Optimal Enforcement of Competition Law

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Proefschrift

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CHAPTER 1

Introduction

1.1 Fining Policies in the US and Europe

The problem of deterring antitrust law violations is well known in the literature and is becoming increasingly important. Antitrust law in Europe consists of the rules on restrictive agreements and abuse of a dominant position laid down in Articles 81 and 82 EC, while in the US the antitrust laws are collected in the Sherman Antitrust Act and in the Clayton Antitrust Act. Despite the recent theoretical developments in this field[1], much still needs to be done in practice by competition authorities in order to prevent collusion and price-fixing in the major industries. In this way competition can be sustained, which increases consumer welfare. We can recollect a lot of examples of recent cases of antitrust law violations in the Netherlands and other European countries, like a tendering procedure in the construction sector or the Cement cartel discovered in 1994. The most striking fining decisions recently made by the European Commission are the large fines imposed on the Vitamins cartel, equal to 855 million euros, and the Organ Peroxide cartel, equal to 1 billion euros. In addition, the Microsoft case is at the moment attracting much attention from the Department of Justice and the European Commission. All these cases show that it is desirable to further develop and refine mechanisms that prevent such violations. Those mechanisms should, ideally, be based

[1] The recent theoretical developments in the US and EC competition policy has been discussed in Motta (2003), Walker and Bishop (2002), Wils (2002), and Rey (2003).

on theoretical models of price-fixing and cartel deterrence.

During recent decades, systems of penalties for antitrust law violations in the US and Europe have been changed several times. There were considerable changes in the number of infringements discovered and the amounts of the fines obtained owing to these changes. However, even after all those changes have been implemented, current rules of antitrust law enforcement still do not comply with the well known result of Becker (1968), which states that the optimal fine should be a multiple of the gains from crime. In most countries, the base fine for a cartel amounts to just 10% of turnover. This scheme provides underdeterrence from an empirical and theoretical point of view, as will be shown in Chapter 2. The main argument here is that, based on expected utility theory, fines set below the gain from the infringement divided by the probability of being punished cannot block the violation. However, taking into account parameters of current penalty schemes in the EU and US, it appears that fines still fall below this value. These considerations seem to suggest that the fines for antitrust violations should be increased. On the other hand, it is shown in Leung (1991) that in a dynamic setting the optimal fine does not necessarily have to be higher than the harm or social cost of the crime. This finding can be a reason to look also for other policy instruments that do not necessarily increase the penalties for cartels.

To give an overview of the current situation, we summarize the results of an OECD report[2] that provides a description of the available sanctions for cartels according to the laws of member countries. Those laws allow for considerable fines against enterprises found to have participated in price-fixing agreements. In most member countries, the fines are expressed either in absolute terms or as a percentage of the overall annual turnover of the firm[3]. In addition, there exists an upper bound for penalties for violations of antitrust law. The fine is constrained from above by the maximum of a certain monetary amount, a multiple of the illegal gains from the cartel, or, if the illegal gain is not known, 10% of the total annual turnover of the enterprise. In some cases, however, the maximum fines determined by these laws may not be sufficiently large to accommodate multiples of the gain to the cartel, as suggested by expected utility theory. Moreover, according to experts' estimations (see OECD, 2002), the best policy is to impose penalties which are a multiple of the illegal gains from price-fixing agreements to the firms. This, of course, would be difficult to estimate in reality, so it is still common practice to use a percentage of turnover as a proxy of the gains from price-fixing

[2]See O.E.C.D. (2002).
[3]See EC (1998).

activities.

We conclude that the current penalty schemes for antitrust law violations are based mainly on the turnover involved in the infringement throughout the entire duration of the infringement, which serves as a proxy of the accumulated illegal gains from cartel or price-fixing activities for the firm. To be more precise, in the laws of most countries, the amount of the fine imposed depends on the gravity and duration of the infringement and on attenuating and aggravating circumstances, such as the willingness of firms to cooperate with authorities by providing information about existing cartels or having a leading part in the infringement.

The main aim of the thesis is to model these features of current penalty systems employing the tools of game theory, dynamic games, and dynamic optimization[4]. It should be stressed that dynamic analysis of competition law enforcement should not be ignored since it captures better both the current antitrust rules and the crime process in general. Application of the above-mentioned tools allowed us to compare current US and EU penalty schemes for violations of antitrust law and to develop policy implications on how existing penalty schemes can be modified in order to increase their deterrence power. This also enables us to answer the main questions addressed in the thesis: What should be the basis for optimal deterrence of violations of competition law? What combination of instruments (fines, rate of law enforcement, leniency programs) should antitrust authorities employ in order to achieve cartel deterrence in the most efficient and least costly way? What is the optimal structure of penalty schemes? This research can also be considered a step towards the solution of the problem of optimal antitrust law enforcement in general.

The introduction is organized as follows. We have already discussed the motivation for the thesis and main questions addressed in the manuscript in section 1.1. Next, in section 1.2 we give an overview of the literature that deals with economics of crime and compare static and dynamic approaches to the solution of the problem of optimal law enforcement. In section 1.3 we move to the discussion of the problem of cartel deterrence and the role of leniency programs in antitrust enforcement. Section 1.4 gives an outline of the thesis. Finally, in section 1.5 we summarize the main results and lessons from the overall work.

[4]Most of these tools are discussed in great details in Fudenberg and Tirole (1991), Dockner et al. (2000), and Feichtinger (1982).

1.2 Static and Dynamic Approaches to the Economics of Crime

The analysis of the economics of optimal antitrust law enforcement is closely related to the general literature on crime and punishment. In his seminal paper, Becker (1968) examined the problem of how many resources and how much punishment should be used to enforce different kinds of legislation. The decision instruments were the expenditures on police and courts influencing the probability that the offender is convicted, and the type and size of punishment for those convicted. The goal was to find those expenditures and punishments that minimize the total social loss. This loss was defined as the sum of damages from offences, the costs of apprehension and conviction, and the costs of carrying out the punishment imposed.

The main contribution of Becker's work is to demonstrate that the best policies to combat illegal behavior are based on an optimal allocation of resources. Becker (1968) investigates this problem using a static economic approach to crime and punishment. He conclude that the optimal fine should be a multiple of the social cost of the crime and inversely related to the probability of detection. So, since an increase in the probability of detection causes an increase in the costs of control, the least costly policy for the antitrust authority would be to decrease the probability of detection and increase the fine itself. However, as described in the previous subsection, legal limitations concerning the upper bound of the fine, which are intended to prevent bankruptcy, are likely to exist. Additional criticism of Becker's work is that his analysis was confined to a static environment, in which it is not possible to take into account many important features of crime and arrest processes like recidivism or intertemporal strategic interactions. Later, in Leung (1991, 1995), Feichtinger (1983, 1995), and Fent et al. (1999, 2002), dynamic (intertemporal) trade-offs between the damages generated from the offences and the costs of the control instruments were studied. These researchers aim to find a mix of policy variables, like prevention, treatment, and law enforcement, which minimizes the discounted stream of total social loss. They argue that, for example, in illicit drug consumption, corruption, or violence, the system dynamics is governed by specific feedback effects. The resulting patterns of uncontrolled processes require the allocation of the policy instruments in a specific way over time. Moreover, dynamic games (rather than single-player maximization problems) should be applied to cope with intertemporal strategic interactions like the symbiosis between bribers and bribees. A similar approach can be applied to the violations of competition law while modelling interactions between the firm and the antitrust authority or between two firms in a cartel.

We provide below a more detailed review of the above-mentioned papers. Leung (1991) introduces a dynamic model of optimal punishment, where the optimal fine is determined by solving an optimal control problem. Leung also stresses that, in many circumstances, the crime process is a dynamic one and the dynamic model is a better description of reality than the static model. Hence, the dynamic analysis of law enforcement should not be ignored.

Leung shows that Becker's findings are no longer valid in a dynamic environment, and the implications of the dynamic model of optimal punishment are found to be considerably different from those of the static model. It was found that the optimal fine was positively related to the social cost of the crime and negatively related to the hazard rate of arrest. Moreover, the author found that the fine, which would block the crime, did not necessarily have to be greater than the harm induced by the infringement, which contradicts the Becker's findings. This is due to the fact that in Leung's dynamic model the flow of the gains from the crime can be sustained only if the offender has not yet been arrested. As a result of this conditioning, the probability of conviction in the static model has to be replaced by the conditional probability (or the hazard rate of arrest) in the dynamic model, which leads to the differences between the implications of the two models[5]. Leung argued that Becker's approach would not generate the optimal outcome, i.e., the outcome which maximizes welfare, in a dynamic environment. In fact, according to Leung (1991), it would cause overcomplience because the fine, which is a multiple of the damages, imposes too heavy a penalty on the offender.

Another important aspect of the dynamic crime process is recidivistic behavior, which was ignored in both Becker's static model and Leung's dynamic model. Fent et al. (1999, 2002) take this aspect into account. They investigate optimal law enforcement strategies where punishment depends not only on the intensity of crime (offence rate) but also on the offender's prior criminal record. This idea was adopted in Fent et al. (1999) in an optimal control model with the aim of discovering the optimal intertemporal strategy of a profit-maximizing offender under a given, static punishment policy. In Feichtinger (1983) and Fent et al. (2002), the framework described above was extended to an intertemporal approach of utility maximization, considering two players, the offender and the authority, with conflicting objectives. The authority aims to minimize the social loss caused by criminal offences, whereas the offending individual aims to maximize the profit

[5] It is essential for the result that conditional density (which is used by Leung to model hazard rate of arrest) is not bounded between 0 and 1.

gained from crime. This leads to a differential game, making it possible to investigate competitive interactions in a dynamic framework. In Fent et al. (2002), the criminal record takes the role of a state variable, where a high record increases the punishment an offender could expect if convicted. Finally, the steady state values of the state and the control variables of the game are derived. The solution implies that full compliance behavior is sustainable in the long run, when the penalty increases with the offender's criminal record.

Another important stream of the literature on optimal law enforcement is concerned with the problem of policy design and research on the optimal structure of penalty schemes. For example, Garoupa (2001) studies the optimal trade-off between probability and severity of punishment. He concludes that when there is substantial underdeterrence (alternatively, when offenders are poor) detection probabilities and fines are complements rather than substitutes. The main assumption that drives this result is that agents are wealth constrained and the fine cannot exceed the offender's wealth. This implies that, in situations when offenders are very poor (the expected fine is significantly less than the social damage caused by the offence) it makes no sense for the authority to spend money on enforcement and, consequently, the rate of law enforcement is also low. However, when wealth goes up, so do the fines. Then it becomes worthwhile for the government to engage in detection and punishment.

Polinsky and Rubinfeld (1991), Dana (2001), and Emons (2003, 2004) investigate the problem of optimal punishment for repeat offenders. The main question addressed in those papers is whether the optimal sanction should decrease or increase with the number of offences. However, full consensus on this topic has not yet been reached, so that this puzzle still requires a deeper investigation in the law and economics literature.

1.3 Leniency Programs and Their Role in Antitrust Law Enforcement

The line of economic thinking about the problems of cartel deterrence[6] and prevention of violations of competition law brings us to the discussion of Leniency Programs, which recently proved to be an effective instrument in the fight against cartels. In the US, for example, the fines collected in 1993 almost doubled those collected in 1992, which can

[6]The most fundamental paper that discusses the structure and forms of collusive agreements and stresses the importance of self-enforcing constraints in cartel formation is Stigler (1964).

be connected with a major modification of leniency programs[7]. Leniency programs have recently been introduced in European antitrust legislation and have quite a long history in the US[8]. Leniency programs grant total or partial immunity from fines to firms that collaborate with the authority. To be more precise, leniency is defined as a reduction of the fine for firms which cooperate with the antitrust authority by revealing information about the existence of the cartel before the investigation has started, or by providing additional information that can help to speed up the investigation. Leniency programs work on the principle that firms which break the law might report their crimes or illegal activities if given proper incentives.

There is some empirical evidence that leniency programs improve welfare by sharply increasing the number of detected cartels and by shortening the investigation. However, there are also other effects of leniency programs, which are now difficult to identify in empirical studies due to the absence of data. For example, questions of how the introduction of leniency programs would influence cartel stability and the duration of cartel agreements, and whether leniency facilitates collusion or reduces it, require further investigation. Chapter 6 gives some insights into these issues.

A number of earlier papers have studied the problem of self-reporting, which is at the heart of leniency schemes. Malik (1993) and Kaplow and Shavell (1994) were the first to identify the potential benefits of schemes which elicit self-reporting by violators. They conclude that self-reporting may reduce enforcement costs and improve risk-sharing, as risk-averse self-reporting individuals may prefer to pay a certain penalty rather than the stochastic penalty faced by non-reporting violators. Focusing on individual wrongdoers committing isolated crimes, Kaplow and Shavell (1994) showed how reducing sanctions against wrongdoers that spontaneously self-report reduces law enforcement costs and increases welfare by lowering the number of wrongdoers to be detected and the risk born by risk-averse wrongdoers. Malik (1993) examined the role of self-reporting in reducing auditing costs in environmental regulation. Innes (1999) investigated a similar problem and highlighted the value of the early prevention of damages that self-reporting allows for. He concludes that switching to this policy leads to less government enforcement activity, and that less deterrence is needed. These papers highlighted important benefits that a lenient treatment of self-reporting wrongdoers brings about, but did not consider its ability to undermine trust in cartels and analogous criminal organizations, which was

[7]This modification implied that the first self-reporting firm could get full immunity. Moreover, full immunity could also be granted if the case was already under investigation.

[8]In the US the first corporate Leniency Program was introduced in 1978. In Europe the first Leniency Programs came into force in 1996.

the main focus of the papers discussed in the next paragraphs.

The use of leniency programs in antitrust has been studied by Motta and Polo (2003). Later, they were followed by Spagnolo (2004), Aubert et al. (2004), and Feess and Walzl (2004). In Motta and Polo (2003), it is shown that such programs can play an important role in the prosecution of cartels provided that firms can apply for leniency after an investigation has started. They conclude that, if given the possibility to apply for leniency, a firm may well decide to give up its participation in the cartel in the first place. They also found that leniency saves resources for the authority. Finally, their formal analysis showed that leniency should only be used when the antitrust authority has limited resources, so that a leniency program is not unambiguously optimal. Motta and Polo's (2003) findings were closely related to those of Spagnolo (2000a) and (2004). In Spagnolo (2000a), it was shown that only courageous leniency programs that reward self-reporting parties may completely and costlessly deter collusion, while moderate leniency programs that reduce or cancel sanctions for the reporting party cannot affect organized crime. In Spagnolo (2004), it was also shown that optimally designed 'courageous' leniency programs should reward the first party that reports sufficient information with the fines to be paid by all other parties. Contrary to Becker's result, Spagnolo's approach allowed to achieve the first best with finitely high fines. Moderate leniency programs that only reduce or cancel sanctions, as implemented in reality, may also destabilize and deter cartels by protecting agents that report from fines; protecting them from other agents' punishment; and increasing the risk of taking part in a cartel.

An important closely related study is that conducted by Aubert, Kovacic, and Rey (2004). They considered rewards in antitrust enforcement in a simpler model that allowed them to focus on important issues complementary to those discussed by Spagnolo (2004). They considered the costs and benefits of creating an agency problem between firms and their individual employees by allowing the latter to benefit directly from cash rewards when they blew the whistle and reported their own firm's collusive behavior. They noted, among other things, that the possibility of employees blowing the whistle reduces the incentives to start a cartel.

A recent paper by Apesteguia, Dufwenberg, and Selten (2004) compared experimentally the performance of moderate leniency and rewards in a one-shot Bertrand setting analogous to that used by Spagnolo (2000b). The findings confirmed that experimental subjects understand and make use of reporting as an instrument that affects the stability of collusive agreements. However, more experimental work in this field appears to be highly needed.

Another attempt to study the efficiency of leniency programs in antitrust law enforcement was made by Feess and Walzl (2004). They compared leniency programs in the EU and the USA. For that purpose they constructed a game with two self-reporting stages, heterogeneous firms with respect to the amount of evidence provided, and ex post asymmetric information. Differences in leniency programs in the US and Europe include the fine reduction granted to first and second self-reporters, the role of the amount of evidence provided, and the impact of whether the case is already under investigation. Feess and Walzl (2004) elaborated on the role of asymmetric information in deriving the optimal degree of leniency and used these findings to compare the programs in the US and the EU.

In conclusion, we would like to stress that only properly designed leniency programs can induce self-reporting, reduce incentives for firms to participate in cartels, and improve welfare. The possibility of counterproductive effects of leniency programs is also discussed in Spagnolo (2000b), Buccirossi and Spagnolo (2001), and Ellis and Wilson (2003). These researchers argue that moderate (in the sense of fine reduction) leniency programs may greatly facilitate the enforcement of long-term illegal cartel agreements. They explain that reduced sanctions for firms that self-report provide the otherwise-missing credible threat necessary to discipline those involved in collusive agreements: they ensure that if a firm unilaterally deviates from collusive strategies, other firms will punish it by reporting information to the antitrust authority. We also argue below that leniency programs that are wrongly designed (too lenient, non-confidential, give a too-generous fine reduction to the second reporter) may worsen the problems rather than solve them. They may give additional incentives for firms to form a cartel in the first place and later also facilitate the stability of the cartel agreement. This implies that particular attention in the economic and legal analysis of leniency programs should be devoted to the problem of optimal design of leniency programs. This problem was the focus of the analysis described in Chapters 5 and 6.

1.4 Outline of the Thesis

The thesis consists of the introduction followed by six chapters. In Chapter 2, we describe the system of penalties for cartels and the effectiveness of sanctions currently used in antitrust law enforcement. In Chapters 3 and 4, we analyze the properties and deterrence power of the current penalty schemes employing tools of optimal control theory and differential games. In Chapters 5 and 6, the effects of leniency programs on cartel stability and optimal design of leniency programs are analyzed. In Chapter 7, an analysis of whether the penalties for repeat offenders should decline or escalate is described.

The analysis reported in Chapters 3 and 4, where we model intertemporal trade-offs, requires the application of tools like dynamic programming, optimal control theory, and, where there is strategic interaction between players, differential games. Most of the papers mentioned in section 2 of the introduction investigate the problem of optimal dynamic law enforcement and minimization of social loss from crime by modelling the interactions between the offender, who commits the crime, and the authority, whose aim is to prevent the crime. In Chapter 3, we suggest a similar approach. We analyze a differential game between a firm and the authority, whose aim is to prevent the crime, to examine the situation of violation of antitrust law by the firm, which illegally fixes prices above the competitive level. Technically, the analysis reported in Chapter 3 is close to the study by Feichtinger (1983), in which he investigated a model of competition between a thief and the police. We extend his framework by allowing the penalty to vary over time. Moreover, we introduce the fine as a function of the current degree of offence and probability of law enforcement at each instant of time. In particular, in this chapter we analyze a differential game describing the interactions between a firm that might be violating competition law and an antitrust authority. The objective of the authority is to minimize social costs (loss in total social welfare) induced by an increase in prices above marginal costs. We found that the penalty schemes which are used now in EU and US legislation appear not to be as efficient as desired from the point of view of minimization of consumer loss from price-fixing activities of the firms. We proved that full compliance behavior (namely, sustaining a competitive price-level) is not sustainable as a Nash Equilibrium in Markovian strategies over the whole planning period, and, moreover, that it will never arise as the long-run steady-state equilibrium of the model. We also investigated which penalty system would enable us to completely deter cartel formation in a dynamic setting. We found that this socially desirable outcome can be achieved if the penalty is an increasing function of the gravity of the offence and is negatively related to the probability of law enforcement.

Chapter 4 of the thesis addresses the problem of whether the fine, determined on the basis of the accumulated turnover of the firm performing price-fixing activities, can provide a complete deterrence outcome. The model of Chapter 4 is an extension of the model used in the study described in Chapter 3 in the sense that, in the former, we relate the penalty not only to the current degree of offence, but also to the accumulated illegal gains from cartel formation. We assume that the fine imposed takes into account the history of the violation. This means that when the violation of antitrust law is discovered, the regulator is able to observe all accumulated rents from cartel formation. Consequently, he will impose the fine that takes into account this information. We also compare the deterrence power of this system with that of a fixed penalty scheme.

Similar to Fent et al. (1999), the set-up of the problem leads to an optimal control model. The main difference between our approach and that of both Fent et al. (1999) and Feichtinger (1983) is that the gain from the cartel accumulated by the firm over the period of infringement takes the role of a state variable, whereas Fent et al. (1999) took the offender's criminal record as a state variable of the dynamic game. An increase in the state variable was thus positively related to the degree of price-fixing by the firm, and increased the fine the firm could expect if convicted. By solving the optimal control problem of the firm under antitrust enforcement, in Chapter 4 we investigate the implications of the different penalty schedules. Recall the result of the model of Chapter 3, where history of the violation is not taken into account and complete deterrence outcome cannot be achieved even in the long run. On the contrary, in the model of Chapter 4, where penalty is related to the accumulated illegal gains from price-fixing, full compliance outcome is sustainable in the long run.

We start the analysis of the effects of leniency programs on the stability of cartel agreements in Chapter 5 and continue it in Chapter 6. The models of Chapters 5 and 6 extend the previous analysis in the sense that we take into account the possibility of strategic interactions between the firms that form a cartel, i.e., the possibility that firms can break the cartel agreement by self-reporting.

As mentioned above, the main contributions in which analysis of optimal policies for the deterrence of violations of antitrust law in the presence of leniency schemes is reported so far are Motta and Polo (2003), Spagnolo (2000), and Aubert et al. (2004). Most of these papers employed a discrete time framework. Though they considered collusive behavior in a dynamic setting with antitrust laws, these papers excluded the important sources of dynamics that were the foci for this thesis: in particular, they did not allow detection and penalties to be sensitive to firms' current and past pricing behavior. However, a number of papers have already looked into this problem, namely, Hinloopen (2003, 2004) and Harrington (2004, 2005). They investigated settings, in which cartel detection probabilities were influenced by firms' behavior and where these probabilities changed over time. However, penalties which are proportional to the degree of offence and change over time, and that most closely reflect current antitrust rules were not analyzed by the above-mentioned researches. Chapter 5 addresses the problem of how the introduction of the leniency program influences the duration of cartels under two different regimes of fines: fixed and proportional. We employ a continuous time dynamic game, in which accumulated gains from price-fixing are the state variable. We investigate intertemporal aspects of this problem using dynamic optimal stopping models and tools of dynamic continuous time preemption games.

In Chapter 5, we suggest a new approach to analyzing the efficiency of the leniency programs that differs from the approaches put forward earlier and that is based on the Reinganum-Fudenberg-Tirole Model. Reinganum (1981) and Fudenberg and Tirole (1985) applied timing games to a technology adoption problem[9]. We apply a similar procedure to a cartel-formation game between two firms in the presence of a leniency program. This framework allows us to investigate not only the duration of cartel agreements, but also the problem of optimal design of leniency programs. One of our aims was to find out whether, in case both firms cooperate with the antitrust authority, they should be treated similarly or whether there should be a difference depending on the timing of application for leniency. In particular, we investigate whether the leniency programs should be stricter and whether the procedure of application for leniency should be open or confidential. We find that the occurrence of cartels would be less likely if the rules of the leniency programs are stricter and the procedure of application for leniency is more confidential. Moreover, we conclude that, when the procedure of application for leniency is not confidential, leniency may in some cases increase the duration of cartel agreements. This occurs when the penalties and the rate of law enforcement are low. Surprisingly, under a fixed penalty scheme, the introduction of a leniency program cannot improve the effectiveness of antitrust law enforcement when the procedure of application for leniency is not confidential.

In Chapter 6, we extend Motta and Polo's (2003) paper by introducing asymmetric firms and by explicitly modelling the effects of the degree of strictness of leniency programs on cartel stability. In general, firms differ in size and operate in several different markets. In our model, they form a cartel in one market only. This asymmetry results in additional costs in case of disclosure of the cartel, caused by an asymmetric reduction of the sales in other markets owing to a negative reputation effect. Moreover, following the rules of existing leniency programs, we analyze the effects of the strictness of the leniency programs, which reflects the likelihood of getting complete exemption from the

[9]For applications of timing games to the problem of investments under uncertainty, see also Huisman (2000) and Huisman et al. (2004).

fine even if many firms self-report simultaneously. Our main findings are that, first, leniency programs work better for small (less diversified) companies, in the sense that a lower rate of law enforcement is needed to induce self-reporting by less diversified firms. At the same time, big (more diversified) firms are less likely to start a cartel in the first place given the possibility of self-reporting in the future. The second important result is that the more cartelized the economy is, the less strict the rules of leniency programs should be.

Chapter 7 of the thesis deals with the general problem of optimal punishments for repeat offenders. That chapter addresses the question whether it is optimal to punish repeat offenders more severely than first-time offenders. In Emons (2003) it is shown that, under certain assumptions, it might be optimal to punish only the first violation. Chapter 7 represents an extension of the two-period analysis by Emons (2003) to a multi-period repeated game. The results obtained in this set-up are similar to those found by Emons. We show that, for wealth-constrained agents who may commit a criminal act several times, the optimal fines, imposed by a cost-minimizing resource-constrained regulator, are equal to the offender's entire wealth for the first crime, and zero for all subsequent crimes. Unfortunately, analogous to Emons (2004), this scheme does not appear to be a time-consistent (subgame perfect) strategy for a government in a multi-period setting. In Chapter 7 we investigate the robustness of the two-period Emons' result in the multi-period repeated game setting.

1.5 Conclusions and Lessons from the Overall Work

In the thesis we try to contribute to the design of optimal enforcement of competition law. We approach this problem from the angle of possible refinements of current penalty schemes for violations of competition law. In particular, we determine the optimal combination of instruments such as the amount of the fine and the rate of law enforcement and the optimal structure and design of penalty schemes. The motivation for this work comes from the well-known fact that the penalties for violations of competition law that are currently used in US and European guidelines are not sufficiently large to accommodate multiples of the gain of a cartel, as suggested by expected utility theory. Although penalties were recently increased considerably and new instruments of cartel deterrence, such as leniency programs, were introduced, still complete deterrence of antitrust law violations has not been achieved.

Properties of penalty schemes, like dependence on the gravity and duration of the infringement, and relation to current and past pricing behavior or to accumulated turnover of the firm, make the history of the violation an important factor for the determination of penalties. This calls for the application of tools of dynamic games for modelling situations of violations of competition law. This was the central idea of the thesis.

The application of these tools allows us to compare current US and EU penalty schemes for violations of antitrust law and to develop policy implications on how existing penalty schemes can be modified in order to increase their deterrence power. The main policy implications that can be drawn from our analysis provide justifications for a further increase of base and maximal penalties for violations of competition law. Given that there are certain legal limitations on maximal fines in Europe, the solution to this problem may come from the further development of the mechanism of private enforcement of competition law and the introduction of individual fines and imprisonment together with already-existing corporate fines in Europe. We also argue that the optimal penalty, i.e., the penalty that allows the achievement of a complete deterrence outcome, should take into account not only the gravity and the duration of the offence, but also the rate of law enforcement (or probability of conviction) by competition authorities.

Another important finding, which was also confirmed in earlier studies, is that only properly designed leniency programs can induce self-reporting, reduce incentives for firms to participate in cartels, and improve welfare. When leniency programs are wrongly designed there is a possibility of counterproductive effects of leniency programs being created. We obtain that cartel occurrence is less likely if the rules of the leniency programs are stricter and the procedure of application for leniency is more confidential. Moreover, we conclude that, when the procedure of application for leniency is not confidential, leniency may in some cases increase the duration of cartel agreements. This occurs especially when penalties and the rate of law enforcement are low.

CHAPTER 2

Optimal Penalties and Effectiveness of Sanctions Used in Antitrust Enforcement

2.1 Introduction

This chapter gives a description of the current EU and US penalty schemes and provides a statistical analysis of the effects of recent historical developments in antitrust law in Europe and the USA on the effectiveness of sanctions used in antitrust law enforcement. As mentioned in Chapter 1, regulations concerning systems of penalties for antitrust law violations in the US and Europe have been changed several times during recent decades. There were considerable changes in the number of infringements discovered and the amounts of the fines obtained owing those changes in the regulations. However, even after the implementation of all those changes, current rules of antitrust law enforcement in the US and Europe[1] still do not comply with the well known result of Becker (1968), which states that the optimal fine should be a multiple of the offender's benefits from crime. Moreover, the current scheme provides underdeterrence from an empirical point of view as well. In this chapter we analyze the main changes of antitrust rules and how they influenced deterrence rates. Unfortunately, complete deterrence of competition law violations has not yet been achieved.

[1] For Europe see Guidelines on the Method of Setting Fines Imposed, PbEG 1998. For the US see Guidelines Manual (Chapter 8: Sentencing of Organizations), 2003, URL: http://www.ussc.gov/2003guid/CHAP8.htm.

This chapter is organized as follows. Section 2 gives a description of the legal framework and introduces some general concepts used in competition law enforcement. To reflect the economic approach to law enforcement of legal behavior, we give an overview of the results found by Becker (1968) and Leung (1991). These are the two seminal papers describing the basics of the static approach and the dynamic approach to the economics of crime, respectively. In section 3, we give a comparative analysis of current European and US systems of fines for violations of competition law. In section 4, recent historical developments in antitrust law are reviewed and statistical data on how those developments influenced the deterrence power of penalty schemes in the US and EU are provided. Finally, section 5 summarizes the results of the analysis of the data on penalties imposed and the number of antitrust cases uncovered and discusses policy implications. The review of recent laws and new enforcement measures for cartel deterrence suggested by the US Department of Justice and OECD will also be discussed.

2.2 General Concepts and Theoretical Approach to the Problem of Fine Imposition

2.2.1 Three Dimensions of Competition Law Enforcement

The EC treaty and secondary legislation based on the Treaty contain three (sets of) competition rules applying to undertakings: Article 81(1) EC Treaty prohibits agreements restricting competition. Article 82 of the Treaty prohibits the abuse of a dominant position on the market and the Merger Regulation deals with merger cases. Articles 81 and 82 of the Treaty do not mention fines, but Article 83 empowers the Council to implement regulations for fine imposition. Similar regulations in the US are fixed in the Sherman Antitrust Act and the Clayton Antitrust Act.

There are three main dimensions of competition law enforcement. The first dimension is the stage of legal intervention. Here, a distinction can be drawn between ex-ante enforcement (prescreening) and ex-post enforcement (deterrence). According to some legal studies[2] and most economic studies, deterrence is dominant in the case of antitrust law enforcement relative to prevention. Therefore, antitrust authorities are more inclined to increase the fine instead of increasing the probability of audit, which is costly.

The second dimension is the form of sanctions. Two main questions must be answered here: who sanctions should be imposed upon (undertakings, companies, individ-

[2] Cf. Wils (2002).

uals) and what form sanctions should take (monetary (fines), non-monetary (imprisonment)). Monetary sanctions would be more appropriate for antitrust enforcement, but non-monetary sanctions are also in practice in the US nowadays.

The third important dimension of competition law enforcement is the choice between public and private enforcement. This concerns the role of private parties (consumers) versus public agents (competition authorities) in enforcement. The choice between public and private enforcement depends to a large extent on how much effort must be expended to obtain information relevant for enforcement. Private enforcement of competition law is quite popular in the US. However, this is still not the case in Europe. Private damage suits are almost non-existent in EC competition law enforcement, but private plaintiffs do, nevertheless, play a significant role in the Commission's enforcement activity. According to EC Regulation 1/2003,[3] the Commission can investigate a case either upon its own initiative or following a complaint. In practice, many cases leading to the imposition of fines involve a complaint. In the discussion below we will concentrate mainly on the first and second dimension of competition law described above.

2.2.2 Economic Approach to Fine Imposition

Two main goals of penalty systems are the deterrence of crime (offence) and compensation of the harm that an infringement inflicts on society. The formal definition of a penalty system is as follows.

Definition 2.1 *A penalty system is a corrective measure established in order to eliminate or reduce costly externalities generated by optimizing economic agents.*

In general, a penalty system consists of a probability of detection and a fine. In case of violations of antitrust law, these two parameters are called the rate of law enforcement by the antitrust authority and the penalty imposed on the firm for either price-fixing activities or participation in the cartel.

In this chapter we concentrate on the following questions: Why do we need to block cartel or price-fixing activities? What should be the basis of deterrence? Which instruments should be used? What is the most efficient way to deter violations of antitrust law?

To illustrate the answer to the first question, we refer to the simple example of the supply / demand diagram shown in Figure 2.1.

[3]Council Regulation (EC) 1/2003 on the implementation of the rules on competition laid down in Articles 81 and 82 of the Treaty, [2003] OJ L001 P001.

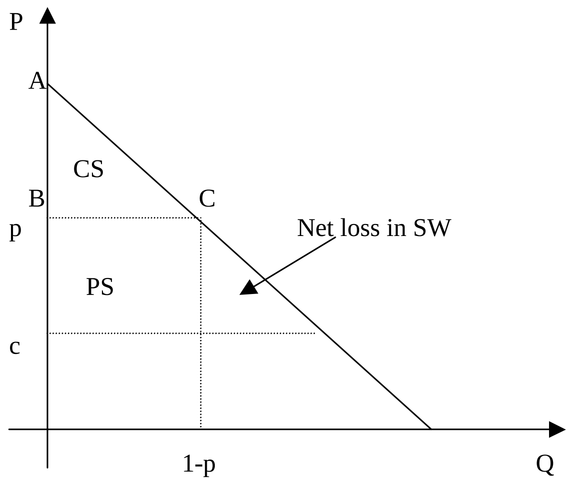


Figure 2.1: Negative effects of price-fixing on Consumer Surplus.

We see that the increase in prices above the competitive level c, induced by a cartel, leads to an increase in profits for the firm that is denoted by PS (Producer Surplus). However, at the same time there are social costs imposed by this change in prices. These social costs are represented by the area of the triangle marked as "Net loss in SW" (Net loss in Total Social Welfare). There is obvious damage to the consumers, since they lose part of the consumer surplus as a consequence of the price-fixing activities of the firm. In addition, there is a clear reduction in total welfare, since as a result of the increase in price above competitive level the reduction of the consumer surplus exceeds the increase in producer surplus. Hence, the answer to the first question is obvious: it is necessary to block the cartel in order to reduce this damage.

Further, and this will be the focus of our analysis, the rest of the questions can be reformulated as follows: What needs to be done in order to deter violation? What is the optimal combination of two instruments (fine and rate of law enforcement)? Should deterrence be focused on cartel benefits or social costs? What is the optimal structure of the penalty scheme?

As mentioned in the introduction, several important contributions to the modeling of fines were made in the literature. In Becker (1968) it is concluded that, in a static environment, the optimal fine should be a multiple of the offender's benefits from crime and negatively related to the probability of detection. Later, Leung (1991) introduced a dynamic model of optimal punishment, where the optimal fine was calculated as a solution to an optimal control problem. Due to the special structure of the dynamic crime process assumed in Leung's paper, the main finding of the study was that the fine which would block the crime can actually be less than the harm induced by infringement, which contradicts Becker's finding.

This provided a puzzle to solve in general and also in relation to violations of competition law. Should the base fines for violations of antitrust law be increased or should another instrument be used, such as increasing the rate of law enforcement, which can improve the efficiency of the penalty system? At the same time, a simple numerical example at the beginning of the next section shows that the fine set in compliance with static economic theory should be at least ten times higher than the current fine level described in the European Sentencing Guidelines.

In the following sections we clarify the answers to the questions addressed above by reporting the results of a comparative analysis of the current European and US penalty schemes for violations of competition law. There are two main goals of antitrust enforcement. They are deterrence of violations of competition law and compensation of damages to consumers in case violation occur. Deterrence is usually accomplished by public antitrust enforcement agencies, while compensation is mainly done through the channels of private enforcement. By comparing US and EU rules we find (and we will show this in more details in the next section) that current European system does not have much room for compensation, while the US system does take into account the fact that the total damage to consumers is usually higher than illegal gains. The main conclusion following from this comparison is that the US system is much closer to what economic theory would suggest, since it takes into account not only the fact that the fine should be related to the illegal gains from price-fixing, which correspond to the area PS in Figure 2.1. The US system also suggests that the fine should compensate the total loss to consumers that higher prices imply. In other words, consumers should be compensated for the total decline in Consumer Welfare that is represented in Figure 2.1 by adding up areas PS and Net loss in SW, which is approximately twice as high as the illegal gains. The second advantage of the US system compared to the EU system is that, in the US, fines imposed on corporations are combined with individual fines as well as imprisonment, while in the EU only corporate fines are available. Next, taking into account recent historical developments in antitrust law and analyzing the data on how those developments influenced the deterrence power of penalty schemes in the US and the EC, we aimed to identify the effects of changes in antitrust laws, such as the introduction of new fining policies and leniency programs, on the effectiveness of deterrence.

2.3 Comparison of Current Penalty Systems in the US and Europe

2.3.1 European System

It is determined in the European Guidelines on the Method of Setting Fines Imposed that the fines must be in proportion to their intended effect in terms of prevention, in proportion to the potential consequences of the prohibited practices in terms of the advantage to the offender and damage to competition, and in proportion to fines imposed on other companies involved in the same infringement. For these reasons, in determining the level of the fine, the turnover involved in the infringement, in principle, is taken into account. In addition, attention is also paid to the importance of the offender in the national economy. In this regard, in determining the level of the fine, the total annual turnover of the undertaking is taken into account.

Calculation of Fines

The general algorithm for setting the fine for competition law violations is as follows. First step is to determine the base fine. Usually, the base fine depends on the type of offence, its gravity, and duration and is set by European Commission. Next, the fine can be changed if there are any aggravating or attenuating circumstances. Finally, the legal upper bound on fines in Europe, which states that the fine cannot exceed 10% of overall annual turnover, is taken into account.

According to the European Sentencing Guidelines, it is recommended that the total fine (F) should be put within the limit of 10% of the overall annual turnover (T) of the organization under investigation:

$$F \leq 0,1T,$$

where T is calculated according to the following rule. If the firm is operating in several markets ($e.g. A, B, C$) and involved in price-fixing in only one of them (market A), then total annual turnover is $T = p^A q^A + p^B q^B + p^C q^C$, where p^i is the price in market i and q^i is the quantity sold in market i. At the same time, turnover involved in the crime (infringement) is given by $t = p^A q^A$. Further, the base fine will be determined on the basis of t and the type of infringement.

As mentioned in section 2.2, one of the main advantages of the US system compared to the EC system is that, in the US, fines on corporations are combined with individual fines as well as imprisonment, while in the EU only corporate fines are available. With an example below we show that exclusive reliance on European corporate sanctions in their current form is unlikely to result in effective deterrence of price cartels and other comparable antitrust infringements, at least in case the firm operates in one market only and forms a cartel in the same market. We will show that, given current parameters of enforcement policies, the fine economic theory would suggest will almost surely exceed the upper bound suggested in the European Sentencing Guidelines ($F \leq 0,1T$)[4]. Expected utility theory suggests[5] that the gain obtained from the infringement by the violator, divided by the probability of being fined, constitutes a floor below which fines can certainly not deter. It does not seem exceptional for a cartel to achieve a 10% price markup and to last for 5 years. Taking the case of a price-cartel, the gain which cartel members obtain from the violation depends on their turnover in the products concerned by the violation, the price increase caused by the cartel, the price elasticity of the demand which the cartel members face, and the life span of the cartel. Assuming a 10% price increase, and a resulting increase in profits of 5% of turnover, a 5-year duration, and a 16% probability of detection and punishment, the floor below which fines would generally not deter price-fixing would be in the order of 150% of the annual turnover in the products concerned by the violation. This is about ten times higher than the current fine level and, if the firm operates and forms a cartel in one market only, this is fifteen times higher than the upper bound suggested in the European Sentencing Guidelines (10% of annual turnover). This calls for either an increase in (or even abolishment of) the upper bound for the fine or the use of some other sanctions, such as individual fines or imprisonment.

We now consider in more detail current EC fining rules. Depending on the gravity of infringement The Commission can distinguish between minor, serious and very serious infringements. Minor infringements are schemes that distort competition to a limited degree, such as vertical schemes, in particular those that do not have prices and sales opportunities as their object, and branch schemes that restrict competition, which do not have prices and sales opportunities as their direct object. The fine in this case will be put within the limit between 1000 and 1 million euros. So, $f_m^g \in [1000, 1.000.000]$, where f_m^g is the fine attributed to minor gravity infringements.

Serious infringements are horizontal schemes, such as discrimination and tied sales, and vertical agreements that exert a direct influence on prices or sales opportunities, such as individual vertical price-fixing and prohibitions on reselling. The fine in this

[4]This numerical example is adopted from Wils (2002).

[5]See Becker (1968).

case will be put within the limit between 1 million and 20 million euros. So, $f_s^g \in [1.000.000, 20.000.000]$, where f_s^g is the fine attributed to serious infringements.

Very serious infringements are horizontal price agreements, collective vertical price-fixing, collective boycotts, horizontal agreements aimed at partitioning markets and quota schemes (including limiting sales and prohibited tendering agreements-'bidrigging'), and forms of abuse of a dominant position aimed at driving or excluding an undertaking from a market. The fine in this case must be higher than 20 million euros. So, $f_v^g \geq 20$ *million*, where f_v^g reflects the fine for the most grave violations.

Depending on the duration we can distinguish between short-duration, medium-duration, and long-duration infringements. For short-duration infringement (less than 1 year), there is no increase in the amount of the fine. And the fine, f^d , is calculated according to the following formula:

$$f^d = f_i^g, \quad i \in \{m, s, v\}.$$

For medium-duration infringements (1-5 years), there is an increase of up to 50% in the amount determined for gravity. The formula in this is as follows:

$$f^d = kf_i^g, k \in [1, 1.5]\,,\ i \in \{m, s, v\}.$$

For long-duration infringements (more than 5 years), there is an increase of up to 10% per year in the amount determined for gravity. The formula in this case is as follows:

$$f^d = Nkf_i^g,\ i \in \{m, s, v\},$$

where $k \in [1, 1.1]$ and N reflects the number of years of existence of the cartel.

Finally, the base fine is calculated as the sum of two amounts established in accordance with the above:

$$f^b = f_i^g + f^d. \tag{2.1}$$

Consider the following examples:

minor infringement, short duration: $f^b = f_m^g + f^d = f_m^g + f_m^g = 2f_m^g$,

serious infringement, medium duration: $f^b = f_s^g + f^d = f_s^g + kf_s^g = (1+k)f_s^g$,

very serious infringement, long duration: $f^b = f_v^g + f^d = f_v^g + Nkf_v^g = (1+Nk)f_v^g$.

So, we can conclude that, in general, the European penalty system for antitrust law violations exhibits linear dependence of the penalty on the level of offense.

Aggravating and Attenuating Circumstances

The basic amount is increased when there are aggravating circumstances such as repeated infringement of the same type by the same undertaking, refusal to cooperate, or having a leading role in the infringement. This corresponds to the variable n in the expression below.

The basic amount is decreased when there are attenuating circumstances such as having a passive role in the undertaking, termination of the infringement as soon as the Commission intervenes, or effective cooperation by the undertaking in the proceedings. This corresponds to the variable s in the expression below.

The final amount of the fine (F) is determined as

$$F = If^b,$$

where $I > 1$ if there are any aggravating circumstances, and $I < 1$ if there are any attenuating circumstances.

To be more precise, the final amount of fine is determined according to the following expression, which is adopted from Wils (2002)[6]:

$$F = f^b * \frac{(100 + i - j)}{100} * \frac{(100 - k)}{100} \tag{2.2}$$

where f^b is the base fine that is determined on the basis of gravity and duration according to expression (2.1), i is the percentage figure reflecting any aggravating circumstances, j is the percentage figure reflecting any attenuating circumstances, k is the percentage figure reflecting the application of the 1996 leniency notice, and f is the final figure of the fine. In this way, Wils translates the various steps contained in the 1998 EC Guidelines for calculating fines into a simple expression (2.2).

In general, according to the 1998 EC Guidelines, the fine is determined as a function of the following form:

$$F = f(g, d, s, n),$$

where g denotes the gravity of the offence, d is duration, s reflects attenuating circumstances, and n reflects aggravating circumstances. The function f is assumed to be strictly decreasing in s and strictly increasing in g, d, and n.

[6]See Wils (2002), supra note 22, p. 252, footnotes 20 and 21.

Fine after each stage in mil. €	**Base fine**
40	Gravity: very serious infringement
62	Duration: 5 years and 9 months, implying an increase of 55% of the amount determined according to gravity and result in a base fine of €62 million.
	Individual fine
99.2	Aggravating factors imply an increase of 60% of the base fine
84.1	Reduction of fine due to the max limit of fines (10% of overall turnover)
50.4	40% reduction due to application of leniency policy
50.4	Total fine

Figure 2.2: Determination of the fine for UCAR according to the EC guidelines.

An Example of a Fining Decision[7]

The determination of the fine by the Commission for the European part of the graphite electrode cartel, UCAR International, is described in Figure 2.2. The nature of the infringement was deemed to be very serious, because UCAR had engaged in market-sharing and price-fixing practices, which were implemented with full knowledge of the illegality of the actions. In considering the actual impact of the infringement, the decision notes that during the time of the cartel agreement, prices nearly doubled. Moreover, the producers represented almost 90% of the worldwide and EEA market for the product and the prices were not only agreed but also announced and implemented. Hence, the amount of the fine according to gravity for the two main producers, UCAR and SGL, was selected to be 40 million euro.

Aggravating factors included UCAR's role as one of the ringleaders and instigators of the cartel and the continuation of the infringement after the investigation started. Although UCAR was not the first company that provided the Commission with decisive evidence, it contributed substantially to establishing important aspects of the case and the Commission, therefore, granted a reduction of 40% of the fine.

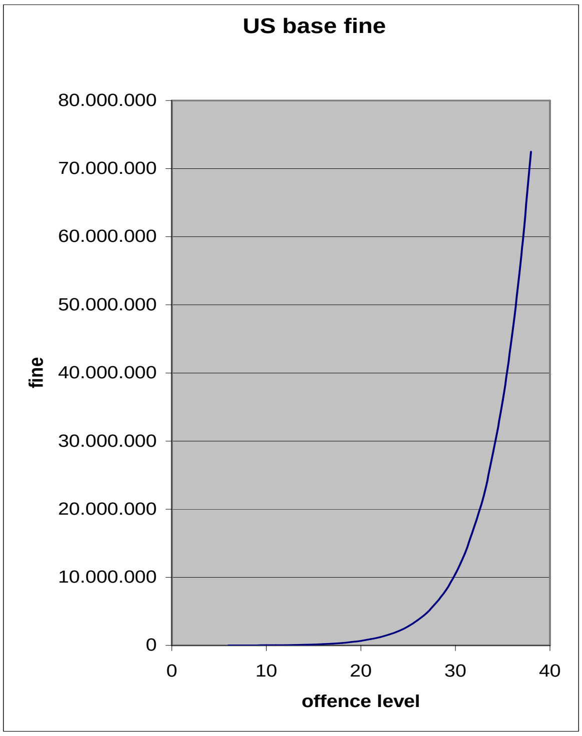
In the next section, we describe the US penalty system, point out its advantages and disadvantages relative to the European system, and summarize the results of the comparison.

2.3.2 US System

According to the US sentencing guidelines, the base fine is the greatest of the amounts of the "level fine" from Figure 2.3 corresponding to the offense level, the pecuniary gain

[7]This example is adopted from Wehmhoerner (2005).

to the organization from the offense, and the pecuniary loss from the offense caused by the organization.



Offense	Level Fine
6	5.000
7	7.500
8	10.000
9	15.000
10	20.000
11	30.000
12	40.000
13	60.000
14	85.000
15	125.000
16	175.000
17	250.000
18	350.000
19	500.000
20	650.000
21	910.000
22	1.200.000
23	1.600.000
24	2.100.000
25	2.800.000
26	3.700.000
27	4.800.000
28	6.300.000
29	8.100.000
30	10.500.000
31	13.500.000
32	17.500.000
33	22.000.000
34	28.500.000
35	36.000.000
36	45.500.000
37	57.500.000
38	72.500.000

Figure 2.3: US base fine as a function of offence level.

Figure 2.3, representing the US fine, implies that the penalty schedule exhibits a convex increasing function of the level of offense. This makes sense, since the higher the level of offense the higher the gains from the cartel for the firms and, at the same time, the higher the harm to consumers in terms of loss of consumer surplus.

The base fine for violations of antitrust law is determined according to the following formula:

$$f^b = \max\{\text{level fine},\ \text{gain from offense, loss from offense}\}$$

According to the US Sentencing Guidelines[8], the gain and the loss from the offense are estimated as follows:

"The fine for an organization is determined by applying Chapter Eight (Sentencing of Organizations). In selecting a fine for an organization within the guideline fine range, the court should consider both the gain to the organization from the offense and the

[8]See §2R1.1, note 3.

harm (loss in consumer surplus) caused by the organization. It is estimated that the average gain from price-fixing is 10% of the selling price. The loss from price-fixing exceeds the gain because, among other things, injury is inflicted upon consumers who are unable or for other reasons do not buy the product at the higher prices. Because the loss from price-fixing exceeds the gain, subsection (d)(1) provides that 20% of the volume of affected commerce is to be used in lieu of the pecuniary loss under §8C2.4(a)(3). The purpose for specifying a percent of the volume of commerce is to avoid the time and expense that would be required for the court to determine the actual gain or loss. In cases in which the actual monopoly overcharge appears to be either substantially more or substantially less than 10%, this factor should be considered in setting the fine within the guideline fine range."

Finally, the base fine is determined according to the expression

$$f^b = \max\{level\ fine, 0.2t^i\},$$

where t^i denotes the volume of affected commerce.

This structure of the penalty can be linked to section 2.2, where we discussed the economic approach to fine imposition. Now we can conclude that the US system is much closer to what economic theory would suggest, since it not only takes into account the fact that the fine should be related to the illegal gains from price-fixing, which correspond to the area PS in Figure 2.1, but also suggests that the fine should compensate the total loss to consumers caused by price-fixing. In other words, consumers should be compensated for the decline in Consumer Surplus that is represented in Figure 2.1 by adding up areas PS and Net loss in SW, which is approximately twice as high as the illegal gains. Hence, although the US approach does not solve all problems, it seems to be at least conceptually better than the European penalty system.

Further, the level of offence is determined by the court according to Chapter Two (offense conduct) of the US Sentencing Guidelines and Chapter Three, Part D (Multiple counts). The court approximates the loss in order to calculate the offense level according to §2R1.1 (note 3) on the basis of the volume of commerce done by the defendant or his principal in goods or services that were affected by the violation.

They start with offense level equal to 10 points.

(1) If the conduct involved participation in an agreement to submit non-competitive bids, increase the offence level by 1 point.

(2) If the volume of commerce attributable to the defendant was more than $400,000, adjust the offense level as follows:

(A) More than $400,000; add 1
(B) More than $1,000,000; add 2
(C) More than $2,500,000; add 3
(D) More than $6,250,000; add 4
(E) More than $15,000,000; add 5
(F) More than $37,500,000; add 6
(G) More than $100,000,000; add 7.

The base fine is increased if the organization has prior history (was recorded in the past). In particular, if the organization committed an offense less than 10 years previously, add 1 point, or if the organization committed an offense less than 5 years previously, add 2 points. The base fine is also increased if the organization conducted a violation of an order. If the organization violated a condition of probation by engaging in similar misconduct, i.e., misconduct similar to that for which it was placed on probation, add 2 points. Finally, the base fine is increased by 3 points if the organization conducted an obstruction of justice.

The base fine is decreased if the firm self-reported, cooperated during the investigation, and accepted responsibility. If the organization, either prior to an imminent threat of disclosure or government investigation or within a reasonably prompt time after becoming aware of the offense, reported the offense to appropriate governmental authorities, fully cooperated in the investigation, and clearly demonstrated recognition and affirmative acceptance of responsibility for its criminal conduct, 5 points are subtracted. If the organization fully cooperated in the investigation and clearly demonstrated recognition and affirmative acceptance of responsibility for its criminal conduct, 2 points are subtracted. Finally, if the organization clearly demonstrated recognition and affirmative acceptance of responsibility for its criminal conduct, 1 point is subtracted.

An Example of a Fining Decision[9]

To provide an overview of the fining method behind the guidelines, the application of the guidelines in the determination of the fine for the US part of the graphite electrode cartel, UCAR International, is described in Figure 2.4. UCAR was accused of price-fixing in the US from 1992 to 1997. The memorandum was filed in April 1998 by the District Court for the Eastern District of Pennsylvania and follows the US guidelines in the calculation of the fine.

[9]This example is adopted from Wehmhoerner (2005).

Fine in US $ mil after each step	**Culpability score**	**Base fine**
142.6		20% of the volume of commerce of US $713 million of UCAR's US sales between July 1992 - June 1997
		Aggravating and attenuating factors
	+5	Starting point as fixed in the guidelines
	+4	1000 employees and high-level personnel involved.
	-2	Acceptance of responsibility and full cooperation.
199.64 to 399.28	=7	Culpability score of 7 implies a min multiplier of 1.40 (40% increase in the base fine) and a maximum multiplier of 2.80 (180% increase in the base fine), yielding a fining range of US $199.64 to US $399.28 million.
110		Alternative fine because of UCAR's inability to pay (15.4% of US volume of commerce)

Figure 2.4: Determination of the fine for UCAR according to the US guidelines.

The fining range is determined by calculating 20% of the volume of affected commerce over the entire duration as a starting fine. Subsequently, for each factor, such as the size of the undertaking in terms of the number of employees, the corresponding points with which to increase or decrease the culpability score can be read from the guidelines. There is a direct quantitative link between these factors and the fining range through the use of the culpability score, which determines the fining range. However, neither the guidelines nor the decision explain how the alternative fine should be determined in case of inability to pay.

2.3.3 Comparison

To summarize the above analysis, we can conclude that the US system is more advanced, since there is no upper bound on the fine as in Europe, where the fine is limited from above by the amount of 10% of the total annual turnover of the firm. However, this upper bound is determined without taking into account dynamic aspects. We believe that the existence of this upper bound and the fact that this boundary is not high enough appears to be one of the main sources of inefficiencies. Other advantages of the US system are that the base fine is a convex and increasing function of the level of offence and depends not only on the illegal gains from the cartel, but also on the estimated loss to consumers from price-fixing, by employing the 20% rule. Moreover, exclusive reliance on corporate sanctions in Europe is unlikely to result in effective deterrence of price cartels due to the existence of the "too-low" upper bound on the fine and also because corporate sanctions do not always guarantee adequate incentives for responsible individuals within the firm. Hence, the introduction of sanctions for individuals and imprisonment in European competition law could also be a part of the solution.

The following table summarizes the main results of the analysis.

Table 2.1: Comparison of the US and EC penalty systems for antitrust violations.

	US
upper bound	no
base fine is determined	as $f^b = \max\{\text{"level fine"}, 0.2t^i\}$
basis for fine	volume of commerce involved in crime
functional form of fine	convex in the level of offense
damage to consumers	taken into account
imprisonment	yes
	Europe
upper bound	$F \leq 0.1T$
base fine is determined	by seriousness and duration, $f^b = f^g + f^d$
basis for fine	decision of the European Commission
functional form of fine	linear in the level of offense
damage to consumers	generally, it is not taken into account
imprisonment	no

2.4 Recent Historical Developments in Antitrust Law and Statistical Overview

2.4.1 Antitrust Law Enforcement in Europe

The Commission used its fining power under the EC Treaty for the first time in July 1969 in the Quinine and the Dyestuffs cases, in which it imposed fines ranging from 10,000 to 210,000 units of account on 6 and 10 companies, respectively. After 25 years, by the end of July 1994, the Commission had taken 81 decisions imposing a total of 346 individual fines for infringements of Articles 81 and 82 EC Treaty.

The number of fining decisions increased over the 1970s and the 1980s, reached its highest level in the second half of the 80s, and declined till the end of the 90s. During the period 1999-2003, the number of fining decisions again increased dramatically.[10] Over

[10]See Figure 2.3.

Period	Number of decisions	Average fine per decision in millions of Euro	Total fine in millions of Euro
1969-1973	7	5,2	36,4
1974-1978	11	0,53	5,83
1979-1983	16	1,5	24
1984-1988	28	8,3	232,4
1989-1993	17	12,1	205,7
1994-1998	17	35,46	602,82
1999-2003	46	84,76	3899,21
total	142	147,85	5006,36

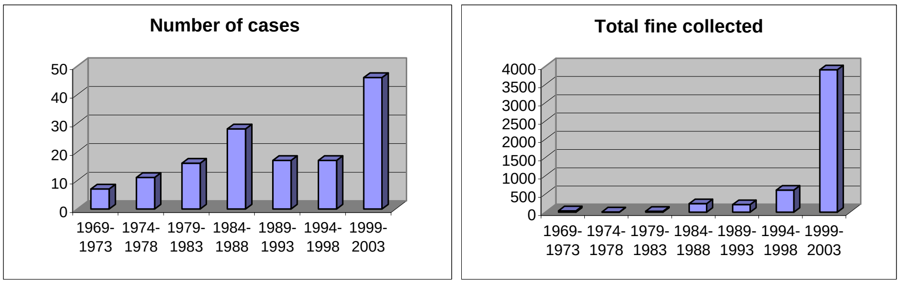


Figure 2.5: Fines imposed by EC in 1969-2003, including all antitrust cases.

the period 1969-1993, 61 decisions were based on Article 81, of which 30 concerned horizontal infringements (including cartels), 29 vertical infringements, and 2 infringements involving intellectual property rights. The highest single fine imposed in the period 1969-1994 was a fine of 75 million ECUs imposed on Tetra Pak for abuse of its dominant position. The highest sum of fines imposed in one single decision amounted to approximately 248 million ECUs, being the sum of the 42 fines imposed in 1994 on the undertakings and associations involved in the Cement cartel.[11] See also Figure 2.5.

Figure 2.6 represents the amounts of the fines imposed on ”cartels only” according to decisions of the European Commission in 1969-2004.[12]

From Figure 2.5 we obtain that the level of the fines in real terms declined dramatically during the 1970s. The ”new, tougher policy” announced in 1979 was really nothing more than a return to the fine levels of the first years of the Commission’s fining practice. Only from the mid-1980s on did the ”new, tougher policy” become visible.

Until 1998, fines were calculated based on the original rules dating from 1969. These rules use a base fine of 2 to 4% of the turnover in the product concerned by infringement, with a few percentage points added in case of infringements of a relatively long duration and serious nature. Starting from 1998, a new system (fine=10% of turnover) was implemented. This system was extensively discussed in section 3.1 above. However, one does not need a sophisticated econometric study to see that such fines are inadequate

[11]Decision of 30 November 1994, Cement,[1994] OJ L343/1.

[12]Data are taken from Jones, Van Der Woude, Lewis (1999) and Geradin and Henry (2005).

year	total fine
1969	0,99
1970	0
1971	0
1972	0
1973	9
1974	0,358
1975	0,1
1976	0
1977	0
1978	0
1979	0
1980	0
1981	0
1982	2,535
1983	1,25
1984	20,425
1985	0
1986	65,535
1987	0
1988	73,9
1989	9,5
1990	18
1991	0
1992	48,636
1993	0
1994	399,296
1995	0
1996	0,645
1997	0
1998	119,59
1999	272,98
2000	202,61
2001	1811,55
2002	1128,34
2003	540,11
2004	372,83

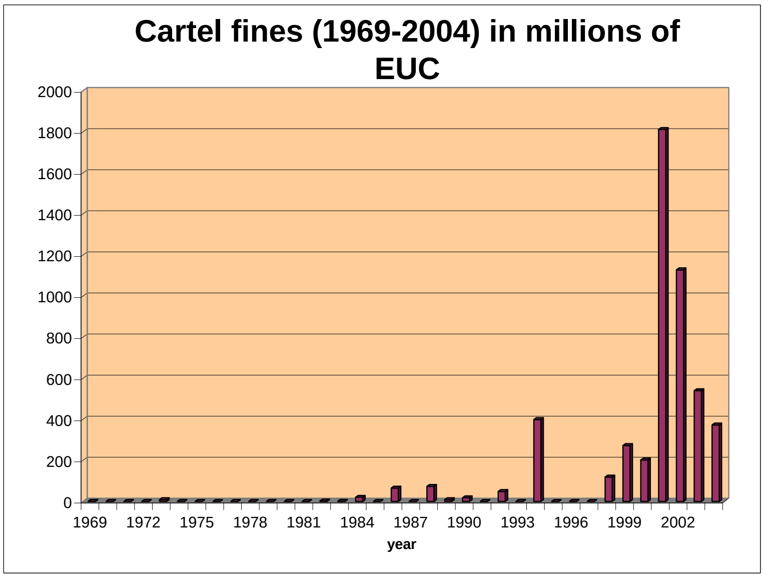


Figure 2.6: Cartel fines in EU (1969-2004).

to deter large categories of the more serious infringements.[13] Hence, the new system introduced in 1998 still requires some changes.

The next considerable step in the historical development of antitrust law was made in 1996, when the European Commission introduced Leniency Programs. Leniency programs give reduced fines to firms which reveal information about the cartel to the antitrust authority. Moreover, the reduction of fines should take place even if firms reveal information after an investigation has started. Leniency programs improve welfare by sharply increasing the probability of collusive practices being interrupted and by shortening the investigation.

However, leniency programs introduced in 1996 were heavily criticized for their lack of transparency and certainty. The vagueness and legal uncertainty embedded in the 1996 notice explained why the notice was not as effective as, for example, the US corporate amnesty program[14], which receives on average two applications per month. For this

[13]Recall the economic approach to fine imposition discussed in section 2.2.2 and the numerical example provided in section 2.3.1. It follows from the above discussion that the gain obtained from the infringement by the violator, divided by the probability of being fined, constitutes a floor below which fines can certainly not deter. Assuming that the probability of being fined is 16%, 10% price markup, and cartel duration of 5 years, it would still mean that deterrence cannot be achieved with fines below 150% of annual turnover in the product concerned.

[14]See D.O.J. (1993).

reason, in 2002, the new EC leniency notice was introduced. With this notice, the Commission intended to increase the effectiveness of the leniency programs by aligning more closely the level of the reduction of fines and the value of a firm's contribution to establishing the infringement. More importantly, the Commission committed itself to guaranteeing immunity from fines if the requisite conditions were fulfilled. Indeed, it would seem that with the entry into force of the new leniency program there has been a substantial increase in leniency applications.[15] One of the greatest improvements in the new rules was the fact that the new notice provides for the opportunity to receive partial immunity even after the Commission has commenced an investigation. According to US Department of Justice officials, approximately one half of all immunity applications are made after the beginning of an investigation in the US.

The currently used system of regulations, which has been discussed extensively in section 2.3, was brought into life in 1998.[16] Fines imposed by the European Commission now appear to be the main method of Competition Law enforcement in Europe, as opposed to the US, where non-monetary punishment schemes are also used.[17]

2.4.2 US Antitrust Law Enforcement

The roots of US Competition Law enforcement go back to 1890, when the Sherman Antitrust Act was introduced. In 1997, certain changes in the approach to the US enforcement of international cartels and clarifications to the Sherman Antitrust Act were developed[18]. These changes influenced dramatically the amounts of the fines obtained. In the 10 years prior to 1997, the Division obtained, on average, $29 million in criminal fines annually. The amounts obtained in the following years were considerably higher, as can be seen in Figure 2.7 below[19].

In Figure 2.7 it can be seen that two points in time, 1993 and 1997, were characterized by a considerable increase in the amounts of fines collected. The increase of the fines

[15] The XXXIIIrd Report on Commission Policy, European Commission, 2003, stated that the Commission has received 34 applications for immunity dealing with at least 30 separate alleged infringements, see para. 30.

[16] Guidelines on the Method of Setting Fines Imposed, PbEG 1998.

[17] Connor and Bolotova (2005) also provide an empirical study of the impact of legal environmental on cartel overcharges.

[18] See D.O.J. (1999).

[19] Data collected by the US Department of Justice: Antitrust Division Workload Statistics FY 1994-2003 (http://www.usdoj.gov/atr/public/12848.htm). See also G.R.Spratling "Are the recent titanic fines in Antitrust cases just the tip of the iceberg?", March 6, 1998.

Year	amount of fines obtained (million $)
1987	17,9
1988	30,8
1989	28,5
1990	23,6
1991	20,4
1992	23,7
1993	42,3
1994	40,2
1995	41,6
1996	26,8
1997	205,2
1998	265
1999	1100
2000	303,2
2001	270,8
2002	93,8
2003	63,6

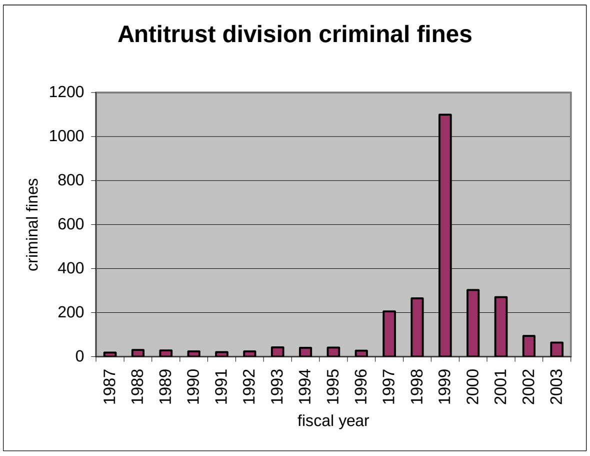


Figure 2.7: Cartel fines in the US obtained in 1987-2003.

collected in 1993 almost doubled those collected in 1992. This can be connected with the major modification of US leniency programs. The dramatic (almost 10 times) increase in the fines obtained in 1997 and later can be connected with the fact that the US started to pay more attention to the prosecution of international cartel activity.

After 1997 the Division has obtained fines of $10 million or more against U.S., Dutch, German, Japanese, Belgian, Swiss, British, and Norwegian-based companies[20]. It is remarkable that during the period before 1997 there were only 5 cases in which the fine imposed was greater than or equal to $10 million. After 1997, there were 28 cases in which a fine greater than $10 million was imposed.

In sum, we conclude that there are three important directions for development of US Department of Justice competition policy: leniency programs, international cartel enforcement, and refinement of the current penalty system. As has already been mentioned, the revised Corporate Amnesty Program[21] has resulted in a surge of amnesty applications. Under the old amnesty policy, the Division obtained roughly one amnesty application per year. Under the new policy, the application rate has been more than one per month. In recent years, cooperation from amnesty applications has resulted in higher scores of convictions and over $1 billion in fines.

The second important direction of the US Department of Justice competition policy

[20]See Table 3 on Sherman Act Violations Yielding a Fine of $10 Million or More, http://www.usdoj.gov/atr/public/speeches/1583.htm.

[21]Three major revisions were made to the program: amnesty is automatic if there is no pre-existing investigation; amnesty may still be available even if cooperation begins after the investigation is underway; and all officers, directors, and employees who cooperate are protected from criminal prosecution.

is the international cartel enforcement. This enforcement emphasis was quite successful in cracking international cartels, securing the convictions of major conspirators, and obtaining high fines. During the year 2000, the Division conducted approximately investigations of suspected international cartel activity. Currently, approximately one-third of the Division's criminal investigations involve suspected international cartel activity.

The third and most straightforward direction is a refinement of the current penalty system. Namely, increasing the level fines and multiplier on the volume of affected commerce[22].

Currently, the main differences between European and US antitrust law enforcement are the successfulness of leniency programs and the types of sanctions used. Firstly, the successfulness of leniency programs, which is highly correlated with the clarity and transparency of those programs, is definitely more obvious in the US. Secondly, criminal prosecution of individuals and imprisonment are prominent components of US antitrust law, whereas these are absent in European legislation. Accordingly, the US leniency program has substantially more to offer to individual cartel members.

2.5 Conclusions

We conclude this chapter by summarizing the analysis of data on penalties imposed, the number of antitrust cases uncovered, and the policy implications. We will also review recent laws and new enforcement measures for cartel deterrence suggested by OECD and discuss their relevance.

Following our comparative analysis of the US and EC antitrust laws and empirical analysis of the performances of current antitrust law enforcement schemes (fines collected and cases discovered), we conclude that the obvious result is that the current sanctions (penalties, sanctions against private persons) are not high enough to completely block cartel formation. A possible remedy is to set the base fine closer to the level of the harm as implied by economic theory. It may be worth investigating whether the increase in resources needed to better estimate the turnover of the affected commerce as well as the overcharge would not be offset by the improved deterrence by attempting to set the base fine closer to the harm. Additionally, the maximum limit should be set to the worst possible harm that could result from an infringement. For the EC guidelines, this would imply reconsidering the limit of 10% of worldwide turnover. Similarly, the alternative

[22]See section 2.3.2.

maximum of doubling the loss or gain in the US[23] may be too low for infringements that are difficult to detect. Given that fining decisions currently do not account for factors such as the probability of detection or the deadweight loss, the trend towards increasing fines appears to be a good development.

At the same time, we found certain trends in the behavior of the amounts of the fines collected and cases discovered connected with the recent changes in regulations. Distinct examples are the two-times increase in the total amounts of fines collected in the US in 1993 after the introduction of leniency programs and the tenfold increase in the total amounts of fines collected in the US in 1997 after the introduction of the strategy of fighting international cartels.

We also should not forget about the other dimension of this problem. This refers to the fact that a cartel is itself a group, and consequently the antitrust authority can influence the internal stability of this group. Leniency programs do this job. They not only increase the number of discovered cartels, but also help to reduce the amount of money spent during investigations. These programs have been introduced in many countries already, but antitrust authorities still need to work on the issues of making those programs clearer and more transparent.

The third perspective direction is international cooperation and sharing of information about cartels. This policy will substantially increase the effectiveness of sanctions against international cartels. Theoretical grounds for such policy implications were given by Levenstein and Suslow (2001), Evenett, Levenstein, and Suslow (2001), and Suslow (2002). They state that the exchange of information about cartels and international cooperation between countries would also increase the effectiveness of leniency programs.

[23] Recall the 20% rule in section 2.3.2.

CHAPTER 3

Determination of Optimal Penalties for Antitrust Violations in a Dynamic Setting

3.1 Introduction

In this chapter we incorporate specific features of antitrust law enforcement, which are in practice now in the US and the European Union, into a dynamic framework of utility maximization with two players having conflicting objectives. In the particular case of violations of antitrust law, those two players are the firm of regulated monopoly type, which rises prices above marginal costs level, or the firm, which participates in cartel agreements, and the Antitrust Authority, whose aim is to prevent price-fixing or cartel formation in the industry.

According to the US Sentencing Guidelines for Organizations and the Guidelines on the Method of Setting Fines Imposed for Violations of Competition Law in Europe, the penalty schemes for antitrust violations are based mainly on the gravity of the violations, which is determined on the basis of the turnover involved in the infringement. To be more precise, in the European regulation the penalty imposed depends on the gravity and duration of the infringement in a linear manner. The level of offence is measured by the turnover involved in the infringement, which is defined as the total sales of the product involved over the whole period of existence of the cartel. In the US sentencing guidelines for organizations the system of fine imposition for antitrust violations is different. There we observe that the penalty schedule for the base fine is represented by a convex increasing function of the level of offence.

In order to investigate the efficiency of the current penalty schemes we incorporate these two features of penalty systems for antitrust law violations into a dynamic model of intertemporal utility maximization by modelling penalty schedule in the stylized form as a linear or quadratic functions of the degree of price-fixing and time. Similar to Feichtinger (1983) the set up of the problem leads to a differential game. The authorities attempt to minimize the social loss caused by price-fixing, whereas the firm wants to maximize the profit gained from price-fixing.

It is found that the stylized form of the existing penalty schemes would not succeed, in the sense that it cannot provide complete deterrence. Therefore, we try to find a more efficient functional form of penalty schedule for violations of antitrust law. Finally, we suggest a new penalty system which is the most efficient from the point of view of complete deterrence of price-fixing in dynamic settings.

We relate our analysis to the general literature on crime and punishment, starting with Becker (1968). In his seminal paper, Becker (1968) studied the problem of how many resources and how much punishment should be used to enforce different kinds of legislation. The decision instruments are the expenditures on police and courts influencing the probability that the offender is convicted, and the type and size of punishment for those convicted. The goal was to find those expenditures and punishments that minimize the total social loss. This loss is the sum of damages from offences, costs of apprehension and conviction, and costs of carrying out the punishment imposed.

The main contribution of Becker's work was to demonstrate that the best policies to combat illegal behavior were part of an optimal allocation of resources. Becker (1968) investigates this problem using a static economic approach to crime and punishment. He derives that in a static environment the optimal fine should be a multiple of the offender's benefits from crime and inversely related to the probability of detection. So, since an increase in the probability of control causes an increase in the costs of detection, the least costly policy for the antitrust authority would be to decrease the probability of control and increase the fine itself. But in this case existence of legal limitations concerning the upper bound of fine would bind fine from above and, hence, least costly policy would not be implementable. Later, in the paper by Souam (2001) optimal crime deterring policies under different regimes of fines were discussed in the static setting with asymmetric information. In Leung (1991), Feichtinger (1983), and Fent et al. (1999, 2002) dynamic (intertemporal) trade-offs between the damages generated from the offences and the costs of the control instruments were studied. More precisely, their papers try to determine a mix of policy variables, like prevention, treatment and law enforcement, which minimize the discounted stream of total social loss.

Now we give a more detailed review of the papers related to the problem, addressed in the current chapter. Leung (1991) introduces a dynamic model of optimal punishment, where the optimal fine is calculated as a solution to an optimal control problem. The results of this model differ a lot from the implications of the static model of Becker (1968). Leung demonstrates that the optimal fine derived from a dynamic model is drastically different from the one obtained from a static model. He finds that the fine which would block the crime can actually be less than the harm induced by the infringement, which contradicts the result of Becker. Leung argues that Becker's approach will not generate the optimal outcome, i.e. the outcome which maximizes welfare, in a dynamic environment. In fact, according to Leung (1991) it would cause overcomplience because the multiple fine imposes too heavy a penalty on the offender.

A considerably different approach was suggested in Fent et al. (1999, 2002). They investigate optimal law enforcement strategies in case punishment is modelled as a function depending not only on the intensity of crime (offence rate) but also on the offender's prior criminal record. This idea was adopted in Fent et al. (1999) in an optimal control model with the aim to discover the optimal intertemporal strategy of a profit maximizing offender under a given, static punishment policy in the model with only one agent. In Fent et al. (2002) the framework described above was extended to an intertemporal approach of utility maximization, considering two players with conflicting objectives. The authorities attempt to minimize the social loss caused by criminal offences, whereas the offending individual wants to maximize the profit gained from offending. This leads to a differential game, which makes it possible to study competitive interactions in a dynamic framework. The criminal record takes the role of a state variable. A high record increases the punishment an offender expects in case of being convicted.

Modeling intertemporal trade-offs requires application of tools like dynamic programming, optimal control theory and, if there is strategic interaction between players, differential games. All the papers mentioned above investigate the problem of optimal dynamic law enforcement and minimization of social loss from crime by modeling the interactions between the offender, who commits the crime, and the authority, whose aim is to prevent the crime. In this chapter we suggest a similar approach. We analyze a differential game between the offender and the authority, whose aim is to prevent the crime, to study the situation of violation of antitrust law by the firm, which fixes the prices above competitive level or participates in a cartel.

Technically our analysis will be close to the paper by Feichtinger (1983), in which he studies violations of criminal law by means of a differential game solution to a model of competition between the thief and the police. We extend his framework by allowing for the penalty for violation to vary over time. Moreover, we introduce the fine as a function of the current degree of offence and probability of law enforcement at each instant of time.

The chapter is organized as follows. In section 3.2 we set up the model describing the intertemporal game played between a firm engaged in price-fixing and the antitrust authority, and recall the modified static microeconomic model of price-fixing. In section 3.3 the differential game will be solved and we show that it is impossible to have complete deterrence under current European and US systems of penalties for antitrust violations. In section 3.4 a new penalty scheme, which gives the desired outcome with no collusion, will be suggested. Section 3.5 provides a summary of our results and outlines possible extensions and generalizations of the model. Finally, in appendixes we provide proofs of the main results of the chapter.

3.2 Description of the Problem

A model is designed to determine optimal penalty schemes for antitrust violations and cartel deterrence in the framework of differential games. There are two types of agents. First, there are the firms, which can perform illegal activities, such as price-fixing and cartel formation or violations of the price limits imposed by the authority on the regulated monopoly. They obtain strictly positive gains from price-fixing in each period that the cartel was present in the market. Second, we have the antitrust authority, which can inspect those firms, and, in case violation is detected, punish them by imposing a fine $s(t)$, where t reflects the time index. The interactions of the agents are modeled as a continuous time problem with finite planning horizon T.

The aim of the firm is to maximize its total expected gain from setting its price above the competitive level by choosing $q(t)$. This variable represents the degree of illegal activities with respect to price-fixing (analogous to the "pilfering rate" in the model of competition between thief and police in Feichtinger (1983)). This variable will be described in more detail in the next subsection, which presents the microeconomic model underlying the problem of fighting price-fixing agreements.

The antitrust authority is modeled as a second decision maker. It also has one instrument, which is the "rate of law enforcement" (or probability of control by the antitrust authority) denoted by $p(t)$. The aim of the antitrust authority is to maximize welfare. This implies that the rents from collusion for the firm need to be reduced. So, the aim of the antitrust authority is to prevent cartel formation at the lowest possible costs.

The profit of the firm in each period or the rent from collusion per period above the competitive profit is $\pi(t) > 0$.[1]

In order to be able to set up the dynamic model and determine the objective functionals of both players, we first describe a static microeconomic model of price-fixing.

3.2.1 Static Microeconomic Model of Price-fixing

Let us consider an industry with M symmetric firms engaged in a price fixing agreement. Assume that they can agree and both increase prices from the competitive level $P^c = c$ to $P > c$, where c is the fixed marginal cost in the industry. Since firms are symmetric, each of them has equal weight in the coalition and consequently total cartel profits will be divided equally among them.[2] Hence, the whole market for the product (in which the price-fixing agreement has been achieved) will be divided equally among M firms, so each firm operates in a specific market in which the inverse demand function equals $P(Q) = 1 - Q$. Demand functions are identical in the submarkets. Under these assumptions we can simplify the setting by considering not the whole cartel (group of violators) but only one representative firm, and apply similar sanctions to all the members of cartel.[3]

Let P^m be the monopoly price in the industry under consideration, and let $P = 1-Q$ be the inverse demand for a particular firm. In order to be able to represent the consumer surplus and extra profits from price fixing for the firm in terms of the degree of collusion, we define the variable q, which denotes the degree of price-fixing, by $q = \frac{P-c}{P^m-c}$, where P is the price level agreed by the firms. In words, q can be interpreted as a ratio of the realized price increase above the competitive level to the maximal price increase that is possible in case of a monopoly. Then it holds that $q \in [0, 1]$ and extra profits from price

[1] Competitive profit (π^c) is assumed to be zero.

[2] We also assume that there is no strategic interaction between the firms in the coalition in the sense that we abstract from the possibility of self-reporting or any other non-cooperative behavior of the firms towards each other.

[3] Of course, in these settings the incentives of the firms to betray the cartel can not be taken into account and the possibility to influence the internal stability of the cartel is not feasible. But this is the topic of Chapters 5 and 6.

fixing for this particular firm will be determined according to the following formula:

$$\pi(q) = q(\frac{(1-c)}{(P^m-c)} - q)(P^m-c)^2.$$

Let $(P^m - c)^2 = A$. With linear demand $P = 1 - Q$ we observe that $P^m = \frac{1+c}{2}$, so that $\frac{1-c}{P^m-c} = 2$ and, consequently, it holds that $A = \frac{(1-c)^2}{4} = \Pi^m$ (monopoly profit in this particular market).

The instantaneous producer surplus, consumer surplus and net loss in social welfare are presented in Figure 3.1.

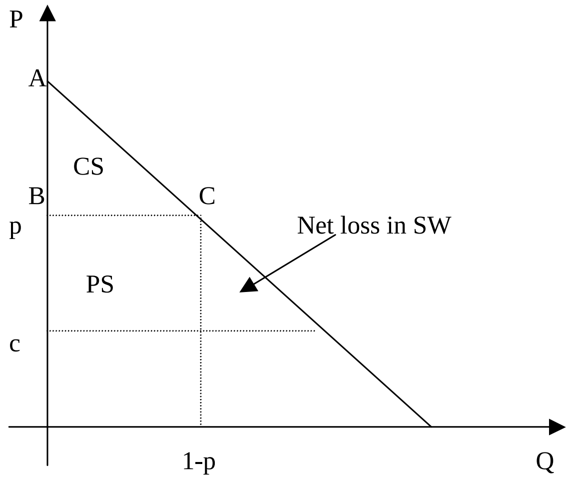


Figure 3.1: Representation of producer surplus, consumer surplus, and net loss in total social welfare in the price-quantity diagram.

Following the above analysis the Producer Surplus equals

$$PS(q) = \pi(q) = \Pi^m q(2-q) \ ,$$

the Net loss in Total Social Welfare is the area of the right triangle

$$NLSW(q) = \frac{1}{2}\Pi^m q^2 \ ,$$

while the Consumer Surplus is determined by the area of triangle ABC:

$$CS(q) = \frac{1}{2}\Pi^m (2-q)^2 \ .$$

Note, that $PS''(q) < 0$, $NLSW''(q) > 0$, $CS''(q) > 0$.

Under the assumption that Π^m is equal to $\frac{1}{4}$ (or $c = 0$), these three functions are presented in Figure 3.2.

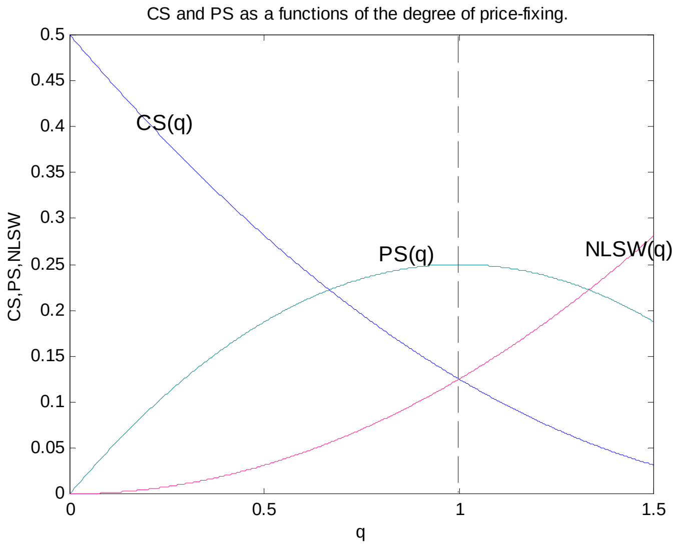


Figure 3.2: Consumer surplus, producer surplus and net loss of social welfare as functions of the degree of price-fixing.

The consumer surplus is lower the higher the degree of collusion. The loss in consumer surplus is higher the higher the degree of collusion, while the rents from cartel for the firm are higher the higher the degree of collusion.

It should be mentioned that in the literature two main objectives of the authority are considered. First, the authority aims to maximize total welfare, i.e. the sum of consumer and producer surpluses. Second, the authority's aim could be to maximize consumer surplus and at the same time minimize the rents from collusion for the firm. The second approach can be justified by the conjecture that the rents obtained through illegal activities are lost for society in most of the cases. So they should not be included in the regulator's maximization function.

Let us consider the first problem in a static setting. The antitrust authority is aiming to maximize $(CS + PS)$, i.e.,

$$maximize \;\; \{\frac{1}{2}\Pi^m(2-q)^2 + \Pi^m q(2-q)\} \;\; s.t. \;\; q \in [0,1].$$

This implies that total welfare is maximized for $q = 0$. Note that this is equivalent to the minimization of $NLSW$.

Let us consider now the second problem in a static setting. The antitrust authority is aiming to maximize CS and at the same time minimize the rents from collusion, i.e. keeping $PS = 0$ equal to competitive profit. In other words, the sum of net loss in social welfare and producer surplus will be minimized. This means that the problem is to minimize $\{\Pi^m q(2-q)+\frac{1}{2}\Pi^m q^2\}$. This is equivalent to minimizing $\{(2q-\frac{1}{2}q^2)\Pi^m\}$, which is equal to the minimization of the total loss from price-fixing for society. Consequently, in the settings where antitrust authority cares only about consumer surplus, the social welfare will be maximized when there is no collusion.

So we can conclude that in the static setting the outcomes of the two problems described above are equivalent in the sense that the antitrust authority should not allow for any collusion irrespective of whether it cares about total welfare of the society or only about the consumer surplus. For this reason we will assume that the aim of the antitrust authority will thus be to achieve zero degree of price-fixing (i.e. $q = 0$) in all the periods of the planning horizon in a dynamic setting as well.

3.2.2 Description of the Dynamic Game

To investigate the interactions between the firm and the antitrust authority we develop a differential game. We consider a firm (player 2) playing against the antitrust authority (player 1). The probability that the firm gets caught at time t, $F(t)$, is influenced by the degree of collusion that is control variable of the firm, $q(t)$, as well as the law enforcement rate that is control variable of the antitrust authority, $p(t)$, in the following manner:

$$\dot{F}(t) = p(t)q(t)[1 - F(t)]. \tag{3.1}$$

Note that $\Phi(t) = \dot{F}(t)[1 - F(t)]^{-1}$ is the hazard rate of the process leading to conviction of the firm. $\Phi(t)$ is the conditional probability of getting caught at time t provided that the firm has not yet been caught. Equation (3.1) says that the hazard rate Φ increases linearly with increasing activities of the firm and antitrust authority, and $F(0) = 0$ is the initial condition.

As usual two types of variables appear in the model: a state variable $F(t)$ (the probability distribution function of the time until the detection of the violation of the firm) and control instruments $q(t)$ (degree of collusion of the firm) and $p(t)$ (law enforcement rate of antitrust authority). Note that the state constraint $0 \leq F(t) \leq 1$ is satisfied automatically. The idea to use $F(t)$ as a state variable is based on Kamien and Schwartz (1971). Assume also that a once convicted firm is not able to collude any more until time T (so punishment is that harsh that the firm needs a lot of time to recover). The parameter r denotes the discount rate.

The objective function for the antitrust authority is given by

$$\max \int_0^T e^{-rt}[-(NLSW(q(t))+C(p(t)))[1-F(t)]+s(q(t),p(t))\dot{F}(t)]dt - e^{-rT}C_1(T)[1-F(T)]. \tag{3.2}$$

The term $C(p(t))$ reflects the costs for the antitrust authority of performing the audit activities (such as the number of inspections, salaries for auditors, etc.). The analysis of the game will be conducted for the case when the costs of law enforcement are quadratic, i.e. $C(p) = Np^2$.[4] As in the previous section, the term $NLSW(q(t))$ reflects the loss in instantaneous total social welfare due to a price increase by the firm. $NLSW(q)$ increases when q increases. The term $s(q(t),p(t))\dot{F}(t)$ reflects the expected revenue for the authority at time t if the cartel is discovered at this particular instant of time. The penalty $s(q,p)$ is a function of both the current degree of offence and the current probability of audit by the antitrust authority. Note that the higher the degree of collusion, $q(t)$, the higher the probability to be caught for the firm, and, consequently, the higher the expected punishment. Later in this chapter we will discuss three different types of penalty functions. The two penalty schemes discussed in section 3.3, namely, stylized EU and stylized US penalty schemes, depend only on the degree of collusion, while the penalty schedule discussed in section 3.4 is a function of both the degree of offence and the probability of audit. $C_1(T)$ is the terminal value (disutility) assessed by the antitrust authority if the firm is not yet caught at time T.[5] Note also that we assume that no additional costs arise after the firm has been caught. This is a reasonable assumption in the context of violations of antitrust law, since it is assumed that only a monetary fine can be imposed and this, contrary to imprisonment, is costless for the authority.

[4]However, qualitatively speaking the results obtained in the paper hold for costs of law enforcement being any increasing convex function of p. The solution of the game for the linear case $C(p) = Np$ is available from the author upon request.

[5]From the underlying static microeconomic model of price-fixing (section 3.2.1) we derive that the maximization of (3.2) is equivalent to a maximization problem of the following form:

$$\max \int_0^T e^{-rt}[(PS(q(t))+CS(q(t))-C(p(t)))[1-F(t)]+CS^{\max}F(t)+s(t)\dot{F}(t)]dt - e^{-rt}C_1(T)[1-F(T)],$$

where the term $PS(q(t))$ reflects the instantaneous producer surplus, $CS(q(t))$ is the instantaneous consumer surplus, and $CS^{\max}F(t)$ denotes the expected instantaneous consumer surplus associated with the time period after the conviction. This relates us back to the discussion in section 3.2.1 and the setting where the authority aims to maximize total social welfare.

The objective function for the firm is given by

$$\max \int_0^T e^{-rt}[PS(q(t))[1-F(t)]+PS^{comp}F(t)-s(q(t),p(t))\dot{F}(t)]dt+e^{-rT}C_2(T)[1-F(T)]. \tag{3.3}$$

Here the term $PS(q(t))$ reflects the instantaneous rents from collusion as defined in section 3.2.1. Expression $-s(q(t),p(t))\dot{F}(t)$ denotes the expected punishment for the firm at time t, i.e. the fine times the probability of being caught. $s(q(t),p(t))$ is the instantaneous penalty at the moment the firm is caught. As discussed above, $s(q,p)$ is assumed to be a function of both control variables. The term $PS^{comp}F(t)$ reflects the profits of the firm during the period after the conviction, when there is no price-fixing. Consequently, the expression PS^{comp} is equal to zero. Finally, $C_2(T)$ is the terminal value (utility) of the firm being not yet convicted in cartel formation at time T.

The corresponding differential game with two players, one state variable $F(t)$, and two control variables, $q(t)$ and $p(t)$, is represented by the expressions (3.1)-(3.3). The state space is $F(t) \in [0,1]$, and the set of feasible controls is $p(t) \in [0,1]$ for player 1 and $q(t) \in [0,1]$ for player 2.

The major difference with earlier papers on crime control (Feichtinger (1983)) is that we introduce $s(t) = s(q(t),p(t))$ being the penalty imposed on the firm as a function of both the degree of offence and the rate of law enforcement, which both can vary over time.

An economically reasonable assumption would be to set salvage values to be nonnegative, i.e. $C_1(T) \geq 0, C_2(T) \geq 0$. Moreover, further, in order to simplify the calculations, we assume a zero discount rate[6] ($r = 0$).

We also assume that players make their choices simultaneously and that the solutions to their control problems correspond to either Markovian or open-loop Nash Equilibrium strategies (see, e.g., Dockner et al. (2000)). Below we provide the formal definitions of Markovian and open-loop Nash Equilibria of a differential game with two players in case each player has only one control variable.

Definition 3.1 *The tuple* (ϕ,ψ) *of functions* $\phi,\psi : [0,1] \times [0,T) \longmapsto R$*, which belong to the sets of feasible controls defined as* $U^1(F(t),q(t),t) \subseteq R$ *and* $U^2(F(t),p(t),t) \subseteq R$*, is a Markovian Nash Equilibrium if for both players optimal control paths* $p(t)$ *and* $q(t)$ *of the control problem (3.1)-(3.3) exist and are given by the Markovian Strategies* $p(t) = \phi(F(t),t)$ *and* $q(t) = \psi(F(t),t)$ *for all* $0 < t < T$*.*

[6]However, most of the results of the chapter would not change if we would relax this assumption.

Definition 3.2 *The tuple* (ϕ, ψ) *of functions* $\phi, \psi : [0, T) \longmapsto R$*, which belong to the sets of feasible controls defined as* $U^1(F(t), q(t), t) \subseteq R$ *and* $U^2(F(t), p(t), t) \subseteq R$*, is an open-loop Nash Equilibrium if for both players optimal control paths* $p(t)$ *and* $q(t)$ *of the control problem (3.1)-(3.3) exist and are given by the open-loop Strategies* $p(t) = \phi(t)$ *and* $q(t) = \psi(t)$ *for all* $0 < t < T$.

It can be shown that for this particular game the set of Markovian Nash Equilibria will coincide with the set of open-loop Nash Equilibria. The proof will be provided in Appendix 2. For this reason it is sufficient to focus on obtaining an open-loop Nash Equilibrium.

3.3 Analysis of the Current EU and US Penalty Schemes

3.3.1 Stylized EU Penalty Scheme

In this section we consider a penalty scheme, which resembles the current European or Dutch systems[7]. We model the main feature of these systems, namely that the base penalty is proportional to the gravity of the infringement or to the turnover involved in the undertaking and does not depend on the rate of law enforcement. It should be mentioned, however, that the functional form described in equation (3.4) does not capture all the properties of the penalty schemes, which are determined in the current "Guidelines for the Setting of the Fines" (such as longer duration of the offence or leading role in the infringement would increase the penalty). That is why we call this scheme a "Stylized EU penalty scheme". Consequently, the penalty in this case is modeled as a linear increasing function of the degree of offence, q:

$$s(q) = K\Pi^m q, \tag{3.4}$$

where K is a positive constant, which determines the steepness of the penalty scheme, and Π^m is the instantaneous monopoly profit to the firm[8].

[7] Guidelines on the method of setting fines imposed for violations of competition law in Europe can be found in PbEG 1998, while guidelines for the setting of fines in the Netherlands are described in Section 57(1) of Competition Act.

[8] The multiplier $K\Pi^m$ is derived from the static optimization problem for the firm. The firm decides on the level of offence given the rate of law enforcement, p, and the functional form of the penalty scheme, which is linear. And the aim of the antitrust authority is to achieve zero price-fixing outcome.

Since our aim is to determine Nash Equilibria in open-loop strategies, we first find a tuple (ϕ, ψ) where $\phi : [0, T] \longmapsto [0, 1]$ and $\psi : [0, T] \longmapsto [0, 1]$ are the fixed strategies for the antitrust authority and the firm, respectively. Hence, ϕ corresponds to the control variable $p(t)$, and ψ corresponds to the control variable $q(t)$.

As the static analysis in the previous section suggests, it is reasonable to assume concavity of the terms $-NLSW(q(t)) - C(p(t))$ and $PS(q(t))$. This allows to obtain the expressions for an interior solution of the differential game (3.1)-(3.3).

The solution of the problem of the firm gives the following expression being the reaction function of the firm at each instant of time[9]:

$$q^*(t) = \begin{cases} 0 \text{ if } \; B \leq 0 \\ B \text{ if } \; 0 < B \leq 1 \\ 1 \text{ if } \; B > 1 \end{cases}, \tag{3.5}$$

where

$$B = \frac{2\Pi^m + \mu(t)\phi(t)}{2\Pi^m + 2\Pi^m K\phi(t)}. \tag{3.6}$$

In (3.6) $\mu(t)$ is the shadow price (costate variable) of the state variable $F(t)$ for the firm.

According to (3.6) the optimal degree of price-fixing for the firm decreases with decreasing shadow price $\mu(t)$. Moreover, the higher the penalty at the instant the firm is caught, the lower will be the optimal rate of price-fixing. The influence of the maximal gains from price-fixing on the optimal degree of price-fixing is determined by taking the derivative of expression (3.6) with respect to Π^m. We get that $\frac{\partial B}{\partial \Pi^m} \geq 0$. So the optimal degree of price-fixing by the firm will increase when the maximal gains from collusion increase. This behavior makes economic sense.

The solution of the problem of the antitrust authority gives us the following expression being the reaction function of the antitrust authority at each instant of time[10]:

$$p^*(t) = \begin{cases} 0 \text{ if } \; D \leq 0 \\ D \text{ if } \; 0 < D \leq 1 \\ 1 \text{ if } D > 1 \end{cases}, \tag{3.7}$$

where

$$D = \frac{(K\Pi^m\psi(t) - \lambda(t))\psi(t)}{2N}. \tag{3.8}$$

In (3.8) $\lambda(t)$ is the shadow price of the state variable $F(t)$ for the antitrust authority.

[9] For a complete derivation of this result see Appendix 1.

[10] For a complete derivation of this result see Appendix 1.

The intuition behind the formula (3.8) is as follows. Since the antitrust authority aims to minimize the total loss, the adjoint variable $\lambda(t)$ measures the shadow costs of one additional unit of probability $F(t)$ imputed by the authority. Thus, $-\lambda(t)$ is the shadow price by which the state variable F is assessed by the authority. From (3.8) we see that a decrease in $\lambda(t)$ results in an increase of the equilibrium rate of law enforcement p. The increase in the absolute value of the penalty $K\Pi^m$ also will cause an increase in the rate of law enforcement, since it becomes more profitable for the antitrust authority to discover more violations. At the same time it holds that the higher the marginal costs of law enforcement N, the lower p.

3.3.2 Determination of the Nash Equilibrium

Based on the expressions (3.8) and (3.6) we can prove the following proposition.

Proposition 3.3 *If the penalty schedule has the form* $s(q) = K\Pi^m q$ *and the costs of law enforcement are a quadratic function of p, then the outcome with no collusion, i.e.* $q(t) = 0$ *for all* $t \in [0, T]$, *cannot arise as equilibrium strategy of the firm.*

Proof : See Appendix 1.

The unique steady state of this problem is given by $q^* = 0$ and $p^* = 0$.[11] By considering the phase diagram of this problem in the (p, q)- plane we conclude that this solution is not stable[12]. This fact provides an additional argument in favor of rejection of the penalty scheme being linear in the degree of price-fixing.

The general result of the analysis of the differential game conducted in this subsection points out the weaknesses of the penalty scheme, which corresponds to the European Sentencing Guidelines for violations of antitrust law.

3.3.3 Stylized US Penalty Scheme

In this section we consider a differential game where the penalty schedule is a convex increasing function of the degree of the offence. This schedule is given by the following expression:

$$s(q) = K\Pi^m q^2. \tag{3.9}$$

[11]For the sake of completeness we also give here the definition of the steady state. In the steady state, state and control variables are constant over time.

[12]See the part on the investigation of stability in Appendix 1.

This resembles the current US system of penalties for violations of antitrust law, where the base penalty imposed by court for the firm convicted in price-fixing will be determined as a convex increasing function of the degree of offence[13]. Again, this system does not exactly capture all the features of the penalties determined in the US guidelines manual (such as dependence on the duration of offence or the role in the infringement). That is why, as in the previous subsection, we call this scheme a "Stylized US penalty scheme". For the convex penalty scheme Proposition 3.4 can be obtained. We refrain from presenting its proof, since it is similar to the linear case[14].

Proposition 3.4 *If the penalty schedule has the form* $s(q) = K\Pi^m q^2$ *and the costs of law enforcement are a quadratic function of* p*, then the outcome with no collusion, i.e.* $q(t) = 0$ *for all* $t \in [0, T]$*, cannot arise as equilibrium strategy of the firm.*

There is some evidence that the deterrence with convex penalty system works slightly better than the deterrence with a linear penalty scheme for more grave offences. When q is sufficiently high, it can be shown that for any given probability of law enforcement the stylized US scheme gives a lower equilibrium degree of price-fixing by the firm than the stylized EU scheme and, consequently, a lower damage for society[15]. Moreover, this result once again gives support to the argument in favor of deterrence focused not only on cartel benefits but also on the harm to the consumers caused by price-fixing. Recall that the net losses in social welfare were proportional to the squared degree of offence[16].

The main implication of the model discussed in this section is that the penalty schemes, which are used now in the EU and US legislation, appear not to be as efficient as desired from the point of view of minimization of consumer loss from price-fixing activities of the firm. The result is that zero collusion (full compliance) behavior is not sustainable as a Nash Equilibrium in Markovian strategies for the whole planning period, and, moreover, there is no stable steady state corresponding to zero collusion. The reason for this is that fines for antitrust violations do not depend in any way on the probability of law enforcement, which should be an important determinant of the

[13]According to the US Guidelines Manual, the base penalty imposed by court for the firm convicted in price-fixing is determined based on the "level fine" that exhibits convex mapping from offence levels into fine levels.

[14]The proof of Proposition 3.4 and investigation of the stability of the system in the long run are available from the author upon request. Also here it holds that the unique steady state given by $p = q = 0$ is not stable.

[15]The detailed proof of this result is available from the author upon request.

[16]Recall from Section 3.2 that *Net Loss in SW* $= \frac{1}{2}\Pi^m q^2$.

efficiency of penalty schemes as has been mentioned in Becker (1968) and Leung (1991). In the next section we pursue this road.

3.4 A Penalty Schedule that Does Prevent Collusion

3.4.1 Solution of the Game

Here the aim is to find an open-loop Nash equilibrium, which is also a Markovian Nash Equilibrium of the game described above[17], when the penalty schedule is determined as follows:

$$s(q,p) = K\Pi^m q + \frac{G}{p} \quad with \quad s(0,0) = 0, \tag{3.10}$$

where G is a positive constant.

The foundation for the penalty schedule determined by expression (3.10) is based on the following considerations. Departing from (3.6), which is the FOC for the firm in case the penalty is linear in q and making use of the fact that $\mu(t) \leq 0$ for all t, we ensure that $q(t) = 0$ for all t by means of adding a strictly negative term, which is in absolute value greater or equal than twice the monopoly profits, to the numerator of expression (3.6). The term $\frac{G}{p}$ in the penalty function (3.10) assures the appearance of this additional term in the expression for the reaction function of the firm. Note that this result has a lot in common with the well known result of Becker (1968).

Searching for the open-loop Nash Equilibria of the game we start by solving the optimal control problem of the firm. If the antitrust authority chooses to play $p(t) = \phi(t)$ then the firm's problem is described by

$$Max \int_0^T e^{-rt}[(\Pi^m q(t)(2-q(t)) - (K\Pi^m q(t) + \frac{G}{\phi(t)})\phi(t)q(t))[1-F(t)]]dt + e^{-rT}C_2(T)[1-F(T)]$$

s.t. $\dot{F}(t) = \phi(t)q(t)[1 - F(t)]$.

The Hamiltonian of this problem equals

$$H(q, F, \mu, t) = [\Pi^m q(t)(2 - q(t)) - (K\Pi^m q(t) + \frac{G}{\phi(t)})\phi(t)q(t) + \mu(t)\phi(t)q(t)][1 - F(t)],$$

where $\mu(t)$ is the costate variable of the problem of the firm.

Solving for $q(t)$ and $\mu(t)$ we get:

$$\dot{\mu}(t) = \Pi^m q(t)(2 - q(t)) - s(t)\phi(t)q(t) + \mu(t)\phi(t)q(t),$$

[17] For verification see Appendix 2.

$$q^*(t) = \begin{cases} 0 \text{ if } & B \leq 0 \\ B \text{ if } & 0 < B \leq 1 \\ 1 \text{ if } & B > 1 \end{cases}, \quad (3.11)$$

where

$$B = \frac{2\Pi^m + \mu(t)\phi(t) - G}{2\Pi^m + 2\Pi^m K\phi(t)}. \quad (3.12)$$

The intuition behind this result is exactly the same as in the previous section. The only difference is that the size of the fixed fine G negatively influences the degree of price fixing.

Now we move to the solution of the optimal control problem of the antitrust authority. If the firm chooses to play $q(t) = \psi(t)$ then the regulator's problem can be described as

$$Min \int_0^T e^{-rt}[(NLSW(t)+C(p(t)))[1-F(t)]-(K\Pi^m\psi(t)+\frac{G}{p(t)})\dot{F}(t)]dt+e^{-rT}C_1(T)[1-F(T)]$$

s.t. $\dot{F}(t) = p(t)\psi(t)[1 - F(t)]$.

The Hamiltonian of this problem equals

$$H(p, F, \lambda, t) = [(\Pi^m\frac{1}{2}\psi^2(t)+Np^2(t))-(K\Pi^m\psi(t)+\frac{G}{p(t)})p(t)\psi(t)+\lambda(t)\psi(t)p(t)]*[1-F(t)],$$

where $\lambda(t)$ is a costate variable of the problem of player 1.

Solving for the optimal $p(t)$ and $\lambda(t)$, and taking into account that the control region for p is constrained by the $[0, 1]-$ interval, we get:

$$\dot{\lambda}(t) = \Pi^m\frac{1}{2}\psi^2(t)) + Np^2(t) - s(t)p(t)\psi(t) + \lambda(t)\psi(t)p(t),$$

$$p^*(t) = \begin{cases} 0 \text{ if } & D \leq 0 \\ D \text{ if } & 0 < D \leq 1 \\ 1 \text{ if } & D > 1 \end{cases}, \quad (3.13)$$

where

$$D = \frac{(K\Pi^m\psi(t) - \lambda(t))\psi(t)}{2N}. \quad (3.14)$$

The intuition behind this result is exactly the same as in the previous section.

Taking into account the assumptions on the terminal values $C_1(T) \geq 0, C_2(T) \geq 0$ we conclude that the transversality conditions will be as follows:

$$\lambda(T) = -C_1(T) \leq 0 \;\; and \;\; \mu(T) = -C_2(T) \leq 0. \quad (3.15)$$

3.4.2 Determination of Nash Equilibrium

Unfortunately, a conventional open-loop Nash Equilibrium of this game does not exist in case the penalty is defined as in (3.10). However, we are able to establish that, under certain conditions on the parameters of the model, an equilibrium with zero degree of collusion in all periods can be sustained as an $\varepsilon-$equilibrium in open-loop or Markovian strategies.

The notion of $\varepsilon-$equilibrium is defined as follows (See Myerson (1991)).

Definition 3.5 *An $\varepsilon-$equilibrium of a strategic-form game is a combination of strategies such that no player could expect to gain more than ε by switching to any of his feasible strategies, instead of following the strategy specified for him.*

Let us investigate the stability of the system and the properties of the last period solution in the case of interior controls. From (3.11)-(3.14) it can be concluded that the system of equations describing the solution of the differential game in terms of reaction functions at the final time of the game has the following form:

$$p^*(T) = \frac{(K\Pi^m q(T) - \lambda(T))q(T)}{2N}, \tag{3.16}$$

$$q^*(T) = \frac{2\Pi^m + \mu(T)p(T) - G}{2\Pi^m + 2\Pi^m Kp(T)}. \tag{3.17}$$

Studying the reaction functions of both players at each instant of time, we can conclude that the following proposition holds

Proposition 3.6 *If the penalty schedule has the form (3.10) where $G \geq 2\Pi^m$, then the unique $\varepsilon-$equilibrium is given by $q(t) = 0$ and $p(t) = \sigma$ for all $t \in [0, T]$ with $\sigma > 0, \sigma(\varepsilon) \to 0$ if $\varepsilon \to 0$.*

Proof.

From expression (3.16) it is obtained that $p^*(T) = 0$ if and only if $q(T) = 0$, since $K\Pi^m q(T) - \lambda(T)$ cannot be equal to zero due to the transversality condition (3.15). This will be situated on an optimal path for the strategy of player 2, given by expression (3.17), when $G \geq 2\Pi^m$ and $\mu(T) \leq 0$. In this case, the best response function $q(p)$ for player 2 is the constant function passing through the point $(0, 0)$, so $q^*(p) = 0$ for any $p \in [0, 1]$.

In Figure 3.3 we sketch time T reaction functions of the firm and antitrust authority in case $\mu(T) < 0$ and $\lambda(T) < 0$. Here solid line, $p(q)$, represents the best response curve of the authority and dashed line, $q(p)$, represents the best response curve of the firm. It is clear from the graph that with $G \geq 2\Pi^m$ there is unique Nash equilibrium with $q^*(T) = p^*(T) = 0$.

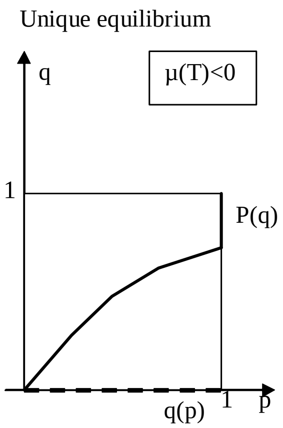


Figure 3.3: Determination of the Nash Equilibrium in the model when the penalty schedule is given by the function $s(q,p) = K\Pi^m q + \frac{G}{p}$ for parameter values $K = 2, \Pi^m = 1, G = 2, N = 1$ and taking $\lambda = -1$.

We can conclude that $q^*(T) = 0$ and $p^*(T) = 0$ will be sustained as an open-loop or Markovian[18] Nash equilibrium at the end of the planning horizon when $G \geq 2\Pi^m$, i.e. the fixed penalty is high enough to make the reaction curve of the firm a horizontal line, passing through the point $q = 0$. Similar arguments hold for $p^*(t)$ and $q^*(t)$ at each instant of time, $t \in [0, T)$.

The problem here is that, according to expression (3.10), the penalty and, consequently, the objective functions become indeterminate when $p(t) = 0$. To overcome this problem we employ the notion of $\varepsilon -$ *equilibrium* (or *almost equilibrium*) as given in Definition 3.5. As a candidate for an $\varepsilon -$ *equilibrium* we consider $q^*(t) = 0$ and $p^*(t) = \sigma$ for all $t \in [0, T]$, where $\sigma > 0$ and $\sigma(\varepsilon) \to 0$ if $\varepsilon \to 0$.

In order to show that $p^*(t) = \sigma$ and $q^*(t) = 0$ for all $t \in [0, T)$ can be sustained as an open-loop or Markovian Nash equilibrium of this game, we need to verify that this solution satisfies the necessary conditions for optimality. Obviously, they are satisfied.

[18]Analogous to the models in section 3, also for this game it holds that the sets of open-loop and Markovian equilibria coincide.

For $F(t) \neq 1$ we can rewrite the differentiated Hamiltonians as follows[19]:

$$\lim_{\sigma \to 0+} \frac{\partial H(p,F,\lambda,t)}{\partial p} = \lim_{\sigma \to 0+} [2Np(t) - K\Pi q^2(t) + \lambda(t)q(t)]_{(p=\sigma, q=0)} = 0,$$

$$\lim_{\sigma \to 0+} \frac{\partial H(q,F,\mu,t)}{\partial q} = \lim_{\sigma \to 0+} [2\Pi^m - 2\Pi^m q(t) + \mu(t)p(t) - 2\Pi^m Kq(t)p(t) - G]_{(p=\sigma, q=0)} = 0.$$

Note that the second equality is satisfied when $G = 2\Pi^m$. However, when $G > 2\Pi^m$, the point with $p = \sigma, q = 0$ is a boundary optimum and, hence, also solves the optimization problem described in section 3.4.1.

Next, we prove that $q(t) = 0$, $p(t) = \sigma$ for all $t \in [0,T]$ is a unique equilibrium. Analysis of the shape of the reaction functions implies that the fact that $\mu(t) \leq 0$ for all $t \in [0,T]$ ensures that $q(t) = 0$, $p(t) = \sigma$ for all $t \in [0,T]$ is a unique solution.

Firstly, $\mu(T) > 0$ can not hold, since according to the transversality condition we have $\mu(T) \leq 0$. Hence, the equilibrium with $q(T) = 0$, $p(T) = \sigma$ is a unique equilibrium in period T given $G \geq 2\Pi^m$.

We can show that the equilibrium with $q(t) = 0, p(t) = \sigma$ will be also unique for all $t \in [0,T)$. In the problem under consideration the necessary condition for uniqueness of the equilibrium $q(t) = 0, p(t) = \sigma$ for all t is the condition $\mu(t) \leq 0$ for any $t \in [0,T]$. Taking into account the transversality condition $\mu(T) \leq 0$ above we now show that $\mu(t) \leq 0$ for $t \in [0,T)$. Assume that there is an arbitrary $t' \in [0,T)$ such that $\mu(t') > 0$. Then from the optimality condition we obtain $\mu = \frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp + G}{p}$[20].

From the costate equation for $\mu(t)$ we obtain that

$$\dot{\mu}(t') = 2\Pi^m q(t') - \Pi^m q(t')^2 - (K\Pi^m q(t') + \frac{G}{p(t')})p(t')q(t') +$$

$$+ (\frac{-2\Pi^m + 2\Pi^m q(t') + 2K\Pi^m q(t')p(t') + G}{p(t')})p(t')q(t') = q^2(t')\Pi^m\,(1 + Kp(t')) \geq 0.$$

Hence, a non-positive terminal value given by $\mu(T) = -C_2(T)$ could never be reached. Thus,

$$\mu(t) \leq 0 \quad for \quad t \in [0,T)$$

Hence we can conclude that with $G = 2\Pi^m$ the outcome with no collusion $q(t) = 0$ for all $t \in [0,T]$ can arise as an open-loop or Markovian Nash Equilibrium solution of the game and this equilibrium is unique[21]. ■

[19] Note that in case $F(t) = 1$ equalities $\frac{\partial H(p,F,\lambda)}{\partial p} = 0$ and $\frac{\partial H(q,F,\mu)}{\partial q} = 0$ are satisfied for any values of p and q.

[20] For a complete derivation see Appendix 3.6.3.

[21] Note that this result also goes through with $r > 0$. The only difference is that $\dot{\mu}(t') = r\mu(t') + q^2(t')\Pi^m\,(1 + Kp(t'))$, which is also greater or equal than zero given that $\mu(t') > 0$.

To summarize the analysis, we stress that this proposition considers the settings, where we model the interactions between the firm and antitrust authority as a differential game. In this game the antitrust authority imposes a penalty of the form $s(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$ at the moment the cartel is discovered and zero penalty if it conducts the audit and does not discover any violation. One important feature of this schedule is that when the cartel is discovered the penalty imposed on the firm must be at least greater than twice the instantaneous monopoly profits from price-fixing in the industry under consideration. It turns out that this penalty scheme is more efficient than the current EU or US penalty schemes, in the sense that this policy leads to the complete deterrence outcome. In particular, the regulator can achieve the outcome with no price-fixing in all periods of the planning horizon at the lowest possible costs.

Finally, consider the infinite horizon problem and let us investigate the stability of the Nash Equilibrium solution in the long run. In the Appendix 3.6.3 we derive the system of differential equations for the Nash-optimal controls and obtain the qualitative behavior of the optimal solution from a phase diagram analysis in the (p,q)- plane for a specific set of parameter values[22]. Studying the phase diagram[23] we can conclude that the following proposition holds.

Proposition 3.7 *The outcome with $q^* = 0$ and $p^* \to 0$ is the unique long run steady state equilibrium of the infinite horizon model, where the penalty is given by the expression $s(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$ and the costs of law enforcement for the antitrust authority are convex.*

Proof: See Appendix 3.

Next, we give a phase portrait in the $(p,q)-$ plane of the system of differential equations which describes the long run dynamics of the system in terms of control variables. In Figure 3.4 the domain of the controls is determined by the square $[0,1] \times [0,1]$. This figure also shows graphically that the solution $(p^* \to 0,\, q^* = 0)$ is the unique stable steady state equilibrium of the game.

Considering the dynamics of the system in this domain, we can distinguish two possible situations. In the first situation we start with initial values of control variables

[22]Numerical examples suggest that these results are robust in the setting with different parameter values. However, a general closed-form solution of the system of the differential equations for the steady state equilibrium of the game still cannot be calculated due to the complicated structure of objective functions.

[23]See Figure 3.6.

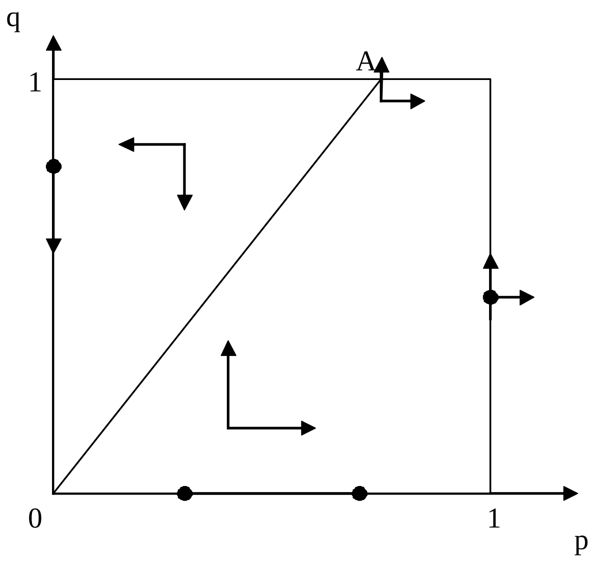


Figure 3.4: Phase portrait in (p,q)-space for the model where penalty schedule is given by $s(q,p) = K\Pi^m q + \frac{G}{p}$ for the set of parameters $K = 2, N = 1, G = 2$, $\Pi^m = 1$.

above the line OA in the Figure 3.4, which denotes the locus where both $\dot{p} = 0$ and $\dot{q} = 0$. In this case, the system will always converge to the point $(0,0)$. In the second situation, starting in any point with initial values of control variables below the line OA (locus $\dot{p} = \dot{q} = 0$), the system arrives at the point $(1,1)$, which is clearly suboptimal compared to the solution $(0,0)$ for the chosen parameter values[24]. So, we can conclude that $q^* = 0, p^* \to 0$ is the unique stable steady state solution of the system of differential equations (3.30), (3.31).

3.5 Conclusions

In this chapter we analyze dynamic interactions between the antitrust authority and a firm involved in a cartel. We develop a model which can be used to study dynamic optimal enforcement of competition law. We can summarize the results of the chapter as follows.

One main result is that the penalty schemes, which are used now in the EU and US legislation, appear not to be as efficient as desired from the point of view of minimization

[24]Note that strictly speaking also the path towards the steady state should be taken into account when comparing objective values.

of consumer loss from price-fixing activities of the firm. In particular, we prove the result that zero collusion (full compliance) behavior is not sustainable as a Nash Equilibrium in Markovian strategies. The reason is that the current penalty schemes do not allow the fine to be high enough to outweigh the accumulated expected gains from price-fixing for colluding firms. An additional reason could be that fines for antitrust violations do not depend in any way on the probability of law enforcement, which should be an important determinant of the efficiency of penalty schemes. The latter result was obtained by Becker (1968) and also by Leung (1991).

Furthermore, we determine a penalty system, that is efficient from the point of view of the possibility of complete deterrence of cartel formation in a dynamic setting. We find that there is a possibility to achieve the socially desirable outcome, i.e. the outcome with no price-fixing in all the periods of planning horizon, and give an example of the penalty scheme with which this outcome can be achieved. The amount of fine should be an increasing function of the degree of offence and it should be negatively related to the probability of law enforcement, which is related to Becker's (1968) result. An interesting implication is that in any case, whatever the degree of offence is, the penalty should be greater than twice the per period maximal gains from price-fixing for the firm. This in some sense confirms the suggestion which has been made in the beginning of the chapter that, indeed, the penalty should be related not only to the gains from price-fixing for the firm but also to the loss in consumer surplus due to price-fixing, which is approximately twice the monopoly profits in case of full collusion.

There is a number of possible extensions of the model described in this chapter. It seems reasonable to assume that the duration of the game is large. Thus it might be interesting to consider also the case of an infinite time horizon in more detail and try to find a more general solution for this setting. In this case the salvage values must be equal to zero and the discount rate must be strictly positive for reasons of convergence of the objective functionals. New insights may be gained by looking at heterogeneity among the violating firms and, consequently, different penalty schedules for offences of different gravity and differentiation between industries can also help to improve the deterrence power of the current penalty schemes for violations of competition law. The introduction of new state variables such as the offender's criminal record or the accumulated gain from cartel formation could give new insights for the determination of optimal penalty schemes for antitrust law violations. This task will be accomplished in the next chapter of the thesis, where we discuss properties of the penalty schemes that take into account history of the violation through the accumulated illegal gains from price-fixing activities.

3.6 Appendix

3.6.1 Appendix 1. Complete Solution of Differential Game with Linear Penalty Schedule

Solution of the Problem of Player 2 (firm)[25]

Let us start by solving the optimal control problem of player 2 defined by (3.18) and (3.19), where we take into account that, according to (3.4), $s(q(t))$ is given by the expression $K\Pi^m q(t)$. If player 1 chooses to play $p(t) = \phi(t)$ then player 2's problem can be written as

$$Max \int_0^T e^{-rt}[(\Pi^m q(t)(2-q(t)) - K\Pi^m \phi(t) q^2(t))(1-F(t))]dt + C_2(T)[1-F(T)] \quad (3.18)$$

$$s.t. \dot{F}(t) = \phi(t)q(t)[1-F(t)],\ F(0)=0,\ F(t)\in[0,1]\ \ and\ \ q(t)\in[0,1]. \quad (3.19)$$

Now $\phi(t)$ is assumed to be a fixed function.

The Hamiltonian of this problem equals

$$H(q,F,\mu,t) = [\Pi^m q(t)(2-q(t)) - K\Pi^m\phi(t)q^2(t) + \mu(t)\phi(t)q(t)] * [1-F(t)].$$

Solving for $q(t)$ and $\mu(t)$ we get:

(i) $\dot{\mu}(t) - r\mu(t) = -\frac{\partial H(q,F,\mu,t)}{\partial F}$. This implies that $\dot{\mu}(t) - r\mu(t) = \Pi^m q(t)(2-q(t)) - K\Pi^m\phi(t)q^2(t) + \mu(t)\phi(t)q(t)$.

(ii) $q^*(t)$ is such that it maximizes $H(q,F,\mu,t)$ on $q\in[0,1]$.

(iii) $F(T)$ is free, which implies a transversality condition of the following form: $\mu(T) = -C_2(T)$.

(ii) implies that

$$\frac{\partial H(q,F,\mu,t)}{\partial q} = 2\Pi^m - 2\Pi^m q(t) + \mu(t)\phi(t) - 2\Pi^m K q(t)\phi(t).$$

The interior solution $2\Pi^m - 2\Pi^m q + \mu\phi - 2\Pi^m K q\phi = 0$ gives us

$$\mu = 2\Pi^m \frac{q-1+Kq\phi}{\phi}.$$

Substituting this expression into the costate equation we obtain that

[25] For the sake of completeness we solve this game under the assumption that $r \geq 0$. Therefore, the results stated in section 3.3.1 (under assumption $r=0$) will hold automatically.

$\dot{\mu} = r\mu + \Pi^m q(2-q) - \Pi^m Kq\phi q + 2\Pi^m(\frac{-1+q+Kq\phi}{\phi})\phi q = r\mu + \Pi^m q^2 + \Pi^m Kq^2\phi.$
Furthermore, $\frac{\partial H(q,F,\mu,t)}{\partial q} = 0$ implies that

$$q(t) = \frac{2\Pi^m + \mu(t)\phi(t)}{2\Pi^m + 2\Pi^m K\phi(t)} = B,$$

and since $q(t) \in [0,1]$ we get that

$$q^*(t) = \begin{cases} 0 \; if \;\; B \leq 0 \\ B \; if \;\; 0 < B \leq 1 \\ 1 \; if \;\; B > 1. \end{cases}$$

Proof of $\mu(t) \leq 0$ **for all** $t \in [0,T]$.

Proof. The transversality conditions are $\lambda(T) = -C_1(T)$ for player 1 (antitrust-authority) and $\mu(T) = -C_2(T)$ for player 2 (firm). Given $C_1(T) \geq 0$ and $C_2(T) \geq 0$ we have that $\lambda(T) \leq 0$ and $\mu(T) \leq 0$, where $\lambda(T)$ and $\mu(T)$ are values of the costate variables of the game in the last period.

Taking into account the conditions above we can show that $\mu(t) \leq 0$ for $t \in [0,T)$.

Assume that there is $t' \in [0,T)$ such that $\mu(t') > 0$. Then, according to the concavity of $PS(q)$ we obtain from the costate equation for $\mu(t)$ that $\dot{\mu}(t') = \mu(t')r + \Pi^m q(t')^2 + \Pi^m Kq(t')^2 p(t') \geq 0$. Hence, a non-positive terminal value given by $\mu(T) = -C_2(T)$ could never be reached. Therefore it holds that

$$\mu(t) \leq 0 \;\; for \;\; t \in [0,T).$$

■

Solution of the Problem of Player 1 (antitrust authority)

Now we move to the solution of optimal control problem of player 1 defined by (3.20) and (3.21). If player 2 chooses to play $q(t) = \psi(t)$ then player 1's problem can be written as

$$Min \int_0^T e^{-rt}(NLSW(\psi(t)) + Np^2(t))[1-F(t)] - K\Pi^m\psi(t)\dot{F}(t)]dt + C_1(T)[1-F(T)] \tag{3.20}$$

$$s.t. \dot{F}(t) = p(t)\psi(t)[1-F(t)], \;\; F(0) = 0, \;\; F(t) \in [0,1] \;\; and \;\; p(t) \in [0,1]. \tag{3.21}$$

Now $\psi(t)$ is assumed to be a fixed function.

The Hamiltonian of this problem equals

$$H(p, F, \lambda, t) = [(\Pi^m \frac{1}{2}\psi^2(t) + Np^2(t)) - s(t)p(t)\psi(t) + \lambda(t)\psi(t)p(t)] * [1 - F(t)].$$

Solving for $p(t)$ and $\lambda(t)$ we get:

(i) $\dot{\lambda}(t) - r\lambda(t) = -\frac{\partial H(p,F,\lambda,t)}{\partial F}$. This implies that $\dot{\lambda}(t) - r\lambda(t) = \frac{1}{2}\Pi^m\psi^2(t) + Np^2(t)) - s(t)p(t)\psi(t) + \lambda(t)\psi(t)p(t)$

(ii) $p^*(t)$ is such that it maximizes $H(p, F, \lambda, t)$ on $p \in [0, 1]$.

(iii) $F(T)$ is free, which implies a transversality condition of the following form: $\lambda(T) = -C_1(T)$.

(ii) implies that

$$\frac{\partial H(p, F, \lambda, t)}{\partial p} = 2Np(t) - K\Pi^m\psi(t)\psi(t) + \lambda(t)\psi(t) = 0,$$

so that

$$p(t) = \frac{(K\Pi^m\psi(t) - \lambda(t))\psi(t)}{2N} = D.$$

Now taking into account the limits of the control region we obtain

$$p^*(t) = \begin{cases} 0 \; if \quad D \leq 0 \\ D \; if \quad 0 < D \leq 1 \\ 1 \; if \quad D > 1. \end{cases}$$

The equation $2Np - K\Pi^m\psi\psi + \lambda\psi = 0$ gives $\lambda = \frac{-2Np + K\Pi^m\psi^2}{\psi}$.

Substituting this expression into the costate equation we obtain that

$\dot{\lambda}(t) - r\lambda(t) = \Pi^m\frac{1}{2}\psi^2 + Np^2 - K\Pi^m\psi p\psi + (-\frac{2Np - K\Pi^m\psi^2}{\psi})\psi p = \frac{1}{2}\Pi^m\psi^2 - Np^2$.

Note that, since the sign of the expression for costate variable of player 1 given by $\dot{\lambda}(t) - r\lambda(t) = \frac{1}{2}\Pi^m q^2 - Np^2$ is ambiguous, it holds that in general $\lambda(t)$ has no unique sign.

Proof of Proposition 3.3

Proof. Consider the value of the control variable of the antitrust authority in the last period of the game given by expression (3.8). It is clear that $p^*(T) = 0$ if and only if $q(T) = 0$. But this contradicts the optimal path for the last period strategy of player 2 given by expression (3.6), which implies that, when $p(T) = 0$, $q^*(T)$ must be equal to 1. We conclude that $p^*(T) = 0$ and $q^*(T) = 0$ do not constitute a Nash equilibrium of the game in the last period for arbitrary salvage value $C_2(T)$. Consequently, the strategy $q(t) = 0$ for all t cannot be sustained as a Nash equilibrium in open-loop or Markovian strategies. ∎

Investigation of Stability of the System when the Penalty is Given by the Expression $s(q) = K\Pi^m q$.

From the solution of the problem of the firm (setting $r = 0$) we obtain that

$$\mu = \frac{-2\Pi^m + 2\Pi^m q(t) + 2K\Pi^m q(t)p(t)}{p(t)}, \tag{3.22}$$

and $\dot{\mu}(t) = 2\Pi^m q - \Pi^m q^2 - (K\Pi^m q)pq + (\frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp}{p})pq = q^2\Pi^m\left(1 + Kp\right).$

From the solution of the problem of the authority (setting $r = 0$) we have that

$$\lambda = \frac{-2Np(t) + K\Pi^m q^2(t)}{q(t)}, \tag{3.23}$$

and $\dot{\lambda}(t) = \frac{1}{2}\Pi^m q^2 + Np^2 - K\Pi^m q^2 p + (\frac{-2Np + K\Pi^m q^2}{q})qp = \frac{1}{2}\Pi^m q^2 - Np^2.$

Differentiating (3.23) and (3.22) with respect to time and equalizing it to $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ respectively we obtain following system of equations:

$$\frac{2p\Pi^m \dot{q} + 2K\Pi^m p^2 \dot{q} + 2\dot{p}\Pi^m - 2\dot{p}\Pi^m q}{p^2} = q^2\Pi^m\left(1 + Kp\right) \tag{3.24}$$

$$\frac{-2qN\dot{p} + K\Pi^m q^2 \dot{q} + 2\dot{q}Np}{q^2} = \frac{1}{2}\Pi^m q^2 - Np^2. \tag{3.25}$$

From (3.25) it follows that

$$\dot{p} = \frac{1}{4}\frac{2K\Pi^m q^2 \dot{q} + 4\dot{q}Np - \Pi^m q^4 + 2Np^2q^2}{qN}.$$

Substituting $\dot{p}$ into (3.24) and solving for $\dot{q}$ we get that

$$\dot{q} = \frac{1}{2}q^2\frac{\Pi^m q^2 - \Pi^m q^3 - 2Np^2 + 4qNp^2 + 2Np^3qK}{2Kp^2qN + K\Pi^m q^2 - K\Pi^m q^3 + 2Np}.$$

From (3.24) it follows that

$$\dot{q} = \frac{1}{2}\frac{-2\dot{p} + 2\dot{p}q + q^2p^2 + q^2p^3K}{p\left(1 + Kp\right)}.$$

Substituting $\dot{q}$ into (3.25) and solving for $\dot{p}$ we get that

$$\dot{p} = \frac{1}{2}q^2p\frac{K^2\Pi^m q^2p^2 + 4Np^2 + 4Np^3K - \Pi^m q^2}{2Kp^2qN + K\Pi^m q^2 - K\Pi^m q^3 + 2Np}.$$

Solving the system of equations above for $\dot{p} = 0$ and $\dot{q} = 0$ we obtain that the solution $p = 0$, $q = 0$ is also a steady state equilibrium of the game described in section 3 of this chapter. But after careful analysis of the phase diagram of this system we can conclude that the equilibrium $p = 0,\ q = 0$ is not stable for some policy relevant values of the parameters of the system.

Given the parameters $\Pi^m = 1, N = 2, K = 0.5$ and given the domain of the control variables is $[0, 1] \times [0, 1]$ we can represent the system dynamics in Figure 3.5.

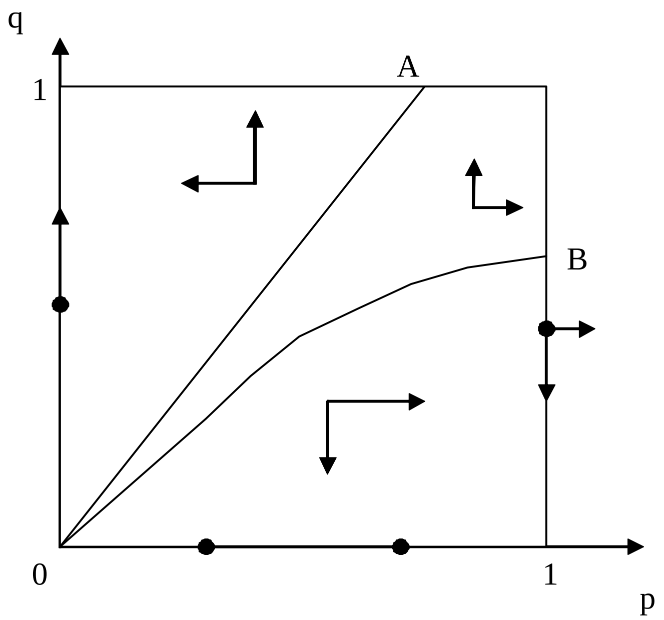


Figure 3.5: Phase portrait in (p,q)-space for the model with linear penalty schedule and convex costs of law enforcement for the set of parameters $K = 0.5, N = 2,\ \Pi^m = 1$.

OA is the locus where the variable p changes its dynamics and OB is the locus where the variable q changes its dynamics.

By studying the phase diagram we can conclude that solution $p^* = 0,\ q^* = 0$ cannot be a stable equilibrium, i.e. an equilibrium to which the system converges in the long run.

3.6.2 Appendix 2

In this appendix we show that for the games described in sections 3, 4 and 5 of this chapter the candidates for an open-loop Nash optimality are also candidates for Markovian Nash optimality and hence the open-loop strategies are also optimal in the set of Markovian strategies.

We have already mentioned that, referring to Feichtinger (1983) we can define the game under consideration as a state-separable game, i.e. the game which has the property that the state variable does not occur in the maximum conditions as well as in the adjoint equations. For such a game the system of differential equations for the Nash-optimal controls can be derived and also the qualitative behavior of the optimal solution can be obtained from a phase diagram analysis in the (p, q)- plane.

According to Feichtinger (1983), in state-separable differential games the candidates for open-loop Nash optimality are also candidates for Markovian Nash optimality. The strategies are independent of the state variable because neither the Hamiltonian-maximizing conditions nor the adjoint equations depend on state variable F. Thus, the open-loop strategies are also optimal in the set of Markovian strategies. Usually it is shown by verifying the sufficient conditions for Markovian Nash equilibrium controls as in Leitmann and Stalford (1974).

For the particular game described in section 3.3 of the chapter the procedure of verifying the sufficient conditions will be as follows.

Recall the definition of Markovian Nash Equilibria given in section 3.2.1. So searching for Markovian equilibria we impose that the choice of the control variable by each player will depend on the realization of state variable and also that both players can observe this realization. In that case the optimal strategies of player 1 (antitrust authority) and player 2 (firm) must be $p(t) = \phi(F(t), t)$ and $q(t) = \psi(F(t), t)$, respectively.

Solving for the open-loop Nash equilibria of the game of section 3 we get $q^*(t) = \frac{2\Pi^m + \mu(t)p(t)}{2\Pi^m + 2\Pi^m Kp(t)}$ and $p^*(t) = \frac{(K\Pi^m q(t) - \lambda(t))q(t)}{2N}$.

Now we substitute $q^*(t)$ and $p^*(t)$ into $H^2(q, F, \mu, t)$ and $H^1(p, F, \lambda, t)$. Then the Maximized Hamiltonians will have the following form:

$$H^{2*}(q, F, \mu, t) = [\Pi^m q^*(t)(2 - q^*(t) - s(q(t), p(t))\phi(t)q^*(t) + \mu(t)\phi(t)q^*(t)][1 - F(t)]$$

$$H^{1*}(p, F, \lambda, t) = [(\Pi^m \frac{1}{2}\psi^2(t) + Np^{*2}(t) - s(q(t), p(t))p^*(t)\psi(t) + \lambda(t)\psi(t)p^*(t)][1 - F(t)].$$

Recall also that in the state-separable game described above the adjoint equations do not depend on the state variable and, consequently, costate variables will not depend on the state variable as well.

Taking the above considerations into account we can notice that the Maximized Hamiltonian functions of both players are linear (and hence concave) with respect to the state variable. So we can conclude that the candidates characterized by $\frac{\partial H^1(p,F,\lambda,t)}{\partial p}$ and $\frac{\partial H^2(q,F,\mu,t)}{\partial q}$ are indeed nondegenerate Markovian Nash Equilibria of the game in section 3.2.2. Since $q^*(t)$ and $p^*(t)$ do not depend on $F(t)$ this open-loop Nash equilibrium of this game could be also regarded as a Nash Equilibrium of a differential game in which both players have full Markovian information.

The same reasoning holds for the model in section 4 of this chapter.

3.6.3 Appendix 3. Calculation of steady states in the model of section 3.4

In this appendix we verify that the equilibrium $(q^* = 0, p^* = 0)$ is also a stable unique steady state equilibrium of the differential game with penalty schedule given by $s(q,p) = K\Pi^m q + \frac{2\Pi^m}{p}$. Firstly, we derive the system of differential equations for the Nash-optimal controls in general. They are given in expressions (3.30) and (3.31). However, since objective functions of this game are quite complicated expressions in terms of control variables, the stability of the system cannot be investigated analytically with the help of general techniques such as an evaluation of the trace and the determinant of the Jacobian matrix. Therefore, in order to investigate the qualitative behavior of the optimal solution we employ a phase diagram analysis in the (p,q)- plane for a specific set of parameter values. Finally, we summarize the results of this analysis in Figure 3.6.

To simplify the calculations we assume that there is no discounting. Unfortunately, we were not able to obtain closed form expressions even for the dynamics of control values in case $r > 0$. However, we can presume that under $r > 0$ the effect of the penalty should be even stronger, since the accumulated expected future gain from price-fixing for the firm would be less.

The solution of the problem of player 2 gives

$$\mu = \frac{-2\Pi^m + 2\Pi^m q + 2K\Pi^m qp + G}{p} \tag{3.26}$$

and

$$\dot{\mu}(t) = q^2\Pi^m (1 + Kp).$$

The solution of the problem of player 1 gives

$$\lambda = \frac{-2Np + K\Pi^m q^2}{q} \tag{3.27}$$

and

$$\dot{\lambda}(t) = \frac{1}{2}\Pi^m q^2 - Np^2 - qG.$$

Differentiating (3.27) and (4.12) with respect to time and equalizing it to $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ respectively we obtain following system of equations:

$$\frac{2p\Pi^m \dot{q} + 2K\Pi^m p^2 \dot{q} + 2\dot{p}\Pi^m - 2\dot{p}\Pi^m q - \dot{p}G}{p^2} = q^2\Pi^m (1 + Kp), \tag{3.28}$$

$$\frac{-2qN\dot{p}+K\Pi^m q^2\dot{q}+2\dot{q}Np}{q^2}=\frac{1}{2}\Pi^m q^2-Np^2-qG. \tag{3.29}$$

From (3.29) it follows that $\dot{q}=$

$$\frac{1}{2}q^2\frac{2\left(\Pi^m\right)^2q^2(1-q)+4\Pi^mNp^2(2q-1)+\Pi^mqG(3q-4)+2q(Np^2-qG)+4\Pi^mqNp^3K}{4K\Pi^mp^2qN+2K\left(\Pi^m\right)^2q^2+4\Pi^mNp-2\left(\Pi^m\right)^2q^3K-GK\Pi^mq^2-2GNp}. \tag{3.30}$$

From (3.28) it follows that

$$\dot{p}=\Pi^mq^2p\frac{K^2\Pi^mq^2p^2+4Np^2+4Np^3K-\Pi^mq^2+2qG+2qGpK}{4K\Pi^mp^2qN+2K\left(\Pi^m\right)^2q^2+4\Pi^mNp-2\left(\Pi^m\right)^2q^3K-GK\Pi^mq^2-2GNp}. \tag{3.31}$$

In order to be able to conduct a more transparent analysis we make assumptions about the parameters of the model. First, we normalize monopoly profits to 1, i.e. $\Pi^m=1$, while the parameter of the penalty scheme is $G=2\Pi^m=2$. Moreover, the costs of law enforcement should be proportional to the amounts of extra gains from price-fixing in every particular industry, since the more the firm has resources, the more efficient it will be in hiding the violation and if violation is found the more fears will be the battle in the court. Consequently the antitrust authority has to spend more resources in order to catch and sew the firm. Taking the above considerations into account we consider $N\cong\Pi^m=1$. Parameter K can be equal to 2 as in static settings or less, this influences neither the location of the steady state, nor the dynamics of the system around steady state.[26]

Given parameters values $K=2, N=1, G=2, \Pi^m=1$ we obtain

$$\dot{q}=\frac{1}{4}q^2\frac{4q-q^2+4p^2+4p^3}{2p^2-q^2} \quad \textit{and} \quad \dot{p}=\frac{1}{4}qp\frac{4q^2p^2+4p^2+8p^3-q^2+4q+8qp}{2p^2-q^2}.$$

Given the complexity of the problem, we cannot derive any stability results from studying the Jacobian matrix. Therefore, we rely on phase plane analysis.

The phase diagram of the above system in the (p,q)-plane is presented in Figure 3.6.

In this diagram the locuses where the variable q changes its sign are

$q=0$, $q=2+2\sqrt{(1+p^2+p^3)}$, $q=2-2\sqrt{(1+p^2+p^3)}$, $q=\sqrt{2}p$, $q=-\sqrt{2}p$.

And the locuses where the variable p changes its sign are $q=0$, $p=0$, $p=-0.5$,

[26] Note that there is one location of steady state which is not influenced by the values of parameters of the system at all, which is in the point $p=0$, $q=0$. This can be seen immediately from the system (3.28), (3.29).

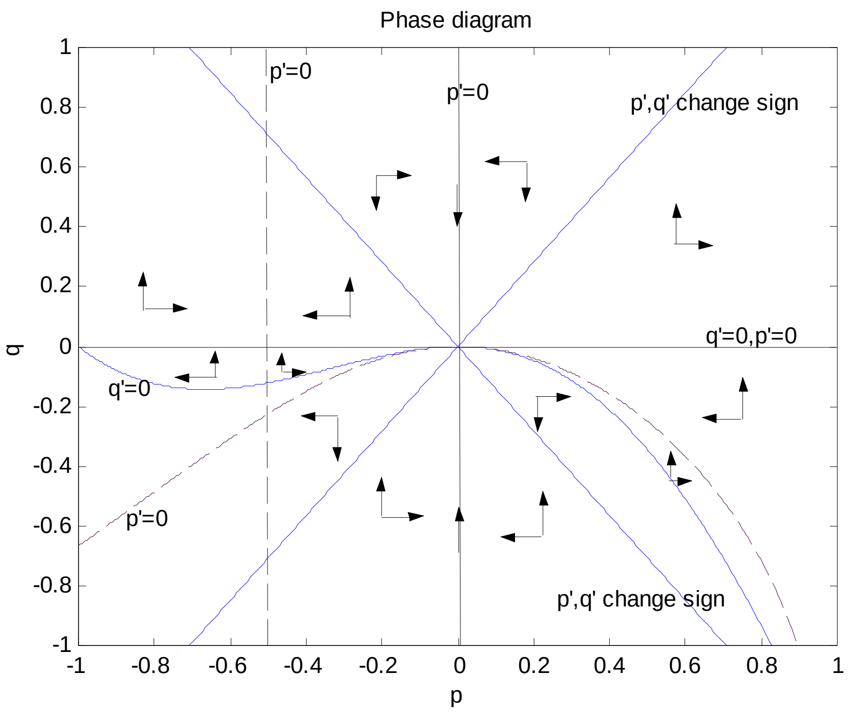


Figure 3.6: Complete phase portrait in (p,q)-space for the model where penalty schedule is given by $s(q,p) = K\Pi^m q + \frac{G}{p}$ for the set of parameters $K = 2, N = 1, G = 2, \Pi^m = 1$.

$q = \frac{1}{2p-1}\left(-2 + 2\sqrt{(1 - 2p^3 + p^2)}\right)$, $q = \frac{1}{2p-1}\left(-2 - 2\sqrt{(1 - 2p^3 + p^2)}\right)$, $q = \sqrt{2}p$, $q = -\sqrt{2}p$.

Recall that the domain of the controls is determined as $(p,q) \in [0,1] \times [0,1]$. Considering the dynamics of the system in this domain, we conclude that for certain initial values of control variables, in particular $q > \sqrt{2}p$, the system will always converge to the point $(0,0)$. Moreover starting in any point with characteristics $q \leq \sqrt{2}p$ will bring the system into the point $(1,1)$, which, as we will prove below, is clearly suboptimal compared to the solution $(0,0)$ given the above parameter values. So we can conclude that $q^* = 0, p^* = 0$ is the stable steady state solution of the system of differential equations (3.30), (3.31).

In order to verify the above statement we consider the values of objective functionals for both players in points $(0,0)$ and $(1,1)$.

In case $p(t) = 0, q(t) = 0$ for all $t \in [0,T]$ we obtain

$$\dot{F}(t) = p(t)q(t)[1 - F(t)]|_{p=0,q=0} = 0.$$

Then, given that $F(0) = 0$, it follows that $F(t) = 0$ for all $t \in [0, T]$.

In case $p(t) = 1, q(t) = 1$ for all $t \in [0, T]$ we obtain

$$\dot{F}(t) = p(t)q(t)[1 - F(t)]|_{p=1,q=1} = 1 - F.$$

Then, given that $F(0) = 0$ it follows that $F(t) = 1 - e^{-t}$ for all $t \in [0, T]$.

Thus, the utility of the firm in both situations can be written as follows $J_2|_{(0,0)} = C_2(T)$ and $J_2|_{(1,1)} = (K + 1)\Pi^m(e^{-T} - 1) + C_2(T)[e^{-T}]$.

Note also that $(K + \Pi^m)(e^{-T} - 1) + C_2(T)[e^{-T}] < C_2(T)$, so that the firm is always better off in point $(0, 0)$.

Now consider the objective function of the authority in both cases:

$$J_1|_{(0,0)} = \int_0^T 2\Pi^m dt = 2\Pi^m T,$$

$$J_1|_{(1,1)} = 2\Pi^m T - ((\frac{3}{2} + K)\Pi^m - N)(e^{-T} - 1).$$

Note that $2\Pi^m T - ((\frac{3}{2} + K)\Pi^m - N)(e^{-T} - 1) < 2\Pi^m T$, when $N < (\frac{3}{2} + K)\Pi^m$. Clearly, this inequality is satisfied given parameters values $K = 2, N = 1, G = 2, \Pi^m = 1$.

CHAPTER 4

Analysis of the Properties of Current Penalty Schemes for Violations of Antitrust Law

4.1 Introduction

This chapter analyzes the optimal policies for the deterrence of violations of antitrust law. We study the effects of penalty schemes, determined according to current US and EU antitrust laws, on the behavior of the firm. We investigate intertemporal aspects of this problem using a dynamic optimal control model of utility maximization by the firm under antitrust enforcement.

This chapter addresses the problem of whether the fine, determined on the basis of accumulated turnover of the firm participating in a cartel, can provide a complete deterrence outcome. We assume that the imposed fine takes into account the history of the violation. This means that when the violation of antitrust law is discovered, the regulator is able to observe all accumulated rents from cartel formation. Consequently, it will impose the fine that takes into account this information. We also compare the deterrence power of this system with the fixed penalty scheme.

The OECD report provides a description of the available sanctions for cartels according to the laws of member countries[1]. Those laws allow for considerable fines against enterprizes found to have participated in price-fixing agreements. In some cases, how-

[1] See O.E.C.D. (2002a) or Wils (2002).

ever, the maximal fines determined by these laws may not be sufficiently large to accommodate multiples of the gain to the cartel, as suggested by expected utility theory. In most of the countries the maximal fines are expressed either in absolute terms or as a percentage (10%) of the overall annual turnover of the firm[2]. However, according to experts' estimations, the best policy is to impose the penalties, which are a multiple of the illegal gains from price-fixing agreements to the firms. This, of course, would be difficult to estimate in real life, so it is still common practice to use the percentage of turnover as a proxy of the gains from price-fixing activities.

Several countries, namely the US, Germany, and New Zealand, have already accommodated this more advanced system. Instead of total turnover, in the US and Germany the maximal fine is stated in terms of unlawful gains. In Germany the maximal fine equals the maximum of the administrative fine of EUR 511518 or three times the additional profit from the cartel. In the US the maximal fine is the maximum of USD 10 million or twice the gain to the cartel[3]. New Zealand has the most advanced system. It provides for three alternatives: the maximum of NZD 10 million, three times the illegal gain, or if the illegal gain is not known, 10% of the total annual turnover of the enterprise. In general, the determination of the final amount of the fine, to be paid by the firm in each particular case, is based on the degree of offence, which is proportional either to the amount of accumulated illegal gains from the cartel or to its proxy, turnover involved throughout entire duration of infringement.

So, we can conclude that the current penalty schemes for antitrust law violations are mainly based on the turnover involved in the infringement throughout the entire duration of the infringement, which serves as a proxy of the accumulated gains from cartel or price-fixing activities for the firm. At the same time there exists an upper bound for the penalties for violations of antitrust law. The fine is constrained from above by the maximum of a certain monetary amount, a multiple of the illegal gains from the cartel, or if the illegal gain is not known, 10% of the total annual turnover of the enterprise. The idea of the current chapter is to incorporate these features of the current penalty systems into a dynamic model of intertemporal utility maximization by a firm, which is subject to antitrust enforcement.

Similar to Fent et al. (1999), the set up of the problem leads to an optimal control model. The main difference compared to Fent et al. (1999) or Feichtinger (1983) is that the gain from the cartel accumulated by the firm over the period of infringement

[2]See EC (1998)
[3]See D.O.J. (2001)

takes the role of a state variable, whereas the idea of Fent et al. (1999) was to take the offender's criminal record as a state variable of the dynamic game. An increase in the state variable is thus positively related to the degree of price fixing by the firm, and increases the fine the firm can expect in case of being convicted. By solving the optimal control problem of the firm under antitrust enforcement, we will investigate the implications of the different penalty schedules. It should also be mentioned that the model of Chapter 4 is an extension of the model used in the study described in Chapter 3 in the sense that, in the former, we relate the penalty not only to the current degree of offence, but also to the accumulated illegal gains from cartel formation. We assume that the fine imposed takes into account the history of the violation. This means that when the violation of antitrust law is discovered, the regulator is able to observe all accumulated rents from cartel formation. Consequently, he will impose the fine that takes into account this information.

Furthermore, this framework allows us to analyze the consequences of two major modifications of the penalty systems for violations of competition law, which have been recently discussed by the OECD and the US Department of Justice (DOJ)[4]. These modifications were concerned with increasing the multiplier for the base fine and raising the legal upper bound for the imposed fines.

The main results are that, for the benchmark case, i.e., when the penalty is fixed, the outcome with complete deterrence of cartel formation is possible but only at the cost of shutting down the firm. In other words, the fixed penalty, which can ensure complete deterrence, is too high, because it leads to immediate bankruptcy. However, the result can be improved by relating the penalty to the illegal gains from price-fixing. The proportional scheme appears to be more appropriate than the fixed penalty, since it can ensure no price-fixing outcome in the long run even in case penalties are moderate[5].

Similar to the Garoupa (1997, 2001) approach, we also study the impact of the main parameters of the penalty scheme (probability and severity of punishment) on the efficiency of deterrence and analyze the optimal trade-off between changes in the scale parameter of the proportional penalty scheme and probability of law enforcement. It turns out that, the higher the probability and severity of punishment, the earlier the cartel formation is blocked. The sensitivity analysis shows that when the penalties are already high, the antitrust policy aiming at a further increase in the severity of

[4]See D.O.J. (1998).

[5]Unfortunately, complete deterrence outcome, i.e. full compliance behavior in all the periods of the planning horizon, is not sustainable even with proportional penalties.

punishment is less efficient than the policy that increases the probability of punishment.

The chapter is organized as follows. In section 4.2 we describe the general setup of an optimal control model of the firm under antitrust enforcement. In section 4.3 we consider the case where the upper bound for the penalty is an exogenously given fixed monetary amount. Moreover, we will derive an analytical expression for this upper bound, which allows to achieve the result of complete deterrence of price-fixing. In section 4.4 we investigate the implications of the penalty being proportional to the accumulated gains from price-fixing. We also conduct sensitivity analysis of the equilibrium values of the variables of the model with respect to the parameters of the penalty scheme. Section 4.5 summarizes the results of the analysis and suggests directions for future work.

4.2 Optimal Control Model. The General Setup.

We introduce the basic ingredients of the intertemporal optimization problem of a profit maximizing firm, which participates in an illegal cartel. The key variable is the accumulated gains from prior criminal offences (in case of a cartel, these offences are price-fixing activities).

Dynamics of the accumulated rents from price-fixing

The accumulated rents from price-fixing, $w(t)$, is the state variable of the model, which increases depending on the degree of offence (price-fixing). Using a continuous time scale the dynamics of the accumulated rents from price-fixing equals[6]

$$\begin{aligned} \dot{w}(t) &= \pi^m q(t)(2 - q(t)), \\ w(0) &= w_0 \geq 0. \end{aligned} \tag{4.1}$$

Here $\dot{w}(t)$ stands for the change in the value of the state variable at time t, π^m denotes the instantaneous illegal gains from cartel, $q(t)$ denotes the degree of price-fixing by the firm at instant t, and w_0 is the initial wealth of the firm before the start of the planning horizon. Expression (4.1) rests on the assumption of the demand function being linear. A complete derivation of expression (4.1) is given in section 3.2.1 of the previous chapter, where $\dot{w}(t)$ is associated with instantaneous producer surplus for the firm caused by fixing price levels above the competitive level. The main idea behind this formulation is that cartel formation leads to higher prices. The "normal" price is c (competitive

[6]To simplify the analysis for the rest of this chapter we assume $w_0 = 0$. However, relaxing this assumption does not change the results stated in propositions of this chapter.

equilibrium) leading to zero profits. Then q denotes the degree of violation, i.e. when the cartel fixes a higher price than "normal". From the definition of q in section 3.2.1 of Chapter 3 it is clear that in case of such a violation, i.e. when price is higher than competitive level, q is positive. Based on the simple linear demand function[7] , profit, or producer surplus, can be expressed as a concave function of q. Now the state variable $w(t)$ adds up the profits over time, and as such $w(t)$ is the total gain from crime (too high prices) from time 0 up to t.

There are strong legal and economic reasons for introduction of the state variable in the form of accumulated gains from price-fixing. It is related to the fact, that in US and EU guidelines for imposition of fines for antitrust violations, the penalty imposed in many cases is based mainly on the turnover involved in the infringement throughout the entire duration of the infringement. Clearly, the accumulated turnover serves as a proxy for accumulated gains from cartel or price-fixing activities for the firm.

In addition, according to the OECD survey, the fines imposed recently, expressed as a percentage of the gain, varied widely, from 3% to 189%. In only four cases the fines were more than 100% of the estimated gain, and in no case the fine was as high as two or three times the gain, as recommended by some experts. So, we can conclude that sanctions actually imposed have not reached the optimal level for deterrence, which, according to a well known Becker's (1968) result, suggests that the fine should be a multiple of illegal gains.

Profit function

The instantaneous illegal gains from price-fixing for the firm equal $\pi^m q(t)(2 - q(t))$. This function has been derived from the microeconomic model underlying the problem of price-fixing[8]. Obviously, this function implies that the marginal profit for the firm is always positive and strictly declining in the interval $q(t) \in [0, 1]$. Moreover, for each positive level of offence the profit is also positive.

The instantaneous profit at time t will also be influenced by accumulated rents from price-fixing. This variable also measures the experience the firm has in cartel formation. The more it has experience, the more efficiently the firm colludes and, consequently, the higher the instantaneous profits from price-fixing[9]. This influence is reflected in the

[7] See section 3.2.1 of Chapter 3.

[8] For complete derivation of this expression see section 3.2.1.

[9] It is reasonable to assume that it is easier to reach an agreement with other members of the cartel when the cartel has already been in place for some time, rather than to reach an agreement while forming a new cartel.

term $\gamma w(t)$ which enters additively the objective function of the firm (see expression (4.4) below)[10].

Law enforcement policy

The goal of the current section is to incorporate the features of the penalty system for antitrust law violations, described above, into the optimal control model of intertemporal utility maximization by the firm in the presence of a benevolent antitrust authority, whose aim is to block any degree of price-fixing which is equivalent to minimization of loss in social welfare due to an increase in price above the competitive level[11]. So, in order to capture the specifics of the sentencing guidelines and current antitrust practice, we model the penalty for violations of antitrust law as a linear increasing function of the accumulated rents from price-fixing for the firm. Therefore, it can be written as

$$s(w) = \alpha w. \tag{4.2}$$

This setup will also allow to study the effects of the changes of the multiplier for the base fine (refinement suggested by OECD) on the deterrence power of the penalty scheme.

According to Becker (1968) *the cost of different punishments to an offender can be made comparable by converting them into their monetary equivalent or worth.* And this is satisfied in our model, since we measure the accumulated rents from price-fixing for the firm in monetary units.

Moreover, our specification of the penalty function satisfies three main conditions specified in Fent et al. (1999), namely:

1. It is strictly increasing in the level of offence (since $w(t)$ is strictly increasing in $q(t)$).

2. Firms which do not collude at all should not be punished: $s(w_0) = 0$.

3. Any detected positive level of offence should lead to a positive amount of punishment: $s(w(t)) > 0$, for any $w(t) > w_0$, which is equivalent to $q(t) > 0$ for some $t \in [0, T]$.

[10]It may be more realistic to express this term as a nonlinear function of w. In particular, a concave formulation may be very tractable since there might be decreasing marginal returns from experience. However, it will not change the results of the paper in a qualitative sense. The solution of the model in case experience gain is modeled as $\gamma\sqrt{w}$ gives the outcome with complete deterrence similar to Proposition 2 and results of sensitivity analysis for the model with proportional penalty still hold. The analysis of the model, where penalty is fixed, with $\gamma\sqrt{w}$ term gives the same qualitative result but the model can only be solved numerically. A complete proof of this statement is available from author upon request.

[11]For verification of this statement see also section 3.2.1.

This implies that, if the firm has been checked, violated the law in the current period and participated in the cartel in some of the previous periods, the fine will be imposed on the basis of the whole accumulated gains from price-fixing, $w(t)$, and thus not only on the basis of the current degree of offence, $q(t)$.

Further, we will compare the efficiency and deterrence power of the penalty systems for a model in which the penalty is given by expression (4.2) and a model in which the penalty is fixed ($s(w) = S^{\max}$), where $S^{\max}$ is the fixed upper bound for the penalty introduced in the sentencing guidelines, which is not related to the level of offence.

Costs of being punished

The cost of being punished at time t equals the expected value of the fine that has to be paid. This will be defined as the probability of being audited by antitrust authority, p (level of law enforcement), $p \in [0, 1]$, times the degree of offence at time t, $q(t)$, times the level of punishment, $s(t)$, which depends on time as well:

$$\text{expected penalty at time t} = s(t)pq(t). \tag{4.3}$$

So, the expected penalty is determined by expression (4.3), where $pq(t)$ is the probability of being punished at time t and $s(t)$ is the fine, which may either be fixed or can be expressed as a function of accumulated gains from price-fixing.

We should stress here that the firm can only be caught at time t if $q(t) > 0$, i.e. the offence is committed exactly at this time. Of course this need not be the case for criminal acts in general: you can convict a thief, if the police has found the stolen things without having caught the burglar in action.[12] However, it does apply to antitrust law practices. According to the US sentencing guidelines (2001) and OECD report (2002), investigation concerning past behavior only starts at the moment it is observed that the current price exceeds the competitive price, thus when $q(t) > 0$. After this has been proved (usually on the basis of empirical analysis of price mark-ups), the antitrust authority will start a more detailed investigation and get access to accounting books and documents that can prove the existence of a cartel agreement. Only after that the gains from price-fixing ($w(t)$) become "perfectly observable", so that the court (or competition authority) can take them into account while determining the amount of fine to be paid.

Here it is also important to realize that the probability of being caught at instant t is $pq(t)$. The firm can only be caught at time t_1 if it does price-fixing on that date, so if $q(t_1) > 0$. Later in time, say at time $t_2 > t_1$, the firm cannot be punished because of the

[12]We thank an anonymous referee who pointed out this difference.

offence at time t_1. At t_2 it can only be caught and punished if $q(t_2) > 0$. At the moment the firm is caught it has to pay a fine, $s(t)$. In one scenario this fine is an increasing function of $w(t)$. So this means that if the firm did a lot of price-fixing in the past, implying that $w(t)$ is large, the fine will be larger. In this sense repeated offenders are more heavily punished, and this is what quite frequently happens in modern democratic societies. So if the firm is caught at time t_2, it is convicted for the crime on t_2, and the level of the fine depends on what the firm did in the past, thus also what it did at time $t_1 < t_2$ as well. In other words, the higher the degree of price-fixing before t_2, the larger the fine will be at t_2. This is independent of how many times the firm was caught in the past: the fine the firm paid before will not be subtracted from w. Since w is non-decreasing over time, it is implicitly taken into account that repeated offenders will be more heavily punished.[13]

Optimization problem

The firm making the decision about the degree of price-fixing faces the following intertemporal decision problem:

$$max\ J(q(t)) := \int_0^\infty e^{-rt}[\pi^m q(t)(2-q(t)) + \gamma w(t) - s(t)pq(t)]dt \tag{4.4}$$

s.t. $\dot{w}(t) = \pi^m q(t)(2-q(t))$ and $q(t) \in [0,1]$.

The parameter r is the discount rate. The objective functional $J(q(t))$ is the discounted profit stream gained from engaging in price-fixing activities. The term $\pi^m q(t)(2-q(t))$ reflects the instantaneous rents from collusion and the term $-s(t)pq(t)$ reflects the possible punishment for the firm, if it is caught. Note that the higher the degree of collusion, the higher the $q(t)$, the higher the expected punishment. $\gamma w(t)$ reflects the experience of the firm in cartel formation which increases future instantaneous gains from cartel formation.

Having made the assumptions of section 2 we define the current value Hamiltonian:

$$H^c(q,w,\mu,t) = \pi^m q(t)(2-q(t)) + \gamma w(t) - s(t)pq(t) + \mu(t)(\pi^m q(t)(2-q(t))) \tag{4.5}$$

[13]In reality it works as follows. If the firm is convicted for the second time its fine is increasing in the amount of price fixing, but compared to the fine for the first conviction, the fine to be paid for the second conviction will be multiplied with a higher number. To model this, ideally after the first conviction the fine is $\alpha w(t1)$, while for the second conviction the fine should be $c\alpha(w(t2) - w(t1))$ with $c > 1$. We did not see a chance to model this in a tractable optimal control framework. Therefore, we decided to approximate this with having a fine equal to $\alpha w(t)$. Since $w(t)$ is non-decreasing over time, it is implicitly taken into account that repeated offenders will be more heavily punished.

where $\mu(t)$ is the current value adjoint variable representing the shadow price of the offence. The Hamiltonian is well-defined and differentiable for all nonnegative values of the state variable $w(t)$ and all values of the control variable $q(t)$ in its domain $[0, 1]$.

4.3 The Model where the Penalty is Represented by a Fixed Monetary Amount

In this section we would like to model the situation where the penalty for violations of antitrust law is represented by a fixed monetary amount. In this case we assume that the fine does not depend on the accumulated gains from price-fixing and is constant over time. This might be a good framework to study the efficiency of antitrust enforcement in an environment where there exists an upper bound for penalties and offences are so grave that punishment always reaches its upper bound, which is true for highly cartelized markets. The analysis of this model is quite essential, since the imposition of the upper bound for penalties for violations of antitrust law is still a current practice in most countries.[14] Only Norway and Denmark do not have this limitation. We modify the model of section 4.2 in such a way that the fine is given by some fixed monetary amount, $S^{\max}$, which denotes the maximal penalty. In other words, the antitrust authority commits to a policy of the following form: the rate of law enforcement is constant $p(t) = p \in (0, 1]$ for all t, and, when the firm is inspected, the penalty is given by $s(t) = \begin{cases} S^{\max} & \text{if } q(t) > 0 \\ 0 & \text{if } q(t) = 0. \end{cases}$

In this section we show that if the fixed penalty (or upper bound for the fine imposed by law) is not high enough, complete deterrence is never possible. Moreover, we will derive an analytical expression for the upper bound, which allows to achieve the result of complete deterrence of price-fixing. For simplicity, we assume that there is no discounting ($r = 0$[15]), the planning horizon is finite ($T < \infty$), salvage values for both players are equal to zero, so that the transversality conditions are $\lambda(T) = 0$, $\mu(T) = 0$ for both players.

[14]There are certain policy reasons that lie behind the provision of an upper bound for fines in competition legislation, namely, to prevent firms from bankruptcy.

[15]To make the analysis more transparent and analytically solvable we assume here that $r = 0$. However, imposing that $r > 0$ does not change the qualitative predictions of the model. Only the dynamics of the costate variable of the firm changes. The equation for $\mu(t)$ becomes $\mu(t) = \frac{\gamma}{r}(1 - e^{(t-T)})$. A complete proof of this statement is available from the author upon request.

We derive the dynamic system for the optimal control $q(t)$ from the following necessary optimality conditions:

$$q(t) = \underset{q}{argmax} H^c(q, w, \mu, t) \tag{4.6}$$

and

$$\dot{\mu}(t) = -\frac{\partial H^c(q, w, \mu, t)}{\partial w}. \tag{4.7}$$

The expression (4.7) gives $\dot{\mu}(t) = -\gamma$. Solving this simple differential equation in case of finite planning horizon, we get $\mu(t) = \gamma(T - t)$. Consequently, $\mu(t) \geq 0$ for all $t \in [0, T]$. This allows us to conclude that the Hamiltonian (4.5) is strictly concave with respect to q. Therefore, condition (4.6) is equivalent to $H^c_q = 0$. It leads to

$$q(t) = 1 - \frac{pS^{\max}}{2\pi^m(1 + \gamma(T - t))} = C. \tag{4.8}$$

However, the control region of the offence rate q is limited by $[0, 1]$, by construction. This implies that the expression for the optimal degree of price-fixing by the firm is given by

$$q^*(t) = \begin{cases} 0 \; if \;\; C \leq 0 \\ C \; if \;\; 0 < C \leq 1 \\ 1 \; if \;\; C > 1. \end{cases} \tag{4.9}$$

We can represent the optimal degree of price-fixing by the firm, q, as a decreasing function of both the penalty for violation and time, which is depicted in Figure 4.1. The first part of this statement is quite intuitive, since a higher expected penalty will, obviously, increase the incentives for the profit maximizing firms to avoid participation in price-fixing agreements and thus reduce the degree of offence, q. The negative relationship between the degree of price-fixing and time is related to the fact that higher gains from price-fixing in the beginning imply that for a longer time period the firm can take an advantage of it, in the sense that due to increased experience profits from price-fixing will be higher. So, incentives to commit crime decrease over time and, hence, the degree of offence falls.

The state-control dynamics

After we substitute (4.8) into (4.1) the differential equation describing the dynamics of the state variable will be as follows:

$$\dot{w}(t) = \pi^m(1 - (\frac{S^{\max}p}{2\pi^m(1 + \gamma(T - t))})^2). \tag{4.10}$$

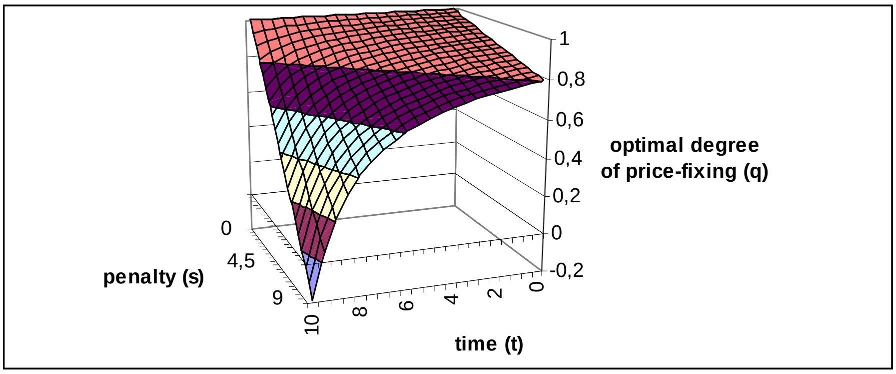


Figure 4.1: Representation of optimal degree of price-fixing as a decreasing function of penalty for violation and time for parameter values: $T = 10, \pi^m = 2, \gamma = \frac{1}{2}, p = \frac{1}{2}$.

The solution of this differential equation in general form will have the following form:

$$w(t) = \pi^m t - \frac{1}{4\pi^m}(S^{\max})^2 \frac{p^2}{\gamma(1 + \gamma T - \gamma t)} + C_1,$$

where C_1 is a constant determined from the initial condition $w(0) = w_0$.

To understand the exact dynamics of the state and control variables over time we consider a numerical example. For parameter values $p = \frac{1}{2}, S^{\max} = 2, \pi^m = 2, \gamma = \frac{1}{2}, T = 10$, the solution of this differential equation in general form will be as follows: $w(t) = \frac{1}{2(-12+t)} + 2t + C$. Taking $w(0) = 1$, we get

$$w(t) = \frac{1}{2(-12+t)} + 2t + \frac{25}{24}.$$

The optimal degree of price-fixing will have the following form: $q(t) = 1 - \frac{1}{24-2t}$ and taking into account the boundaries of the control region we obtain

$$q^*(t) = \begin{cases} 0 \text{ if } \ 1 - \frac{1}{24-2t} \leq 0 \\ 1 - \frac{1}{24-2t} \ \text{ if } 0 < 1 - \frac{1}{24-2t} \leq 1 \\ 1 \text{ if } \ 1 - \frac{1}{24-2t} > 1. \end{cases}$$

The results for different values of $S^{\max}$ are summarized in the Table 4.1.

Table 4.1.

Penalty	Accumulated gains from collusion	Degree of price-fixing
2	$w(t) = \frac{1}{2(-12+t)} + 2t + \frac{25}{24}$, $w(T) = 20$	$q^*(t) = 1 - \frac{1}{24-2t}, \to q(T) = \frac{3}{4}$
10	$w(t) = \frac{25}{2(-12+t)} + 2t + \frac{49}{24}$, $w(T) = 15$	$q^*(t) = 1 - \frac{5}{24-2t}, \to q(T) = 0$
20	$w(t) = \frac{50}{(-12+t)} + 2t + \frac{31}{6}$, $w(T) \approx 0.1$	$q^*(t) = 1 - \frac{10}{24-2t}, \to q(T) = 0$

Consequently, when all the parameters of the model are fixed, $w(t)$ is increasing over time and the degree of offence is a decreasing function of time. Unfortunately, we must conclude that, for example, when the fixed penalty equals 2, which is the instantaneous monopoly profit for the firm for these parameter values, it does not allow to achieve complete deterrence even in the last period. On the contrary, the last period degree of price-fixing is quite high (75% out of 100%).

We can conclude that the policies with fixed penalty appear to be highly inefficient, since to achieve $q^*(t) = 0$ for all $t \in [0, T]$ we should have $1 - \frac{s(t)p}{2\pi^m(1+\gamma(T-t))} \leq 0$, which implies $s(t) \geq \frac{2\pi^m(1+\gamma(T-t))}{p}$. In the example with parameter values $T = 10, \pi^m = 2, \gamma = \frac{1}{2}, p = \frac{1}{2}$ we get $s(0) \geq 48 = 24\pi^m$ and $s(T) = s(10) \geq 8 = 4\pi^m$. This enormous penalty will drive the firm bankrupt immediately[16]. Moreover, this result is counterintuitive and unfair, since the firm colluding for one period will obtain less extra gain than a firm colluding for ten periods, and, consequently, should be punished less.

The main result of the analysis of the model with fixed penalty is represented in the following proposition.

Proposition 4.1 *In the optimal control model, where $p(t) = p > 0$ for all $t \in [0, T]$, the no collusion outcome (i.e. complete deterrence of price-fixing) occurs when $S^{\max} \geq \frac{2\pi^m(1+\gamma(T-t))}{p}$ for all $t \in [0, T]$, thus when $S_0^{\max} \geq \frac{2\pi^m(1+\gamma T)}{p}$.*

It follows from this proposition that if the regulator would be able to take into account the duration of the infringement while setting the fixed maximal upper bound for the fine, $S^{\max}$, then this fixed upper bound would have to be decreasing function of time and rate of law enforcement and positively related to possible monopoly profits in the industry and total length of the planning horizon. One of the implications of this result is that the penalty for antitrust violation, which potentially can provide complete deterrence, should be imposed by the antitrust authority (thus, not by the court), i.e. by the authority which has complete information about the probability of law enforcement. Another interesting implication is that the fine should be inversely related to the probability of investigation (similar to Becker (1968)). Moreover, the penalty should be based mainly on the instantaneous monopoly profits in the industry. Of course, this value is different for each industry, so the specifics of the industry also should be taken into account when the optimal fine for antitrust violations is determined. The length of the planning horizon should also play a role in determining the optimal penalty.

[16]Here we define bankruptcy as a situation where the fine imposed on the firm is higher than the maximal possible gain from the violation.

However, in real life the implementation of this scheme is problematic, since the court (not the antitrust authority) imposes the penalty and, consequently, the parameter p cannot be verified.

Unfortunately, the fixed penalty system does not always work. For $S^{\max} < \frac{2\pi^m(1+\gamma(T-t))}{p}$ for some t, the result with no price-fixing outcome during the whole planning period is not possible. For reasonable parameter values (such as $p = \frac{1}{5}, \pi^m = \$1\ million, \gamma = \frac{1}{5}, T = 10$) we obtain that, for example, for violation of 1 million gravity discovered in the fifth period of planning horizon (impling 5 years duration) the fine S^{max} must be at least 8 million. This is clearly higher than the current fine levels determined by European Sentencing Guidelines. In this situation the new policy sugession to increase the legal upper bound or completely remove it from the rules, as it has been done in Norway, Denmark, or US, can be a good solution.

Moreover, this result resembles the result of Emons (2003), where the subgame perfect punishment for repeated offenders in a repeated game setting was investigated. The final conclusion of his paper is that if the regulator's aim is to block violation at the lowest possible cost, the penalty should be a decreasing function of time. Moreover, he concludes that the first period penalty (penalty for the first detected violation) should be the highest and should extract the entire wealth of the offender. So, another drawback of this system is that it does not explain escalating sanctions based on offense history which are embedded in many penal codes and sentencing guidelines.

Another problem with this result is that the fixed penalty, which can ensure complete deterrence, is too high. It is clearly unbearable for the firm and leads to immediate bankruptcy. Already for the first violation we have to punish twenty times more than the maximal per-period monopoly profit. To resolve this "impossibility result" we look for another scheme. Again we take an example from current legislation. This other system relates the penalty to the illegal gains from price-fixing. Moreover, it has already been implemented in a number of developed countries and the US.

In particular, in the next section we introduce the penalty as a linear increasing function of accumulated gains from price-fixing for the firm given by the expression (4.2) above. The proportional scheme appears to be better than the fixed penalty, since it can ensure the no collusion outcome in the long run even in the case where penalties are moderate.

4.4 Analysis of the Model with a Proportional Penalty

This setup reflects another important feature of the penalty systems for violations of antitrust law suggested by current sentencing guidelines. Namely, that the fine is proportional to the illegal gains from cartel formation. This more advanced system has already been implemented in the US, Germany, New Zealand and some other countries.

Utility maximization.

As before, we derive the optimal control $q(t)$ from the following necessary optimality conditions:

$$q(t) = \underset{q}{argmax} H^c \tag{4.11}$$

$$\dot{\mu}(t) - r\mu(t) = -\gamma + \alpha p q(t). \tag{4.12}$$

Since the control region of the offence rate q is limited by $[0, 1]$, the maximization condition (4.11) is equivalent to:

$$q^*(t) = \begin{cases} 0 \; if \;\; C \leq 0 \\ C \; if \;\; 0 < C \leq 1 \\ 1 \; if \;\; C > 1 \end{cases}, \tag{4.13}$$

where

$$C = 1 - \frac{\alpha w(t) p}{2\pi^m (1 + \mu(t))}. \tag{4.14}$$

We conclude that the optimal degree of price-fixing by the firm is a decreasing function of both the penalty for violation and the probability of law enforcement. This is also quite intuitive from an economic point of view. The profit maximizing firm will reduce their optimal degree of price-fixing in response to the increase in the rate of law enforcement, since it makes conviction more likely. Secondly, increase in accumulated illegal gains from collusion also rises the expected penalty, and this gives an additional incentive for the firm to reduce the degree of price-fixing. This allows the system to gradually converge to the socially desirable outcome with no price-fixing.

The analysis of the state-costate dynamics

Substituting (4.14) into (4.1) and (4.12) gives the following system of differential equations:

$$\begin{cases} \dot{w}(t) = \pi^m (1 - (\frac{\alpha w p}{2\pi^m (1+\mu)})^2) = 0 \\ \dot{\mu}(t) = -\gamma + \alpha p (1 - \frac{\alpha w p}{2\pi^m (1+\mu)}) + r\mu = 0. \end{cases} \tag{4.15}$$

A stationary point can be obtained by intersecting the locuses $\dot{w} = 0$ and $\dot{\mu} = 0$. The $\dot{w} = 0$ isocline is given by $w(\mu) = 2\pi^m(\frac{\mu+1}{p\alpha})$ and the $\dot{\mu} = 0$ isocline satisfies $w(\mu) = 2\pi^m(\frac{-\gamma\mu-\gamma+p\alpha\mu+p\alpha+\mu^2r+r\mu}{\alpha^2p^2})$.

The steady state of the system (4.15), being located in the positive orthant, is given by

$(\mu^* = \frac{\gamma}{r}, w^* = \frac{2\pi^m(1+\frac{\gamma}{r})}{\alpha p} \rightarrow q^* = 0)$.

Existence of stationary points

Both the $\dot{w} = 0$ and $\dot{\mu} = 0$ locuses are increasing functions of μ. From the expression for $\dot{\mu} = 0$ we obtain $w(0) = 2\pi^m(\frac{-\gamma+p\alpha}{\alpha^2p^2})$ and $lim_{\mu\rightarrow\infty}\ w(\mu) = \infty$. Similarly, from the expression for $\dot{w} = 0$ it follows $w(0) = \frac{2\pi^m}{p\alpha}$ and $lim_{\mu\rightarrow\infty}\ w(\mu) = \infty$.

Now we see that one condition for the existence of a stationary point in the positive orthant (where $w \geq 0$) is $\frac{2\pi^m}{p\alpha} \geq 0$, which is always true. Another necessary condition for the existence of a stationary point in the positive orthant is $2\pi^m(\frac{-\gamma+p\alpha}{\alpha^2p^2}) \leq \frac{2\pi^m}{p\alpha}$. This is always true. The final condition, that has to be satisfied in order to obtain the existence of a unique point of intersection of the locuses $\dot{w} = 0$ and $\dot{\mu} = 0$ in the positive orthant, is that the slope of the line that corresponds to $\dot{\mu} = 0$ in the (μ, w)- plane is greater than the slope of $\dot{w} = 0$. $\dot{w} = 0$ gives $w'(\mu) = \frac{2\pi^m}{p\alpha}$ and $\dot{\mu} = 0$ implies that $w'(\mu) = 2\pi^m(\frac{-\gamma+p\alpha+2\mu r+r}{\alpha^2p^2})$. Comparing these two expressions, we can conclude that the final condition for existence of stationary points in the positive orthant is satisfied for any non-negative μ only in case that $\gamma < r$. This means that, when the extra benefits for the firm from cartel formation do not increase much with the experience of the firm in cartel formation, the outcome with no collusion is more likely to be sustained in the long run, since it is less attractive for the firm to participate in the cartel agreements. So a unique stationary point in the positive orthant always exists, except when $p = 0$ (i.e the probability to be caught is zero) or when $\gamma > r$ (i.e. the extra benefits for the firm from cartel formation increase very fast when the experience of the firm in cartel formation increases). The optimal control problem does not have a stable solution in these cases.

The solution procedure and construction of the phase portrait is illustrated via the next example.

Example. We construct the phase portrait when the parameters are $\gamma = 0.5, \pi^m = 1, \alpha = 2, p = 0.2, r = 0.2$. The $\dot{w} = 0$ isocline is given by $1 - (\frac{\frac{2}{5}w}{2(1+\mu)})^2 = 0$, which implies that $\mu = -1 + \frac{1}{5}w$. Similarly, the $\dot{\mu} = 0$ isocline is given by $-\frac{1}{2} + \frac{2}{5}(1 - \frac{\frac{2}{5}w}{2(1+\mu)}) + \frac{1}{5}\mu = 0$, so that $\mu = -\frac{1}{4} + \frac{1}{20}\sqrt{(225 + 160w)}$. The stationary point then satisfies $-1 + \frac{1}{5}w = -\frac{1}{4} + \frac{1}{20}\sqrt{(225 + 160w)}$. This implies that $w^* = \frac{35}{2}$ and $\mu^* = 2.5$.

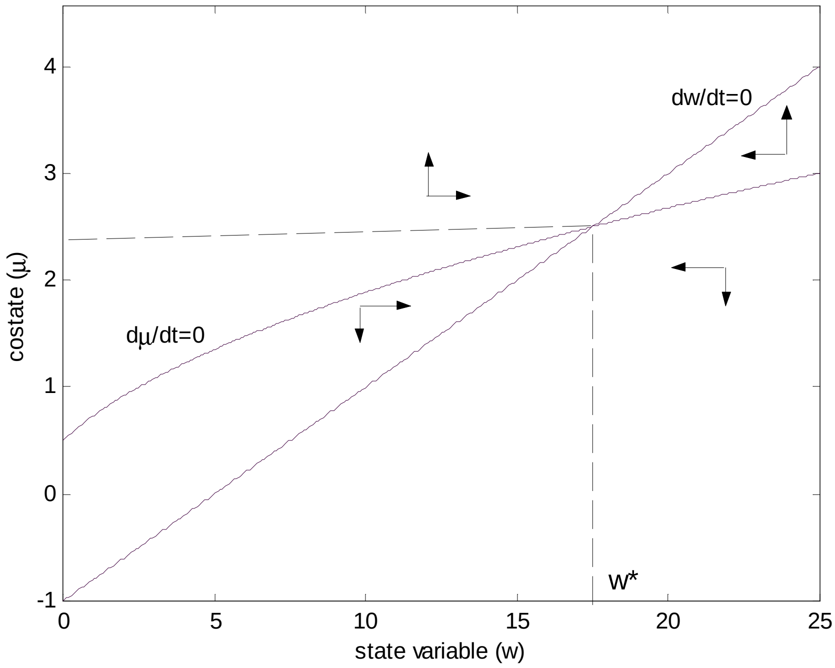


Figure 4.2: Phase portrait in the (w, μ)-space for the optimal control model for the set of parameter values $\gamma = 0.5, \pi^m = 1, \alpha = 2, p = 0.2, r = 0.2$, where the penalty schedule is given by $s(w) = \alpha w$.

Studying the stability of the steady state equilibrium $w^* = \frac{35}{2}$ and $\mu^* = 2.5$ we obtain the following expressions for the values of the trace and determinant of the Jacobian matrix of system (4.15):

$trace\ J = -(\frac{\frac{2}{5}}{(\frac{7}{2})})^2 \frac{35}{4} + \frac{\frac{35}{2}\left(\frac{2}{5}\right)^2}{2(\frac{7}{2})^2} + \frac{1}{5} = \frac{1}{5} > 0,$

$det\ J = -(\frac{\frac{2}{5}}{(\frac{7}{2})})^2 \frac{35}{4}(\frac{\frac{35}{2}\left(\frac{2}{5}\right)^2}{2(\frac{7}{2})^2} + \frac{1}{5}) + \frac{2(7)^2}{4(\frac{7}{2})^3}\frac{\left(\frac{2}{5}\right)^2}{7} = -\frac{4}{175} < 0.$

This allows us to conclude that the point with $w^* = \frac{35}{2}, \mu^* = 2.5, q^* = 0$ is a saddle point.

Stability analysis

Starting with the system dynamics (4.15) in the state-costate space, we can calculate the Jacobian matrix

$$J = \begin{pmatrix} -\left(\frac{\alpha p}{(1+\mu)}\right)^2 \frac{2w}{4\pi^m} & \frac{2(\alpha p w)^2}{4\pi^m(1+\mu)^3} \\ -\frac{(\alpha p)^2}{2\pi^m(1+\mu)} & \frac{(\alpha p)^2 w}{2\pi^m(1+\mu)^2} + r \end{pmatrix}.$$

Obviously, the determinant has to be evaluated in the steady state (μ^*, w^*, q^*). It turns out that *trace* $J > 0$ and *det* $J < 0$, so that the steady state is a saddle point.

In general, with arbitrary values of the parameters and arbitrary equilibrium values the matrix J has two real eigenvalues of opposite sign and the steady state has the local saddle-point property. This means that there exists a manifold containing the equilibrium point such that, if the system starts at the initial time on this manifold and at the neighborhood of the equilibrium point, it will approach the equilibrium point at $t \to \infty$.

This proves the following proposition.

Proposition 4.2 *The outcome with complete deterrence is sustainable in the long run, provided that the parameter p is strictly greater than zero. The steady state with $\mu^* = \frac{\gamma}{r}$, $w^* = \frac{2\pi^m(1+\frac{\gamma}{r})}{\alpha p}$ and $q^* = 0$ is a saddle point.*

The proposition implies that in the long run the full compliance behavior arises in a sense that the outcome with $q^* = 0$ is the saddle point equilibrium of the model. This means that one can always choose the initial value for the adjoint variable such that the equilibrium trajectory starts on the stable manifold and converges to the steady state. Economically speaking, the firm which maximizes profits over time under a proportional penalty scheme will gradually reduce the degree of violation to zero. However there is one exception: for $p = 0$ the degree of offence is maximal. The parameter α influences only the speed of convergence to the steady state value, not the steady state value of the control variable. Clearly, a higher α increases incentives for the firm to stop the violation earlier. Basically, deciding on the time of stopping the violation the firm compares the expected punishment and expected benefits from crime. Consequently, since in the setup with proportional penalty the expected punishment also rises when the benefits from price-fixing rise, in the long run the system will end up in the equilibrium with full compliance.

Trajectories of the state, control and costate variables of the model

It is also illuminating to investigate the behavior of the variables of the model over time and with respect to the main parameter of the penalty scheme.

We can obtain analytical solutions for control, state and costate variables of the model only in case $p = 0$ for all t:

$p = 0 \Longrightarrow q^*(t) = 1$ for all $t \in [0, T] \Longrightarrow \mu(t) = \gamma(T - t)$.

Substituting this result into the state dynamics (4.1) we obtain that $w(t) = \pi^m t + w_0$.

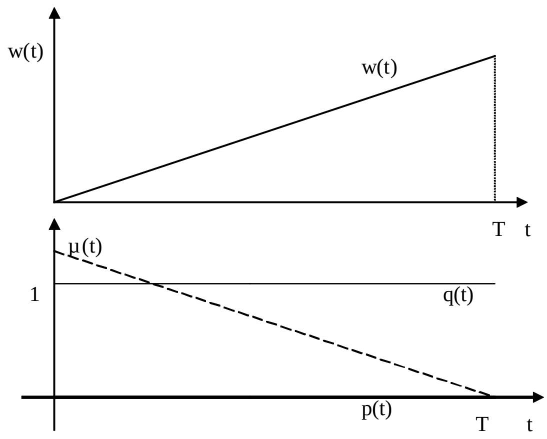


Figure 4.3: Trajectories of the state, control and co-state variables of the model, where $p = 0$ and $\alpha = 2, \pi^m = 2, w_0 = 0, \gamma = \frac{1}{2}, T = 10, r = 0$.

Note that $p = 0$ never leads to complete deterrence, since (4.14) implies that the best response of the firm in this case is $q^* = 1$.

Now consider the situation where $p > 0$. Combining (4.15) and (4.14) we obtain that

$$\dot{w}(t) = \pi^m(1 - (\frac{\alpha w(t) p}{2\pi^m(1 + \ \mu(t))})^2), \ w(0) = w_0. \tag{4.16}$$

Even if we have the information about the dynamics of $\mu(t)$ we cannot obtain an analytical solution for the differential equation (4.16). We can only conclude that in the model, where the penalty is determined by $s(w) = \alpha w$, the antitrust authority, whose aim is to achieve no price-fixing outcome at least by the end of the planning period will have to commit to the following policy:

$$s(t) = \alpha w(t) \ for \ all \ t \in [0, T] \ and \ \ p(t) = \begin{cases} \dfrac{1 \ \ for \ all \ t \in [0, t^{**}]}{min\{1, \frac{2\pi^m(1+\mu(t))}{\alpha w(t^{**})}\} \ \ for \ all \ t \in [t^{**}, T]} \end{cases}$$

Where t^{**} is the root of the equation $q(t) = 1 - \frac{\alpha w(t)}{2\pi^m(1+\mu(t))} = 0$ (see (4.14)).

Note, that $\mu(t) > -1$ for all t is an additional condition for the existence of the root of this equation. Since $\mu(T) = 0$, this will be ensured by the condition $\dot{\mu}(t) = -\frac{\partial H^2}{\partial w} = -\gamma + \alpha p q(t) < 0$ for all $t \in [0, T]$.

So the trajectories of the state, control and costate variables of the firm together with the most cost efficient policy of the antitrust authority will have the following form. When the firm is subject to antitrust enforcement with proportional penalty, the degree of offence by the firm gradually declines and finally reaches its steady state value. This happens because the expected penalty rises over time as well when the firm commits offence more often. Consequently, the accumulated rents from price-fixing activities to the firm increase over time, but the speed of this increase declines when the system approaches the steady state equilibrium level. The aim of the antitrust authority is to block the violation as fast as possible. In this case the most cost efficient policy of the antitrust authority in response to this behavior of the firm would be to keep the probability of law enforcement at the highest possible level until the state variable reaches its steady state value and then reduce the efforts gradually keeping expression (4.14) $q^*(t) = 1 - \frac{\alpha w(t^{**}) p(t)}{2\pi^m(1+\mu(t))}$ equal to zero (see Figure 4.4).

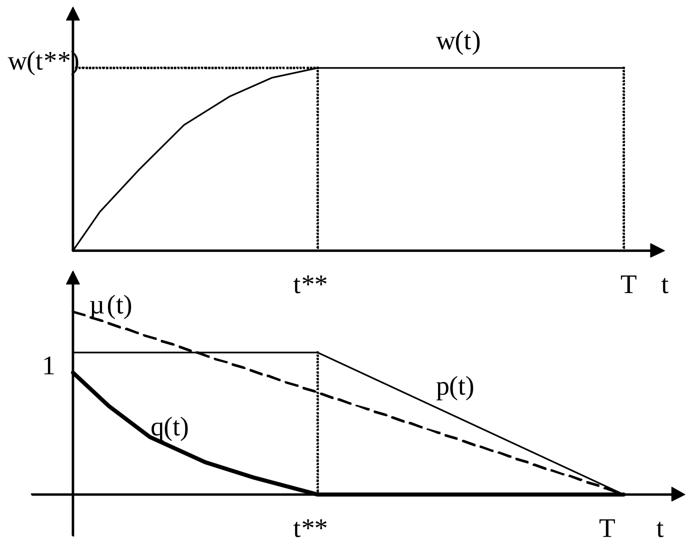


Figure 4.4: Trajectories of the state, control and co-state variables of the model, with optimally chosen p and $\alpha = 2, \pi^m = 2, w_0 = 0, \gamma = \frac{1}{2}, T = 10, r = 0$.

A no price-fixing outcome ($q(t) = 0$) can be sustained, but it occurs only at the end of the planning period. To be more precise, the dynamics of the optimal behavior of the firm is such that, given the parameters of the penalty system (p and α), the firm gradually reduces the degree of offence to zero, which happens at time t^{**}. After that no more collusion will take place. Consequently, accumulated gains from price-fixing will gradually increase and after $t = t^{**}$ will stay at the level $w(t^{**})$. The parameters of the penalty system (p and α) have an impact on the optimal behavior of the firm and consequently on the deterrence power of the penalty system, which is measured by the timing of optimal deterrence or, in other words, by the value of t^{**}. The higher α and p the closer t^{**} to the origin, and consequently the earlier the cartel formation is blocked.

Sensitivity analysis

Here we investigate in which direction the saddle point equilibrium moves if the set of parameter values changes. Analyzing the properties of the proportional penalty scheme ($s(t) = \alpha w(t)$), the main parameters of our interest are the scale parameter of the penalty schedule, α, and the parameter which determines the certainty of punishment, p. They appear to be also quite important parameters for the firm, whose objective is to maximize the expected rents from price-fixing in the presence of antitrust enforcement. Clearly, the firm will condition its behavior on the parameters of the penalty scheme, chosen by the regulator (see expression (7.1)). Moreover, the result obtained below will provide hints on how to choose the optimal enforcement policy to minimize the steady state degree of price-fixing by the firms.

As a result of the necessary optimality conditions, in the steady state equilibrium it holds that

$$\dot{w}(t) = f(q, w, \mu, \alpha, p) = \pi^m q(2 - q) = 0,$$
$$\dot{\mu}(t) = r\mu(t) - H_w(q, w, \mu, \alpha, p) = r\mu - \gamma + \alpha pq = 0,$$
$$H_q(q, w, \mu, \alpha, p) = (2\pi^m - 2\pi^m q)(1 + \mu) - \alpha wp = 0.$$

Computing the total derivative of the above equations with respect to α we get

$$\begin{pmatrix} f_\mu & f_w & f_q \\ f_w - r & H_{ww} & H_{wq} \\ f_q & H_{qw} & H_{qq} \end{pmatrix} \begin{pmatrix} \frac{\partial \mu}{\partial \alpha} \\ \frac{\partial w}{\partial \alpha} \\ \frac{\partial q}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} -f_\alpha \\ -H_{w\alpha} \\ -H_{q\alpha} \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 0 & 0 & 2\pi^m(1-q) \\ -r & 0 & -\alpha p \\ 2\pi^m(1-q) & -\alpha p & 2\pi^m(1+\mu) \end{pmatrix} \begin{pmatrix} \frac{\partial \mu}{\partial \alpha} \\ \frac{\partial w}{\partial \alpha} \\ \frac{\partial q}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ pq \\ wp \end{pmatrix}.$$

Performing the same exercise for parameter p we obtain that

$$\begin{pmatrix} 0 & 0 & 2\pi^m(1-q) \\ -r & 0 & -\alpha p \\ 2\pi^m(1-q) & -\alpha p & 2\pi^m(1+\mu) \end{pmatrix} \begin{pmatrix} \frac{\partial \mu}{\partial p} \\ \frac{\partial w}{\partial p} \\ \frac{\partial q}{\partial p} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha q \\ \alpha w \end{pmatrix}.$$

The next step is solving this system of linear equations with Cramer's rule. In order to determine the signs of $\frac{\partial \mu}{\partial \alpha}, \frac{\partial w}{\partial \alpha}, \frac{\partial q}{\partial \alpha}$, and $\frac{\partial \mu}{\partial p}, \frac{\partial w}{\partial p}, \frac{\partial q}{\partial p}$ we first observe the sign of the determinant $\triangle$ of the matrix of the coefficients:

$\triangle = 2\pi^m(1-q)r\alpha p > 0.$

The sign of $\frac{\partial \mu}{\partial \alpha}$ can now be derived by determining the fraction $\frac{\triangle_\mu}{\triangle}$, with

$$\triangle_\mu := \det \begin{pmatrix} 0 & 0 & 2\pi^m - 2\pi^m q \\ pq & 0 & -\alpha p \\ wp & -\alpha p & 2\pi^m(1+\mu) \end{pmatrix} = -2\pi^m(1-q)\alpha q p^2 < 0.$$

So we can conclude that $\frac{\partial \mu}{\partial \alpha} = \frac{\triangle_\mu}{\triangle} = \frac{-2\pi^m(1-q)\alpha q p^2}{2\pi^m(1-q)r\alpha p} = -\frac{qp}{r} < 0$. The same result holds for behavior of the costate variable with respect to a change in the probability of law enforcement, $\frac{\partial \mu}{\partial p} = \frac{-2\pi^m(1-q)\alpha^2 q p}{2\pi^m(1-q)r\alpha p} = -\frac{\alpha q}{r} < 0$. This means that the equilibrium steady state value of the shadow price decreases when the slope of the penalty function (α) increases or the rate of law enforcement increases. The reason is that with higher α or p a higher accumulated wealth increases the expected punishment much faster than in the case when α or p are low.

In the same way we can derive the sign of $\frac{\partial w}{\partial \alpha}$ and $\frac{\partial w}{\partial p}$, which we get through computing $\frac{\triangle_w}{\triangle}$, where

$$\triangle_w := \det \begin{pmatrix} 0 & 0 & 2\pi^m(1-q) \\ -r & pq & -\alpha p \\ 2\pi^m(1-q) & wp & 2\pi^m(1+\mu) \end{pmatrix} = -2\pi^m(1-q)rwp - 4pq(\pi^m)^2(1-q)^2 < 0.$$

This implies that $\frac{\partial w}{\partial \alpha} = \frac{\triangle_w}{\triangle} = \frac{-2\pi^m(1-q)rwp - 4pq(\pi^m)^2(1-q)^2}{2\pi^m(1-q)r\alpha p} = -\frac{w}{\alpha} - \frac{2\pi^m(1-q)q}{r\alpha} < 0.$

Similar calculations for the parameter p give that $\frac{\partial w}{\partial p} = \frac{-2\pi^m(1-q)rw\alpha - 4\alpha q(\pi^m)^2(1-q)^2}{2\pi^m(1-q)r\alpha p} = -\frac{w}{p} - \frac{2\pi^m(1-q)q}{rp} < 0.$

This means that either an increase in the scale parameter of the penalty scheme or an increase in the certainty of punishment would cause a reduction of the equilibrium accumulated rents from collusion, so that the firms will try to reduce their gains in order to be punished less.

Finally, we have a look at the change of the offence level caused by a change in the slope of the punishment function or a change in the rate of law enforcement. That means we are now interested in the signs of $\frac{\partial q}{\partial \alpha}$ and $\frac{\partial q}{\partial p}$. Computing the determinants we find that $\frac{\partial q}{\partial \alpha} = \frac{\partial q}{\partial p} = 0$.

So we can conclude that the effect of either change in certainty or in severity of the penalty on the equilibrium value of the degree of offence is absent. It follows logically from the model, since $q^* = 0$ is a steady state solution of the model and its absolute value and existence does not depend on the size of the parameters α and p.

The change in α or in p only influences the t^{**} value in Figure 4.5[17]. Numerical analysis of the behavior of the state and control variables of the model with respect to the main parameters of the penalty scheme (α and p) shows that a higher α or p leads to earlier deterrence, i.e. t^{**} moves closer to the origin (see Figure 4.5). Consequently, the degree of price fixing is lower at each instant of time and total accumulated gains from price-fixing by the colluding firm are lower. Moreover, this policy allows to reduce the costs for society as well, since we can block violation earlier and hence reduce the control efforts earlier.

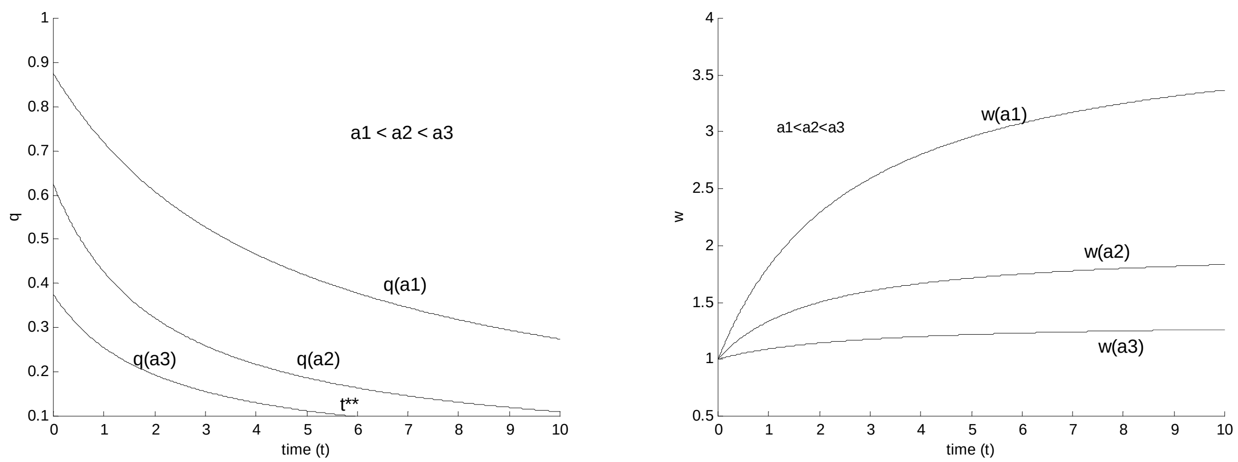


Figure 4.5: Numerical analysis of the behavior of the state and control variables of the model with respect to the scale parameter of the penalty scheme (α) when parameter values are $\gamma = 0.5, \pi^m = 1, p = 0.2, r = 0.2$.

Looking at the partial derivatives of the state variable of the model with respect to the main parameters of the penalty scheme we obtain the following proposition.

Proposition 4.3 *a) Under the policies that provide underdeterrence, i.e. when α is low, i.e. $\alpha = p \in [0, 1]$ and $\left|\frac{\partial w}{\partial \alpha}\right| = \left|\frac{\partial w}{\partial p}\right|$, the effects of detection probability and severity of punishment on the deterrence power of the penalty scheme in the steady state are equal.*

[17]Recall also Figure 4.4 in the part where we describe optimal trajectories of the state, control and costate variables of the model.

b) When α is high, i.e. $\alpha > 1$ and, hence, $\left|\frac{\partial w}{\partial \alpha}\right| < \left|\frac{\partial w}{\partial p}\right|$, i.e. under the policies that can potentially provide more efficient deterrence, the effect of an increase of the probability of punishment on the deterrence power of the penalty scheme in the steady state is much stronger.

Proof.

Consider the partial derivatives of the state variable of the model with respect to the main parameters of the penalty scheme. Following the above analysis they are

$$\frac{\partial w}{\partial \alpha} = -\frac{w}{\alpha} - \frac{2\pi^m(1-q)q}{r\alpha} \tag{4.17}$$

$$\frac{\partial w}{\partial p} = -\frac{w}{p} - \frac{2\pi^m(1-q)q}{rp}. \tag{4.18}$$

Now we can show that, when α is higher than p, thus, for instance, when $\alpha > 1$, the decrease in w, in absolute terms, when α increases, is less than the decrease in w, in absolute terms, when p increases. When $\alpha > 1$, then from (4.17) we obtain $\left|\frac{\partial w}{\partial \alpha}\right| < \frac{wr+2\pi^m(1-q)q}{r}$. Similarly, keeping in mind that $p \in [0,1]$ by construction, from (4.18) we obtain that $\left|\frac{\partial w}{\partial p}\right| > \frac{wr+2\pi^m(1-q)q}{r}$. ■

The general conclusion of this subsection is that, when $w_0 = 0$, only partial deterrence is feasible. But nevertheless, $q(t) = 0$ for some $t \in [t^{**}, T]$ can be achieved in the model if $p(t) > 0$ for all $t \in [0, T]$ and the equilibrium with $q^* = 0$ can be sustained as the long run saddle point steady state equilibrium of the model with penalty system given by $s(t) = \alpha w(t)$ and $p > 0$ under certain additional conditions on the parameters of the model.

Moreover, studying the sensitivity of the steady state values of the main variables of the model with respect to the parameters of the penalty scheme we found an interesting result, which gives new insights into the problem of optimal trade-off between the probability and severity of punishment. This problem has been studied quite extensively in a static setting by Polinsky and Shavell (1979) and later by Garoupa (1997) and (2001). The result, stated in Proposition 4.3, shows that, when the penalty is high a further increase in the severity of punishment is less efficient than an increase in probability of punishment.

4.5 Conclusions

The main problem addressed in this chapter is how the fine, which takes into account the history of the violation, i.e. determined on the basis of accumulated turnover of the firm participating in cartel, affects the efficiency of the deterrence. To motivate this problem, we refer to two main features of penalty systems for violations of the antitrust law prescribed by the current sentencing guidelines. Firstly, there exists an upper bound for the fine. The penalty is constrained from above by either a certain monetary amount or by the amount of 10% of the total annual turnover of the firm. Secondly, the penalty is based on the accumulated gains from cartel or price-fixing activities for the firm. These regulations suggest to model the penalty as an increasing function of the accumulated illegal gains from price-fixing to the firm.

The main innovation of the chapter compared to the existing literature, e.g. Fent et al. (1999) or Feichtinger (1983), is the idea that the accumulated wealth of the firm takes the role of the state variable in the optimal control model. This modification allows to incorporate two main features of the current penalty systems for antitrust law violations, discussed above, into a dynamic model of intertemporal utility maximization by the firm under antitrust enforcement. In particular, this modification allows to develop a framework, in which the penalty for antitrust violations can be constructed in such a way that it can capture the history of the violation. In order to capture the history, we model the penalty for price-fixing as an increasing function of the accumulated gains from price-fixing throughout the entire duration of the infringement (which is the state variable of the model).

First, we look at the case where the penalty is fixed. We derive an analytical expression for this penalty, which allows to achieve the result of complete deterrence of price-fixing, given a strictly positive rate of law enforcement by the antitrust authority. Numerical calculations show that the policy aiming at increasing or even abolishing the legal upper bound for fine might be successful. But, unfortunately, this system does not solve the problem of optimal deterrence as well, since the penalties in this case, which allow to achieve complete deterrence, are too high, and thus unbearable for the firm because they can drive the firm to immediate bankruptcy.

We also analyze the optimal control model, in which the penalty is determined as a linear increasing function of the accumulated rents from price-fixing. On the basis of this analysis we conclude that the parameters of the penalty system have an impact on the optimal behavior of the firm and consequently on the deterrence power of the penalty system, which is measured by the timing of optimal deterrence. The higher the probability and severity of punishment the earlier the cartel formation is blocked. Moreover, a proportional system seems to be more fair than one with a fixed penalty and allows to achieve a full compliance outcome in the long run. The analysis of this model also confirms that modification of penalty systems suggested recently by OECD (which, in terms of our model, implies an increase of multiplier α) is quite promising, since it will lead to earlier deterrence.

In addition, we conduct sensitivity analysis of the equilibrium values of the main variables of the model with respect to the changes in the scale parameter of the proportional penalty scheme and probability of law enforcement. Studying the sensitivity of the steady state values of the main variables of the model with respect to the parameters of the penalty scheme we found an interesting result, which gives new insights into the problem of optimal trade-off between the probability and severity of punishment. This result states that when the penalties are high a further increase in the severity of punishment is less efficient than the increase in certainty of punishment. This implies that in order to achieve improvements in deterrence when penalties are already high, it is more efficient to spend resources and increase the probability of punishment rather than simply raise the upper bound for the fine.

We can also suggest a number of possible extensions of the model. One possibility is to introduce a second state variable (offender's criminal record) into the model in addition to accumulated gains from price-fixing. This will allow to relate penalty to both important factors: gravity of the violation and past reputation of the offender (recidivistic behavior). This extension will help to explain escalating sanctions based on offence history which are embedded in many penal codes and sentencing guidelines. Another interesting direction is to extend the analysis to two players case and consider a similar problem in the framework of differential games. One would say that a dynamic game situation would be more appropriate to describe the problem at hand. A pursuit-evasion game of the Feichtinger (1983) type would help to reflect the idea that competition authority can also act strategically and not rule based. However, the scope of the current chapter, which is aiming to compare the effects of fixed and proportional penalties on the behavior of the firms that violate competition law, does not require a competition authority acting strategically. Although, the differential game framework would be an interesting extension of the problem at hand in case we want to find an optimal combination of both instruments of antitrust authority (fine and rate of law enforcement), which allows to achieve the result of complete deterrence.

CHAPTER 5

Effects of Leniency Programs on Cartel Stability

5.1 Introduction

This chapter analyzes the effects of leniency programs by employing a game between two firms, which participate in a cartel agreement and decide on the optimal time of revealing the information about the cartel to the antitrust authority. The enforcement problem we study has several ingredients. Firstly, we analyze the design of self-reporting schemes, where we have a group of defendants. Secondly, we consider a dynamic set-up, where accumulated benefits and losses from crime are taken into account. Leniency programs allow for complete or partial exemption from the fine for firms that reveal information about the cartel to the antitrust authority. It is intuitively clear that a legally sanctioned opportunity for costless self-reporting changes the nature of the game played between the antitrust authority and the group of firms. To analyze the impact of this opportunity on cartel stability we apply tools of timing games. In particular, we study a dynamic game of the preemption type.

Leniency programs have been recently introduced in the European antitrust legislation and have quite a long history in the US. "Leniency programs" grant total or partial immunity from fines to firms that collaborate with the authority. To be more precise, leniency is defined as a reduction of the fine for firms, which cooperate with the antitrust authority by revealing information about the existence of the cartel before the investigation has started, or by providing additional information that can help to speed up the investigation. Leniency programs work on the principle that firms, who break the law, might report their crimes or illegal activities if given proper incentives and, therefore, reduce harm for society by reducing the number of cartels and their duration.

In the US the first Corporate Leniency Program was introduced in 1978. Then it was refined and extended in August 1993. Later the Antitrust Division of the US Department of Justice revised its Corporate Leniency Program to make it easier for and more attractive to companies to come forward and cooperate with the Division[1]. Three major revisions were made to the program, namely, amnesty is automatic if there is no pre-existing investigation, amnesty may still be available even if cooperation begins after the investigation is underway, and all officers, directors, and employees who cooperate are protected from criminal prosecution. As a result of these changes, the Amnesty Program is the Division's most effective generator of international cartel cases. Moreover, the revised Corporate Amnesty Program has resulted in a surge in amnesty applications. Under the old amnesty policy the Division obtained roughly one amnesty application per year. Under the new policy, the application rate has been more than one per month. In the last few years, cooperation resulting from amnesty applications led to scores of convictions of over $1 billion in fines[2].

In Europe the first Leniency Programs were introduced in 1996. The modified Leniency program introduced by the EC in 2002 gives complete immunity from fines to firms, which were the first to submit evidence about the cartel to the antitrust authority. Moreover, partial reduction of fines (approximately by 50%) takes place even if firms reveal information after an investigation has started. Similar programs have been introduced in 2002 in the UK and other European countries.

There is some empirical evidence that Leniency programs improve welfare by sharply increasing the probability of interrupting collusive practices and by shortening the investigation. In the US, for example, the fines collected in 1993 almost doubled the ones in 1992, which can be connected with the major modification of leniency programs. However, there are also other effects of leniency programs, which are now difficult to identify in empirical studies due to the absence of data. For example, questions of how the introduction of leniency programs would influence cartel stability and duration of cartel agreement, or whether leniency facilitates collusion or reduces it, still require deeper investigation. In this chapter we give some insights by analyzing these problems.

Other contributions that analyze optimal policies for the deterrence of violations of

[1] See Spratling Gray R. (1998).

[2] See O.E.C.D. report (2002a).

antitrust law in the presence of leniency schemes are Motta and Polo (2003), Spagnolo (2000), or Malik (1993). Most papers on leniency employ a discrete time framework. However, proportional penalty schemes that most closely reflect current antitrust rules were not analyzed in the discrete time repeated games models so far. Though they considered collusive behavior in a dynamic setting with antitrust laws, these papers excluded the important sources of dynamics that are the foci for this thesis: in particular, they did not allow detection and penalties to be sensitive to firms' current and past pricing behavior. However, a number of papers have already looked into this problem, namely, Hinloopen (2003, 2004) and Harrington (2004, 2005). They investigated settings, in which cartel detection probabilities were influenced by firms' behavior and where these probabilities changed over time. However, penalties which are proportional to the degree of offence and change over time, and that most closely reflect current antitrust rules were not analyzed by the above-mentioned researches. In this chapter we study the problem of how an additional enforcement instrument, such as a leniency program, influences the stability of cartels under two different regimes of fines, fixed and proportional. We analyze a setting with a proportional penalty scheme employing a continuous time dynamic preemption game, in which accumulated gains from price-fixing is the state variable. We investigate intertemporal aspects of this problem using dynamic optimal stopping models and tools of dynamic continuous time preemption games and obtain that the strength of incentives to preempt is the driving force of success of leniency programs. In this way we extend the existing literature.

It should be stressed that a legally sanctioned opportunity for costless whistle-blowing changes the game played between the antitrust authority and the group of firms, compared to a setting where leniency is not available. Intuitively, this opportunity should reduce cartel stability and increase the incentives for firms to reveal the cartel. In this chapter we investigate the effects of leniency programs on the behavior of firms participating in price-fixing agreements. The main finding of this chapter is that depending on the design of leniency programs, they may reduce duration of cartel agreements but this result is not unambiguous. This problem has already been discussed in a number of earlier papers (see for example, Spagnolo (200b) or Ellis and Wilson (2003)). They found that wrongly designed leniency programs may facilitate collusion rather than break cartels. In this chapter we obtain a similar result. The analysis of this chapter implies that under strict antitrust enforcement, i.e. when only the first reporter can obtain complete immunity from the fine and penalties and the rate of law enforcement are high, the possibility to self-report and be exempted from the fine increases the incentives for firms to stop cartel formation. This reduces the duration of cartels. However, when both first and second reporters are treated similarly, and penalties and rate of law enforcement are low, introduction of leniency programs may, on the contrary, facilitate collusion. Under a fixed penalty scheme, even in the presence of leniency, the efficiency of cartel deterrence (in terms of reduction of duration of cartel agreements) depends only on the amount of the fine and the probability of law enforcement. We also show that "too lenient " leniency programs may facilitate collusion, when penalties are fixed and fall below a certain threshold.

We distinguish two regimes with respect to the rules of leniency programs and the application procedure. The first regime corresponds to more strict enforcement, i.e. only the firm, which is the first to self-report, is eligible for complete exemption from the fine and the application procedure is strictly confidential. The second firm bears either the full fine or, if it provides sufficient evidence, it can be exempted from up to 50% of the fine. This set up most closely reflects the rules of current guidelines for reduction of fines for firms that cooperate with antitrust authorities and reveal information about existing cartels[3]. The second regime corresponds to the case where the rules of antitrust enforcement are not too strict (more lenient). In this case also the firm, which is the second to self-report, obtains partial exemption from the fine. This implies that the first and second reporter are treated similarly and the antitrust authority makes the application procedure publicly observable. Comparison of these two regimes implies that, if the rules of leniency programs and the procedure of application for leniency are more strict (in the sense that only the first self-reporting firm can obtain complete exemption from fine), cartel occurrence is less likely.

A number of earlier papers have studied the problem of self-reporting. Malik (1993) and Kaplow and Shavell (1994) were the first to identify the potential benefits of schemes which elicit self-reporting by violators. They focus on the set-up with individual wrong-doers and conclude that self-reporting may reduce enforcement costs and improve risk-sharing, as risk-averse self-reporting individuals face a certain penalty rather than the stochastic penalty faced by non-reporting violators. A similar paper in this field is Innes (1999), who considers environmental self-reporting schemes.

The use of leniency programs in antitrust has been extensively studied by Motta and Polo (2003). They show that such programs might play an important role in the prosecution of cartels provided that firms can apply for leniency after an investigation has started. They conclude that, if given the possibility to apply for leniency, the firm

[3]See O.E.C.D. report (2002).

might well decide to give up its participation in the cartel in the first place. They also find that leniency saves resources for the authority. Finally, their formal analysis shows that leniency should only be used when the antitrust authority has limited resources, so that a leniency program is not unambiguously optimal. The paper by Motta and Polo (2003) is closely related to the paper by Spagnolo (2000). He shows that only courageous leniency programs that reward self-reporting parties may completely and costlessly deter collusion, while moderate leniency programs that reduce or cancel sanctions for the reporting party cannot affect organized crime.

A next attempt to study the efficiency of leniency programs in antitrust enforcement was made in Feess and Walzl (2003). They compare leniency programs in the EU and the USA. For that purpose they construct a stage-game with two self-reporting stages, heterogeneous types with respect to the amount of evidence provided, and ex post asymmetric information. Their analysis shows that self-reporting schemes are much more promising for criminal teams than for single violators, since strategic interactions between team members lead to increased expected fines, and reduce the frequency of violations. Hence, their model once again confirms the effectiveness of leniency programs in the fight against cartels.

The chapter is organized as follows. Section 5.2 describes the basic model. In section 5.3 we consider the benchmark model and solve the timing game of cartel formation in the situation where leniency is not available. There are two symmetric firms that participate in a cartel agreement and decide on the optimal stopping time, i.e. the moment of quitting the cartel in the absence of leniency programs. Further, in section 5.4 we study a timing game with two identical firms forming a cartel after the leniency programs are introduced. We suggest a new approach to analyze the efficiency of the leniency programs that differs from the earlier papers and that is based on the Reinganum-Fudenberg-Tirole Model. Reinganum (1981), Fudenberg and Tirole (1985), and Huisman (2001) applied timing games to a technology adoption problem. We apply a similar procedure to the cartel formation game between two firms in the presence of a leniency program. Section 5.5 analyzes the effects of leniency while the leniency program is less strict. In section 5.6 we solve the game in case penalty is fixed and compare it with the result under proportional penalty. Section 5.7 deals with an extension of the model of section 5.4 by including dynamic price competition and tacit collusion. The last section summarizes the results and suggests directions for future work.

5.2 The Model: Formal Description and Assumptions

We introduce the basic ingredients of the intertemporal optimization problem of an expected profit-maximizing firm, which participates in an illegal cartel. The key variable is the accumulated gains from prior criminal offences at time t, $w(t)$. In case of a cartel, these offences are price-fixing activities. Further we call $w(t)$ the value of collusion at time t.

Let us consider an industry with two symmetric firms engaged in a price-fixing agreement. Assume that they can agree and increase prices from $p^c = c$ to $p^m > c$ each, where c is the constant marginal cost in the industry and p^m is the monopoly price. Since firms are symmetric, each of them has equal weight in the coalition and, consequently, total cartel profits will be divided equally among them. In a game theoretic model we assume that there is a possibility of strategic interaction between the firms in the coalition in the sense that they can break the cartel agreement by self-reporting. By doing this we allow for the possibility for the firms to betray the cartel and this influences the internal stability of the cartel.

The instantaneous monopoly profit in the industry under consideration is denoted by π^m. Consequently, since the firms are assumed to be symmetric, the instantaneous profit per firm will be $\frac{\pi^m}{2}$.

We consider two cases: the case where the penalty, s, is constant over time, i.e. $s(t) = F^n$,[4] and the case where the penalty is a fraction of the accumulated gains from price-fixing activities for the firms. In the latter case the penalty is represented by the expression $s(t) = \alpha w(t)$, where α is the scale parameter of the penalty scheme. This setup will also allow us to compare the efficiency of fixed and proportional penalty schemes. Both of them are currently used in the sentencing guidelines of different countries[5].

The main feature of a leniency program is the reduction of the fine (or complete exemption from the fine) for the firm that first reveals the information about the existence of the cartel. To be more precise, in the model we assume the following set-up. If one of the firms reports the cartel, then this firm pays no fine, $s^L = 0$, while the other firm will pay the normal fine, s^n, that (according to current sentencing guidelines for violations of antitrust law) can be approximated by the amount of 10% of overall turnover of the enterprise. Following the current rules, we also assume that, if the second firm

[4] F^n denotes the normal fine level in the absence of leniency.

[5] See O.E.C.D. (2002a).

decides to cooperate, the fine for this firm will be reduced by the amount in the range between 0% and 50%, i.e. $s^F \in (\frac{1}{2}s^n, s^n]$. Next, we assume that if both firms report the cartel simultaneously, then each of them pays the reduced fine, $s^M = 0.5s^n$. These rules are roughly consistent with partial immunity clauses that often apply if more than one cartelist reports[6].

The rate of law enforcement by the antitrust authority equals $\lambda \in (0, 1]$. This variable denotes the instantaneous probability that the firm is checked by antitrust authority and found guilty.

Given this set-up, firms, participating in the cartel agreement, decide on the optimal stopping time, i.e. the moment of revelation of information about the violation to the authority. An alternative way of stopping may be described in terms of quitting the cartel without reporting to the antitrust authority. We assume that, after the cartel has been discovered due to self-reporting by one or both firms or simply due to the efforts of antitrust authority, collusion stops forever and, consequently, the stream of illegal gains also stops. It is quite realistic to assume that firms would not renew the agreement, if one of them betrayed the other.

The expected penalty if the firm, which was participating in the cartel, is caught at date t is given by $\lambda s(t)$. The discount rate is denoted by r. We assume that there are two identical firms that form a cartel and restrict analysis to the symmetric pure strategy[7] equilibria. The infinite planning horizon is considered, on which the risk-neutral firms maximize their value at discount rate r.

5.3 Benchmark: Timing Game without Leniency

To study the effects of leniency programs on cartel stability, we, first, consider a benchmark case, where leniency is not available, i.e. the firms act without taking into account the possibility of self-reporting. Second, in section 4 we move to the setting where the

[6]Moreover, Apesteguia, Dufwenberg and Selten (2003) use a similar mechanism to design one of the treatments in their experimental paper, which studies the effects of leniency on the stability of a cartel. Feess and Walzl (2003) also consider partial reduction of fines for both firms in case of simultaneous self-reporting.

[7]Taking into account that we employ timing games for analysis of the problem described above, we restrict our attention to search only for pure strategy equilibria. This is motivated by the fact that mixed strategy equilibria for timing games, where firms self-report at a certain point in time only with some probability, do not make economic sense and would not give any reasonable policy implications for antitrust enforcement.

antitrust authority introduces leniency. In that case the dynamic interactions between two firms, which form a cartel but can also betray it, are modeled by employing tools of preemption games.

To analyze a benchmark case, where leniency is not available, we consider a simple timing game[8] with two symmetric firms. Each firm's choice is when to choose the action "stop cartel", and once one of the players chooses this action cartel formation stops forever. That is, if the game has not stopped at any $\tau < t$, the action set for each player i, $i \in \{1, 2\}$ is $A_i(t) = \{stop, continue\}$; if game has stopped at any $\tau < t$, then $A_i(t)$ is a null action "*don't move*". We will consider here only two player timing games, and restrict our attention to the subgame-perfect equilibria in pure strategies. Following Fudenberg and Tirole (1991), we are able to express both players payoffs as functions of the time

$T = min\{t : a_i^t = stop\ cartel\ for\ at\ least\ one\ i\}$

at which the first player stops; if no player ever stops we set $T = \infty$. Here a_i^t denotes action of player i at time t. Fudenberg and Tirole (1991) describe players payoffs using the functions L_i, F_i and B_i : if one player i stops at T, then he is the leader and receives $L_i(T)$, and his opponent receives follower payoff $F_j(T)$. If both players stop simultaneously at T, the payoffs are $B_1(T)$ and $B_2(T)$. They also assume that

$$lim_{T\to\infty} L_i(T) = lim_{T\to\infty} F_i(T) = lim_{T\to\infty} B_i(T), \tag{5.1}$$

which will be the case if payoffs are discounted.

In our set-up, since, after cartel formation stops, profits are assumed to be zero and also the value of being leader is the same as the value of simultaneous stopping, we have that $L_i(T)$, $F_j(T)$ and $B_i(T)$ coincide. The only relevant value is the value of stopping the cartel at time T, denoted by $St(T)$.

The last step is to specify the strategy space. The history at date t is simply the fact that the game is still going on then. Thus, pure strategies s_i are simply maps from the set of dates t to $\{stop, continue\}$. So, pure strategies are simply stopping times. Here, as we have already mentioned, we will restrict attention to pure strategies, since mixed strategies do not make sense from the point of view of policy implications.

Now we determine the payoff function of the game described in section 5.2. The value of collusion changes according to the following law:

$dw = \frac{\pi^m}{2} e^{-rt} dt$ and $w(0) = 0$. This implies that $w(t) = \int_0^t \frac{\pi^m}{2} e^{-rs} ds = \frac{\pi^m}{2r}(1 - e^{-rt})$.

[8]For general definition and analysis of simple timing games see Fudenberg and Tirole (1991, p. 117).

The value of stopping the cartel at time T, $St(T)$, is determined as an integral over time of instantaneous expected gains from collusion before time T. It should be positively related to the instantaneous profits from price-fixing before reporting, $\frac{\pi^m}{2}$, and negatively related to the instantaneous expected penalty, $\lambda s(t)$. We assume that cartel formation stops only in case firms decide to quit the cartel or self-report to the antitrust authority, while firms always renew collusive agreement after they are caught and punished by the antitrust authority without cooperation of cartel members. So, the value of stopping the cartel at time T for each firm in the absence of leniency programs is determined according to the following formula:

$$St(T) = \int_0^T (\frac{\pi^m}{2} - \lambda s(t))e^{-rt}dt. \tag{5.2}$$

In case the fine is proportional to the accumulated illegal gains from price-fixing, expression (5.2) will have the following form

$$St(T) = \int_0^T (\frac{\pi^m}{2} - \lambda \alpha w(t))e^{-rt}dt, \tag{5.3}$$

where $w(t) = \int_0^t \frac{\pi^m}{2} e^{-rl}dl$. Clearly, the limiting conditions (5.1) is also satisfied in this setting.

To find the optimal time of stopping the cartel, we differentiate (5.3) with respect to T and obtain $\frac{\partial St(w)}{\partial T} = \frac{\pi^m}{2} e^{-rT} - \frac{\alpha\lambda\pi^m}{2r} e^{-rT} + \frac{\alpha\lambda\pi^m}{2r} e^{-2rT} = 0$. This implies that in the symmetric equilibrium the optimal stopping time for each firm, which takes a decision whether to quit the cartel or to continue collusion, is given by

$$T^* = \frac{\ln(\frac{\alpha\lambda}{\alpha\lambda - r})}{r}. \tag{5.4}$$

It is easy to show that given that both firms stop at time T^*, neither of them can gain by deviation from T^*. Assume that firm i decides to stop cartel at time $t' < T^*$. Then both firms get $St(t') < St(T^*)$, and, hence, deviation to t' is not profitable. The same happens for any $t' > T^*$. This proves that simultaneous stopping at T^* is a subgame-perfect equilibrium of the timing game that describes the situation of cartel formation in the absence of leniency programs.

Expression (5.4) shows that the optimal time of stopping the cartel decreases when both the probability and the severity of punishment increases. The higher the expected penalty, the earlier the firm decides to quit the cartel agreement, since $\frac{\partial T^*}{\partial \alpha} < 0$ and $\frac{\partial T^*}{\partial \lambda} < 0$.

5.4 Preemption Game with Leniency

Now we describe a timing game of the preemption type played between two symmetric firms. The leader in this game (i.e., the firm which is the first to self-report) has the advantage of complete exemption from the fine, i.e. $s^L = 0$. Moreover, since firms are identical it seems natural to consider symmetric strategies.

First, we consider a setting where firms cannot respond immediately to the actions of their rivals. Following the rules of application for leniency currently used by most antitrust authorities, the information about applications is kept confidential. This information normally does not become public immediately after the firm has applied for leniency. That is why in this section we analyze a setting where it is not possible to react instantaneously. The firm, which self-reports as second, can be exempted only for less than 50% of the fine, while the leader gets complete immunity from fine.

Next, in section 5.5 we compare the regime described above with the case where the rules of leniency programs are less strict and the procedure of application for leniency is less confidential. We model this by relaxing the assumption that instantaneous reaction is not possible. That is, we consider a setting where firms can respond immediately to their rival's decisions. This implies that actions of the firms are perfectly observable and the procedure of self-reporting is instantaneous (does not take any time). Clearly, in this case simultaneous self-reporting is possible.

We study a continuous time preemption game and employ the feedback equilibrium solution concept in order to solve it. First, we determine the payoffs and the objective functions for the first mover (leader), the second mover (follower), and in case of simultaneous self-reporting. Next, we determine optimal stopping times for each case. Finally, we derive the feedback equilibrium of the preemption game with leniency.

The definition of feedback equilibrium solution concept as it is employed in the current chapter is taken from Huisman (2001, p. 77-79). He states that in a feedback equilibrium the leader (player that moves first) takes into account that its decision to self-report affects the decision of the follower. To determine the equilibrium we should plot the leader's payoff as a function of its own stopping time and take the stopping time of the follower equal to its optimal reaction. The feedback equilibrium is given by the stopping time at which the leader's payoff is at its maximum and optimal reaction of the follower on that stopping time[9].

[9]The feedback equilibrium is also, sometimes, referred as a closed loop equilibrium (as in Fudenberg and Tirole (1991, p. 131)) or perfect equilibrium concept for timing games (as in Fudenberg and Tirole (1985)). Some authors, for instance, Fudenberg and Tirole (1991), also mention that feedback equilibria

We assume that after the first firm has reported about the existence of the cartel to the authority, the cartel stops, and consequently, the stream of illegal gains also stops. In case of complete information about the actions of the rival the best response of the second firm would be to cooperate and reveal the cartel immediately after the first firm (the leader) does so. In addition, our approach represents a quite extreme form of preemption in that the follower loses entirely its chance to be completely exempted from the fine if it is forestalled by the leader. In the general setting the leader reports at the same time or before the follower, i.e. $0 \leq T_L \leq T_F$, where T_L and T_F are the optimal stopping times for the leader and follower, respectively.

Given the times T_L and T_F, and due to the special structure of the game, the value of the leader equals the integral over time of the instantaneous illegal gains from price-fixing, $\frac{\pi^m}{2}$, less the instantaneous expected penalty, $\lambda s(t)$. By construction of the game the fine the leader has to pay equals zero, $s^L = 0$. Hence, the value of the leader when $T_L < T_F$ is given by

$$V_L(T_L, T_F) = \int_0^{T_L} (\frac{\pi^m}{2} - \lambda s(t))e^{-rt}dt - s(T_L)e^{-rT_L} = \int_0^{T_L} (\frac{\pi^m}{2} - \lambda s(t))e^{-rt}dt. \quad (5.5)$$

After time T_L , i.e. after the cartel was reported to the authority, the flow of illicit gains stops, so the exact value of T_F is not relevant for the determination of $V_L(T_L, T_F)$ and $V_F(T_L, T_F)$, since it does not directly influence the payoff. However, the optimal reaction of the follower can still influence the decision of the leader.

In the same way the value of the follower, $V_F(T_L, T_F)$, can be derived. The follower value is given by the integral over time of the instantaneous illegal gains from price-fixing, $\frac{\pi^m}{2}$, less the instantaneous expected penalty, $\lambda s(t)$. The term $-s^n(T_L)$ reflects the value of the normal (full) fine that has to be paid by the follower after the cartel is discovered[10]. Hence, the follower value, when $T_L < T_F$, is given by

$$V_F(T_L, T_F) = \int_0^{T_L} (\frac{\pi^m}{2} - \lambda s(t))e^{-rt}dt - s^n(T_L)e^{-rT_L} = V_L(T_L, T_F) - s^n(T_L)e^{-rT_L}. \quad (5.6)$$

Similarly, the value of the firm in case of simultaneous self-reporting is determined by expression (5.7) below. Recall that in case of simultaneous self-reporting both firms

in continuous-time games can be related to subgame perfect equilibria of discrete-time timing games.

[10]Note that the results of the analysis below are valid for any $s^F \in (\frac{1}{2}s^n, s^n]$. This is in line with current leniency rules (see OECD report 2002).

pay 50% of the normal fine.

$$V_M(T_L,T_F)=\int_0^{T_c}(\frac{\pi^m}{2}-\lambda s(t))e^{-rt}dt-\frac{1}{2}s^n(T_c)e^{-rT_c},\quad \text{iff}\quad T_L=T_F. \tag{5.7}$$

Now we define the optimal stopping times for the leader (T_L^*) and in case of simultaneous self-reporting (T_c^*). First, we define $T_c^*=\arg\max_{T_c}V_M(T_c,T_c)$ and $T_L^*=\arg\max_{T_L\,:\,(T_L\leq T_F)}V_L(T_L,T_F)$. Note also that $V_L(T_L,T_F)=V_M(T_L,T_F)$, when $T_L=T_F$. From expressions (5.5) - (5.7) it is clear that $V_F(T_L,T_F)<V_M(T_L,T_F)<V_L(T_L,T_F)$ for any $T_L<T_F$.

5.4.1 Confidential Leniency Programs

In this subsection we analyze a model, where it is not possible for firms to react instantaneously to the actions of their rivals, i.e. the rules and procedure of application for leniency are very strict. This corresponds to the first regime mentioned in the introduction to the chapter, namely the regime with more strict enforcement, i.e. only the firm, which formally self-reports the first (even if second firm also does it voluntary), is eligible for complete exemption from the fine. The second firm bears the full fine (even it is several seconds later to self-report than the first firm), and the application procedure is strictly confidential.

The objective functions of the firms can be described as follows. In a feedback equilibrium[11] the leader takes into account that its stopping decision affects the decision of the follower. However, for this particular problem it holds that the decision of the follower does not influence the value of the leader's payoff after he decides to reveal the cartel, see (5.5) or (5.8). This implies that the expressions (5.8) and (5.9) below do not depend on the reaction of the follower.

Now in the set-up with proportional penalty, we can define the following three functions

$$L(T)=\int_0^{T}(\frac{\pi^m}{2}-\lambda\alpha w(t))e^{-rt}dt, \tag{5.8}$$

$$F(T)=\int_0^{T}(\frac{\pi^m}{2}-\lambda\alpha w(t))e^{-rt}dt-\alpha w(T)e^{-rT}, \tag{5.9}$$

[11]See Huisman (2001).

$$M(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt - \frac{1}{2}\alpha w(T)e^{-rT}. \qquad (5.10)$$

The function $L(T)$ $(F(T))$ is equal to the expected discounted value at time $t = 0$ of the leader (follower) when the leader reports at time T. $M(T)$ is the discounted value at time $t = 0$ of the firm when there is simultaneous self-reporting at time T.

Here we assume that the firms cannot react instantaneously, i.e. only a lagged reaction is possible. The implication is that the payoff of $M(t)$ is no longer available for the follower. Therefore, given the expressions (5.8), (5.9) and (5.10), in equilibrium the following inequalities hold

$$L(t) > M(t) > F(t) \quad for\ all \quad t \in (0, \infty). \qquad (5.11)$$

Note that $L(0) = F(0) = M(0)$.

To find feedback equilibria of this model we consider the dynamic timing game[12].At each instant of time t the following simultaneous move matrix game is played (see Table 5.1).

	Self-report(S)	Not self-report(N)
Self-report(S)	$(M(t), M(t))$	$(L(t), F(t))$
Not self-report(N)	$(F(t), L(t))$	repeat game

Table 5.1. Payoffs and strategies of the matrix game played at time t.

We denote by π the value of the infinitely lasting cartel. In case of proportional penalty this value is given by the following expression: $\pi = \int_0^\infty (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt$. This value will be extensively used in the proof of Proposition 5.1.

The game described in Table 5.1 is played at time t if no firm has reported about the existence of the cartel so far. Playing the game costs no time and if firm 1 chooses action Not Self-report (N) and firm 2 also the game is repeated. If necessary the game will be repeated infinitely many times[13]. Clearly, in this matrix game the outcomes,

[12] For the definition of feedback equilibrium as it is employed in the proof of the main results of the models of this chapter, we refer to Huisman (2001) or Fudenberg and Tirole (1985). Note also that in Fudenberg and Tirole (1985) this equilibrium is called a perfect equilibrium for timing games. It is still subject to discussion whether a subgame perfect equilibrium of an arbitrary discrete-time dynamic game would converge to a feedback equilibrium of the continuous-time game. However, for the particular game described in our model this property holds, when you let the length of the time period of the discrete-time game approach zero. The result of Proposition 5.1 confirms this statement.

[13] This representation is borrowed from Huisman et al. (2004).

simultaneous self-reporting by both firms - (S, S) and the decision not to reveal the cartel by both players - (N, N), can arise as a Nash Equilibrium in pure strategies. The result depends on the magnitude of the maximal simultaneous self-reporting value and the value of the profits in case of infinitely lasting cartel.

The results of the analysis in the setting with proportional penalty are summarized in the next proposition. Later, in section 5.6, we compare these results to the solution of the model with fixed penalty.

Proposition 5.1 *For the setting with proportional penalty, the unique feedback equilibrium (and unique SPNE) of the game with leniency, where firms cannot react instantaneously to the actions of their rivals, is*

immediate simultaneous self-reporting, i.e. $(t_1^*, t_2^*) = (0, 0)$, *if* $\alpha\lambda > r$,

or cartel forever, i.e. $(t_1^*, t_2^*) = (\infty, \infty)$, *if* $\alpha\lambda < r$.

Proof: See Appendix 1.[14]

Although the proposition has been proven using the subgame perfect equilibrium of a discrete-time preemption game and the property of its convergence to the equilibrium of continuous-time game, it also holds that this outcome arises as a special case of Proposition 2 of Fudenberg and Tirole (1985), where the perfect equilibrium concept for continuous-time timing games is employed. In particular, the special case of Fudenberg and Tirole's model with $L(t) > M(t)$ implies that perfect equilibria exhibit rent equalization, i.e. firms stop when $T = min\{t \in (0, T_L) : L(t) = F(t)\}$. In our model this implies simultaneous stopping by self-reporting at $t = 0$, at which the leader value equals the value of the follower.

The result of this proposition suggests that after the introduction of leniency programs antitrust enforcement appears to be more efficient than in the absence of leniency. Even in combination with moderate penalties it leads to immediate self-reporting by both firms in the beginning of the game. Depending on the severity of punishment, two possible outcomes can arise. Either both firms report the cartel simultaneously in the beginning of the game, or the cartel lasts forever.

Let us compare this result with the conclusion of the model, where firms take decisions in the absence of leniency programs. Recall that from expression (5.4) we obtain that if the optimal stopping time of the model without leniency (T^*) exists, then $T^* > 0$ for any values of parameters of the model ($\alpha \in (0, \infty), \lambda \in (0, 1], r \in (0, 1]$ such that $\alpha\lambda > r$), since expression $\frac{\alpha\lambda}{\alpha\lambda - r}$ is always greater than one, when $\alpha\lambda > r$. In the game

[14] Here t_i^* denotes the optimal time of self-reporting by firm i.

with leniency we have immediate self-reporting by both firms in the beginning of the game, when $\alpha\lambda > r$. This result suggests that antitrust enforcement after introduction of leniency programs is more efficient than in the absence of leniency. Hence, strictly confidential leniency programs improve upon the situation without leniency. Moreover, clearly preemption mechanism is the reason for the strength of leniency programs.

5.5 Non-confidential Leniency Programs

In this section we discuss the preemption game with leniency under the assumption that firms can react instantaneously to the actions of their rivals. In particular, this implies that here we study the second regime, mentioned in the introduction, namely, where the rules of antitrust enforcement are not too strict and the procedure of application for leniency is less confidential. However, this could be a too strong assumption for the model that describes leniency programs, since in most cases the procedure of application for leniency is very confidential. Nevertheless, we consider it in order to compare the results of the two regimes described above and show that if the rules of leniency programs and the procedure of application for leniency were stricter, cartels would be less likely. One can also think about the economic justification for this case by considering situation of collusion between two symmetric firms that are threatened by possibility of conviction and punishment. It is realistic to assume that they agree and come forward with the application for leniency simultaneously as soon as the expected penalty, which also takes into account reduced fine, exceeds expected future benefits. In this case it would be reasonable that both firms obtain equal reduction of fine.

First, we determine the objective functions of both players in case there is a first mover (leader) and a second mover (follower), and in case of simultaneous self-reporting for proportional penalty setting. Next, we find optimal stopping times for each case. This will be necessary for further analysis in order to obtain equilibria of the model and derive policy implications. Finally, we derive the feedback equilibrium of the preemption game with leniency under the assumption that instantaneous reaction is possible. We will employ the definition of feedback equilibrium given in Huisman (2001, p. 77-79).

Now, we describe in more detail the derivation of the optimal stopping times for the leader and in case of simultaneous self-reporting in a setting where the penalty is proportional to the amount of illicit gains, i.e. $s(t) = \alpha w(t)$ for all $t \in [0, \infty)$.

In this case the value of the leader is obtained by substituting $s(t) = \alpha w(t)$ into expression (5.5):

$$L(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt. \tag{5.12}$$

Similarly, after substitution of $s(t) = \alpha w(t)$ into expression (5.6) the value of the follower equals

$$F(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt - \alpha w(T)e^{-rT}. \tag{5.13}$$

Finally, the value of simultaneous self-reporting is determined by

$$M(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt - \frac{1}{2}\alpha w(T)e^{-rT}, \tag{5.14}$$

where $w(t) = \int_0^t \frac{\pi^m}{2}e^{-rs}ds$, $w(0) = 0$. For further analysis, similarly to section 5.4, we define T_c to be the optimal time of simultaneous self-reporting and $T_L(T_F)$ the optimal time of self-reporting by the leader (follower).

Now we move to the derivation of $T_L = \arg\max_T \ L(T)$ and $T_c = \arg\max_T \ M(T)$ that are used further for determination of an equilibrium of the preemption game and also in the discussion of policy implications of the model. Taking the derivative of (5.12) with respect to T and equalizing it to zero, we obtain

$$T_L = \frac{\ln(\frac{\alpha\lambda}{\alpha\lambda - r})}{r}. \tag{5.15}$$

Note that $T_L = T^* = \arg\max_T \ St(T)$. Recall expression (5.4).

The necessary condition for a maximum is satisfied since $\frac{\partial L^2(T_L)}{\partial^2 T} < 0$.

From expression (5.15) we obtain, that the earliest time of revelation (i.e. breaking the cartel agreement) by one of the firms will decrease when either α or λ increases. This result is quite intuitive, because it means that the cartel stability should be reduced when either severity or probability of punishment increases. At the same time, the effect of an increase in the discount rate on the optimal time of self reporting gives $\frac{\partial T_L}{\partial r} < 0$. Hence, the firms will find it more attractive to stop earlier if the discount rate is higher, since future illicit gains become less valuable.

Similarly to the above analysis we take the derivative of (5.14) with respect to T and equalize it to zero. In this way we obtain the optimal stopping time in case both firms report the cartel simultaneously

$$T_c = \frac{\ln(\frac{2\alpha(\lambda - r)}{2\alpha\lambda - 2r - \alpha r})}{r}. \tag{5.16}$$

The necessary condition for the existence of maximum is satisfied since $\frac{\partial M^2(T_c)}{\partial^2 T} < 0$.

From this expression we obtain, that the earliest time of revelation (i.e. breaking the cartel agreement) by both firms simultaneously will decrease when either α or λ increase. So, the cartel stability is lower when either severity or probability of punishment increases.[15]

Moreover, the solution of this problem exists only when $\lambda > r$ (i.e. the rate of law enforcement is higher than the discount rate) and $\alpha\lambda > \frac{r(2+\alpha)}{2}$ (i.e. the coefficient of expected penalty is greater than the sum of the discount rate and half of the product of the scale parameter and discount rate)[16]. In other words, the expected penalty is high enough to outweigh the current benefits from crime compared to the future penalties. Comparison of expressions (5.15) and (5.16) implies the following lemma.

Lemma 5.2 *Given $\lambda > r$ and $\alpha\lambda > \frac{r(2+\alpha)}{2}$, there exist $T_L = \arg\max_T L(T) = T^* = \arg\max_T St(T)$ and $T_c = \arg\max_T M(T)$ such that*

$T^* < T_c$, when $r < \alpha\lambda < 2r$ and $T^* > T_c$, when $2r < \alpha\lambda$.

Proof: See Appendix 2.

This result shows that when the multiplier of the expected penalty is lower than twice the discount rate, in the absence of leniency programs the firm stops cartel formation sooner than in case of simultaneous self-reporting after introduction of leniency. And vice versa, when the instantaneous expected penalty is high enough, the firm that decides about the optimal time of quitting the cartel on its own, in the absence of leniency programs, will choose to report later than in case the firms coordinate their actions after introduction of the leniency program. The result of this lemma will also be used later when we consider the implications of the feedback equilibrium of the preemption game with leniency.

5.5.1 Derivation of the Feedback Equilibrium

The above described preemption game has a special feature in that the leader payoff is not influenced by the decision of the follower. However, in the feedback equilibrium the reaction of the follower should influence the decision of the leader about the optimal

[15]It also should be mentioned that $\ln(\frac{2\alpha(\lambda-r)}{2\alpha\lambda-2r-\alpha r}) > 0$ only if $\alpha < 2$. For any $\alpha \geq 2$, we obtain $\ln(\frac{2\alpha(\lambda-r)}{2\alpha\lambda-2r-\alpha r}) \leq 0$, consequently, $T_c = 0$, since $T_c \in [0, \infty)$.

[16]Note, that $\alpha\lambda > \frac{r(2+\alpha)}{2}$ implies $\alpha\lambda > r$. Hence, existence of non-negative value for optimal stopping time of simultaneous self-reporting in the presence of leniency implies existence of non-negative optimal stopping time in case when leniency is not available.

time of self-reporting. The leader should take into account that the second firm can react instantaneously to the actions of the leader. This implies that the second firm will choose the same action as the leader at each instant of time due to the fact that its fine will be halved in this way. Hence, $T_F = T_L$ for any $T_L \in [0, \infty)$. This implies that the firm that moves first maximizes the value of simultaneous self reporting, $M(T)$, at each instant of time. Hence, $T_c = \arg\max_{T \geq 0} M(T)$, and $T_L = \arg\max_{T \geq 0} L(T)$ satisfy $L(T_L) > M(T_c)$.

Due to the assumptions of symmetry and the possibility of instantaneous reaction for the second firm, from the expressions (5.12)-(5.14) it is clear that in equilibrium, with equilibrium stopping time of simultaneous self-reporting denoted by t^*, the following condition is satisfied $L(t^*) = F(t^*) = M(t^*)$.

To find feedback equilibria of this model we represent the matrix game played at each instant of time as in Table 5.2.

	SR	Not SR
SR	$(M(t^*), M(t^*))$	$(L(t^*), F(t^*))$
Not SR	$(F(t^*), L(t^*))$	repeat

$\rightarrow$

	SR	Not SR
SR	$(M(t^*), M(t^*))$	$(M(t^*), M(t^*))$
Not SR	$(M(t^*), M(t^*))$	repeat

Table 5.2. Payoffs and strategies of the matrix game played at time t under the assumption of possibility of instantaneous reaction.

We again denote by π the value of the infinitely lasting cartel. In case of proportional penalty this value is given by :

$$\pi = \int_0^\infty (\frac{\pi^m}{2} - \lambda \alpha w(t)) e^{-rt} dt.$$

So the equilibrium where both firms self-report, (S, S), arises as a pure strategy Nash Equilibrium of the matrix game described above in case $M(t^*) > \pi$. On the other hand, the decision not to reveal cartel by both players, (N, N), is a pure strategy Nash Equilibrium of this matrix game in case $\pi > M(t^*)$ or $\pi > M(t)$ for all $t \in [0, \infty)$. Recall that the maximal payoff in case of simultaneous self-reporting equals

$$M(T_c) = \int_0^{T_c} (\frac{\pi^m}{2} - \lambda \alpha w(t)) e^{-rt} dt - \frac{1}{2} \alpha w(T_c)) e^{-rT_c},$$

where T_c is the equilibrium time of simultaneous self-reporting.

After we simplify these expressions, we obtain

$$\pi = \frac{\pi^m}{2r}(1 - \frac{\alpha\lambda}{2r}) \tag{5.17}$$

$$M(T_c) = \pi - \frac{\pi^m}{2r}e^{-rT_c}(1 - \frac{\alpha\lambda}{r} + \frac{\alpha}{2} + \frac{\alpha\lambda}{2r}e^{-rT_c} - \frac{\alpha}{2}e^{-rT_c}). \tag{5.18}$$

Based on expressions (5.17) and (5.18) we conclude that $\pi > M(t)$ for all $t \in [0, \infty)$ only in case $\alpha\lambda < r + \frac{\alpha r}{2}$. Hence, when $\alpha\lambda < r + \frac{\alpha r}{2}$ the unique SPNE of the game is $(N, N)_t$ for all $t \in [0, \infty)$. This means that self-reporting is never optimal when $\alpha\lambda < r + \frac{\alpha r}{2}$, because firms prefer to keep the cartel forever. In this case, introduction of leniency programs does not have any effect on cartel stability.

To complete the analysis, we consider the setting where $\alpha\lambda > r + \frac{\alpha r}{2}$. In this case, self-reporting occurs at the moment, T_c , when $M(t)$ reaches its maximum. Hence, the unique SPNE of the game is to play $(N, N)_t$ for all $t \in [0, T_c)$ and to play $(S, S)_t$ when $t = T_c$. Hence, the game stops after period T_c. In this case, introduction of leniency programs can influence the cartel stability. We will study these effects in more detail in the next proposition.

In case $\alpha\lambda > r + \frac{\alpha r}{2}$, according to Lemma 5.2 two possible outcomes can arise: $T^* < T_c$ or $T^* > T_c$. The first inequality implies that the result, obtained in case we consider the game with leniency, leads to a later time of self-reporting compared to the solution of the simple timing game when leniency is not available. And the latter case implies an earlier stopping time after introduction of leniency. In both cases the result described in the following proposition holds[17].

Proposition 5.3 *In the feedback equilibrium of the game both firms report simultaneously at time* $T_c = \frac{\ln(\frac{2\alpha(\lambda - r)}{2\alpha\lambda - 2r - \alpha r})}{r}$.

Proof: See Appendix 3.

In short, the intuition behind the proof of this proposition is as follows. There exists a continuum of simultaneous self-reporting equilibria, from which simultaneous self-reporting at time T_c Pareto dominates all other equilibria. In this Pareto dominant (or payoff dominant) equilibrium, firms "tacitly cooperate" by keeping the cartel until time T_c and then reveal it simultaneously and pay half of the fine, which is most beneficial for both of them.

Clearly, in contrast with the benchmark case, in the preemption game, which takes into account the possibility of leniency, the antitrust authority can influence the outcome of the game, i.e. the decision about the time of breaking the cartel agreement by both firms, not only by changing the fine and the probability of law enforcement. The

[17] This proposition has been derived using the definition of feedback equilibrium as it is described in Huisman (2001). However, we believe that employing the notion of SPNE we will obtain the same result.

introduction of leniency programs also appears to be an important factor that may either reduce cartel stability or facilitate collusion.

The above result also states that in the model without possibility of instantaneous reaction, leniency programs appear to be more efficient, since they enforce immediate self-reporting for lower expected fines compared to the model where instantaneous reaction is possible. Recall the model without possibility of instantaneous reaction. There we got that self-reporting became a dominant strategy already when $\alpha\lambda > r$. On the other hand, in the model, where instantaneous reaction is possible, self-reporting becomes a dominant strategy only when $\alpha\lambda > r + \frac{\alpha r}{2}$. This comparison clearly gives the result of earlier self-reporting in case the rules are more strict, i.e. there is no possibility of instantaneous reaction. This implies that the incentives for the firms to break the cartel are stronger under the assumption that they cannot react instantaneously to the actions of their rivals.

We conclude that if the rules of leniency programs are stricter and the procedure of application for leniency is more confidential, cartel occurrence is less likely. This can happen due to the fact that the absence of the possibility to react to the actions of a rival instantaneously increases the expected future losses if the cartel is revealed, since the payoff of $M(t)$ is no longer available for the follower.

5.5.2 Effects of Leniency Programs when Instantaneous Reaction is Possible

The equilibrium of the game with leniency may lead to either earlier or later deterrence than in case the firms take the decision about the stopping cartel agreement in the absence of leniency programs.

Earlier deterrence happens if $2r < \alpha\lambda$, while the result of later deterrence arises if $r < \alpha\lambda < 2r$. A special case occurs when $r > \alpha\lambda$. In this case maxima of $M(t)$ and $St(t)$ in the positive orthant do not exist and $M'(t) > 0$ and $St'(t) > 0$ for all $t \in [0, \infty)$. Hence, the best strategy is cartel forever, since self-reporting is never profitable. This situation is depicted in Figure 5.1.

Moreover, for $\alpha \geq 2$ we obtain from expression (5.16) that $T_c \leq 0$. This means that cartel formation stops immediately, $T_c = 0$, i.e. in the equilibrium of the preemption game with leniency when instantaneous reaction is possible it is optimal for both firms to reveal the cartel immediately after the introduction of the leniency program. So, we can conclude that,- for proportional penalty similarly to fixed penalty,- in combination with a strict enforcement policy (when α is high, $\alpha \geq 2$) leniency programs appear to be quite efficient. They allow to achieve immediate deterrence.

If we compare the impact of the penalty of the form $s(t) = \alpha w(t)$ (with $\alpha \geq 2$) in the absence of the leniency programs, we do not observe the outcome with complete deterrence in the beginning of the planning horizon for any parameter values, whereas with the introduction of leniency programs this result becomes unambiguous[18].

Moreover, for any $\alpha < 2$, thus when penalties are low, introduction of leniency does not lead to the outcome with immediate complete deterrence, since $T_c > 0$.

To illustrate the above analysis, in Figures 5.2 and 5.3 the two functions , $M(t)$ and $St(t)$, are plotted for cases $r < \alpha\lambda < 2r$ and $2r < \alpha\lambda$, respectively. The solid lines correspond to the value of stopping in situation without leniency, and the dotted line represents the value of simultaneous self-reporting that is relevant value in the game with leniency where instantaneous reaction is possible.

In case $r < \alpha\lambda < 2r$, see Figure 5.2., we get that $T_c > T^*$, where $T_c = \arg\max_{t \geq 0} M(t)$ and $T^* = \arg\max_{t \geq 0} St(t)$. This implies that the result, obtained in case we consider equilibria of the preemption game with leniency, leads to a later optimal stopping time. Hence, compared to the benchmark case where no leniency is available, greater harm is done to the consumers. Recall that $T^* = \arg\max_{t \geq 0} St(t)$ reflects the optimal time of stopping the cartel formation in the benchmark model, where firms do not take strategic considerations into account, see expression (5.4). So, the fact that the firms take into account the reaction of the other firm clearly increases the stability of the cartel for intermediate values of α and λ, i.e. $r < \alpha\lambda < 2r$, compared to the optimum of a single decision maker in the situation without leniency. However, in case $2r < \alpha\lambda$, the equilibrium of the game with leniency (T_c, T_c) leads to an earlier stopping time than in the benchmark model. In this case, see Figure 5.3, in the solution of the game with leniency the duration of cartel agreement is reduced, since $\arg\max_{t \geq 0} M(t) < \arg\max_{t \geq 0} St(t)$ or $T_c < T^*$.

The main conclusion of the above analysis is that, when the procedure of application for leniency is not confidential, leniency may still reduce duration of cartel agreements, but not in all cases. When leniency programs are non-confidential and penalties and rate of law enforcement are low, introduction of leniency programs may, on the contrary, facilitate collusion. In other words, when incentives to preempt are reduced, i.e. leniency programs are less confidential, the effects of introduction of leniency programs on cartel stability will be weaker or even adverse.

[18] For a complete derivation of this result see chapter 4 that is based on Motchenkova and Kort (2004).

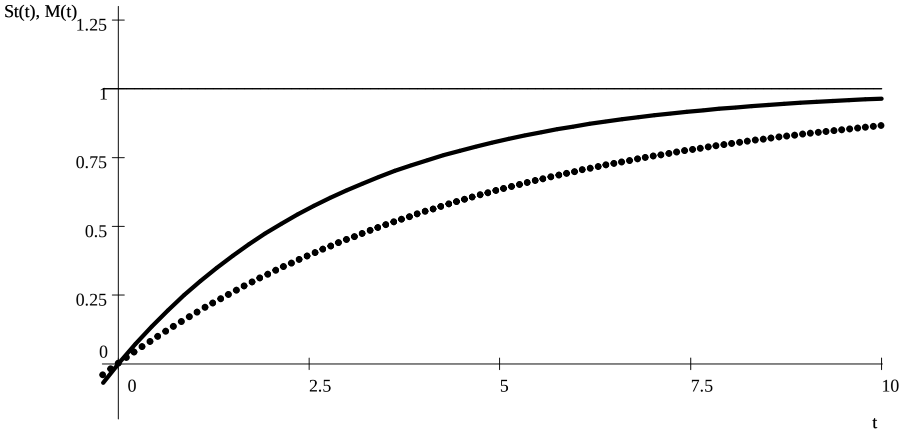


Figure 5.1: Graphs of π (horizontal line), St(t) (solid line), and M(t) (dashed line) for the case $\alpha\lambda < r$. Parameter values are α=1.5,λ=0.2,r=0.1,π^m=1.

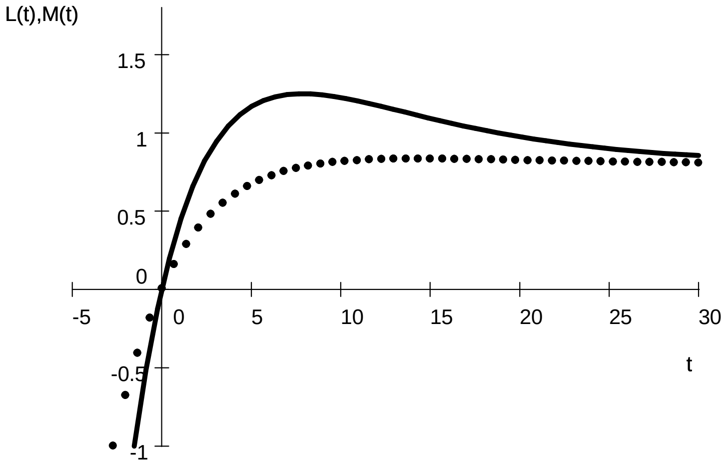


Figure 5.2: Graphs of St(t) (solid line) and M(t) (dashed line) for the case $r < \alpha\lambda < 2r$. Parameter values are α=1,λ=0.2,r=1/8,π^m=1.

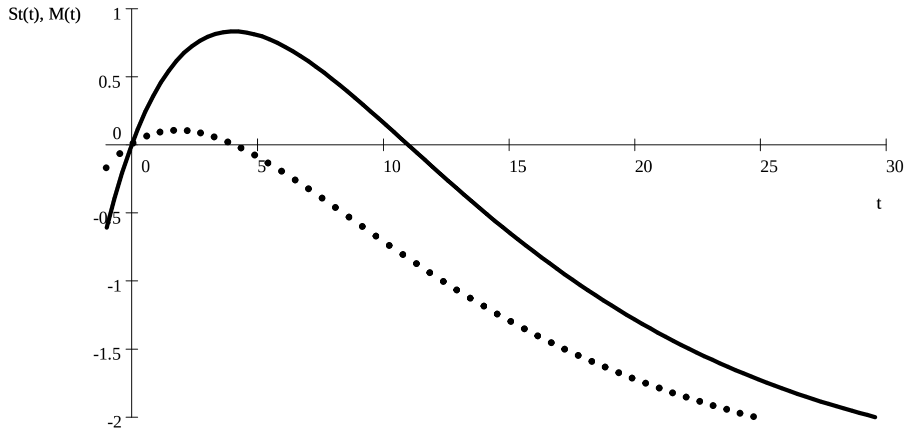


Figure 5.3: Graphs of St(t) (solid line) and M(t) (dashed line) for the case $\alpha\lambda > 2r$. Parameter values are α=1.5,λ=0.2,r=0.1,π^m=1.

5.6 Analysis of the Model with Fixed Penalty

5.6.1 Benchmark Model without Leniency

In case the penalty is fixed the value of stopping the cartel formation is determined as in expression (5.2) with $s(t) = F^n$, so that it has the following form:

$$St(T) = \max_T \int_0^T (\frac{\pi^m}{2} - \lambda F^n)e^{-rt}dt. \tag{5.19}$$

Following the same arguments as in section 5.3, in order to find the optimal time of stopping the cartel agreement, we maximize (5.19) with respect to time. This implies that the optimal time of quitting the cartel agreement for firm i is given by

$$\begin{aligned} T_i^* &\rightarrow \infty \text{ if } \frac{\pi^m}{2} > \lambda F^n \\ T_i^* &= 0 \text{ if } \frac{\pi^m}{2} \leq \lambda F^n. \end{aligned} \tag{5.20}$$

So, we can conclude that, while taking the decision about the optimal time of quitting the cartel agreement, the firms just compare expected instantaneous benefits from price-fixing and expected punishment. Moreover, from expression (5.20) it follows that when the expected penalty is high enough, i.e. $\lambda F^n > \frac{\pi^m}{2}$, cartel formation stops immediately at time zero.

Expression (5.20) shows that the optimal decision is either to stop collusion immediately or never. The higher the expected penalty the more likely that cartel formation stops immediately. On the other hand, the higher the instantaneous illegal gains the more likely that the cartel will last forever.

5.6.2 Analysis of the Game with Leniency

Regime with Confidential Procedure of Application for Leniency

Depending on the severity of punishment, two possible outcomes can arise in a feedback equilibrium of a preemption game with leniency where instantaneous reaction is not possible. Either both firms report the cartel simultaneously in the beginning of the game or the cartel will last forever.

Proposition 5.4 *The feedback equilibria of the game, when penalty is fixed and equals F^n, are immediate simultaneous self-reporting, i.e. $(t_1^*, t_2^*) = (0, 0)$, if $\lambda F^n > \frac{\pi^m}{2}$ or cartel forever, i.e. $(t_1^*, t_2^*) = (\infty, \infty)$, if $\lambda F^n \leq \frac{\pi^m}{2}$.*

We refrain from presenting the proof of Proposition 5.4, since it is similar to the proof of Proposition 5.1 with a number of simplifications. Clearly, in case firms cannot react instantaneously to the actions of their rivals, under the fixed penalty scheme the solution of the game with leniency coincides with the outcome of the benchmark model, where leniency is not available (see expression (5.20)).

Non-confidential Procedure of Application for Leniency

In this subsection we consider a situation with less strict leniency programs, where firms can react instantaneously to the actions of their rivals. If we compare the optimal stopping time in a setting without leniency and the equilibrium of the preemption game with leniency we can conclude that for any positive discount rate the optimal time of simultaneous self-reporting in case of leniency is more likely to be greater than the optimal stopping time, which maximizes the individual payoff when leniency is not available. To be more precise, due to the discontinuity result of the model with fixed penalty, in case of leniency the outcome of infinitely lasting cartel is more likely than the outcome of immediate simultaneous self-reporting compared to the benchmark case. These results are summarized in the following proposition.

Proposition 5.5 *Consider the situation where the penalty is fixed and $(\lambda - \frac{r}{2})F^n < \frac{\pi^m}{2} < \lambda F^n$.*

1. *In the setting without leniency both firms report at $t_1^* = t_2^* = 0$.*

2. If we consider the equilibrium of the game with non-confidential leniency, immediate self-reporting does not occur: both firms report at $T_c = t_c^* \to \infty$.

Proof: See Appendix 4.

So, with a fixed penalty scheme, even in the presence of leniency, the efficiency of deterrence depends only on the amount of the fine and the probability of law enforcement. Moreover, we show that, when penalties are fixed and fall below a certain threshold, leniency programs may well facilitate collusion.

Hence, if we consider the setting where self-reporting is not possible, we can conclude that cartel formation stops immediately, at the beginning of the planning horizon, only when the penalty is fixed and high enough to outweigh the expected benefits from collusion. However, in case the government introduces leniency that is not confidential, even an expected penalty being greater than instantaneous gains from price-fixing, cannot ensure immediate success of the leniency program. Only the condition $\frac{\pi^m}{2} \leq (\lambda - \frac{r}{2})F^n$ which implies $\lambda F^n > \frac{\pi^m}{2} + \frac{r}{2}F^n$, can ensure immediate self-reporting in case firms take into account the possibility of leniency and are able to react instantaneously to the actions of their rivals. So, when penalties are fixed and the procedure of application for leniency is not confidential, the introduction of leniency programs reduces the effectiveness of antitrust enforcement. This implies that the authority will have to increase either the amount of the penalty or the rate of law enforcement in order to achieve whistle-blowing by both firms immediately in the beginning of the planning horizon. Otherwise, when the penalty is low, i.e. $F^n < \frac{\pi^m}{2\lambda - r}$, introduction of leniency makes the cartel more stable. This is a very surprising result, since intuitively leniency should increase the incentives for firms to betray the cartel and, hence, reduce cartel stability. However, when the penalty is low, and does not depend on the amount of illegal gains, it may be the case that the reduced (as a consequence of leniency) net expected fine is, actually, less than the instantaneous gains from price-fixing, and this drives the result.

5.7 Effects of Leniency in the Model of Dynamic Price Competition and "Tacit Collusion"

In this section we study the effects of leniency programs on the behavior of the firms in the model of dynamic price competition where "tacit collusion" may arise[19]. In the previous sections the situation of a formalized cartel was considered. In particular, we

[19] See, e.g., Tirole (1988), chapter 6.

analyzed a model where there is a formal cartel agreement. This can be discovered by the antitrust authority and punished on the basis of official documents, which provide evidence of illegal price-fixing agreement. However, it is often the case that firms do not form an explicit cartel, but sustain high prices by means of "tacit collusion", which harms consumers. This is also an illegal activity and can be punished according to the Article 81 of the EC Treaty. Recall, for example, the Soda-Ash case. In that case the Commission decided that tacit collusion between ICI, a British company, and Solvay, a Belgian company, was an infringement of Article 81 (ex-85). The Commission motivated the decision by the fact that the term "concerted practices" mentioned in Article 81 among the prohibited practices also covered the case of tacit collusion between these two companies.

The situation of "tacit collusion" assumes that when there is no formal agreement between two firms, but they still keep prices above competitive level, both of them have incentives to undercut and obtain monopoly profits. Hence, this situation involves the possibility of undercutting. This special feature makes this case different from the assumptions of the preemption game described above[20].

In this section we incorporate the possibility of undercutting into the model of leniency without instantaneous reaction[21]. We consider a game between two symmetric firms that may cooperate by charging the monopoly price, and obtain half of the monopoly profits in the industry, $\frac{\pi^m}{2}$, each period. However, there is a threat that this violation will be discovered by the antitrust authority. There are two other options for the firms: self-reporting or undercutting. The second option is to self-report to the authority and obtain leniency (reduction of the fine). The third option is to undercut by reducing the monopoly price by a minimal amount. Then it obtains monopoly profits, π^m, for one or more periods. We also assume that after one of the firms betrayed and another firm discovers it, collusion stops forever. We define here an information lag, which delays the punishment phase and allows the firm to enjoy extra profits for several periods, by ε. This setup gives us a number of interesting results that differ from the model where undercutting is not possible. We can summarize these results in the following proposition.

[20] In the case of an explicit cartel undercutting is not possible. In addition, in case of an explicit cartel there should be no possibility of renegotiation in order to sustain an agreement. So, if we assume that renegotiation is either impossible or very costly, then we can include an additional strategy in the form of "possibility of undercutting" in the model of explicit collusion as well.

[21] Clearly, the model of dynamic price competition and "tacit collusion", as it is described in Tirole (1988) rests on the assumption that instantaneous reaction is not possible.

Proposition 5.6 *The feedback equilibria of the game with proportional penalty are*

1. *immediate stopping by undercutting, i.e.* $(t_1^*, t_2^*) = (0, 0)$, *if* $\alpha\lambda > 2r - re^{r\varepsilon}$;
2. *cartel forever, i.e.* $(t_1^*, t_2^*) = (\infty, \infty)$, *if* $\alpha\lambda \leq 2r - re^{r\varepsilon}$.

We refrain here from presenting the proof of this proposition, since it uses the same methods as the proof of Proposition 5.1. We sketch the main arguments of the proof here. First, we describe the objective function of the firm that chooses the undercutting option. If firm 1 decides to undercut at instant T, it obtains half of the cartel profits, $\frac{\pi^m}{2}$, from the initial period until T and full monopoly rents, π^m, from T until $T + \varepsilon$. At instant $T + \varepsilon$, the second firm discovers that firm 1 betrayed the cartel and, hence, collusion stops forever. However, there is a threat of expected punishment throughout periods 0 to $T+\varepsilon$. Hence, the value of undercutting at instant T is given by the following expression

$$U(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt + \int_T^{T+\varepsilon} (\pi^m - \lambda\alpha w(t)e^{-rt}dt,$$

while the value of the firm that is undercutted is given by $U_F(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt - \frac{\pi^m}{2}e^{-rT} - \int_T^{T+\varepsilon} \lambda\alpha w(t)e^{-rt}dt.$

At the same time the leader value or the value of self-reporting at instant T has the form $L(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt$. Clearly, the value of undercutting intersects the leader value from above in the point where $\int_T^{T+\varepsilon} (\pi^m - \lambda\alpha w(t)e^{-rt}dt = 0$. This point is given by the following expression[22]:

$$T^{**} = \frac{\ln(\frac{\lambda\alpha(1+e^{-r\varepsilon})}{2(\lambda\alpha - 2r)})}{r}.$$

This implies that the best option and, hence, the option that is chosen by both firms (due to symmetry) up to the time T^{**} is the undercutting option, if an additional condition, $\lambda\alpha > 2r$, for existence of T^{**} is satisfied. Hence, up to T^{**} the undercutting value, $U(T)$, will be compared to $\pi = \int_0^\infty (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt$, being the value of the infinitely lasting cartel. After time T^{**} the leader value, $L(T) = \int_0^T (\frac{\pi^m}{2} - \lambda\alpha w(t))e^{-rt}dt$,

[22] For a graphical representation see also Figure 5.4.

will be compared to the value of the infinitely lasting cartel.[23]

Firstly, for any $\alpha\lambda < 2r - re^{r\varepsilon}$, the value of an infinitely lasting cartel gives the highest payoff to both firms and, hence, collusion forever will arise in equilibrium. It is also instructive to show that in case $\alpha\lambda > 2r - re^{r\varepsilon}$ undercutting at $(0,0)$ is an equilibrium since no firm can gain by deviating. In the intermediate case, thus when $2r - re^{r\varepsilon} < \alpha\lambda < 2r$, it holds that $U(t) > L(t)$ for all $t \in [0, \infty)$ and $U(t) > \pi$ for some $t \in [0, \infty)$. This implies that the outcome of immediate stopping at $(0,0)$ by undercutting arises. A more difficult case arises when $\alpha\lambda > 2r$. Then there exists a point of intersection of $U(t)$ and $L(t)$. In that case collusion forever, which gives (π, π) to both players, cannot be an equilibrium. The reason is that if one of the firms deviates and undercuts at $t = 0$, it gains $\pi^m > \pi$. [24] On the other hand, undercutting at T_u^* by both firms also cannot be an equilibrium, since then both firms get only $(St(T_u^*), St(T_u^*))$[25], which is less than what one of the firms can obtain if it deviates by preemption and undercuts at $T_u^* - \epsilon$. Recall that $U(T_u^* - \epsilon) > St(T_u^*) = L(T_u^*)$, due to the fact that $U(t) > St(t) = L(t)$ for all $t \in [0, T^{**})$ given $\alpha\lambda > 2r$. We illustrate these considerations in Figure 5.4.

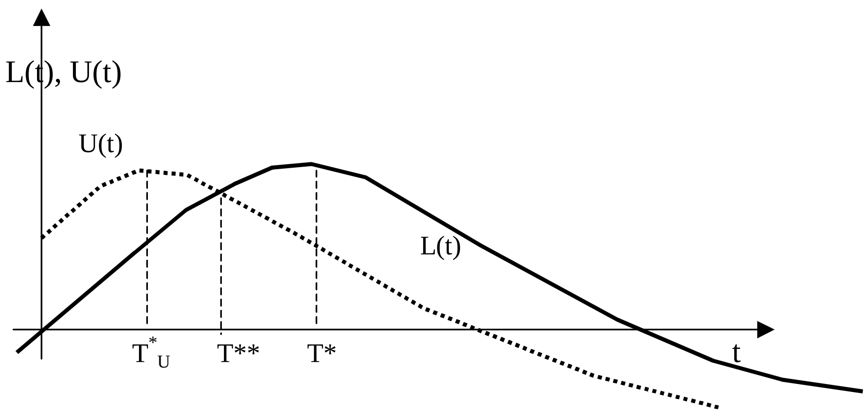


Figure 5.4: Graphs of L(t) (solid line) and U(t) (dotted line) for the case $\alpha\lambda > 2r$.

Proposition 5.6 states that in the preemption game with leniency, where firms illegally fix the prices above competitive level and can undercut each other, the no collusion (or immediate stopping) outcome arises when the coefficient of the expected penalty, $\alpha\lambda$, is greater than $2r - re^{r\varepsilon}$. Now, if we compare the result of Proposition 5.6 to the result of

[23]See the proof of Proposition 5.1. The complete proof of proposition 5.6 is available from the author upon request.

[24]Note that for $\alpha\lambda > 2r$ we have $\pi = \frac{\pi^m}{2r}(1 - \frac{\alpha\lambda}{2r}) < 0$. Hence, $\pi < \pi^m$.

[25]Recall that by construction of objective functions $St(T_u^*) = L(T_u^*)$.

Proposition 5.1, we can conclude that, since $r > 2r - re^{r\varepsilon}$ for any $r \in (0, 1)$, the possibility of undercutting improves the result: a smaller expected penalty, $\alpha\lambda > 2r - re^{r\varepsilon}$, insures immediate stopping compared to the model where only the self-reporting option is available to the firms. Hence, the antitrust authority has to put less efforts into control in order to achieve the outcome of complete deterrence. This result is also quite intuitive, since the possibility of undercutting increases the incentives for the firms to betray the cartel and, hence, it reduces the stability of price-fixing agreements.

Another interesting observation is connected with the influence of the size of the informational lag in case of undercutting on the stability of cartel agreement. From Proposition 5.6 it follows that the bigger the ε (information lag) the easier for the antitrust authority to block the violation, since a smaller expected penalty insures immediate stopping. Moreover, for $\varepsilon > \frac{\ln 2}{r}$ collusion will never arise in equilibrium. This can be explained by the fact that, when the information lag is bigger, the cartel is less stable due to the fact that undercutting brings benefits for a longer period and, hence, it is a more attractive option.

However, it should also be mentioned that introduction of leniency does not influence stability of collusive agreements in case undercutting is possible[26]. To give an intuitive explanation of this result we compare the model described above to the model of dynamic price competition without leniency. In the latter case only the three following options are available for each firm: just stopping the cartel agreement at T^*, undercutting at $T < \infty$ or collusion forever. Analogous to the above reasoning, we have to determine whether the value of undercutting intersects the value of stopping and whether the value of undercutting exceeds the value of the infinitely lasting cartel for some $t \in [0, \infty)$ or not. This leads to the following result, which exactly coincides with Proposition 5.6.

Proposition 5.7 *The feedback equilibria of the game without leniency, where firms can undercut each other in prices, and the penalty is proportional are*

immediate stopping by undercutting, i.e. $(t_1^*, t_2^*) = (0, 0)$, *if* $\alpha\lambda > 2r - re^{r\varepsilon}$

or cartel forever, i.e. $(t_1^*, t_2^*) = (\infty, \infty)$, *if* $\alpha\lambda \leq 2r - re^{r\varepsilon}$.

Hence, introduction of leniency in the game where undercutting is possible would not influence the stability of collusive agreements. On the other hand, harshness of penalty, rate of law enforcement, and size of information lag do affect cartel stability. The bigger the fine, rate of law enforcement, or information lag, the more likely that the

[26] Formal proof of this result uses the same arguments as Proposition 5.6 above and is available from the author upon request.

outcome with immediate stopping by undercutting will arise and, hence, the less stable the collusive agreement is.

5.8 Conclusions

The main problem addressed in this chapter is how leniency programs influence the stability of cartels under two different regimes of fines. First, we study the effects of leniency in case the penalty is an increasing function of the accumulated illegal gains from price-fixing to the firm. Next, we look at the case where the penalty is fixed. We denote the former system by proportional penalty scheme. The enforcement problem we study has several ingredients. We analyze the design of self-reporting incentives, having a group of (and not a single) defendants. Moreover, we consider a dynamic setup, where accumulated benefits and losses from crime are taken into account.

For this purpose we use the tools of optimal stopping and timing games. In particular, the preemption game is studied in order to identify the advantages of being the leader in the race to the court game between the members of the existing cartel after the introduction of leniency programs. The approach, we use, is based on the Reinganum-Fudenberg-Tirole approach, who applied the concept of timing games to a technology adoption problem. We apply a similar procedure to a cartel formation game between two firms in the presence of leniency programs, which allows taking into account the possibility to influence the internal stability of the cartel.

Comparison of results in the situations with and without leniency suggests that antitrust enforcement after introduction of leniency programs is more efficient than in the absence of leniency. Hence, leniency improves upon the situation without leniency.

We also obtain that in the settings with a strictly confidential procedure of application for leniency, the outcome is immediate self-reporting by both firms in case the expected penalty is sufficiently high (but still below the threshold of the model where instantaneous reaction is possible). This implies that strict leniency programs unambiguously increase the efficiency of antitrust enforcement and reduce cartel stability. The reason is that the impossibility to react instantaneously to the actions of the rival increases expected future losses in case the cartel is revealed. This happens due to the fact that it is no longer possible for the follower to obtain reduction of the fine from simultaneous self-reporting. Hence, we conclude that if the rules of leniency programs are stricter and the procedure of application for leniency is more confidential, cartel occurrence is less likely.

We find that in most cases leniency reduces duration of cartel agreements but this result is not unambiguous. In case leniency programs are not too strict and fines are proportional to the accumulated illegal gains from price-fixing the result is as follows. Under strict antitrust enforcement[27], the possibility to self-report and be exempted from the fine increases the incentives for the firms to stop cartel formation, and, hence, reduces the duration of cartels. However, when penalties and rate of law enforcement are low, introduction of leniency programs may, on the contrary, facilitate collusion.

Under a fixed penalty scheme, even in the presence of leniency, the efficiency of deterrence depends only on the amount of the fine and the probability of law enforcement. Moreover, we have shown that in some cases, when penalties are fixed and fall below a certain threshold, less strict leniency would programs facilitate collusion.

The main conclusion of the chapter is that the strength of the preemption mechanism is the main determinant for the successfulness of leniency programs. Cartel stability is reduced if the rules of the leniency programs are stricter and the procedure of application for leniency is more confidential, i.e. when incentives to preempt by self-reporting are stronger.

Another interesting conclusion comes from a numerical comparison of the efficiency of antitrust enforcement under proportional penalty in the absence of leniency and after introduction of leniency. In the earlier case we do not observe the outcome of complete deterrence in the beginning of the planning horizon for any relevant (from the legislation point of view) parameter values, whereas after the introduction of leniency programs this result becomes unambiguous for sufficiently high (but still in the range of legally acceptable) values of the scale parameter of the penalty scheme.

We also study the effects of leniency programs on the behavior of firms in the model of dynamic price competition and "tacit collusion", where firms can undercut each other in prices. The result of this model implies that in environments where undercutting is possible it is easier for competition authority to prevent price-fixing, since a smaller expected penalty insures immediate stopping. This implies that the antitrust authority has to put less efforts into control in order to achieve the result of complete deterrence. However, this result is purely due to the possibility of undercutting and leniency does not play a key role in this setting. Only severity of punishment and rate of law enforcement can influence stability of collusive agreements in this case.

Another interesting extension would be to study the behavior of asymmetric firms. They may differ either in costs or size. This extension would make the model much closer

[27]Here by strict antitrust enforcement we mean high fines and rate of law enforcement.

to real world situation but solution of dynamic games with asymmetric information it not a trivial task. This task will be accomplished in the next chapter.

5.9 Appendix

5.9.1 Appendix 1: Proof of Proposition 5.1

Following the book by Fudenberg and Tirole (1991), which suggests to use subgame perfect equilibria for the solution of two-player timing games (see pages 117-128), in this proof we restrict attention to subgame perfect equilibria in pure strategies[28] of a discrete-time preemption game. Fudenberg and Tirole (1991) also claim that for a continuous time formulation of the similar game (as it has been described in the main text of the chapter) symmetric continuous-time perfect equilibrium of timing game can be obtained as a limit of symmetric discrete-time equilibrium (or SPNE) when time intervals between periods tend to zero. Then the feedback equilibrium of continuous-time dynamic game, as stated in the main text, can be approximated by an SPNE of the discrete time repeated game considered in this appendix.

Proof. Let the penalty be proportional to the accumulated illegal gains from cartel formation $s(t) = \alpha w(t)$. In this case two possible outcomes can arise depending on the parameters of the model. Either both firms report the cartel simultaneously in the beginning of the game or the cartel will last forever.

1. Consider the case $\alpha\lambda \leq r$.

In this case we compare the value of infinitely lasting cartel $\pi = \int_0^\infty (\frac{\pi^m}{2} - \lambda s(t))e^{-rt}dt$, which can be rewritten as $\pi = \frac{\pi^m}{2r}(1 - \frac{\alpha\lambda}{2r}) > 0$, with the leader's value that can be rewritten as $L(t) = \pi - \frac{\pi^m}{2r}e^{-rt}(1 - \frac{\alpha\lambda}{r} + \frac{\alpha\lambda}{2r}e^{-rt})$. Hence, given $\alpha\lambda \leq r$, we obtain $\pi > L(t)$ for all $t \in [0, \infty)$. This implies that the matrix game played at each instant t, which has been described in section 5.4.1, has two pure strategy Nash Equilibria, and the equilibrium $(N, N)_t$ Pareto dominates $(S, S)_t$ for all $t \in [0, \infty)$. This implies that, when $\alpha\lambda \leq r$ the unique SPNE of the dynamic game is $(N, N)_t$ for all $t \in [0, \infty)$. Hence, $(t_1^*, t_2^*) = (\infty, \infty)$ is the unique feedback equilibrium of the preemption game if $\alpha\lambda \leq r$.

The above considerations imply that in case $\alpha\lambda \leq r$ the cartel will last forever, and

[28] We restrict our attention to search only for pure strategy equilibria, since mixed strategy equilibria for timing games where firms self-report at a certain point in time only with some probability do not make economic sense and and would not give any reasonable policy implications for antitrust enforcement.

self-reporting is never a dominant strategy for any of the firms. See also Figure 5.1 above.

End of the proof of part 1.

2. Consider the setting with $\alpha\lambda > r$. Two possible sub-cases can arise here.

a) $\pi < 0$, so that $\pi < L(t)$ for all $t \in [0, \infty)$ This can hold only if $\alpha\lambda > 2r$. See Figure 5.3.

b) $\pi < L(t)$ for some $t \in [0, \infty)$ holds when $r < \alpha\lambda < 2r$. This situation is depicted in Figure 5.2.

In both cases the dominant strategy for each firm is to play S_t (self-report at time t) at each instant of time. This implies that $(t_1^*, t_2^*) = (0, 0)$ is the unique feedback equilibrium of the preemption game if $\alpha\lambda > r$.

We prove this statement by backward induction.

We can show that in both cases, $\alpha\lambda > 2r$ and $r < \alpha\lambda < 2r$, the function $L(t)$ approaches π from above when t tends to infinity, i.e. there exists a finite number $\widehat{t}$ such that $L(t) > \pi$ for all $t > \widehat{t}$,[29] where $\widehat{t}$ satisfies $L(t) - \pi = 0$. This implies that $\widehat{t} = \frac{\ln(\frac{\alpha\lambda}{2(\alpha\lambda - r)})}{r}$. It is clearly finite for any finite values of α, λ and r when $\alpha\lambda > r$. Moreover, it is easily verified that $L(t) - \pi = -\frac{\pi^m}{2r}e^{-rt}(1 - \frac{\alpha\lambda}{r} + \frac{\alpha\lambda}{2r}e^{-rt}) > 0$ for all $t > \widehat{t}$.

Since $M(t) > F(t)$ for all $t \in (0, \infty)$, given $s^F \in (\frac{1}{2}s^n, s^n]$ and $s^m = \frac{1}{2}s^n$, and $L(t) > \pi$ for $t > \widehat{t}$, we can conclude that for both firms the strategy S_t (self-report at t) strictly dominates strategy N_t for all $t > \widehat{t}$. Hence, for any $t \in [\widehat{t}, \infty)$, there is a unique Nash Equilibrium of simultaneous move matrix game played at instant t of a dynamic game, which is described by $(S, S)_t$.

Now we apply the backward induction argument. We look at the matrix game played at instant t and assume that, if the game continues for one more period, the equilibrium of the game at t^+ will be $(S, S)_{t^+}$, since simultaneous self-reporting should be part of the subgame perfect strategy given the result above. Then the payoff matrix at t will have form as in Table 5.3.

	Self-report	Not self-report
Self-report	$(M(t), M(t))$	$(L(t), F(t))$
Not self-report	$(F(t), L(t))$	$(M(t^+), M(t^+))$

Table 5.3. Payoffs and strategies of the matrix game played at time t.

By assumption of the model, the function $L(t)$ is always above the function $M(t)$ and, hence, $L(t) > M(t^+)$ for any t, $t^+ \in (0, \infty)$. This inequality implies that the strategy S_t

[29] Recall Figure 5.2 and Figure 5.3.

(self-report at t) is dominant for both firms. This implies that the matrix game at t has a unique pure strategy Nash Equilibrium: (S, S). Repeating this argument backwards to the initial period of the game, and taking into account the fact that $L(t) > M(t^+)$ for any t, $t^+ \in (0, \infty)$, we obtain that self-reporting is a dominant strategy for both players at each instant of time. Consequently, immediate simultaneous self-reporting at $t = 0$ is an SPNE of the infinitely repeated game. And, hence, $(t_1^*, t_2^*) = (0, 0)$ is the unique feedback equilibrium of the preemption game when $\alpha\lambda > r$.

In other words, when $\alpha\lambda > r$, both firms want to become leader and report at $T_L = \arg\max L(t)$. As a result a firm will try to preempt the other firm by reporting at time $T_L - \varepsilon$. But then the other will try to preempt by reporting at $T_L - 2\varepsilon$ and so forth and so on. This process stops at time $t = 0$, where both values $L(t)$ and $F(t)$ are equal.

End of the proof of part 2.

■

5.9.2 Appendix 2: Proof of Lemma 5.2

Proof. First, we reformulate expressions (5.15) and (5.16) in order to make them comparable. Expresson (5.15) gives $T_L = \frac{\ln(\frac{1}{1-\frac{r}{\alpha\lambda}})}{r}$. Similarly, expression (5.16) gives $T_c = \frac{\ln(\frac{1-\frac{r}{\lambda}}{1-\frac{r}{\alpha\lambda}-\frac{r}{2\lambda}})}{r}$.

Second, we consider the difference

$$\frac{1}{1-\frac{r}{\alpha\lambda}} - \frac{1-\frac{r}{\lambda}}{1-\frac{r}{\alpha\lambda}-\frac{r}{2\lambda}} = \alpha r \frac{\alpha\lambda - 2r}{(\alpha\lambda - r)(2\alpha\lambda - 2r - r\alpha)}. \tag{5.21}$$

It is clear that expression (5.21) is negative when $r < \alpha\lambda < 2r$. This implies that $T_c > T_L$ when $r < \alpha\lambda < 2r$.

In case $2r < \alpha\lambda$ expression (5.21) becomes strictly positive and, consequently, we obtain that $T_L > T_c$ when $2r < \alpha\lambda$. ■

5.9.3 Appendix 3: Proof of Proposition 5.3

Proof. For derivation of this proposition we employ the definition of feedback equilibrium as it is given in Huisman (2001).

If the leader reports at time T_L, then the best response of the follower, given the possibility to react instantaneously, is to report at time T_L as well. But this means simultaneous self-reporting and, consequently, the payoffs for both firms will be $M(T_L)$, which is less than $M(T_c)$ by definition. So, the rational leader will anticipate this and take into account this best response of the follower. Consequently, his optimal strategy would be to wait until T_c and then both firms report simultaneously at time T_c.

Here $(t_1^*, t_2^*) = (T_c, T_c)$ is a feedback equilibrium of the preemption game with leniency, since no one of the firms has incentives to deviate either by waiting with self-reporting till $t > T_c$ or by preempting the other firm by playing $t < T_c$. In both cases, given the assumption that firms can react instantaneously and, hence, the second firm will also self-report immediately after the first firm does so, both firms obtain lower payoffs: $M(t) < M(T_c)$ for any $t \neq T_c$, since by definition $T_c = \arg\max_{t\geq 0} M(t)$. ■

Note, that this result holds only under the assumptions that firms are completely symmetric and can react instantaneously to the actions of their opponents. In case when we relax the assumption that firms can react instantaneously, the feedback equilibrium of the game is $(t_1, t_2) = (0, 0)$ if $\alpha\lambda > r$ or cartel forever, i.e. $(t_1, t_2) = (\infty, \infty)$ if $\alpha\lambda < r$.

5.9.4 Appendix 4: Proof of Proposition 5.5

Proof. To prove the result of proposition 5.5, we first derive the optimal stopping time for the firm when there is no leniency and the optimal stopping time in case of simultaneous self-reporting with leniency in the setting where the penalty is fixed.

Following the result of the benchmark model, we obtain that the optimal stopping time in the model without leniency is given by

$$\begin{aligned} T_L &\to \infty \text{ if } \frac{\pi^m}{2} > \lambda F^n, \\ T_L &= 0 \text{ if } \frac{\pi^m}{2} \leq \lambda F^n. \end{aligned} \tag{5.22}$$

In the game with leniency, following the reasoning similar to Proposition 5.3, we conclude that $(t_1, t_2) = (T_c, T_c)$ is a feedback equilibrium of the preemption game with leniency, where $T_c = \arg\max_{t\geq 0} M(t)$. In this case the firms have no incentives to deviate either by waiting with self-reporting till $t > T_c$ or by preempting the other firm by playing $t < T_c$. In both cases, given the assumption that firms can react instantaneously and, hence, the second firm will also self-report immediately after the first firm does so, both firms obtain lower payoffs: $M(t) < M(T_c)$ for any $t \neq T_c$.

The value of simultaneous self-reporting in the setting with fixed penalty is given by

$$M(T_c) = \int_0^{T_c} (\frac{\pi^m}{2} - \lambda F^n) e^{-rt} dt - \frac{1}{2} F^n e^{-rT_c}. \tag{5.23}$$

Next, we derive exact formulas for the feedback equilibrium of the game with leniency where the penalty is fixed. Recall the game described in Table 5.1. The outcome, i.e.

whether (S,S) or (N,N) will occur, depends on the magnitude of gains from cartel formation and the expected fine. Maximizing expression (5.23) with respect to time we conclude that $T_c \to \infty$ if $\frac{\pi^m}{2} > (\lambda - \frac{r}{2})F^n$ and $T_c = 0$ if $\frac{\pi^m}{2} \leq (\lambda - \frac{r}{2})F^n$. Hence, $(S,S)_t$ with $t = 0$ is an SPNE when $\frac{\pi^m}{2} \leq (\lambda - \frac{r}{2})F^n$, while $(N,N)_t$ is an SPNE for all $t \in [0,\infty)$ when $\frac{\pi^m}{2} > (\lambda - \frac{r}{2})F^n$. We conclude that two outcomes can arise as an equilibrium in feedback strategies: one is immediate self-reporting at $T_c = 0$, i.e. $(t_1^*, t_2^*) = (0,0)$, and the other equilibrium is never self-report, i.e. $T_c \to \infty$, so that $(t_1^*, t_2^*) = (\infty,\infty)$. ■

CHAPTER 6

Strictness of Leniency Programs and Cartels of Asymmetric Firms

6.1 Introduction

The main question we address is whether the leniency rules work (and if so – for what types of companies) given that there are other costs to admitting an infraction of competition law other than the fine. As an example of those costs we will consider reduction of sales due to the reputation effect after cartel is discovered. We define effective leniency programs in the sense that they achieve the objective of voluntary applications for leniency which in return results in the break-up of illegal cartels. Since the leniency rules only offer a reduction in the fine calculated on the basis of the affected turnover, but do not take into account the expected costs of admitting illegal behavior and ignore other costs which can potentially outweigh the fine, one can question the effect that may be expected from the leniency rules. The literature has already noted these other negative effects on the expected number of requests for leniency (see, for example, Motta and Polo (2003) or Spagnolo (2000b)). We add to the literature a specific notion: companies are diversified to a certain extent and the measure of diversification is not identical for the firms. We will present a model that takes into account fines that result from a conviction by the competition authority and also other costs resulting from affected sales in markets other than those involved in illegal behavior. Earlier papers on leniency do not take this aspect into account.[1] We will investigate how companies will react to a leniency rule given that firms are diversified to a specific extent, which is unique for each firm, and that the leniency rules do not take into account the effects on markets other than the markets on which the cartel was proven to operate.

In general, introducing a legally embedded sanctioned opportunity for whistle-blowing rewarded by fine reduction or even exemption changes the game played between the antitrust authority and the group of firms. Intuitively, this opportunity should reduce cartel stability by increasing the incentives for firms to reveal the cartel. This conclusion is well established in the literature (see, again, Motta and Polo (2003)). However, in the presence of asymmetries these incentives change. In the symmetric situation where the cartel members are identical in all respects, the firms all have identical incentives and will all apply for leniency at the same moment in time. This, however, is a very theoretical situation and, therefore, we introduce asymmetries on the basis of the measure of diversification of the firm. We will show that the effectiveness of Leniency Programs is different for different types of companies and depends crucially on the number of markets in which a firm operates relative to the number of markets covered by the cartel if one takes into account a reputation effect.

A number of earlier papers have studied effects of leniency programs in antitrust enforcement without taking into account possible asymmetries between the firms and the reputation effect. Recall Motta and Polo (2003), Spagnolo (2000, 2004), or Hinloopen (2003). Malik (1993) and Kaplow and Shavell (1994) were the first to identify the potential benefits of schemes, which elicit self-reporting by violators. They conclude that self-reporting may reduce enforcement costs and improve risk-sharing, as risk-averse self-reporting individuals face a certain penalty rather than the stochastic penalty faced by non-reporting violators. The difference is that they consider individual violators rather than a group of violators. A similar paper in this field is Innes (1999), who considers environmental self-reporting schemes. Direct applications of leniency programs in antitrust policy have been studied by Motta and Polo (2003). They show that such programs might have an important role in the prosecution of cartels provided that firms can apply for leniency after an investigation has started. They conclude that, if given the possibility to apply for leniency, the firm might well decide to give up its participation in the cartel in the first place. They find also that leniency saves resources for the authority. Finally, their formal analysis shows that leniency should only be used

[1]Our set-up can be linked to Kobayashi (1992). There he considered deterrence with multiple defendants in the framework of asymmetric bargaining.

when the antitrust authority has limited resources, so that a leniency program is not unambiguously optimal.

Another attempt to study the efficiency of leniency programs in antitrust enforcement was made in Feess and Walzl (2004). They compare leniency programs in the EU[2] and the USA[3]. For that purpose they constructed a stage-game with two self-reporting stages, heterogeneous firms with respect to the amount of evidence provided, and ex post asymmetric information. Differences in leniency programs in the US and Europe include the fine reduction granted for first and second self-reporters, the role of the amount of evidence provided, and the impact of whether the case is already under investigation. The paper by Feess and Walzl (2004) elaborates on the role of asymmetric information to derive the optimal degree of leniency and uses these findings to compare the programs in the US and the EU.

Another line of literature we will touch upon is the literature on reputation. One of the conditions for the functioning of the reputation mechanism is that there should be information on the performance of the company, see Graafland and Smit (2004). Miles and Covin (2000) find empirical support that a reputation advantage enhances marketing and financial performance. Whereas they investigate the proof for an environmental reputation, we will consider the reputation of an offender of competition law. Graafland and Smit mention several ways in which reputation loss due to admitting of having been a member of a cartel may result in additional costs. A good reputation may attract highly qualified workers, it could benefit the company on the goods market and on the financial market. In this chapter, we restrict ourselves to the effects of reputation on the goods market. We explicitly introduce the notion that the goods market is divided into several markets, where the reputation in one market may to some extent carry over to the reputation in another market. Of course, these effects in goods markets may directly affect the financial position of the firm, but we will not discuss these links.

This chapter contributes to this literature[4] by studying the effectiveness of leniency programs for companies, which are not symmetric. We take into account that a conviction by a competition authority results in costs other than the fine. The additional costs we will single out are the cost associated with reduced sales in all markets the convicted

[2]For description of European system see Guidelines on the method of setting fines imposed (PbEG 1998) and the report of EC "Commission adopts new leniency policy for companies which give information on cartels", press release, Brussels, Feb. 13, 2002.

[3]For description of US system see Guidelines manual (chapter 8-Sentencing of organizations) and The twelfth annual report (DOJ 1998).

[4]Our model also can be seen as an extension of Spagnolo (2000, 2004) or Motta and Polo (1999).

cartel participant operates in. This, what we call reputation effect, depends on the size of the firms. The effectiveness of a leniency program largely depends on markets outside the market corrupted by the cartel agreement.

The structure of the chapter is as follows. In section 6.2 we give a qualitative description of the problem followed by a summary of the system of leniency rules adopted by the Dutch national competition authority, the Nederlandse Mededingingsautoriteit (NMa). Section 6.3 provides formal description of the model. In section 6.4, we solve the model and find subgame perfect equilibrium of the game. Section 6.5 outlines the optimal enforcement strategies of antitrust authority and strategies that allow to implement the no collusion outcome. Finally, section 6.6 concludes the analysis.

6.2 Outline of the Model

6.2.1 Qualitative Analysis

The model consists of three groups of actors - firms, consumers, and the competition authority. We assume that companies are asymmetric in the sense that they are diversified to different extents. We intend this to imply that firms operate in several distinct relevant markets. The competition authority has the power to scrutinize all markets. However, practical constraints (resources) imply that it cannot investigate all markets to the same degree. The general public only gets proof on the existence of a cartel when the findings of the competition authority result in a formal report. Enterprises only have knowledge about the cartels in which they are involved. For the markets in which they are not present they have no information advantage over the general public.

We assume that both the public and enterprises react to cartel findings in that they reduce purchases from the enterprises that are fined. The intuitive explanation for this behavior is that buyers have an instinctive desire to punish the cartel members and reduce purchases from the offenders. The consumers thus apply a tit-for-tat strategy; as soon as they discover that they have been deceived by producers they reduce their purchases from the producers involved. This negative reputation effect can be modeled as a factor $R \in (0, 1)$ that denotes the reduction of the sales of the cartel members that were part of the illegal agreement. Notice that if all producers are involved, the consumers have a desire to reduce consumption, but they are not able to find alternative suppliers. In this situation, we assume that the buyers will temporarily withhold purchases whenever possible[5].

Another reasoning that is based more on rational behavior rather than introducing reactions not normally associated with the "homo economicus" but reaching the same conclusion involves uncertainty. Assuming that consumers (a) have a preference for goods with a higher quality ($dU/dQual > 0$) but (b) cannot perfectly observe the quality of a good even after purchasing the good (because reliability and durability can only be estimated after some time), one can argue that consumers take the relative prices as an indication of relative quality. Profit margins are implicitly considered identical and all firms all equally efficient resulting in a direct relation between production costs and product quality. The consumers then choose the product with the optimal price/quality ratio given the budget constraints. After the discovery of a cartel by the competition authority, buyers know that they have paid a mark-up over and above normal profit margins to the enterprises that are involved in the cartel, implying that the cost component (related to expected quality) must have been lower than that of non-cartel competitors. Thus, the message to buyers is that the quality of the product of the cartel-members must have been lower than expected. Since they have no reason to assume that other, competing enterprises in the same market have applied the same mark-up, these products gain a quality reputation and buyers redress their buying patterns optimizing the price/quality level incorporating the new information.

The crucial observation to make here is that consumers do not fine-tune their retaliatory actions to the relevant market(s) affected by the cartel, but rather reduce expenditure on all the products and services produced by firms that were convicted for participating in the cartel. Most of the public, and perhaps to some extent even private enterprises, are unable to distinguish between organizational divisions within the offending companies and the application of very specific defined relevant product and geographic markets used by the competition authorities. Thus the subtleties of the fact that an operational unit of a company is named as the participant of an illegal cartel in a specific relevant market are lost and the company is considered to have participated in a cartel increasing the price of (all) its products. This implies that the reduction is

[5]An indication of this behavior is clear in the building sector in the Netherlands. Following the discovery and publication of an extensive documentation on widespread illegal transactions by hundreds of construction companies, there have been a significant reduction in sales. The sector considers this discovery as one of the causes of the economic downturn of the sector. The publication of this case was made by a specific parlamentary inquiry board, whereas, in general only the information published by the Competition authority will be available. However, this economic downturn of the sector can possibly also be attributed to the business circle effect.

applied to the sales of all products of the company (not only on the markets involved).

We consider the assumption that all sales are reduced to be very plausible. In markets for final goods, this holds especially when sales on many independent relevant markets are made under a common brand or company name and consumers simply associate the brand name with the cartel. In markets for immediate goods, where the buyers are professionals, there are two effects which may cancel out so that the level of reduction, R, may be identical to that associated with final consumers. The first effect is that the professional buyers may be better able to make a distinction between the part of the seller's organization that was part of the cartel and the part of the organization that was not involved, thus resulting in an overall lower R. The second effect is that the professional buyers may be better able to determine the origin of products even when these are marketed using separate brand names, thus resulting in an overall higher level of R. For simplicity, we will not make a distinction between markets for final consumers and markets for intermediate consumers and therefore use a single value for R.

With enterprises that have issued publicly held and traded shares, one additional source of loss of wealth is that the expectation of lower sales immediately results in a lower valuation of the shares. If consumers act in the way that we model – i.e. they reduce purchases from the companies involved in all markets where these enterprises operate – the loss in share value may well be more significant than one would expect on the basis of the reduction in sales in the markets where the cartel has been proven to exist. Indeed there are such empirical indications that support the plausibility of the existence of a reduction on all sales rather than on affected sales only. Soppe (2000), considering the effects on companies after discovery of participation in a cartel, concludes that the loss in investor returns is normally much bigger than what would be expected on the basis of expected fines and compensations. Archer and Wesolowsky (1996) find that owners of durable goods do not seem to tolerate more than one incident without consequences for not only product loyalty but also manufacturer loyalty. This could point to the existence of expectation of reduction of sales in other than the affected markets.

6.2.2 The Leniency Policy of the NMa

In the EC competition law, a leniency policy was introduced in 1996. The Dutch national competition authority (NMa) adopted leniency rules on July 2, 2002. The current version we will discuss is the "Richtsnoeren Clementietoezegging", last amended on April 28, 2004. The text on the leniency rules is numbered in the margin, and we will refer to the numbers in the margin for ease of reference.

The purpose of the leniency rules is to give the NMa a choice of whether to impose a fine or not when it discovers and proves breach of either article 6 Mw or article 81 EC (2). The objective of leniency is to gain information on cartels and to make discovery, punishment and termination of cartels more effective (1). Thus, the NMa can offer reduction of fines to members of a cartel that wish to terminate their involvement in behavior that is illegal under cartel legislation. The leniency rules make a distinction between several situations. The biggest reductions are obtained when a cartel member informs the NMa of a cartel before the NMa has started a formal investigation. The first cartel member to approach the NMa with sufficient information to start an investigation obtains 100% immunity for resulting fines unless this company forced other companies to participate in the cartel or the company does not fully cooperate during the investigation (5). Who is first depends on the exact time the companies first contacted a leniency officer (14). The first cartel member to help the NMa with additional information after starting an investigation of this cartel will receive a reduction of between 50 and 100% of the fine given that this company has not forced other companies to participate in the cartel and that the company cooperates during the investigation. Additional information is defined as information that the NMa did not have before and without which the case cannot be proven (9). Companies that were either not the first to approach the NMa with additional information with respect to a specific cartel or that were the first to contact the NMa with additional information but that had forced other companies to participate in the cartel can obtain fine reduction ranging from 10 to 50% (7). In addition, the companies providing additional information do receive 100% immunity for information resulting in an increase in the fine on which the reduction is applied (10) (e.g. because the cartel was in operation longer than assumed on the basis of the investigation of the NMa or the information of the informer that resulted in the start of the investigation).

6.3 The Model (Formal Analysis)

We consider a group of firms, which may form a cartel, taking into account the enforcement activity of the antitrust authority. The antitrust authority commits to a certain enforcement policy, which uses leniency programs. Leniency programs grant either complete or partial reduction of fines to the firms, which reveal the existence of a cartel to the antitrust authority. The main innovation of this model, compared to the earlier papers on leniency by Motta and Polo (1999) and (2003) or Feess and Walzl (2003), is that we consider asymmetric firms that have different size and operate in several different markets, but form a cartel only in one market. This gives rise to additional costs in case of disclosure of cartel that are caused by a reduction of the sales in other markets due to a negative reputation effect. This effect is asymmetric: firm 1 bears additional costs of Rh_1, while firm 2 suffers additional costs of Rh_2. Here h_1 and h_2 are the total sales in other markets, in which the relevant company does not form a cartel. The second innovation of the model is that the leniency policy of antitrust authority is not only limited to the option of fine reduction for self-reporting firm, but also takes into account possibility of different treatment of the first and second reporter that is imbedded in the current leniency rules of many competition authorities worldwide.

First, we describe the policy choices of the antitrust authority. Second, we specify the firms' strategies. And, finally, we describe the timing of the game.

Enforcement policy: The main goal of the antitrust authority is to prevent cartel formation in the first place. However, if the cartel has already been formed, the antitrust authority aims to discover it at the lowest possible cost. Following Becker (1968), we distinguish two main parameters of enforcement policy: penalty and probability of detection. Hence, the antitrust policy in the presence of leniency programs can be described by the following three parameters.

- The full fines $F \in [0, F^{\max}]$ for firms that are proved guilty and that have not cooperated with the antitrust authority, where $F^{\max}$ reflects the upper bound for the fine that is exogenously given by the law[6]. Following the Becker's argument, in this set-up, the fixed fine F will generally be set at $F^{\max}$.

- The reduced fines $f \in [0, F)$ specified by leniency programs. In particular, if only one of the firms reports the cartel, then this firm pays no fine, while the other firm will pay the normal fine, F. Moreover, we consider the set-up in which all the firms that cooperate can be granted reduced fines f. However, the amount of reduction depends on the circumstances, especially the order of self-reporting and the "value" of additional information. Applying the rules of current Dutch leniency practices discussed in section 6.2[7], the possibility of simultaneous self-reporting by the firms should be ruled out. However, the model, described in this chapter, is richer and can also predict in the situation where firms self-report simultaneously. To simplify the analysis, we consider

[6] According to the NMa, the maximum fine depends on the turnover in all markets, not only the market where the illegal agreement applied to.

[7] Richtsnoeren Clementietoezegging, last amended on April 28, 2004.

a two-firms' game. The first firm to self-report gets complete exemption from the fine, while the second pays the reduced fine, $f = \frac{1}{2}F$.[8] This set-up describes the most strict adherence to the Leniency rules. However, we will also consider an alternative set-up, where the antitrust authority is less strict and grants partial immunity to both firms in case they self-report almost at the same time. This possibility will be captured by an additional (non-traditional) instrument of antitrust authority, which we call "strictness" of leniency rules, and which is denoted by α. This parameter reflects the estimated probability that the firm, which self-report almost simultaneously with its rival, gets zero fine.

- The probability of law enforcement by the antitrust authority equals $p \in (0, 1]$. This variable can be thought as an instantaneous probability that the firm is checked by antitrust authority and found guilty. Contrary to Motta and Polo (2003), we assume that whenever the antitrust authority checks the guilty firm, the violation is successfully discovered. Moreover, we assume that p is determined by an exogenous budget of the antitrust authority financed by the government that can be used to promote enforcement, so that p reflects the costs of efforts of antitrust authority put into law enforcement activities.

Firms' strategies: We analyze two different **collusive strategy profiles** of the firms "Enter Cartel and Self-report" and "Enter Cartel and Not Self-report" and one **competitive strategy profile** "Not Enter the Cartel in the first place".

First, we consider the strategy Enter Cartel and Self-report (E S). The firms decide to enter a cartel agreement. This may give them per period profits π_m if the cartel is stable. At the next stage of the game one or both firms choose to report the existence of the cartel to the authority. This allows them to obtain a reduced fine. However, they loose not only extra profits from cartel formation, but also a fraction of the sales in other markets, since information about cartel becomes publicly available. The second collusive strategy is Enter Cartel and Not Self-report (E NS). In this case the payoff is determined as an expectation of the monopoly gains, π_m (if cartel is not found), and competitive profits, π_n, less the fine and losses due to the reputation effect (if violation is discovered by antitrust authority).

[8] These rules are roughly consistent with partial immunity clauses that often apply if more than one cartelist reports. Moreover, Apesteguia, Dufwenberg and Selten (2003) use a similar mechanism to design one of the treatments in their experimental paper, which studies the effects of leniency on the stability of cartel. Feess and Walzl (2003) consider partial reduction of fines for both firms in case of simultaneous self-reporting.

The competitive strategy profile is "Not Enter the Cartel in the first place": (NE), which implies that the two stage game is reduced to one stage. In this case both firms obtain competitive profits π_n forever. Note that $0 \leq \pi_n < \pi_m$.

Timing of the game: The two asymmetric firms play the two stage game without knowing the action of the rival. At time $t = 0$ the antitrust authority sets parameters of enforcement policy: F and p. Here we assume that leniency program is not yet into existence at time $t = 0$ and, hence, no reduction of fine is possible in case the firm cooperates with the antitrust authority. Consequently, self-reporting is not an option at this stage. This set up resembles the policy of, for example, Dutch Competition Authority (NMa) before the year 2001, when the leniency programs were introduced in The Netherlands[9].

Next, at time $t = 1$ "the cartel formation subgame" is played. At $t = 1$ both firms decide whether to participate in the cartel or stay out and realize the per-period associated payoff, respectively π_m and π_n. If both firms agree to participate, the cartel is formed and the game continues into second stage. If at least one of the firms decides to stay out, the game stops and both firms obtain competitive profits, π_n, forever. We assume that the existence of a collusive outcome in the industry cannot be observed by the antitrust authority until it starts an investigation in this market.

Further, at time $t = 2$ (an analogy of the year 2001 in The Netherlands) the antitrust authority introduces leniency programs, which allow firms to be exempted from the fine in case of self-reporting. Now those firms, who already formed a cartel, have the choice either to keep it secret or report it to the antitrust authority. Hence, at $t = 2$ "the revelation subgame" is played, where both firms simultaneously decide whether to report the existence of the cartel to the authority or not. If at least one of them does so, cartel formation stops and both firms obtain π_n. If no firm reveals, the antitrust authority is able to prove them guilty and punish with probability $p \in (0, 1]$ in any subsequent period. We assume here again, differently from Motta and Polo (1999), that a firm proved guilty does not collude any more, so after being punished firms do not go back

[9] When leniency programs are already present, then ((E S),(E S)) equilibrium is dominated by Not Entering ((NE),(NE))equilibrium. In that case the game played is not a two stage game anymore but can be considered as a simultaneous move game and there are no additional cartel profits realized in the first stage. This implies that in the situations where the structure of the penalty scheme and leniency programs are both introduced in the beginning of the game, as it is at the moment in most developed economies, the solution of the game would follow the same lines as described in section 6.4 with the simplification that the strategy (E,S) will not be played in equilibrium any more (for any possible parameter values), since it's strictly dominated by the strategy not to enter the cartel in the first place.

to collusion, while in case the cartel has not been revealed or discovered, firms sustain the collusive strategy for at least one more period and obtain monopoly profits, π_m.

The antitrust authority does not take an active part in the game. It only sets policy parameters, F, f , p, α, and the rules of leniency programs. As said before, the strictness of the leniency rules is modelled through parameter α. A "strict" antitrust authority would give complete exemption from the fine only to the self-reporting firm, which is literally the first to self-report. In this case the parameter α is close to zero and the firm that cooperates will almost surely get only partial exemption. Hence, it pays the reduced fine, $f = \frac{1}{2}F$. A "mild" antitrust authority can give complete exemption from the fine to all the firms that cooperated. In this case the parameter α is equal to 1 and every cooperating firm gets zero fine. It speaks for itself that in our model α is only relevant when both firms self-report at the same stage of the game.

It should also be mentioned that under a regular antitrust policy without a leniency program, collusion can be sustained only when the short run gain from an unilateral deviation from collusive agreement by undercutting in prices is smaller than the expected loss triggered by the deviation. This loss follows from the fact that cartel profits, π_m, will be replaced by competitive profits, π_n. Hence, collusion under a regular antitrust policy (i.e. when leniency is not available, and only rate of law enforcement and fine are instruments of competition authority) takes place only when the following inequality is satisfied for each firm

$$\frac{\pi_m - p(F + Rh_i)}{1-\delta} > 2\pi_m - p(F + Rh_i) + \frac{\delta\pi_n}{1-\delta} \quad for \quad i = 1, 2. \tag{6.1}$$

Here $2\pi_m$ reflects the extra profits from undercutting, since we assume there are only two firms in the market. This inequality implies that collusion can arise only when the discount factor is large enough, namely, $\delta \geq \frac{\pi_m}{2\pi_m - \pi_n - pF - pRh_i} \quad for \quad i = 1, 2.$ This condition states that the discount factor required to induce collusion is smaller if either the difference between monopoly profit and competitive profit (the gains of cartel) increases and/or the expected fine (the expected costs following discovery of the cartel) decreases. If this condition is not met, it is more attractive for either of the firms to deviate from the collusive strategy, and obtain monopoly profits for one period and then compete for the rest of the game. For the further analysis we restrict our attention to the case where this condition is met for both firms, which implies that in the absence of leniency programs, the equilibrium state is collusion. Hence, inequality (6.1) represents a necessary condition for the second stage of the game ("revelation subgame" played at $t = 2$) to be reached. Another important restriction on the discount factor is

$\delta \geq \frac{\pi_m}{2\pi_m - \pi_n}$, which implies that in the absence of the antitrust policy, collusion would arise in equilibrium[10]. Note that in this case the second stage of the game ("revelation subgame") is also automatically reached, since it is implied by $\delta \geq \frac{\pi_m}{2\pi_m - \pi_n - pF - pRh_i}$.

It should be stressed that for any $t > 2$, the decisions of both players do not change and payoffs obtained at $t = 2$ will be discounted into the future. This is due to the fact that the penalty is fixed and, hence, the environment does not change.

We summarize the above description of the game as follows:

Stage 0: Antitrust authority announces parameters of the penalty scheme, p and F.

Stage 1: Firms decide whether to be in a cartel or not (once and for all decision).

Stage 2a: Antitrust authority introduces leniency program.

Stage 2b: Firms decide whether to self-report or not (once and for all decision). If no self-reporting by both firms is chosen then repeated game between authority and firms, where authority can discover violation with probability p in each period, is played till infinity.

The discount factor is denoted by $\delta = \frac{1}{1+r}$, where r is the interest rate. The game tree and players' payoffs are described in Figure 6.1.

We now proceed to establish the subgame perfect equilibria of the two stage game, which is described in Figure 6.1, played among firms once the policy parameters are set.

6.4 Solution of the Game

6.4.1 Solution of "Revelation Subgame"

To find the subgame perfect equilibria of the game, consider first the "revelation subgame", which is played in stage 2. In case of simultaneous self-reporting, a firm i gets a payoff of $\frac{\pi_n}{1-\delta} - Rh_i - \frac{(1-\alpha)}{2}F$. This expression reflects the rules of current sentencing guidelines that the first firm to self-report gets complete exemption from the fine, while the second pays the reduced fine, $f = \frac{1}{2}F$. Given that the other firm self-reports at approximately the same time, the probability to be the first to report and get zero fine is α. However, there is also a chance $(1 - \alpha)$ that another firm is the leader in the "race to the court". If a firm i does not self-report but the other firm does, then this firm

[10]In the absence of any antitrust enforcement, i.e. when neither fines nor rate of law enforcement can be used, collusion can be sustained only when the short run gain from an unilateral deviation from collusive agreement by undercutting in prices together with competitive profits thereafter is smaller than the payoff from sustaining collusive strategy forever: $\frac{\pi_m}{1-\delta} > 2\pi_m + \frac{\delta\pi_n}{1-\delta} \quad for \quad i = 1, 2$.

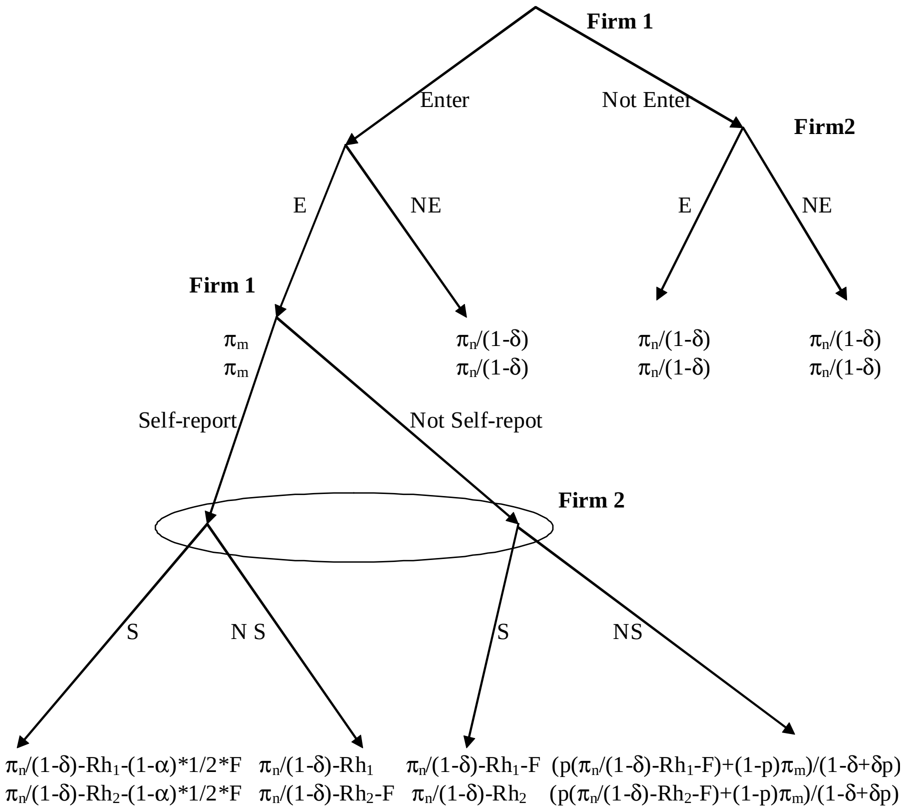


Figure 6.1: Game tree and players' payoffs.

receives a payoff of $\frac{\pi_n}{1-\delta} - Rh_i - F$, while the other firm is granted complete leniency and obtains $\frac{\pi_n}{1-\delta} - Rh_j$. Recall that there is still a negative reputation effect, because information about the cartel becomes public. Finally, if no firm self-reports, each firm receives an expected payoff $\frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p}$[11]. The normal form of the simultaneous move "revelation subgame" is given in Table 6.1.

[11]The complete derivation of this expression is quite easy to show using the recursive formula. Value of the strategy (*Enter* and *Not self-report*) can be written as $V = (1-p)[\pi_m + \delta V] + p[(\pi_n - Rh_i - F) + \frac{\pi_n \delta}{1-\delta}]$. Here, with probability $(1-p)$ firms can continue collusion in the next period and with probability p cartel will be discovered in the next period which causes fine and losses due to the reputation effect. Now, solving for V, we obtain that $V = \frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p}$.

firm 1 \ firm 2	Self-report (S)	Not Self-report (NS)
Self-report (S)	$\frac{\pi_n}{1-\delta} - Rh_1 - \frac{(1-\alpha)}{2}F$, $\frac{\pi_n}{1-\delta} - Rh_2 - \frac{(1-\alpha)}{2}F$	$\frac{\pi_n}{1-\delta} - Rh_1$, $\frac{\pi_n}{1-\delta} - Rh_2 - F$
Not Self-report (NS)	$\frac{\pi_n}{1-\delta} - Rh_1 - F$, $\frac{\pi_n}{1-\delta} - Rh_2$	$\frac{p(\frac{\pi_n}{1-\delta} - Rh_1 - F) + (1-p)\pi_m}{1-\delta+\delta p}$, $\frac{p(\frac{\pi_n}{1-\delta} - Rh_2 - F) + (1-p)\pi_m}{1-\delta+\delta p}$

Table 6.1: The normal form of the simultaneous move "revelation subgame".

It is easily verified that the tuple (*Self-report, Self-report*), that we denote as (S, S), in which all firms choose to cooperate with Antitrust Authority obtaining a reduction of fines, is always a Nash Equilibrium. The tuple (*Not Self-report, Not Self-report*), or (NS, NS), is a Nash equilibrium if $\frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p} \geq \frac{\pi_n}{1-\delta} - Rh_i$, $i = 1, 2$. Note also, that the (NS, NS) would also be Pareto dominant or payoff dominant equilibrium if $\frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p} \geq \frac{\pi_n}{1-\delta} - Rh_i - \frac{(1-\alpha)}{2}F$.[12] This implies that Not to Self-Report can be sustained in equilibrium if the following condition holds:

$$p \leq \frac{\pi_m - \pi_n + Rh_i(1-\delta) + \frac{(1-\delta)(1-\alpha)}{2}F}{\pi_m - \pi_n + Rh_i(1-\delta) + (1 - \frac{\delta(1-\alpha)}{2})F} = p^*(F, h_i, \alpha) \text{ for } i = 1, 2. \tag{6.2}$$

It is easily verified that for player i the payoff in the (NS, NS) equilibrium is strictly greater than the payoff in (S, S) equilibrium, when $p \leq p^*(F, h_i, \alpha)$[13]. Therefore, following the Pareto-dominance criterion, firms self-report only if $p > p^*(F, h_i, \alpha)$ for $i = 1$ or $i = 2$. This gives us the first incentive compatibility constraint. We represent it in Figure 6.2 by the line p^*, which plots $\alpha(p)$ as a convex decreasing function of p in the $(p, \alpha) - plane$.

In addition, comparative statics of the behavior of $p^*(F, h_i, \alpha)$ with respect to the main parameters of the model shows that

$$\frac{\partial p^*(F, h_i, \alpha)}{\partial h_i} > 0, \ \frac{\partial p^*(F, h_i, \alpha)}{\partial F} < 0, \ \frac{\partial p^*(F, h_i, \alpha)}{\partial \alpha} < 0. \tag{6.3}$$

The first inequality is a result of incorporating the level of diversification (or asymmetries) of the firms. The more diversified the firm is (strictly speaking, the higher

[12]The notion of Pareto-dominant equilibrium is well established in the literature (see, for example, Fudenberg and Tirole (1991) pp. 20-22). Following their arguments, players will coordinate on the Pareto-dominant equilibrium if they are able to talk to one another before the game is played and agree to play highest payoff equilibrium in case of multiple Nash Equilibria. And, since firms are perfectly rational payoff maximizing agents, there is no reason for them to deviate from this agreement later on.

[13]Given $p \leq p^*(F, h_i, \alpha)$, we obtain $\frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p} \geq \frac{\pi_n}{1-\delta} - Rh_i$. Hence, $\frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1-\delta+\delta p} > \frac{\pi_n}{1-\delta} - Rh_1 - \frac{(1-\alpha)}{2}F$, since $\frac{(1-\alpha)}{2}F > 0$.

the turnover from markets not cartelized compared to total turnover) the bigger the incentives for the firm to abstain from self-reporting. In other words, this implies that the bigger the size of the firm (or the greater the amount of "honest" sales) in other markets, the higher the incentives for this firm to keep the cartel secret, since a bigger threshold probability $p^*(F, h_i, \alpha)$ implies that greater efforts from antitrust authority, in terms of increasing the rate of law enforcement, are needed in order to induce the self-reporting by this firm. The second inequality in (6.3) reflects the usual trade-off between the probability and severity of punishment extensively discussed in Becker (1968) and in Garoupa (1997, 2001). The third inequality in (6.3) implies that the uncertainty of the firms about getting the first price (or, in other words, strictness of the rules for leniency[14], which can grant the complete exemption from the fine only to one firm) actually reduces the incentives for both types of the firms to self-report.

6.4.2 Solution of "Cartel Formation Subgame"

Now we move on to the decision taken by the firms in stage 1 of the game. For each firm, we have to calculate the discounted sum of profits if firms form a cartel and compare it with the discounted sum of profits in case the cartel is not formed. This comparison has to be done for both cases, either when firms decide to self-report in the second stage of the game, and when they prefer to continue the cartel.

First, we consider the betrayal scenario where both firms choose the strategy Enter Cartel and Self-report, which we denote (E S). According to the analysis of the previous section this strategy can arise when $p > p^*(F, h_i, \alpha)$ for $i = 1$ or $i = 2$, or both. In case both firms self-report in the second stage of the game, the expected payoff for each firm includes the collusive profits obtained at $t = 1$ plus the expected payoff from simultaneous self-reporting at $t = 2$, derived in the previous section, and is given by the following expression[15]:

$$V_{ES} = \frac{\pi_m}{\delta} + \frac{\pi_n}{1-\delta} - Rh_i - \frac{(1-\alpha)}{2}F. \tag{6.4}$$

[14] We refer here to the current Leniency rules of NMa. These rules correspond to low α in our setting, which means that there is very high uncertainty for the firms about getting the first prize. From the third inequality in (6.3) it follows that in this case the threshold probability $p^*(F, h_i, \alpha)$ is maximal and, hence, the incentives for the firms to self-report are reduced.

[15] To simplify the calculations we evaluate all the payoffs at time $t = 2$. So, we discount payoffs obtained at $t = 1$ into second period with the factor $(1 + r) = \frac{1}{\delta}$, and payoffs obtained in periods $t > 2$ into second period with discount factor $\frac{1}{(1+r)^{t-2}} = \delta^{t-2}$.

However, when no agreement about cartel formation is reached the discounted payoffs for both firms, evaluated at $t = 2$, are given by $V_{NE} = \frac{\pi_n}{\delta(1-\delta)}$.

Collusion and self-reporting will arise if $V_{ES} > V_{NE}$, that is, if the following condition is satisfied:

$$\frac{\pi_m - \pi_n}{\delta} - Rh_i - \frac{(1-\alpha)}{2}F > 0. \tag{6.5}$$

This implies that the value of the parameter α that is necessary in order to ensure that the cartel is not formed, should satisfy:

$$\alpha < \alpha^*(h_i, F) = \frac{2Rh_i\delta + F\delta - 2(\pi_m - \pi_n)}{\delta F}, i = 1 \text{ or } 2. \tag{6.6}$$

This expression provides the second incentive compatibility constraint, which is represented in Figure 6.2 by the horizontal line α^*. Note, that three considerably different solutions can arise depending on the parameter values of the model. When $2Rh_i\delta + F\delta > 2(\pi_m - \pi_n) > 2Rh_i\delta$, we obtain from (6.6) that $0 < \alpha^*(h_i, F) < 1$ and then the graph in the right part of Figure 6.2 applies. When $2(\pi_m - \pi_n) < 2Rh_i\delta$ we obtain from (6.6) that $\alpha^*(h_i, F) > 1$ and then the incentive compatibility constraints and SPNEs of the game are represented by the graph in the left part of Figure 6.2. The third possibility is when $2Rh_i\delta + F\delta \leq 2(\pi_m - \pi_n)$, so that $\alpha^*(h_i, F) \leq 0$. In this case the equilibrium with no collusion will be lost. The competitive outcome will not arise in equilibrium for any parameter values. The intuition behind this result refers to the fact that when the losses to the firm both due to the fine imposed and due to the reduction of sales caused by the reputation effect are not high enough, the leniency programs can, actually, have a perverse effect. Too low fines can lead to an outcome were all the firms will participate in a cartel agreement and then depending on the size of relative gains and losses reveal it or keep it secret.

The expression (6.5) implies that the higher the h_i, the less likely this inequality will hold. Hence a bigger firm, which operates in many markets, would be less willing to enter the cartel agreement in the first place. In other words, for bigger firms the strategy to form a cartel and then self-report is more likely to be dominated by a strategy of not entering the cartel agreement in the first stage of the game, than for a smaller firm, for which h_i is low.

We may also notice that the decision of both firms when they choose between the strategy Enter Cartel and Self-report or Not Enter the cartel at all does not depend on the value of p (rate of law enforcement). However, it does depend on other parameters of the model, such as F and α. In particular, a higher fine reduces the value of the strategy Enter Cartel and Self-report, and increases the incentives for the firms to stay out of the cartel. At the same time the lower the parameter α, which reflects the perceived probability for the firm to be the first to report, or the higher the uncertainty about getting the first prize, the greater the incentives for the firms to stay out of the cartel.

Looking at the first and second incentive compatibility constraint simultaneously[16], we obtain that for $\alpha < \alpha^*(h_i, F), i = 1, 2$ firms choose not to enter the cartel in the first place, and for all $\alpha > \alpha^*(h_i, F), i = 1, 2$ and $p > p^*(F, h_i, \alpha), i = 1, 2$ firms prefer to collude and then self-report in the second stage of the game. This proves the following lemma.

Lemma 6.1 *For given policy parameters (F, f, p, α), a subgame perfect equilibrium in which firms enter the cartel and self-report exists if $p > p^*(F, h_i, \alpha)$ and $\alpha > \alpha^*(h_i, F)$ for $i = 1, 2$.*

The outcome of this lemma is depicted in the right part of Figure 6.2 by the shaded trapezium. In the left part of Figure 6.2 this equilibrium is absent, since for more diversified firms the value of α^* is likely to be greater than 1. The right part of Figure 6.2 shows that for $p > p^*$ and $\alpha > \alpha^*$ both firms decide to enter the cartel in the first stage and then, because of the high probability of conviction and the fact that rules of leniency programs are not too strict, so that almost surely every cooperating firm gets complete immunity from fine, firms choose to reveal the violation.

Next, we look at the second possible outcome of the stage 2 of the game, where both firms choose not to self-report. This outcome arises under condition $p \leq p^*(F, h_i, \alpha)$ for both $i = 1, 2$. In this case, firms anticipate that neither of them will reveal any information. The expected payoff from playing this strategy for each firm includes the collusive profits obtained at $t = 1$ plus the expected payoff from non-cooperation with antitrust authority at $t = 2$ and is given by the following expression[17]:

$$V_{ENS} = \frac{\pi_m}{\delta} + \frac{p(\frac{\pi_n}{1-\delta} - Rh_i - F) + (1-p)\pi_m}{1 - \delta + \delta p}.$$

Again, when no agreement about cartel formation is reached, the discounted payoffs for both firms evaluated at $t = 2$ are given by expression $V_{NE} = \frac{\pi_n}{\delta(1-\delta)}$.

Collusion will arise if $V_{ENS} > V_{NE}$, that is if the following condition is satisfied:

$$p \leq \frac{\pi_m - \pi_n}{\delta(F + Rh_i)} = p^{**}(F, h_i) \quad for\ i = 1, 2. \tag{6.7}$$

[16]See the right part of Figure 6.2, which reflects the case where the critical value of α^* is less than one.

[17]Recall section 6.4.1.

Comparative statics of the expression (6.7) with respect to the main parameters of the model shows that

$$\frac{\partial p^{**}(F, h_i)}{\partial h_i} < 0, \frac{\partial p^{**}(F, h_i)}{\partial F} < 0. \tag{6.8}$$

The first inequality implies that the bigger the size of the firm (or the greater the amount of "honest" sales) in other markets, the smaller the threshold probability $p^{**}(F, h_i)$, and, hence, the easier for antitrust authority to prevent the firm from entering the cartel agreement in the first stage of the game. The second inequality, as above, reflects the usual trade-off between the probability of detection and the severity of punishment discussed in Becker (1968) and Garoupa (1997) and (2001).

Expression (6.7) provides the third incentive compatibility constraint, which implies that the strategy "Enter cartel and Not Self-report" is preferred to not entering by both firms when $p \leq p^{**}(F, h_i), i = 1, 2$, see also Figure 6.2. Further, recall the first incentive compatibility constraint, which implies that not self-reporting is preferred to self-reporting in the second stage if $p \leq p^{*}(F, h_i, \alpha), i = 1, 2$. Combining these two constraints we obtain the following lemma.

Lemma 6.2 *For given policy parameters* (F, f, p, α)*, a subgame perfect equilibrium in which firms enter the cartel and do not self-report exists if* $p \leq p^{*}(F, h_i, \alpha)$ *and* $p \leq p^{**}(F, h_i), i = 1, 2$.

The result of this lemma is quite intuitive. For low values of rate of law enforcement, the worst outcome for society may arise, i.e. firms collude and keep the cartel secret, even when leniency is introduced. However, looking at the right part of Figure 6.2, we conclude that for high values of α, when leniency programs are not too strict, the efficiency of antitrust enforcement can be improved more easily, since then a lower rate of law enforcement is necessary in order to obtain the second best outcome, namely, "Enter and Self-report".

Finally, on the basis of Lemmas 6.1 and 6.2 we can conclude that the following proposition holds.

Proposition 6.3 *Once the policy parameters* (F, f, p, α) *are set, in the repeated game played by the firms from* $t = 1$ *on, we can describe the Subgame Perfect Equilibrium (SPE) in the* $(p, \alpha)-$*space as follows:*

1. When $\alpha^{*}(h_i, F) \geq 1$ *for* $i = 1, 2$*, i.e. when* $(\pi_m - \pi_n) < Rh_i\delta$ *for both firms, the Pareto dominant SPE is ((E NS),(E NS)) for* $p < min_{i=1,2}p^{**}(F, h_i))$*, while the unique SPE is (NE, NE) otherwise.*

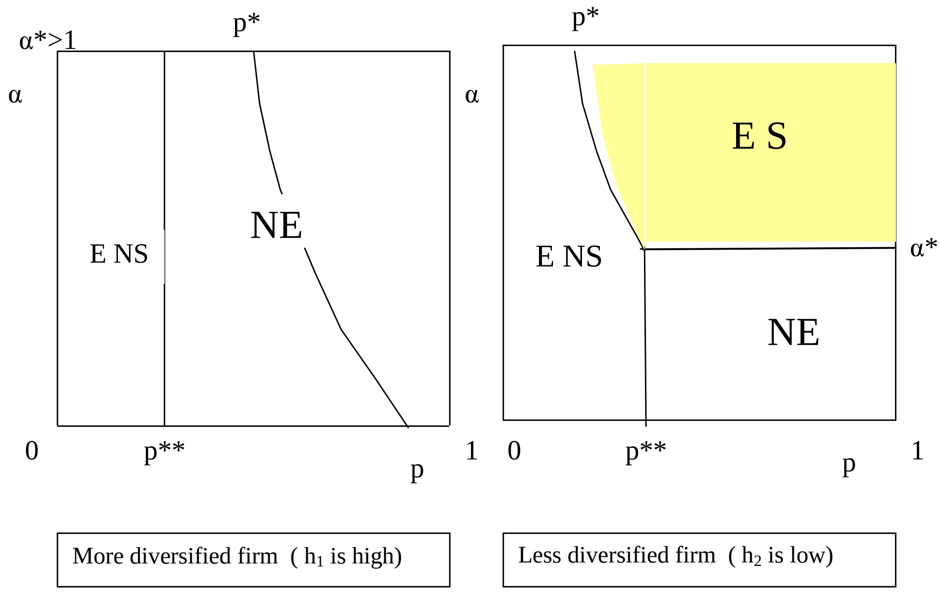


Figure 6.2: Incentive compatibility constraints for two types of firms.

2. When $0 \leq \alpha^*(h_i, F) < 1$ *for* $i = 1, 2$*, i.e. when* $Rh_i\delta + \frac{1}{2}F\delta > (\pi_m - \pi_n) > Rh_i\delta$ *for both firms, the Pareto dominant SPE is ((E NS),(E NS)) for* $p < min_{i=1,2}p^{**}(F, h_i))$ *and* $p < min_{i=1,2}p^*(F, h_i, \alpha)$*, it is ((E S),(E S)) for* $p > max_{i=1,2}p^*(F, h_i, \alpha)$ *and* $\alpha > max_{i=1,2}a^*(F, h_i)$*, while the unique SPE is (NE, NE) otherwise.*

3. When $\alpha^*(h_i, F) < 0$ *for* $i = 1, 2$*, i.e. when* $(\pi_m - \pi_n) > Rh_i\delta + \frac{1}{2}F\delta$ *for both firms, the Pareto dominant SPE is ((E NS),(E NS)) for* $p < min_{i=1,2}p^*(F, h_i, \alpha)$*, while the outcome with self-reporting arises otherwise.*

Proof: See Appendix 1.

This proposition identifies the regions where the (Enter and Self-report), (Enter and Not Self-report), and (Not Enter) equilibria exist. Clearly, both parameters p and α influence the choice of the non-collusive strategy. Moreover, all three possible outcomes can arise in equilibrium only for intermediate range of profits, i.e. when $Rh_i\delta + \frac{1}{2}F\delta > (\pi_m - \pi_n) > Rh_i\delta$ for $i = 1, 2$. For low gains from collusion, when $(\pi_m - \pi_n) < Rh_i\delta, i = 1, 2$, a SPE where both firms choose to enter and self-report does not exist. While, when gains from collusion are high, $(\pi_m - \pi_n) > Rh_i\delta + \frac{1}{2}F\delta, i = 1, 2$, a pure competitive SPE does not exist.

6.5 Optimal Enforcement with Asymmetric Firms (Implementing the No Collusion Outcome)

This section provides an analysis of the enforcement strategies of an antitrust authority, which has the aim to prevent cartel formation in the industry. Here, we study the optimal enforcement policy in the game described in Section 6.4. The objective of antitrust authority is to maximize the discounted consumer surplus and the amount of collected fines minus the costs of control. The costs of control and amount of fines are completely determined by parameter p. Hence, the enforcement strategies are determined mainly through the rate of law enforcement, p. Further, we assume that the fine is fixed and equals its legal upper bound. However, in our setting there are two additional instruments that the antitrust authority can use to achieve the no-collusion outcome. One of them is leniency, i.e. the possibility of fine reduction if firms self-report; and the second is the strictness of leniency programs, or the possibility of getting complete exemption from the fine even in case simultaneous self-report occurs. As the amount of collected fines also depend on the strictness of leniency programs, the antitrust authority maximizes the following objective function: $W(p,\alpha) = \max_{p,\alpha}\{\frac{CS}{1-\delta} - C(p) + p\sum f_i(\alpha)\}$.[18] Here the aim of the authority is to maximize discounted stream of consumer benefits. The authority also wants to minimize the costs of control that are reflected in the term $C(p)$, which serves as a generalized notation for accumulated costs of audit, where discounting is already taken into account. Finally, we assume that the regulator's aim is to maximize the amount of fines. This is reflected in the term $p\sum f_i(\alpha)$, which serves as a generalized notation for expected accumulated collected fines.

The specific characteristic of our model is the fact that we consider asymmetric firms, in the sense that they are diversified to different extends. We point out the following regularities for the threshold probabilities which have been derived above. Assume $h_1 > h_2$, i.e. firm 1 is more diversified, then for the threshold probability determined in the "revelation subgame" the following inequality holds: $p^*(F,h_2,\alpha) < p^*(F,h_1,\alpha)$. However, for the threshold probability determined in the "cartel formation subgame" the opposite holds: $p^{**}(F,h_1) < p^{**}(F,h_2)$. Hence, it is more difficult to enforce self-reporting by bigger (more diversified) firms, but at the same time a smaller rate of law

[18] We will also consider another form of objective function where the regulator is benevolent and does not have as a direct aim maximization of collected fines. In this case the objective function of the authority is as follows: $W(p,\alpha) = \max_{p,\alpha}\{\frac{CS}{1-\delta} - C(p)\} = \frac{CS}{1-\delta} + \min_p\{C(p)\}$. This set-up gives similar but less general results.

enforcement (less policing) is necessary in order to prevent the bigger firm from entering the cartel agreement in the first place.

First, we specify the enforcement technology and calculate welfare gains from implementing outcomes that are most desirable for society. These outcomes maximize the sum of consumer surplus and collected fines less the costs of control. We assume that imposing the monetary fines and determining the strictness of leniency programs is not costly, while increasing the probability of discovery involves costs. In general we expect a trade-off not only between the rate of law enforcement (policing) and the amount of imposed fines (fining), but also between the rate of law enforcement (policing) and the rules of leniency programs: increasing the strictness of leniency rules would imply a reduction in the level of policing required to reach a desired level of cartel formation and discovery. However, we will see that this intuitive trade-off does not always work in this direction.

In the further analysis deadweight losses will approximate losses of consumer surplus due to the fact that the market outcome does not coincide with competitive one. The traditional deadweight loss (DWL) measures the welfare gains associated with a successful intervention that induces a more competitive market equilibrium. We evaluate the welfare gains of antitrust enforcement by comparing the equilibrium outcomes where both firms "do Not Enter", "Enter and Self-report", and "Enter and Not Self-report" to the situation with collusion. Note that the antitrust authority will rank the regions as follows: (NE) gives higher welfare gains than (E S); and (E S) gives higher welfare gains than (E NS). Cartels entail an allocative efficiency loss, and, therefore, the antitrust authority aims to deter or break them if they are already formed. In the first case, (NE), cartels are deterred; in the second case, (E S), cartels are broken in the second stage if they happen to be formed in the first stage; in the third case, (E NS), only those cartels, which are investigated, will be broken.

6.5.1 Optimal Enforcement in the Two Stage Game

In this subsection we identify the optimal policies of the antitrust authority. Recall that the antitrust authority changes its policy throughout the planning horizon in the sense that leniency is introduced later in time than the penalty scheme. We first characterize the optimal policy when the antitrust authority wants to implement each of the three outcomes (NE), (E S), or (E NS). Then we compare the implementable outcomes and select the best one.

As a general point in all the equilibrium outcomes, it is always optimal to set the fine equal to its legal upper bound since increasing the fines is not costly and allows to obtain more favorable (lower) boundaries for the threshold probabilities for the rate of law enforcement.

In the model described above savings of dead weight loss $\frac{SDWL}{1-\delta}$ are the welfare gains from the "Not Enter" equilibrium. The welfare gains, in case of the "Enter and Self-report" equilibrium are $\frac{SDWL}{1-\delta} - SDWL = \frac{\delta SDWL}{(1-\delta)}$[19]. In the "Enter and Not Self-report" equilibrium the antitrust authority interrupts collusion only with probability p, hence the welfare gains are $\frac{p\delta SDWL}{(1-\delta)}$. Note that the following inequality holds: $\frac{SDWL}{1-\delta} > \frac{\delta SDWL}{(1-\delta)} > \frac{p\delta SDWL}{(1-\delta)}$. Hence, the most favorable for society outcome is no cartel formation, second best is when firms collude and then reveal the cartel after leniency programs are introduced. The worst for society outcome is "Collude and Not Reveal". Of course, this information is not enough for determination of the equilibrium that maximizes welfare, since costs of enforcement and revenues from collecting fines are not taken into account yet.

Figure 6.3 illustrates the optimal policies to implement each of the three outcomes discussed above, if $h_1 > h_2$. The solid lines p_1^* , p_1^{**}, and α_1^* represent the incentive compatibility constraints for the more diversified firm, while the dashed lines p_2^* , p_2^{**}, and α_2^* represent the incentive compatibility constraints for the less diversified firm. Moreover, in Proposition 6.4 we state the optimal policies that implement the "Not Enter", "Enter and Self-report", and "Enter and Not Self-report" outcomes.

Proposition 6.4 *Let $h_1 > h_2$. Given the objective function of antitrust authority: $W(p,\alpha) = \max_{p,\alpha}\{\frac{SDWL}{1-\delta} - C(p) + p\sum f_i(\alpha)\}$, in the repeated game played by the firms from $t = 1$ on, the optimal policies are:*

-Implementing (NE) outcome[20] *sets $p = p_1^{**}$ and $\alpha \in [0, \alpha^*(F, h_1)]$.*

-Implementing (E S) outcome[21] *picks up the point that satisfies the equation $(p_2^*(\alpha))'_\alpha = -F$, if at this solution $p < p_1^{**}$. Otherwise, if $p > p_1^{**}$, then the optimal policy to implement (E S) sets $p = p_2^*(1)$ and $\alpha = 1$ or $p = p_1^{**}$ and $\alpha = \alpha^*$.*

-Implementing ((E NS), (E NS))[22] *sets $p = 0$ and $\alpha \in [0, 1]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} + 2F < 0$, or $p = p_1^{**}$ and $\alpha \in [0, \alpha^*]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} + 2F \geq 0$.*

[19] Here we substract the DWL in the first stage of the game, when the cartel is formed, from the total savings of DWL from period $t = 1$ on, given by $\frac{SDWL}{1-\delta}$.

[20] The (NE) outcome will arise already when it is profitable for at least one firm not to enter the cartel agreement.

[21] The (E S) outcome will arise already when it is attractive to self-report for at least one of the firms.

[22] The (E NS) outcome can only arise when entering the cartel agreement and keeping it secret is attractive for both firms.

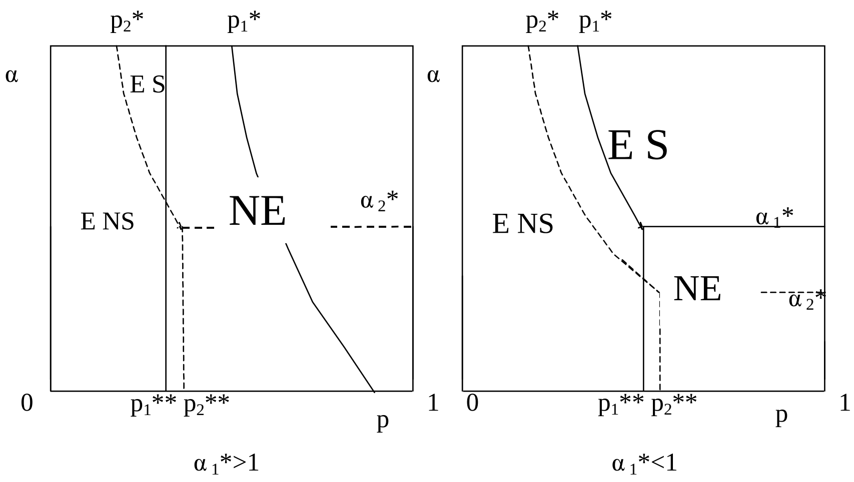


Figure 6.3: The optimal policies that implement the (NE), (E S), and (E NS) outcomes. Solid lines represent incentive compatibility constraints for the more diversified firm and dashed lines represent incentive compatibility constraints for the less diversified firm.

Proof.

To simplify the calculations, throughout the proof of this proposition we assume that $C(p)$ is a linear increasing function of the following form $C(p) = \frac{p}{1-\delta}$. Clearly, this implies per period costs of control will be equal to p. However, qualitative results would not change, if we assume any increasing and convex functional form.

1. The proof of the first part of the proposition follows directly from Figure 6.3. The social welfare in case cartel formation does not occur is given by $W_{(NE)}(p,\alpha) = \frac{SDWL}{(1-\delta)} - p + 0$. It does not depend on α. Hence, the optimal policy to implement (NE) would just minimize p and, hence, sets $p = p_1^{**}$ and $\alpha \in [0, \alpha_1^*]$.

2. The proof of the second part of this proposition is based on the idea that the solution of the maximization problem:

$\{\max_{p,\alpha}\{\frac{\delta SDWL}{1-\delta} - p + (1-\alpha)F\}$ s.t. $V_{ES} > V_{NE}$ and $V_{ES} > V_{ENS}\}$ is given by the tangency point of the iso-welfare curve in case the (E S) outcome is implemented with the lowest incentive compatibility constraint for self-reporting to be profitable, i.e. $p_2^*(\alpha)$. See Figure 6.4.

In this situation two cases can arise:

Firstly, if at the tangency point $p < p_1^{**}$, the welfare, in case the (E S) outcome is implemented, is given by $W_{(ES)}(p,\alpha) = \frac{\delta SDWL}{(1-\delta)} - p + (1-\alpha)F$. Hence, the slope of the iso-welfare curve will be equal to $-\frac{\partial W_{(ES)}(p,\alpha)/\partial \alpha}{\partial W_{(ES)}(p,\alpha)/\partial p} = -F$, implying that the tangency point is determined by the solution of the following equation: $(p_2^*(\alpha))'_\alpha = -F$. See point A in Figure 6.4, where the dashed negatively sloped straight lines represent iso-welfare curves.

Secondly, if at the tangency point $p > p_1^{**}$, we consider two corner solutions.

The first is given by $p = p_2^*(1)$ and $\alpha = 1$. This is illustrated by point B in Figure 6.4. The welfare in this case is given by $W_{(ES)}(p_2^*(1), 1) = \frac{\delta SDWL}{(1-\delta)} - p_2^*(1) + 0$

The second is given by $p = p_1^{**}$ and $\alpha = \alpha^*$, where $\alpha^* : p_2^* = p_1^{**}$. This is illustrated by point C in Figure 6.4. The welfare in this case is given by $W_{(ES)}(p_1^{**}, \alpha^*) = \frac{\delta SDWL}{(1-\delta)} - p_1^{**} + (1 - \alpha^*)F$.

3. The third part of the proposition follows directly from Figure 6.3 and the objective function of the antitrust authority: $W_{(E,N)}(p, \alpha) = \max_{p,\alpha}\{\frac{p\delta SDWL}{(1-\delta)} - \frac{p}{1-\delta} + p2F\}$.

Then there are only corner solutions given by $p = 0$ and $\alpha \in [0, 1]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} + 2F < 0$; and by $p = p_1^{**}$ and $\alpha \in [0, \alpha^*]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} + 2F \geq 0$. So, if gains to the society from conviction are low, it is reasonable not to control at all. And vise versa, if gains due to the savings of DWL and fines that can be collected are high, it is desirable for the antitrust authority to impose a strictly positive rate of law enforcement, $p = p_1^{**}$.
■

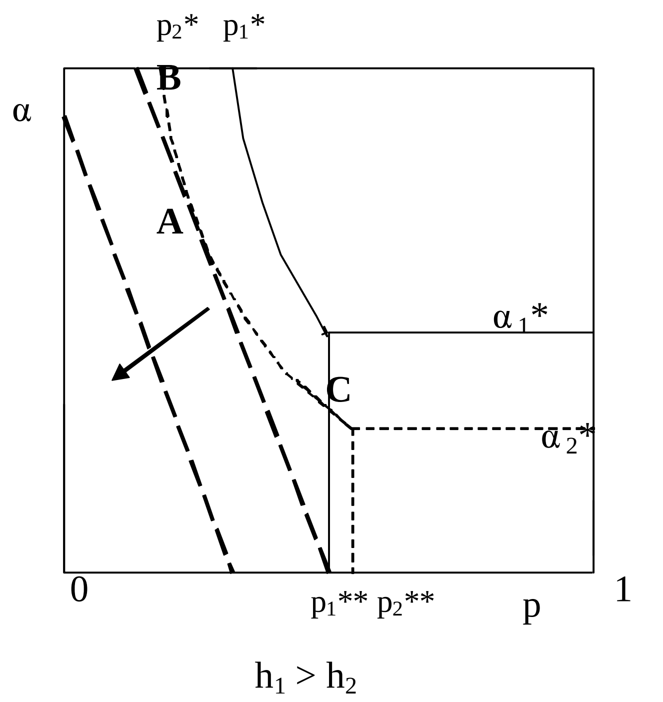


Figure 6.4: Implementation of (E S) outcome.

We can conclude that the first best outcome, i.e. when the cartel is not formed, can be achieved only with a sufficiently high rate of law enforcement, i.e. $p \geq p_1^{**}$, and when the rules of leniency programs are strict enough, $\alpha < \alpha^*(F, h_1)$, in other words, in case of simultaneous self-reporting both firms almost certainly get no exemption from the fine. However, if the cartel has already been formed in the first stage, before the leniency program was introduced, the optimal policy that can ensure the second best outcome, i.e. self-reporting in the second stage, should impose a lower rate of law enforcement, $p_2^*(1)$, and less strict rules of leniency programs, $\alpha = 1$. Hence, in general the enforcement that aims at stopping formation of already existing cartels should be less strict.

In the next proposition we state a similar result for the less general case, where the objective function of the antitrust authority is given by $W(p, \alpha) = \max_{p,\alpha}\{\frac{SDWL}{1-\delta} - C(p(\alpha))\} = \frac{SDWL}{1-\delta} + \min_{p,\alpha}\{C(p(\alpha))\}$. However, it must be noted that the former case is more relevant for the current objectives of antitrust enforcement, since in most cases the authority takes into account the objective of maximizing the amount of fines collected. While, ideal benevolent antitrust authority takes into account only the objective of minimizing dead weight loss and reducing the costs of law enforcement. In this case the following proposition holds.

Proposition 6.5 *Let $h_1 > h_2$ and the objective function of the antitrust authority is given by $W(p, \alpha) = \max_{p,\alpha}\{\frac{SDWL}{1-\delta} - C(p(\alpha))\}$.*

In the repeated game played by the firms the optimal policies of antitrust authority that implement the (NE), (E S), and (E NS) outcomes are:

-The optimal policy to implement (NE) sets $p = p_1^{**}$ and $\alpha \in [0, \alpha^*(F, h_1))$.

-The optimal policy to implement (E S) sets $p = p_2^*(1)$ and $\alpha = 1$.

-The optimal policy to implement (E NS) sets $p = 0$, $\alpha \in [0, 1]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} < 0$, or $p = p_1^{**}$, $\alpha \in [0, \alpha^*]$ if $\frac{\delta SDWL}{(1-\delta)} - \frac{1}{1-\delta} \geq 0$.

Proof: See Appendix 2.

We conclude that under a different objective function still the result is qualitatively the same. An interesting implication of this analysis is that the regulation by a benevolent authority would not only lead to lower fines for firms and less strict leniency programs, but will also reduce the costs of law enforcement in some scenarios.

6.6 Conclusions

This chapter studies the effects of antitrust enforcement and leniency programs on the behavior of firms participating in cartel agreements. The main innovation of our analysis, compared to the earlier papers on leniency by Motta and Polo (1999, 2003), Spagnolo (2000, 2004), or Feess and Walzl (2003), is that we consider asymmetries between firms. In general, firms have different size and operate in several different markets. However, they form a cartel in one market only. This gives rise to additional costs in case of disclosure of cartel caused by a reduction of sales in other markets due to a negative reputation effect. This effect is asymmetric for firms that are diversified to different extends, that is the smaller the percentage of turnover in markets covered by the cartel in relation to total turnover of a firm. The same modelling framework can be applied to the case of international cartels, where firms that form a cartel come from different countries and, consequently, will be subject to different punishment procedures. The most striking example of this asymmetry concerns international cartels of European and US firms. In this situation, due to the fact that in US consumers engage in private law suits more often than in Europe, the actual penalty for the US firm would be greater than for the European firm in case the cartel is discovered and information about its existence becomes public. Hence, following the terminology introduced in this chapter, US firms will correspond to more diversified firms, or the firms that suffer higher costs other than fines in case of disclosure of cartel.

In the chapter we study the situation where the antitrust authority changes its policy throughout the planning horizon in the sense that leniency is introduced later in time and not simultaneously with the penalty scheme[23]. This reflects the situation, for example, in The Netherlands before and after the year 2001, when the leniency programs were introduced in Dutch Competition Law. This model can also be used for analyzing the economic implications of the introduction of leniency programs in countries, where these programs have not yet been introduced, such as developing or countries in transition.

Another feature of our approach is that the enforcement strategies of antitrust authority are determined not only through the rate of law enforcement, but also through an additional instrument (called "strictness" of leniency programs), i.e. introducing the possibility of getting complete exemption from the fine even in case many firms self-

[23]The same framework can be applied to study the effects of leniency programs on the behavior of firms participating in cartel agreements in case, where the structure of the penalty schemes and leniency programs are introduced at the beginning of the game. For more details see subsection on timing of the game.

report simultaneously. We study the impact of the "strictness" of leniency programs on the effectiveness of antitrust enforcement and derive the optimal enforcement strategies.

First, we describe the general results, which come from the analysis of the behavior of asymmetric firms. We found that the bigger the size of the firm (or the more the firm is diversified), the higher the incentives for this firm to keep the cartel secret. Then greater efforts from the antitrust authority, in terms of increasing the rate of law enforcement, are needed in order to induce self-reporting by this firm. So, leniency programs work better for small (less diversified) companies in the sense that they result in self-reporting by small firms while the rate of law enforcement is lower, which implies lower costs for society.

Furthermore, we can conclude that for bigger firms the strategy of not entering the cartel agreement in the first stage of the game is more likely to be preferred over the strategy to form a cartel and then self-report, than for smaller (less diversified) firms. The bigger the size of the firm (or the higher its losses due to the reputation effect), the easier it is for the antitrust authority to prevent the firm from entering the cartel agreement in the first stage of the game. Hence, big firms (or firms for which costs other than fines are higher) are more reluctant to start a cartel in the first place.

Next, we proceed by describing the optimal combination of instruments of antitrust authority: rate of law enforcement and "strictness" of leniency programs. Uncertainty of the firms about getting the first prize (or, in other words, strictness of the rules for leniency, which can grant the complete exemption from the fine only to one firm) reduces the incentives for both types of firms to self-report. Therefore, in a highly cartelized economy, where a lot of cartels are already formed, the best strategy for the antitrust authority is to concentrate on policies that increase the incentives to self-report, in particular, increase the fine or reduce the strictness of leniency programs. In other words, the more cartelized the economy, the less strict the rules of leniency programs should be, or in other words, complete exemption from fine should be granted to all self-reporters.

On the other hand, when there are not too many cartels and leniency is not yet introduced, the antitrust authority should implement the policy that reduces the incentives to enter the cartel agreements in the first place. In this case both, the fine and the strictness of the leniency programs, should be increased. Hence, when the economy is not highly cartalized the rules of leniency programs should be more strict, i.e. complete leniency should be granted only to the first self-reporting firm.

Finally, we conclude that the optimal enforcement can implement the no collusion outcome only when the rate of law enforcement is sufficiently high and the rules of leniency programs are sufficiently strict. Moreover, the second best outcome, i.e. "Enter cartel and Self-report", can be implemented when the rate of law enforcement is sufficiently high and the leniency programs grant complete exemption from fines to all the firms that cooperate with antitrust authority.

To conclude the discussion, it is worthwhile to mention that the framework developed in this chapter can also be used for an analysis of the effectiveness of leniency programs in situations where disclosure of cartel can lead to additional costs for the firms different from the fine for violations of competition law itself. Those costs can result from the threat that tax authorities will conduct additional control and possibly frauds connected with cartel agreements will be discovered, or consumers will challenge firms in the courts applying for private law damages. Obviously, the threat of all these additional losses would reduce incentives for the firms to self-report and diminish the effectiveness of leniency programs for already existing cartels. However, the positive feature is that at the same time this would also reduce the incentives for the firms to enter new cartel agreements.

6.7 Appendix

6.7.1 Appendix 1: Proof of Proposition 6.3

Proof. The result of this proposition follows directly from Lemmas 6.1 and 6.2 and the fact that all three locuses $p^*(F, h_i, \alpha), p^{**}(F, h_i)$, and $a^*(F, h_i)$ intersect in the same point. Simple algebraic calculations confirm that $p^*(F, h_i, \alpha^*) = p^{**}(F, h_i)$.

In order to prove this fact we substitute $\alpha^*(h_i, F) = \frac{2Rh_i\delta + F\delta - 2(\pi_m - \pi_n)}{\delta F}$ into the expression for p^* in (6.2) and show that $p^*(F, h_i, \alpha^*) = p^{**}(F, h_i)$.

Recall that $p^*(F, h_i, \alpha) = \frac{\pi_m - \pi_n + Rh_i(1-\delta) + \frac{(1-\delta)(1-\alpha)}{2}F}{\pi_m - \pi_n + Rh_i(1-\delta) + (1 - \frac{\delta(1-\alpha)}{2})F}$.

Hence, $p^*(F, h_i, \alpha^*) = \frac{\pi_m - \pi_n + Rh_i(1-\delta) + \frac{(1-\delta)(1 - \frac{2Rh_i\delta + F\delta - 2(\pi_m - \pi_n)}{\delta F})}{2}F}{\pi_m - \pi_n + Rh_i(1-\delta) + (1 - \frac{\delta(1 - \frac{2Rh_i\delta + F\delta - 2(\pi_m - \pi_n)}{\delta F})}{2})F} = \frac{\pi_m - \pi_n}{\delta(F + Rh_i)} = p^{**}(F, h_i)$.

■

6.7.2 Appendix 2: Proof of Proposition 6.5

Again, in order to simplify the calculations, throughout the proof of this proposition we assume that $C(p)$ is a linear increasing function of the following form $C(p) = \frac{p}{1-\delta}$.

Clearly, this implies per period costs of control will be equal to p. However, qualitative results would not change, if we assume any increasing and convex functional form.

Proof.

1. The proof of the first part of the proposition follows straightforwardly from Figure 6.4.

2. The second part of the proposition says that a combination of policy instruments of the form $p = p_2^*(1)$ and $\alpha = 1$ would minimize the costs of law enforcement in case the (E S) outcome has to be implemented and, hence, it would maximize the social welfare $W(p, \alpha) = \frac{\delta DWL}{1-\delta} - p(\alpha)$.

Indeed, recall expression (6.3), which says that $\frac{\partial p^*(F,h_i,\alpha)}{\partial \alpha} < 0$. This implies that $\min_\alpha p^*(\alpha) = p^*(1)$. Now looking at Figure 6.4, we conclude that the optimal policy to implement (E S) sets $\alpha = 1$ and $p = \min_\alpha p^*(\alpha) = p_2^*(1)$.

3. The third part of the proposition follows directly from Figure 6.4 and from the objective function of the antitrust authority $W(p, \alpha) = \max_p p(\frac{\delta DWL-1}{1-\delta})$. This implies that $p = 0$, $\alpha \in [0, 1]$ if $\frac{\delta DWL}{1-\delta} - \frac{1}{1-\delta} < 0$, and $p = p_1^{**}$, $\alpha \in [0, \alpha^*]$ if $\frac{\delta DWL}{(1-\delta)} - \frac{1}{1-\delta} \geq 0$.
■

CHAPTER 7

Cost Minimizing Sequential Punishment Policies for Repeat Offenders

7.1 Introduction

In this chapter we concentrate on policies for crime control that are not only aimed at reducing the number of violations but are also cost minimizing from the point of view of the regulator. Unfortunately, these two objectives conflict with each other. Reduction of expenditures on crime control will lead to a lower deterrence rate and vice versa. However, both objectives seem to be very important for society. Society is better off when both the number of violations and the costs of crime control are reduced.

Another important question addressed in this chapter is whether the optimal sanction scheme should be decreasing or increasing in the number of offenses. For the law and economics literature on optimal law enforcement, escalating sanction schemes, embedded in most sentencing guidelines, are still a puzzle. Garoupa (1997) or Polinsky and Shavell (2000) give excellent surveys of this literature.

The purpose of this study is to find the optimal penalty scheme which takes into account the two objectives, mentioned above. We study the problem of optimal sanctions for repeat offenders in a multi-periods model employing the two-periods framework suggested in Emons (2003). We assume that agents may commit a crime several times. The criminal act is inefficient, it causes harm for society; the agents are thus to be deterred.

An important assumption of the model is that the agents are wealth constrained so that increasing the fine for the first offence means a reduction in the possible sanction for the subsequent offences and vice versa. A simplification compared to Emons (2003) is that in the forward looking solution we consider only history independent strategies of the agents. The government seeks to minimize the probability of apprehension and the number of crimes, since it is costly for society.

The main result is that the optimal penalty scheme is decreasing in the number of offenses. We find that it is optimal to set the sanction for the first detected offense equal to the entire wealth of the agent while the sanctions for all the subsequent offenses equal zero.

In this chapter we discuss a general set up with representative offender and regulator whose aim is to block violations of law. However, it is clear that a similar framework can be applied in case of an antitrust authority dealing with a group of firms that form an illegal cartel. Antitrust law violations often are committed repeatedly by the same firm. Remarkably, sentencing guidelines in both US and Europe attach a higher gravity factor to recidivistic violations and, hence, prescribe to punish repeated offenders more heavily. Clearly, this does not go in line with the main results of the Emons (2003) work and our analysis. This puzzle still requires deep investigation in the law and economics literature. From the other point of view, our model, where offenders are wealth constrained, captures another important feature of current penalty schemes, namely, the existence of upper bound for the fine. Usually, this upper bound is given by either 10% of overall turnover of enterprise or by fixed monetary amount. The motivation for existence of this rule can be connected to the fact that antitrust authorities should not force firms to go bankrupt, in other words, the fact that firms are wealth constrained is taken into account.

We start the discussion with a review of the related literature. Rubinstein (1980) considers a setup where an agent can commit two crimes. A high penalty for the second crime is exogenously given. Rubinstein shows that for any set of parameters there exists a utility function such that deterrence is higher if the sanction for the first crime is lower than the sanction for the second crime. Landsberger and Meilijson (1982) develop a dynamic model with repeated offenses. They studied how prior offenses should affect the probability of detection rather than the level of punishments. In Polinsky and Rubinfeld (1991) it was found that it may be optimal (for some parameter values) to punish repeat offenders more severely, when the government cannot observe illicit gains from criminal activities.

In Burnovski and Safra (1994) agents decide on the optimal number of crimes. They show that reducing the sanctions on subsequent crimes while increasing the penalty on previous crimes will reduce the overall criminal activity, if the probability of detection is sufficiently small. Our analysis is very similar to their paper. They also consider an n-periods framework. The main difference is that they search for the most deterring sequential policy for repeat offenders without taking into account that the regulator also has an objective to minimize the enforcement costs.

In Polinsky and Shavell (1998) agents live for two periods and can commit a crime twice. Their result is that young first-time offenders and old-second time offenders are penalized with the maximum sanction. Dana (2001) argues that, contrary to what is frequently assumed in the literature, probabilities of detection increase for repeat offenders. As a result, the optimal deterrence model a la Becker dictates declining, rather than escalating, penalties for repeat offenders.

Finally, the paper by Emons (2003) studies a two-period model, where agents may commit a crime two times. One important assumption of his model is that the agents are wealth constrained so that increasing the fine for the first offence means a reduction in the possible sanction for subsequent offences and vice versa. He also assumes that, besides crime deterrence, the main objective of the regulator is minimization of enforcement costs. The paper concludes that the optimal penalty structure should be declining in the number of offenses.

To summarize the results of earlier papers we conclude that the main argument in favor of decreasing penalty schemes is that probabilities of detection, usually, increase for repeat offenders and then Becker's model implies declining sanctions. The main policy implication from this analysis would be that sanctions should be declining when the regulator is resource constrained and offenders are wealth constrained. On the other hand, it is intuitively clear that recidivistic behavior should be punished more severely than first time offences or crimes committed by accident, since it usually signals a more grave criminal intent. With respect to policy implications, this scheme should be applied when the government cannot observe illicit gains from criminal activities[1].

In this chapter we first analyze an n-period repeated game, where the regulator's main objective is to block any violations of law and, at the same time, to minimize the costs of crime control. We describe a forward looking solution, i.e. the regulator can commit to a certain policy from the beginning of the game and does not change the parameters of the penalty scheme (fine and probability of control) till the end of

[1]See Polinsky and Rubinfeld (1991).

the planning horizon. The solution of this problem gives the desired result of complete deterrence. Even the first crime never happens, unless benefits from crime are much higher than the initial wealth of the offender. In the model of section 7.2 we rule out this possibility. The main intuition that drives this result comes from the fact that the agent pays the sanction for the first offence with probability p (rate of law enforcement). While any further sanction will be paid with lower probability, since the second offence can be detected only conditional on the fact that the first offence has been discovered. Hence, since paying the first fine is more likely than paying any subsequent fine, shifting resources from the last periods to the first increases deterrence for given rate of law enforcement. Consequently, as in Emons (2003), p is minimized by putting all scarce resources into the penalty for the first detected offence.

However, the outcome will be different in case the regulator follows a time consistent (subgame perfect) strategy. This implies that the government is able to change its policy every period, conditioning its choice on the outcome of the preceding periods. In this case the regulator chooses the optimal subgame perfect action in the beginning of each period. In section 7.3 we show that the scheme derived as a forward looking solution in case of full commitment is not a time consistent (subgame perfect) strategy for the regulator in a multi-period setting. Section 7.4 concludes.

7.2 Multi-period Model, Forward Looking Solution (Full Commitment Case)

We consider a multi-period optimization problem of a cost minimizing regulator (antitrust authority or police) whose aim it is to block violations of law (for example, violations of antitrust law, violations of criminal law, violations of pollution standards).

We consider a continuum of potential offenders which has measure 1. Individuals or firms live for n periods. In each period the agents can engage in an illegal activity, such as polluting the environment, evading taxes, or violating competition law. If an agent commits the act in either period he receives a monetary benefit $b > 0$. Following Polinsky and Rubinfeld (1991) b is the illicit gain and the crime creates no acceptable gain. The act causes a monetary harm $h > 0$ to society and, thus, has to be deterred. We assume that the following inequality is satisfied, $h > b$. So, the act is not socially desirable.

To achieve deterrence the government chooses sanctions and a probability of apprehension. The regulator cannot tell in which period of its life the individual is. It can only observe the information after the crime has been discovered. Hence, the regulator only observes whether the crime is the first or second or n^{th} one. Accordingly, the government applies fines $s_1, s_2, ..., s_n \geq 0$, where s_i is the penalty in case the offense by this particular agent is recorded by the authority already i times. Moreover, the government chooses a rate of law enforcement, p, which can also be seen as the probability of conviction. We assume that p is the same for all (first time and repeated) offenses. Since apprehension is costly, the government wishes to minimize p and reduce the number of crimes. The overall objective of the regulator is to minimize the number of crimes. Subject to that objective being reached, the regulator aims to minimize costs of control, p. So, the objective function of the regulator can be written as $max - (p + Hk)$, where p is the probability of control (or rate of law enforcement), k is the number of crimes, and H is the disutility from crime for the regulator, which is assumed to be a large positive number.

The agents are risk neutral and maximize expected income. They have initial wealth W and hold it over all n periods unless the government interferes with sanctions. Benefits from crime b are consumed immediately, and the maximum of what the government can extract from the agents is W.[2] Moreover, based on Becker's (1968) maximum fine result, we assume that in order to minimize p the government will use the agent's entire wealth for sanctions. This implies that the fines $s_1, s_2, ..., s_n$ have to satisfy the "budget constraint" $\sum\limits_{i=1}^{n} s_i = W$. To simplify the analysis we also assume no discounting.

An agent chooses the number of crime that can be committed or, in other words, he (she) can choose between following strategies:

Not to commit a criminal act (i.e. not to participate in a cartel) at all. Then the utility from this strategy for the "offender" has the following form $U(0, 0, ..., 0) = W$.

Commit crime (collude) only once in any of the periods.

The utility from this strategy for the offender equals $U(1, 0, ..., 0) = W + b - ps_1$.

Commit crime in any two periods: $U(1, 1, 0, ..., 0) = W + b - ps_1 + b - p(1-p)s_1 - p^2 s_2$.

Commit crime in any three periods: $U(1, 1, 1, ..., 0) = U(0, 1, 1, 1, ..., 0) = U(0, 0, ..., 0, 1, 1, 1) = W + b - ps_1 + b - p(1 - p)s_1 - p^2 s_2 + b - (1 - p)^2 ps_1 - 2p^2(1 - p)s_2 - p^3 s_3$.

....

Commit crime in all n periods:

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[2]This assumption seems to be not quite realistic. In most of the cases, for example in case of tax evasion or illegal price-fixing activities, the penalty takes in to account not only initial wealth of the firm but also accumulated rents from illegal activities. However, this assumption is adopted here to focus on obtaining analytical results with respect to establishing an optimal sequence of sanctions.

$U(1,1,1,...,1) = W + b - ps_1 + b - p(1-p)s_1 - p^2 s_2 + b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 + ... + b - (C_{n-1}^0 (1-p)^{n-1} ps_1 + C_{n-1}^1 (1-p)^{n-2} p^2 s_2 + C_{n-1}^2 (1-p)^{n-3} p^3 s_3 + ... + C_{n-1}^{n-2}(1-p)p^{n-1}s_{n-1} + C_{n-1}^{n-1} p^n s_n)$,

where coefficients of these polynomials are formed according to the following formula:

$$C_h^k = \frac{h!}{k!(h-k)!}, \quad h \geq k.$$

To clarify the notation:

$b - ps_1$ is the expected benefit from the first detected crime

$b - p(1-p)s_1 - p^2 s_2$ is the expected benefit from the second detected crime

$b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3$ is the expected benefit from the third detected crime

$b - (C_{n-1}^0 (1-p)^{n-1} ps_1 + C_{n-1}^1 (1-p)^{n-2} p^2 s_2 + C_{n-1}^2 (1-p)^{n-3} p^3 s_3 + ... + C_{n-1}^{n-2}(1-p)p^{n-1}s_{n-1} + C_{n-1}^{n-1} p^n s_n)$ is the expected benefit from n^{th} detected crime.

We impose the following assumptions on the parameters $0 < p < 1, b > 0, W > 0$. The possibility $p = 0$ does not make sense, since then there is no threat for the agent to be convicted and no way to prove the criminal to be guilty.

We also assume here that agents have enough wealth so that deterrence is always possible, i.e., $nb < \sum_{i=1}^{n} s_i \leq W$. Further, we derive sanctions that give the agents the proper incentives not to engage in criminal activities in either period. This means, we derive a penalty scheme which ensures $U(1,0,...,0) < U(0,0,...,0)$, $U(1,1,...,0) < U(0,0,...,0)$,..., $U(1,1,...,1) < U(0,0,...,0)$. These are included as constraints in the optimization model. The main objective of the regulator is crime prevention and minimization of costs of law enforcement, i.e. minimization of p. This leads to the following model.

The aim of the regulator to prevent crime and to minimize the enforcement costs is reflected in the objective function (7.1) below, while the aim to provide incentives for the agents not to commit any crime is reflected in incentive constraints (7.2)-(n+1).

$$min \;\; p + Hk \tag{7.1}$$

s.t.

$$b - ps_1 \leqq 0 \tag{7.2}$$

$$2b - ps_1 - p(1-p)s_1 - p^2 s_2 \leqq 0 \tag{7.3}$$

$$3b - ps_1 - p(1-p)s_1 - p^2 s_2 - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 \leqq 0 \tag{7.4}$$

....

$$lb - \sum_{h=1}^{l}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k \leqq 0 \qquad \text{(l+1)}$$

....

$$nb - \sum_{h=1}^{n}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k \leqq 0 \qquad \text{(n+1)}$$

$$s_1 + s_2 + ... + s_{n-1} \leqq W \qquad \text{(n+2)}$$

$$s_1 \geq 0, s_2 \geq 0, ..., s_{n-1} \geq 0, p > 0. \qquad \text{(n+3)}$$

The Lagrangian for this problem has the following form:

$$L = -p - \sum_{j=1}^{n} \lambda_j [jb - \sum_{h=1}^{j}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k] - \lambda^* (\ s_1 + s_2 + ... + s_{n-1} - w) \qquad (7.5)$$

Using Kuhn-Tucker conditions to solve the minimization problem (7.1)-(n+3), we obtain the result stated in Proposition 7.1.

Proposition 7.1 *The optimal cost minimizing sanction scheme sets the penalty for the first detected violation equal to the entire wealth of the agent and for all subsequent violations the penalties will be equal to zero, i.e.* $s_1^* = W$ *and* $s_2^* = ... = s_n^* = 0$. *The probability of law enforcement is constant over time and equals* p^*, *which represents the smallest positive solution of the polynomial of order* n *in* p, *given by expression (7.19).*

The proof of Proposition 7.1 consists of several steps: first, we derive FOCs and complementary slackness conditions of the minimization problem described above; second, based on the FOCs we prove Lemma 7.2, which states that inequality $\frac{\partial L}{\partial s_l} > \frac{\partial L}{\partial s_{l+1}}$ holds for any time period $l \in \{1, ..., n-1\}$; finally, applying Lemma 7.2 and the complementary slackness conditions we obtain the optimal penalty schedule with $s_1^* = W$ and $s_2^* = ... = s_n^* = 0$ and $p > 0$.

Proof. To derive the FOCs we take partial derivatives of expression (7.5) with respect to all $n-1$ variables, which denote the penalties in the corresponding periods. Recall that, taking into account that the budget constraint must be binding, s_n can be expressed through all the unknowns and initial wealth as follows $s_n = W - \sum_{i=1}^{n-1} s_i$. So, differentiating and simplifying the expressions, we obtain $n-1$ FOCs with respect to penalties in corresponding periods (7.6)-(7.10) and one FOC with respect to the probability of law enforcement (7.15). We also write down $n+1$ complementary-slackness conditions in expressions (7.11)-(7.14) below.

$$\frac{\partial L}{\partial s_1} = p(1-p)^0 \sum_{i=k+1}^{n} \lambda_i + \sum_{k=1}^{n-1} [C_k^0 p(1-p)^k (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \le 0 \quad (= 0 \ if \ s_1 > 0) \tag{7.6}$$

$$\frac{\partial L}{\partial s_2} = \sum_{k=1}^{n-1} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \le 0 \quad (= 0 \ if \ s_2 > 0) \tag{7.7}$$

$$\frac{\partial L}{\partial s_3} = \sum_{k=2}^{n-1} [C_k^2 p^3 (1-p)^{k-2} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \le 0 \quad (= 0 \ if \ s_3 > 0) \tag{7.8}$$

....

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \le 0 \quad (= 0 \ if \ s_l > 0) \tag{7.9}$$

....

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1} [C_k^{n-2} p^{n-1} (1-p)^{k-(n-2)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \le 0 \quad (= 0 \ if \ s_{n-1} > 0) \tag{7.10}$$

Complementary slackness conditions are:

$$\lambda_1 \ge 0 \quad (\quad \lambda_1 * (7.2) = 0 \quad) \tag{7.11}$$

$$\lambda_2 \ge 0 \quad (\quad \lambda_2 * (7.3) = 0 \quad) \tag{7.12}$$

....

$$\lambda_n \ge 0 \quad (\quad \lambda_n * (n+1) = 0 \quad) \tag{7.13}$$

$$\lambda^* \ge 0 \quad (\quad \lambda^* * (\sum_{i=1}^{n-1} s_i - W) = 0 \ ) \tag{7.14}$$

$$\frac{\partial L}{\partial p} = 0. \tag{7.15}$$

Next, we prove the following lemma.

Lemma 7.2 *For any* $l \in \{1, ..., n-1\}$, $\frac{\partial L}{\partial s_l} > \frac{\partial L}{\partial s_{l+1}}$.

Proof. The proof of this lemma is based on mathematical induction.

1. First, we show that the result stated in Lemma 7.2 holds in case the number of periods equals to three, i.e. $n = 3$.

We take $n = 3$, which implies that $k \in \{1, 2, 3\}$ and $l \in \{1, 2\}$.

Consequently, for $l = 1$, we obtain from (7.6) and (7.7) that

$$\frac{\partial L}{\partial s_1} - \frac{\partial L}{\partial s_2} = p(1-p)^0 \sum_{i=1}^{3} \lambda_i + \sum_{k=1}^{2} [C_k^0 p(1-p)^k (\sum_{i=k+1}^{n} \lambda_i)] - \sum_{k=1}^{2} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{3} \lambda_i)] =$$

$$= p\lambda_1 + 2p(1-p)\lambda_2 + (p^2 - 2p + 1)\lambda_3 > 0$$

Similarly, for $l = 2$, we obtain from (7.7) and (7.8) that

$$\frac{\partial L}{\partial s_2} - \frac{\partial L}{\partial s_3} = \sum_{k=1}^{2} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{3} \lambda_i)] - \sum_{k=2}^{2} [C_k^2 p^3 (1-p)^{k-2} (\sum_{i=k+1}^{3} \lambda_i)] =$$

$$= C_1^1 p^2 (\sum_{i=2}^{3} \lambda_i) + C_2^1 p^2 (1-p)^1 \lambda_3 - C_2^2 p^3 \lambda_3 = p^2 (\sum_{i=2}^{3} \lambda_i) + 2p^2 \lambda_3 - 3p^3 \lambda_3 = p^2 \lambda_2 + 3p^2 (1-p) \lambda_3 > 0.$$

2. Next, we show that the result stated in Lemma 7.2 holds for any arbitrary number of periods. For that purpose we show that if the result of Lemma 7.2 holds for $n = m$, then it also holds for $n = m + 1$.

Now, assume that

$$\frac{\partial L}{\partial s_l} - \frac{\partial L}{\partial s_{l+1}} = \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m} \lambda_i)] > 0 \tag{7.16}$$

is true for any $1 < l \leq n$ when $n = m$.

Based on this we have to prove that

$$\sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \sum_{k=l}^{n-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{n} \lambda_i)] > 0$$

is true for any $1 < l \leq n$ when $n = m + 1$. Clearly,

$$\sum_{k=l-1}^{m} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m+1} \lambda_i)] - \sum_{k=l}^{m} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m+1} \lambda_i)] =$$

$$= \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m+1} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m+1} \lambda_i)] + C_m^{l-1} p^l (1-p)^{m-(l-1)} \lambda_{m+1}) -$$

$$- C_m^l p^{l+1} (1-p)^{m-l} \lambda_{m+1} =$$

$$= \sum_{k=l-1}^{m-1}[C_k^{l-1}p^l(1-p)^{k-(l-1)}(\sum_{i=k+1}^{m}\lambda_i)]-\sum_{k=l}^{m-1}[C_k^lp^{l+1}(1-p)^{k-l}(\sum_{i=k+1}^{m}\lambda_i)]+\lambda_{m+1}\sum_{k=l-1}^{m-1}[C_k^{l-1}p^l(1-p)^{k-(l-1)}]-$$

$$-\lambda_{m+1}\sum_{k=l}^{m-1}[C_k^lp^{l+1}(1-p)^{k-l}] + C_m^{l-1}p^l(1-p)^{m-(l-1)}\lambda_{m+1} - C_m^lp^{l+1}(1-p)^{m-l}\lambda_{m+1} =$$

$$= \sum_{k=l-1}^{m-1}[C_k^{l-1}p^l(1-p)^{k-(l-1)}(\sum_{i=k+1}^{m}\lambda_i)] - \sum_{k=l}^{m-1}[C_k^lp^{l+1}(1-p)^{k-l}(\sum_{i=k+1}^{m}\lambda_i)]+ \qquad (7.17)$$

$$+\lambda_{m+1}\left\{\sum_{k=l-1}^{m}[C_k^{l-1}p^l(1-p)^{k-(l-1)}] - \sum_{k=l}^{m}[C_k^lp^{l+1}(1-p)^{k-l}]\right\} > 0. \qquad (7.18)$$

Expression (7.17) is positive due to (7.16), while (7.18) will be strictly positive for any $p < \frac{1}{2}$, which corresponds to current rates of law enforcement for major types of economic crimes. ■

Next, using the result of Lemma 7.2, we derive the optimal penalty schedule.

We start by showing that it is impossible that constraint (n+2) is not binding.

In case this constraint is not binding, there are three possibilities:

1. $\sum_{i=1}^{n-1} s_i < W$ and $s_i > 0$ for all $i \in \{1, ..., n-1\}$,
2. $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for all $i \in \{1, ..., n-1\}$,
3. $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for some $i \in \{1, ..., n-1\}$.

The result of Lemma 7.2 immediately implies that the solution with $s_i > 0$ for all $i \in \{1, ..., n-1\}$ is impossible.

Consider $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for all $i \in \{1, ..., n-1\}$. Then the first order conditions (7.6)-(7.10) imply that $\frac{\partial L}{\partial s_1} < 0, \frac{\partial L}{\partial s_2} < 0, ..., \frac{\partial L}{\partial s_{n-1}} < 0$. Moreover, it holds that $\lambda^* = 0$. This implies that (7.9) becomes

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1}[C_k^{l-1}p^l(1-p)^{k-(l-1)}(\sum_{i=k+1}^{n}\lambda_i)] - \lambda_n p^n < 0 \quad \text{for all } l \in \{1, ..., n-1\}.$$

However, take the last period $l = n-1$, then

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1}[C_k^{n-2}p^{n-1}(1-p)^{k-(n-2)}(\sum_{i=k+1}^{n}\lambda_i)]-\lambda_n p^n = p^{n-1}\lambda_{n-1}+np^{n-1}\lambda_n(1-p) > 0.$$

Hence, condition (7.10) cannot be strictly negative. This implies that the outcome with $s_i = 0$ for all $i \in [1, n-1]$ and $\lambda^* = 0$ cannot arise as a solution of the minimization problem of the regulator.

Next, consider $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for some $i \in \{1, ..., n-1\}$. Assume $s_l = 0$ for $l < n-1$. This means that (7.9) must be non-positive, i.e.

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n < 0.$$

But we have just shown that

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1} [C_k^{n-2} p^{n-1} (1-p)^{k-(n-2)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n > 0$$

and, hence, using Lemma 7.2, we can conclude that this outcome also cannot be a solution.

The outcome with $\sum_{i=1}^{n-1} s_i = W$ and $s_i = 0$ for $i < k \in \{1, ..., n-1\}$ and $s_l > 0$ for $l > k \in \{1, ..., n-1\}$ is impossible due to the result of Lemma 7.2.

Moreover, the outcome with $\sum_{i=1}^{n-1} s_i = W$ and $s_1 > 0$, $s_2 > 0$ and $s_i = 0$ for all $i \in \{3, ..., n-1\}$ cannot arise. Consider $s_1 > 0$, $s_2 > 0$. Using (7.6) and (7.7) we obtain that $\frac{\partial L}{\partial s_1} = \frac{\partial L}{\partial s_2} = 0$. But this contradicts the result of Lemma 7.2, which states that $\frac{\partial L}{\partial s_1} > \frac{\partial L}{\partial s_2}$.

We conclude that only the following is possible: $s_1^* > 0$, $s_2^* = ... = s_n^* = 0$ and $\sum_{i=1}^{n-1} s_i = W$, which implies that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$.

Finally, optimal behavior implies that only condition (n+1) on the benefits from crime will be binding, so that $\lambda_1 = \lambda_2 = ... = \lambda_{n-1} = 0$ and $\lambda_n \geq 0$. Hence, the expressions for the optimal probability of law enforcement, p^*, λ^*, and λ_n will be determined from condition (n+1), $\frac{\partial L}{\partial s_1} = 0$, and $\frac{\partial L}{\partial p} = 0$.

In particular, p^* is represented as a solution of the polynomial of order n (7.19) with $s_1 = W$, $s_2 = 0, ..., s_n = 0$.

$$nb - \sum_{h=1}^{n} \sum_{k=1}^{h} C_{h-1}^{k-1} (1-p)^{h-k} p^k s_k = 0 \tag{7.19}$$

Next we present the proof of of the fact that only constraint (n+1) on the benefits from crime will be binding.

Proof. We can show that for the penalty scheme given by $s_1^* = W$, $s_2^* = ... = s_n^* = 0$, only condition $(n+1)$ can be binding and, hence, p^* is found as a solution of the polynomial of order n given in (7.19).

The main intuition for the proof of this result is the observation that only constraint $(n+1)$ can be binding due to the construction of the problem. Assume, for example, that constraint $(l+1)$ is binding for some $l \in \{2, ..., n-1\}$, then it follows that the LHS of the constraint $(l+2)$ has to be strictly positive, which is impossible by construction of the problem.

Now we prove this statement using rigorous mathematical tools. First, we consider the situation where constraint $(l+1)$ is binding. It can be written as follows:

$b - ps_1 + b - p(1-p)s_1 - p^2 s_2 + b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 + ... + b - (C_{l-1}^0 (1-p)^{l-1} ps_1 + C_{l-1}^1 (1-p)^{l-2} p^2 s_2 + ... + C_{l-1}^{l-1} p^n s_l) = 0.$

At the same time constraint $(l+2)$ can be written as follows

$$(l+1) + b - (C_l^0 (1-p)^l ps_1 + C_l^1 (1-p)^l p^2 s_2 + ... + C_l^l p^n s_{l+1}).$$

Now, taking into account that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$ and $(l+1) = 0$, constraint $(l+2)$ can be rewritten as

$$b - C_l^0 (1-p)^l pW = b - (1-p)^l pW. \tag{7.20}$$

Moreover, using the formula for finite geometric series and the fact that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$, constraint $(l+1)$ can be rewritten as follows.

$$(l+1) = bl - pW(1 + (1-p) + (1-p)^2 + ... + (1-p)^{l-1}) = bl - W(1 - (1-p)^l).$$

Recall that constrained $(l+1)$ is binding. This implies that $b = \frac{W(1-(1-p)^l)}{l}$. Now expression for constraint $(l+2)$ in (7.20) becomes

$$\frac{W(1-(1-p)^l)}{l} - (1-p)^l pW = \frac{W}{l}(1 - (1-p)^l - pl(1-p)^l)$$

It is easy to show that the first derivative of this expression with respect to p is strictly positive for any $0 < p < 1$, $l > 0$, and $W > 0$. Hence, this function is strictly increasing in p for any $0 < p < 1$, $l > 0$, and $W > 0$. At the same time function $\frac{W}{l}(1 - (1-p)^l - pl(1-p)^l) = 0$ when $p = 0$. Hence, this expression is strictly positive for any $0 < p < 1$.

This proves the fact that given that constraint $(l+1)$ is binding, it must hold that the LHS of the constraint $(l+2)$ in the problem (7.1)-(n+1) must be strictly positive, but this would contradict the construction of optimization problem. ■

End of the proof of Proposition 7.1. ■

The main intuition behind this proposition is very simple. It follows immediately from any of the incentive constraints (7.3)-(n+1). The agent pays the sanction s_1 with probability p, while any further sanction will be paid with lower probability : s_2 with probability p^2, s_3 with probability p^3, and s_n only with probability p^n. In other words, the agent is charged s_2 with probability p only if he has paid already s_1 . Hence, since paying the first fine is more likely than paying any subsequent fine, shifting resources from the last periods to s_1 increases deterrence for given p. Consequently, as in Emons (2003), p is minimized by putting all scarce resources into s_1.

Example 7.3 *Figure 7.1 illustrates the proof graphically in the* (p, s_1)*-diagram for the two-period case. The game in this case is described as follows. A strategy of player 1 (regulator) is given by* $\sigma = (p, s_1, s_2)$*, while the strategy set of player 2 (offender) is given by* $\{0, 1, 2\}$.

In case $n = 2$*, the optimization problem of the regulator will be as follows:*

$$min \quad p + Hk$$

s.t.

$$b - ps_1 \leqq 0 \qquad (1)$$

$$2b - ps_1 - p(1-p)s_1 - p^2 s_2 \leqq 0 \qquad (2)$$

$$s_1 + s_2 \leqq W \qquad (3)$$

$$0 < p \leqq 1.$$

Suppose $b \geqq 0, \;\; W > 2b, \; s_2 = W - s_1$.

Graphically, the solution of this problem, which has the form $s_1^* = W$*,* $s_2^* = 0$*,* $p = p^*$*, is represented in Figure 7.1, where the parameter values are* $b = 1$*,* $W = 3$.

The solution of the problem is represented by point A in Figure 7.1, with $s_1 = W$*,* $s_1 = 0$*,* $p = p^* > 0$*. In general, for the n-period case this diagram will be an n-dimensional* $(p, s_1, ..., s_{n-1})$ *and the solution of the problem will be represented by the point of the n-dimensional cone which is the closest to the vertical axis and satisfies all the incentive constraints.*

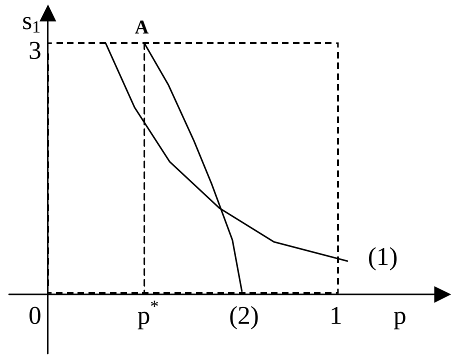

Figure 7.1: Graphical illustration of the solution in two-period case.

7.3 Optimal Sanctions if Government cannot Commit

In this section we investigate under which conditions the sanction scheme described in Proposition 7.1 is sub-game perfect. This means: Does the government really implement these sanctions once the agent has committed a crime? To do so, we study the subgame starting when the agent has been apprehended for the first time.

In the setting, where the regulator can change its strategy once the crime has occurred, the scheme described in section 7.2 will no longer be optimal. To show this we consider the subgame starting when the agent has been apprehended for the first time. If the government sticks to the penalty scheme described in Proposition 7.1, the agent will commit the second offence for sure because it comes for free. At the same time, in this setting the government's payoff is much higher in case it does not try to prevent the next period crime, since in this way it saves on costs of control. So, clearly, an equilibrium with the authority playing $p = 0$ and the agent committing the crime will be chosen in each period after the first conviction and, hence, can emerge as an SPNE of the multi-period repeated game. This implies that the scheme of Proposition 7.1 does not appear to be a time consistent (subgame perfect) strategy for a government in an n-period setting. Moreover, an argument given below shows that, if the government cannot commit, equal rather than decreasing sanctions will be optimal.

Derivation of an SPNE in No-Commitment Case[3]

[3]Unfortunately, we were only able to find one possible SPNE penalty scheme for which it holds that zero-crime outcome is sustained in equilibrium. However, we believe there exist many more SPNE penalty schemes, where some of them could have positive levels of crime in equilibrium as well. Hence,

Let us consider a finitely repeated game where objective functions and participation constraints of the players have exactly the same form as in the model of section 7.2. However, here we assume that the regulator can change its strategy in any period of the game, hence, also once the crime has occurred. The game in this case will be described as follows. A strategy of player 1 (regulator) is given by $\sigma = (p_1, ..., p_n, s_1, ..., s_n)$, while a strategy of player 2 (offender) is given by $k \in \{0, 1, 2, ..., n\}$. We also assume that agents have enough wealth so that deterrence is always possible, i.e., $nb < \sum_{i=1}^{n} s_i \leq W$.

Here we aim to check whether the outcome with penalty scheme set by the regulator (p_i and s_i for all $i \in \{1, ..., n\}$) such that any strategy for the firm except of strategy $(0, 0, ..., 0)$ will be blocked can arise as an SPNE of this game. Solving this game backwards we get the following results.

Consider the optimal strategy for the antitrust authority in the last period. Irrespective of what had happen before, in the beginning of period n the anti-trust authority solves the following problem, where p_n is probability of control as before, H reflects disutility of crime for the regulator, and I is an indicator function that is equal to 1 in case crime occurs in period n and 0 in case crime is blocked,

$$min \;\; p_n + H * I \tag{7.21}$$

s.t.

$$b - p_n s_n \leqq 0 \tag{7.22}$$

$$s_1 + ... + s_n \leqq W \tag{7.23}$$

$$0 \leqq p_n \leqq 1. \tag{7.24}$$

This problem shows that the primary aim of the regulator in the beginning of period n is to block the n^{th} period crime and this has to be achieved at the lowest possible cost. So, at time n the regulator chooses s_n and p_n such that $I = 0$ is achieved in the period n, but also such that the wealth that is left to the offender after the penalty s_n is paid is enough to block crimes in all the preceding periods. The $I = 0$ outcome in period n is ensured if constraint (7.22) is satisfied. Moreover, constraints (7.23) and (7.24) on the parameters of the penalty scheme must also be satisfied.

This implies that a possible solution of this problem has the following form: $p_n = \frac{b}{s_n}$ and $s_n = W - \sum_{j=1}^{n-1} s_j$.

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we could not characterize in general the set of SPNEs of the game in question. That is why, we state that the "equal" sanctions scheme is only an example of a possible policy that can reach full compliance behavior (k=0) in case the government cannot commit.

Looking for an SPNE, now given that we have blocked the crime in period n, we will try to find the optimal strategy for antitrust authority in the period $n-1$. Again the solution boils down to finding the optimum of the following problem:

$$min \;\; p_{n-1} + H * I \tag{7.25}$$

s.t.

$$b - p_{n-1}s_{n-1} \leqq 0 \tag{7.26}$$

$$s_1 + ... + s_n \leqq W \tag{7.27}$$

$$0 \leqq p_{n-1} \leqq 1. \tag{7.28}$$

Which is also given by $p_{n-1} = \frac{b}{s_{n-1}}$ and $s_{n-1} = W - \sum\limits_{j \neq n-1} s_j$.

The same solution we get for every period. Hence, in the beginning of the first period antitrust authority again solves a similar problem:

$$min \;\; p_1 + H * I \tag{7.29}$$

s.t.

$$b - p_1 s_1 \leqq 0 \tag{7.30}$$

$$s_1 + ... + s_n \leqq W \tag{7.31}$$

$$0 \leqq p_1 \leqq 1. \tag{7.32}$$

A solution is $p_1 = \frac{b}{s_1}$ and $s_1 = W - \sum\limits_{j=2}^{n} s_j$.

Consequently, a possible SPNE strategy of the regulator that satisfies conditions $p_i = \frac{b}{s_i}$ and $s_i = W - \sum\limits_{j \neq i} s_j$ is given in expression (7.33).

$$s_i = \frac{W}{n} \;\; \text{and} \;\; p_i = \frac{bn}{W} \quad \text{for all} \;\; i \in \{1, ..., n\} \tag{7.33}$$

In this SPNE the firm chooses not to commit any offence in any of the periods and the regulator sets penalty and rate of law enforcement that are uniform over time.

The only condition for existence of this solution is $bn < W$, which is also respected in the model of section 7.2.

The above analysis implies that in the repeated game setting the optimal penalty scheme, which is the part of SPNE strategy, can be given by $s_i = \frac{W}{n}$ and $p_i = \frac{bn}{W}$ for all $i \in \{1, ...n\}$. In this SPNE of the repeated game both penalties and rate of law enforcement are uniform over time.

7.4 Conclusions

The main conclusion of this chapter is the result that, when offenders are wealth constrained and the government is resource constrained and can commit to a certain policy throughout the whole planning horizon, cost minimizing deterrence is decreasing, rather than increasing, in the number of offenses. We prove that for the agents who may commit an act several times, optimal sanctions are such that the fine for the first crime equals the offender's entire wealth, and the fines are zero for all the subsequent crimes. Since the agent can only be a repeat offender if he has been a first-time offender, there are no further offenses if we completely deter the first one. This conclusion completely supports the result obtained by Emons (2003) for a two-period model.

This result contradicts the widely prevailing escalating penalties imbedded in many penal codes and sentencing guidelines. This puzzle still requires deep investigation in the law and economics literature. However, we should be careful to make too strong conclusions and policy implications on the basis of the model of Section 7.2, since, unfortunately, analogous to Emons (2004), this scheme does not appear to be a time consistent (subgame perfect) strategy for the government in an n-periods setting.

Finally, we suggest some extensions of the model described above. Introduction of history dependent strategies will make the analysis more complete but at the moment it does not seem to be analytically solvable. However, it seems that the main result, namely a declining penalty scheme, will arise as a solution of optimization problem in that case as well. Another possibility is to introduce the opportunity for both players to react to the actions of the rival. This suggests to extend this model to a repeated n-period game between the regulator and the offender. This also allows to consider the case when full commitment is not possible and the set of strategies for the firm will automatically include all history dependent and history independent strategies. A third extension would be to introduce discounting. But this will only increase incentives for the cost minimizing regulator to extract the fine as soon as possible, so that the arguments in favor of a declining penalty scheme will be even stronger.

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Nederlandse Samenvatting

In dit proefschrift proberen we bij te dragen aan het probleem van optimale handhaving van de mededingingswet. We beschouwen dit probleem vanuit de hoek van mogelijke verfijningen van de bestaande strafmaatregelen voor overtredingen van de mededingingswet. In het bijzonder bepalen we een optimale combinatie van instrumenten zoals de hoogte van de boete, de mate van wetshandhaving en de optimale structuur en samenstelling van sanctiepakketten. De motivatie voor deze studie komt voort uit het feit dat de in Europa en VS gehanteerde strafmaatregelen tegen overtredingen van de mededingingswet niet voldoende zwaar zijn om de voordelen van kartelvorming te compenseren. Hoewel de straffen aanzienlijk zwaarder zijn geworden en nieuwe instrumenten voor het ontmoedigen van kartelvorming, zoals clementieprogramma's, werden ingevoerd, is volledige uitbanning van overtredingen van de antitrust wet nog steeds niet bereikt.

Door de eigenschappen van sanctieschema's, zoals de afhankelijkheid van de ernst en de duur van de overtreding, het verband tussen eerdere en huidige prijsvorming en de gecumuleerde omzet van de onderneming, wordt de historie van de overtreding een belangrijke factor bij het bepalen van strafmaatregelen. Dit vraagt om toepassing van dynamische speltheorie in het modelleren van situaties waarin de mededingingswet wordt overtreden. Dit is het kernidee van deze dissertatie. Het dient ook te worden benadrukt dat de dynamische analyse van de handhaving van de medingingswet niet buiten beschouwing mag worden gelaten, omdat het de antitrust regels en het overtredingsproces in het algemeen beter beschrijft.

Door het toepassen van dynamische speltheorie kunnen we de huidige strafmaatregelen op het gebied van overtredingen van de antitrust wet in de VS en de EU met elkaar vergelijken. Bovendien kunnen we aangeven hoe de huidige sancties kunnen worden aangepast zodanig dat kartelvorming zoveel mogelijk wordt ontmoedigd. De belangrijkste implicaties van ons onderzoek met betrekking tot het beleid ten aanzien van overtreding van de mededingingswet zijn dat de basisstraf en de maximale straf verhoogd dienen te worden. Gegeven dat het maximum voor boetes wettelijk is begrensd in Europa, kan de oplossing voor dit probleem gezocht worden in de verdere ontwikkeling en de invoering van individuele boetes in combinatie met de reeds bestaande boetes in Europa. Verder beredeneren we ook dat bij vaststelling van de optimale sanctie, rekening moet worden gehouden met de ernst en de duur van de overtreding, maar ook met de de mate van wetshandhaving (of de kans op veroordeling) door mededingingsautoriteiten.

Een volgende belangrijke bevinding, welke wordt bevestigd in eerdere papers, is dat alleen zorgvuldig samengestelde clementieprogramma's kan leiden tot self-reporting, tot vermindering van beweegredenen om te participeren in een kartel en tot bevordering van de welvaart. Wanneer clementieprogramma's verkeerd worden samengesteld, is er een kans dat contraproductieve effecten van clementieprogramma's ontstaan. We vinden dat kartelvorming minder waarschijnlijk wordt wanneer de regels voor clementieprogramma's strikter zijn en wanneer de aanvraagprocedure voor clementie vertrouwelijker is. Bovendien concluderen we dat wanneer de aanvraagprocedure voor clementie niet vertrouwelijk is, clementie in bepaalde gevallen kan leiden tot een langere duur van karteloveréénkomsten. Dit komt vooral voor wanneer sancties en de mate van handhaving van de wetten laag is.

Het proefschrift bestaat uit een inleiding die wordt gevolgd door zes hoofdstukken. Hoofdstuk 2 geeft een beschrijving van het sanctiesysteem voor kartelvorming en van de effectiviteit van de sancties die hedendaags worden gebruikt in de handhaving van de antitrust wetten. In de hoofdstukken 3 en 4 maken we gebruik van optimal control theorie en differential games om de eigenschappen en de ontmoedigingskracht van de huidige sanctiepakketten te analyseren. De hoofdstukken 5 en 6 behandelen we de optimale samenstelling clementieprogramma's en de effecten van deze programma's op kartelstabiliteit. Hoofdstuk 7, tenslotte, bestudeert de vraag of sancties voor veelplegers moeten worden verzwaard of verlicht.

De analyse van de hoofdstukken 3 en 4, waarin we intertemporele trade-offs modelleren, vereist toepassing van instrumenten als dynamisch programmeren, optimal control theorie en, als er strategische interactie tussen agenten plaatsvindt, differential games. De meeste van de papers die genoemd worden in sectie 2 van de introductie onderzoeken de vraagstukken van optimale dynamische wetshandhaving en van minimalisatie sociale verlies als gevolg van delicten. Dit wordt gedaan door het modelleren van de interacties tussen de overtreder van de wet en de autoriteit die is aangesteld om de wet te handhaven. In hoofdstuk 3 gebruiken we een vergelijkbare aanpak. Technisch gezien sluit de analyse van hoofdstuk 3 aan bij Feichtinger (1983) waarin een competitie model tussen politie en dief bestudeerd wordt. We breiden dit raamwerk uit door een tijdsvariërende sanctie toe te staan. Bovendien, introduceren we een boete die afhankelijk is van de ernst van de overtreding en van de kans op wetshandhaving op ieder tijdstip. In dit hoofdstuk analyseren we in het bijzonder een differential game dat de interacties tussen de antitrust-autoriteit en een onderneming die mogelijkerwijs de mededingswet overtreedt. De doelstelling van deze autoriteit is om de sociale kosten te minimaliseren (verlies in totale sociale welvaart) die worden veroorzaakt doordat prijzen hoger zijn dan de marginale kosten. Als het criterium van consumentenverlies door prijszettingsactiviteiten van de onderneming geminimaliseerd wordt dan blijkt dat de strafmaatregelen die nu worden gebruikt in de EU en de VS wetgeving niet zo efficient zijn als gewenst. We tonen in het bijzonder aan dat volledige naleving van de mededingingswet (dat wil zeggen, een competitief prijsniveau) geen Nash-evenwicht is en bovendien zal dit gedrag nooit optreden als het lange termijn steady-state evenwicht van het model. Ook de vraag welk sanctiesysteem leidt tot volledig ontmoediging van kartelvorming in een dynamische setting wordt bestudeerd. We vinden dat deze sociaal gewenste uitkomst kan worden verkregen als de strafmaat een stijgende functie is van het gewicht van de overtreding en negatief gerelateerd is met de kans op wetshandhaving.

In hoofdstuk 4 wordt onderzocht of een boete die bepaald wordt op basis van gecumuleerde omzet van de onderneming die zich bezighoudt met prijsafspraken, kan leiden tot volledige ontmoediging van kartelvorming. Het model van hoofdstuk 4 is een uitbreiding van het model in hoofdstuk 3 in de zin dat we de sanctie niet alleen relateren aan de zwaarte van de overtreding maar ook aan de illegale winsten die zijn verkregen door kartelvorming. We nemen hierbij aan dat bij de vaststelling van de opgelegde boete rekening wordt gehouden met de historie van de overtreding. Dit houdt in dat wanneer een overtreding van de antitrust wet wordt geconstateerd, de autoriteit in staat is om alle gecumuleerde opbrengsten van de kartelvorming te bepalen. Daarom zal de autoriteit een sanctie opleggen die gebaseerd is op deze informatie. We vergelijken ook de ontmoedigingskracht van dit systeem met die van een vast boetesysteem.

De structuur van dit probleem leidt, net als in Fent et al. (1999) tot een optimal control model. Het grootste verschil tussen onze aanpak en die van Fent et al. (1999) of Feichtinger (1983) is dat de winst die door de onderneming wordt verworven als gevolg van kartelvorming wordt gezien als toestandsvariabele in onze aanpak terwijl in Fent et al. (1999) het crimineel verleden van de overtreder als toestandsvariabele wordt opgenomen. Een stijging van de toestandsvariabele is dus positief gerelateerd aan de mate van prijszetting door de onderneming en leidt daardoor tot een stijging van de boete in het geval de onderneming veroordeeld wordt. Door het oplossen van het optimal control probleem van de onderneming worden in hoofdstuk 4 de implicaties van verschillende boeteschema's onderzocht.

In hoofdstuk 5 en hoofdstuk 6 analyseren we de effecten van clementieprogramma's op de stabiliteit van kartelovereenkomsten. De modellen in de hoofdstukken 5 en 6 zijn een uitbreiding van voorgaande analyses in de zin dat rekening wordt gehouden met de mogelijkheid dat er strategische interactie op kan treden tussen de ondernemingen die een kartel vormen. Ondernemingen kunnen namelijk de kartelovereenkomst verbreken door middel van self-reporting.

Motta en Polo (2003), Spagnolo (2000), Malik (1993), en Aubert et al. (2004) zijn de belangrijkste bijdragen in de literatuur op het gebied van optimaal beleid ten aanzien van het overtreden van de antitrust wetten in de aanwezigheid van clementieprogramma's. De meeste van deze papers gebruiken een setting in discrete tijd. Hoewel in deze papers het samenwerkingsgedrag van ondernemingen wordt beschouwd in een dynamische omgeving, worden de bronnen van de onderliggende dynamiek, die de speerpunten zijn van deze dissertatie, buiten beschouwing gelaten. Deze referenties laten bijvoorbeeld niet toe dat de vaststelling van een kartel en de sanctie afhankelijk zijn van hoe de onderneming prijzen vaststelt, nu en in het verleden. Er zijn al wel een aantal papers dat zich hebben beziggehouden met dit probleem, te weten Hinloopen (2004), Harrington (2003) en Harrington (2004). Deze papers bestuderen settings waarin de kans dat een kartel wordt ontmanteld afhankelijk is van het gedrag van de onderneming. Bovendien wordt bekeken hoe deze kansen over de tijd veranderen. Echter, sancties die tijdsvariërend zijn en proportioneel zijn met de mate van de overtreding en daardoor het meest aansluiten bij de huidige antitrust regels, worden buiten beschouwing gelaten in de zojuist genoemde papers. Hoofdstuk 5 behandelt het probleem van de invloed van de invoering van het clementieprogramma op de duur van kartels onder twee verschillende boeteregimes (te weten, vast en proportioneel). In dit hoofdstuk gebruiken we een continue-tijd dynamic game, waarin de gecumuleerde winsten van prijszetting de toestandsvariabele is. We onderzoeken intertemporele aspecten van dit probleem waarbij gebruik gemaakt wordt van optimal stopping modellen en het instrument van dynamic continue-tijd pre-emption games.

In wezen wordt in hoofdstuk 5 een nieuwe aanpak voorgesteld om de efficiëntie van van leniency programs te analyseren. Deze aanpak verschilt van eerdere papers en is gebaseerd op het Reiganum-Fudenberg-Tirole model. Reiganum (1981) en Fudenberg en Tirole (1985) pasten timing games toe op het probleem van de acceptatie van technische ontwikkelingen. Wij passen een vergelijkbare methode toe op het gebied van kartelvormingsgame tussen twee ondernemingen in de aanwezigheid van een clementieregels. In deze setting kunnen we niet alleen de duur van kartelovereenkomsten onderzoeken, maar ook de optimale samenstelling van clementieprogramma's. Een van onze doelen is om uit te zoeken of, in het geval dat beide ondernemingen samenwerken met de antitrust-autoriteit, deze ondernemingen gelijkwaardig behandeld moeten worden of dat er een verschil in behandeling moet zijn, gebaseerd op het moment dat clementie wordt aangevraagd. In het bijzonder vragen we ons af of de clementieprogramma's strikter moeten zijn en of de aanvraagprocedure voor leniency open of vertrouwelijk moet zijn. We vinden dat kartelvorming minder waarschijnlijk zal zijn als de regels voor clementie strikter zijn en de aanvraagprocedure voor clementie vertrouwelijker is. Bovendien concluderen we dat wanneer de aanvraagprocedure voor clementie niet vertrouwelijk is, de duur van kartelovereenkomsten langer wordt in bepaalde gevallen. Dit gebeurt wanneer boetes en de mate van wetshandhaving laag is. Een verrassend resultaat is dat onder een systeem van vaste boetes, de invoering van een clementieprogramma de effectiviteit van handhaving van de antitrust wetgeving niet zal verbeteren wanneer de aanvraagprocedure niet vertrouwelijk is.

In hoofdstuk 6 van de dissertatie breiden we Motta en Polo (2003) uit door het introduceren van asymmetrische ondernemingen en door het expliciet modelleren van de effecten van de mate van striktheid van clementieprogramma's op kartelstabiliteit. In het algemeen verschillen ondernemingen in omvang en opereren ze in verschillende markten. In ons model vormen ze een kartel in slechts n markt. Deze asymmetrie leidt tot additionele kosten in geval het kartel ontmanteld wordt. Deze additionele kosten worden veroorzaakt door de asymmetrische reductie van verkopen in andere markten. Dit is te wijten aan aan het negatieve reputatie-effect. Bovendien analyseren we de effecten van de striktheid van clementieprogramma's waarbij de bestaande regels van de bestaande leniency programs in acht worden genomen. De striktheid van clementieprogramma's geeft de waarschijnlijkheid van volledige vrijstelling van de boete weer, zelfs in het geval veel ondernemingen gelijktijdig self-reporten. Onze belangrijkste resultaten zijn dat, ten eerste, clementieprogramma's beter werken voor kleine (minder gediversificeerde) bedrijven in de zin dat een lagere mate van wetshandhaving nodig is om kleinere ondernemingen tot self-report te bewegen. Tegelijkertijd is het voor grote ondernemingen minder waarschijnlijk dat ze een kartel opstarten vanwege mogelijke self-reporting in de toekomst. Het tweede belangrijke resultaat is dat hoe meer kartelvorming er optreedt in een economie hoe minder strikt de regels van clementieprogramma's moeten zijn.

Hoofdstuk 7 behandelt de vraag hoe veelplegers optimaal gesanctioneerd kunnen worden. Moeten ze strenger worden bestraft of is het beter om alleen de eerste overtreding te bestraffen? In Emons (2003) wordt aangetoond dat het onder bepaalde voorwaarden optimaal is om alleen bij een eerste overtreding te straffen. Hoofdstuk 7 geeft een uitbreiding van de 2-perioden analyse in Emons (2003) naar een n-perioden repeated game. De resultaten die op deze manier worden verkregen zijn vergelijkbaar met die van Emons (2003). We tonen aan dat voor agenten met een begrensd financieel vermogen, die een criminele activiteit herhaaldelijk kunnen uitvoeren, de optimale sanctie bij een eerste overtreding het gehele vermogen is en niets bij iedere herhaling. Het blijkt echter, analoog aan Emons (2004) dat dit santieschema niet een strategie is die tijd-consistent is voor een beleidsbepaler in een n-perioden setting.