposition:

> Proposition #4: The leading country is worse off with capital mobility and the lagging country gains from capital mobility if spillovers within the country are more important than spillovers between countries ($\alpha_h > \alpha_f$).
>
> Proof: the sign of the expression in (28) depends on the sign of $-(\alpha_h - \alpha_f)\,\tilde{h}_0^R$.

Numerical results are again presented in Table E. The welfare premium of capital mobility for the country that lags 1 per cent behind amounts to about 0.01 per cent of consumption. This number turns out to be relatively sensitive to the growth rate and domestic spillovers.

It might come as a surprise that the introduction of capital mobility does not improve welfare for both countries. One might expect capital mobility to be welfare improving as it opens the possibility of consumption smoothing. However, this effect is of second order in the linear approximations around a symmetrical steady state.[12] What causes gains and losses

[12] This result also echos some results from the static literature on capital mobility (see Wong 1995). If the number of traded goods equals the number of factors and production functions are the same in both countries, then, starting from free goods trade, opening up to international capital markets does not affect welfare since factor prices are already equalised by goods trade. In our model, the issue is also whether capital mobility affects welfare, but in a dynamic setting with intrasectoral (rather than intersectoral) trade. Without capital mobility,

from capital mobility is the impact of knowledge spillovers in both countries. These spillovers create externalities in research and leads to suboptimal investment decisions, since the market for public knowledge is missing. Domestic knowledge spillovers imply underinvestment from a welfare point of view. Under capital mobility, productivity in the leading country grows more slowly than under balanced trade and in the lagging country it grows faster, since capital flows to the lagging country. Therefore, capital mobility mitigates the underinvestment effect in the lagging country but aggravates it in leading country. Cross-country spillovers have an opposite effect. The returns to innovation undertaken by one country accrue partly to its trading partner, thereby deteriorating its competitive position. Hence, foreign spillovers result in overinvestment from the point of national welfare. Capital mobility aggravates overinvestment in country one. The proposition states that if the national externality is more severe than the international externality, there is on balance underinvestment in each country. Since capital mobility speeds up investment in the lagging country, it is this country that gains from capital mobility.

6. Conclusions

In the two-country growth model in this chapter, initial productivity differences between countries are ultimately eliminated since the lagging country grows faster. In the long run, international growth rates fully converge. The long-run growth rate does not depend on the degree of monopolistic competition, as measured by the inverse of the elasticity of substitution between product varieties.[13] However, since elasticity of substitution between product varieties shapes competition between home and foreign producers, it affects the incentives to invest in home relative to foreign firms. Hence, monopolistic competition is a key determinant of the speed of convergence. With low intertemporal substitution ($1/\rho < 1$), we have found that lower monopoly profits speed up convergence under capital mobility but reduce the convergence speed in the absence of capital mobility. For a given degree of monopolistic competition, capital mobility allows for faster convergence, but it improves welfare only in the receiving country if domestic knowledge spillovers are larger than international knowledge spillovers. Externalities in research and development make the leading country worse off international capital flows. It remains to be seen how the results

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factor prices (wages and rates of return) are equalised only in the long-run. Production functions (for goods and knowledge) are the same.

[13] This independency allows us to focus on the role of export demand elasticities. In the chapter by Smulders and Van de Klundert in this book it is explained that this property arises because the number of firms is fixed.

change when national governments subsidize R&D to correct these externalities, and how the result change in the case of coordinated R&D policies. This is left for future research.

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Appendix

Proof of proposition 2

Total differentiation of (24) gives $d\lambda_k = -[dD_k + \lambda_k dT_k]/(2\lambda_k + T_k)$. Differentiating with respect to ε, we find:

$$\mathrm{d}\lambda_{CM}/\mathrm{d}\varepsilon = [2\alpha_f\xi - \lambda_{CM}(\alpha_h - \alpha_f)]\xi L/(2\lambda_{CM} + T_{CM}) \text{ and}$$

$$\mathrm{d}\lambda_{BT}/\mathrm{d}\varepsilon = [2\alpha_f\xi - \lambda_{BT}(\alpha_h - \alpha_f)](1-\rho)[\rho(\varepsilon - 1) + 1]^{-2}\xi L/(2\lambda_{BT} + T_{BT}).$$

- If $\alpha_h < \alpha_f$, it follows immediately that $\mathrm{d}\lambda_{CM}/\mathrm{d}\varepsilon > 0$ and *sign* $\mathrm{d}\lambda_{CM}/\mathrm{d}\varepsilon$ = *sign* $(1-\rho)$.

- If $\alpha_h > \alpha_f$, $\mathrm{d}\lambda_{CM}/\mathrm{d}\varepsilon > 0$ if and only if $\lambda_{CM} < 2\alpha_f\xi/(\alpha_h - \alpha_f)$. This latter inequality always holds, since using the expression of λ_{CM} above, we find (i) $\lambda_{CM} = 0$ if $\varepsilon = 0$, (ii) $\lim_{\varepsilon\to\infty}\lambda_{CM} = 2\alpha_f\xi/(\alpha_h - \alpha_f)$, and (iii) λ_{CM} is a continuous function of ε.
- If $\alpha_h > \alpha_f$ and $\rho > 1$, $\mathrm{d}\lambda_{BT}/\mathrm{d}\varepsilon < 0$ if and only if $\lambda_{BT} < 2\alpha_f\xi/(\alpha_h - \alpha_f)$. This latter inequality holds, since (i) from (8) and (16), it follows that $\lambda_{CM} = \lambda_{BT}$ if $\varepsilon = 1$ and (ii) we have already proved that $\lambda_{CM} < 2\alpha_f\xi/(\alpha_h - \alpha_f)$. Hence, $\lambda_{BT} < \lambda_{CM}$ for all $\varepsilon > 1$ and $\rho > 1$.
- If $\alpha_h > \alpha_f$ and $\rho < 1$, $\mathrm{d}\lambda_{BT}/\mathrm{d}\varepsilon > 0$ if and only if $\lambda_{BT} < 2\alpha_f\xi/(\alpha_h - \alpha_f)$. This latter inequality holds for all $\varepsilon > 1$, since (i) $\lambda_{BT} = \lambda_{CM} < 2\alpha_f\xi/(\alpha_h - \alpha_f)$ if $\varepsilon = 1$ and (ii) λ_{BT} is a continuous function of ε.

Proof of proposition 1

If $\varepsilon = 1$, $\lambda_{CM} = \lambda_{BT}$. From proposition 2 we conclude that for $\rho > 1$, λ_{CM} increases with ε and λ_{BT} decreases with ε. Hence $\lambda_{CM} > \lambda_{BT}$ for all $\varepsilon > 1$ and $\rho > 1$.

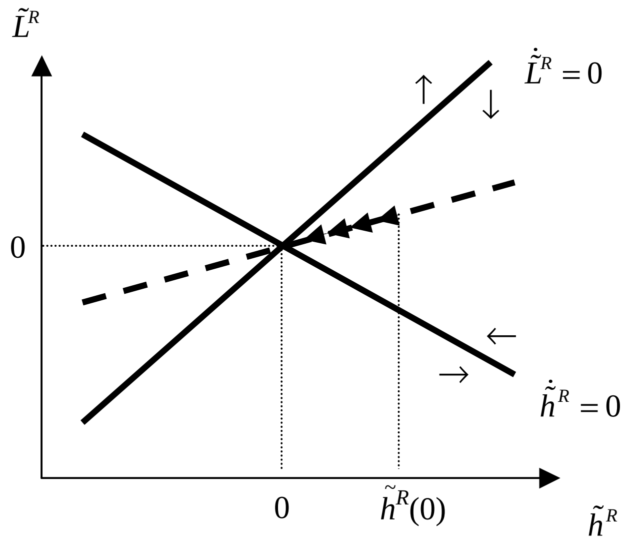

Figure 1 Phase diagram

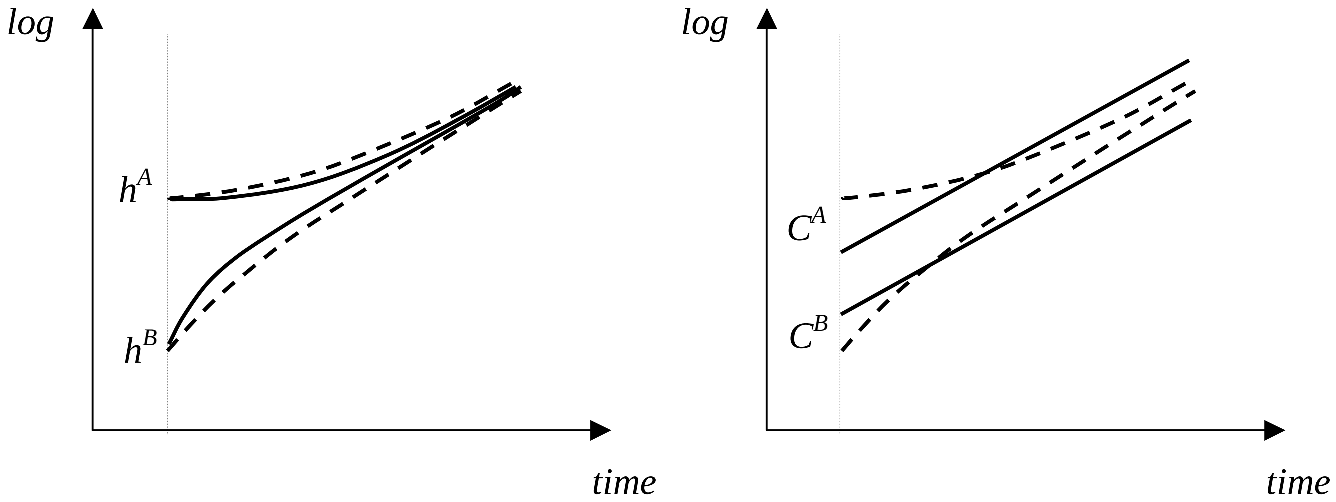

Figure 2 Log Productivity and Log Consumption over time
(solid lines: capital mobility; broken lines: balanced trade)